3

Square and Cube Roots

Square

- The square of a number is the product of the number with itself, i.e. $a \times a = a^2$

Perfect Square

- A given number is said to be a perfect square if it can be expressed as the product of two equal factors.
- A natural number 'n' is a perfect square if $n = m^2$ for any natural number m.

  e.g. $4 = 2^2$ or 2×2

  and $25 = 5^2$ are perfect squares.

Properties of Square

(i) The number of zeroes at the end of a perfect square is always even.

(ii) Squares of even numbers are always even.

(iii) Squares of odd numbers are always odd.

(iv) Square of a negative number is always positive.

Squares from 1 to 20 numbers

Number	Square
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400

Square Root

- The square root of the number x is the number which when multiplied by itself gives x as the product.
- If $y = x^2$, then we call, x is the square root of y, i.e. $x = \pm\sqrt{y}$.

Properties of Square Root

(i) If the unit digit of a number is 2, 3, 7 or 8, then it does not have a square root.

(ii) Square root of even number is even.

(iii) Square root of odd number is odd.

Square Root of a Perfect Square by the Prime Factorisation Method

The steps are given below

I. Resolve the given number into prime factors.

II. Make pairs of similar factors.

III. Choose one prime from each pair and multiply all primes.

Thus, the product obtained is the square root of given number.

Example 1 The square root of 1764 is

(a) 41 (b) 43 (c) 42 (d) 40

Sol. (*c*) By prime factorisation method,

2	1764
2	882
3	441
3	147
7	49
7	7
	1

$$\therefore \quad \sqrt{1764} = \sqrt{2 \times 2 \times 3 \times 3 \times 7 \times 7}$$
$$= 2 \times 3 \times 7 = 42$$

Square Root of a Perfect Square by the Long Division Method

The steps are given below

(i) Group the digits in pairs starting with the digit in the units place and place a bar on every pair of digit.

(ii) Find the largest number whose square is equal to or less than the first bar. Take this number as the divisor and also the quotient.

(iii) Subtract the product of the divisor and the quotient from the first bar and bring down the next bar to the right of the remainder. This number becomes the new dividend.

(iv) Now, the new divisor is obtained by doubling the quotient and annexing with it a suitable digit which is also taken as the next digit of the quotient, chosen in such a way that the product of the new divisor and this digit is equal to or just less than the new dividend.

Repeat steps (ii), (iii), and (iv) till all the bars have considered.

Hence, the quotient is the required square root of the given number.

Example 2 Find the square root of 390625.

(a) 635 (b) 625

(c) 645 (d) 615

Sol. (*b*) By long division method,

	625
6	$\overline{39}\ \overline{06}\ \overline{25}$
	36
122	306
	244
1245	6225
	6225
	×

$$\therefore \quad \sqrt{390625} = 625$$

Formula for Finding the Number of Digits in the Square Root of a Perfect Square

If any perfect square number contains 'n' digits. then, its square root will contain

$\frac{n}{2}$ digits, when n is even

and $\frac{n+1}{2}$ digits, when n is odd.

e.g. Square root of 64 is 8. [$\because$ n = 2 i.e. even]

Also, square root of 144 is 12. [$\because$ n = 3 i.e. odd]

Square Root of Number in Decimal Form

Make the number of decimal places even by affixing zero, if necessary. Now, mark bars and find out the square root by the long division method. Put the decimal point in the square root as soon as the integral part is exhausted.

Example 3 Find the square root of 176.252176.

(a) 13.276 (b) 13.801

(c) 13.295 (d) 13.218

Sol. (*a*) By long division method,

	13.276
1	1 76.25 21 76
	1
23	76
	69
262	725
	524
2647	20121
	18529
26546	159276
	159276
	×

$\therefore \quad \sqrt{176.252176} = 13.276$

Square Root of Fraction

For any positive numbers a and b, we have

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b} \text{ and } \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Pythagorean Triplet

In a triplet (m, n, p) of three natural numbers m, n and p is called a Pythagorean triplet, if $m^2 + n^2 = p^2$.

Example 4 2025 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each now.

(a) 55, 45 (b) 45, 45 (c) 35, 35 (d) 36, 36

Sol. (*b*) Let the number of rows be x.

