2.2 CHAPTER

Progression

Talking about a series 1, 3, 5, 7, 9....... . In this series every term is made by adding a fixed value in the previous term. So it can be stated as the series is in the form of a, (a + d), (a + 2d), ... This sort of series is called as Arithmetic Series and Term 'a' is first term while 'd' is the common difference b/w two consecutive terms.

Basics of Arithmetic Progression

- Finding out the n^{th} term of an AP
  $T_n = a + (n - 1)d \rightarrow n^{th}$ term,
  $T_n = a + (n - 1)d$
- Sum of n terms of on AP

$$S_n = \Sigma T_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$= \frac{n}{2}\left[a+a+(n-1)d\right]$$

$$= \frac{n}{2}\left[a+T_n\right]$$

$$\text{So } S_n = \frac{n}{2}\left[a+T_n\right] = \frac{n}{2}\left[2a+(n-1)d\right]$$

Properties of Arithmetic Progression

- Application of numbers which are in AP
  (a) If three numbers are in AP then they can be assumed as a – d, a, a + d
  (b) If four numbers have to be assumed then they can be a – 3d, a – d, a + d, a + 3d
  (c) If five numbers have to be assumed then they will be (a – 2d), (a – d), a, (a + d), (a + 2d)

  If in an AP sum of n terms is 'm' and sum of 'm' terms is 'n' then sum of (m + n) terms will be "–(m + n)"

Example 1.

For an AP sum of 2 terms is '4' and sum of '4' terms is '2'. Find out the sum of 6 term of that 'AP.

Solution.

Let the first term be 'a' and common difference be 'd' the

$$\frac{2}{2}\left[2a+(2-1)d\right] = 4$$

$$\Rightarrow \quad 2a + d = 4 \quad \text{...(i)}$$

$$\text{and } \frac{4}{2}\left[2a+(4-1)d\right] = 2$$

$$\Rightarrow \quad 4a + 6d = 2 \quad \text{...(ii)}$$

By solving (i) & (ii) we get,

$$d = \frac{3}{2} \text{ and } a = \frac{11}{4}$$

So sum of 6 terms

$$= \frac{6}{2}\left[2\times\frac{11}{4}+(6-1)\left(\frac{-3}{2}\right)\right]$$

$$= \frac{6}{2}\left[\frac{11}{2}-\frac{15}{2}\right]$$

$$= -6$$

- If in an AP m^{th} term is 'n' & n^{th} term is 'm' then
  (a) The common difference of such AP will be '–1'
  (b) The $(m + n)^{th}$ term of the AP will be 'zero'
  (c) The first term of the AP will be '(m + n –1)'

Example 2.

Talk about any AP of common difference – 1.

7, 6, 5, 4, 3, 2, 1, 0
Its 2^{nd} term is 6
and 6^{th} term is 2
then 8^{th} term will be '0'
and the first term is 6 + 2 – 1 = '7'

- If same number is added or subtracted from each term of the AP. The new series will again be an AP.

Example 3.

If 2, 6, 10, 14, 18, ... is an AP and a number 5 is either added or subtracted, then the new series formed are

7, 11, 15, 19, 23, ...

–3, 1, 5, 9, 13, ... [Both series are AP]

- If same number is multiplied in all terms of AP, the new series formed will be an AP.

Example 4.

Take series 1, 3, 5, 7, 9.......

Multiplying it by 2 we get

2, 6, 10, 14, 18, ... This series is again an AP

- If sum of m terms of an AP is equal to sum of n terms of the same AP then the AP will have

$\left(\frac{m+n+1}{2}\right)^{th}$ term as 'zero'

It is valid only when (m + n) is odd

When m + n is even

There is no possibility that the term will be zero i.e. in that A.P. No term can be zero. Let us understand it with one example and discuss both the cases.

Example 5.

The sum of first 8 terms of an AP is equal to the sum of first (a) 15 terms (b) 16 terms then which term in the AP will have 'zero' value.

(a) $S_8 = S_{15}$

$$\Rightarrow \frac{8}{2}[2a+7d] = \frac{15}{2}[2a+14d]$$

$\Rightarrow$ 8a + 28d = 15a + 105d

$\Rightarrow$ 7a = –77d

$\Rightarrow$ a = –11d

So –11d + (m – 1)d = 0

$\Rightarrow$ n = 12

(b) $S_8 = S_{16}$

$$\Rightarrow \frac{8}{2}[2a+7d] = \frac{16}{2}[2a+16d]$$

$\Rightarrow$ 2a + 7d = 4a + 30d

$$\Rightarrow a = \frac{-23}{2}d$$

$$\text{So } \frac{-23}{2}d + (n-1)d = 0$$

$$\Rightarrow n = \frac{25}{2}$$

Which is not possible.

- If two APs are compared & the ratio of T_m of first AP to T_n of the second AP is P : q then they will have the ratio of sums determinable only in one case & that will be

$$\frac{S_{2m-1}}{S_{2n-1}} = \frac{2m-1}{2n-1}\left(\frac{T_m}{T_n}\right)$$

Some Important Applications

- Sum of first n natural numbers

$$S_n = \Sigma n = \frac{n(n+1)}{2}$$

- Sum of the squares of first n natural numbers

$$S_{n^2} = \frac{n(n+1)(2n+1)}{6}$$

- Sum of the cubes of first n natural no.

$$S_{n^3} = \left\{\frac{n(n+1)}{2}\right\}^2$$

So $S_{n^3} = (S_n)^2$

Geometric Series

Consider a series 2, 4, 8, 16, 32... . In this series, we see every term in the series is obtained by multiplying the precedent term by a fixed value. This sort of series are called as geometric series. So a geometric series is in the form.

a, ar, ar^2, ar^3, ar^4 .... & so on.

Here r is the common ratio & a is the first term.

