

Atomic Physics

Topics Covered

- Atoms and Atomic Models
- Rutherford's α -particles Scattering Experiment
- Rutherford's Model of the Atom
- Bohr's Model of the Atom
- Bohr's Theory of Hydrogen Atom
- Total Energy of Electron in Stationary Orbits
- Origin of Spectral Lines

Atoms and Atomic Models

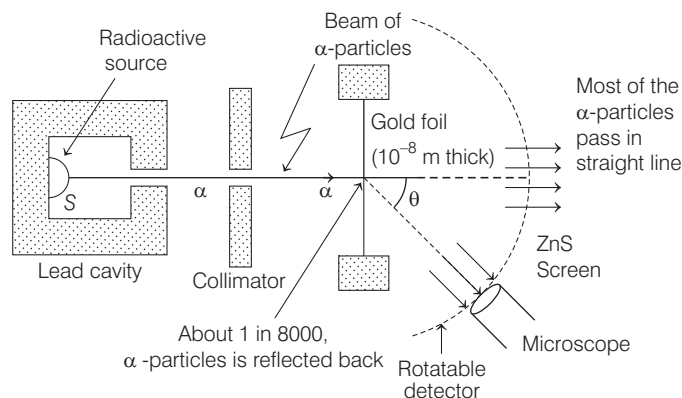
All elements consist of very small invisible particles, called **atoms**. Every atom is a sphere of radius of the order of 10^{-10} m, in which entire positive charge and mass is uniformly distributed and negatively charged electrons revolve around the nucleus.

The first model was proposed by JJ Thomson called **Plum pudding model** of atom. Later, Rutherford worked on it and named his model as Rutherford's planetary model of atom.

Rutherford's α -particles Scattering Experiment

In 1911, Rutherford suggested an experiment known as α -particles scattering experiment, which led to the discovery of atomic nucleus.

Rutherford's α -particles scattering experimental set up is as given below:

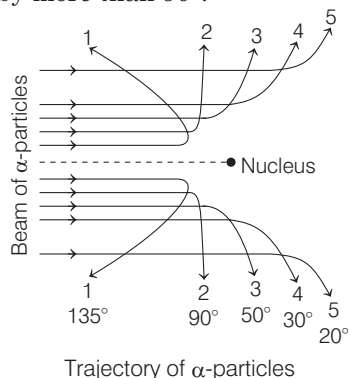


A collimated beam of α -particles was allowed to fall on a thick gold foil. This resulted in scattering of α -particles in different directions, which produced scintillations on zinc sulphide screen. These scintillations were counted at different angles from the direction of incident beam.

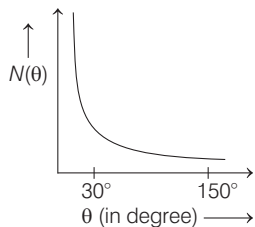
Observations from Rutherford's Experiment

Rutherford made the following observations from his experiment:

- (i) Most of the α -particles passed through the gold foil undeflected.
- (ii) Only about 0.14% of incident α -particles scatter by more than 1° .
- (iii) About one α -particle in every 8000 α -particles deflects by more than 90° .



- (iv) The number of α -particles scattered per unit area $N(\theta)$ at scattering angle θ varies inversely as $\sin^4 \frac{\theta}{2}$.



- (v) Force between α -particle and nucleus is

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{(2e)(Ze)}{r^2}$$

where, r = distance between α -particle and nucleus.

The magnitude and direction of this coulombic force on α -particle continuously changes as it approaches the nucleus and recedes away from it.

Terms Related to Rutherford's Experiment

- (i) **Impact Parameter (b)** It is the perpendicular distance of initial velocity vector of α -particle from central line of nucleus of atom.

$$b = \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2 \cot \frac{\theta}{2}}{\text{KE}}$$

- (ii) **Distance of Closest Approach (r_0)** At r_0 , whole of KE of α -particle converts into electrostatic PE and α -particle cannot go further close to nucleus.

$$r_0 = \frac{1}{4\pi\epsilon_0} \cdot \frac{4Ze^2}{mv^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2Ze^2}{\text{KE}}$$

Rutherford's Model of the Atom

- (i) Atom consists of small central core, called atomic nucleus in which whole mass and positive charge is assumed to be concentrated.
- (ii) The size of nucleus is much smaller than the size of the atoms. It is of the order of 10^{-15} m \approx 1 fermi.
- (iii) The nucleus is surrounded by electrons and atom is electrically neutral.
- (iv) The electrons revolve around the nucleus in various orbits. The centripetal force required for their revolution is provided by the electrostatic attraction between the electrons and the nucleus.

Limitations of Rutherford's Atomic Model

Rutherford's atomic model limitations are given below:

- (i) Could not explain stability of atom clearly.
- (ii) Unable to explain line spectrum.
- (iii) Unable to explain the similarity between atoms of an element.

Bohr's Model of the Atom

In 1913, Bohr combined classical and early quantum concepts and gave his theory in the form of three postulates.

These are as given below:

- (i) **Bohr's first postulate** was that an electron in an atom could revolve in certain stable orbits without the emission of radiant energy, contrary to the predictions of electromagnetic theory.
- (ii) **Bohr's second postulate** defines these stable orbits. This postulate states that the electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of $h/2\pi$, where h is the Planck's constant ($= 6.6 \times 10^{-34}$ J-s). Thus, the angular momentum (L) of the orbiting electron is quantised, i.e. $L = \frac{nh}{2\pi}$.

