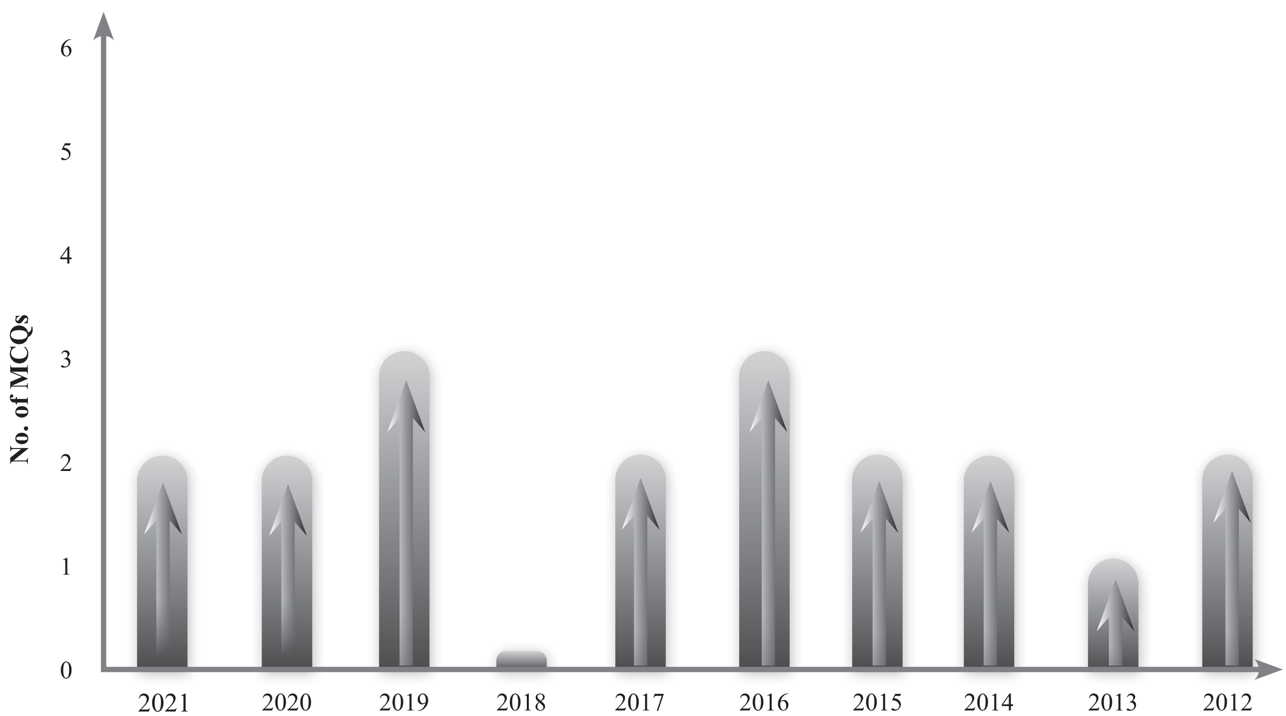
3

Motion in a Plane

Past Years NEET Trend



Investigation Report

TARGET EXAM	PREDICTED NO. OF MCQs	CRITICAL CONCEPTS
NEET	0-1	• Relative motion, projectile motion, circular motion

Perfect Practice Plan

Topicwise Questions	Learning Plus	Multiconcept MCQs	NEET Past 10 Years Questions	Total MCQs
57	28	18	19	122

PHYSICAL QUANTITIES

A physical quantity is a property of a material or system that can be quantified by measurement. A physical quantity can be expressed as the combination of a magnitude and a unit.

Ex: Length, Mass, Time, Velocity, Force, etc..

Physical quantities are mainly classified into three types. a) Scalars b) Vectors c) Tensors

Scalar quantities are those which have only magnitude. Example : mass, time, distance, speed, area, volume, density, work, power, energy, frequency, temperature, electric charge, electric current, potential, resistance, capacity, velocity of light, intensity of sound, etc..

Vectors are physical quantities which have both magnitude and direction and obey laws of addition of vectors. Example: velocity, force, displacement, acceleration, impulse, angular displacement, angular velocity, angular acceleration, moment of a force, torque, magnetic moment, magnetic induction field, intensity of electric field, etc..

Tensors are those quantities with one magnitude in any directions. These do not obey the law of vector addition. Example: Moment of inertia stress and strain etc..

VECTORS

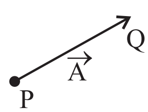
Definition

Phyical quantities that have magnitude as well as a sense of direction, and follow vector law of addition, are called vector quantities. Examples are displacement, force, velocity etc.

Representation of a vector:

A vector is represented by a line segment headed with an arrow. Its length is proportional to its magnitude. $\vec{A}$ is a vector.

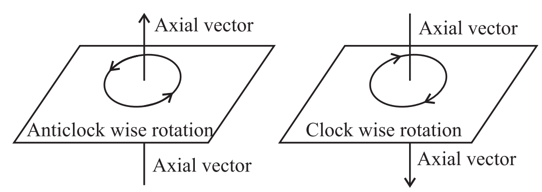
$\vec{A} = \overrightarrow{PQ}$ Magnitude of $\vec{A} = |\vec{A}|$ or $A = |\overrightarrow{PQ}|$



Magnitude of a vector is always positive.

Types of Vector

1. **Equal vectors :** Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal when they have equal magnitudes and have same direction.
2. **Parallel vector :** Two vectors $\vec{A}$ and $\vec{B}$ are said to be parallel when
   (*i*) Both have same direction.
   (*ii*) One vector is scalar (positive) non-zero multiple of another vector.
3. **Anti-parallel vectors :** Two vectors $\vec{A}$ and $\vec{B}$ are said to be anti-parallel when
   (*i*) Both have opposite direction.
   (*ii*) One vector is scalar non-zero negative multiple of another vector.
4. **Collinear vectors:** Two vectors are collinear if they have the same direction or are parallel or anti-parallel. They can be expressed in the form *a* = *Kb* where *a* and *b* are vectors and '*K*' is a scalar quantity.
5. **Zero vector** $(\vec{0})$ **:** A vector having zero magnitude and arbitrary direction (not known to us) is a zero vector.
6. **Unit vector :** A vector divided by its magnitude is a unit vector. Unit vector for $\vec{A}$ is $\hat{A}$ (read as *A* cap / *A* hat). Since, $\hat{A} = \frac{\vec{A}}{A} = \vec{A} = A\hat{A}$.
   Thus, we can say that unit vector gives us the direction.
7. **Orthogonal unit vectors :** $\hat{i}, \hat{j}$ and $\hat{k}$ are called orthogonal unit vectors. These vectors must form a Right Handed Triad (It is a coordinate system such that when we Curl the fingers of right hand from x to y then we must get the direction of z along thumb). Then
   $\hat{i} = \frac{\vec{x}}{x}, \hat{j} = \frac{\vec{y}}{y}, \hat{k} = \frac{\vec{z}}{z}$
   $\therefore \vec{x} = x\hat{i}, \vec{y} = y\hat{j}, \vec{z} = z\hat{k}$
8. **Axial Vectors :** These represent rotational effects and are always along the axis of rotation in accordance with right hand screw rule. Angular velocity, torque and angular momentum, etc., are example of physical quantities of this type.

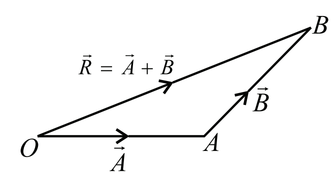


9. **Coplanar vector :** Three (or more) vectors are called coplanar vector if they lie in the same plane. Two (free) vectors are always coplanar.

Triangle Law of Vector Addition of Two Vectors

If two non zero vectors are represented by the two sides of a triangle taken in same order then the resultant is given by the closing side of triangle in opposite order. i.e. $\vec{R} = \vec{A} + \vec{B}$

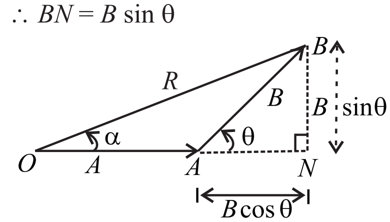
$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$



(1) Magnitude of resultant vector

In ΔABN, $\cos\theta = \frac{AN}{B}$ $\therefore$ AN = B cos θ

$\sin\theta = \frac{BN}{B}$ $\therefore BN = B \sin\theta$



In ΔOBN, we have $OB^2 = ON^2 + BN^2$

$\Rightarrow R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$

$\Rightarrow R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$

$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta \Rightarrow R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

(2) Direction of resultant vectors:

If θ is angle between $\vec{A}$ and $\vec{B}$ then

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

If $\vec{R}$ makes an angle α with $\vec{A}$, then in ΔOBN,

$$\tan\alpha = \frac{BN}{ON} = \frac{BN}{OA + AN}$$

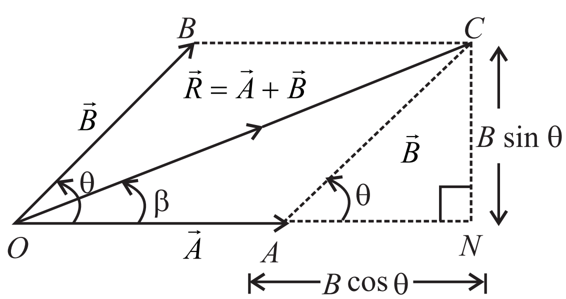
$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

Parallelogram Law of Vector Addition of Two Vectors

If two non zero vector are represented by the two adjacent sides of a parallelogram then the resultant is given by the diagonal of the parallelogram passing through the point of intersection of the two vectors.

(1) Magnitude

Since, $R^2 = ON^2 + CN^2$



$\Rightarrow R^2 = (OA + AN)^2 + CN^2$

$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$

$$\therefore \; R = |\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Special cases : $R = A + B$ when $\theta = 0^\circ$

$R = A - B$ when $\theta = 180^\circ$

$R = \sqrt{A^2 + B^2}$ when $\theta = 90^\circ$

(2) Direction

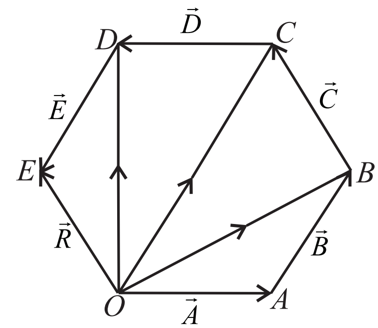
$$\tan\beta = \frac{CN}{ON} = \frac{B\sin\theta}{A + B\cos\theta}$$

Polygon Law of Vector Addition

If a number of non zero vectors are represented by the $(n - 1)$ sides of an n-sided polygon then the resultant is given by the closing side or the n^{th} side of the polygon taken in opposite order. So,

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$$

$$\overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{OE}$$



✍ KEY NOTE

- Resultant of two unequal vectors can not be zero.
- Resultant of three co-planar vectors may or may not be zero
- Resultant of three non co-planar vectors can not be zero

Subtraction of Vectors

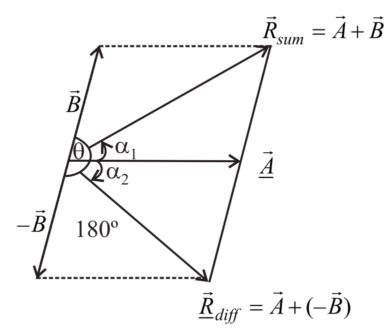
Since $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ and $|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\Rightarrow |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos(180^\circ - \theta)}$

Since, $\cos(180^\circ - \theta) = -\cos\theta$

$\Rightarrow |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$$\tan\alpha_1 = \frac{B\sin\theta}{A + B\cos\theta} \text{ and } \tan\alpha_2 = \frac{B\sin(180 - \theta)}{A + B\cos(180 - \theta)}$$

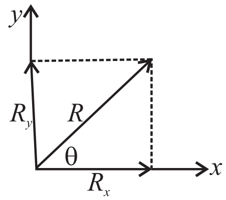


But $\sin(180^\circ - \theta) = \sin\theta$ and $\cos(180^\circ - \theta) = -\cos\theta$

$$\Rightarrow \tan\alpha_2 = \frac{B\sin\theta}{A - B\cos\theta}$$

Resolution of Vector Into Components

Consider a vector $\vec{R}$ in *x-y* plane as shown in fig. If we draw orthogonal vectors $\vec{R}_x$ and $\vec{R}_y$ along *x* and *y* axes respectively, by law of vector addition, $\vec{R} = \vec{R}_x + \vec{R}_y$



Now as for any vector $\vec{A} = A\hat{n}$ so, $\vec{R}_x = \hat{i}R_x$ and $\vec{R}_y = \hat{j}R_y$

so $\vec{R} = \hat{i}R_x + \hat{j}R_y$ .....(i)

But from fig $R_x = R\cos\theta$ .....(ii)

and $R_y = R\sin\theta$ .....(iii)

Since R and θ are usually known, Equation (ii) and (iii) give the magnitude of the components of $\vec{R}$ along *x* and *y*-axes respectively.

Here it is worthy to note once a vector is resolved into its components, the components themselves can be used to specify the vector as –

(1) The magnitude of the vector $\vec{R}$ is obtained by squaring and adding equation (ii) and (iii), i.e.

$$R = \sqrt{R_x^2 + R_y^2}$$

(2) The direction of the vector is obtained by dividing equation (iii) by (ii), i.e.

$\tan\theta = (R_y/R_x)$ or $\theta = \tan^{-1}(R_y/R_x)$

TRAIN YOUR BRAIN

Q. A particle moves in xy plane with a velocity given by $\vec{v} = (8t - 2)\hat{i} + 2\hat{j}$. If it passes through the point (0, 0) at t = 0 sec., then give equation of the path.

(*a*) y(y – 1) = x (*b*) y(x – 1) = y

(*c*) x(y – 1) = x (*d*) x(x – 1) = y

Sol (*a*) $v_x = 8t - 2$ integrating 0 to x, we have displacement as: $x = 4t^2 - 2t$

$v_y = 2$ integrating from 0 to y, we have displacement as: $y = 2t$

so, $x = y^2 - y = y(y - 1)$

Rectangular Components of 3-D Vector

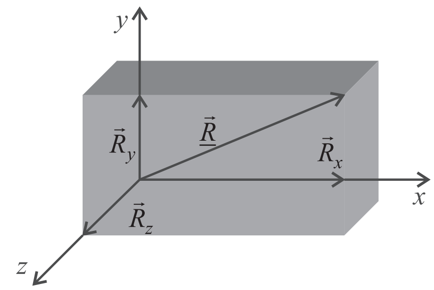
$\vec{R} = \vec{R}_x + \vec{R}_y + \vec{R}_z$ or $\vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$

If $\vec{R}$ makes an angle α with *x* axis, β with *y* axis and γ with *z* axis, then

$$\Rightarrow \cos\alpha = \frac{R_x}{R} = \frac{R_x}{\sqrt{R_x^2 + R_y^2 + R_z^2}} = l$$

$$\Rightarrow \cos\beta = \frac{R_y}{R} = \frac{R_y}{\sqrt{R_x^2 + R_y^2 + R_z^2}} = m$$

$$\Rightarrow \cos\gamma = \frac{R_z}{R} = \frac{R_z}{\sqrt{R_x^2 + R_y^2 + R_z^2}} = n$$



where *l*, *m*, *n* are called Direction Cosines of the vector $\vec{R}$

$$l^2 + m^2 + n^2 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{R_x^2 + R_y^2 + R_z^2}{R_x^2 + R_y^2 + R_z^2} = 1$$

or $(1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$

or $3 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 1$ or $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

TRAIN YOUR BRAIN

Q. A particle of mass m moves along a curve $y = x^2$. When particle has x –co–ordinate as 1/2 and x–component of velocity as 4m/s. Then pick the incorrect statements:

(*a*) The position coordinate of particle are (1/2, 1/4).

(*b*) The velocity of particle will be along the line 4x – 4y –1 = 0.

(*c*) The magnitude of velocity at that instant is $4\sqrt{2}$ m/s

(*d*) The magnitude of angular momentum of particle about origin at that position is 0.

Sol (*d*) $y = x^2$

at $x = \frac{1}{2}$, $y = \frac{1}{4}$; so position is $\left(\frac{1}{2}, \frac{1}{4}\right)$

$V_x = 4$, $\frac{dy}{dt} = 2x\frac{dx}{dt}$

$V_y = 2xV_x$

at $V_x = 4$

$V_y = 2\left(\frac{1}{2}\right)4$

$V_y = 4$

$\left(\frac{1}{2}, \frac{1}{4}\right)$

Scalar Product of Two Vectors

(1) Definition:

The scalar product (or dot product) of two vectors is defined as the product of the magnitude of two vectors with cosine of angle between them.

Thus if there are two vectors $\vec{A}$ and $\vec{B}$ having angle θ between them, then their scalar product written as $\vec{A}\cdot\vec{B}$ is defined as $\vec{A}\cdot\vec{B} = AB\cos\theta$

(2) Properties:

(*i*) It is always a scalar which is positive if angle between the vectors is acute (i.e., $\theta < 90°$) and negative if angle between them is obtuse (i.e. $90° < \theta < 180°$).

(*ii*) It is commutative, i.e. $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

(*iii*) It is distributive, i.e. $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

(*iv*) As by definition $\vec{A}\cdot\vec{B} = AB\cos\theta$

The angle between the vectors $\theta = \cos^{-1}\left[\frac{\vec{A}\cdot\vec{B}}{AB}\right]$

(*v*) cos θ = max = 1, i.e. θ = 0º, i.e., vectors are parallel $(\vec{A}\cdot\vec{B})_{max} = AB$

(*vi*) Scalar product of two vectors will be minimum when |cos θ| = min = 0, i.e. θ = 90º $(\vec{A}\cdot\vec{B})_{min} = 0$

(*vii*) The scalar product of a vector by itself is termed as self dot product and is given by $(\vec{A})^2 = \vec{A}\cdot\vec{A} = AA\cos\theta = A^2$ i.e., $A = \sqrt{\vec{A}\cdot\vec{A}}$

(*viii*) In case of unit vector $\hat{n}$ $\hat{n}\cdot\hat{n} = 1\times 1\times\cos 0 = 1$

so $\hat{n}\cdot\hat{n} = \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

(*ix*) In case of orthogonal unit vectors $\hat{i},\hat{j}$ and $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i}$ = 1×1 cos 90 = 0

(*x*) In terms of components $\vec{A}.\vec{B} = (\hat{i}A_x + \hat{j}A_y + \hat{k}A_z).(\hat{i}B_x + \hat{j}B_y + \hat{k}B_z)$

$= [A_xB_x + A_yB_y + A_zB_z]$

(3) Application :

(*i*) Work (W): In physics for constant force work is defined as,

$W = \vec{F}\cdot\vec{s} = Fs\cos\theta$ i.e. work is the scalar product of force with displacement.

(*ii*) Power (*P*) : As $W = \vec{F}\cdot\vec{s}$

or $\frac{dW}{dt} = \vec{F}\cdot\frac{d\vec{s}}{dt}$ [As $\vec{F}$ is constant]

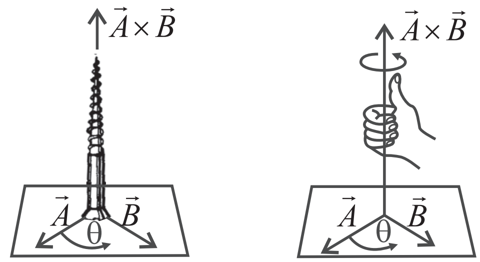
or $P = \vec{F}\cdot\vec{v}$ i.e., power is the scalar product of force with velocity. $\left[As\frac{dW}{dt} = P \; and \; \frac{d\vec{s}}{dt} = \vec{v}\right]$

Vector Product of Two Vector

(1) Definition:

The vector product or cross product of two vectors is defined as a vector having a magnitude equal to the product of the magnitudes of two vectors with the sine of angle between them, and direction perpendicular to the plane containing the two vectors in accordance with right hand screw rule. $\vec{C} = \vec{A}\times\vec{B}$

Thus, if $\vec{A}$ & $\vec{B}$ are two vectors, then their vector product written as $\vec{A}\times\vec{B}$ is a vector $\vec{C}$ defined by $\vec{C} = \vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$



The direction of $\vec{A}\times\vec{B}$ i.e. $\vec{C}$ is perpendicular to the plane containing both vectors $\vec{A}$ and $\vec{B}$ and in the sense of advance of a right handed screw rotated from $\vec{A}$ (first vector) to $\vec{B}$ (second vector) through the smaller angle between them. Thus, if a right handed screw whose axis is perpendicular to the plane framed by $\vec{A}$ and $\vec{B}$ is rotated from $\vec{A}$ to $\vec{B}$ through the smaller angle between them, then the direction of advancement of the screw gives the direction of $\vec{A}\times\vec{B}$ i.e. $\vec{C}$

(2) Properties:

(*i*) Vector product of any two vectors is always a vector perpendicular to the plane containing these two vectors, i.e., orthogonal to both the vectors $\vec{A}$ and $\vec{B}$ though the vectors $\vec{A}$ and $\vec{B}$ may or may not be orthogonal.

(*ii*) Vector product of two vectors is not commutative, i.e., $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$ [but $= -\vec{B}\times\vec{A}$]

Here it is worthy to note that

$|\vec{A}\times\vec{B}| = |\vec{B}\times\vec{A}| = AB\sin\theta$

i.e., in case of vector $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ magnitudes are equal but directions are opposite.

(*iii*) The vector product is distributive when the order of the vectors is strictly maintained, i.e.

$\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$

(*iv*) As by definition of vector product of two vectors $\vec{A}\times\vec{B}$ = AB sin $\theta\hat{n}$

So $|\vec{A}\times\vec{B}| = AB\sin\theta$ i.e. $\theta = \sin^{-1}\left[\frac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|}\right]$

(*v*) The vector product of two vectors will be maximum when sin θ = max = 1, i.e., θ = 90º

$[\vec{A}\times\vec{B}]_{max} = AB\,\hat{n}$

i.e., vector product is maximum if the vectors are orthogonal.

(*vi*) The vector product of two non-zero vectors will be minimum when $|\sin\theta| =$ minimum = 0, i.e., θ = 0º or 180º

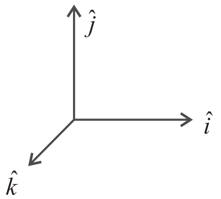
$[\vec{A}\times\vec{B}]_{min} = 0$

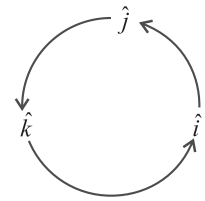
i.e. if the vector product of two non-zero vectors vanishes, the vectors are collinear.