Basics of GP

- N^{th} term of the GP is = ar^{n-1}
- Sum of n terms

$$S_n = a\left(\frac{r^n-1}{r-1}\right) \qquad \text{when } r > 1$$

$$S_n = a\left(\frac{1-r^n}{1-r}\right) \qquad \text{when } r < 1$$

Infinite GP

This is a geometric series with its number of terms as infinite. The sum of infinite terms of GP can be calculated as

$$S_\infty = \infty \qquad \text{(when } r > 1)$$

$$S_\infty = \frac{a}{1-r} \qquad \text{(Wen } r < 1)$$

Basic Properties of GP

Assumption of number of terms in geometric progression is same as of the AP for three terms will be

assume $\frac{a}{r}$, a, ar, for four terms $\frac{a}{r^3}$, $\frac{a}{r}$, ar, ar^3.

Harmonic Series

If all the terms of an arithmetic series are reversed then the series obtained will be harmonic series. Let us take a series 1, 3, 5, 7, 9, ... It is an arithmetic series, if we are going to reverse the term it means the series $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \ldots$ This is harmonic series.

Basics of HP

The n^{th} term of a harmonic series can be calculated only by finding out the n^{th} term of the arithmetic series obtained by inversing the harmonic series and then again reversing the term.

Example.

Find out the n^{th} term for the series

$\frac{1}{5}, \frac{1}{11}, \frac{1}{17}, \frac{1}{23}, \ldots$

The terms are reversed then the AP obtained is 5, 11, 17, 23, ...

So n^{th} term of HP is $\frac{1}{6n-1}$

Basic Means

For the given terms there are three types of means

- Arithmetic mean
- geometric mean
- Harmonic mean

Arithmetic Mean

For n terms $a_1, a_2, a_3, a_4, \ldots a_n$ the AM will be

$$\frac{a_1 + a_2 + a_3 + \ldots a_n}{n}$$

Geometric Mean

Geometric mean is the term which when included, makes all the term in GP. Let us say there are two terms a and b then GM will be in such a way that a, GM, be will be in GP

So $GM = \sqrt{(a, b)}$

or it can be written as

- GM (for two terms = $(a.b)^{1/2}$

If there are n terms $a_1, a_2, a_3, a_4, \ldots a_n$ then

- $GM = (a_1.a_2.a_3, \ldots . a_n)^{1/n}$

Harmonic Mean

Harmonic mean is the term when included between the terms make all the terms in harmonic progression. The harmonic mean for the two terms a and b is

$$HM = \frac{2ab}{a+b}$$

Relation between AM, GM, HM

When compared for two fixed term, the arithmetic mean is having the highest value.

$AM \geq GM \geq HM$

- When taken for two terms a and b, arithmetic mean, geometric mean and harmonic mean
  $HM \times AM = GM^2$

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Solved Examples

1. If in an AP 10^{th} term is 86 and 86^{th} term is 10, then what will be the 100^{th} term?

(a) 1 (b) 3
(c) 4 (d) –4

Ans. (d)

$$a + 9d = 86$$

$$\Rightarrow \quad -76d = 76 \text{ or } d = -1$$

$$a + 85d = 10$$

substituting the values

We get $a = 5, d = -1$

So 100^{th} term $T_{100} = 95 + (100 - 1)(-1)$

$\Rightarrow \quad T_{100} = -4$

hence option (d)

Alternate: For this series from the property of AP. We can say that 96^{th} term is 0 and $d = -1$ so 100^{th} term = –4.

2. Given that a function S ={2, 3, 4,..... 2n + 1} and X is average of all the odd integers of S while Y is average of all the even integers of S, find out what is (X – Y)

(a) 1 (b) n – 1
(c) n (d) None of these

Ans. (a)

$$X = \frac{3+5+7+\ldots.2n+1}{\text{No. of terms}}$$

$$= \frac{3+(2n+1)}{2} = n+2$$

$$Y = \frac{2+4+6+\ldots.2n}{\text{No. of terms}}$$

$$= \frac{2+2n}{2} = n+1$$

So X – Y = 1

Hence option (a)

Alternate: Difference of 2 consecutive terms i.e. odd term and even term = 1 so difference of avg. of odd and avg. of even = 1

i.e. $x - y = 1$.

3. The number of terms between 30 to 530, which are divisible by 11 are

(a) 32 (b) 35

(c) 36 (d) 46

Ans. (d)

The first term divisible by 11 = 33

The last term divisible by 11 = 528

No. of terms = $\frac{528-33}{11} + 1 = 46$

Hence option (d).

4. Split 74 into four parts in such a manner that all parts are in AP and the product of first part and last part is 2 less than the product of second part & third part.

Sol.

Let the four parts be

$a - 3d, a - d, a + d, a + 3d$

no $a - 3d + a - d + a + d + a + 3d = 74$

$\Rightarrow a = \frac{37}{2}$

and $(a - 3d)(a + 3d) + 2 = (a - d)(a + d)$

$\Rightarrow a^2 - 9d^2 + 2 = a^2 - d^2$

$\Rightarrow 8d^2 = 2 \Rightarrow d = \pm 1/2$

So the parts are 17, 18, 19, 20

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Progression

I Practice Exercise: I

1. How many terms are there in the AP 20, 25, 30, ...130.

(a) 22 (b) 23

(c) 21 (d) 24

2. Find the 1st term of an AP whose 8th and 12th terms are respectively 39 and 59.

(a) 5 (b) 6

(c) 4 (d) 3

3. Find the 15th term of the sequence 20, 15, 10, ...

(a) –45 (b) –55

(c) –50 (d) 0

4. A number 15 is divided into three parts which are in AP and the sum of their squares is 83. Find the smallest number.

(a) 5 (b) 3

(c) 6 (d) 8

5. The sum of the first 16 terms of an AP whose first term and third are 5 and 15 respectively is

(a) 600 (b) 765

(c) 640 (d) 680

6. A boy agrees to work at the rate of one rupee on the first day, two rupees on the second day, four rupees on the third day and so on. How much will the boy get if he starts working on the 1st of February and finishes on the 20th of February?

