Chapter – 4

Work, Energy and Power

Multiple Choice Questions

Question 1.

A uniform force of $(2\hat{i} + \hat{j})$ + N acts on a particle of mass 1 kg. The particle displaces from position $(3\hat{j} + \hat{k})$ m to $(5\hat{i} + 3\hat{j})$ m. Th e work done by the force on the particle is
[AIPMT model 2013]

(a) 9 J
(b) 6 J
(c) 10 J
(d) 12 J

Answer:
(c) 10 J

Question 2.
A ball of mass 1 kg and another of mass 2 kg are dropped from a tall building whose height is 80 m. After, a fall of 40 m each towards Earth, their respective kinetic energies will be in the ratio of [AIPMT model 2004]

(a) $\sqrt{2}$: 1

(b) 1 : $\sqrt{2}$

(c) 2 : 1

(d) 1 : 23

Answer:
(d) 1 : 23

Question 3.
A body of mass 1 kg is thrown upwards with a velocity 20 m s^{-1}. It momentarily comes to rest after attaining a height of 18 m. How much energy is lost due to air friction?

(Take g = 10 ms^{-2}) [AIPMT 2009]
(a) 20 J
(b) 30 J
(c) 40 J
(d) 10 J

Answer:
(a) 20 J

Question 4.
An engine pumps water continuously through a hose. Water leaves the hose with a velocity v and m is the mass per unit length of the water of the jet. What is the rate at which kinetic energy is imparted to water ? [AIPMT 2009]

(a) $\frac{1}{3}mv^2$ (b) mv^3 (c) $\frac{1}{3}mv^2$ (d) $\frac{1}{4}mv^2$

Answer:
(a) $\frac{1}{2}mv^2$

Question 5.
A body of mass 4 m is lying in xv-plane at rest. It suddenly explodes into three pieces. Two pieces each of mass m move perpendicular to each other with equal speed v the total kinetic energy generated due to explosion is [AIPMT 2014]

(a) mv^2 (b) $\frac{3}{2}mv^2$ (c) $2mv^2$ (d) $4\,mv^2$

Answer:
(b) $\frac{3}{2}mv^2$

Question 6.
The potential energy of a system increases, if work is done
(a) by the system against a conservative force
(b) by the system against a non-conservative force
(c) upon the system by a conservative force
(d) upon the system by a non-conservative force

Answer:
(a) by the system against a conservative force

Question 7.
What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop?

(a) $\sqrt{2gR}$ (b) $\sqrt{3gR}$ (c) $\sqrt{5gR}$ (d) $\sqrt{gR}$

Answer:
(c) $\sqrt{5gR}$

Question 8.
The work done by the conservative force for a closed path is
(a) always negative
(b) zero
(c) always positive
(d) not defined

Answer:
(b) zero

Question 9.
If the linear momentum of the object is increased by 0.1 %, then the kinetic energy is increased by
(a) 0.1%
(b) 0.2%
(c) 0.4%
(d) 0.01%

Answer:
(b) 0.2%

Question 10.
If the potential energy of the particle is $\alpha - \frac{\beta}{2}x^2$, then force experienced by the particle is

(a) $\beta = \frac{\beta}{2}x^2$ (b) $F = \beta x$ (c) $F = -\beta x$ (d) $F = -\frac{\beta}{2}x^2$

Answer:
(c) $F = -\beta x$

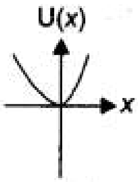
Question 11.
A wind-powered generator converts wind energy into electric energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to
(a) v
(b) v^2
(c) v^3
(d) v^4

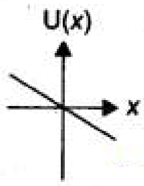
Answer:
(c) v^4

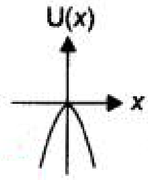
Question 12.
Two equal masses m_1 and m_2 are moving along the same straight line with velocities 5 ms^{-1} and -9 ms^{-1} respectively. If the collision is elastic, then calculate the velocities after the collision of Wj and m2, respectively
(a) -4 ms^{-1} and 10 ms^{-1}
(b) 10 ms^{-1} and 0 ms^{-1}
(c) -9 ms^{-1} and 5 ms^{-1}
(d) 5 ms^{-1} and 1 ms^{-1}