As, angular momentum of electron (L) = mvr .

\therefore For any permitted (stationary) orbit, $mvr = \frac{nh}{2\pi}$

where, n = any positive integer 1, 2, 3,

It is also called **principal quantum number**.

(iii) **Bohr's third postulate** states that an electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final states. The frequency of the emitted photon is then given by

$$h\nu = E_i - E_f$$

where, E_i and E_f are the energies of the initial and final states and $E_i > E_f$.

Bohr's Theory of Hydrogen Atom

A hydrogen atom consists of a tiny positively charged nucleus (with one proton) and an electron revolves in a stable circular orbit around the nucleus.

$$mv_n^2 = \frac{Ze^2}{4\pi\epsilon_0 r_n}$$

From the second postulate, the angular momentum of the electron is

$$mv_n r_n = n \frac{h}{2\pi}$$

where, $n = (1, 2, 3, \dots)$ is **principal quantum number**.

The equation for the radii of the permitted orbits is given by

$$r_n = n^2 \frac{h^2 \epsilon_0}{\pi m Z e^2}$$

1. Bohr's Radius

The radius of the first orbit ($n = 1$) of hydrogen atom ($Z = 1$) will be

$$r_1 = \frac{h^2 \epsilon_0}{\pi m e^2}$$

This is called Bohr's radius and its value is 0.53 \AA . Since $r \propto n^2$, the radius of second orbit of hydrogen atom will be $(4 \times 0.53) \text{ \AA}$ and that of the third orbit $(9 \times 0.53) \text{ \AA}$ and so on.

2. Velocity of Electron in Stationary Orbits

The velocity of electron in the first orbit ($n = 1$) of hydrogen atom ($Z = 1$) is

$$v_1 = \frac{e^2}{2h \epsilon_0} = \frac{c}{137} \quad [\because c = 3 \times 10^8 \text{ ms}^{-1}]$$

and in n th orbit, $v_n = \left(\frac{e^2}{2h \epsilon_0} \right) \frac{1}{n}$

Total Energy of Electron in Stationary Orbits

The energy E of an electron in an orbit is the sum of kinetic and potential energies.

$$\begin{aligned} E_n &= -\frac{me^4 Z^2}{8\epsilon_0 h^2 n^2} \\ &= -\frac{RheZ^2}{n^2} \end{aligned}$$

where, $n = 1, 2, 3, \dots$

This is the expression for the energy of the electron in the n th orbit.

For hydrogen atom $Z = 1$, substituting the standard

values, we get $E_n = \frac{-13.6}{n^2} \text{ eV}$, negative energy of the

energy shows that the electron is bound to the nucleus and is not free to leave it.

Ground State and Excited State

Ground State of the atom refers to the lowest permitted energy level, i.e. for which $n = 1$. The state higher than ground state (i.e., $n > 1$) are known as **excited states**.

The energy required to bring an electron from its ground state to the continuum is defined as the **ionisation energy** of the atom.

Origin of Spectral Lines (Hydrogen Spectrum)

Hydrogen spectrum consists of discrete bright lines on a dark background and it is specifically known as **hydrogen emission spectrum**.

There is one more type of hydrogen spectrum, where we get dark lines on the bright background. It is known as **absorption spectrum**.

Balmer found an empirical formula by the observation of a small part of this spectrum and it is represented by

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

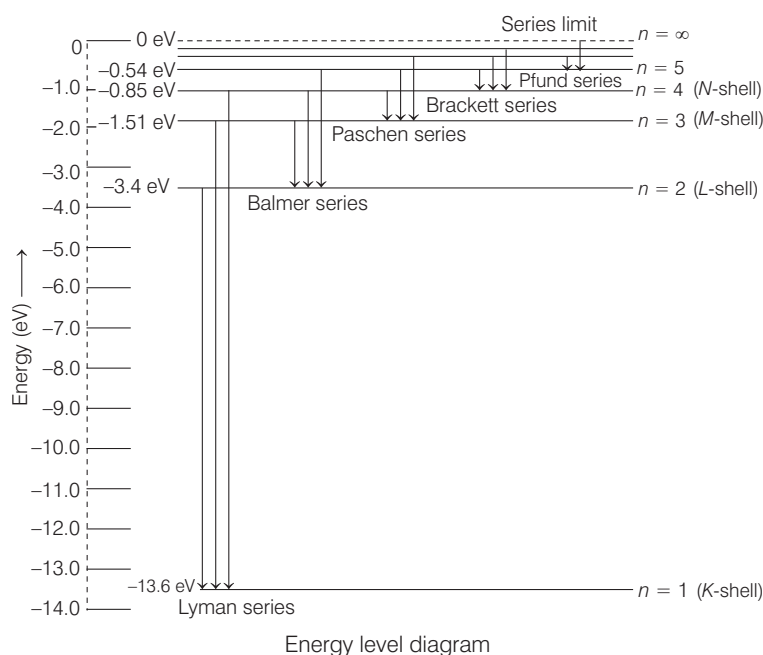
where, $n = 3, 4, 5, \dots$

where, R is a constant called **Rydberg constant** and its value is $1.097 \times 10^7 \text{ m}^{-1}$.

So, $\frac{1}{\lambda} = 1.522 \times 10^6 \text{ m}^{-1}$
 $= 656.3 \text{ nm for } n = 3$

Explanation

The atomic hydrogen emits a line spectrum consisting of various series.