(a) 2^{20} (b) $2^{20} - 1$

(c) $2^{19} - 1$ (d) 2^{19}

7. The seventh term of a GP is 8 times the fourth term. What will be the first term when it's fifth term is 48?

(a) 4 (b) 3

(c) 5 (d) 2

8. The sum of three numbers on a GP is 14 and the sum of their squares is 84. Find the largest number.

(a) 8 (b) 6

(c) 4 (d) None of these

9. How many natural numbers between 300 to 500 are multiples of 7?

(a) 29 (b) 28

(c) 27 (d) None of these

10. The 4th and 10th term of an GP are 1/3 and 243 respectively. Find the 2nd term.

(a) 3 (b) 1

(c) 1/27 (d) 1/9

11. The 7th and 21st terms of an AP are 6 and –22 respectively. Find the 26th term.

(a) –34 (b) –32

(c) –12 (d) –10

12. Find the number of terms of the series 1/81, –1/27, 1/9, ... –729.

(a) 11 (b) 12
(c) 10 (d) 13

13. 'a' and 'b' are two number whose AM is 25 and GM is 7. Which of the following may be value of 'a'?
(a) 10 (b) 20
(c) 49 (d) 25

14. Two number A and B are such that their GM is 20% lower than their AM. Find the ratio between the numbers.
(a) 3 : 2 (b) 4 : 1
(c) 2 : 1 (d) 3 : 1

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Solutions

1. Ans. (b)
In series 20, 25, 30 ....130.
$a = 20$, $d = 5$
n^{th} term is 130
$\Rightarrow 20 + (n-1)5 = 130$
$\Rightarrow 5n = 115, n = 23$

2. Ans. (c)
Here $a + 7d = 39$ ...(i)
$a + 11d = 59$ ...(ii)
So, $4d = 20$
$d = 5$
$a + (7 \times 5) = 39$
So, $a = 4$

3. Ans. (c)
AP 20, 15, 10
Here $a = 20$
$d = -5$
15^{th} term will be $a + 14d$ which is equal to
$20 + (-5) \times 14 = -50$

4. Ans. (b)
Let numbers are
$a - d$, a and $a + d$
now it is given that
sum = 15 & sum of there square is 83 i.e.
$3a = 15$ ...(i)
$(a-d)^2 + a^2 + (a+d)^2 = 83$ ...(ii)
$3a^2 + 2d^2 = 83$
$75 + 2d^2 = 83$
$2d^2 = 8$
$d^2 = 4, d = 2$
So, least term is $a - d = 3$

5. Ans. (d)
$a = 5$, $a + 2d = 15$
So $d = 5$
now sum of 16 term will be
$16/2\,[10 + 15 \times 5] = 680$

6. Ans. (b)
This sequence is in GP 1, 2, 4, 8, ...
Now sum of first 20 term will be
$$\frac{a(r^n - 1)}{r-1} = 2^{20} - 1$$

7. Ans. (b)
It is given that
$ar^6 = 8\,ar^3$
$r^3 = 8$, $r = 2$
Now $ar^4 = 48$
$a \times 2^4 = 48$, $a = 3$

8. Ans. (a)
a, ar, ar^2
now $a + ar + ar^2 = 14$ ...(i)
also $a^2 + a^2r^2 + a^2r^4 = 84$ ...(ii)
only suitable combination is 2, 4, 8
So largest term is 8

9. Ans. (a)
Between 300 and 500 the first term divisible by 7 is 301 and last term divisible by 7 is 497
So, here $a = 301$, $d = 7$
$L = a + (n-1)\,d$
$497 = 301 + (n-1) \times 7$
$\Rightarrow 7n = 203$
$\Rightarrow n = 29$

10. Ans. (c)
It is given that
$ar^3 = 1/3$ ...(i)
$ar^9 = 243$ ...(ii)
From (i) and (ii) we get
$r^6 = 729$
$r = 3$ ...(iii)
From (i) and (iii) we get
$$a \times 3^3 = \frac{1}{3}, \qquad a = \frac{1}{81}$$
second term will be $\frac{1}{27}$

11. Ans. (b)

It is given that

$a + 6d = 6$ and

$a + 20d = -22$

$\Rightarrow 14d = -28, \quad d = -2,$

$\Rightarrow a = 18$

26^{th} term will be –32

12. Ans. (a)

It is given that

GP is $\frac{1}{81}, \frac{-1}{27}, \frac{7}{9}, \ldots\ldots\ldots\ldots -729$

Here $a = \frac{1}{81}, \quad r = -3$

Now $ar^{n-1} = 729$

$\frac{1}{81} \times (-3)^{n-1} = -729$

$(-3)^{n-1} = 3^{10}, n = 11$

13. Ans. (c)

$\frac{a+b}{2} = 25$

$a + b = 50$

also $\sqrt{ab} = 7$

$ab = 49$

here $a = 1$ & $b = 49$ or vice versa

14. Ans. (b)

$\frac{a+b}{2} = AM$

$\sqrt{ab} = GM$

Here it is given that

GM = 20% less than AM, that is

$GM = \frac{4}{5} AM$

$\Rightarrow \sqrt{ab} = \frac{4}{5} \times \frac{a+b}{2}$

only possibility is $a = 1$ & $b = 4$

or $a = 4$, $b = 1$

So, ratio is 4 : 1

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Progression

II Practice Exercise: II

1. Determine k so that $\frac{2}{3}$, k, and $\frac{5}{8}$k are three consecutive terms of an A.P.
   (a) 16/33 (b) 14/33
   (c) 12/33 (d) 18/33

2. If 7 time the 7^{th} term of an A.P. is equal to 11 times its, 11^{th} term, then the 18^{th} term of the A.P. is
   (a) 1 (b) 2
   (c) 0 (d) 3

3. If the p^{th}, q^{th} and rth terms of an A.P. are a, b, c, respectively, find the value of
   $a(q - r) + b(r - p) + c(p - q)$
   (a) 2 (b) 1
   (c) 0 (d) 3

4. A ball rolling up an incline covers 36 metres during the first second, 32 metres during the second, 28 metres during the next and so on. How much distance will it travel during the 8^{th} second?
   (a) 8 metres (b) 6 metres
   (c) 7 metres (d) 9 metres

5. The sum of a series in A.P. is 525. Its first term is 3 and last term is 39. Find the common difference.
   (a) 3/2 (b) 3/3
   (c) 2/3 (d) 1/3