(*vii*) The self cross product, i.e., product of a vector by itself vanishes, i.e., null vector $\vec{A}\times\vec{A} = AA\sin 0^{\circ}\,\hat{n} = \vec{0}$

(*viii*) In case of unit vector $\hat{n}\times\hat{n} = \vec{0}$ so that $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{0}$

(*ix*) In case of orthogonal unit vectors, $\hat{i},\hat{j},\hat{k}$ in accordance with right hand screw rule :





$\hat{i}\times\hat{j} = \hat{k},\quad \hat{j}\times\hat{k} = \hat{i}$ and $\hat{k}\times\hat{i} = \hat{j}$

And as cross product is not commutative,

$\hat{j}\times\hat{i} = -\hat{k}\quad \hat{k}\times\hat{j} = -\hat{i}$ and $\hat{i}\times\hat{k} = -\hat{j}$

(*x*) In terms of components

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}(A_yB_z - A_zB_y) + \hat{j}(A_zB_x - A_xB_z) + \hat{k}(A_xB_y - A_yB_x)$$

TRAIN YOUR BRAIN

Q. The vector $\hat{i} + x\hat{j} + 3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i} + (4x-2)\hat{j} + 2\hat{k}$. The values of x is

(*a*) $-\frac{2}{3}$ (*b*) $\frac{1}{3}$

(*c*) $\frac{2}{3}$ (*d*) None

Sol (***a***) $\sqrt{4^2 + (4x-2)^2 + 2^2} = 2\sqrt{1^2 + x^2 + 3^2}$

$5 + (2x-1)^2 = 10 + x^2$

$x = -\frac{2}{3}, 2$

(3) Application:

Since vector product of two vectors is a vector, some of the vector physical quantities (particularly representing rotational effects) like torque, angular momentum, velocity and force on a moving charge in a magnetic field can be expressed as the vector product of two vectors. It is well - established in physics that:

(*i*) Torque $\vec{\tau} = \vec{r}\times\vec{F}$

(*ii*) Angular momentum $\vec{L} = \vec{r}\times\vec{p}$

(*iii*) Velocity $\vec{v} = \vec{\omega}\times\vec{r}$

(*iv*) Force on a charged particle q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by $\vec{F} = q(\vec{v}\times\vec{B})$

(*v*) Torque on a dipole in a field $\vec{\tau_E} = \vec{p}\times\vec{E}$ and $\vec{\tau_B} = \vec{M}\times\vec{B}$

PROJECTILES

A body which is moving under the influence of gravity in air in two dimensions is called a projectile. It is of two types

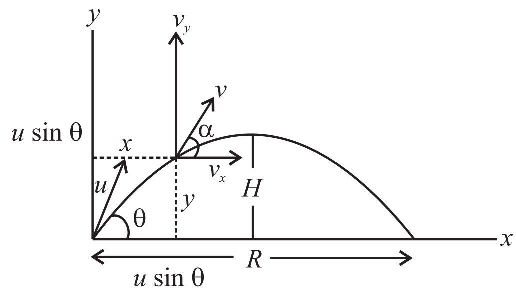
(1) Oblique projectile (2) Horizontal projectile

Oblique Projectile:

Any body projected into air with some velocity at angle of 'θ' [θ ≠ 90º and 0º] with the horizontal is called an oblique projectile.

Horizontal component of velocity $u_x = u\cos\theta$, remains constant throughout the journey.

Vertical component of velocity $u_y = u\sin\theta$, varies at the rate of '*g*'.



At the Point of Projection

(*a*) Horizontal component of velocity $u_x = u\cos\theta$

(*b*) Vertical component of velocity $u_y = u\sin\theta$

(*c*) Velocity vector $\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

(*d*) Angle between Velocity and acceleration is (90 + θ)

(*e*) Horizontal acceleration, $a_x = 0$

(*f*) vertical acceleration $a_y = -g$

At an instant 't'

(*a*) Horizontal acceleration, $a_x = 0$

(*b*) Vertical acceleration $a_y = -g$

(*c*) Horizontal component of velocity

$v_x = u\cos\theta$

(*d*) Vertical component of velocity

$v_y = u_y + a_y t = u\sin\theta - gt$

(*e*) Resultant Velocity

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$$

(*f*) Direction of velocity is given by

$$\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{u\sin\theta - gt}{u\cos\theta}\right)$$

(*g*) Horizontal displacement during a time *t*

$x = u_x t = (u\cos\theta)t$

(*h*) Vertical displacement during a time *t*

$$y = u_y t - \frac{1}{2}gt^2 = (u\sin\theta)t - \frac{1}{2}gt^2$$

(*i*) displacement $\vec{s} = x\hat{i} + y\hat{j}$

Net displacement of the body, $|\vec{s}| = \sqrt{x^2 + y^2}$

(*j*) Equation of a projectile y

$= (\tan\theta)\,x - \left(\dfrac{g}{2u^2\cos^2\theta}\right)x^2 = Ax - Bx^2$

Where A and B are constants

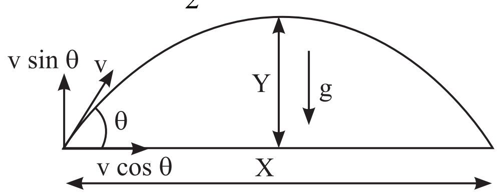
$A = \tan\theta, B = \dfrac{g}{2u^2\cos^2\theta}$

From the above equation $\theta = \tan^{-1}$ (A)

DERIVE THE EQUATION OF THE PATH OF A PROJECTILE.

Before deriving the equation of the path of a projectile, let's discuss projectile motion in brief. A motion where the body travels in an arc when thrown with a certain velocity at a given angle to the horizontal is known as projectile motion. The projectile motion can be said to be the sum of two different types of motion, where in the vertical component of the motion is an accelerated motion but the horizontal component of the motion is a motion with uniform velocity. Let us proceed with the derivation of the path of the projectile.

Formula Used: $s = ut + \frac{1}{2}at^2$, displacement = velocity × time



Since the acceleration is acting along the y-axis, we'll apply the second equation of motion to find the displacement.

The second equation of motion is given as $s = ut + \frac{1}{2}at^2$

$$Y = v\sin\theta(t) + \frac{1}{2}(-g)t^2$$

$$\Rightarrow Y = v\sin\theta t - \frac{gt^2}{2} \quad \text{....................(1)}$$

The motion along x-axis is a motion with constant velocity so displacement can be given as

displacement = velocity × time

Substituting the values, we get

X = v cos θ × t – ....(2)

Now we have to eliminate the time variable from the two equations. This is done by substituting the value of time from the second equation into the first equation. The equation can now be given as

$$Y = v\sin\theta\left(\frac{X}{v\cos\theta}\right) - \frac{1}{2}g\left(\frac{X}{v\cos\theta}\right)^2$$

$$\Rightarrow Y = X\tan\theta - \frac{gX^2}{2v^2\cos^2\theta}$$

This is the required equation of projectile motion.

Note: The equation of projectile motion is the equation of its trajectory. So if we know the x-component of the position of an object, we can find the y-component of the position by using the equation of the projectile motion. All the quantities involved in projectile motion can be directly or indirectly obtained from the equation of the trajectory of projectile motion.

Time of ascent (t_a) = Time of descent (t_d) = $\dfrac{u\sin\theta}{g}$

Time of flight $T = t_a + t_d = \dfrac{2u\sin\theta}{g}$

Maximum height $H = \dfrac{u^2\sin^2\theta}{2g}$

At the maximum height

(*a*) The vertical component of velocity becomes zero.

(*b*) The velocity of the projectile is minimum at the highest point and equal to $u\cos\theta$.

(*c*) Acceleration is equal to acceleration due to gravity 'g', and it always acts vertically downwards.

(*d*) The angle between minimum velocity and acceleration is 90º.

Horizontal range $R = (u\cos\theta)T$

$$R = u\cos\theta \times \frac{2u\sin\theta}{g} = \frac{2u_x u_y}{g}$$

$$R = \frac{u^2\sin 2\theta}{g}$$

(*a*) Range is maximum when θ = 45º

(*b*) Maximum range, $R_{Max} = \dfrac{u^2}{g}$

(*c*) When 'R' is maximum, $H_{Max} = \dfrac{R_{Max}}{4} = \dfrac{u^2}{4g}$

(*d*) $R = \dfrac{gT^2}{2\tan\theta}$ and if $\theta = 45^\circ$ then $R = \dfrac{gT^2}{2} \Rightarrow T = \sqrt{\dfrac{2R}{g}}$

Relation between H, T and R

(*a*) $\dfrac{H}{T^2} = \dfrac{g}{8}$

(*b*) $\dfrac{H}{R} = \dfrac{\tan\theta}{4}$

(*c*) $\dfrac{R}{T^2} = \dfrac{g}{2\tan\theta}$

Complimentary angles of projection

(*a*) When two bodies are projected with same initial velocity but at two different angles of projection θ and (90 – θ) then range is same.

(*b*) If T_1 are T_2 the times of flight for complimentary angles of projection then

(*i*) $\dfrac{T_1}{T_2} = \tan\theta$

(*ii*) $T_1T_2 = \dfrac{2R}{g} \Rightarrow T_1T_2 \propto R$

(*c*) If H_1 and H_2 are maximum heights for complimentary angles of projection then

(*i*) $\frac{H_1}{H_2} = \tan^2\theta$

(*ii*) $H_1 + H_2 = \frac{u^2}{2g}$

(*iii*) $R = 4\sqrt{H_1 H_2}$

(*iv*) $R_{max} = 2(H_1 + H_2)$

Relation between range and maximum height is $4H = R\tan\theta$ and if $R = H$ then $\theta = \tan^{-1}(4) = 76^o$

If a man throws a body to a maximum distance 'R' then he can project the body to maximum vertical height $R/2$.

(*a*) The angle between velocity and acceleration during the rise of projectile is $180^o > \theta > 90^o$

(*b*) The angle between velocity and acceleration during the fall of projectile is $0^o < \theta < 90^o$

In terms of range, equation of trajectory is given by $y = x\tan\theta\left(1 - \frac{x}{R}\right)$

At the point of striking the ground

(*a*) Horizontal component of velocity = $u\cos\theta$

(*b*) Vertical component of velocity = $-u\sin\theta$

(*c*) Velocity of projection is equal to striking velocity of projectile in magnitude.

(*d*) Angle of projection is equal to the striking angle of projectile

(*e*) Direction between velocity and acceleration is $(90 - \theta)$

(*f*) If the angle of projection with the horizontal is θ then angle of deviation is 2θ

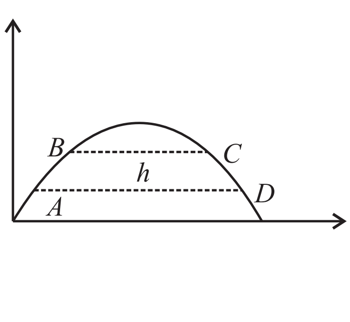
(*g*) Change in the velocity of the projectile along horizontal direction, $\Delta v_x = 0$ and along vertical direction,

$\Delta v_y = v_f - v_i = u\sin\theta - (-u\sin\theta) = 2u\sin\theta$

(*h*) Change in momentum of the projectile along horizontal direction, $\Delta p_x = 0$ and along vertical direction, $\Delta p_y = 2mu\sin\theta$

The projectile crosses the points A, D in t_1 seconds and B, C in t_2 then

$$t_1^2 - t_2^2 = \frac{8h}{g}$$



If a body is projected with a velocity u making an angle θ with the horizontal, the time after which direction of velocity is perpendicular to the initial velocity is $t = \frac{u\,\mathrm{cosec}\,\theta}{g} = \frac{u}{g\sin\theta}$ and its velocity at that time is $v = u\cot\theta$

TRAIN YOUR BRAIN

Q. A ball is projected so as to pass a wall at a distance a from the point of projection at an angle of 45° and falls at a distance b on the other side of the wall. If h is height of wall then:

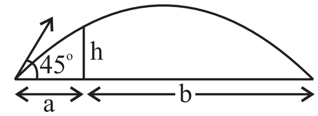
(*a*) $h = a\sqrt{2}$ (*b*) $h = b\sqrt{2}$

(*c*) $h = \frac{\sqrt{2}\,ab}{a+b}$ (*d*) $h = \frac{ab}{a+b}$

Sol (*d*) $\tan\alpha + \tan\beta = \tan\theta$

$$\frac{h}{a} + \frac{h}{b} = \tan 45^o$$

$$h = \frac{ab}{a+b}$$



Projectile As Seen From Another Projectile :

Suppose two bodies A and B are projected simltaneously from same point with initial velocities u_1 and u_2 at angles θ_1 and θ_2 with horizontal.

The instantaneous positions of the two bodies are given by

Body A : $x_1 = u_1\cos\theta_1 t$,

$$y_1 = u_1\sin\theta_1 t - \frac{1}{2}gt^2$$

Body B : $x_2 = u_2\cos\theta_2\, t$,

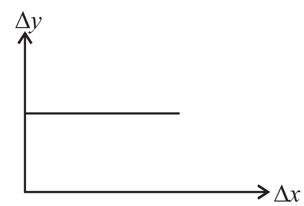
$$y_2 = u_2\sin\theta_2 t - \frac{1}{2}gt^2$$

$\Delta x = (u_1\cos\theta_1 - u_2\cos\theta_2)t$

$\Delta y = (u_1\sin\theta_1 - u_2\sin\theta_2)t$

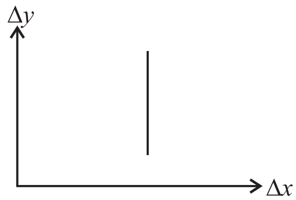
$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{u_1\sin\theta_1 - u_2\sin\theta_2}{u_1\cos\theta_1 - u_2\cos\theta_2}$$

(*i*) If $u_1\sin\theta_1 = u_2\sin\theta_2$. (initial vertical components) then slope $\frac{\Delta y}{\Delta x} = 0$



The path is a horizontal straight line

(*ii*) If $u_1\cos\theta_1 = u_2\cos\theta_2$ (initial horizontal components) Then slope $\frac{\Delta y}{\Delta x} = \infty$



The path is a vertical straight line

(*iii*) If $u_1 \sin\theta_1 > u_2 \sin\theta_2$, $u_1 \cos\theta_1 > u_2 \cos\theta_2$
Then the path. is a straight line with +Ve slope

(*iv*) If $u_1 \sin\theta_1 > u_2 \sin\theta_2$, $u_1 \cos\theta_1 > u_2 \cos\theta_2$
(or)
$u_1 \sin\theta_1 < u_2 \sin\theta_2$, $u_1 \cos\theta_1 > u_2 \cos\theta_2$
The path is straight line with - ve slope.

For a projectile, '*y*' component of velocity at half the maximum height is $\dfrac{u\sin\theta}{\sqrt{2}}$

at maximum height, resultant velocity $v = u\sqrt{\dfrac{1+\cos^2\theta}{2}}$

> ✍ **KEY NOTE**
> - For a projectile, w.r.t stationary frame path or trajectory is parabola
> - Path of projectile w.r.t frame of another projectile is a straight line
> - Acceleration of a projectile relative to another projectile is zero

A body is projected vertically up from a topless car relative to the car which is moving horizontally relative to earth

(*a*) If car velocity is constant, ball will be caught by the thrower.

(*b*) If car velocity is constant, path of ball relative to the ground is a parabola and relative to this car is straight up and then straight down

(*c*) If the car accelerates, ball falls back relative to the car

(*d*) If the car retards ball falls forward relative to the car

TRAIN YOUR BRAIN

Q. A projectile can have the same range R for two angles of projection θ and (90° – θ). If t_1 and t_2 be the times of flights in the two cases, then the ratio of the two times of flights is

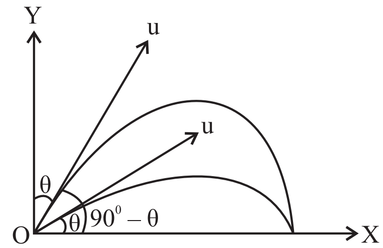
(*a*) $\dfrac{\cos^2\theta}{2g}$ (*b*) $\dfrac{\sin\theta}{g}$

(*c*) $\cos\theta$ (*d*) $\tan\theta$

Sol (*d*) $R = \dfrac{2u^2\sin\theta\cos\theta}{g}$

$t_1 = \dfrac{2u\sin\theta}{g}$ ...(*i*)

And $t_2 = \dfrac{2u\sin(90°-\theta)}{g} = \dfrac{2u\cos\theta}{g}$ ...(*ii*)



From Eqs. (i) and (ii), we get

$\dfrac{t_1}{t_2} = \dfrac{\sin\theta}{\cos\theta}$
$= \tan\theta$

The physical quantities which remains constant during projectile motion are

(*i*) acceleration due to gravity

(*ii*) Total energy $E_0 = \dfrac{1}{2}mu^2$

(*iii*) The horizontal component of the velocity '$u\cos\theta$'

The physical quantities which change during projectile motion are

(*i*) speed (*ii*) velocity

(*iii*) linear momentum (*iv*) *K.E.*

(*v*) *P.E.*

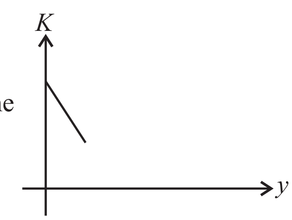
If E is the energy of the body when it is projected at an angle θ with horizontal then, the *KE* of the body at maximum height $E\cos^2\theta$ and *PE* is $\sin^2\theta$

A particle is projected up from a point at an angle 'θ' with the horizontal. At any time '*t*' if *p* = linear momentum, *y* = vertical displacement *x* = horizontal displacement, then the *K.E.* of the particle (*K*) plotted against these parameters can be

(*i*) *K – y* graph

$K = K_i - mgy$

It is a straight line

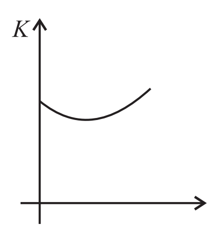


(*ii*) *K – t* graph

$K = K_i - mg\left(u_y t - \dfrac{1}{2}gt^2\right)$

$\because y = u_y t - \dfrac{1}{2}gt^2$

It is a parabola

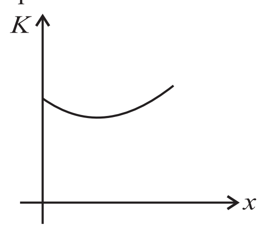


(***iii***) $\boldsymbol{K-x}$ **graph**

$$K = K_i - mg\left(x\tan\theta - \frac{g\,x^2}{2u_x^2}\right)$$

$$\therefore \quad y = (\tan\theta)x - \left(\frac{g}{2u_x^2}\right)x^2$$

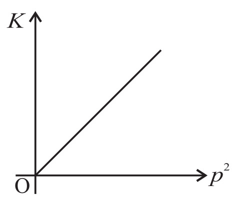
It is also a parabola



(***iv***) $\boldsymbol{K-p^2}$ **graph**

$OP^2 = 2\text{m}.K.$

$p^2 \alpha K$



It is a straight line passing through origin.

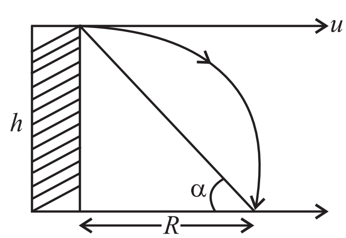
A particle is projected with a velocity $\vec{u} = a\hat{i} + b\hat{j}$ then the radius of curvature of the trajectory of the particle at the

(*i*) point of projection is $r = \dfrac{\left(a^2+b^2\right)^{3/2}}{ga}$

(*ii*) Highest point is $r = \dfrac{a^2}{g}$

HORIZONTAL PROJECTILE

When a body is projected horizontally with a velocity from a point above the ground level, it is called a *Horizontal Projectile*. When a stone is projected horizontally with a velocity 'u' from the top of a tower of height 'h' it describes a parabolic path as shown in figure.



(*a*) Time of descent $t = \sqrt{\dfrac{2h}{g}}$ (independent of u)

(*b*) The horizontal displacement (or) range $R = u\sqrt{\dfrac{2h}{g}}$

(*c*) The velocity with which it hits the ground

$$V = \sqrt{u^2 + 2gh} = \sqrt{u^2 + g^2t^2}$$

(*d*) The angle at which it strikes the ground

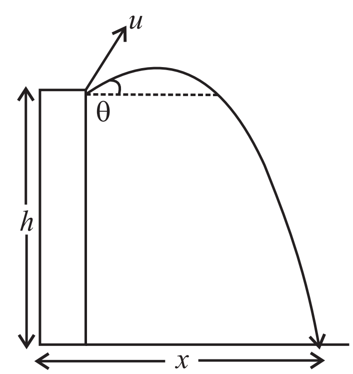
$$\theta = \tan^{-1}\left(\frac{gt}{u}\right) = \tan^{-1}\left(\frac{\sqrt{2gh}}{u}\right)$$

(*e*) If α is angle of elevation of point of projection from the point where body hits the ground then

$$\tan\alpha = \frac{h}{R} = \frac{gt^2/2}{ut} = \frac{gt}{2u}$$

$$\Rightarrow \quad \tan\alpha = \frac{\tan\theta}{2}$$

Case (*i*) : If the body is projected at an angle θ in upward direction from the top of the tower, then



(*a*) $h = (-u\sin\theta)t + \dfrac{1}{2}gt^2$

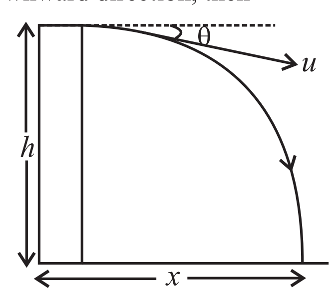
(*b*) $x = u\cos\theta \times t$

(*c*) The velocity with which it strikes the ground $v = \sqrt{u^2 + 2gh}$

(*d*) The angle at which it strikes the ground $\alpha = \tan^{-1}\left[\dfrac{-u\sin\theta + gt}{u\cos\theta}\right]$

(or) $\alpha = \tan^{-1}\left[\dfrac{\sqrt{u^2\sin^2\theta + 2gh}}{u\cos\theta}\right]$

Case (*ii*) : If the body is projected at angle θ from top of the tower in the downward direction, then



(*a*) $h = (u\sin\theta)t + \dfrac{1}{2}gt^2$

(*b*) $x = u \cos\theta \cdot t$

(*c*) The velocity with which it strikes the ground $v = \sqrt{u^2 + 2gh}$

(*d*) The angle at which it strikes the ground

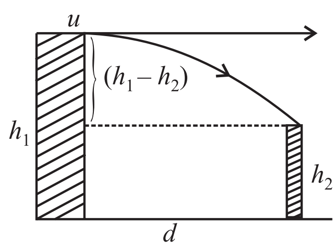
$$\alpha = \tan^{-1}\left[\frac{\sqrt{u^2 \sin^2\theta + 2gh}}{u\cos\theta}\right]$$

When an object is dropped from an aeroplane moving horizontally with constant velocity

(*a*) Path of the object relative to the earth is parabola

(*b*) Path of the object relative to pilot is a straight line vertically down.

Two tall towers having heights h_1 and h_2 are separated by a distance d. A person throws a ball horizontally with velocity u from the top of the first tower to reach the top of the second tower then



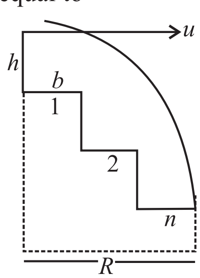
(*a*) Time taken $t = \sqrt{\dfrac{2(h_1 - h_2)}{g}}$

(*b*) Horizontal distance traveled $d = ut$

A ball rolls off from the top of a stair case with a horizontal velocity u. If each step has a height 'h' and width 'b' then the ball will just hit the n^{th} step if n equal to

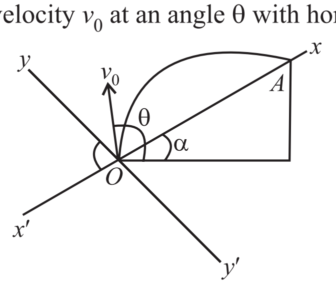
$nb = ut$ and $nh = \dfrac{1}{2}gt^2$

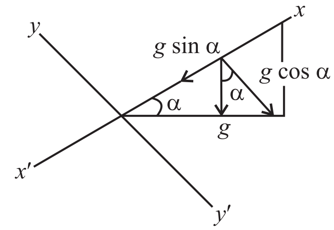
$$n = \frac{2hu^2}{gb^2}$$



Motion of a Projected Body on an Inclined Plane:

A body is projected up the inclined plane from the point O with an initial velocity v_0 at an angle θ with horizontal.