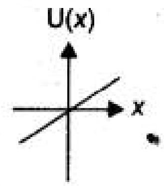
Answer:
(c) -9 ms^{-1} and 5 ms^{-1}

Question 13.
A particle is placed at the origin and a force $F = kx$ is acting on it (where k is a positive constant). If $U(0) = 0$, the graph of $U(x)$ versus x will be (where U is the potential energy function) [IIT 2004]

(a) 

(b) 

(c) 

(d) 

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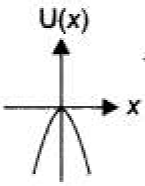
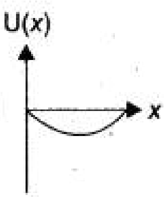
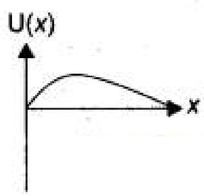
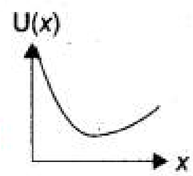
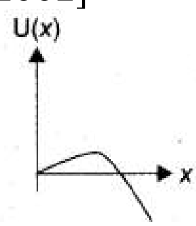
(c) 

Question 14.
A particle which is constrained to move along x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here, k and a are positive constants. For $x \geq 0$, the functional form of the potential energy U(x) of the particle is [IIT 2002]

(a) 

(b) 

(c) 

(d) 

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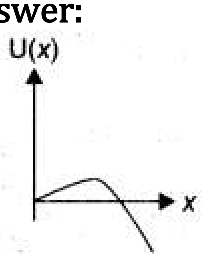
(d) 

Question 15.
A spring of force constant k is cut into two pieces such that one piece is double the length of the other. Then, the long piece will have a force constant of

(a) $\frac{2}{3}k$ (b) $\frac{3}{2}k$ (c) $3k$ (d) $6k$

Answer:
(b) $\frac{3}{2}k$

Short Answer Questions

Question 1.
Explain how the definition of work in physics is different from general perception.

Answer:
The term work is used in diverse contexts in daily life. It refers to both physical as well as mental work. In fact, any activity can generally be called as work. But in Physics, the term work is treated as a physical quantity with a precise definition. Work is said to be done by the force when the force applied on a body displaces it.

Question 2.
Write the various types of potential energy. Explain the formulae.

Answer:
(a) U = mgh
U – Gravitational potential energy
m – Mass of the object,
g – acceleration due to gravity
h – Height from the ground,
(b) $u = \frac{1}{2} K x^2$
u – Elastic potential energy
k – String constant; x-displacement.
(c) $U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$
U – electrostatic potential energy
ε_0 = absolute permittivity
q_1, q_2 – electric charges

Question 3.
Write the differences between conservative and non-conservative forces. Give two examples each.

Answer:

Comparison of conservative and non-conservative forces		
S.No	Conservative forces	Non-conservative forces
1.	Work done is independent of the path	Work done depends upon the path
2.	Work done in a round trip is zero	Work done in a round trip is not zero
3.	Total energy remains constant	Energy is dissipated as heat energy
4.	Work done is completely recoverable	Work done is not completely recoverable.
5.	Force is the negative gradient of potential energy	No such relation exists.
	Example: (*i*) Elastic spring force (*ii*) Electrostatic force	*Examples:* (*i*) The force due to air resistance (*ii*) Viscous force.

Question 4.
Explain the characteristics of elastic and inelastic collision.

Answer:
In any collision process, the total linear momentum and total energy are always conserved whereas the total kinetic energy need not be conserved always. Some part of the initial kinetic energy is transformed to other forms of energy. This is because, the impact of collisions and deformation occurring due to collisions may in general, produce heat, sound, light etc. By taking these effects into account, we classify the types of collisions as follows:

(a) Elastic collision
(b) Inelastic collision

(a) Elastic collision: In a collision, the total initial kinetic energy of the bodies (before collision) is equal to the total final kinetic energy of the bodies (after collision) then, it is called as elastic collision, i.e.,
Total kinetic energy before collision = Total kinetic energy after collision