6. The sum of p terms of an A.P. is $3p^2+4p$. Find the n^{th} term?
   (a) 5n + 2 (b) 6n + 1
   (c) 8n + 3 (d) 7n + 3

7. The 5^{th} term of a G.P. is 2, find the product of first 9 terms.
   (a) 508 (b) 512
   (c) 504 (d) 516

8. The 3^{rd} term of a G.P. is the square of the first term. If the second term is 8, determine the 6^{th} term.
   (a) 136 (b) 132
   (c) 128 (d) 124

9. The sum of first three terms of a G.P. is to the sum of first six terms is 125 : 152. Find the common ratio of G.P.

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

(c) $\frac{3}{5}$ (d) $\frac{1}{5}$

10. Evaluate $\sum_{j=1}^{11}(2+3^j)$

(a) $22+\frac{3}{2}(3^{11}-1)$ (b) $11+\frac{3}{2}(3^{11}-1)$

(c) $22+\frac{3}{2}(3^{10}-1)$ (d) None of these

11. The common ratio of a G.P. is $-\frac{4}{5}$ and the sum to infinity is $\frac{80}{9}$. Find the first term.

(a) 14 (b) 16
(c) 12 (d) 10

12. Sum the series to infinity

$\frac{3}{4}-\frac{5}{4^2}+\frac{3}{4^3}-\frac{5}{4^4}+\frac{3}{4^5}-\frac{5}{4^6}+$............

(a) $\frac{8}{15}$ (b) $\frac{7}{17}$

(c) $\frac{7}{15}$ (d) $\frac{8}{17}$

13. Find the 9th term of the H.P. 6, 4, 3, ...
(a) 7/5 (b) 6/5
(c) 5/6 (d) None of these

14. If $x > 1$, $y > 1$, $z > 1$ are in G.P., then $\frac{1}{1+\log x}, \frac{1}{1+\log y}, \frac{1}{1+\log z}$ are in
(a) A.P. (b) H.P.
(c) G.P. (d) None of these

15. If a be the first term of a G.P., l the nth term and P the product of first n terms then P =
(a) $(al)^{n/2}$ (b) $(a-l)^{n/2}$
(c) $(a+l)^{n/2}$ (d) None of these

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Solutions

1. Ans. (a)

$\because \frac{2}{3}, k, \frac{5}{8}k$ are in A.P.

$\therefore k-\frac{2}{3}=\frac{5}{8}k-k \Rightarrow \frac{5k}{8}-2k=\frac{-2}{3}$

$\Rightarrow \frac{-11k}{8}=\frac{-2}{3} \Rightarrow k=\frac{16}{33}.$

2. Ans. (c)

Let a be the first term and d, the common difference of an A.P.

$\therefore a_7 = a + 6d$

$a_{11} = a + 10d \quad \therefore 7a_7 = 11a_{11}$

$7(a + 6d) = 11(a + 10d)$

$\Rightarrow 7a + 42d = 11a + 110d$

$\Rightarrow 4a + 68d = 0$

$\Rightarrow a + 17d = 0$

$\Rightarrow a_{18} = 0$

3. Ans. (c)

Let A be the first term and D, the common difference of A.P.

$a_p = a, \therefore A + (p-1)D = a$ ....(i)

$a_{q=b}, \therefore A + (q-1)D = b$ ....(ii)

$a_r = c, \therefore A+(r-1)D = c$ ....(iii)

$\therefore a(q-r) + b(r-p) + c(p-q)$

$= [A + (p-1)D](q-r) + [A+(q-1)D](r-p)$

$+[A+(r-1)D](p-q)$

$=(q-r+r-p+p-q)A+[(p-1)(q-r)$

$+(q-1)(r-p)+(r-1)(p-q)]D$

$= 0\cdot A + 0\cdot D = 0$

4. Ans. (a)

Distance covered during the Ist second = 36 m

Distance covered during the 2nd second = 32 m

Distance covered during the 3rd second = 28 m.

The distance covered form an A.P.

= 36 + 32 + 28 ..... in which

a = 36, d = –4

∴ Distance covered in 8th second

$= 8^{th}$ term of the A.P.
$= a + 7d = 36 + 7(-4)$
$= 36 - 28 = 8$ metres.

5. Ans. (a)
If n be the number of terms, then
$a_n = a + (n-1)d$,
where a is the first term and d the common difference.
$\therefore\ 39 = 3 + (n-1)d.$

Also $S_n = \frac{n}{2}[a_1 + a_n]$

$\therefore\ 525 = \frac{n}{2}[3+39] \Rightarrow 1050 = n(42)$

or, $n = \frac{1050}{42} = 25$

putting n = 25 in (i), we get

$(25-1)d = 36 \Rightarrow d = 36 \div 24 = \frac{3}{2} = 1\frac{1}{2}$

6. Ans. (b)
Here, $S_p = 3p^2 + 4p$
Putting p = n, we have
$S_n = 3n^2 + 4n$
Changing n to (n − 1), we get
$S_{n-1} = 3(n-1)^2 + 4(n-1)$
$= 3(n^2 - 2n + 1) + 4n - 4$
$= 3n^2 - 2n - 1$
$\therefore\ a_n = s_n - S_{n-1}$
$= 3n^2 + 4n - 3n^2 + 2n + 1 = 6n + 1.$

7. Ans. (b)
Let a be the first term and r the common ratio
$\therefore\ a^5 = 2 \Rightarrow ar^4 = 2$ ...(i)
Now, product of first 9 terms
$= a \times ar \times ar^2 \times \ldots\ ar^8$
$= a^9 r^{1+2+\ldots+8} = a^9 r^{36}$
$= (ar^4)^9 = 2^9 = 512.$

8. Ans. (c)
Let a be the first term and r be the common ratio of G.P.
We have $a_3 = (a_1)^2 \Rightarrow ar^2 = a^2$
$\Rightarrow r^2 = a$ ...(i)
Also, $a_2 = 8$
$\Rightarrow ar = 8$ ...(ii)
Multiplying (i) and (ii) we get
$ar^3 = 8 \times a$
$\therefore\ r^3 = 8 \Rightarrow r = 2$
From (i) $a = (2)^2 = 4$
Hence, $a^6 = ar^5 = (4)(2)^5$
$= 4 \times 32 = 128.$

9. Ans. (c)

Here, $\frac{S_3}{S_6} = \frac{125}{152}, \frac{a(r^3-1)/(r-1)}{a(r^6-1)/(r-1)} = \frac{125}{152}$

$\Rightarrow \frac{r^3-1}{r^6-1} = \frac{125}{152},\ \therefore\ \frac{r^3-1}{(r^3-1)(r^3+1)} = \frac{125}{152}$

$\Rightarrow \frac{1}{r^3+1} = \frac{125}{152},\ \therefore\ 152 = 125r^3 + 125$

$\Rightarrow 125r^3 = 27, \Rightarrow r^3 = \frac{27}{125}.$

or, $r^3 = \left(\frac{3}{5}\right)^3,\ \therefore\ r = \frac{3}{5}.$

Hence, the common ratio of G. P. is $\frac{3}{5}$.