The wavelength of line in these series can be expressed by formulae:

(i) **Lyman series,**

$$\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{n^2} \right); n = 2, 3, 4 \dots$$

where, R is Rydberg's constant which is equal to $1.097 \times 10^7 \text{ m}^{-1}$.

The wavelength range is from 1216 Å-912 Å, which corresponds to ultraviolet region of the spectrum.

(ii) **Balmer series,**

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right); n = 3, 4, 5, \dots \infty.$$

The wavelength range is from 6561 Å-3646 Å, which corresponds to visible region of spectrum.

(iii) **Paschen series,**

$$\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{n^2} \right); n = 4, 5, 6, \dots \infty.$$

The wavelength range is from 18752 Å-8204 Å which corresponds to visible region of spectrum.

(iv) **Brackett series,**

$$\frac{1}{\lambda} = R \left(\frac{1}{4^2} - \frac{1}{n^2} \right); n = 5, 6, 7, \dots \infty.$$

The wavelength range is from 40514 Å-14585 Å, which also corresponds to infrared region of spectrum.

(v) **Pfund series,**

$$\frac{1}{\lambda} = R \left(\frac{1}{5^2} - \frac{1}{n^2} \right); n = 6, 7, 8, \dots \infty.$$

The wavelength range is from 74583 Å-22789 Å, which corresponds to infrared region.

Limitations of Bohr's Atomic Model

- This model is applicable only to a simple atom like hydrogen having $Z = 1$. This theory fails, if $Z > 1$.
- It does not explain the fine structure of spectral lines in H-atom.
- This model does not explain why orbits of electrons are taken as circular, whereas elliptical orbits are also possible.

PRACTICE QUESTIONS

Exam', Textbook's & Other Imp. Questions

1 MARK Questions

Exams' Questions

Q.1 Which series of hydrogen spectrum lies in the visible region? [2019]

Sol. The Balmer series lies in the visible region. (1)

Q.2 The angular momentum of the electron in the second orbit of the hydrogen atom is $2h/\pi$. (Correct the sentence, if required, without changing the underlined words). [2017]

Sol. The angular momentum of the electron in the second orbit of the hydrogen atom is h/π . (1)

Q.3 If E_n and L_n represent respectively, the total energy and angular momentum of the electron in the n th orbit of hydrogen atom, then [2015]

- (a) $E_n \propto L_n$ (b) $E_n \propto L_n^2$
(c) $E_n \propto 1/L_n$ (d) $E_n \propto 1/L_n^2$

Sol (d) $E_n \propto \frac{1}{L_n^2}$ [As, $E_n \propto \frac{1}{(n)^2}$ and $L_n \propto n$] (1)

Q.4 The radius of the third Bohr orbit for hydrogen is 4.5 Å. The radius of its fourth Bohr orbit is

- (a) 7.2 Å (b) 8 Å
(c) 9.6 Å (d) 11.3 Å [2014]

Sol (b) Since for H-atom, $r_n \propto n^2$

$$\therefore \frac{r_1}{r_2} = \left(\frac{n_1}{n_2}\right)^2$$

$$\Rightarrow \frac{4.5}{r_2} = \left(\frac{3}{4}\right)^2 \Rightarrow \frac{4.5}{r_2} = \frac{9}{16}$$

$$\Rightarrow r_2 = 8.0 \text{ Å} \quad (1)$$

Q.5 The ratio of radii of 1st and 3rd Bohr orbits of the electron in hydrogen atom is

- (a) 1 : 3 (b) 1 : 6
(c) 1 : 9 (d) 1 : 27 [2014]

Sol (c) 1 : 9, since for H-atom $r_n \propto n^2$ (1)

Q.6 The orbital angular momentum of an electron in the third Bohr's orbit is

- (a) $\frac{9h}{2\pi}$ (b) $\frac{7h}{4\pi}$ (c) $\frac{3h}{2\pi}$ (d) $\frac{h}{4\pi}$ [2013]

Sol (c) $\frac{3h}{2\pi}$, since $L = \frac{nh}{2\pi}$ (1)

Q.7 From Bohr's theory, when electrons jump from higher energy orbits to second orbit, the spectral lines that occur belong to brackett series. (Correct the underlined word, if necessary) [2013]

Sol From Bohr's theory, when electrons jump from higher energy orbits to second orbit, the spectral lines that occur belong to Balmer series. (1)

Q.8 In hydrogen atom, the electron jumps from orbits specified by $n = 1$ to $n = 2$. Write the change in its angular momentum. [2013 Instant, Textbook]

Sol $\therefore \Delta L = L_2 - L_1 = \frac{2h}{2\pi} - \frac{h}{2\pi} = \frac{h}{2\pi}$ (1)

Q.9 The ground state energy of hydrogen atom is -13.6 eV. Hence, the energy of the electron of a doubly ionised lithium atom in its 1st orbit will be [2012]

- (a) -4.5 eV (b) -13.6 eV
(c) -40.8 eV (d) -122.4 eV

Sol (d) -122.4 eV, since $E_n = -\frac{13.6Z^2}{n^2}$, $Z = 3$ for He. (1)

Q.10 The spectral lines in the brackett series arise due to transition of electrons in hydrogen atom from higher orbits to the orbit with [2010]

- (a) $n = 1$ (b) $n = 2$
(c) $n = 3$ (d) $n = 4$

Sol (d) $n = 4$, because for brackett series, electron will jump from higher orbit to orbit number 4. (1)

Q.11 What is the relation between orbit radius r and orbit number n of electron in an atom according to Bohr's theory? [2009]