(b) Inelastic collision: In a collision, the total initial kinetic energy of the bodies (before collision) is not equal to the total final kinetic energy of the bodies (after collision) then, it is called as inelastic collision, i.e.,
Total kinetic energy before collision ≠ Total kinetic energy after collision

$$\begin{pmatrix}\text{Total kinetic energy}\\ \text{after collision}\end{pmatrix}-\begin{pmatrix}\text{Total kinetic energy}\\ \text{before collision}\end{pmatrix}=\begin{pmatrix}\text{loss in energy}\\ \text{during collision}\end{pmatrix}=\Delta Q$$

Even though kinetic energy is not conserved but the total energy is conserved. This is because the total energy contains the kinetic energy term and also a term ΔQ, which includes all the losses that take place during collision. Note that loss in kinetic energy during collision is transformed to another form of energy like sound, thermal, etc. Further, if the two colliding bodies stick together after collision such collisions are known as completely inelastic collision or perfectly inelastic collision. Such a collision is found very often. For example when a clay putty is thrown on a moving vehicle, the clay putty (or Bubblegum) sticks to the moving vehicle and they move together with the same velocity.

Question 5.
Define the following
(a) Coefficient of restitution
(b) Power
(c) Law of conservation of energy
(d) Loss of kinetic energy in inelastic collision.

Answer:

(a) The ratio of velocity of separation after collision to the velocity of approach before collision

i.e. $$e=\frac{\text{velocity of separation (after collision)}}{\text{velocity of approach (before collision)}}=\frac{(v_2-v_1)}{(u_1-u_2)}$$

(b) Power is defined as the rate of work done or energy delivered

$$P=\frac{\text{Work done}}{\text{Time taken}}$$

Its unit is watt.

(c) The law of conservation of energy states that energy can neither be created nor destroyed. It may be transformed from one form to another but the total energy of an isolated system remains constant.

(d) In perfectly inelastic collision, the loss in kinetic energy during collision is transformed to another form of energy like sound, thermal, heat, light etc. Let KE_i be the total kinetic energy before collision and KE_f be the total kinetic energy after collision.

Total kinetic energy before collision,

$$KE_i = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 \quad ...(1)$$

Total kinetic energy after Collision,

$$KE_f = \frac{1}{2} (m_1 + m_2) v^2 \quad ...(2)$$

Then the loss of kinetic energy is

$$\text{Loss of KE, } \Delta Q = KE_f - KE_i = \frac{1}{2}(m_1 + m_2) v^2 - \frac{1}{2} m_1 u_1^2 - \frac{1}{2} m_2 u_2^2 \quad ...(3)$$

Substituting equation $V = \frac{m_1 u_1 + m_2 u_2}{(m_1 + m_2)}$ in equation (3), and on simplifying (expand v by using the algebra $(a + b)^2 = a^2 + b^2 + 2ab$, we get

$$\text{Loss of KE, } \Delta Q = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(u_1 - u_2)^2 \quad ...(4)$$

Long Answer Questions

Question 1.
Explain with graphs the difference between work done by a constant force and by a variable force.

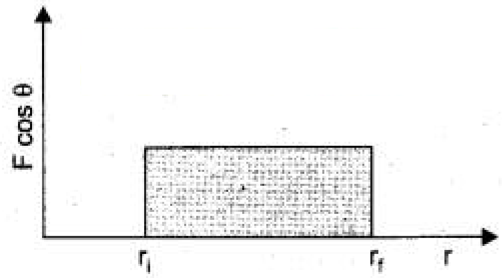
Answer:
Work done by a constant force: When a constant force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation,
$dW = (F \cos \theta)\, dr$ ..(1)

The total work done in producing a displacement from initial position r_i to final position r_f is,

$$W = \int_{r_i}^{r_f} dW \qquad ...(2)$$

$$W = \int_{r_i}^{r_f} (F \cos\theta)\, dr = (F \cos\theta) \int_{r_i}^{r_f} dr = (F \cos\theta)(r_f - r_i) \qquad ...(3)$$

The graphical representation of the work done by a constant force is shown in figure given below. The area under the graph shows the work done by the constant force.