10. Ans. (a)
$(2+3^1) + (2+3^2) + (2+3^3) + .. + (2+3^{11})$
$= 2 + 2 + 2 + \ldots$ up to 11 terms
$+(3 + 3^2 + 3^3 + \ldots$ up to 11 terms)

$= 11 \times 2 + \frac{3(3^{11}-1)}{3-1} = 22 + \frac{3}{2}(3^{11}-1)$

11. Ans. (b)

$S_\infty = \frac{a}{1-r} \Rightarrow \frac{80}{9} = \frac{a}{1-\left(\frac{-4}{5}\right)} \Rightarrow \frac{80}{9} = \frac{a}{9/5}$

$\Rightarrow a = \frac{80}{9} \times \frac{9}{5} = 16$

Hence, the first term is 16.

12. Ans. (c)

$\left(\frac{3}{4} + \frac{3}{4^3} + \frac{3}{4^5} + \ldots\text{to } \infty\right)$

$-\left(\frac{5}{4^2} + \frac{5}{4^4} + \frac{5}{4^6} + \ldots\text{to } \infty\right)$

$= \frac{3/4}{1-\left(\frac{1}{4}\right)^2} - \frac{5/4^2}{1-\left(\frac{1}{4}\right)^2}$

$$= \frac{3}{4} \times \frac{16}{15} - \frac{5}{16} \times \frac{16}{15} = \frac{4}{5} - \frac{1}{3}$$

$$= \frac{12-5}{15} = \frac{7}{15}.$$

13. Ans.(b)

The given sequence is 6, 4, 3, ... which is H..P. The sequence of reciprocals of its terms is

$\frac{1}{6}, \frac{1}{4}, \frac{1}{3}$........ which is an A,.P

Here, $a = \frac{1}{6}$, $d = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

$\therefore$ a_9 of A.P. = a + 8d

$$= \frac{1}{6} + 8 \times \frac{1}{12} = \frac{1}{6} + \frac{4}{6} = \frac{5}{6}$$

$\therefore$ 9^{th} term of H.P. is $\frac{6}{5}$.

14. Ans. (b)

$\because$ x, y, z are in G.P.

$\because$ $y^2 = xz$

Taking log on both sides

$2 \log y = \log x + \log z$

$\Rightarrow$ $2 + 2\log y = (1+\log x) + (1+\log z)$

$\Rightarrow$ $2(1+\log y) = (1+\log x) + (1 + \log z)$

$\Rightarrow$ $1+\log x$, $1+\log y$, $1+\log z$, are in A.P.

$\Rightarrow$ $\frac{1}{1+\log x}, \frac{1}{1+\log y}, \frac{1}{1+\log z}$ are in H.P.

15. Ans. (a)

If r is the common ratio of G.P., then

$l = a r^{n-1}$ ...(i)

The first n terms of the G.P. are

$a, ar, ar^2, ar^3, \ldots\ldots ar^{n-1}$

$P = a \times ar \times ar^2 \times \ldots \times ar^{n-1}$

$= a^n \times r^{1+2+3\ldots+(n-1)}$

$$= a^n \times r^{(n-1)\times\frac{n}{2}} = \left(a^2\right)^{n/2} \times \left(r^{n-1}\right)^{n/2}$$

$$= \left(a^2 r^{n-1}\right)^{n/2} = \left(a.\ ar^{n-1}\right)^{n/2} = (al)^{n/2}.$$

❑❑❑❑

III Practice Exercise: III

1. If the sum of the 6^{th} and the 15^{th} elements of an arithmetic progression is equal to the sum of the 7^{th}, 10^{th} and 12^{th} elements of the same progression, then which element of the series should necessary be equal to zero?
   (a) 10^{th} (b) 8^{th}
   (c) 1^{st} (d) None of these

2. If p, q, r, s are in harmonic progression and p > s, then
   (a) $\frac{1}{ps} < \frac{1}{qr}$ (b) $q + r = p + s$
   (c) $\frac{1}{q} + \frac{1}{r} = \frac{1}{p} + \frac{1}{s}$ (d) None of these

3. If $\log_x a$, $a^{x/2}$ and $\log_b x$ are in GP, then x is
   (a) $\log_a(\log_b a)$
   (b) $\log_a(\log_e a) + \log_a(\log_e b)$
   (c) $-\log_a(\log_a b)$
   (d) $\log_a(\log_e b) - \log_a(\log_e a)$

4. A person pays Rs. 975 in monthly installments, each monthly installment being less than the former by Rs. 5. The amount of the first installment is Rs. 100. In what tune, will the entire amount be paid?
   (a) 12 months (b) 26 months
   (c) 15 months (d) 18 months

5. Let S_n denote the sum of the first 'n' terms of an A.P. $S_{2n} = 3S_n$. Then, the ratio S_{3n}/S_n is equal to
   (a) 4 (b) 6
   (c) 8 (d) 10

6. Three numbers are in G.P. Their sum is 28 and their product is 512. The numbers are
   (a) 6, 9 and 13 (b) 4, 8 and 16
   (c) 2, 8 and 18 (d) 2, 6 and 18

7. The sum of the series:
   $1^2 + 2^2 + 3^2 + 4^2 + \ldots + 15^2$ is
   (a) 1080 (b) 1240
   (c) 1460 (d) 1620

8. If $\frac{1}{b} - \frac{1}{c} = \frac{1}{a} - \frac{1}{b}$, then a, b, c form a/an
   (a) Arithmetic progression
   (b) Geometric Progression