Then, number of plants in a row = x

So, number of plants to be planted in a garden

$$x \times x = x^2$$

According to the question,

Total number of plants to be planted = 2025

$\therefore \quad x^2 = 2025$

3	2025
3	675
3	225
3	75
5	25
5	5
	1

$\Rightarrow \quad x = \sqrt{2025} = \sqrt{3 \times 3 \times 3 \times 3 \times 5 \times 5}$

$= 3 \times 3 \times 5 = 45$

Hence, the number of rows is 45 and the number of plants in each row is 45.

Example 5 Find the smallest square number that is divisible by each of the numbers 4, 9 and 10.

(a) 900 (b) 9250 (c) 9100 (d) 9003

Sol. (*a*) The smallest number divisible by each one of 4, 9 and 10 is equal to the LCM of 4, 9 and 10.

2	4, 9, 10
2	2, 9, 5
3	1, 9, 5
3	1, 3, 5
5	1, 1, 5
	1, 1, 1

The prime factorisation of 180 = $2 \times 2 \times 3 \times 3 \times 5$

Here, prime factor 5 is unpaired.

Clearly, to make it a perfect square, it must be multiplied by 5.

Therefore, the required number $= 180 \times 5 = 900$

Hence, the smallest square number is 900.

Cube

- The cube of a number is the product of the number with itself twice.
- e.g. If x is a non-zero number, then $x \times x \times x = x^3$ is called cube of x.
- The cube of rational number is the cube of the numerator divided by the cube of denominator.

  e.g. cube of $\frac{4}{5}$ is $\frac{64}{125}$.

Perfect Cube

- A natural number n is said to be a perfect cube if there is an integer m such that $n = m \times m \times m$.

Properties of Cube of Numbers

The cubes of numbers have some interesting properties, given below

(i) Cubes of all even natural numbers are always even.

(ii) Cubes of all odd natural numbers are always odd.

(iii) Cubes of negative integers are always negative.

Cube from 1 to 15 number

Number	Cube
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000
11	1331
12	1728
13	2197
14	2744
15	3375

Important Points

(i) Number that have 1, 4, 5, 6 and 9 in the unit place have cube with the same digit in the unit place.

(ii) 3 in the unit place have cube with 7 in the unit place.

(iii) 7 in the unit place have cube with 3 in the unit place.

(iv) 2 in the unit place have cube with 8 in the unit place.

(v) 8 in the unit place have cube with 2 in the unit place.

Cube Root

If n is perfect cube i.e. $n = m^3$, for any integer m, then m is called the cube root of n and it is denoted by $m = \sqrt[3]{n}$.

Cube Root of a Perfect Cube by Prime Factorisation

The steps are given below

I. Factorise the given number into prime factors.

II. Make triplets of similar factors, or arrange them in group of three equal factors at a time.

III. Take the product of prime factors choosing one out of every triplet.

The product is the required cube root of the given number.

Example 6 Find the cube root of 74088.

(a) 40 (b) 47 (c) 42 (c) 45

Sol. (*c*) Resolving the given number, we get

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$74088 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7}$

$\therefore \quad \sqrt[3]{74088} = 2 \times 3 \times 7 = 42$

Cube Root of Decimals

Cube root of decimal values can be computed through the given steps.

***Step* I** Make 3 digits group separately for the portions on the left and the right of decimal point.

***Step* II** Ignore the decimal point and find the cube root.

***Step* III** Put the decimal point such that the number of decimal places in the cube root is one-third of the number of decimal places in the given number.

Example 8 Cube root of 30.371328 is

(a) 3.12 (b) 31.2

(c) 0.312 (d) None of these

Sol. *(a)* Setp **I** $\overline{30}\,.\,\overline{371}\,\overline{328}$

Setp **II** Ignoring decimal, the cube root of 30371328

$= \sqrt[3]{30371328} = 312$

Setp **III** Since, the given number has six decimal places, so its cube root will have 1/3rd of 6 i.e. two decimal places.

So, $\sqrt[3]{30.371328} = 3.12$

Cube Root of Fraction

Cube root of fraction can be computed through the given steps.