Work done by the constant force

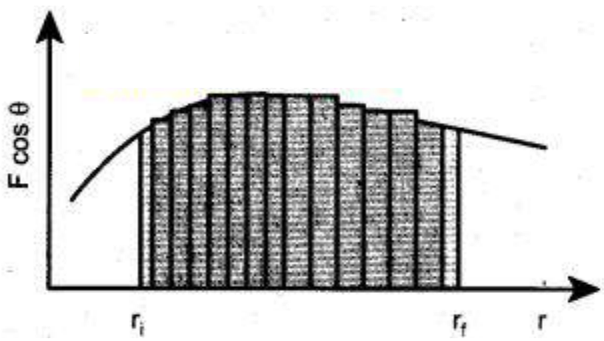
Work done by a variable force: When the component of a variable force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation

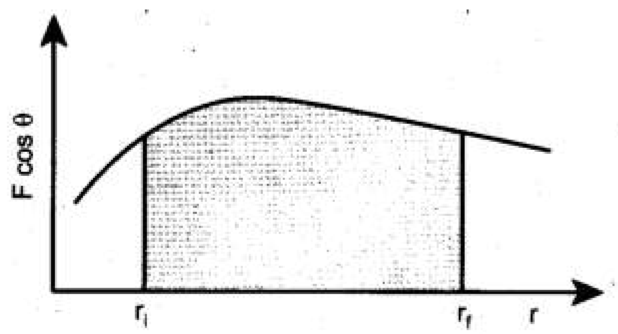
dW = F cos θ dr [F cos θ is the component of the variable force F]

where, F and θ are variables. The total work done for a displacement from initial position r_i to final position r_f is given by the relation,

$$W = \int_{r_i}^{r_f} dW = \int_{r_i}^{r_f} F \cos\theta\, dr \qquad ...(4)$$

A graphical representation of the work done by a variable force is shown in figure given below. The area under the graph is the work done by the variable force.





Work done by a variable force

Question 2.
State and explain work energy principle. Mention any three examples for it.

Answer:
(i) If the work done by the force on the body is positive then its kinetic energy increases.
(ii) If the work done by the force on the body is negative then its kinetic energy decreases.
(iii) If there is no work done by the force on the body then there is no change in its kinetic energy, which means that the body has moved at constant speed provided its mass remains constant.
(iv) When a particle moves with constant speed in a circle, there is no change in the kinetic energy of the particle. So according to work energy principle, the work done by centripetal force is zero.

Question 3.
Arrive at an expression for power and velocity. Give some examples for the same.

Answer:
The work done by a force $\overrightarrow{F}$ for a displacement $d\vec{r}$ is

$$W = \int \vec{F} . d\vec{r} \quad \text{...(i)}$$

Left hand side of the equation (i) can be written as

$$W = \int dW = \int \frac{dW}{dt} dt \text{ (multiplied and divided by } dt\text{)} \quad ...(ii)$$

Since, velocity is $\vec{v} = \frac{d\vec{r}}{dt}; d\vec{r} = \vec{v}\, dt$. Right hand side of the equation (i) can be written as dt

$$\int \vec{F} . d\vec{r} = \int \left(\vec{F} . \frac{d\vec{r}}{dt} \right) dt = \int (\vec{F} . \vec{v})\, dt \left[\vec{v} = \frac{d\vec{r}}{dt} \right] \quad ...(iii)$$

Substituting equation (ii) and equation (iii) in equation (i), we get

$$\int \frac{dW}{dt} dt = \int (\vec{F} \cdot \vec{v}) dt$$

$$\int \left(\frac{dW}{dt} - \vec{F} \cdot \vec{v} \right) dt = 0$$

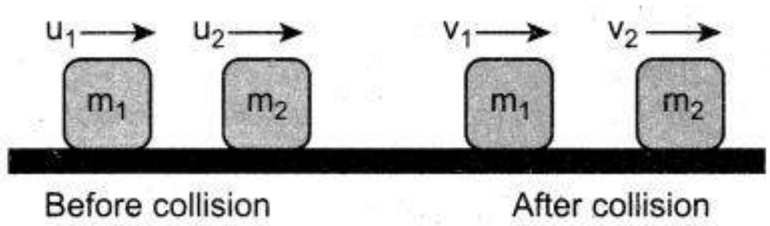
This relation is true for any arbitrary value of dt. This implies that the term within the bracket must be equal to zero, i.e.,

$$\frac{dW}{dt} - \vec{F} \cdot \vec{v} = 0 \quad \text{Or} \quad \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

Hence power $P = \overrightarrow{F} \cdot \vec{v}$

Question 4.
Arrive at an expression for elastic collision in one dimension and discuss various cases.

Answer:
Consider two elastic bodies of masses m_1 and m_2 moving in a straight line (along positive x direction) on a frictionless horizontal surface as shown in figure given below.