- (a) $r \propto n^{-1}$ (b) $r \propto n$
(c) $r \propto n^{-2}$ (d) $r \propto n^2$

Sol (d) $r \propto n^2$ (1)

Q.12 Write the formula for Balmer series. [2009]

Sol $\frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{n^2} \right]$, $n = 3, 4, \dots$
where, R = Rydberg's constant. (1)

- Q.13** The energy of an electron of hydrogen atom in the ground state is -13.6 eV . What will be the energy in the second excited state? [2008]

Sol $\therefore E_3 = \frac{-13.6}{3^2} = -1.51\text{ eV}$ (1)

- Q.14** Velocity of an electron in the first orbit of hydrogen atom is $2.2 \times 10^6\text{ ms}^{-1}$. What is its velocity in the second orbit? [2007 Instant]

Sol Velocity of an electron in an orbit is

$$v = \frac{Ze^2}{2\epsilon_0 n h} = \frac{e^2}{2\epsilon_0 h} \left(\frac{Z}{n} \right)$$

$$\Rightarrow \frac{v_2}{v_1} = \frac{1}{2}$$

$$\Rightarrow v_2 = \frac{v_1}{2} = \frac{2.2 \times 10^6}{2} = 1.1 \times 10^6\text{ ms}^{-1}$$
 (1)

Important Questions

- Q.15** The dimension of an atom is of the order of [Textbook]

(a) 10^{-6} m (b) 10^{-8} m (c) 10^{-10} m (d) 10^{-14} m

- Sol** (c) The dimension or radius of an atom is of order of 10^{-10} m . (1)

- Q.16** In which of the orbits will the electron of a hydrogen atom have maximum energy [Textbook]

(a) first (b) second
(c) third (d) completely detached

- Sol** (d) When electron is completely detached from the atom, then it has maximum energy. (1)

- Q.17** Which is quantised in Bohr's atomic model? [Textbook]

(a) Total energy of the electron
(b) Angular momentum of the electron
(c) Linear velocity of the electron
(d) Angular velocity of the electron

- Sol** (b) The angular momentum of the orbiting electron is quantised in Bohr's atomic model. (1)

- Q.18** For a given value of n , the number of electrons in an orbit is [Textbook]

(a) n (b) n^2 (c) $2n^2$ (d) $2n$

- Sol** (c) Here, n gives the number of orbits in an atom. So, the number of electrons are $2n^2$. (1)

- Q.19** The potential energy of the electron in hydrogen atom is $-ke^2/r$. Its kinetic energy is [Textbook]

(a) $-\frac{ke^2}{2r}$ (b) $+\frac{ke^2}{2r}$ (c) $-\frac{ke^2}{r}$ (d) $\frac{ke^2}{r}$

- Sol** (b) The potential energy of the electron in an orbit of radius r is given by

$$\text{PE} = -\frac{ke^2}{r}$$

$$\text{As, KE (kinetic energy)} = -\frac{\text{PE (potential energy)}}{2}$$

$$\Rightarrow \text{KE} = +\frac{ke^2}{2r} \quad (1)$$

- Q.20** In a black and white television, pictures on the screen are produced due to bombardment of [Textbook]

(a) X-ray photons on a white screen
(b) X-ray photons on a white fluorescent screen
(c) electrons on a white screen
(d) electrons on a fluorescent white screen

- Sol** (d) The pictures on the screen of a black and white television are produced due to the bombardment of electrons on a fluorescent white screen. (1)

- Q.21** The first spectral line of sodium atom is 5890 \AA . the first excitation potential of sodium is [Textbook]

(a) 3.7 V (b) 2.1 V
(c) 10.2 V (d) 4.1 V

- Sol** (b) The excitation energy for sodium atom is

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{5890 \times 10^{-10}}$$

$$= 3.36 \times 10^{-19}\text{ J} \quad (1/2)$$

\therefore Excitation potential,

$$V = \frac{E}{e} = \frac{3.36 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.1\text{ V} \quad (1/2)$$

- Q.22** Of the following transitions in the hydrogen atom, the one which gives an emission line of highest frequency is [Textbook]

(a) $n = 1$ to $n = 2$ (b) $n = 3$ to $n = 10$
(c) $n = 10$ to $n = 3$ (d) $n = 2$ to $n = 1$

- Sol** (d) The emission of frequency takes place when electron jump from higher orbit to a lower orbit.

In following question it is possible for (c) and (d).

$$\text{For (c), } v = \frac{c}{\lambda} = cR \left[\frac{1}{3^2} - \frac{1}{100^2} \right]$$

$$= cR \times \frac{91}{900}$$

$$\text{For (d), } v = \frac{c}{\lambda} = cR \left[\frac{1}{1^2} - \frac{1}{2^2} \right] = cR \times \frac{3}{4}$$

Therefore, for transition $n = 2$ to $n = 1$, the emission frequency is highest. (1)

- Q.23** The kinetic energy of the electron in an orbit of radius r in hydrogen atom is (e = electronic charge) [Textbook]

(a) $\frac{e^2}{r^2}$ (b) $\frac{e^2}{2r}$
(c) $\frac{e^2}{r}$ (d) $\frac{2e^2}{r}$

Sol (b) The kinetic energy of the electron in an orbit is

$$KE = \frac{1}{8\pi\epsilon_0} \frac{Ze^2}{r_n}$$

Here, $Z = 1$ (for hydrogen atom) and $r_n = r$

$$\therefore KE = \frac{ke^2}{2r} \Rightarrow KE = \frac{e^2}{2r} \quad [\text{if } k = 1] \quad (1)$$