***Step* I** Convert the mixed fraction into an improper fraction first.

***Step* II** Find the cube root of numerator and the denominator with the fraction.

***Step* III** Put this in the form $\frac{p}{q}$, to get required result.

Example 8. The value of $\sqrt[3]{\frac{729}{1331}}$ is

(a) $\frac{9}{11}$ (b) $\frac{11}{9}$ (c) $\frac{5}{6}$ (d) $\frac{81}{121}$

Sol. (*a*) The value of $\sqrt[3]{\frac{729}{1331}} = \frac{\sqrt[3]{729}}{\sqrt[3]{1331}}$

$= \frac{9}{11}$

Cube Root of a Negative Perfect Cube

If a is a positive integer, then –a is a negative integer.

We know that, $(-a)^3 = -a^3$

So, $\sqrt[3]{-a^3} = -a$

In general, we have $\sqrt[3]{-x} = -\sqrt[3]{x}$

Cube Root of Product of Integers

The cube root of product of integers is the cube root of integer taking separately.

For any two integer a and b, we have

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$$

Cube Root of Rational Number $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

Example 9 Show that

$$\sqrt[3]{125 \times 64} = \sqrt[3]{125} \times \sqrt[3]{64}$$

Sol. $125 \times 64 = \underline{5 \times 5 \times 5} \times \underline{4 \times 4 \times 4}$

$\therefore$ LHS $= \sqrt[3]{125 \times 64} = (5 \times 4) = 20$

Now, $\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5$

and $\sqrt[3]{64} = \sqrt[3]{4 \times 4 \times 4} = 4$

$\therefore$ RHS $= \sqrt[3]{125} \times \sqrt[3]{64} = (5 \times 4) = 20$

$\Rightarrow$ LHS = RHS

$\Rightarrow \sqrt[3]{125 \times 64} = \sqrt[3]{125} \times \sqrt[3]{64}$ **Hence proved.**

Example 10 Difference of two perfect cubes is 189. If the cube root of the smaller of the two numbers is 3, then find the cube root of the larger number.

(a) 5 (b) 6 (c) 7 (d) 8

Sol. (*b*) Given difference of two perfect cubes = 189

and cube root of the smaller number = 3

$\therefore$ Cube of smaller number $= (3)^3 = 27$

Let cube root of the larger number be x.

Then, cube of larger number $= x^3$

According to the question,

$$x^3 - 27 = 189 \Rightarrow x^3 = 189 + 27$$

$$\Rightarrow x^3 = 216 \Rightarrow x = \sqrt[3]{216} = \sqrt[3]{6 \times 6 \times 6}$$

$$\therefore x = 6$$

Hence, the cube root of the larger number is 6.

Practice Exercise

1. The value of $(301)^2 - (300)^2$ is

(a) 1 (b) 601 (c) 106 (d) 100

2. If a number is increased by two times, then the square of the number will increase

(a) two times (b) three times
(c) four times (d) five times

3. Which of the following cannot be a digit in the unit place of a perfect square ?

(a) 0 (b) 1 (c) 5 (d) 7

4. $\sqrt{12} + \sqrt{24}$ is equal to

(a) $2\sqrt{3} + 3\sqrt{2}$ (b) $4\sqrt{3} + \sqrt{6}$
(c) $\sqrt{7} + 2\sqrt{3}$ (d) $2\sqrt{3} + 2\sqrt{6}$

5. The value of $\dfrac{\sqrt{80} - \sqrt{112}}{\sqrt{45} - \sqrt{63}}$ is

(a) $\frac{3}{4}$ (b) $1\frac{3}{4}$ (c) $1\frac{1}{3}$ (d) $1\frac{7}{9}$

6. If $x = \sqrt{3018 + \sqrt{36 + \sqrt{169}}}$, then the value of x is

(a) 55 (b) 44
(c) 63 (d) 42

7. What is that fraction which when multiplied by itself gives 227.798649 ?

(a) 15.093 (b) 15.099 (c) 14.093 (d) 9.0019

8. The number of digits in the square root of 298116 is

(a) 4 (b) 5 (c) 3 (d) 6

9. The square root of 73.96 is

(a) 8.6 (b) 86
(c) 0.86 (d) None of these

10. The value of $\sqrt{\dfrac{16}{36} + \dfrac{1}{4}}$ is

(a) $\frac{2}{5}$ (b) $\frac{1}{3}$ (c) $\frac{5}{3}$ (d) $\frac{5}{6}$

11. In a triplet (6 , a , 10) what value of 'a' will make it a Pythagorean triplet ?

(a) 4 (b) 16 (c) 8 (d) 5

12. The value of $\sqrt{6 + \sqrt{6 + \sqrt{6 + ..}}}$ is

(a) $6\frac{2}{3}$ (b) $3\frac{1}{2}$ (c) 3 (d) 6

13. A General arranges his soldiers in rows to form a perfect square. He finds that in doing so, 60 soldiers are left out. If the total number of soldiers be 8160. Then, the number of soldiers in each row is