Elastic collision in one dimension

Mass	Initial velocity	Final velocity
Mass m_1	u_1	v_1
Mass m_2	u_2	v_2

In order to have collision, we assume that the mass m] moves faster than mass m_2 i.e., $u_1 > u_2$. For elastic collision, the total linear momentum and kinetic energies of the two bodies before and after collision must remain the same.

	Momentum of mass m_1	Momentum of mass m_2	Total linear momentum
Before collision	$p_{i1} = m_1u_1$	$p_{i2} = m_2u_2$	$p_i = p_{i1} + p_{i2}$ $p_i = m_1u_1 + m_2u_2$
After collision	$p_{f1} = m_1v_1$	$p_{f2} = m_2v_2$	$p_f = p_{f1} + p_{f2}$ $p_f = m_1v_1 + m_2v_2$

From the law of conservation of linear momentum,

Total momentum before collision (p_i) = Total momentum after collision (p_f)
$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ ...(i)

Or $\quad m_1(u_1 - v_1) = m_2(v_2 - u_2)$ ...(ii)

Further,

	Kinetic energy of mass m_1	Kinetic energy of mass m_2	Total kinetic energy
Before collision	$KE_{i1} = \frac{1}{2}m_1u_1^2$	$KE_{i2} = \frac{1}{2}m_2u_2^2$	$KE_i = KE_{i1} + KE_{i2}$ $KE_i = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$
After collision	$KE_{f1} = \frac{1}{2}m_1v_1^2$	$KE_{f2} = \frac{1}{2}m_2v_2^2$	$KE_i = KE_{i1} + KE_{i2}$ $KE_f = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

For elastic collision,

Total kinetic energy before collision KE_i = Total kinetic energy after collision KE_f

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \qquad \text{...(iii)}$$

After simplifying and rearranging the terms,

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

Using the formula $a^2 - b^2 = (a + b)(a - b)$, we can rewrite the above equation as

$$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2) \qquad \text{...(iv)}$$

Dividing equation (iv) by (ii) gives,

$$\frac{m_1(u_1 + v_1)(u_1 - v_1)}{m_1(u_1 - v_1)} = \frac{m_2(v_2 + u_2)(v_2 - u_2)}{m_2(v_2 - u_2)}$$

$$u_1 + v_1 = v_2 + u_2$$

$$u_1 - u_2 = v_2 - v_1 \qquad \text{Rearranging, (v)}$$

Equation (v) can be rewritten as

$$u_1 - u_2 = -(v_1 - v_2)$$

This means that for any elastic head on collision, the relative speed of the two elastic bodies after the collision has the same magnitude as before collision but in opposite direction. Further note that this result is independent of mass.

Rewriting the above equation for v_1 and v_2,

$v_1 = v_2 + u_2 - u_2$ ...(vi)

Or $v_2 = u_1 + v_1 - u_2$ ...(vii)

To find the final velocities v_1 and v_2:

Substituting equation (vii) in equation (ii) gives the velocity of as m_1 as

$m_1 (u_1 - v_1) = m_2(u_1 + v_1 - u_2 - u_2)$

$m_1 (u_1 - y_1) = m_2 (u_1 + + v_1 - 2u_2)$

$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1 + 2m_2u_2$

$m_1u_1 - m_2u_1 + 2m_2u_2 = m_1v_1 + m_2v_1$

$(m_1 - m_2) u_1 + 2m_2u_2 = (m_1 + m_2) v_1$

Or
$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \qquad \text{...(viii)}$$

Similarly, by substituting (vi) in equation (ii) or substituting equation (viii) in equation (vii), we get the final velocity of m_2 as

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 \quad \text{...(ix)}$$

Case 1: When bodies has the same mass i.e., $m_1 = m_2$,

equation (viii) $\Rightarrow$ $$v_1 = (0)\, u_1 + \left(\frac{2m_2}{2m_2}\right)u_2$$

$$v_1 = u_2 \quad \text{...(x)}$$

equation (ix) $\Rightarrow$ $$v_2 = \left(\frac{2m_1}{2m_1}\right)u_1 + (0)\, u_2$$

$$v_2 = u_1 \quad \text{...(xi)}$$

The equations (x) and (xi) show that in one dimensional elastic collision, when two bodies of equal mass collide after the collision their velocities are exchanged.