(c) Harmonic Progression
(d) None of these

9. The value of
$(1^3 + 2^3 + 3^3 + ... 15^3) - (1 + 2 + 3 + ... 15)$ is
(a) 14280 (b) 14400
(c) 12280 (d) 13280

10. The mean of the cubes of the first n natural numbers is

(a) $\frac{n(n+1)^2}{4}$ (b) n^2

(c) $\frac{n(n+1)(n+2)}{8}$ (d) (n^2+n+1)

11. If $\frac{3+5+7+....+n \text{ terms}}{5+8+11+...+10 \text{ terms}} = 7$, then the value of n is
(a) 35 (b) 36
(c) 37 (d) 40

12. If the sum of first n natural numbers is one-fifth of the sum of their squares, then *n* is
(a) 5 (b) 6
(c) 7 (d) 8

13. What is the least value of *n* such that $(1 + 3 + 3^2 + ... + 3^n)$ exceeds 2000?
(a) 7 (b) 5
(c) 8 (d) 6

14. The sum of 12 terms of an A.P., whose first term is 4, is 252. What is the last term?
(a) 35 (b) 36
(c) 37 (d) 38

15. If log 2, $\log(2^x - 1)$ and $\log(2^x + 3)$ (all to the base 10) be three consecutive terms of an Arithmetic Progression, then the value of x is equal to
(a) 0 (b) 1
(c) $\log_2 5$ (d) $\log_{10} 2$

16. The third term of a Geometric Progression is 4. The product of the first five terms is
(a) 4^3 (b) 4^5
(c) 4^4 (d) None of these

□□□□

Solutions

1. **Ans. (b)**
Let a be the first term and d be the common difference of an A.P.

$\therefore \quad (a+5d)+(a+14d)$
$= (a + 6d) + (a + 9d) + (a + 11d)$
$\Rightarrow a + 7d = 0$
$\Rightarrow$ 8th term = 0.

2. **Ans. (c)**
p, q, r, s are in harmonic progression

$\Rightarrow \frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ and $\frac{1}{s}$ are in A.P.

$\Rightarrow \frac{1}{q} - \frac{1}{p} = \frac{1}{s} - \frac{1}{r}$

$\Rightarrow \frac{1}{q} + \frac{1}{r} = \frac{1}{p} + \frac{1}{s}$.

3. **Ans. (a), (c)**
Given statement

$\Rightarrow \left(a^{x/2}\right)^2 = (\log_b x)\times(\log_x a)$

$= a^x = \log_b a$

$\Rightarrow x \log a = \log_a[\log_b a]$

$\Rightarrow x = \log_a[\log_b a]$.

4. **Ans. (c)**
Let n be the number of months in which all the installments can be paid
First Installment = Rs. 100
Common Difference = –5
$\Rightarrow$ Sum of the series with *n* terms whose first term is 100 or common difference is (–5) = 975

i.e. $\frac{n}{2}[2a+(n-1)d] = 975$

i.e. $\frac{n}{2}[2\times 100+(n-1)(-5)] = 975$

i.e. $n^2 - 41n + 390 = 0$
i.e. n = 26 or n = 15
For n=15, total amount paid

$= \frac{15}{2}[2\times 100+(15-1)(-5)]$

$= \frac{15}{2}[200-70] = 975.$

5. **Ans.(b)**

$$S_n = \frac{n}{2}[a+(n-1)d]$$

[where a is the first term and d is the common difference]

$$S_{2n} = \frac{2n}{2}[a+(2n-1)d]$$

$$S_{3n} = \frac{3n}{2}[a+(3n-1)d]$$

Given, $S_{2n} = 3S_n$

$$\Rightarrow \quad n[a+2nd-d] = 3\left[\frac{n}{2}(a+nd-d)\right]$$

$$\Rightarrow \quad d = \frac{a}{1+n}$$

$$\therefore \quad \frac{S_{3n}}{S_n} = \frac{\frac{3n}{2}[a+3nd-d]}{\frac{n}{2}[a+nd-d]}$$

$$= \frac{3\left[a+\frac{3na}{1+n}-\frac{a}{1+n}\right]}{a+\frac{na}{1+n}-\frac{a}{1+n}} = 6.$$

6. **Ans. (b)**

Let the three numbers be a, ar, ar^2, where r is common ratio.

$$\therefore \quad a+ar+ar^2 = 28 \text{ and } a^3r^3 = 512$$

$$\therefore \quad ar = 8 \Rightarrow a+ar^2 = 20$$

$$\Rightarrow \quad 8r^2 - 20r + 8 = 0$$

$$\Rightarrow \quad r = 2, \quad r = \frac{1}{2}.$$

if r = 2, a = 4. Therefore, the three numbers are 4, 8, 16

7. **Ans. (b)**

The sum of the squares of the first n natural numbers is

$$\frac{n(n+1)(2n+1)}{6}$$

Put n = 15, we have, $1^2 + 2^2 + 3^2 + 4^2 + ... + 15^2$

$$= \frac{15(15+1)(30+1)}{6} = 1240.$$

8. **Ans. (c)**

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

$$\Rightarrow \quad b = \frac{2ac}{a+c} \Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ are in A.P.}$$

$\Rightarrow$ a, b, c are in H.P.