Q.24 Band spectrum is produced by [Textbook]

- (a) H (b) H_2 (c) He (d) Na

Sol (b) A single hydrogen atom produced a line spectrum while a double hydrogen molecule or H_2 produces band spectrum. (1)

Q.25 Ionisation potential of hydrogen atom is 13.6 eV. Hydrogen atoms in the ground state are excited by monochromatic radiation of photon energy 12.1 eV. The spectral lines emitted by the hydrogen atoms according to Bohr's theory will be [Textbook]

- (a) one (b) two (c) three (d) four

Sol (c) Using the relation of ionisation potential,

$$\Delta E = 13.6 \left(\frac{1}{1^2} - \frac{1}{n^2} \right)$$

Here, $\Delta E = 12.1$ eV as it is used to excite electrons.

$$\therefore 12.1 = 13.6 \left(1 - \frac{1}{n^2} \right)$$

$$\Rightarrow \frac{1}{n^2} = 0.11 \quad \text{or} \quad n = \sqrt{9} = 3$$

Hence, three spectral lines are emitted. (1)

Q.26 What is deduced from the fact that when α -particles are fired at a fine gold leaf, most of them pass through with little or no deflection? [Textbook]

- (a) α -particles have a high penetrating power
(b) α -particles are positively charged
(c) gold atoms are nearly all empty space
(d) α -particles causes gold atoms to disintegrate

Sol (c) As most of the α -particles pass through the gold leaf with little or no deflection, so the gold atom have nearly all empty space in it. (1)

Q.27 In Rutherford's experiment, the number of α -particles scattered through an angle of 60° by a silver foil is 200 per minute. When the silver foil is replaced by a copper foil of the same thickness, the number of α -particles scattered through an angle of 60° [Textbook]

- (a) $\frac{200 \times Z_{Cu}}{Z_{Ag}}$ (b) $200 \left(\frac{Z_{Cu}}{Z_{Ag}} \right)^2$
(c) $200 \times \frac{Z_{Ag}}{Z_{Cu}}$ (d) $200 \times \left(\frac{Z_{Ag}}{Z_{Cu}} \right)^2$

Sol (b) The number of α -particles scattered at an angle θ ,

$$N = \frac{kZ^2}{\sin^4 \left(\frac{\theta}{2} \right)}$$

$$\therefore \frac{N_{Cu}}{N_{Ag}} = \left(\frac{Z_{Cu}}{Z_{Ag}} \right)^2 \times \frac{\sin^4 \left(\frac{60^\circ}{2} \right)}{\sin^4 \left(\frac{60^\circ}{2} \right)}$$

$$\Rightarrow N_{Cu} = N_{Ag} \left(\frac{Z_{Cu}}{Z_{Ag}} \right)^2 = 200 \left(\frac{Z_{Cu}}{Z_{Ag}} \right)^2 \quad (1)$$

Q.28 A continuous band of radiation having all wavelength from about 1000 \AA to 10000 \AA is passed through a gas of monoatomic hydrogen. In the emission spectrum, one can observe the entire [Textbook]

- (a) Lyman series (b) Balmer series
(c) Paschen series (d) Pfund series

Sol (b) As the range of wavelength 1000 \AA to 10000 \AA corresponds to the visible part of the spectrum, so the emission spectrum observed is of Balmer series. (1)

Q.29 Rydberg constant is [Textbook]

- (a) same for all elements
(b) different for different elements
(c) a universal constant
(d) is different for lighter elements but same for heavier elements

Sol (a) The Rydberg constant (R) is a constant having value of $1.097 \times 10^7 \text{ m}^{-1}$ and is same for all elements. (1)

Q.30 When an electron jumps from a higher energy state to a lower energy state with an energy difference of ΔE electron volt, then the wavelength of the line emitted is give approximately by [Textbook]

- (a) $\frac{12375}{\Delta E} \text{ cm}$ (b) $\frac{12375}{\Delta E} \text{ metre}$
(c) $\frac{12375}{\Delta E} \text{ angstrom}$ (d) $\frac{12375}{\Delta E} \text{ micron}$

Sol (c) The wavelength of emitted line is given by

$$\lambda = \frac{12375}{\Delta E} \text{ angstrom}$$

where, ΔE = energy difference of two states in eV. (1)

Q.31 Which of the following sources give discrete emission spectrum? [Textbook]

- (a) Incandescent electric bulb (b) Sun
(c) Mercury vapour lamp (d) Candle

Sol (c) Among given sources, mercury vapour lamp gives discrete emission spectrum. (1)

Q.32 While electron in hydrogen atom revolves in a stationary orbit it [Textbook]

- (a) radiates light but its velocity is unchanged
- (b) does not radiate light but its velocity remains unchanged
- (c) does not radiate though its velocity changes
- (d) radiates light with the change of energy

Sol (c) According to Bohr's postulate, an electron in hydrogen atom revolving in a stationary orbit does not radiate though its velocity is changing at every point on the circular orbit, due to change in direction. (1)

Q.33 Generally the approximate limits of visible spectrum are [Textbook]

- (a) 1000 to 4000 Å
- (b) 4000 to 7000 Å
- (c) 7000 to 10000 Å
- (d) 10000 to 13000 Å

Sol (b) The approximate limits of visible spectrum are 4000 to 7000 Å. (1)

Q.34 If the mass of an electron is reduced to half, the Rydberg constant [Textbook]

- (a) remains unchanged
- (b) becomes half
- (c) becomes double
- (d) becomes one-fourth

Sol (b) The Rydberg's constant, $R = \frac{me^2}{8\varepsilon_0^2ch^3} \Rightarrow R \propto m$

So, if mass of an electron is reduced to half, then the value of R also becomes half. (1)

Q.35 Given the electronic charge, $e = 1.6 \times 10^{-19}$ C, electronic mass, $m = 9.1 \times 10^{-31}$ kg, $h = 6.6 \times 10^{-34}$ J-s and $c = 3 \times 10^8$ m/s.