Case 2: When bodies have the same mass i.e., $m_1 = m_2$ and second body (usually called target) is at rest ($u_2 = 0$),
By substituting $m_1 = m_2 =$ and $u_2 = 0$ in equations (viii) and equations (ix) we get,

from equation (viii) $\Rightarrow v_1 = 0$ ...(xii)
from equation (ix) $\Rightarrow v_2 = u_1$ ..... (xiii)
Equations (xii) and (xiii) show that when the first body comes to rest the second body moves with the initial velocity of the first body.

Case 3: The first body is very much lighter than the second body

$\left(m_1 << m_2, \frac{m_1}{m_2} << 1\right)$ then the ratio $\frac{m_1}{m_2} \approx 0$ and also if the target is at rest ($u_2 = 0$)

Dividing numerator and denominator of equation (viii) by m_2, we get

$$v_1 = \left(\frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1}\right) u_1 + \left(\frac{2}{\frac{m_1}{m_2} + 1}\right)(0)$$

$$v_1 = \left(\frac{0 - 1}{0 + 1}\right) u_1$$

$$v_1 = -u_1$$

Similarly the numerator and denominator of equation (ix) by m_2, we get

$$v_2 = \left(\frac{2\frac{m_1}{m_2}}{\frac{m_1}{m_2} + 1}\right) u_1 + \left(\frac{1 - \frac{m_1}{m_2}}{\frac{m_1}{m_2} + 1}\right)(0)$$

$$v_2 = (0)u_1 + \left(\frac{1 - \frac{m_1}{m_2}}{\frac{m_1}{m_2} + 1}\right)(0)$$

$$v_2 = 0 \qquad \text{...(xv)}$$

The equation (xiv) implies that the first body which is lighter returns back (rebounds) in the opposite direction with the same initial velocity as it has a negative sign. The equation (xv) implies that the second body which is heavier in mass continues to remain at rest even after collision. For example, if a ball is thrown at a fixed wall, the ball will bounce back from the wall with the same velocity with which it was thrown but in opposite direction.

Case 4: The second body is very much lighter than the first body $\left(m_2 \ll m_1, \frac{m_2}{m_1} \ll 1\right)$ then the ratio $\frac{m_2}{m_1} \approx 0$ and also if the target is at rest $u_2 = 0$

Dividing numerator and denominator of equation (viii) by m_1, we get

$$v_1 = \left(\frac{1-\frac{m_2}{m_1}}{1+\frac{m_2}{m_1}}\right)u_1 + \left(\frac{2\frac{m_2}{m_1}}{1+\frac{m_2}{m_1}}\right)(0)$$

$$v_1 = \left(\frac{1-0}{1+0}\right)u_1 + \left(\frac{0}{1+0}\right)(0)$$

$$v_1 = u_1 \quad \text{...(xvi)}$$

Similarly,

Dividing numerator and denominator of equation (xiii) by m_1, we get

$$v_2 = \left(\frac{2}{1+\frac{m_2}{m_1}}\right)u_1 + \left(\frac{\frac{m_2}{m_1}-1}{1+\frac{m_2}{m_1}}\right)(0)$$

$$v_2 = \left(\frac{2}{1+0}\right)u_1$$

$$v_2 = 2u_1 \quad \text{...(xvii)}$$

The equation (xvi) implies that the first body which is heavier continues to move with the same initial velocity. The equation (xvii) suggests that the second body which is lighter will move with twice the initial velocity of the first body. It means that the lighter body is thrown away from the point of collision.

Question 5.

What is inelastic collision? In which way it is different from elastic collision. Mention few examples in day to day life for inelastic collision.

Answer:

Inelastic collision: In a collision, the total initial kinetic energy of the bodies (before collision) is not equal to the total final kinetic energy of the bodies (after collision) then, it is called as inelastic collision, i.e.,

Total kinetic energy before collision ≠ Total kinetic energy after collision

$$\begin{pmatrix}\text{Total kinetic energy}\\ \text{after collision}\end{pmatrix} - \begin{pmatrix}\text{Total kinetic energy}\\ \text{before collision}\end{pmatrix} = \begin{pmatrix}\text{loss in energy}\\ \text{during collision}\end{pmatrix} = \Delta Q$$

Even though kinetic energy is not conserved but the total energy is conserved.