9. **Ans. (a)**

Given expression

$$= \left[\frac{15\times16}{2}\right]^2 - \frac{15\times16}{2} = (120)^2 - 120$$

$$= 120 \times 119 = 14280.$$

10. **Ans. (a)**

Sum of the cubes of first n natural numbers

$$= \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2(n+1)^2}{4}$$

$$\therefore \quad \text{Mean} = \frac{n(n+1)^2}{4}.$$

11. **Ans. (a)**

S_n = Sum of n terms of an A.P.

$$= \frac{n}{2}[2a+(n-1)d]$$

where a = first term,

d = common difference

$$\therefore \quad \frac{3+5+7+...+n \text{ terms}}{5+8+11+...+10 \text{ terms}} = 7$$

$$\Rightarrow \quad \frac{\frac{n}{2}[2\times3+(n-1)\times2]}{\frac{10}{2}[2\times5+(10-1)\times3]} = 7$$

$$\Rightarrow \quad \frac{n(2n+4)}{370} = 7$$

$$\Rightarrow \quad 2n^2 + 4n - 2590 = 0$$

$$\Rightarrow \quad n^2 + 2n - 1295 = 0$$

$$\Rightarrow \quad n^2 + 37n - 35n - 1295 = 0$$

$$\Rightarrow \quad n(n+37) - 35(n+37) = 0$$

$$\Rightarrow \quad (n-35)(n+37) = 0$$

$$\Rightarrow \quad n = 35.$$

12. **Ans. (c)**

Sum of the first n natural numbers $= \frac{n(n+1)}{2}$

Sum of the squares of the first n natural numbers

$$= \frac{n(n+1)(2n+1)}{6}$$

$$\therefore \quad \frac{n(n+1)}{2} = \frac{1}{5}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$\Rightarrow \quad 2n+1 = 15 \Rightarrow n = 7.$$

13. Ans. (c)

$$\frac{3^n - 1}{3-1} > 2000 \Rightarrow 3^n > 4001 \Rightarrow n = 8.$$

14. Ans. (d)

$$S_n = \frac{n}{2}[\text{First term + Last term}]$$

Where a = 4, n = 12, L = ?

$$\therefore \quad S_{12} = 256 = \frac{12}{2}[4 + L]$$

$\Rightarrow$ 4 + L = 42

$\Rightarrow$ L = 38

15. Ans. (c)

log 2, $\log(2^x - 1)$,

$\log(2^x + 3)$ are in A.P.

$$\Rightarrow \quad 2\left[\log\left(2^x - 1\right)\right] = \log 2 + \log\left(2^x + 3\right)$$

$$= \log\left[2 \times \left(2^x + 3\right)\right]$$

$$\Rightarrow \quad \log\left(2^x - 1\right)^2 = \log\left[2^{x+1} + 6\right]$$

$$\Rightarrow \quad \left(2^x - 1\right)^2 = 2^{x+1} + 6 = 2^x.2 + 6$$

Let $2^x = y$

$$\therefore \quad (y-1)^2 = 2y + 6 \Rightarrow y^2 - 2y + 1 = 2y + 6$$

$$\Rightarrow \quad y^2 - 4y - 5 = 0$$

$$\Rightarrow \quad (y-5)(y+1) = 0$$

$$\Rightarrow \quad y = 5, -1$$

If $y = 5 \Rightarrow 2^x = 5 \Rightarrow x\log 2 = \log 5$

$$\Rightarrow \quad x = \frac{\log 5}{\log 2} \Rightarrow x = \log_2 5.$$

15. Ans.(b)

Let a be the first term of a G. P. and r be the common ratio

$\therefore$ First five terms of a G.P. are a, ar, ar^2, ar^3, ar^4.

$\therefore$ Third term = $ar^2 = 4 \Rightarrow (ar^2)^5 = 4^5$.

□□□□

IV Practice Exercise: IV

1. If for an AP m^{th}, n^{th}, p^{th} terms are a, b, c respectively then c(m − 1) + a(n − p) + b(p − m) will have the values as

   (a) 0 (b) $\frac{1}{2}$

   (c) −1 (d) $\frac{abc}{mnp}$

2. The sum of all the odd 3 digit integers which are divisible by 9 is

   (a) 23400 (b) 27900

   (c) 35100 (d) 55350

3. If first, third 4 last term of an AP are a, b, 2a respectively then its sum is

   (a) ab (b) $\frac{3ab}{a-b}$

   (c) $\frac{3ab}{2(b-a)}$ (d) $\frac{3ab}{2(b-a)}$

4. If $|a| < 1$ and $S = a^2 + a^4 + a^6 + \ldots\ldots$ then a =

   (a) o (b) aS

   (c) $\frac{S}{1-S}$ (d) $\left(\frac{S}{1+S}\right)^{1/2}$

5. A car moves 25 m is 3^{rd} second, 65 meters in 7^{th} second, 75 meters in 8^{th} second and so on. How for it will move in 15^{th} second

   (a) 165 (b) 145

   (c) 155 (d) 175

6. The sum of n terms of an AP is $n^2 + 2n$, the common difference will be

   (a) 1 (b) 3

   (c) 2 (d) −1

7. If a, b, c, d are four real numbers what will be the minimum possible value of

   $$[a^2 + b^2 + c^2 + d^2]\left[\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{d^2}\right]$$

   (a) 4 (b) 2

   (c) abcd (d) 16

8. Take an AP 2, 3, 4, ... and a GP 3, 6, 12 ...., if taken upto 100. How many terms will be there which are common in both.

(a) 23 (b) 33
(c) 6 (d) 13

9. If second term of an AP is equal to the 9^{th} term of another AP. What will be the sum of 17 terms of second AP as the sum of first 3 terms of first AP is given as 9.
(a) 85 (b) 102
(c) 204 (d) 51

10. There are three numbers is AP. Which have their sum as 45 and product as 1875. The highest of the number is
(a) 15 (b) 25
(c) 35 (d) 21

11. A ball is dropped from a height of 50 m and it rebounces to 1/3 of the initial height. If it continues to rebound in the same manner. What is the distance travelled by the ball in the air
(a) 75 (b) 60
(c) 150 (d) 100

12. Three numbers P, q, r are in GP if P & q are increased by 4 and r is decreased by 4 then they forms and AP. If sum of the numbers is 56 then highest among P, q, r will be
(a) 24 (b) 18
(c) 36 (d) 32

13. Find the sum to infinite term of the series

$$\frac{1}{56}+\frac{1}{72}+\frac{1}{90}+\frac{1}{110}+....$$

(a) $\frac{1}{8}$ (b) $\frac{1}{7}$

(c) $\frac{1}{6}$ (d) $\frac{1}{28}$

14. Let {1, 2, 3, ...541}. Find out how many APs can be formed by taking the element of S starting from 1 and ending at 541. Which have atleast 3 elements
(a) 12 (b) 13
(c) 23 (d) 24