(*a*) Acceleration along x – axis $a_x = -g \sin\alpha$

(*b*) Acceleration along y – axis $a_y = -g \cos\alpha$

(*c*) Component of velocity along x – axis $u_x = v_0 \cos(\theta - \alpha)$

(*d*) Component of velocity along y – axis $u_y = v_0 \sin(\theta - \alpha)$

(*e*) Time of flight $T = \dfrac{2v_0 \sin(\theta - \alpha)}{g\cos\alpha}$

(*f*) Range of projectile

$$R = \frac{v_0^2}{g\cos^2\alpha}\left[\sin(2\theta - \alpha) - \sin\alpha\right]. \text{ (or)}$$

$$R = \frac{2v_0^2 \sin(\theta - \alpha)\cos\theta}{g\cos^2\alpha}$$

For maximum range $(2\theta - \alpha) = \dfrac{\pi}{2}$

$$\therefore \quad R_{max} = \frac{v_0^2(1 - \sin\alpha)}{g\cos^2\alpha}$$

(g) $T^2 g = 2R_{\max}$

RELATIVE VELOCITY

Consider two bodies A & B and A is moving with a velocity V_A *w.r.t.* ground and B is moving with velocity V_B *w.r.t.* ground

Relative velocity of body 'A' *w.r.t.* 'B' is given by $\vec{V}_R = \vec{V}_A - \vec{V}_B$

Relative velocity of body 'B' *w.r.t.* 'A' is given by $\vec{V}_R = \vec{V}_B - \vec{V}_A$

$\overline{V}_A - \overline{V}_B$ and $\overline{V}_B - \overline{V}_A$ are equal in magnitude but opposite in direction.

$$|\overline{V}_R| = |\overline{V}_A - \overline{V}_B| = \sqrt{V_A{}^2 + V_B{}^2 - 2.V_A V_B \cdot \cos\theta}$$

For two bodies moving in the same direction, magnitude of relative velocity is equal to the difference of magnitude of velocities.

($\theta = 0^o$, $\cos 0 = 1$) $\therefore |\overline{V}_R| = V_A - V_B$

For two bodies moving in opposite direction, magnitude of relative velocity is equal to the sum of the magnitude of their velocities.

($\theta = 180^o$; $\cos 180^o = -1$) $\therefore |\overline{V}_R| = V_A + V_B$

Relative displacement A w.r.t. B is $\vec{X}_{AB} = \vec{X}_{AG} - \vec{X}_{BG}$

Where $\vec{X}_{AG}$ = displacement of 'A' w.r.t ground and $\vec{X}_{BG}$ displacement of 'B' w.r.t ground

Relative velocity A w.r.t. B is $\vec{V}_{AB} = \vec{V}_{AG} - \vec{V}_{BG}$

Relative acceleration of A w.r.t. B is

$$\vec{a}_{AB} = \vec{a}_{AG} - \vec{a}_{BG}$$

TRAIN YOUR BRAIN

Q. A particle starts with initial velocity of $\left(2\hat{i}+\hat{j}\right)$ m/s. Uniform acceleration $\left(-\hat{i}+3\hat{j}\right)$ m/s^2. What is *y* component of velocity at the instant when *x* component of velocity becomes zero?

(*a*) 5 m/s (*b*) 7 m/s

(*c*) 6 m/s (*d*) 10 m/s

Sol. (*b*) At any instant 't'

$v_x = 2 - t$

$v_y = 1 + 3t$

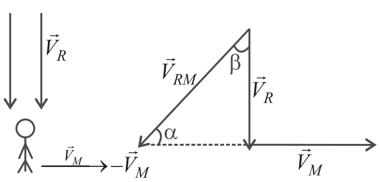
when $v_x = 0$, $t = 2$

$v_y = 7$ m/s

Rain umbrella Concept:

If the rain is falling with a velocity $\vec{V}_R$ and a man moves with a velocity $\vec{V}_M$ relative to ground, he will observe the rain falling with a velocity $\vec{V}_{RM} = \vec{V}_R - \vec{V}_M$.

Case – I : If rain is falling vertically with a velocity $\vec{V}_R$ and an observer is moving horizontally with velocity $\vec{V}_M$, the velocity of rain relative to observer will be:



$\vec{V}_{RM} = V_R - V_M$

The magnitude of velocity of rain relative to man is

$$V_{RM} = \sqrt{V_R^2 + V_M^2}$$

If α is the angle made by the umbrella with horizontal, than,

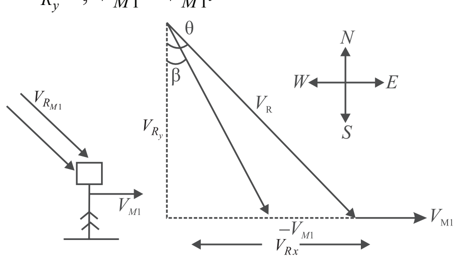
$$\tan\alpha = \frac{V_R}{V_M}$$

If β is the angle made by the umbrella with vertical, then,

$$\tan\beta = \frac{V_M}{V_R}$$

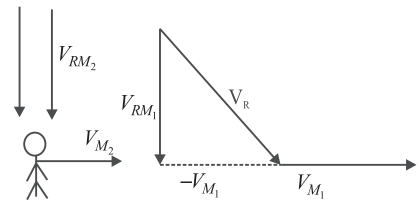
Case – II: If a man is moving with a velocity V_{M_1} relative to ground towards east(positive x-axis), and the rain is falling with a velocity $\vec{V}_R$ relative to ground by making an angle θ with vertical(negative z-axis). Then the velocity of rain relative to man $\vec{V}_{RM_1}$ is as shown in figure.

$$\vec{V}_R = V_{R_x}\hat{i} - V_{R_y}\hat{k}\ ;\ \vec{V}_{M1} = V_{M1}\hat{i}$$

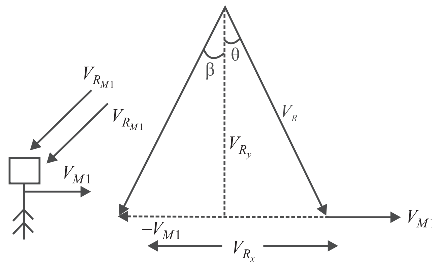


and $\tan\beta = \dfrac{V_{R_x} - v_{M_1}}{V_{R_y}}$ ...(2)

Case – III : If the man speeds up, at a particular velocity $\vec{V}_{M_2}$, the rain will appear to fall vertically with $\vec{V}_{RM_2}$, then $\vec{V}_{RM_2} = \vec{V}_R - \vec{V}_{M_2}$ as shown in figure.



Case - IV : If the man increases his speed further, he will see the rain falling with a velocity as shown in figure.



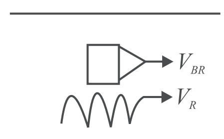
$$\vec{V}_{RM_3} = \vec{V}_R - \vec{V}_{M_3}$$

$$\tan\beta = \frac{V_{M_3} - V_{R_x}}{V_{R_y}}$$

Motion of a Boat in the River

Boat motion is classified into three categories based on the angle between V_{BR} and V_R

1. **For down stream:** θ = 0º



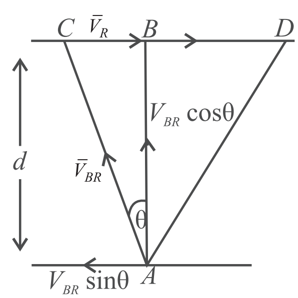
Resultant velocity of the boat = $V_{BR} + V_R$

2. **For Up stream:** θ = 180º

Resultant velocity of the boat = $V_{BR} - V_R$

3. **For Cross stream** θ = 90º

Suppose the boat starts at point *A* on one bank with velocity V_{BR} and reaches the other bank at point *D*



The component of velocity of boat parallel to the flow of water is V_{BR} sin θ

The component of velocity of boat perpendicular to the flow of water is $V_{BR}\cos\theta$

The time taken by the boat to cross the river is, $t = \dfrac{d}{V_{BR}\cos\theta}$

Along the flow of water, distance travelled by the boat or drift is $= (V_R - V_{BR}\sin\theta)t$

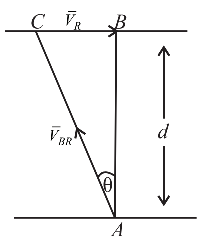
$$x = (V_R - V_{BR}\sin\theta)\left[\frac{d}{V_{BR}\cos\theta}\right]$$

(*a*) The boat reaches the other end of the river to the right of B if $V_R > V_{BR}\sin\theta$

(*b*) The boat reaches the other end of the river to the left of B if $V_R < V_{BR}\sin\theta$

(*c*) The boat reaches the exactly opposite point on the bank if $V_R = V_{BR}\sin\theta$

A Boat Crossing the River in Shortest Distance



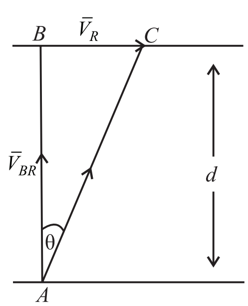
(*i*) The boat is to be rowed upstream making some angle θ with normal to the bank of the river which is given by θ $= \theta = \sin^{-1}\left(\dfrac{V_R}{V_{BR}}\right)$

(*ii*) The angle made by boat with the river flow (or) bank is $= 90^o + \theta$

(*iii*) Velocity of boat wrt ground has a magnitude of $V_B = \sqrt{V_{BR}^2 - V_R^2}$

> ✍ **KEY NOTE**
>
> The time taken to cross the river is $t = \dfrac{d}{\sqrt{V_{BR}^2 - V_R^2}}$
>
> V_{BR} = Relative velocity of the boat w.r.t river (or) velocity of boat in still water.

A Boat Crossing the River in Shortest Time:



If $\bar{V}_{BR}$, $\bar{V}_R$ are the velocities of a boat and river flow respectively then to cross the river in shortest time, the boat is to be rowed across the river i.e., along normal to the banks of the river.

(*i*) Time taken to cross the river, $t = \dfrac{d}{V_{BR}}$ where d = width of the river. This time is independent of velocity of the river flow

(*ii*) Velocity of boat *w.r.t* ground has a magnitude of $V_B = \sqrt{V_{BR}^2 + V_R^2}$

(*iii*) The direction of the resultant velocity is $\theta = \tan^{-1}\left(\dfrac{V_R}{V_{BR}}\right)$ with the normal.

(*iv*) The distance (BC) traveled downstream $= V_R \times \dfrac{d}{v_{BR}}$

CIRCULAR MOTION

Radius Vector: Consider a particle moving along a circular path. The line joining the centre of circle to position of particle is called radius vector

Angular displacement: The angle turned by the radius vector in a given time interval is called angular displacement α

$$d\theta = \frac{arc\ length}{radius}$$

SI unit radian

Small angular displacements are vectors

Large angular displacements are scalar as it does not obey commutative law

The direction of angular displacement is along the axis of rotation and it is given by right hand screw rule.

When a particle completes one revolution the angular displacement is $\theta = 2\pi$ radian

When a particle completes N revolutions in a circle the angular displacement is $\theta = 2\pi N$

When an object moves in circular path at a constant speed, the motion is in uniform circular motion

In uniform circular motion, uniform means constant speed. But direction of velocity remains changing

Angular Velocity ω:

The time rate of angular displacement of particle is called angular velocity

If $\Delta\theta$ is angular displacement in small interval of time Δt then

Average angular velocity $\omega_{av} = \dfrac{\Delta\theta}{\Delta t}$

Instantaneous angular velocity is $\omega = \underset{\Delta t\to 0}{Lt}\left[\dfrac{\Delta\theta}{\Delta t}\right] = \dfrac{d\theta}{dt}$

SI Unit rad S^{-1}

Dimensional Formula T^{-1}

Angular velocity is a axial vector.

Its direction is given by right hand screw rule

Its direction is along axis of rotation

> ✍ **KEY NOTE**
> - When a body makes 'N' revolutions in 't' sec's then its average angular velocity is
>
> $$\omega = \frac{2\pi N}{t}$$
>
> - If a particle makes 'n' rotations per sec its angular velocity is
>
> $$\omega = \frac{2\pi}{T} = 2\pi n$$

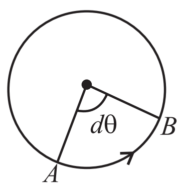
TRAIN YOUR BRAIN

Q. An insect trapped in a circular groove of radius 14 cm moves along the groove steadily and completes 10 revolutions in 100 s. The linear speed of the insect is

(*a*) 4.3 cm s^{-1} (*b*) 8.8 cm s^{-1}

(*c*) 6.3 cm s^{-1} (*d*) 7.3 cm s^{-1}

Sol (*b*) Here, r = 14 cm



Frequency, $\nu = \frac{10}{100}$ Hz

The angular speed of the insect is

$$\omega = 2\pi\nu = 2\pi \times \frac{10}{100} = 2\pi \times 0.1$$

The linear speed of the insect is

$v = \omega r = 2\pi \times 0.1 \times 14 = 8.8$ cm s^{-1}

Angular velocity of hands of a clock:

Angular velocity of seconds hand $\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = \frac{\pi}{30}\ rad\ s^{-1}$

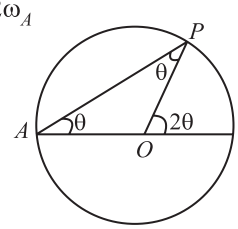
Angular velocity of minutes hand $\omega = \frac{2\pi}{60 \times 60} = \frac{\pi}{1800}\ rad\ s^{-1}$

Angular velocity of hours hand $\omega = \frac{2\pi}{12 \times 3600} = \frac{\pi}{21600}\ rad\ s^{-1}$

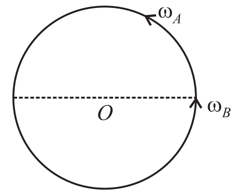
In case of self rotation of earth about its own axis

$$\omega = \frac{2\pi}{24 \times 60 \times 60}\ rad/\text{sec}$$

In the below fig. Let the angular velocity of particle (P) about the point 'O' is ω_0, Let the angular velocity of particle about A is ω_A then $\omega_0 = 2\omega_A$

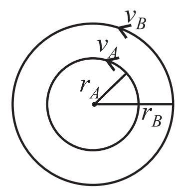


In the below fig. if two particles A and B are moving in same circular path in the same direction, for a person at the centre of the circle $\omega_{BA} = \omega_B - \omega_A$



Time taken by one particle to complete one rotation with respect to another particle is $T = \frac{2\pi}{\omega_{rel}} = \frac{2\pi}{\omega_B - \omega_A} = \frac{T_A T_B}{T_A - T_B}$

If two particles A and B are moving in concentric circles as shown in the fig., if they are nearer to each other.



$r_{rel} = r_B - r_A$, $v_{rel} = v_B - v_A$

$$\omega_{rel} = \frac{v_{rel}}{r_{rel}} = \frac{v_B - v_A}{r_B - r_A}$$

> ✍ **KEY NOTE**
> - In general the relative angular velocity of A with respect to B is determine by using the formula
> - $\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{\text{relative velocity of } A \text{ w.r.t } B \text{ perpendicular to line } AB}{\text{Seperation between } A \,\&\, B}$

TRAIN YOUR BRAIN

Q. What is the magnitude of average velocity after half rotation, if a particle is moving with constant speed v in a circle?

(*a*) 2v (*b*) $2\frac{v}{\pi}$

(*c*) $\frac{v}{2}$ (*d*) $\frac{v}{2\pi}$

Sol (*b*) The time taken to cover the complete circle would be equal to $T = \frac{2\pi r}{v}$,

Hence the time taken to cover half the rotation would be equal to $T_{1/2} = \frac{2\pi r}{2v} = \frac{\pi r}{v}$.

Hence the magnitude of average velocity after half rotation would be equal to $v_{av} = \frac{2r\nu}{\pi r} = \frac{2v}{\pi}$

Angular acceleration (a)

The time rate of change of angular velocity of a particle is called angular acceleration

If $\Delta\omega$ be the change in angular velocity of the particle in time interval 't' to $t + \Delta t$, while moving on a circular path, then

Average angular acceleration $\alpha_{av} = \frac{\Delta\omega}{\Delta t}$

Instantaneous angular acceleration $\alpha_{inst} = \underset{\Delta t \to 0}{Lt} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega\frac{d\omega}{d\theta}$$

S.I. units rad. sec^{-2}

Dimensional of angular acceleration $[T^{-2}]$

Its direction is in the direction of change in angular velocity and it is given by right hand screw rule. When angular velocity increases the direction of angular acceleration is in the direction of angular velocity

When angular velocity decreases the direction of angular acceleration is in the opposite direction of angular velocity.

If a particle rotates with uniform angular velocity then $\alpha = 0$

If the particle has constant angular acceleration that is if α = constant , in this case we use following equations of motion

$$\omega = \omega_0 + \alpha t, \; \theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 - \omega_0^2 = 2\alpha\theta$$

> ✍ **KEY NOTE**
>
> - In general we can also use the following equations to solve the problems
>
> $$\omega_{av} = \frac{\Delta\theta}{\Delta t}, \alpha_{av} = \frac{\Delta\omega}{\Delta t}$$
>
> $$\omega = \frac{d\theta}{dt}, \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega\frac{d\omega}{d\theta}$$
>
> $$\int d\theta = \int \omega dt, \int d\omega = \int \alpha dt, \int \alpha\, d\theta = \int \omega d\omega$$

Relation between linear and angular variables

Relation between linear and angular displacement is $ds = rd\theta$

Relation between linear and angular velocities is $v = r\omega$, $\vec{v} = \omega \times \vec{r}$

Relation between tangential and angular acceleration is $a_t = r\alpha$, a instead of alpha $= \vec{\alpha} \times \vec{r}$

Linear acceleration of a particle moving in a circle

we know $\vec{v} = \vec{\omega} \times \vec{r}$

diff. w.r.t time, we get

$$\frac{d\vec{v}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$

$$\vec{a} = (\vec{\alpha} \times \vec{r}) + (\vec{\omega} \times \vec{v})$$

but $\vec{\alpha} \times \vec{r} =$ a, it is tangential acceleration

$\vec{\omega} \times \vec{v} = \vec{a}_c$, it is centripetal acceleration

Due to change in direction of velocity there is an acceleration and is always directed towards the centre. This is called centripetal or radical acceleration and the corresponding force acting towards the centre is called centripetal force

Centripetal Acceleration (a_r or a_c)

If v is the linear velocity and ω is angular velocity then $\vec{v} = \vec{\omega} \times \vec{r}$ & $v = r\omega$.

When a paticle is moving along a circle of radius r with a uniform speed v, then the centripetal acceleration is a_r.

$$\vec{a}_r = \vec{\omega} \times \vec{v} = \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$a_r = \omega v = r\omega^2 = \frac{v^2}{r}$$

Position vector is always perpendicular to velocity vector. i.e, $\vec{r} \times \vec{v} = 0$

The directions of $\vec{a}_r, \vec{\omega}$ and $\vec{v}$ are mutually perpendicular.

Velocity vector is always perpendicular to the centripetal acceleration vector is $\vec{v} \cdot \vec{a}_c = 0$

Position vector $(\vec{r})$ and centripetal acceleration $(\vec{a}_r)$ are always antiparallel.

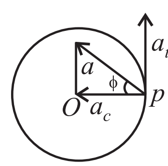
Momentum goes on changing in direction but its magnitude is constant.

Due to change in magnitude of velocity (speed) of a particle in circular motion, it has tangential acceleration and the corresponding force is called tangential force

$$a_t = \frac{dv}{dt} \text{ also } a_t = r\alpha \; (\vec{a}_t = \vec{\alpha} \times \vec{r})$$

$$F_t = ma_t = m\frac{dv}{dt} = mr\alpha$$

Net linear acceleration of particle in circular motion is $a = \sqrt{a_c^2 + a_t^2}$



If ϕ is the angle made by 'a' with a_c then $\tan\phi = \frac{a_t}{a_c}$

Net force $F = \sqrt{F_c^2 + F_t^2}$

Uniform circular motion:

In the above case if $a_c = 0$, $a_t = 0$, , then the particle under go uniform circular motion

or

When a particle moves in a circular path with constant speed then it is said be in uniform circular motion. in this case the acceleration of the particle is $a = v(\omega) = v^2/r = r\omega^2$,

In uniform circular motion

(*a*) magnitude of velocity does not change
(*b*) velocity changes
(*c*) direction of velocity changes
(*d*) angular velocity is constant
(*e*) centripetal acceleration changes (only in direction)
(*f*) linear momentum changes
(*g*) angular momentum w.r.t to center does not change

Non Uniform Circular motion:

In a circular motion if $a_c \neq 0$, $a_t \neq 0$ then the particle undergo non uniform circular motion , in this case the acceleration of particle is given by $a = \sqrt{a_c^2 + a_t^2}$

If ϕ is the angle made by 'a' with a_c then $\tan\phi = \dfrac{a_t}{a_c}$

For a particle in non uniform circular motion, the resultant force on the particle is $F = \sqrt{F_c^2 + F_t^2}$

In non uniform circular motion

(a) both magnitude and direction of velocity changes
(b) angular velocity ω changes
(c) linear momentum and angular momentum are not conserved

Note: In circular motion

$a_c \rightarrow$ is towards centre

$v, a_t \rightarrow$ are along tangential direction

$d\theta, \omega, \alpha \rightarrow$ are along axis of rotation

TRAIN YOUR BRAIN

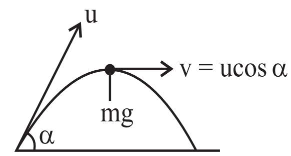
Q. A projectile is projected at an angle 60° with horizontal with speed 10 ms^{-1}. The minimum radius of curvature of trajectory is:

(*a*) 2.55 m (*b*) 2 m
(*c*) 10 m (*d*) None of these

Sol (*a*) The radius of curvature is given by

$$\rho = \frac{mv^3}{|F \times v|} = \frac{mv^3}{Fv\sin\theta} = \frac{mv^3}{mgv\sin\theta}$$

$(\because F = mg)$



$$\Rightarrow \quad \rho = \frac{v^2}{g\sin\theta}$$

$$\therefore \rho_{min} = \frac{v^2}{g(\sin\theta)_{max}} \quad \text{[For } p_{min}, \theta \text{ should be maximum]}$$

$$\therefore \rho_{min} = v^2/g \qquad [\because (\sin\theta)_{max} = 1]$$

i.e. $\theta = 90°$ is possible only when the projectile is at the maximum height.

At the maximum height the velocity,

$v = u\cos\alpha = 10\cos 60° = 5$ ms^{-1}

$$\therefore \rho_{min} = \frac{5 \times 5}{9.8} = 2.55\text{m}$$

ILLUSTRATIONS

1. There are N coplanar vectors each of magnitude V. Each vector is inclined to the preceding vector at angle $\dfrac{2\pi}{N}$. What is the magnitude of their resultant?

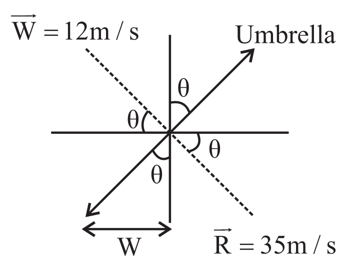
(*a*) $\dfrac{V}{N}$ (*b*) V
(*c*) Zero (*d*) $\dfrac{N}{V}$

Sol. (*c*) Zero. As each vector cancels out the opposite vector i.e. opposite in direction.

2. Rain is falling vertically with a speed of 35 ms^{-1}. Winds starts blowing after sometime with a speed of 12 ms^{-1} in east to west direction. At what angle with the critical should a boy waiting at a bus stop hold his umbrella to protect himself from rain?

(*a*) $\sin^{-1}\left(\dfrac{12}{15}\right)$ (*b*) $\cos^{-1}\dfrac{12}{35}$
(*c*) $\tan^{-1}\dfrac{12}{35}$ (*d*) $\cot^{-1}\dfrac{12}{35}$

Sol. (*c*)

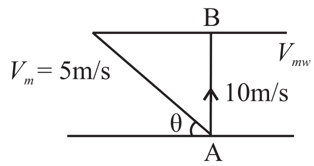


$$\tan\theta = \frac{12}{35}, \ \theta = \tan^{-1}\left(\frac{12}{35}\right)$$

3. A river is flowing from west to east with a speed 5 ms^{-1}. A swimmer can swim in still water at speed of 10 ms^{-1}. If he wants to start from point A on south bank and reach opposite point B on north bank, in what direction should he swim?