The radius of the first orbit of electron in hydrogen atom is [Textbook]

- (a) 5.3×10^1 Å
- (b) 5.3×10^0 Å
- (c) 5.3×10^{-1} Å
- (d) 5.3×10^{-2} Å

Sol (c) The radius of first orbit is given as

$$r_1 = \frac{h^2 \varepsilon_0}{\pi m e^2}$$

Substituting given values, we get

$$r_1 = \frac{(6.6 \times 10^{-34})^2 \times (8.85 \times 10^{-12})}{3.14 \times (9.1 \times 10^{-31}) \times (1.6 \times 10^{-19})^2} = 0.53 \text{ Å or } 5.3 \times 10^{-1} \text{ Å} \quad (1)$$

Q.36 The velocity of the electron in the first Bohr's orbit is [Textbook]

- (a) 2.2×10^4 m/s
- (b) 2.2×10^5 m/s
- (c) 2.2×10^6 m/s
- (d) 2.2×10^7 m/s

Sol (c) The velocity of the electron in the first Bohr's orbit is given as

$$v_1 = \frac{e^2}{2h\varepsilon_0} = \frac{c}{137} = \frac{3 \times 10^8}{137} = 0.022 \times 10^8 \text{ ms}^{-1} = 2.2 \times 10^6 \text{ ms}^{-1} \quad (1)$$

Q.37 The time period of revolution of the electron in the first Bohr's orbit is [Textbook]

- (a) 1.5×10^{-4} s
- (b) 1.5×10^{-16} s
- (c) 1.5×10^{-18} s
- (d) 1.5×10^{-20} s

Sol (b) The time period of revolution of electron in first Bohr's orbit is given as

$$T_1 = \frac{hr_1}{ke^2}$$

$$\begin{aligned} \text{where, } k &= \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N} \cdot \text{m}^2 \text{C}^{-2} \\ &= \frac{6.6 \times 10^{-34} \times 0.53 \times 10^{-10}}{9 \times 10^9 \times (1.6 \times 10^{-19})^2} \\ &= 0.15 \times 10^{-15} \text{ s or } 1.5 \times 10^{-16} \text{ s} \quad (1) \end{aligned}$$

Q.38 The frequency of revolution of the electron in the first Bohr's orbit is [Textbook]

- (a) 6.6×10^{15} Hz
- (b) 6.6×10^{13} Hz
- (c) 6.6×10^{17} Hz
- (d) 6.6×10^{19} Hz

Sol (a) The frequency of revolution of the electron in the first Bohr's orbit is given by

$$\begin{aligned} v_1 &= \frac{ke^2}{hr_1} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{(6.6 \times 10^{-34}) \times (0.53 \times 10^{-10})} \\ &= 6.6 \times 10^{15} \text{ s}^{-1} \text{ or Hz} \quad (1) \end{aligned}$$

Q.39 In an atom, two electrons move round the nucleus in circular orbits of radii R and $4R$. The ratio of the times taken by them to complete one revolution is [Textbook]

- (a) 1/4
- (b) 4/1
- (c) 8/1
- (d) 1/8

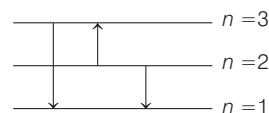
Sol (a) The time period of a revolving electron is given by

$$\begin{aligned} T_n &= \frac{nh r_n}{kZe^2} \Rightarrow T_n \propto r_n \\ \therefore \frac{T_1}{T_2} &= \frac{r_1}{r_2} = \frac{R}{4R} = \frac{1}{4} \text{ or } 1:4 \quad (1) \end{aligned}$$

Q.40 The hydrogen atoms in a sample are in excited state, described by $n = 3$. The number of spectral lines in emission spectrum will be [Textbook]

- (a) 1
- (b) 2
- (c) 3
- (d) 6

Sol (c) The number of spectral line in emission spectrum is shown by the diagram below



Hence, 3 spectral lines are possible for $n = 3$. (1)

Q.41 The atomic nucleus scatters α -particles at large angles but not the electrons. Explain. [Textbook]

Sol Atomic nucleus is positively charged due to this reason when as α -particle comes near it. It experience force. (1)

Q.42 Explain, why the emission spectra of hydrogen possesses many lines even though it has only one electron. [Textbook]

Sol Each hydrogen atom which is a source of hydrogen spectrum has many stationary states. All possible transitions can occur from any higher level to lower level. This gives rise to large number of spectral lines. (1)

Q.43 What is the trajectory of scattered α -particles? [Textbook]

Sol The trajectory of α -particles after scattering is hyperbolic. (1)

Q.44 The total energy of the electron in an atom is always negative (True/False). [Textbook]

Sol (b) True, the total energy of electron in an atom is always negative. (1)

Q.45 The ratio of the energies of hydrogen atom in its first and second excited states is

- (a) 1 : 4 (b) 4 : 1
(c) 9 : 4 (d) 4 : 9

Sol (c) Since, $\frac{E_2}{E_3} = \frac{-13.6/4}{-13.6/9} = \frac{9}{4}$ (1)