(a) 90 (b) 91 (c) 92 (d) 80

14. The greatest six-digit number which is a perfect square is

(a) 998004 (b) 998006
(c) 998049 (d) 998001

15. The least number to be added to 269 to make it a perfect square is

(a) 31 (b) 16 (c) 17 (d) 20

16. The least number which is added to 17420 will make it a perfect square is

(a) 3 (b) 5 (c) 9 (d) 4

17. The smallest square number divisible by each of the numbers, 8, 12, 15 is

(a) 3600 (b) 9000 (c) 4200 (d) 100

18. Which of the following perfect cube is the cube of an even number ?

(a) 343 (b) 2197
(c) 216 (d) 1331

19. The value of $\sqrt[3]{\dfrac{27}{125}}$ is

(a) $\frac{3}{5}$ (b) $\frac{3}{25}$
(c) $\frac{9}{25}$ (d) not appropriate data

20. $(-216 \times 729)^{1/3}$

(a) 54 (b) −54
(c) −45 (d) 45

21. The cube root of $\dfrac{-343}{1331}$ is

(a) $\frac{7}{11}$ (b) $\frac{-7}{11}$ (c) $\frac{11}{7}$ (d) $\frac{-11}{7}$

22. The value of $\dfrac{\sqrt[3]{1728}-\sqrt[3]{729}}{\sqrt[3]{2197}+\sqrt[3]{2744}}$ is

(a) $\frac{21}{27}$ (b) $-\frac{1}{9}$ (c) $\frac{1}{9}$ (d) $\frac{27}{21}$

23. The value of $\sqrt[3]{0.064}+\sqrt[3]{27}-\sqrt[3]{729}$ is

(a) 12.4
(b) −2
(c) 5.6
(d) −5.6

Answers

1	*(b)*	2	*(c)*	3	*(d)*	4	*(d)*	5	*(c)*	6	*(a)*	7	*(a)*	8	*(c)*	9	*(a)*	10	*(d)*
11	*(c)*	12	*(c)*	13	*(a)*	14	*(d)*	15	*(d)*	16	*(d)*	17	*(a)*	18	*(c)*	19	*(a)*	20	*(b)*
21	*(b)*	22	*(c)*	23	*(d)*														

Hints & Solutions

1. (b) $(301)^2-(300)^2=(301+300)(301-300)$

$[\because a^2-b^2=(a+b)(a-b)]$

$=(601)\times 1=601$

2. (c) Let the number be y.

If the number is increased by two times it becomes 2y.

Square of the number $=(2y)^2=4y^2$

∴ The square of the number will be increased by four times.

3. (d) Digit 7 cannot be at the unit place of a perfect square.

4. (d) $\sqrt{12}+\sqrt{24}=\sqrt{2\times2\times3}+\sqrt{2\times2\times6}$

$=2\sqrt{3}+2\sqrt{6}$

5. (c) $\dfrac{\sqrt{80}-\sqrt{112}}{\sqrt{45}-\sqrt{63}}=\dfrac{4\sqrt{5}-4\sqrt{7}}{3\sqrt{5}-3\sqrt{7}}$

$=\dfrac{4(\sqrt{5}-\sqrt{7})}{3(\sqrt{5}-\sqrt{7})}$

$=\dfrac{4}{3}=1\dfrac{1}{3}$

6. (a) Now, $\sqrt{3018+\sqrt{36+\sqrt{169}}}$

$=\sqrt{3018+\sqrt{36+13}}$

$=\sqrt{3018+7}$

$=\sqrt{3025}$

$=55$

7. (a) Let the fraction be x.