(*a*) 30° east of north
(*b*) 60° east of north
(*c*) 30° west of north
(*d*) 60° west of north

Sol. (*c*) $\cos\theta = \frac{V_w}{V_{mw}} = \frac{5}{10} = \frac{1}{2}$



$\theta = 60°$

∴ Angle is 90° – 60° west of north.

4. A unit vector in the direction of resultant vector of $\vec{A} = -2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} - 4\hat{k}$ is:

(*a*) $\frac{-2\hat{i} - \hat{j} + \hat{k}}{\sqrt{6}}$ (*b*) $\frac{2\hat{i} + \hat{i} + \hat{k}}{\sqrt{6}}$

(*c*) $\frac{-\hat{i} + 5\hat{j} - 5\hat{k}}{\sqrt{51}}$ (*d*) $\frac{2\hat{i} - \hat{j} + \hat{k}}{\sqrt{6}}$

Sol. (*c*) $\vec{A} = -2\hat{i} + 3\hat{j} - \hat{k}, \vec{B} = \hat{i} + 2\hat{j} - 4\hat{k}$

$\bar{R} = -\hat{i} + 5\hat{j} - 5\hat{k}$

$\hat{n} = \frac{\bar{R}}{|\bar{R}|} = \frac{-\hat{i} + 5\hat{j} - 5\hat{k}}{\sqrt{1+25+25}} = \frac{-\hat{i} + 5\hat{j} - 5\hat{k}}{\sqrt{51}}$

5. If R and H represent horizontal range and maximum height of the projectile, then the angle of projection with the horizontal is:

(*a*) $\tan^{-1}\left(\frac{H}{R}\right)$ (*b*) $\tan^{-1}\left(\frac{2H}{R}\right)$

(*c*) $\tan^{-1}\left(\frac{4H}{R}\right)$ (*d*) $\tan^{-1}\left(\frac{4R}{H}\right)$

Sol. (*c*) $H = \frac{u^2 \sin^2\theta}{2g}, R = \frac{u^2 \sin 2\theta}{g}$

$R = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin\theta\cos\theta}{g}$

$\frac{H}{R} = \frac{\sin\theta}{2 \times 2\cos\theta}$

$\frac{H}{R} = \frac{\tan\theta}{4}$ $\tan\theta = \frac{4H}{R}$

6. Two constant Force, $F_1 = 2\hat{i} - 3\hat{j} + 3\hat{k}$ and $F_2 = \hat{i} + \hat{j} - 2\hat{k}$ acts on a body and displace it from the position $r_1 = \hat{i} + 2\hat{j} - 2\hat{k}$ to $r_2 = 7\hat{i} + 10\hat{j} + 5\hat{k}$. Find the work done:

(*a*) 9 J (*b*) 4 J

(*c*) –3 J (*d*) 8 J

Sol. (*a*) $W = \left(\vec{F}_1 + \vec{F}_2\right).\left(\vec{r}_2 - \vec{r}_1\right)$

$= (3\hat{i} - 2\hat{j} + \hat{k}) . (6\hat{i} + 8\hat{j} + 7\hat{k})$

W = 18 – 16 + 7 = 9 J

7. If the vectors $A = \hat{i} + 2\hat{j} + 4\hat{k}$ and $B = 5\hat{i}$ represents the two sides of a triangle, then the third side of the triangle has length?

(*a*) $\sqrt{56}$ (*b*) $\sqrt{21}$

(*c*) 5 (*d*) 6

Sol. (*a*) $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$|A| = \sqrt{21}, |B| = 5$

A.B = |A| |B| cosθ

$5 = \sqrt{21} \times 5\cos\theta \therefore \cos\theta = \frac{5}{5\sqrt{21}}$

$R = \sqrt{21 + 25 + 2 \times 5 \times \sqrt{21} \times \frac{1}{\sqrt{21}}}$

$R = \sqrt{21 + 25 + 10} = \sqrt{56}$

8. A ball is thrown at angle of 45° with horizontal with kinetic energy E. The kinetic energy at the highest point during the flight is:

(*a*) Zero (*b*) $E/2$

(*c*) E (*d*) $2^{1/2}E$

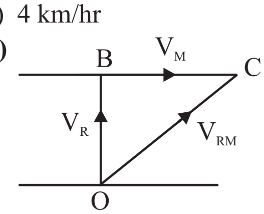
Sol. (*b*) $V = V\cos\theta = V\cos 45° = \frac{V}{\sqrt{2}}$

$E_2 = \frac{1}{2}m\left(\frac{V}{\sqrt{2}}\right)^2 = \frac{E}{2}$

9. It is raining vertically downward with velocity of 3 km/hr. A man walks in the rain with a velocity of 4 km/hr. The rain drop will fall on the man with a relative velocity of:

(*a*) 1 km/hr (*b*) 3 km/hr
(*c*) 4 km/hr (*d*) 5 km/hr

Sol. (*d*)



$V_{R/M} = 4\hat{i} - 3\hat{j}$

$|V| = \sqrt{4^2 + 3^2} = 5 \text{ km/hr}$

10. An arrow is shot into air. Its range is 200 m and time of flight is 5 sec. Horizontal component of velocity of the arrow is:

(*a*) 12.5 m/s

(*b*) 25.0 m/s

(*c*) 31.25 m/s

(*d*) 40 m/s

Sol. (*d*) $R = \dfrac{u^2 \sin 2\theta}{g} = 200$

$$T = \frac{2u \sin\theta}{g} = 5$$

$$\frac{R}{T} = \frac{200}{5} = \frac{2u^2 \sin\theta\cos\theta}{2u\sin\theta}$$

$40 = u \cos\theta = u_x$

$\therefore u_x = 40$ m/s

11. A boat takes 2 hr to travel 8 km and back in still water. If velocity of water is 4 km/hr, the time taken for going upstream 8 km and coming back is:

(*a*) 2 hr (*b*) 2 hr 40 min

(*c*) 1 hr 20 min (*d*) None of the above

Sol. (*b*) Total Distance travel = 8 + 8 = 16 km in 2 hr in still water

$$\therefore V_b = \frac{16}{2} = 8 \text{ km / hr}$$

Effective velocity of boat in upstream = $V_b - V_w$ = 8 – 4 = 4 km/hr

$\therefore$ Time taken to travel upstream, $v = \dfrac{8}{4} = 2$ hrs

Effective velocity downstream = 8 + 4 = 12 km/s

Time taken downstream = $\dfrac{8}{12} = 40$ min

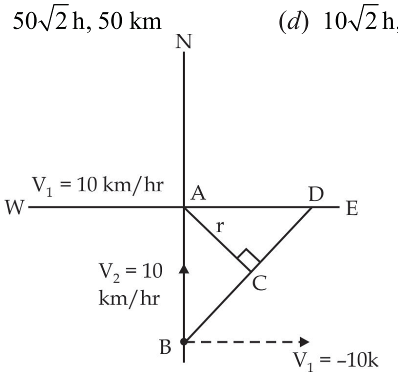
$\therefore$ Total Time = 2 hr 40 min.

12. A ship 'A' moving towards west with a speed of 10 km/hr and ship 'B' 100 km south of 'A' is moving northward with a speed of 10 km/hr. The time after which the distance between them is shortest and the shortest distance between them are:

(*a*) 0h, 100 km (*b*) 5 hr, $50\sqrt{2}$ km

(*c*) $50\sqrt{2}$ h, 50 km (*d*) $10\sqrt{2}$ h, $50\sqrt{2}$ km

Sol. (*b*)



$$V_{BA} = V_r = \sqrt{V_1^2 + V_2^2} = \sqrt{10^2 + 10^2} = 10\sqrt{2} \text{ km / hr}$$

along BC

Shortest distance f = AC = AB sin 45°

$$\frac{100}{\sqrt{2}} = 50\sqrt{2} \text{ km}$$

Shortest time, $t = \dfrac{d}{V_r} = \dfrac{50\sqrt{2}}{10\sqrt{2}} = 5$ hr

13. Two stones are projected with same velocity ***v*** at an angle θ and (90° – θ). If H and H_1 are the greatest height in the two paths, what is the relation between R, H and H_1?

(*a*) $R = 4\sqrt{HH_1}$ (*b*) $R = \sqrt{HH_1}$

(*c*) $R = HH_1$ (*d*) None of these

Sol. (*a*) $H_1 = \dfrac{u^2 \sin^2\theta}{2g}, H_2 = \dfrac{u^2 \sin^2(90° - \theta)}{2g}$

For H & H_1 to be max θ = 45^0

$$\therefore H_1 = H = \frac{u^2}{4g}$$

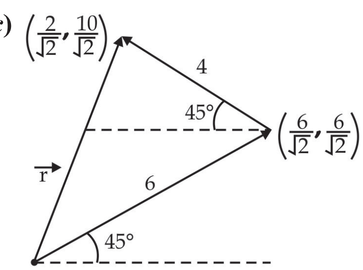
$$R = \frac{u^2 \sin 2\theta}{g} = 4\sqrt{H_1 H}$$

14. A car travels 6 km towards north at an angle of 45° to the east and then travels distance of 4 km towards north at an angle 135° to east. How far is the point from the starting point? What angle does the straight line joining its initial and final positions makes with the east?

(*a*) $\sqrt{50}$ km and $\tan^{-1} 10$ (*b*) 10 km and $\tan^{-1}(\sqrt{5})$

(*c*) $\sqrt{52}$ km and $\tan^{-1}(5)$ (*d*) $\sqrt{52}$ km and $\tan^{-1}(\sqrt{5})$

Sol. (*c*)



$$r = \sqrt{\frac{4}{2} + \frac{100}{2}} = \sqrt{\frac{104}{2}}$$

r = $\sqrt{52}$ km

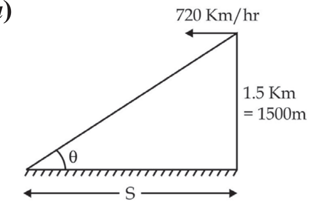
$$\tan\theta = \frac{10}{\sqrt{2}} \times \frac{\sqrt{2}}{2} = 5 \Rightarrow \theta = \tan^{-1}(5)$$

15. A fighter plane is flying horizontally at an altitude of 1.5 km with speed 720 km/hr. At what angle of sight (w.r.t. horizontal) when the target is seen, should the pilot drop the bomb in order to attack the target? (Take g = 10 m s^{-2}, tan23° = 0.43)

(*a*) 23° (*b*) 32°

(*c*) 12° (*d*) 42°

Sol. (*a*)



Time taken by bomb to reach the ground

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1500}{10}}$$

$$t = \sqrt{300} = \sqrt{3} \times 10 \text{ s}$$

Now, for horizontal motion

$V = 720$ km/hr $= 200$ m/s $\hat{i}$

$$S = 200 \times 10\sqrt{3} = 2000\sqrt{3}$$

$$\therefore \tan\theta = \frac{1500}{S} = \frac{1500}{2000\sqrt{3}} = \frac{\sqrt{3}}{4}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{4}\right) \sim = 23°$$

16. A stone is thrown from the top of a tower of height of 50 m with a velocity of 30 m/s at angle of 30° above horizontal. Find the time during which the stone will be in air and the distance from the tower base to where the stone will hit the ground:

(*a*) 3 sec, 50 m (*b*) 4 sec, 60 m

(*c*) 5 sec, 130 m (*d*) None of these

Sol. (***c***) For vertical motion.

$$V_X = V\sin\theta = 15 \text{ m/s}$$

$$S = ut + \frac{1}{2}at^2$$

S = +ve, u = –ve, y = +ve

$$50 = -15t + \frac{1}{2}gt^2$$

$$t^2 - 3t - 10 = 0$$

$\therefore$ t = 5, –2 $\therefore$ t = 5 sec

$$R = 15\sqrt{3} \times t \qquad [\because V_x = V\cos\theta = 15\sqrt{3} \text{ m/s}]$$

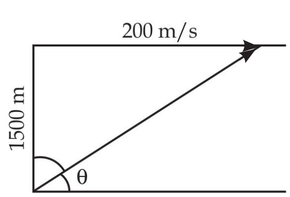
$$= 75 \times \sqrt{3} = 130 \text{ m}$$

17. A fighter plane flying horizontally at an altitude of 1.5 km with speed 720 km h^{-1} passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600 m s^{-1} to hit the plane:
(Take g = 10 m s^{-2})

(*a*) $\sin^{-1}\left(\frac{1}{3}\right)$ (*b*) $\sin^{-1}\left(\frac{2}{3}\right)$

(*c*) $\cos^{-1}\left(\frac{1}{3}\right)$ (*d*) $\cos^{-1}\left(\frac{2}{3}\right)$

Sol. (***a***)



$V_x = V\cos\theta$

$V_y = V\sin\theta$

Time taken by gun to reach 500 m

$$t = \frac{1500}{600\sin\theta} = \frac{5}{2\sin\theta}$$

Distance travelled horizontally by gun

$$S = 600\cos\theta \times \frac{5}{2\sin\theta}$$

Distance travelled by plane in same time

$$S = 200 \times \frac{5}{2\sin\theta} \text{ m}$$

$\therefore$ Since horizontal distance are equal

$$600\cos\theta \times \frac{5}{2\sin\theta} = 200 \times \frac{5}{2\sin\theta}$$

$$\cos\theta = 1/3$$

$$\therefore \theta = \cos^{-1} 1/3$$

With vertical $\rightarrow \frac{\pi}{2} - \theta = \frac{\pi}{2} - \cos^{-1} 1/3 = \sin^{-1} 1/3$

18. A ball rolls off the top of a stair case with a horizontal velocity of u m/s. If the steps are h m high and b m wide, the ball will hit the edge of the n^{th} step where n is:

(*a*) $\frac{2hu}{gb^2}$ (*b*) $\frac{2hu^2}{gb^2}$

(*c*) $\frac{2hu^2}{gb}$ (*d*) $\frac{hu^2}{gb^2}$

Sol. (***b***) Vertical distance travelled by ball = nh = $\frac{1}{2}gt^2$

$$nh = \frac{1}{2}gt^2 \qquad ...(i)$$

Horizontal distance traveled by ball = nb = ut

$$t = \frac{nb}{u} \qquad ...(ii)$$

$$\therefore nh = \frac{1}{2}g\left(\frac{nb}{u}\right)^2 \quad \text{or} \quad n = \frac{2u^2h}{gb^2}$$

19. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s. If g = 10 m/s^2. Find the equation of Trajectory:

(*a*) $Y = X - 5X^2$ (*b*) $Y = 2X - 5X^2$

(*c*) $4Y = 2X - 5X^2$ (*d*) $4Y = 2X - 25X^2$

Sol. (***b***) $u = \hat{i} + 2\hat{j}$

$$|u| = \sqrt{1+4} = \sqrt{5} \text{ m/s}$$

$$Y = X\tan\theta - \frac{gX^2}{2u^2}(1 + \tan^2\theta) \qquad \left[\tan\theta = \frac{Y}{X} = \frac{2}{1} = 2\right]$$

$$Y = X \times 2 - \frac{10X^2}{2 \times (\sqrt{5})^2}[1 + 2^2]$$

$$Y = 2X - \frac{10X^2}{10}[5] = 2X - 5X^2$$

Topicwise Questions

INTRODUCTION OF VECTORS, VECTORS ADDITION & SUBTRACTION

1. Two equal forces have their resultant equal to either of the force, at what angle are they inclined?

(*a*) 120° (*b*) 360°

(*c*) 90° (*d*) 60°

2. A particle starting from the origin (0,0) moves in a straight line in the (x,y) plane. Its coordinates at a later time are $(\sqrt{3},3)$. The path the particle makes with x-axis, is inclined at an angle of:

(*a*) 45° (*b*) 60°

(*c*) 30° (*d*) 90°

3. A unit vector parallel to the resultant of the vectors $\vec{A}=4\hat{i}+3\hat{j}+6\hat{k}$ and $\vec{B}=-\hat{i}+8\hat{j}-8\hat{k}$

(*a*) $\frac{3\hat{i}+11\hat{j}-2\hat{k}}{2}$ (*b*) $\frac{\hat{i}+2\hat{j}-3\hat{k}}{\sqrt{166}}$

(*c*) $\frac{3\hat{i}+11\hat{j}-2\hat{k}}{\sqrt{134}}$ (*d*) $\frac{4\hat{i}+6\hat{j}+8\hat{k}}{\sqrt{11}}$

4. Two forces whose magnitude are in the ratio 9 : 11 give a resultant of 38 N. If the angle of their inclination is 60° then what will be the magnitude of each force?

(*a*) 19.8 N, 24.2 N (*b*) 20 N, 24N

(*c*) 25 N, 30 N (*d*) None of these

5. If $\vec{A}=4\hat{i}-3\hat{j}$ and $\vec{B}=5\hat{i}+18\hat{j}$ then magnitude and direction of $\vec{A}+\vec{B}$ will be:

(*a*) 17.49, $\tan^{-1}$ (2) (*b*) 15, $\tan^{-1}$ (4/3)

(*c*) 20, $\tan^{-1}$ (4/5) (*d*) 17.49, $\tan^{-1}$ (5/3)

6. Two equal forces act at a point inclined to each other at an angle of 60°. The magnitude of their resultant is:

(*a*) $\sqrt{3}A$ (*b*) 2A

(*c*) 3A (*d*) $\sqrt{2}A$

7. The resultant of two vectors $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If Q is doubled; the new resultant is perpendicular to P, then R equals to:

(*a*) P (*b*) Q

(*c*) P + Q (*d*) P – Q

8. If vectors $\hat{i}-3\hat{j}+5\hat{k}$ and $\hat{i}-3\hat{j}-a\hat{k}$ are equal vectors then the value of a is:

(*a*) 5 (*b*) 4

(*c*) –4 (*d*) –5

9. If the magnitudes of vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 4 , 3 and 5 units respectively and $\vec{A}+\vec{B}=\vec{C}$, the angle between vectors $\vec{A}$ and $\vec{B}$ is:

(*a*) 90° (*b*) $\cos^{-1}\left(\frac{5}{16}\right)$

(*c*) $\tan^{-1}(5)$ (*d*) $\tan^{-1}\left(\frac{12}{5}\right)$

10. At what angle the two vectors of magnitudes (A+B) and (A–B) must act, so that resultant is $\sqrt{A^2+B^2}$?

(*a*) $\cos^{-1}\frac{(A+B)}{A-B}$ (*b*) $\cos^{-1}\left(\frac{(A^2+B^2)}{2(B^2-A^2)}\right)$

(*c*) $\cos^{-1}\left(\frac{A^2+B^2}{A^2-B^2}\right)$ (*d*) None of these

VECTOR PRODUCTS & RESOLUTION

11. The vector projection of a vector $3\hat{i}+5\hat{k}$ on y-axis is:

(*a*) 5 (*b*) 4

(*c*) 3 (*d*) Zero

12. If $\vec{A}=3\hat{i}+5\hat{j}-7\hat{k}$, the direction of cosines of the vector $\vec{A}$ are:

(*a*) $\frac{2}{\sqrt{83}},\frac{5}{\sqrt{83}},\frac{7}{\sqrt{83}}$

(*b*) $\frac{3}{\sqrt{83}},\frac{5}{\sqrt{83}},\frac{7}{\sqrt{83}}$

(*c*) $\frac{1}{\sqrt{83}},\frac{2}{\sqrt{83}},\frac{5}{\sqrt{83}}$

(*d*) $\frac{3}{\sqrt{83}},\frac{5}{\sqrt{83}},\frac{-7}{\sqrt{83}}$

13. The component of a vector is:

(*a*) Always less than its magnitude

(*b*) Always greater than its magnitude

(*c*) Always equal to its magnitude

(*d*) None of the above

14. If $\left|\vec{A}\times\vec{B}\right|=\frac{(\vec{A}.\vec{B})}{\sqrt{3}}$ then the value of angle between $\vec{A}$ and $\vec{B}$ is:

(*a*) 0^0 (*b*) π

(*c*) $\frac{\pi}{2}$ (*d*) $\frac{\pi}{6}$

15. If a vector $2\hat{i}+4\hat{j}+6\hat{k}$ is perpendicular to the vector $5\hat{i}+6\hat{j}-a\hat{k}$. Then the value of a is:

(*a*) 14/3 (*b*) 17/9

(*c*) 14/5 (*d*) 17/3

16. A particle moves from position $3\hat{i}+5\hat{j}-8\hat{k}$ to $15\hat{i}-20\hat{j}+8\hat{k}$ due to uniform force of $(3\hat{i}+\hat{j}+4\hat{k})$N. If the displacement is in metre then work done will be:

(*a*) 70 J (*b*) 75 J
(*c*) 72 J (*d*) 80 J

17. Consider two vectors $\vec{F_1}=2\hat{i}+5\hat{k}$ and $\vec{F_2}=3\hat{j}+4\hat{k}$. The magnitude of the scalar product of these vectors is:

(*a*) 26 (*b*) 28
(*c*) 30 (*d*) 20

18. The linear velocity of a rotating body is given by $\vec{v}=\vec{\omega}\times\vec{r}$. The angular velocity of body is $\vec{\omega}=5\hat{i}-4\hat{j}+9\hat{k}$ and the radius vector $\vec{r}=8\hat{i}-6\hat{j}+3\hat{k}$ then $|\vec{v}|$ is:

(*a*) 70.8 units (*b*) 90 units
(*c*) $\sqrt{149}$ units (*d*) $\sqrt{2017}$ units

19. Let $\vec{A}=\hat{i}A\cos\theta+\hat{j}A\sin\theta$, be any vector. Another vector $\vec{B}$ which is normal to $\vec{A}$ is:

(*a*) $\hat{i}B\cos\theta+\hat{j}B\sin\theta$ (*b*) $\hat{i}B\sin\theta+\hat{j}B\cos\theta$
(*c*) $\hat{i}B\sin\theta-\hat{j}B\cos\theta$ (*d*) $\hat{i}A\cos\theta-\hat{j}A\sin\theta$

20. What is the angle between $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}$?

(*a*) $\frac{\pi}{6}$ (*b*) $\frac{\pi}{4}$
(*c*) $\frac{\pi}{3}$ (*d*) None of these

21. If the angle between the vector $\vec{A}$ and $\vec{B}$ is 0, the value of the product $(\vec{B}\times\vec{A}).(\vec{A})$ is equal to:

(*a*) $BA^2\cos\theta$ (*b*) 1
(*c*) $A^2B\cos\theta$ (*d*) Zero

22. The value of P, so that the vectors $2\hat{i}-\hat{j}+\hat{k}, \hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}+P\hat{j}+5\hat{k}$ are coplanar should be

(*a*) 16 (*b*) -4
(*c*) 4 (*d*) -8

23. A variable force given by the two - dimensional vector $3\hat{i}+2\hat{j}+\hat{k}$ acting at the point $r=8\hat{i}+2\hat{j}+3\hat{k}$. What is the torque?

(*a*) $14\hat{i}-38\hat{j}+16\hat{k}$ (*b*) $4\hat{i}-\hat{j}-10\hat{k}$
(*c*) $-4\hat{i}+\hat{j}+10\hat{k}$ (*d*) $4\hat{i}+7\hat{j}+10\hat{k}$

HORIZONTAL PROJECTILE, OBLIQUE PROJECTION FROM HEIGHT

24. A body of mass m is projected horizontally with a velocity v from the top of a tower of height h and it reaches the ground at a distance x from the foot of the tower. If a second body of mass 4 m is projected horizontally from the top of a tower of height 4 h, it reaches the ground at a distance 4x from the foot of the tower. The horizontal velocity of the second body is:

(*a*) 6v (*b*) $\sqrt{2}$v
(*c*) 2v (*d*) 5v

25. Two tall buildings are 100 m apart. With what speed must a ball be thrown horizontally from the window 500 m above the ground in one building, so that it will enter a window 100 m above the ground in the other?

(*a*) 11 m/s (*b*) 11.2 m/s
(*c*) 34 m/s (*d*) 15 m/s

26. A stone is dropped from the window of a bus moving at 80 km/h. If the window is 150 cm high, find the distance along the track which the stone moves before striking the ground.