15. Find the sum to first 10 terms of the series
12 + 35 + 70 + 117 + ...
(a) 2595 (b) 2575
(c) 2945 (d) 2495

□□□□

Solutions

1. Ans. (a)
Let first term be 'x'

$$x + (m-1)d = a$$
$$\Rightarrow (m-n) = a-b$$
$$x + (n-1)d = b$$
$$\Rightarrow (n-p) = b-c$$
$$x + (p-1)d = c$$
$$\Rightarrow (p-m) = c-a$$
$$\Rightarrow c(m-n) + a(n-p) + b(p-m) = c(a-b) + a(b-c) + b(c-a) = 0$$

2. Ans. (d)
First '3' digit integer divisible by '9' = 108
Last '3' digit integer divisible by '9' = 999
Let 999 be n^{th} number

$$\Rightarrow 999 = 108 + (n-1)9$$

So $\frac{100}{2}[108+999] = 50 \times 1107 = \text{sum}$

$$\Rightarrow \text{Sum} = 55350$$

3. Ans. (d)
Common difference = (b – a)

$$2a = a + (n-1)b - a$$
$$\Rightarrow \frac{a}{b-a} = n-1$$
$$\Rightarrow n = \frac{b}{b-a}$$
$$\frac{1}{2}\left(\frac{b}{b-a}\right)[2a+a] = \text{sum} = \frac{3ab}{2(b-a)}$$

4. Ans. (d)

$$S = \frac{a^2}{1-a^2}$$
$$\Rightarrow s(1-a^2) = a^2$$
$$\Rightarrow s - sa^2 = a^2$$
$$\Rightarrow a^2 = \frac{s}{1+s}$$
$$\Rightarrow a = \left(\frac{s}{1+s}\right)^{1/2}$$

5. Ans. (b)
Let speed be x and the car covers distance in A.P.
Then the distance covered in
3^{rd} sec. = $x + 2d = 25$
$x + 6d = 65$ so d = 10 and $x = 5$.
So the distance covered in
15^{th} sec. = 5 + 14 × 10 = 145 m.

6. **Ans. (c)**

$T_n = s_n - s_{n-1} = \{n^2 + 2n\} - \{(n-1)^2 + 2(n-1)\}$

$= (2n - 1) + 1 = 2n$

$T_{n-1} = 2(n-1) = 2n - 2$

Common diff. $= 2n - (2n - 2) = 2$

7. **Ans. (d)**

Applying A.M. ≥ G.M

$$\frac{[a^2 + b^2 + c^2 + d^2]}{4} \geq (a^2 . b^2 . c^2 . d^2)^{1/4}$$

$$\& \frac{\left[\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{d^2}\right]}{4} \geq \left(\frac{1}{a^2} . \frac{1}{b^2} . \frac{1}{c^2} . \frac{1}{d^2}\right)^{1/4}$$

$$\text{So } \frac{1}{16}(a^2 + b^2 + c^2 + d^2) . \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{d^2}\right) \geq 1$$

or the min. possible value = 16

8. **Ans. (b)**

In the A.P. all the multiples of 3 will be common with G.P. and since the common diff. is '1' every term will be there in consecutive 3 terms.

So net common terms $= \frac{100}{3} = 33$.

9. **Ans. (d)**

$$T_2 = T_9' \Rightarrow \frac{T_2}{T_9'} = \frac{1}{1}$$

$$\text{So } \frac{s_{(2.2-1)}}{s'_{(2.9-1)}} = \frac{(2.2-1)}{(2.9-1)} \frac{T_2}{T_9'}$$

$$\Rightarrow \frac{s_3}{s_{17}'} = \frac{3}{17} \times \frac{1}{1} = \frac{3}{17}$$

i.e. $s_3 = 9$, so $s'_{17} = 51$.

10. **Ans. (b)**

$a - d + a + a + d = 45 \Rightarrow a = 15$

and $15(15 - d)(15 + d) = 1875$

So $d = \pm 0$. Then highest number

$= 15 + 10 = 25$.

11. **Ans. (d)**

This will be a infinite G.P. with common ratio as $\frac{1}{3}$

So net distance covered

$$= 50 + \frac{50}{3} + \frac{50}{3} + \frac{50}{9} + \frac{50}{9} + \frac{50}{27} + \frac{50}{27} + \ldots$$

↓ up ↓ down

$$= 100\left(1 + \frac{1}{3} + \frac{1}{9} + \ldots\right) - 50$$

$$= 100\left(\frac{1}{1 - \frac{1}{3}}\right) - 50 = 100m$$

12. **Ans. (d)**

$q^2 = pr$ ...(i)

$2(q + 4) = (p + 4) + (r - 4)$

$\Rightarrow 2q + 8 = p + r$ ...(ii)

$p + q + r = 56$ ...(iii)

by solving the eq. we get

$p = 8, q = 16, r = 32$

13. **Ans. (d)**

$$\text{The series} = \frac{1}{7.8} + \frac{1}{8.9} + \frac{1}{9.10} + \ldots.$$

$$= \frac{1}{7} - \frac{1}{8} + \frac{1}{8} - \frac{1}{9} + \frac{1}{9} - \frac{1}{10} + \ldots$$

So the required

$$\text{sum} = \frac{1}{7} - 0 = \frac{1}{7}$$

14. **Ans. (c)**

In this case $541 = 1 + (n - 1)d$

So the possible value of n will be the number of factors of 540.

So no. of factors of $540 = 3 \times 4 \times 2 = 24$

in this we can not include 'l' as $n - 1 = 1$.

So $n = 2$.

Which is not acceptable. So net possibilities $= 24 - 1 = 23$

15. **Ans. (a)**

The series

$s = 3.4 + 5.7 + 7.10 + 9.13 + \ldots..$

So for n terms

$T_n = \{3 + (n - 1)2\}\{4 + (n - 1)3\}$

$= (2n + 1)(3n + 1)$

$= 6n^2 + 5n + 1$

$S_n = 6\Sigma n^2 + 5\Sigma n + \Sigma 1$

$$= 6\frac{(n)(n+1)(2n+1)}{6} + 5\frac{(n)(n+1)}{2} + n$$

For n = 10

$$S_{10} = (10 \times 11 \times 21) + \frac{5}{2} \times 10 \times 11 + 10$$

$S_{10} = 2595$