Then, $x^2=227.798649$

∴ $x=\sqrt{227.798649}=15.093$

	15.093
1	9227.798649
	1
25	127
	125
3009	27986
	27081
30183	90549
	90549
	×

8. (c) Here, n = 6 (even)

∴ Number of digits in the square root

$=\dfrac{n}{2}=\dfrac{6}{2}=3$

9. (a) $73.96=\dfrac{7396}{100}=\dfrac{86\times86}{100}$

$\Rightarrow \sqrt{73.96}=\sqrt{\dfrac{86\times86}{100}}=\dfrac{86}{10}=8.6$

10. (d) $\sqrt{\dfrac{16}{36}+\dfrac{1}{4}}=\sqrt{\dfrac{16+9}{36}}=\sqrt{\dfrac{25}{36}}=\dfrac{5}{6}$

11. (c) $6^2=36,\ a^2=a^2, 10^2=100$

By Pythagorean triplet,

$6^2+a^2=10^2\Rightarrow a^2=10^2-6^2$

$\Rightarrow\quad ta^2=100-36=64\ \Rightarrow a=\sqrt{64}=8$

12. (c) Let $y=\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}$

$\Rightarrow\quad y=\sqrt{6+y}\Rightarrow y^2-y-6=0$

$\Rightarrow\quad (y-3)(y+2)=0\quad\Rightarrow y=3,-2$

$\Rightarrow\quad y=3\quad [\because$ y cannot be negative$]$

13. (a) Total number of soldiers arranged

$= 8160 - 60 = 8100$

Since, the number of soldiers in each row is equal to number of rows.

$\therefore$ Number of soldiers in each row

$= \sqrt{8100} = \sqrt{9 \times 9 \times 10 \times 10} = 90$

14. (d) The greatest six-digit number = 999999

	999
9	99 99 99
	81
189	1899
	1701
1989	19899
	17901
	1998

$\therefore$ The greatest number of six digit which is a perfect square $= 999999 - 1998 = 998001$

15. (d) We know, $256 < 269 < 289$

$\Rightarrow \quad (16)^2 < 269 < (17)^2$

$\therefore$ Number to be added $= (17)^2 - 269$

$= 289 - 269 = 20$

16. (d) Since, 17420 lies between 131^2 and 132^2.

Now, $\quad (132)^2 = 17424$

Hence, 4 should be added.

17. (a) Smallest number divisible by each one of 8, 12, 15 is equal to LCM of 8, 12, 15.

Then,

2	8, 12, 15
2	4, 6, 15
2	2, 3, 15
3	1, 3, 15
5	1, 1, 5
	1, 1, 1

$\therefore$ LCM of 8, 12, 15 = 120

Here, prime factor 2, 3, 5 are unpaired.

So, perfect square $= 120 \times 2 \times 3 \times 5 = 3600$

$\therefore$ Smallest square number is 3600.

18. (c) 216 is the cube of an even number because cube of an even number is always even.

19. (a) $\sqrt[3]{\frac{27}{125}} = \sqrt[3]{\frac{3 \times 3 \times 3}{5 \times 5 \times 5}} = \frac{3}{5}$

20. (b) $(-216 \times 729)^{1/3} = (-216)^{1/3} \times (729)^{1/3}$

$= -\left(\underline{6 \times 6 \times 6}\right)^{1/3} \times \left(\underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}\right)^{1/3}$

$= -6 \times 3 \times 3 = -54$

21. (b) $\sqrt[3]{\frac{-343}{1331}} = \frac{\sqrt[3]{-343}}{\sqrt[3]{1331}} = \frac{\sqrt[3]{-7 \times 7 \times 7}}{\sqrt[3]{11 \times 11 \times 11}} = \frac{-7}{11}$

22. (c)

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

3	729
3	243
3	81
3	27
3	9
3	3
	1

13	2197
13	169
13	13
	1

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$\therefore \quad \frac{\sqrt[3]{1728} - \sqrt[3]{729}}{\sqrt[3]{2197} + \sqrt[3]{2744}} = \frac{12 - 9}{13 + 14} = \frac{3}{27} = \frac{1}{9}$

23. (d) $\sqrt[3]{0.064} + \sqrt[3]{27} - \sqrt[3]{729}$

$\because \quad \sqrt[3]{0.064} = \sqrt[3]{0.4 \times 0.4 \times 0.4} = 0.4$

$\sqrt[3]{27} = \sqrt[3]{3 \times 3 \times 3} = 3$

$\sqrt[3]{729} = \sqrt[3]{\underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}} = 3 \times 3 = 9$

$\therefore \quad \sqrt[3]{0.064} + \sqrt[3]{27} - \sqrt[3]{729} = 0.4 + 3 - 9 = -5.6$