(*a*) 15.26 m (*b*) 13 m
(*c*) 14.16 m (*d*) 12 m

27. A circular disc of radius r = 5m is rotating in horizontal plane about y-axis. Y-axis is vertical axis passing through the centre of disc and x-z is the horizontal plane at ground. The height of disc above ground is h = 5m. Small particles are ejecting from disc in horizontal direction with speed 12 m/s from the circumference of disc then the distance of these particles from origin when they hits the x-z plane is -

(*a*) 12 m (*b*) 13 m
(*c*) 5 m (*d*) None of these

28. A ball is projected upwards from the top of the tower with a velocity 50 ms^{-1} making an angle 30° with the horizontal. The height of tower is 70 m. After how many seconds the ball will strike the ground ?

(*a*) 3 sec (*b*) 5 sec
(*c*) 7 sec (*d*) 9 sec

29. A particle starts with an initial velocity 2.5 m/s along the positive x-direction and it accelerates uniformly at the rate 0.50 m/s^2. Time taken to reach the velocity 7.5 m/s will be-

(*a*) 5 s (*b*) 2 s
(*c*) 10 s (*d*) 15 s

30. A stuntman plans to run across a roof top and then horizontally of it to land on the roof of the next building. The roof of the next building is 4.9 m below the first one and 6.2 m away from it. The minimum roof top speed for successful jump is -

(*a*) 6.2 m s^{-1} (*b*) 8.7 m s^{-1}
(*c*) 9.8 m s^{-1} (*d*) 10 m s^{-1}

31. The relation between the time of flight of a projectile, T_f and time to reach the maximum height, t_m is

(*a*) $T_f=2t_m$ (*b*) $T_f=t_m$
(*c*) $T_f=\frac{t_m}{2}$ (*d*) $T_f=\sqrt{2}(t_m)$

GROUND TO GROUND PROJECTILE

32. The maximum range of a gun on horizontal terrain is 15 km. If g = 10 m/s^2. What must be the muzzle velocity of the shell?

(*a*) 387.2 m/s (*b*) 390 m/s
(*c*) 380 m/s (*d*) 350 m/s

33. Ratio between maximum range and square of time of flight in projectile motion is:

(*a*) 1 (*b*) 2
(*c*) 4 (*d*) 5

34. A man projects a coin upwards from the gate of a uniformly moving train. The path of coin for the man will be:

(*a*) Parabolic (*b*) Inclined straight line
(*c*) Vertical straight line (*d*) Horizontal straight line

35. A person can throw a stone to a maximum distance of 500 m. The greatest height to which he can throw the stone is:

(*a*) 50 m (*b*) 250 m
(*c*) 260 m (*d*) 1000 m

36. In a projectile motion, where is the angular momentum minimum?

(*a*) At the starting point
(*b*) On the landing point
(*c*) Highest point of projectile
(*d*) As no such position

37. Two particles A and B are projected with same speed so that the ratio of their maximum heights reached is 3 : 1. If the speed of A is tripled without altering other parameters, the ratio of the horizontal ranges attained by A and B is:

(*a*) 10 : 1 (*b*) 9 : 1
(*c*) 2 : 5 (*d*) 6 : 13

38. A bullet is fired with a velocity u making an angle of 30^0 with the horizontal plane. The horizontal component of the velocity of the bullet when it reaches the maximum height is:

(*a*) $\frac{u}{\sqrt{2}}$ (*b*) u
(*c*) $u\sqrt{3}$ (*d*) $\frac{\sqrt{3}}{2}u$

39. A man standing on the edge of a cliff throws a stone straight up with initial speed u and then throws another stone straight down with the same initial speed and from the same position. Find the ratio of the speed the stones would have attained when they hit the ground at the base of the cliff.:

(*a*) 4 : 1 (*b*) 1 : 3
(*c*) 1 : 1 (*d*) 1 : 2

40. A gun fires two bullets at 60^0 and 30^0 with the horizontal. The bullets strikes at some horizontal distance. The ratio of maximum heights for the two bullets is in the ratio:

(*a*) 1/4 (*b*) 3/4
(*c*) 3 (*d*) 2

RAIN-MAN, RIVER-BOAT PROBLEMS, RELATIVE MOTION IN 2D

41. Suppose that two objects A and B are moving with velocities $\overrightarrow{V_A}$ and $\overrightarrow{V_B}$ (each with respect to some common frame of reference). Let $\overrightarrow{V_{AB}}$ represents the velocity of A with respect to B then:

(*a*) $\vec{V}_{AB} + \vec{V}_{BA} = 0$
(*b*) $\vec{V}_{AB} - \vec{V}_{BA} = 0$
(*c*) $\vec{V}_{AB} \neq \vec{V}_A + \vec{V}_B$
(*d*) $|\vec{V}_{AB}| \neq |\vec{V}_{BA}|$

42. The speed of boat is 18 km/h in still water. It crosses a river of width 2 km along the shortest path in 7 minutes. The velocity of the river is:

(*a*) 5.5 km/h (*b*) 6 km/h
(*c*) 7 km/h (*d*) 10 km/h

43. An aeroplane is flying with the speed of 100 m/s towards east. If the wind is blowing with the velocity 25 m/s w.r.t ground southwards total speed of the plane is:

(*a*) V > 100 m/s (*b*) V = 101 m/s
(*c*) V ≥ 100 m/s (*d*) V = 125 m/s

44. A boat man could row his boat with a speed 10m/sec. He wants to take his boat from P to a point Q just opposite on the other bank of the river flowing at a speed 4m/sec. He should row his boat:

(*a*) At right angle to the stream
(*b*) At an angle of $\sin^{-1}$ (2/5) with PQ up the stream
(*c*) At an angle $\sin^{-1}$ (2/5) with PQ down the stream
(*d*) At an angle $\cos^{-1}$ (2/5) with PQ down the stream

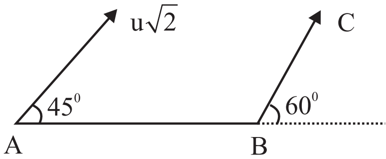
45. A boat moves relative to water with a velocity which is 1/n times the river flow velocity. At what angle to the stream direction must the boat move to minimize drifting?

(*a*) $\pi / 2$ (*b*) $\sin^{-1}(1/n)$
(*c*) $\frac{\pi}{2} + \sin^{-1}(1/n)$ (*d*) $\frac{\pi}{2} - \sin^{-1}(1/n)$

46. A man standing on a road has to hold his umbrella at 30^0 with the vertical to keep the rain away. He thrown the umbrella and starts running at 10 km/h. He finds that rain drop are hitting his head vertically. Find the speed of rain w.r.t. road:

(*a*) 10 km/s (*b*) 20 km/h
(*c*) $10\sqrt{3}$ km / s (*d*) $20\sqrt{3}$ km / s

47. A particle is projected from a point A with velocity $u\sqrt{2}$ at an angle of 45º with horizontal as shown in figure. It strikes the plane BC at right angles. The velocity of the particle at the time of collision is :



(*a*) $\frac{\sqrt{3}u}{2}$ (*b*) $\frac{u}{2}$

(*c*) $\frac{2u}{\sqrt{3}}$ (*d*) u

48. The path of one projectile in motion as seen from another moving projectile is -

(*a*) A straight line (*b*) A circle

(*c*) An ellipse (*d*) A parabola

CIRCULAR MOTION KINEMATICS

49. In a two dimensional motion, instantaneous speed V_0 is a positive constant. Then which of the following are necessarily true?

(*a*) The average velocity is not zero at any time

(*b*) Average acceleration must always vanish

(*c*) Displacement in equal time intervals are equal

(*d*) Equal path lengths are traversed in equal intervals

50. In a ground, a cyclist follows a track that turn to his right by an angle of 60° after every 100 m. The displacement of the cyclist from the starting point to the fourth turn:

(*a*) $200\sqrt{10}$ (*b*) $100\sqrt{3}$

(*c*) $2\sqrt{2}$ (*d*) $4\sqrt{6}$

51. If a body is moving in a circle of radius r with a constant speed v, its angular velocity is:

(*a*) $\frac{v^2}{r}$ (*b*) vr

(*c*) $\frac{v}{r}$ (*d*) $\frac{r}{v}$

52. A particle moving on a circular path with constant speed then its acceleration will be:

(*a*) Zero

(*b*) External radial acceleration

(*c*) Internal radial acceleration

(*d*) Constant acceleration

53. The angular speed of seconds needle in a mechanical watch is:

(*a*) π rad/s (*b*) π × 30 rad/s

(*c*) $\frac{\pi}{2}$ rad / s (*d*) $\frac{\pi}{30}$ rad / s

54. If a_r and a_t represents radial and tangential accelerations, the motion of a particle will be uniformly circular if:

(*a*) $a_r = 0$ and $a_t = 0$

(*b*) $a_r = 0$ and $a_t \neq 0$

(*c*) $a_r \neq 0$ and $a_t = 0$

(*d*) $a_r \neq 0$ and $a_t \neq 0$

55. The earth moves round the sun in a near circular orbit of radius 1.8×10^{11} m. Its centripetal acceleration is:

(*a*) 6×10^{-3} m/s^2

(*b*) 10×10^{-3} m/s^2

(*c*) 12×10^{-3} m/s^2

(*d*) 10^{-3} m/s^2

56. Which of the following statements is not correct in uniform circular motion?

(*a*) The speed of the particle remains constant

(*b*) The linear velocity remains constant

(*c*) The acceleration always points towards the centre

(*d*) The angular speed remains constant

57. The angle turned by a body undergoing circular motion depends on time as $\theta = \theta_0 + \theta_1 t + \theta_2 t^2 + \theta_3 t^3$. The angular acceleration will be proportional to:

(*a*) θ_3 (*b*) θ_3 and t

(*c*) t (*d*) θ_3 and 1/t

Learning Plus

1. The vector sum of two forces is perpendicular to their vector differences. In that case, the forces:

(*a*) Cannot be predicted

(*b*) Are perpendicular to each other

(*c*) Are equal to each other in magnitude

(*d*) Are not equal to each other in magnitude

2. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. If magnitudes of $\vec{A}+\vec{B}$ is equal to n times the magnitude of $\vec{A}-\vec{B}$, then the angle between $\vec{A}$ and $\vec{B}$ is:

(*a*) $\cos^{-1}\left(\frac{n+n^2}{n-n^2}\right)$ (*b*) $\cos^{-1}\left(\frac{n^2+1}{n^2-1}\right)$

(*c*) $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$ (*d*) $\cos^{-1}\left(\frac{2+n}{2-n}\right)$

3. A ball is thrown horizontally from the top of a tower of height 8 m. It touches the ground at a distance of 90 m from the foot of the tower. The initial velocity of the body is:

(*a*) 60 m/s (*b*) 80 m/s

(*c*) 70 m/s (*d*) 60 m/s

4. A body is thrown horizontally with a velocity $\sqrt{2gh}$ from the top of a tower of height h. It strikes the level ground through the foot of the tower at a distance x from the tower. The value of x is -

(*a*) h (*b*) $\frac{h}{2}$

(*c*) 2h (*d*) $\frac{2h}{3}$

5. A projectile fired with initial velocity u at some angle θ has a range R. If the initial velocity be doubled at the same angle of projection, then the range will be:

(*a*) 4R (*b*) 8R

(*c*) R/4 (*d*) R

6. For an object thrown at 30° to horizontal, the maximum height (H) and horizontal range (R) are related as:

(*a*) R = 16H

(*b*) $R = 4\sqrt{3}H$

(*c*) $R = 2\sqrt{2}H$

(*d*) R = 2H

7. Two projectiles A and B thrown with speeds in the ratio $1:\sqrt{2}$ acquired the same heights. If A is thrown at an angle of 60° with the horizontal, the angle of projection of B will be:

(*a*) $\sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{2}}\right)$ (*b*) 30°

(*c*) 45° (*d*) 90°

8. Rain is falling vertically with speed of 35 m/s. A woman rides a bicycle with a speed of 15 m/s in East to West direction. What is the direction in which she should hold her umbrella?

(*a*) $\tan^{-1}\left(\frac{3}{7}\right)$ with the vertical towards west

(*b*) $\tan^{-1}\left(\frac{4}{7}\right)$ with the vertical towards east

(*c*) $\tan^{-1}\left(\frac{5}{7}\right)$ with the vertical towards east

(*d*) Towards North downward

9. A man walks in rain with a velocity of 10 km/h. The rain drops strikes at him at an angle of 45^0 with the horizontal. What will be the downward velocity of the raindrops and relative velocity of rain w.r.t man?

(*a*) $\vec{V_r}$ = 10 km/h, $\vec{V_{rm}}$ = 10 m/s

(*b*) $\vec{V_r}$ = 20 km/h, $\vec{V_{rm}} = 10\sqrt{2}$ km/h

(*c*) $\vec{V_r}$ = 10 km/h, $\vec{V_{rm}} = 10\sqrt{2}$ km/h

(*d*) $\vec{V_r}$ = 1 km/h, $\vec{V_{rm}}$ = 1 km/h

10. A boat which has a speed of 5 km/hr in still water crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in km/hr is:

(*a*) 1 (*b*) 3

(*c*) 4 (*d*) $\sqrt{41}$

11. A swimmer crosses a river of width d flowing at velocity V while swimming he heads himself always at an angle of 120° with the river flow and on reaching the other end he find a drift of $\frac{d}{2}$ in the direction of flow of river. The speed of the swimmer with respect to river is:

(*a*) $\left(2-\sqrt{3}\right)V$ (*b*) $2\left(2-\sqrt{3}\right)V$

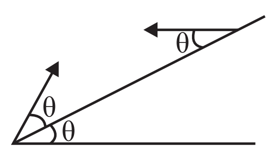
(*c*) $4\left(2-\sqrt{3}\right)V$ (*d*) $\sqrt{41}$

12. A swimmer can swim in still water with speed v and the river flowing with velocity *v*/2. To cross the river in shortest distance, he should swim making angle θ with the upstream. What is the ratio of the time taken to swim across in the shortest time to that in swimming across over shortest distance?

(*a*) cos θ (*b*) sin θ

(*c*) tan θ (*d*) cot θ

13. From an inclined plane two particles are projected with same speed at same angle θ, one up and other down the plane as shown in figure. Which of the following statement is correct ?



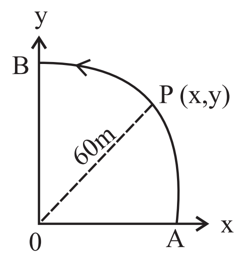
(*a*) The particles will collide the plane with same speed

(*b*) The times of flight of each particle are same

(*c*) Both particles strikes the plane perpendicularly

(*d*) None of these

14. A point P moves in counterclockwise direction on a circular path as shown in figure. The movement of P is such that it sweeps out a length $S = t^3 + 8$, where S in metre and t is in seconds. The radius of the path is 60 m. The acceleration of P when t = 4s is nearby:



(*a*) 45.28 m/s^2 (*b*) 60 m/s^2

(*c*) 50.2 m/s^2 (*d*) 30 m/s^2

15. A car is travelling with linear velocity v on a circular road of radius R. If its speed is increasing at the rate of a m/s^2, net acceleration of the car, is

(*a*) $\frac{v^2}{R}+a$ (*b*) $\frac{v^2}{R}-a$

(*c*) $\sqrt{\left(\frac{v^2}{R}\right)^2+a^2}$ (*d*) $\sqrt{\left(\frac{v^2}{R}\right)-a^2}$

16. A proton in a cyclotron changes its velocity from 30 kmh^{-1} north to 45 kmh^{-1} east in 20 s. What is the magnitude of average acceleration during the same?

(*a*) 2.5 kms^{-2} (*b*) 12.5 kms^{-2}

(*c*) 22.5 kms^{-2} (*d*) 32.5 kms^{-2}

17. Raindrops are falling vertically with a velocity of 10 m/s. To a cyclist moving on a straight road the raindrops appear to be coming with a velocity of 20 m/s. The velocity of cyclist is:

(*a*) 10 m/s (*b*) $10\sqrt{3}$ m/s

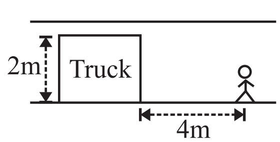
(*c*) 20 m/s (*d*) $20\sqrt{3}$ m/s

18. A man starts running along a straight road with uniform velocity $u\hat{i}$ observes that the rain is falling vertically downward. If he doubles his speed, he finds that the rain is coming at an angle θ to the vertical. The velocity of rain with respect to the ground is :

(*a*) $u\hat{i}-u\tan\theta\hat{j}$ (*b*) $u\hat{i}-u\cot\theta\hat{j}$

(*c*) $u\hat{i}+u\cot\theta\hat{j}$ (*d*) $\frac{u}{\tan\theta}\hat{i}-u\hat{j}$

19. A 2 m wide truck is moving with a uniform speed of 8 m/s along a straight horizontal road. A pedestrian starts crossing the road at an instant when the truck is 4 m away from him. The minimum constant velocity with which he should run to avoid an accident is:



(*a*) $1.6\sqrt{5}$ m/s (*b*) $1.2\sqrt{5}$ m/s

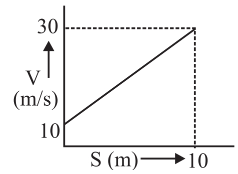
(*c*) $1.2\sqrt{7}$ m/s (*d*) $1.6\sqrt{7}$ m/s

20. A particle starts with velocity v_0 at time t = 0 and is decelerated at a rate proportional to the square root of its speed at time t with constant of proportionality α. The total time for which it will move before coming to rest is

(*a*) $\sqrt{v_0}$ (*b*) $\frac{v_0^{3/2}}{\alpha}$

(*c*) $\frac{2v_0^{3/2}}{\alpha}$ (*d*) $\frac{2\sqrt{v_0}}{\alpha}$

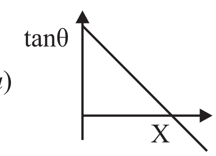
21. Velocity of a particle changes with position according to following curve. Acceleration of the particle at S = 1 m

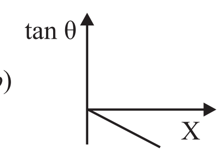


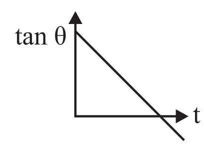
(*a*) 24 m/s^2 (*b*) 2 m/s^2

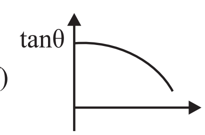
(*c*) 20 m/s^2 (*d*) 3 m/s^2

22. A particle is dropped from a tower in a uniform gravitational field at t = 0. The particle is blown over by horizontal wind with constant velocity. Slope of trajectory of particle (tan θ) with horizontal varies according to

(*a*) 

(*b*) 

(*c*) 

(*d*) 

23. A body falling freely from a given height H hits an inclined plane in its path at a height 'h'. As a result of this impact the direction of the velocity of the body becomes horizontal. Find the total time the body will take to reach the ground.

(*a*) $\sqrt{\frac{2}{g}}\left(\sqrt{h}+\sqrt{H-h}\right)$

(*b*) $\sqrt{2gh}$

(*c*) $\sqrt{\frac{2}{g}}\left(\sqrt{H-h}\right)$

(*d*) None of the above

24. Two bodies fall freely from the same height, but the second body starts falling T seconds after the first. The time (after which the first body begins to fall) when the distance between the bodies equals L is:

(*a*) $\frac{1}{\sqrt{gh}}$ (*b*) $\frac{1}{\sqrt{2gh}}$

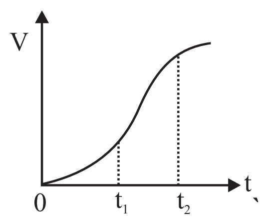
(*c*) $\frac{1}{gh}$ (*d*) None of these

25. A particle rotates along a circle of radius $R = \sqrt{2}$m with an angular acceleration $\alpha = \frac{\pi}{4} \text{rad/s}^2$ starting from rest. Calculate the magnitude of average velocity of the particle over the time it rotates a quarter circle.

(*a*) 0.5 (*b*) 1

(*c*) 2 (*d*) 1.5

26. The velocity-time graph of a particle in one dimensional motion is shown in the figure. Which of the following formula is correct for describing the motion of the particle over the time interval t_1 to t_2?



(*a*) $x(t_2) = x(t_1) + v(t_1)(t_2 - t_1) + \left(\frac{1}{2}\right)a(t_2 - t_1)^2$

(*b*) $v(t_2) = v(t_1) + a(t_2 - t_1)$

(*c*) $v_{average} = \frac{[x(t_2) + x(t_1)]}{(t_2 - t_1)}$

(*d*) $a_{average} = \frac{[v(t_2) - v(t_1)]}{(t_2 - t_1)}$

27. A radius vector of point A relative to the origin varies with time t as $\vec{r} = at\hat{i} - bt^2\hat{j}$ where a and b are constants. The equation of point's trajectory is

(*a*) $y = -\frac{b}{a^2}x^2$ (*b*) $y = \frac{b}{a^2}x^2$

(*c*) $y = -\frac{2b}{a^2}x^2$ (*d*) $y = \frac{2b}{a^2}x^2$

28. A car is moving with speed 40 m/s on a circular path of radius 500 m. Its speed is increasing at the rate of 4 m/s^2. What is the acceleration of the car:

(*a*) 6.12 m/s^2 (*b*) 4×10^2 m/s^2

(*c*) 5.122 m/s^2 (*d*) 3.9 m/s^2

Multiconcept MCQs

1. A cricket ball thrown across a field is at heights h_1 and h_2 from the point of projection at times t_1 and t_2 respectively. After the throw. The ball is caught by a fielder at the same height as that of projection. The time of flight of the ball in this journey is

(*a*) $\left(\frac{h_1t_2^2 - h_2t_1^2}{h_1t_2 - h_2t_1}\right)$ (*b*) $\left(\frac{h_1t_2^2 - h_2t_1^2}{h_1t_1 - h_2t_2}\right)$

(*c*) $\left(\frac{h_1t_2^2 + h_2t_1^2}{h_2t_1 + h_1t_2}\right)$ (*d*) $\left(\frac{h_1t_2^2 - h_2t_1^2}{h_2t_1 - h_1t_2}\right)$

2. A body is projected with an angle θ. The maximum height reached is h. If the time of flight is 4s and g is 10 m/s^2, then value of h is

(*a*) 40 m (*b*) 20 m

(*c*) 5 m (*d*) 10 m

3. The equation of trajectory of a projectile is $y = 10x - \left(\frac{5}{9}\right)x^2$. If we assume g = 10 ms^{-2}. What will be the range of projectile?

(*a*) 36 m (*b*) 24 m

(*c*) 18 m (*d*) 9 m

4. A body is projected with a speed u m/s at an angle β with the horizontal. The kinetic energy at the highest point is $\frac{3}{4}$th of the initial kinetic energy. The value of β is

(*a*) 30° (*b*) 45°

(*c*) 60° (*d*) 120°

5. A particle is projected with a velocity v such that its range on the horizontal plane is twice the greatest height attained by it. What is the range of the projectile ?

(*a*) $\frac{4v^2}{5g}$ (*b*) $\frac{4g}{5v^2}$

(*c*) $\frac{v^2}{g}$ (*d*) $\frac{4v^2}{\sqrt{5g}}$

6. A ball is projected from the ground at angle θ with horizontal. After 1 s it is moving at angle 45° with the horizontal and after 2 s it is moving horizontally. What is the velocity of projection of the ball?

(*a*) $10\sqrt{3}ms^{-1}$

(*b*) $20\sqrt{3}ms^{-1}$

(*c*) $10\sqrt{5}ms^{-1}$

(*d*) $20\sqrt{2}ms^{-1}$

7. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 ms^{-1} at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground?