Q.46 The ionisation energy of hydrogen atom is [Textbook]

- (a) 8.24 eV (b) 10.36 eV (c) 13.6 eV (d) 14.24 eV

Sol (c) 13.6 eV (1)

Q.47 The hydrogen atom is said to be in its ground state when the electron revolving in an orbit

- (a) is at rest (b) has escaped from the atom
(c) spirals into nucleus (d) is in its lowest energy level

Sol (d) The hydrogen atom is said to be in its ground state when the electron revolving in an orbit is in its lowest energy level. (1)

Q.48 The angular momentum of electron in the hydrogen atom varies as the principal quantum number n . (Fill in the blank)

Sol directly (1)

Q.49 Rutherford atomic model failed to explain the.....of atom. (Fill in the blank)

Sol stability (1)

Q.50 One of the assumptions made by Bohr is that the atoms never radiate. (Check whether the sentence is correct or incorrect)

Sol The sentence is incorrect. (1)

Q.51 Electrons can revolve in **stationary** orbits in Bohr's atomic model. (Correct the sentence, if necessary)

Sol The sentence is correct. (1)

Q.52 Paschen series belongs to which region?

[Textbook]

Sol Infrared region (1)

Q.53 Calculate the potential energy of revolving electron when the total energy of an electron in first Bohr's orbit is -13.6 eV.

Sol Potential energy of electron = Total energy $\times 2$
 $= -13.6 \times 2 = -27.2$ eV (1)

2 MARKS Questions

Exams' Questions

Q.54 When a photon corresponding to the third line of Paschen series of hydrogen spectrum is emitted, determine the change in angular momentum of the electron associated with the process. [2018]

Sol Angular momentum of moving electron is given by a relation is

$$mvr = \frac{nh}{2\pi}$$

For third line of Paschen series,

$$n_i = 3 \text{ and } n_f = 1 \quad (1)$$

\therefore Change in angular momentum (ΔL) = $L_i - L_f$

$$\begin{aligned} &= \frac{3h}{2\pi} - \frac{h}{2\pi} \\ &= \frac{2h}{2\pi} = \frac{6.6 \times 10^{-34}}{3.14} = 2.10 \times 10^{-34} \text{ kg-m}^2\text{s}^{-1} \quad (1) \end{aligned}$$

Q.55 Calculate the ratio of the longest to the shortest wavelength of Balmer series of hydrogen spectra. [2017]

$$\text{Sol } \therefore \frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n_i^2} \right) = R \left(\frac{1}{2^2} - \frac{1}{3^2} \right)$$

$$= R \left(\frac{1}{4} - \frac{1}{9} \right) = R \left(\frac{5}{36} \right)$$

$$\Rightarrow \lambda_{\text{longest}} = \frac{36}{5R}$$

$$\text{and } \lambda_{\text{shortest}} = \left[R \left(\frac{1}{2^2} - \frac{1}{\infty^2} \right) \right]^{-1} = \frac{4}{R}$$

$$\therefore \frac{\lambda_{\text{longest}}}{\lambda_{\text{shortest}}} = \frac{36}{5R} \times \frac{R}{4} = 9:5 \quad (2)$$

Q.56 Calculate the longest wavelength of the Paschen series spectrum. [2016]

(Take, Rydberg's constant = $1.097 \times 10^7 \text{ m}^{-1}$)

$$\text{Sol } \text{For hydrogen spectrum, } \frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For Paschen series, $n_1 = 3$

$$\Rightarrow \frac{1}{\lambda} = R_H \left(\frac{1}{9} - \frac{1}{n_2^2} \right) \quad (1)$$

For maximum wavelength, $n_2 = 4$

$$\Rightarrow \frac{1}{\lambda_{\max}} = R_H \left(\frac{1}{9} - \frac{1}{16} \right) = 1.097 \times 10^7 \times \left(\frac{16-9}{144} \right)$$

$$\frac{1}{\lambda_{\max}} = 1.097 \times 10^7 \times \frac{7}{144}$$

$$\Rightarrow \lambda_{\max} = \frac{144 \times 10^{-7}}{1.097 \times 7} = 18.752 \times 10^{-7} \text{ m}$$

$$\Rightarrow \lambda_{\max} = 18.752 \times 10^{-7} \text{ m} \quad (1)$$

Q.57 Calculate the maximum wavelength in Å, emitted in the Balmer series of hydrogen spectrum.
(Take, Rydberg constant = $1.097 \times 10^7 \text{ m}^{-1}$)
[2013 Cancelled]

Sol For Balmer series,

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right), \text{ where } n = 3, 4, 5, \dots$$

$$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{3^2} \right) \quad (1)$$

$$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{9} \right)$$

$$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \times \frac{5}{36}$$

$$\Rightarrow \lambda = 6563 \times 10^{-10} \text{ m} = 6563 \text{ Å}$$

This is the maximum wavelength emitted in Balmer series. (1)

Q.58 Explain Balmer series of hydrogen spectrum.
[2011 Instant]

Sol From Bohr's theory, when electrons jump from higher energy orbits to second orbit, the spectral lines that occur belong to Balmer series.
The wavelength of spectral lines in Balmer series is given by

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right), \text{ where } n = 3, 4, 5, \dots$$

The spectral lines in Balmer series are in the visible region. (2)

Q.59 Write the postulates of Bohr's atom model which differ from those of Rutherford's model.
[2011 Instant]