This is because the total energy contains the kinetic energy term and also a term ΔQ, which includes all the losses that take place during collision. Note that loss in kinetic energy during collision is transformed to another form of energy like sound, thermal, etc. Further, if the two colliding bodies stick together after collision such collisions are known as completely inelastic collision or perfectly inelastic collision.

Such a collision is found very often. For example when a clay putty is thrown on a moving vehicle, the clay putty (or Bubblegum) sticks to the moving vehicle and they move together with the same velocity.

Difference between Elastic & in elastic collision

S.No.	Elastic	Inelastic
1.	Total kinetic energy is conserved	Total kinetic energy is not conserved
2.	Forces involved are conservative forces	Forces involved are non-conservative forces
3.	Mechanical energy is not dissipated.	Mechanical energy is dissipated into heat, light, sound etc.

Numerical Problems

Question 1.
Calculate the work done by a force of 30N in lifting a load of 2 Kg to a height of 10m($g = 10\ ms^{-1}$)

Answer:
Given: F = 30 N, load (m) = 2 kg; height = 10 m, $g = 10\ ms^{-2}$
Gravitational force F = mg = 30 N
The distance moved h = 10 m
Work done on the object W = Fh = 30 × 10 = 300 J.

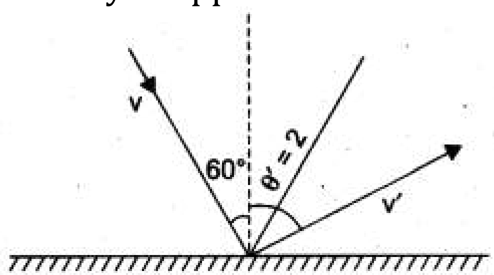
Question 2.
A ball with a velocity of $5\ ms^{-1}$ impinges at angle of 60° with the vertical on a smooth horizontal plane. If the coefficient of restitution is 0.5, find the velocity and direction after the impact.

Answer:
Given: Velocity of ball: $5\ ms^{-1}$

Angle of inclination with vertical: 60°
Coefficient of restitution = 0.5.

Note: Let the angle reflection is θ’ and the speed after collision is v’. The floor exerts a force on the ball along the normal during the collision. There is no force parallel to the surface. Thus, the parallel component of the velocity of the ball remains unchanged. This gives
v’ sin θ’ = v sin θ …… (i)

Vertical component with respect to floor = v’ cos θ’ (velocity of separation)
Velocity of approach = v cos θ



$$\text{Coefficient of restitution } e = \frac{\text{velocity of separation}}{\text{velocity of approach}}$$

$$e = \frac{v' \cos\theta'}{v\cos\theta} \quad \therefore \quad v'\cos\theta' = ev\cos\theta \qquad \text{...(ii)}$$

from (i) and (ii)

(ii) $$v'\sqrt{(1-\sin'^2\theta)} = ev\cos\theta$$

$$v'^2(1-\sin'^2\theta) = e^2v^2\cos^2\theta$$

$$v'^2 - v'^2\sin'^2\theta = e^2v^2\cos^2\theta$$

$$v'^2 = v^2\sin^2\theta + e^2v^2\cos^2\theta \qquad [v'\sin\theta' = v\sin\theta]$$

$$v' = \sqrt{v^2\sin^2\theta + e^2v^2\cos^2\theta}$$

$$v' = v\sqrt{\sin^2\theta + e^2\cos^2\theta}$$

The speed after collision $v' = v\sqrt{\sin^2\theta + e^2\cos^2\theta}$

$$v' = 5\sqrt{\sin^2(60) + (0.5)^2\cos^2 60} = 5\sqrt{\frac{3}{4} + 0.25 \times \frac{1}{4}}$$

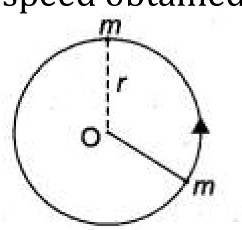
$$= \frac{5}{2}\sqrt{3.25} = 2.5 \times 1.8 = 4.5\ \text{ms}^{-1}$$

Angle of reflection $\theta' = \tan^{-1}\left(\frac{\tan\theta}{e}\right) = \tan^{-1}\left(\frac{\tan 60°}{0.5}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{0.5}\right)$

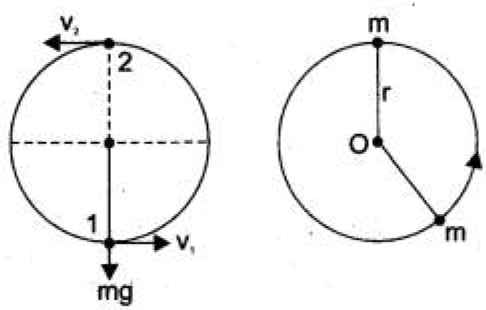
$$= \tan^{-1}(3.464) = 73.9°.$$

Question 3.
A bob of mass m is attached to one end of the rod of negligible mass and length r, the other end of which is pivoted freely at a fixed center O as shown in the figure.