(take, $g = 10\ ms^{-2}$, $\sin 30° = ½$, $\cos 30° = \sqrt{3}/2$)

(*a*) 5.20 m (*b*) 4.33 m

(*c*) 2.60 m (*d*) 8.66 m

8. A body moving along a circular path of radius R with velocity v, has centripetal acceleration a. If its velocity is made equal to 2v. What will be the centripetal acceleration ?

(*a*) 4a (*b*) 2a

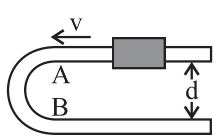
(*c*) $\frac{a}{4}$ (*d*) $\frac{a}{2}$

9. A projectile is projected at an angle (α > 45°) with an initial velocity u. The time t, at which its magnitude of horizontal velocity will equal the magnitude of vertical velocity is :

(*a*) $t = \frac{u}{g}(\cos\alpha - \sin\alpha)$ (*b*) $t = \frac{u}{g}(\cos\alpha + \sin\alpha)$

(*c*) $t = \frac{u}{g}(\sin\alpha - \cos\alpha)$ (*d*) $t = \frac{u}{g}(\sin^2\alpha - \cos^2\alpha)$

10. A U-shaped smooth wire has a semicircular bending between A and B as shown in the figure. A bead of mass m moving with uniform speed v through the wire enters the semicircular bend at A and leaves at B. The average force, exerted by the bead on the part AB of the wire is:



(*a*) 0 (*b*) $\frac{4mv^2}{\pi d}$

(*c*) $\frac{2mv^2}{\pi d}$ (*d*) None of these

11. A particle moves along the positive part of the curve $y = \frac{x^2}{2}$, where $x = \frac{t^2}{2}$. What will be velocity of particle at *t* = 2 sec.

(*a*) $2\hat{i} - 4\hat{j}$ (*b*) $2\hat{i} + 4\hat{j}$

(*c*) $4\hat{i} + 2\hat{j}$ (*d*) $4\hat{i} - 2\hat{j}$

12. The horizontal range and maximum height attained by a projectile are R and H respectively. If a constant horizontal acceleration a = g/4 is imparted to the projectile due to wind, then its horizontal range and maximum height will be

(*a*) $(R + H), \frac{H}{2}$ (*b*) $\left(R + \frac{H}{2}\right), 2H$

(*c*) (R + 2H), H (*d*) (R + H), H

13. A point moves along a circle with a velocity v = at, where a = 0.50 m/s^2. Find the total acceleration of the point at the moment when it has covered the n^{th} (n = 0.10) fraction of the circle after beginning of the motion.

(*a*) 0.8 (*b*) 0.6

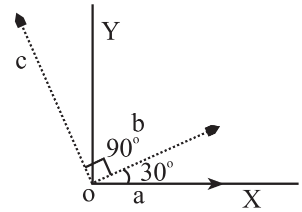
(*c*) 0.7 (*d*) 0.9

14. Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T minutes. A man cycling with a speed of 20 kmh^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. The period T of the bus service is:

(*a*) 4.5 min (*b*) 9 min

(*c*) 12 min (*d*) 24 min

15. Three vectors as shown in the fig have magnitudes $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = 10$. Find the numbers p and q such that $\vec{c} = p\vec{a} + q\vec{b}$.



(*a*) $-\frac{20}{3}, \frac{5\sqrt{3}}{2}$ (*b*) $\frac{20}{3}, \frac{5\sqrt{3}}{2}$

(*c*) $-\frac{10}{3}, \frac{\sqrt{3}}{2}$ (*d*) None

16. Find a unit vector perpendicular to both the vectors $(2\hat{i} + 3\hat{j} + \hat{k})$ and $(\hat{i} - \hat{j} + 2\hat{k})$.

(*a*) $\frac{1}{\sqrt{8}}(4\hat{i} + 3\hat{j} - 5\hat{k})$ (*b*) $\frac{1}{\sqrt{83}}(7\hat{i} - 3\hat{j} - 5\hat{k})$

(*c*) $\frac{1}{\sqrt{8}}(4\hat{i} + 3\hat{j} + 1\hat{k})$ (*d*) None of these

17. Rain water is falling vertically downwards with velocity v, when the velocity of wind is zero, water is collected at a rate R. When the wind starts blowing horizontally at a speed u, the rate of collection of water in the same vessel is:

(*a*) $\sqrt{u^2 + v^2 R}$ (*b*) $\frac{v}{u}R$

(*c*) $\frac{uR}{v}$ (*d*) R

18. The magnitude of the displacement of a particle moving in a circle of radius a with constant angular speed ω varies with time t as :

(*a*) a sin ωt (*b*) $2a \sin\frac{\omega t}{2}$

(*c*) a cos ωt (*d*) $2a \cos\frac{\omega t}{2}$

NEET Past 10 Years Questions

1. A particle moving in a circle of radius R with a uniform speed takes a time T to complete one revolution.

If this particle were projected with the same speed at an angle θ to the horizontal, the maximum height attained by it equals 4R. The angle of projection, θ is then given by: ***(2021)***

(*a*) $\theta = \cos^{-1}\left(\frac{\pi^2 R}{gT^2}\right)^{1/2}$

(*b*) $\theta = \sin^{-1}\left(\frac{\pi^2 R}{gT^2}\right)^{1/2}$

(*c*) $\theta = \sin^{-1}\left(\frac{2gT^2}{\pi^2 R}\right)^{1/2}$

(*d*) $\theta = \cos^{-1}\left(\frac{gT^2}{\pi^2 R}\right)^{1/2}$

2. A car starts from rest and accelerates at 5 m/s^2. At t = 4 s, a ball is dropped out of a window by a person sitting in the car.

What is the velocity and acceleration of the ball at t = 6s? ***(2021)***

(*a*) 20 m/s, 0 (*b*) $20\sqrt{2}$m/s,0

(*c*) $20\sqrt{2}$m/s,10m/s^2 (*d*) 20 m/s, 5 m/s^2

3. A ball is thrown vertically downward with a velocity of 20 m/s from the top of a tower. It hits the ground after some time with a velocity of 80m/s. The height of the tower is : (g = 10 m/s^2) ***(2020)***

(*a*) 340 m (*b*) 320 m

(*c*) 300 m (*d*) 360 m

4. A person sitting in the ground floor of a building notices through the window, of height 1.5 m, a ball dropped from the roof of the building crosses the window in 0.1 s. What is the velocity of the ball when it is at the topmost point of the window? (g = 10 m/s^2) ***(2020 Covid Re-NEET)***

(*a*) 14.5 m/s (*b*) 4.5 m/s

(*c*) 20 m/s (*d*) 15.5 m/s

5. The speed of a swimmer in still water is 20 m/s. The speed of river water is 10 m/s and is flowing due east. If he is standing on the south bank and wishes to cross the river along the shortest path, the angle at which he should make his strokes w.r.t. north is given by : ***(2019)***

(*a*) 30° west (*b*) 0°

(*c*) 60° west (*d*) 45° west

6. Two particles A and B are moving in uniform circular motion in concentric circles of radii r_A and r_B with speed v_A and v_B respectively. Their time period of rotation is the same. The ratio of angular speed of A to that of B will be : ***(2019)***

(*a*) $r_A : r_B$ (*b*) $v_A : v_B$

(*c*) $r_B : r_A$ (*d*) 1 : 1

7. When an object is shot from the bottom of a long smooth inclined plane kept at an angle 60° with horizontal, it can travel a distance x_1 along the plane. But when the inclination is decreased to 30° and the same object is shot with the same velocity, it can travel x_2 distance. Then $x_1 : x_2$ will be: ***(2019)***

(*a*) $1:\sqrt{2}$ (*b*) $\sqrt{2}:1$

(*c*) $1:\sqrt{3}$ (*d*) $1:2\sqrt{3}$

8. The angle between $\vec{A}-\vec{B}$ and $\vec{A}\times\vec{B}$ is $(\vec{A}\neq\vec{B})$: ***(2017-Gujarat)***

(*a*) 60° (*b*) 90°

(*c*) 120° (*d*) 45°

9. A ball of mass 1 kg is thrown vertically upwards and returns to the ground after 3 seconds. Another ball, thrown at 60° with vertical also stays in air for the same time before it touches the ground. The ratio of the two heights are: ***(2017-Gujarat)***

(*a*) 1 : 3 (*b*) 1 : 2

(*c*) 1 : 1 (*d*) 2 : 1

10. If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is: ***(2016 - I)***

(*a*) 1° (*b*) 90°

(*c*) 45° (*d*) 180°

11. A particle moves so that its position vector is given by $\vec{r} = \cos\omega t\hat{x} + \sin\omega t\hat{y}$ where ω is a constant Which of the following is true? ***(2016 - I)***

(*a*) Velocity and acceleration both are perpendicular to $\vec{r}$

(*b*) Velocity and acceleration both are parallel to $\vec{r}$

(*c*) Velocity is perpendicular to $\vec{r}$ and acceleration is directed towards the origin

(*d*) Velocity is perpendicular to $\vec{r}$ and acceleration is directed away from the origin

12. A uniform circular disc of radius 50 cm at rest is free to turn about an axis which is perpendicular to its plane and passes through its center. It is subjected to a torque which produces a constant angular acceleration of 2.0 rad s^{-2}. Its net acceleration in ms^{-2} at the end of 2.0 s is approximately: ***(2016 - I)***

(*a*) 8.0 (*b*) 7.0

(*c*) 6.0 (*d*) 3.0

13. A ship A is moving Westwards with a speed of 10 km/h and a ship B 100 km South of A, is moving Northwards with a speed of 10 km/h. The time after which the distance between them becomes shortest, is: (*2015*)

(*a*) 5 h (*b*) $5\sqrt{2}$ h

(*c*) $10\sqrt{2}$ h (*d*) 0 h

14. The position vector of a particle $\vec{R}$ as a function of time is given by:

$$\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$$

Where R is in metres, t is in seconds and $\hat{i}$ and $\hat{j}$ denote unit vectors along x and y-direction, respectively. Which one of the following statements is wrong for the motion of particle? (*2015*)

(*a*) Path of the particle is a circle of radius 4 metre

(*b*) Acceleration vectors is along $-\vec{R}$

(*c*) Magnitude of acceleration vector is $\frac{v^2}{R}$ where v is the velocity of particle.

(*d*) Magnitude of the velocity of particle is 8 metre/second

15. A particle is moving such that its position coordinates (x, y) are:

(2 m, 3 m) at time t = 0,

(6 m, 7 m) at time t = 2 s and

(13 m, 14 m) at time t = 5 s

Average velocity vector $(\vec{v}_{av})$ from t = 0 to t = 5 s is: (*2014*)

(*a*) $\frac{1}{5}(13\hat{i} + 14\hat{j})$ (*b*) $\frac{7}{3}(\hat{i} + \hat{j})$

(*c*) $2(\hat{i} + \hat{j})$ (*d*) $\frac{11}{5}(\hat{i} + \hat{j})$

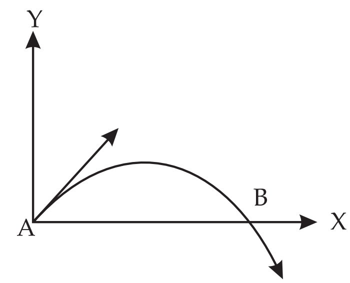
16. A projectile is fired from the surface of the earth with a velocity of 5 ms^{-1} and angle θ with the horizontal. Another projectile fired from another planet with a velocity of 3 ms^{-1} at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in ms^{-2}) is:

(given g = 9.8 ms^{-2}) (*2014*)

(*a*) 3.5 (*b*) 5.9

(*c*) 16.3 (*d*) 110.8

17. The velocity of a projectile at the initial point A $(2\hat{i} + 3\hat{j})$ m/s. Its velocity (in m/s) at point B is: (*2013*)



(*a*) $2\hat{i} + 3\hat{j}$ (*b*) $-2\hat{i} - 3\hat{j}$

(*c*) $-2\hat{i} + 3\hat{j}$ (*d*) $2\hat{i} - 3\hat{j}$

18. A particle has initial velocity $(2\hat{i} + 3\hat{j})$ and acceleration $(0.3\vec{i} + 0.2\vec{j})$. The magnitude of velocity after 10 sec will be: (*2012 Pre*)

(*a*) $9\sqrt{2}$ units (*b*) $5\sqrt{2}$ units

(*c*) 5 units (*d*) 9 units

19. The horizontal range and the maximum height of a projectile are equal. The angle of projection of the projectiles is: (*2012 Pre*)

(*a*) $\theta = \tan^{-1}\left(\frac{1}{4}\right)$ (*b*) $\theta = \tan^{-1}(4)$

(*c*) $\theta = \tan^{-1}(2)$ (*d*) θ = 45°

ANSWER KEY

Topicwise Questions

1. (*a*)	**2.** (*b*)	**3.** (*c*)	**4.** (*a*)	**5.** (*d*)	**6.** (*a*)	**7.** (*b*)	**8.** (*d*)	**9.** (*a*)	**10.** (*b*)
11. (*d*)	**12.** (*d*)	**13.** (*d*)	**14.** (*d*)	**15.** (*d*)	**16.** (*b*)	**17.** (*d*)	**18.** (*a*)	**19.** (*c*)	**20.** (*d*)
21. (*d*)	**22.** (*b*)	**23.** (*c*)	**24.** (*c*)	**25.** (*b*)	**26.** (*d*)	**27.** (*b*)	**28.** (*c*)	**29.** (*c*)	**30.** (*a*)
31. (*a*)	**32.** (*a*)	**33.** (*d*)	**34.** (*c*)	**35.** (*b*)	**36.** (*a*)	**37.** (*b*)	**38.** (*d*)	**39.** (*c*)	**40.** (*c*)
41. (*a*)	**42.** (*a*)	**43.** (*a*)	**44.** (*b*)	**45.** (*c*)	**46.** (*b*)	**47.** (*c*)	**48.** (*a*)	**49.** (*d*)	**50.** (*b*)
51. (*c*)	**52.** (*c*)	**53.** (*d*)	**54.** (*c*)	**55.** (*a*)	**56.** (*b*)	**57.** (*b*)			

Learning Plus

1. (*c*)	**2.** (*c*)	**3.** (*c*)	**4.** (*c*)	**5.** (*a*)	**6.** (*b*)	**7.** (*a*)	**8.** (*a*)	**9.** (*c*)	**10.** (*b*)
11. (*c*)	**12.** (*b*)	**13.** (*b*)	**14.** (*a*)	**15.** (*c*)	**16.** (*a*)	**17.** (*b*)	**18.** (*c*)	**19.** (*a*)	**20.** (*d*)
21. (*a*)	**22.** (*b*)	**23.** (*a*)	**24.** (*d*)	**25.** (*b*)	**26.** (*d*)	**27.** (*a*)	**28.** (*c*)		

Multiconcept MCQs

1. (*a*)	**2.** (*b*)	**3.** (*c*)	**4.** (*a*)	**5.** (*a*)	**6.** (*c*)	**7.** (*d*)	**8.** (*a*)	**9.** (*c*)	**10.** (*b*)
11. (*b*)	**12.** (*d*)	**13.** (*a*)	**14.** (*b*)	**15.** (*a*)	**16.** (*b*)	**17.** (*d*)	**18.** (*b*)		

NEET Past 10 Years Questions

1. (*c*)	**2.** (*c*)	**3.** (*c*)	**4.** (*a*)	**5.** (*a*)	**6.** (*d*)	**7.** (*c*)	**8.** (*b*)	**9.** (*c*)	**10.** (*b*)
11. (*c*)	**12.** (*a*)	**13.** (*a*)	**14.** (*d*)	**15.** (*d*)	**16.** (*a*)	**17.** (*d*)	**18.** (*b*)	**19.** (*b*)	

Solution

Topicwise Questions

1. (*a*) As we know that-

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

A = R = B = F (according to question)

$$F = \sqrt{F^2 + F^2 + 2F^2\cos\theta}$$

$F^2 = 2F^2 + 2F^2 \cos\theta$ $\quad$ $F^2 = 2F^2 (1 + \cos\theta)$

$\frac{1}{2} - 1 = \cos\theta$ $\quad$ $-\frac{1}{2} = \cos\theta \Rightarrow \theta = 120°$

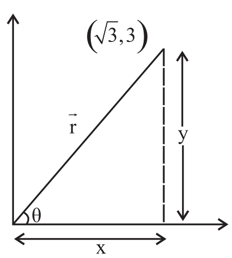
2. (*b*) Position vector of a particle $\vec{r} = \sqrt{3}\hat{i} + 3\hat{j}$

Angle with x-axis =

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{3}{\sqrt{3}}\right)$$

$$\theta = \tan^{-1}\left(\sqrt{3}\right)$$

$$\theta = 60°$$



3. (*c*) Resultant of vectors $\vec{A}$ and $\vec{B}$

So, $\vec{R} = \vec{A} + \vec{B}$

$$\vec{R} = 4\hat{i} + 3\hat{j} + 6\hat{k} - \hat{i} + 8\hat{j} - 8\hat{k}$$

$$\hat{R} = 3\hat{i} + 11\hat{j} - 2\hat{k}$$

$$\left|\vec{R}\right| = \sqrt{(3)^2 + (11)^2 + (-2)^2}$$

$$\hat{R} = \frac{3\hat{i} + 11\hat{j} - 2\hat{k}}{\sqrt{9 + 121 + 4}} = \frac{3\hat{i} + 11\hat{j} - 2\hat{k}}{\sqrt{134}}$$

4. (*a*) Let the two forces A and B $\quad \therefore \frac{A}{B} = \frac{9}{11}$

A = 9x, B = 11x θ = 60°

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$38 = \sqrt{(9x)^2 + (11x)^2 + 2 \times 9x \times 11x \times \frac{1}{2}}$$

$$38 = \sqrt{81x^2 + 121x^2 + 99x^2} \qquad 38 = \sqrt{301x^2}$$

$$38 = 17.3x \quad \Rightarrow x = \frac{38}{17.34} = 2.19 \approx 2.2$$

Thus = A = 9 × x = 9 × 2.2 = 19.8 N

B = 11 x = 11 × 2.2 = 24.2N

5. (*d*) $\left|\vec{A} + \vec{B}\right| = 4\hat{i} - 3\hat{j} + 5\hat{i} + 18\hat{j} = 9\hat{i} + 15\hat{j}$

$$|\vec{A} + \vec{B}| = \sqrt{(9)^2 + (15)^2} = \sqrt{81 + 225} = \sqrt{306} = 17.49$$

$$\tan\theta = \frac{15}{9} = \frac{5}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{5}{3}\right)$$

6. (*a*) Let the two forces each of magnitude A then the resultant is

$$R = \sqrt{A^2 + A^2 + 2A \times A \times \frac{1}{2}} = \sqrt{2A^2 + A^2} = \sqrt{3A^2}$$

$$R = \sqrt{3}A$$

7. (*b*) $\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$ $\qquad$ $\tan 90^0 = \frac{Q\sin\theta}{P + 2Q\cos\theta}$

$$\infty = \frac{Q\sin\theta}{P + 2Q\cos\theta} \Rightarrow \frac{1}{0} = \frac{Q\sin\theta}{P + 2Q\cos\theta}$$

$$P + 2Q\cos\theta \Rightarrow \cos\theta = \frac{-P}{2Q}$$

$$R = \sqrt{(P)^2 + (Q)^2 + 2PQ \times \cos\theta} = Q$$

8. (*d*) Vectors are equal

$$\hat{i}-3\hat{j}+5\hat{k}=\hat{i}-3\hat{j}-a\hat{k}$$

so,

$$5\hat{k}=-a\hat{k}$$

$$a=-5$$

9. (*a*) Given condition is that

$$\vec{C}=\vec{A}+\vec{B}$$

5 is the resultant or modulus of a vector $|\vec{A}+\vec{B}|$

$$|\vec{C}|^2=|\vec{A}+\vec{B}|^2$$

$$25=16+9+2\times4\times3\cos\theta$$

$$\Rightarrow 25-25=24\cos\theta \quad \Rightarrow \cos\theta=0 \quad \Rightarrow \theta=90°$$

10. (*b*) $R=\sqrt{P^2+Q^2+2PQ\cos\theta}$

$$R=\sqrt{A^2+B^2}\ ,\ P=(A+B)\ ,\ Q=(A-B)$$

$$\sqrt{A^2+B^2}$$

$$=\sqrt{(A+B)^2+(A^2-B^2)+2(A+B)(A-B)\cos\theta}$$

Squaring both sides

$A^2+B^2=(A+B)^2+(A-B)^2+2(A^2-B^2)\cos\theta$

$A^2+B^2=A^2+B^2+2AB+A^2+B^2-2AB+2(A^2-B^2\cos\theta$

$-A^2-B^2=2(A^2-B^2)\cos\theta$

$-(A^2+B^2)=2(A^2-B^2)\cos\theta$

Multiply by (–)

$A^2+B^2=2(B^2-A^2)\cos\theta$

$$\cos^{-1}\left(\frac{A^2+B^2}{2(B^2-A^2)}\right)=\theta$$

11. (*d*) There is no y-component in the vector.

12. (*d*) $\vec{A}=3\hat{i}+5\hat{j}-7\hat{k}$

The direction cosines are

$$\cos\alpha=\frac{A_x}{|A|}\ ,\ \cos\beta=\frac{A_y}{|A|}$$

$$\cos\gamma=\frac{A_z}{|A|} \qquad |\vec{A}|=\sqrt{(A_x)^2+(A_y)^2+(A_z)^2}$$

$$=\sqrt{9+25+49}=\sqrt{83}$$

$$\cos\alpha=\frac{3}{\sqrt{83}}\ ,\ \cos\beta=\frac{5}{\sqrt{83}}\ ,\ \cos\gamma=\frac{-7}{\sqrt{83}}$$

13. (*d*) Magnitude of vector depends upon the magnitudes of component vectors and their orientations with respect to each other, so that's why we can say the components of vectors may be equal to or less than or greater than the magnitude of vector.

14. (*d*) $|\vec{A}\times\vec{B}|=\dfrac{(\vec{A}.\vec{B})}{\sqrt{3}}$

$$AB\sin\theta=\frac{AB\cos\theta}{\sqrt{3}}$$

$$\frac{\sin\theta}{\cos\theta}=\frac{1}{\sqrt{3}} \qquad \theta=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\Rightarrow\theta=\frac{\pi}{6}$$

15. (*d*) When the vectors are perpendicular to each other then their dot product is equal to zero

$$(2\hat{i}+4\hat{j}+6\hat{k}).(5\hat{i}+6\hat{j}-a\,\hat{k})=0$$

$$10+24-6a=0 \ \Rightarrow 34=6a \ \Rightarrow a=\frac{17}{3}$$

16. (*b*) Displacement of particle $=15\hat{i}-20\hat{j}+8\hat{k}-(3\hat{i}+5\hat{j}-8\hat{k})$

$$\Delta S=15\hat{i}-20\hat{j}+8\hat{k}-3\hat{i}-5\hat{j}+8\hat{k}$$

$$\Delta S=12\hat{i}-25\hat{j}+16\hat{k}$$

Work done = F . ΔS

$=(3\hat{i}+\hat{j}+4\hat{k})(12\hat{i}-25\hat{j}+16\hat{k})$

= 36 – 25 + 64 = 75 Joule

17. (*d*) For the scalar product, $\vec{F_1}.\vec{F_2}=(2\hat{i}+5\hat{k})(3\hat{j}+4\hat{k})$

$$\vec{F_1}.\vec{F_2}=20$$

18. (*a*) As, $\vec{V}=\vec{\omega}\times\vec{r}$, then

$$\vec{V}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 5 & -4 & +9\\ 8 & -6 & 3\end{vmatrix}$$

$$\vec{V}=\hat{i}\big(-12-(-54)-\hat{j}(15-72)+\hat{k}(-30-(-32))\big)$$

$$\vec{V}=42\hat{i}-57\hat{j}+2\hat{k}$$

$$|\vec{V}|=\sqrt{(42)^2+(57)^2+(2)^2}$$

$$\Rightarrow |\vec{V}|=\sqrt{5017}=70.8\,\text{unit}$$

19. (*c*) For perpendicular condition, $\vec{A}.\vec{B}=0$

$$\vec{A}.\vec{B}=(\hat{i}A\cos\theta+\hat{j}A\sin\theta)(\hat{i}B\sin\theta-\hat{j}B\cos\theta)$$

$=A\cos\theta\ B\sin\theta-A\sin\theta\ B\cos\theta$

$=AB\sin\theta\cos\theta-AB\sin\theta\cos\theta \quad =0$

$\vec{A}.\vec{B}=0$ (condition satisfied)

20. (*d*) Let $\vec{A}$ be $\hat{i}+\hat{j}+\hat{k}$ and $\vec{B}$ be $\hat{i}$

According to the relation $\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \cos\theta$

$$\therefore \frac{(\hat{i}+\hat{j}+\hat{k}).(\hat{i})}{\sqrt{1^2+1^2+1^2}\times\sqrt{1^2}} = \frac{1}{\sqrt{3}}$$

$$\cos\theta = \frac{1}{\sqrt{3}} = \text{None of these}$$

21. (*d*) $\cos\theta = \frac{(\vec{B}\times\vec{A}).(\vec{A})}{|\vec{B}\times\vec{A}||\vec{A}|}$

$$(\vec{B}\times\vec{A}).(\vec{A}) = |\vec{B}\times\vec{A}||\vec{A}|\cos 90^0$$

$$(\vec{B}\times\vec{A}).(\vec{A}) = 0$$

22. (*b*) For three vectors to be coplanar,

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & P & 5 \end{vmatrix} = 0$$

$\Rightarrow 2\,(10 + 3P) + 1(5 + 9) + 1\,(P - 6) = 0$

$\Rightarrow 20 + 6P + 5 + 9 + P - 6 = 0 \Rightarrow 7P + 28 = 0$

$\Rightarrow P = -\frac{28}{7} = -4$

23. (*c*) $\vec{\tau} = \vec{r}\times\vec{F}$ $\qquad \vec{\tau} = -4\hat{i}+\hat{j}+10\hat{k}$

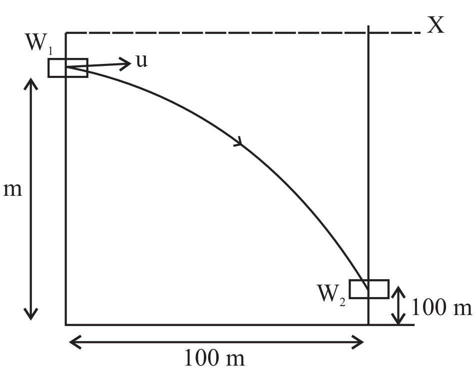
24. (*c*) For the first body, $h = \frac{1}{2}gt^2 \quad (u = 0)$

$t = \frac{x}{v} \qquad \therefore h = \frac{1}{2}g\left(\frac{x^2}{v^2}\right) \qquad$ ...... (1)

$h \propto \frac{x^2}{v^2} \qquad \therefore \frac{h}{4h} = \frac{x^2}{v^2}\times\frac{v'^2}{(4x)^2}$

$4v^2 = v'^2 \qquad 2v = v'$

25. (*b*)



Height of W_1 = 500 m

Height of W_2 = 100 m

Net vertical height which attains by ball = 500 – 100 = 400 m

Time taken by the ball to enter W_2 is

$$S = ut + \frac{1}{2}gt^2 \Rightarrow 400 = 0 + \frac{1}{2}\times 10t^2$$

$$\frac{400}{5} = t^2 \Rightarrow t = \sqrt{80} = 8.9\,\text{sec}.$$

Motion of the ball from W_1 to W_2

$$S = u_x t + \frac{1}{2}a_x t^2 \qquad 100 = u\times 8.9 + \frac{1}{2}\times(0)\times(8.9)^2$$

u = 11.2 m/s

Short trick: Horizontal Range = 100 m

Vertical height = 500 – 100 = 400 m

$$R = u\sqrt{\frac{2h}{g}} \Rightarrow u = R\sqrt{\frac{g}{2h}}$$

$$u = 100\times\sqrt{\frac{10}{2\times 400}} = \frac{100}{8.9} = 11.2\,\text{m/s}$$

26. (*d*) As we know the relation,

$$u = 80\times\frac{5}{18} = 22.22\,\text{m/s} \qquad R = u\sqrt{\frac{2h}{g}}$$

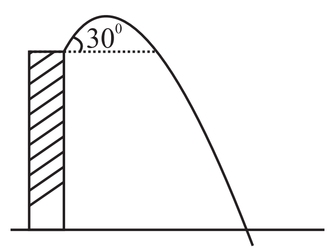
$h = 150\times 10^{-2} = 1.5$ m

$$R = 22.22\times\sqrt{\frac{2\times 1.5}{10}} = 22.22\sqrt{\frac{3}{10}} = 22.22\times 0.54$$

$= 11.99 \approx 12$

27. (*b*) $R = u\sqrt{\frac{2h}{g}} = 12\sqrt{\frac{10}{10}} = 12\,\text{m} \quad \therefore S = \sqrt{R^2+r^2} = 13\,\text{m}$

28. (*c*)



$$\bar{Y} = \bar{v}_y t + \frac{1}{2}\bar{a}_y t^2$$

$$70 = -(50\sin 30^0)t + \frac{1}{2}\times 10\times t^2$$

$$\Rightarrow 70 = -25t + 5t^2$$

or $t^2 - 5t - 14 = 0 \Rightarrow t = -2$sec (–ve time is not possible)

or t = 7sec.

29. (*c*) We have v = u + at or 7.5 = 2.5 + 0.50 t $\Rightarrow$ t = 10 s

Hence correct answer is (c)

30. (*a*) $x = u\sqrt{\frac{2H}{g}}, 6.2 = u\sqrt{\frac{2(4.9)}{9.8}} \Rightarrow u = 6.2\ \text{ms}^{-1}$

31. (*a*) Time of flight is 2 times that of time taken to reach the maximum height, i.e., $T_f = 2t_m$

32. (a) Maximum range $= \frac{u^2}{g}$ $\quad 10 \times 15000 = u^2$

$150000 = u^2 \quad \Rightarrow \quad 100\sqrt{15} = 387.2\,\text{m/s}$

33. (*d*) $\frac{R}{T^2} = \frac{u^2 \sin 2\theta}{g} \times \frac{g^2}{4u^2 \sin^2 \theta} \Rightarrow \frac{R}{T^2} = \frac{g\cos\theta}{2\sin\theta} = 4.9$

34. (*c*) Relative horizontal displacement will be zero because horizontal component of velocity of coin is same for coil & observer/person. No angular projection has given to coin so no parabolic path.

35. (*b*) According to the relation, $H = \frac{u^2}{2g}$

$R = \frac{u^2}{g} = 500 \Rightarrow u^2 = 500\text{ g} \qquad H = \frac{500g}{2g} = 250\text{ m}$

36. (*a*) Angular momentum = mvr

at starting point r = 0

37. (*b*) $H_1 = \frac{u^2 \sin^2 \theta_1}{2g}$, $H_2 = \frac{u^2 \sin^2 \theta_2}{2g}$

$\frac{H_1}{H_2} = \frac{\sin^2\theta_1}{\sin^2\theta_2} \Rightarrow \frac{\sqrt{3}}{1} = \frac{\sin\theta_1}{\sin\theta_2} \Rightarrow \frac{\cos\theta_1}{\cos\theta_2} = \frac{1}{\sqrt{3}}$

We know that:

$\frac{R_1}{R_2} = \frac{(3u^2)\sin 2\theta_1}{u^2 \sin 2\theta_2} = \frac{9\sin\theta_1\cos\theta_1}{\sin\theta_2\cos\theta_2} = \frac{9}{1}$

$\frac{R_1}{R_2} = \frac{9}{1}$ other parameters are same ($\theta_1 = \theta_2$)

38. (*d*) Horizontal component of the velocity does not change in over all motion of projectile.

So, at highest point it will be.

$u\cos\theta = u\cos 30° = \frac{u\sqrt{3}}{2}$

39. (*c*) As the stone thrown vertically up will come back to the point of projection with same speed, both the stones will move downward with same initial velocity, so both will hit the ground with velocity.

$v^2 = u^2 + 2gh$ i.e., $v = \sqrt{(u^2 + 2gh)}$

So, the ratio of speeds attained when they hit the ground is 1 :1

Hence correct answer is (c).

40. (*c*) $H = \frac{u^2 \sin^2\theta}{2g} \Rightarrow H \propto \sin^2\theta$

$\frac{H_1}{H_2} = \frac{\sin^2 60°}{\sin^2 30°} = \frac{3}{4} \times \frac{4}{1} \qquad \frac{H_1}{H_2} = 3$

41. (*a*) Velocity of object A relative to that of object B is

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

Velocity of object B relative to that of A is

$\therefore \vec{V}_{BA} = \vec{V}_B - \vec{V}_A \quad \therefore \vec{V}_{BA} = -\vec{V}_{AB}$ and $|\vec{V}_{AB}| = |\vec{V}_{BA}|$

42. (*a*) When a boat tends to cross a river of width along a shortest path, relative velocity of boat is-

$V_R = \sqrt{V_B^2 - V_r^2}$

Resultant velocity of the boat and river $= \frac{2\,\text{km}}{\frac{7}{60}\text{hr}}$

$= \frac{2 \times 60}{7} = 17.14$

$V_R^2 = V_B^2 - V_r^2 \Rightarrow V_r^2 = V_B^2 - V_R^2$

$V_r^2 = \sqrt{V_B^2 - V_R^2} = \sqrt{(18)^2 - (17.14)^2}$

$= \sqrt{324 - 293.7} = \sqrt{30.3} = 5.5\,\text{km/h}$

43. (*a*) $V_p = 100$ m/s $\qquad V_w = 25$ m/s

Total speed of the plane $V = \sqrt{V_p^2 + V_w^2}$

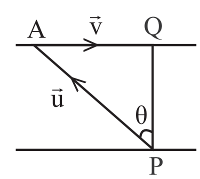
$V = \sqrt{(100)^2 + (25)^2} = \sqrt{10000 + 625}$

$\Rightarrow V > 100\,\text{m/s}$

44. (*b*) He should row his boat at an angle θ such that

$\sin\theta = \frac{v}{u} = \frac{4}{10} = \frac{2}{5}$

$\therefore \theta = \sin^{-1}\left(\frac{2}{5}\right)$



45. (*c*) Let width of the river be d speed of stream be v and the speed of the boat relative to water be u and the angle with the verticle at which the boat must move for minimum drifting is θ.

Time taken to cross the river $= \frac{d}{u\cos\theta}$

Drift of the boat is (v – usinθ) (d/ucosθ)

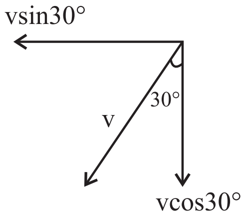
Differentiating this w.r.t time and equating to zero we get the angle θ for minimum drifting as $\sin^{-1}\left(\frac{v}{u}\right)$

Angle with the direction of the stream is

$90° + \sin^{-1}\left(\frac{v}{u}\right)$

Here $u = \frac{v}{n}$ $\qquad \therefore$ Angle $= \frac{\pi}{2} + \sin^{-1}\left(\frac{1}{n}\right)$

46. (*b*)

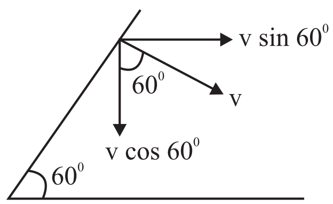


Rain drop will hit vertical when

$v \sin 30° = 10$ km/h $\Rightarrow v = 20$ km/h

47. (*c*) Let v be the velocity at the time of collision.

Then, $u\sqrt{2}\,\cos 45^0 = v\,\sin\,60^0$

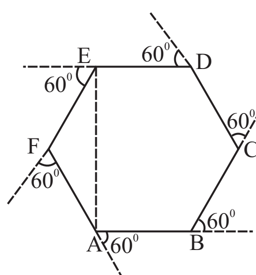


$\left(u\sqrt{2}\right)\left(\dfrac{1}{\sqrt{2}}\right) = \dfrac{\sqrt{3}v}{2}$ $\qquad \therefore\ v = \dfrac{2}{\sqrt{3}}u$

48. (*a*) Since relative acceleration is zero, therefore relative velocity will be constant.

49. (*d*) If instantaneous speed is constant that means motion is uniform such that equal path length / distance are covered in equal time intervals.

50. (*b*) Cyclist starts from A and reaches a point E



So their displacement is AE

51. (*c*) As we know that, $v = r\omega \quad \therefore \omega = \dfrac{v}{r}$

52. (*c*) In uniform circular motion, acceleration is produce due to change in direction, and in circular motion direction continues changes. Acceleration is internally radial due to centripetal acceleration because centripetal force act towards centre.

53. (*d*) $\omega = \dfrac{2\pi}{T} = \dfrac{2\times\pi}{60} = \dfrac{\pi}{30}$ rad/s

54. (*c*) In a uniform circular motion radial acceleration remains constant and tangential acceleration remains zero.

55. (*a*) Centripetal acceleration, $a_c = \omega^2 R$

$$= \frac{(2\pi)^2 \times R}{T^2} = \frac{4\times(3.14)^2\times 1.8\times 10^{11}}{(365\times 24\times 60\times 60)^2}$$

$$= \frac{4\times 10\times 1.8\times 10^{11}}{\left(31536\times 10^3\right)^2} = 6\times 10^{-3}\,\text{m/s}^2$$

56. (*b*) In uniform circular motion speed and angular speed remains constant. Centripetal acceleration acts towards the center (centre seeking) because centripetal force acts towards the centre but velocity does not remains constant because direction is continuously changing.

57. (*b*) Angular acceleration $\alpha = \dfrac{d^2\theta}{dt^2}$

$$\alpha = \frac{d^2}{dt^2}\left(\theta_0 + \theta_1 t + \theta_2 t^2 + \theta_3 t^3\right)$$

$\alpha = 2\theta_2 + 6\,\theta_3 t$

Learning Plus

1. (*c*) Let the two vectors $\vec{A} = \vec{F}_1 + \vec{F}_2$ and $\vec{B} = \vec{F}_1 - \vec{F}_2$

As two vectors are perpendicular to each other so-

$\vec{A}.\vec{B} = 0 \qquad \left(\vec{F}_1 + \vec{F}_2\right).\left(\vec{F}_1 - \vec{F}_2\right) = 0$

$\left(\vec{F}_1\right)^2 - \left(\vec{F}_2\right)^2 = 0 \Rightarrow \left|\vec{F}_1\right|^2 - \left|\vec{F}_2\right|^2 = 0$

$\left|\vec{F}_1\right|^2 = \left|\vec{F}_2\right|^2 \qquad \Rightarrow \left|\vec{F}_1\right| = \left|\vec{F}_2\right|$

2. (*c*) According to the question

$\left|\vec{A} + \vec{B}\right| = n\left|\vec{A} - \vec{B}\right|$

Squaring both side

$A^2 + B^2 + 2AB\cos\theta = n^2\,(A^2 + B^2 - 2AB\cos\theta)$

$A^2 + B^2 + 2AB\cos\theta = n^2A^2 + n^2B^2 - n^2 \times 2AB\cos\theta$

$2A^2 + 2A^2\cos\theta = 2n^2A^2 - 2n^2A^2\cos\theta$

$2A^2\,(1 + \cos\theta) = 2n^2A^2\,(1 - \cos\theta)$

$1 + \cos\theta = n^2 - n^2 \cos\theta$

$1 + \cos\theta + n^2 \cos\theta = n^2$

$$\left|\vec{A}+\vec{B}\right| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\left|\vec{A}-\vec{B}\right| = \sqrt{A^2 + B^2 - 2AB\cos\theta} \qquad \left|\vec{A}\right| = \left|\vec{B}\right|$$

$$\cos\theta = \frac{n^2-1}{n^2+1} \qquad \theta = \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$$

3. (*c*) According to the relation

$$R = u\sqrt{\frac{2h}{g}} \Rightarrow u = R\sqrt{\frac{g}{2h}} = 90\sqrt{\frac{10}{2\times 8}}$$

$$u = 90\sqrt{\frac{10}{16}} \Rightarrow u = \frac{90}{4}\sqrt{10} \Rightarrow u = \frac{90}{4}\times 3.1$$

$\Rightarrow u = 69.75$

4. (*c*) $x = v\sqrt{\frac{2h}{g}} = \sqrt{2gh}\times\sqrt{\frac{2h}{g}} = 2h$

5. (*a*) $\text{Range}(R) = \frac{u^2 \sin 2\theta}{g}$

$\text{Range} \propto u^2 \qquad \therefore \frac{R}{R'} = \frac{u^2}{4u^2} \Rightarrow R' = 4R$

6. (*b*) According to the relations

$$R = \frac{u^2 \sin 2\theta}{g},\ H = \frac{u^2 \sin^2\theta}{2g}$$

$$H_{max} = \frac{u^2 \sin^2 30^0}{2g} = \frac{u^2}{8g} \qquad R = \frac{u^2 \sin 60^\circ}{g} = \frac{u^2\sqrt{3}}{2g}$$

$$\therefore \frac{H}{R} = \frac{u^2}{8g}\times\frac{2g}{u^2\sqrt{3}} \Rightarrow R = 4\sqrt{3}H$$

7. (*a*) For projectile A, Max. Height $H_A = \frac{u_A{}^2 \sin^2 60^\circ}{2g}$

For projectile B $\quad H_B = \frac{u_B{}^2 \sin^2\theta}{2g}$

According to question $\quad \frac{u_A{}^2 \sin^2 60^\circ}{2g} = \frac{u_B{}^2 \sin^2\theta}{2g}$

$$\frac{u_A{}^2}{u_B{}^2} = \frac{\sin^2\theta}{\sin^2 60^\circ}$$

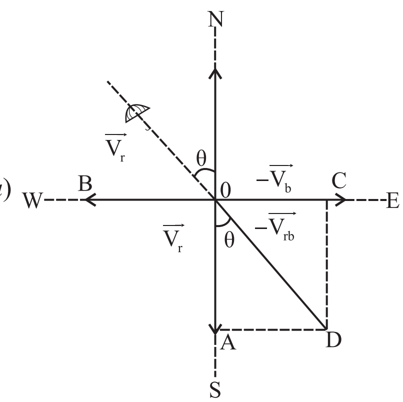
$$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{\sin^2\theta}{\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \frac{3}{4}\times\frac{1}{2} = \sin^2\theta$$

$$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2\sqrt{2}}$$

$$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{2}}\right)$$

8. (*a*)

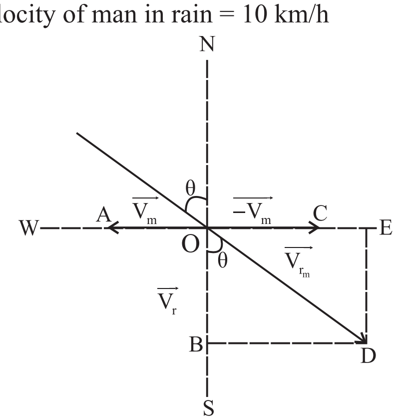


$\vec{V}_r = 35\,m/s \qquad \vec{V}_b = 15\,m/s$

$$\tan\theta = \frac{AD}{OA} \Rightarrow \tan\theta = \frac{\vec{V}_b}{\vec{V}_r} = \frac{15}{35} = \frac{3}{7} = 0.4285$$

$\theta = \tan^{-1}(0.4285) = 23.19^0$

9. (*c*) Velocity of man in rain = 10 km/h



As we know that the following relation

$$\vec{V}_{r_m} = \sqrt{V_r{}^2 + V_m{}^2 + 2V_rV_m\cos 90^0}$$

$\vec{V}_{r_m}$ = relative velocity of rain w.r.t the man

$\vec{V}_m$ = velocity of man $\qquad \tan\theta = \frac{BD}{OB} = \frac{V_m}{V_r}$

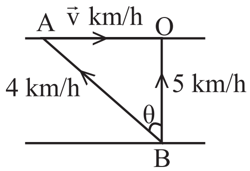
θ is the angle which V_{rm} makes with the vertical direction

$$\tan 45^0 = \frac{10}{V_r} \Rightarrow V_r = 10\,km/h$$

relative velocity of rain w.r.t the man

$$\vec{V}_{r_m} = \sqrt{(10)^2 + (10)^2} = \sqrt{100+100} = 10\sqrt{2}\,km/h$$

10. (*b*)

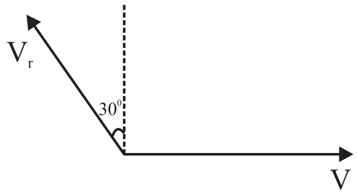


Velocity of the river is along AO

By applying triangle law of vector addition, value of AB comes to be 3 km/hr.

11. (*c*) $t = \frac{d}{V_r \cos 30} = \frac{2d}{\sqrt{3}V_r}$

Now, drift $\frac{d}{2} = (V - V_r \sin 30)t \quad = \left[V - \frac{V_r}{2}\right]\left[\frac{2d}{\sqrt{3}V_r}\right]$



$\sqrt{3}V_r = 4V - 2V_r$

$V_r = \left[\frac{4}{2+\sqrt{3}}\right]V = 4\left(2-\sqrt{3}\right)V$

12. (*b*) d = width of the river

θ = angle made with upstream

90 – θ = angle made with normal to the stream.

Shortest time $t = \frac{d}{v}$

Time of shortest distance $= t' = \frac{d}{v\cos(90° - \theta)}$

Ratio of time in two cases $\quad \frac{t}{t'} = \frac{d}{v} \div \frac{d}{v\sin\theta} = \sin\theta.$

13. (*b*) Apply formula of time of flight on inclined plane.

14. (*a*) $S = t^3 + 8$

$\therefore$ speed, $v = \frac{ds}{dt} = 3t^2 \qquad \therefore a = \frac{dv}{dt} = 6t$

tangential acceleration $a_t = 6 \times 4 = 24$ m/s^2

Centripetal acceleration $a_c = \frac{v^2}{R} = \frac{\left(3t^2\right)^2}{R} = \frac{9t^4}{R}$

$a_c = \frac{9\times16\times16}{60} = \frac{3\times16\times16}{20} \qquad a_c = 38.4$ m/s^2

$\therefore$ Net acceleration $a = \sqrt{{a_c}^2 + {a_t}^2} = \sqrt{(38.4)^2 + (24)^2}$

$a = \sqrt{1474.56 + 576} = \sqrt{2050.56} \Rightarrow a = 45.28$ m/s^2

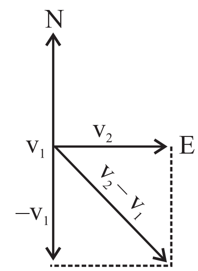
15. (*c*) Tangential acceleration $= a_t = a$

radial acceleration $= a_r = \frac{v^2}{R}$

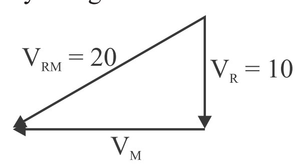
net acceleration $= \sqrt{a_t^2 + a_r^2} = \sqrt{a^2 + \left(\frac{v^2}{R}\right)^2}$

16. (*a*) Change in velocity $= \sqrt{(40)^2 + (30)^2} = 50\text{kms}^{-1}$

Average acceleration $= \frac{50}{20} = 2.5\text{kms}^{-2}$

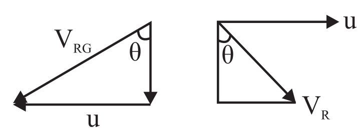


17. (*b*) By using addition of vector in vector form



$V_m = \sqrt{(20)^2 - (10)^2} \quad = \sqrt{400 - 100} \quad = \sqrt{300}$

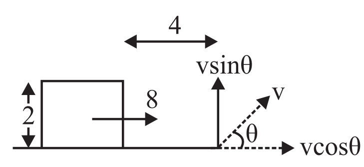
18. (*c*) Velocity of rain along x-axis is v_0



$V_R = u\hat{i} + u\cot\theta\hat{j}$

19. (*a*) (v sin θ) $t = 2$

(8 – vcosθ) t = 4



$\frac{v\sin\theta}{8 - v\cos\theta} = \frac{1}{2}$

2v sin θ = 8 –v cos θ

2v sin θ + v cosθ = 8

$v = \frac{8}{2\sin\theta + \cos\theta}$

$v_{min} = \frac{8}{\sqrt{2^2 + 1^2}}$

$v_{min} = \frac{8}{\sqrt{5}} = \frac{8\sqrt{5}}{\sqrt{5}\sqrt{5}} = 1.6\sqrt{5}\text{ m/s}$

20. (*d*) $u = v_0, \quad a = -\alpha\sqrt{v}$

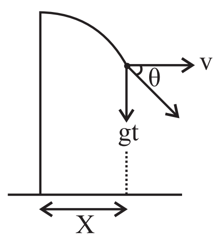
so, $\frac{dv}{dt} = -\alpha\sqrt{v} \Rightarrow \int_{v_0}^{0} \frac{1}{\sqrt{v}} dv = \int_0^t -\alpha\, dt \Rightarrow t = \frac{2\sqrt{v_0}}{\alpha}$

21. (*a*) From the graph $\quad v = \frac{20}{10}s + 10 \Rightarrow v = 2s + 10$

$a = v.\frac{dv}{ds} = 2v = 4s + 20 \quad \Rightarrow a = 24\ m/s^2$

22. (*b*) Let v be the horizontal speed provided by wind then at any instant 't'.