What initial speed must be given to the object to reach the top of the circle? (Hint: Use law of conservation of energy). Is this speed. less or greater than speed obtained in the section 4.2.9?



Answer:
To get the vertical speed given to the object to reach the top of the circle, law of conservation of energy can be used at a points (1) and (2)
Total energy at 1 = Total energy at 2
∴ Potential energy at point 1 = 0



from eqn (i)

$$0 + \frac{1}{2}mv_1^2 = 2mgr + \frac{1}{2}mv_2^2$$

$$\frac{1}{2}v_1^2 = 2gr + \frac{1}{2}v_2^2$$

$$v_1^2 - v_2^2 = 4gr \quad ...(2)$$

In this case bob of mass m is connected with a rod of negligible mass, so the velocity of bob at highest point can be equal to zero i.e. $v_2 = 0$

$\therefore$ eqn. (2) becomes $v_1^2 = 4gr$; $v_1 = \sqrt{4gr}$ ms^{-1} or $v_1 = 2\sqrt{gr}$ ms^{-1}

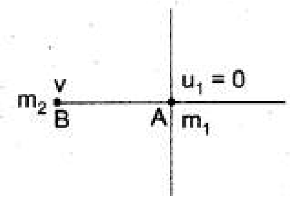
The speed of bob obtained here is lesser than the speed obtained in section 4.2.9. It is only because of string is replaced by a massless rod here.

Question 4.
Two different unknown masses A and B collide. A is initially at rest when B has a speed v. After collision B has a speed v/2 and moves at right angles to its original direction of motion. Find the direction in which A moves after collision.

Answer:

Given: $u_2 = v, \quad u_1 = 0$

$$v_2 = \frac{v}{2}, \quad \theta = ?$$

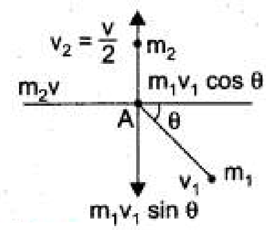


Component along x-axis

$$m_1 v_1 \cos\theta = m_2 v \quad \text{...(1)}$$

Component along y-axis

$$m_1 v_1 \sin\theta = m_2\left(\frac{v}{2}\right) \quad \text{...(2)}$$



from (1) and (2)

$$\tan\theta = \frac{\left(\frac{v}{2}\right)}{v} = \frac{1}{2};$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}(0.5);\ \theta = 26°33'.$$

Question 5.
A bullet of mass 20 g strikes a pendulum of mass 5 kg. The centre of mass of pendulum rises a vertical distance of 10 cm. If the bullet gets embedded into the pendulum, calculate its initial speed.

Answer:
Given: $m_1 = 20 \text{ g} = 20 \times 10^{-3}$ kg; $m_2 = 5$ kg; $s = 10 \times 10^{-2}$ m.
Let the speed of the bullet be v. The common velocity of bullet and pendulum bob is V. According to law of conservation of linear momentum.

$$V = \frac{m_1 v}{(m_1 + m_2)} = \frac{20\times10^{-3} v}{5 + 20\times10^{-3}} = \frac{0.02}{5.02} v = 0.004\, v$$

The bob with bullet go up with a deceleration of $g = 9.8$ ms^{-2}. Bob and bullet come to rest at a height of 10×10^{-2} m.

from III rd equation of motion

$$v^2 = u^2 + 2as \text{ here}$$

$$v^2 - 2gs = 0$$

$$v^2 = 2gs$$

$$(0.004\, v)^2 = 2 \times 9.8 \times 10 \times 10^{-2}$$

$$v^2 = \frac{2 \times 9.8 \times 10 \times 10^{-2}}{(0.004)^2}$$

$$v = 350 \text{ ms}^{-1}.$$