$\tan\theta = \frac{gt}{v}$ also $X = vt$



23. (*a*) $(H - h) = \frac{1}{2}gt_1^2$

$h = \frac{1}{2}gt_2^2$

$t = t_1 + t_2$

24. (**d**) Separation between two bodies after T seconds

$= \frac{1}{2}gT^2$

Relative velocity = gT

so time when separation becomes 'L' is $\frac{L - \frac{1}{2}gT^2}{gT}$

25. (*b*) $v_{av} = \frac{\sqrt{2}R}{t} \qquad t = \sqrt{\frac{2\left(\pi/2\right)}{\pi/4}} = 2s \quad v_{av} = 1\ m/s$

26. (*d*) In the given interval the slope of v-t graph (i.e. acceleration) neither constant nor uniform. Therefore relations (a), (b) and (c) are incorrect but (d) is correct.

27. (*a*) $\vec{r} = at\hat{i} - bt^2\hat{j} \qquad x = at$ and $y = -bt^2$

$\Rightarrow t = \left(\frac{x}{a}\right)$ and $y = -b\left(\frac{x}{a}\right)^2 \Rightarrow y = \frac{-b}{a^2}x^2$

28. (*c*) Net acceleration of the car $a = \sqrt{(a_t)^2 + (a_c)^2}$

$\therefore a = \sqrt{(4)^2 + \frac{v^4}{r^2}} \Rightarrow a = \sqrt{16 + \frac{(40)^4}{(500)^2}}$

$\therefore a = \sqrt{16 + \frac{256\times10^4}{25\times10^4}} \Rightarrow a = \sqrt{16 + 10.24} = 5.122\ m/s^2$

Multiconcept MCQs

1. (*a*) For motion along vertical axis,

$h_1 = u\sin\theta t_1 - \frac{1}{2}gt_1^2$ (for h_1)

$$\Rightarrow \qquad t_1 = \frac{h_1 + \frac{1}{2}gt_1^2}{u\sin\theta} \qquad ...(i)$$

$\Rightarrow h_2 = u\sin\theta t_2 - \frac{1}{2}gt_2^2$ (for h_2)

$$\Rightarrow t_2 = \frac{h_2 + \frac{1}{2}gt_2^2}{u\sin\theta} \qquad ...(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{t_1}{t_2} = \frac{\left(h_1 + \frac{1}{2}gt_1^2\right)/u\sin\theta}{\left(h_2 + \frac{1}{2}gt_2^2\right)/u\sin\theta}$$

$\Rightarrow \quad h_1t_2 - h_2t_1 = \frac{g}{2}(t_1t_2^2 - t_1^2t_2)$

The time of flight of the ball

$T = \frac{2u\sin\theta}{g} = \frac{2}{g}(u\sin\theta)$

$= \frac{2}{g}\left(\frac{h_1 + \frac{1}{2}gt_1^2}{t_1}\right) = \frac{2}{t_1}\left(\frac{h_1}{g} + \frac{t_1^2}{2}\right)$ [from Eq. (i)]

$= \frac{h_1}{t_1}\times\frac{2}{g} + t_1 = \frac{h_1}{t_1}\times\left(\frac{t_1t_2^2 - t_1^2t_2}{h_1t_2 - h_2t_1}\right) + t_1$

$= \frac{h_1t_1t_2^2 - h_1t_1^2t_2 + h_1t_1^2t_2 - h_2t_1^3}{t_1(h_1t_2 - h_2t_1)} = \frac{h_1t_2^2 - h_2t_1^2}{(h_1t_2 - h_2t_1)}$

2. (*b*) The projectile is shown below

$t = \frac{2u\sin\theta}{g} \Rightarrow 4 = \frac{2u\sin\theta}{g} \quad ...(i)$

Also, $v^2 = u^2 + 2as \Rightarrow 0^2 = u^2\sin^2\theta - 2gh$

$\frac{u^2\sin^2\theta}{2g} = h$

From equation (i)

$$\frac{u\sin\theta}{g} = 2 \qquad \Rightarrow \frac{4g^2}{2g} = h$$

$h = 2g \quad \Rightarrow \qquad h = 20$ m

3. (*c*) Equation of projectile, $y = 10x - \left(\frac{5}{9}\right)x^2$

Equation of trajectory is give by

$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}.x^2$$

On comparing, $\tan\theta = 10$ and $\frac{g}{2u^2\cos^2\theta} = \frac{5}{9}$

or $10u^2\cos^2\theta = 9g \quad \therefore \quad u^2\cos^2\theta = 9$

Range of projectile,

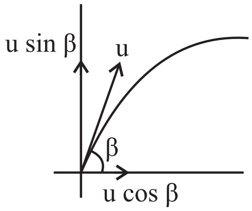
$$R = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{2u^2\tan\theta\cos^2\theta}{g}$$

$(\because \sin\theta = \tan\theta\cos\theta)$

$$= \frac{2(u^2\cos^2\theta).\tan\theta}{g}$$

$$= \frac{2\times9\times10}{10} = 18\text{m}$$

4. (*a*) The kinetic energy at the highest point would be equal to $\frac{1}{2}m(u\cos\beta)^2 = \frac{1}{2}mu^2\cos^2\beta$ (as the vertical component of the velocity is zero).



The initial kinetic energy is the maximum kinetic energy.

So, $(KE)_H = (KE)_i\cos^2\beta$ where, $(KE)_i = \frac{1}{2}mv^2$

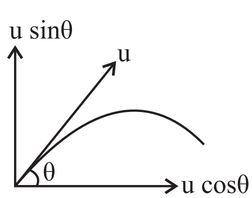
Thus, $KE_i\cos^2\beta = \frac{3}{4}(KE)_i \Rightarrow \cos\beta = \frac{\sqrt{3}}{2} \Rightarrow \beta = 30°$

5. (*a*) Given, R = 2H

We know that, R = 4H cot θ

2H = 4H cotθ

$\cot\theta = \frac{1}{2}$



$\Rightarrow \sin\theta = \frac{2}{\sqrt{5}}$ and $\cos\theta = \frac{1}{\sqrt{5}}$

∴ Range of projectile,

$$R = \frac{2v^2\sin\theta\cos\theta}{g} = \frac{2v^2}{g}\times\frac{2}{\sqrt{5}}\times\frac{1}{\sqrt{5}} = \frac{4v^2}{5g}$$

6. (*c*) Let the angle made by the instantaneous velocity with the horizontal be α. Then,

$$\tan\alpha = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$$

Given, α = 45°, when t = 1 s

(i.e., tan 45° = 1)

This gives u cos θ = u sin θ – g ...(i)

α = 0 °, when t = 2 s (i.e. tan θ° = 0)

And u sin θ – 2g = 0 ... (ii)

On solving Eqs. (i) and (ii), we have

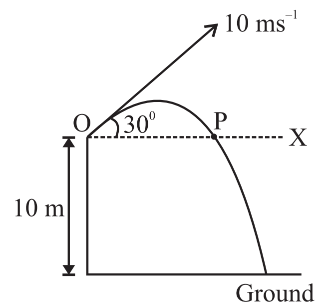
u sin θ = 2g ⇒ u cos θ = g

squaring and adding

$$\Rightarrow \quad u = \sqrt{5}g = 10\sqrt{5}\text{ms}^{-1}$$

7. (*d*) Say, the ball is at point P when it is at a height of 10 m from the ground. So, we have to find distance OP, which can be calculated direct considering it is a projectile on a level OX at a height h from the horizontal level.

$$OP = R = \frac{u^2\sin2\theta}{g}$$



$$\Rightarrow \qquad R = \frac{10^2\times\sin(2\times30°)}{10} = 8.66\text{m}$$

8. (*a*) We know that centripetal acceleration is given by

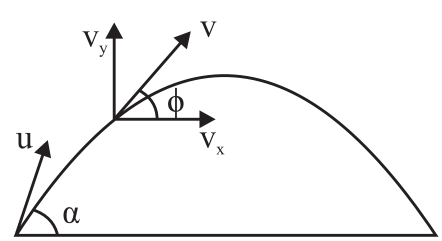
$$a = \frac{V^2}{R}$$

For constant R. $\qquad a \propto V^2$ or $\frac{a_1}{a_2} = \frac{V_1^2}{V_2^2}$

$$\therefore \qquad \frac{a_1}{a_2} = \frac{v^2}{(2v)^2} = \frac{1}{4}$$

$a_2 = 4a_1$ or $a_2 = 4a \quad (\because a_2 = a)$

9. (*c*)



$v_x = v_y$

i.e. $\phi = 45°$

$$\tan\phi = \frac{v_y}{v_x} = \frac{u_y + a_y t}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$$

$$\tan\phi = \tan\theta - \frac{gt}{u\cos\theta} \qquad \tan 45^0 = \tan\alpha - \frac{gt}{u\cos\alpha}$$

$$1 = \frac{\sin\alpha - \left(\frac{gt}{u}\right)}{\cos\alpha} \qquad \cos\alpha = \sin\alpha - \frac{gt}{u}$$

$$\frac{gt}{u} = (\sin\alpha - \cos\alpha) \qquad t = \frac{u}{g}(\sin\alpha - \cos\alpha)$$

10. (*b*) Change in momentum is 2 mv, time taken to go around

$$t_{AB} = \frac{\pi\left(\frac{d}{2}\right)}{v} \qquad \text{Force} = \frac{\text{change in momentum}}{\text{time}}$$

11. (*b*) $\nu = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$

$$\frac{dx}{dt} = t,\ \frac{dy}{dt} = x.\frac{dx}{dt} = \frac{t^3}{2} \qquad \text{so, } \nu = 2\hat{i} + 4\hat{j}$$

12. (*d*) There will be no impact on maximum height & time of flight.

Range will increase by = $\frac{1}{2}\left(\frac{g}{4}\right)T^2$ as $H = \frac{1}{2}g\left(\frac{T}{2}\right)^2$

So, new range = R + H

13. (*a*) $S = \frac{1}{10}(2\pi r) = \frac{1}{2}(0.5)t^2 \quad \Rightarrow t = \sqrt{\frac{4}{10}(2\pi r)}$

$$a_{CP} = \frac{v^2}{r} = \frac{a^2t^2}{r} = \frac{\left(\frac{1}{4}\right)(0.4)(2\pi r)}{r}$$

$$a_{CP} = 0.2\pi$$

$$a_T = 0.5 \qquad a_{Net} = \sqrt{a_{CP}^2 + a_T^2} \approx 0.8$$

14. (*b*) Let v km h^{-1} be the constant speed with which the buses ply between the towns A and B.

Relative velocity of the bus from A to B with respect to the cyclist = (v – 20) km h^{-1}

Relative velocity of the bus from B to A with respect to the cyclist = (v + 20) km h^{-1}

Distance travelled by the bus in time T (minutes) = vT

As per question

$$\frac{vT}{v-20} = 18$$

or vT = 18v – 18 × 20 .....(i)

and $\frac{vT}{v+20} = 6$

or vT = 6v + 20 × 6 ....(ii)

Equating (i) and (ii), we get

18v – 18 × 20 = 6v + 20 × 6

or 12v = 20 × 6 + 18 × 20 = 480 or v = 40 km h^{-1}

Putting this value of v in (i), we get

40T = 18 × 40 – 18 × 20 = 18 × 20

or $T = \frac{18\times 20}{40} = 9\,\text{min}$

15. (*a*) $\vec{a} = 3\hat{i}, \vec{b} = x\hat{i} + y\hat{i}$

$$a.b = |a||b|\cos\theta$$

$$\vec{a} = 3\hat{i} \qquad \vec{b} = 2\sqrt{3}\,\hat{i} + 2\hat{j}$$

$$\vec{c} = p(3\hat{i}) + q(2\sqrt{3}\,\hat{i} + 2\hat{j})$$

$$= (3p + q2\sqrt{3})\hat{i} + 2q\hat{j}$$

$$|\vec{c}| = \sqrt{(3p + 2q\sqrt{3})^2 - (2q)^2} = 10$$

only (1) satisfying it.

16. (*b*) Let $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{B} = \hat{i} - \hat{j} + 2\hat{k}$ unit vector perpendicular to both $\vec{A}$ and $\vec{B}$ is

$$n = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & -1 & 2 \end{vmatrix}$$

$$= \hat{i}(6+1) - \hat{j}(4-1) + \hat{k}(-2-3) = 7\hat{i} - 3\hat{j} - 5\hat{k}$$

$$\therefore |\vec{A}\times\vec{B}| = \sqrt{7^2 + (-3)^2 + (-5)^2} = \sqrt{83}\,\text{unit}$$

$$\therefore \hat{n} = \frac{1}{\sqrt{83}}(7\hat{i} - 3\hat{j} - 5\hat{k})$$

17. (*d*) Rate of collection (R) = vA cosθ (θ is the angle between the velocity of the rain and area of vessel)

R = vA cos0° = vA

When the wind blows R = v′A cosθ, (v is the new velocity of rain)

$$v' = \sqrt{v^2 + u^2}$$

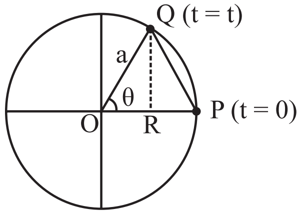
$$\cos\theta = \frac{v}{\sqrt{v^2 + u^2}}$$

∴ R = v′A cosθ = vA = R

18. (*b*) In time t, particle has rotated an angle,

$\theta = \omega t$

$s = PQ = \sqrt{QR^2 + PR^2}$



or $\quad s = \sqrt{(a\sin\omega t)^2 + (a - a\cos\omega t)^2}$

or $\quad s = 2a\sin\dfrac{\omega t}{2}$

NEET Past 10 Years Questions

1. (*c*) $T = \dfrac{2\pi R}{V}$

$u = \dfrac{2\pi R}{T}$

$H = \dfrac{u^2\sin^2\theta}{2g}$

$4R = \dfrac{4\pi^2R^2\sin^2\theta}{T^2 2g}$

$\sin^2\theta = \dfrac{8RT^2g}{4\pi^2R^2}$

$\sin\theta = \sqrt{\dfrac{2T^2g}{\pi^2R}}$

$\theta = \sin^{-1}\left(\dfrac{2T^2g}{\pi^2R}\right)^{1/2}$

2. (*c*) $u = 0$

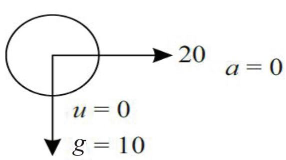
$a = 5$ m/s^2

$t = 4$ sec

$V = u + at$

$V = 0 + 5 \times 4$

$V = 20$ m/s



$V_x = 20$ m/sec

$V_y = u + gt \qquad [\because t = 6 - 4 = 2 \text{ sec}]$

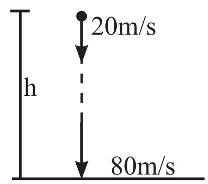
$= 10 \times 2$ m/s

$V_y = 20$ m/sec

$V = 20\sqrt{2}$

and $a = 10$ m/sec^2

3. (*c*)



$v^2 = u^2 + 2gh$

$80^2 = 20^2 + 2 \times 10h$

$h = 300$m

4. (*a*) From equation of motion

$S = ut + \dfrac{1}{2}at^2$

here S = 1.5 m $\quad$ t = 0.1s

$1.5 = u(0.1) + \dfrac{1}{2}(10)(0.1)(0.1)$

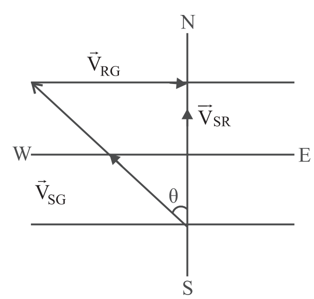
$\Rightarrow u = 14.5$ m/s

5. (*a*) $V_{SG} = 20$m/s

$V_{RG} = 10$m/s

For shortest path

$\vec{V}_{SG} + \vec{V}_{RG} = \vec{V}_{SR}$



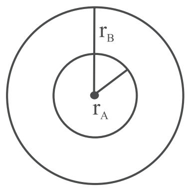
From vector triangle :-

$\sin\theta = \left|\dfrac{\overline{V}_{RG}}{\overline{V}_{SG}}\right|$

$$\sin\theta = \frac{10}{20}$$

$$\sin\theta = \frac{1}{2} = \theta = 30°\text{west}$$

6. (*d*)



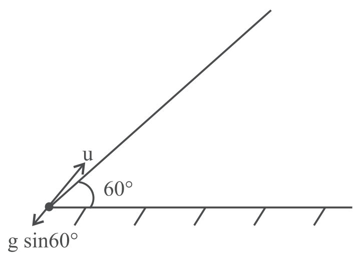
$$T_A = T_B = T$$

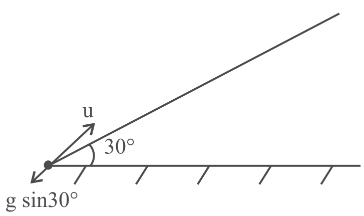
$$\omega_A = \frac{2\pi}{T_A}$$

$$\omega_B = \frac{2\pi}{T_B}$$

$$\frac{\omega_A}{\omega_B} = \frac{T_B}{T_A} = \frac{T}{T} = 1$$

7. (*c*)





(Stopping distance) $x_1 = \frac{u^2}{2g\sin 60°}$

(Stopping distance) $x_2 = \frac{u^2}{2g\sin 30°}$

$$\Rightarrow \frac{x_1}{x_2} = \frac{\sin 30°}{\sin 60°} = \frac{1\times 2}{2\times\sqrt{3}} = 1:\sqrt{3}$$

8. (*b*) $\vec{A}-\vec{B}$ will give us a new vector whose direction will be in the plane of A and B.

$\vec{A}\times\vec{B}$ will give us a new vector whose direction will be perpendicular to A and B.

Then the angle between $\vec{A}-\vec{B}$ and $\vec{A}\times\vec{B}$ will be 90°

9. (*c*) $T_1 = T_2$

V_y = Same for both cases

$$H = \frac{V_y^2}{2g}$$

$H_1 = H_2$ since all are same for both cases

10. (*b*) $\left|\vec{A}+\vec{B}\right| = \left|\vec{A}-\vec{B}\right|$

$A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$

$4AB\cos\theta = 0$

$\cos\theta = 0$

$\theta = 90°$

11. (*c*) $\vec{r} = \cos\omega t\hat{x} + \sin\omega t\hat{y}$

$\hat{v} = -\omega\sin\omega t\hat{x} + \omega\cos\omega t\hat{y}$

$\vec{a} = -\omega^2\cos\omega t\hat{x} + \omega^2(-\sin\omega t)\hat{y}$

$= -\omega^2\vec{r}$

$\vec{r}.\vec{v} = 0$ hence $\vec{r}\perp\vec{v}$

12. (*a*) Particle at periphery will have both radial and tangential acceleration

$a_t = R\alpha = 0.5\times 2 = 1\ \text{ms}^{-2}$

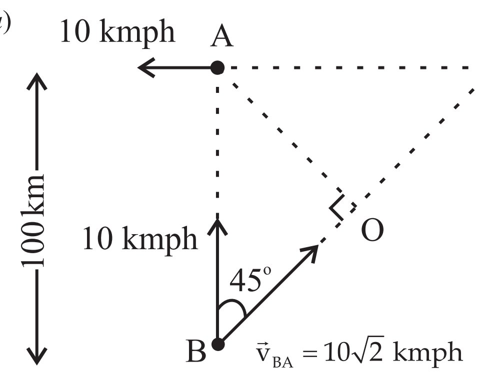
$\omega = \omega_0 + \alpha t$

$\omega = 0 + 2\times 2 = 4\ \text{rad/sec}$

$a_c = \omega^2 R = (4)^2\times 0.5 = 16\times 0.5 = 8\ \text{ms}^{-2}$

$a_{total} = \sqrt{a_t^2 + a_c^2} = \sqrt{1^2+8^2} \approx 8\ \text{ms}^{-2}$

13. (*a*)



$$\left|\vec{v}_{BA}\right| = \sqrt{10^2+10^2} = 10\sqrt{2}\ \text{kmph}$$

Distance $OB = 100\cos 45° = 50\sqrt{2}\ \text{km}$

Time taken to reach the shortest distance between

$$A\ \&\ B = \frac{50\sqrt{2}}{\left|\vec{v}_{BA}\right|} = \frac{50\sqrt{2}}{10\sqrt{2}} = 5\ \text{h}$$

14. (*d*) $\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$

$$\vec{v} = \frac{d\vec{R}}{dt} = 8\pi\cos 2\pi t\hat{i} - 8\pi\sin 2\pi t\hat{j}$$

$$|\vec{v}| = \sqrt{[8\pi\cos(2\pi t)]^2 + [-8\pi\sin(2\pi t)]^2}$$
$$= \sqrt{64\pi^2}$$
$$= 8\pi \text{ m/s}$$

$\therefore$ statement in option (d) is wrong

15. (*d*)

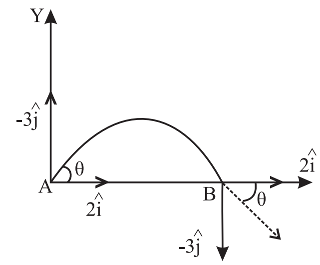
$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(13-2)\hat{i} + (14-3)\hat{j}}{5-0} = \frac{11}{5}(\hat{i} + \hat{j})$$

16. (*a*) As Range $= \dfrac{u^2\sin^2\theta}{g}$ so, $g \propto u^2$

Therefore $g_{planet} = \left(\dfrac{3}{5}\right)^2 (9.8 \text{ ms}^{-2}) = 3.5 \text{ ms}^{-2}$

17. (*d*) In a projectile vertical component of velocity keeps on changing with time.

While horizontal velocity component remains constant



$\therefore$ Velocity is $2\hat{i} - 3\hat{j}$

18. (*b*) $\vec{v} = \vec{u} + \vec{a}t = (2\hat{i} + 3\hat{j}) + (0.3\hat{i} + 0.2\hat{j})(10) = 5\hat{i} + 5\hat{j}$

Hence $|\vec{v}| = 5\sqrt{2}$ units

19. (*b*) $R = H_{max} \Rightarrow \dfrac{2u^2\sin\theta\cos\theta}{g} = \dfrac{u^2\sin^2\theta}{2g}$

$\Rightarrow \tan\theta = 4 \;\Rightarrow\; \theta = \tan^{-1}(4)$