Sol Bohr's first, second, third postulates Refer to text on pages 259 and 260.

$$\left(\frac{1}{2} + 1 + \frac{1}{2} \right)$$

Q.60 How much energy will be released when an electron jumps from $n = 3$ to $n = 2$ orbit in hydrogen atom? Energy of electron in the first orbit = -13.6 eV .
[2009]

Sol Using $E_n = \frac{-13.6}{n^2} \text{ eV}$

$$\therefore E_3 = \frac{-13.6}{3^2} \text{ eV} = -1.51 \text{ eV} \quad (1/2)$$

and $E_2 = \frac{-13.6}{2^2} \text{ eV} = -3.40 \text{ eV} \quad (1/2)$

$$\therefore \text{Energy release} = E_3 - E_2 = [-1.51 - (-3.40)] \text{ eV} = 1.89 \text{ eV} \quad (1)$$

Q.61 Calculate the wavelength of the second member of Balmer series.
(Take, Rydberg's constant, $R = 1.097 \times 10^7 \text{ m}^{-1}$)
[2008]

Sol For Balmer series, $\frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{n^2} \right]$

where, $n = 3, 4, 5, \dots$ (1/2)

For second member, $n = 4$

$$\frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{4^2} \right] \quad (1/2)$$

$$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left[\frac{1}{4} - \frac{1}{16} \right]$$

$$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \times \frac{3}{16}$$

$$\Rightarrow \lambda = 4.862 \times 10^{-7} = 4862 \text{ Å} \quad (1)$$

Q.62 If first line of Balmer series has wavelength 6563 Å, calculate wavelength of first line of Lyman series.
[2007 Instant]

Sol As $\frac{1}{\lambda} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$

For Lyman series,

$$\frac{1}{\lambda_L} = R \left[\frac{1}{1^2} - \frac{1}{2^2} \right] = R \times \frac{3}{4} \Rightarrow \frac{1}{\lambda_L} = \frac{3}{4} R \quad (1/2)$$

For Balmer series,

$$\frac{1}{\lambda_B} = R \left[\frac{1}{2^2} - \frac{1}{3^2} \right] = R \times \frac{5}{36}$$

$$\Rightarrow \frac{1}{\lambda_B} = \frac{5}{36} R \quad (1/2)$$

Now, $\frac{\lambda_B}{\lambda_L} = \frac{36}{5R} \times \frac{3R}{4} = \frac{27}{5}$

$$\Rightarrow \lambda_L = \frac{5}{27} \lambda_B = \frac{5}{27} \times 6563 = 1215 \text{ Å} \quad (1)$$

Q.63 The second member of Lyman series in hydrogen atom has wavelength 5400 Å. What is the wavelength of the first member?
[2001]

Sol For Lyman series, $\frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{n^2} \right]$

where, $n = 2, 3, \dots$

For first member, $n = 2$

$$\frac{1}{\lambda_1} = R \left[\frac{1}{1^2} - \frac{1}{2^2} \right]$$

$$\frac{1}{\lambda_1} = R \left[\frac{1}{1} - \frac{1}{4} \right] \text{ or } \frac{1}{\lambda_1} = \frac{3}{4} R \Rightarrow \lambda_1 = \frac{4}{3R} \quad (1/2)$$

For second member, $n = 3$

$$\frac{1}{\lambda_2} = R \left[\frac{1}{1^2} - \frac{1}{3^2} \right] = \frac{8}{9} R \Rightarrow \lambda_2 = \frac{9}{8R} \quad (1/2)$$

Now, $\frac{\lambda_1}{\lambda_2} = \frac{4}{3R} \times \frac{8R}{9} = \frac{32}{27}$

$\Rightarrow \lambda_1 = \frac{32}{27} \times \lambda_2 = \frac{32}{27} \times 5400 = 6400 \text{ \AA}$ (1)

Important Questions

Q.64 An electron in a hydrogen atom moves with a uniform speed in its stationary circular orbit of radius $5.3 \times 10^{-11} \text{ m}$. Find

- its speed and
- energy in its ground state. [Textbook]

Sol (i) Given, Bohr's radius, $r_1 = 5.3 \times 10^{-11} \text{ m}$

\therefore Speed, $v_1 = \frac{e^2}{2h\epsilon_0} = \frac{c}{2h\epsilon_0} = \frac{c}{137} = 2.2 \times 10^6 \text{ m/s}$ (1)

- Energy in the ground state,

$E_n = -\frac{13.6}{n^2} \text{ eV} = -13.6 \text{ eV}$ (1)

Q.65 Why is Rutherford model for an atom of electron orbiting around the nucleus not able to explain the atomic structure?

Sol The classical (Rutherford) model could not explain the atomic structure as the electrons revolving around the nucleus are accelerated and according to Rutherford's model, emit energy. As a result, the radius of the electron's circular orbits goes on decreasing. Ultimately, electrons fall into the nucleus, which is not true for an atom. (2)

Q.66 Mention the drawbacks of Rutherford's model of an atom. [Textbook]

- Sol** (i) Could not explain stability of atom clearly. (1)
- (ii) Unable to explain line spectrum. (1)

Q.67 The energy of an electron in ground state of hydrogen atom is -13.6 eV . (i) What does the negative sign signify? and (ii) How much energy is required to take an electron in this atom from ground state to first excited state?

Sol (i) Negative sign implies that electrons are bound to the nucleus by means of electrostatic force of attraction. (1)

(ii) As, $E_n = \frac{-13.6}{n^2}$

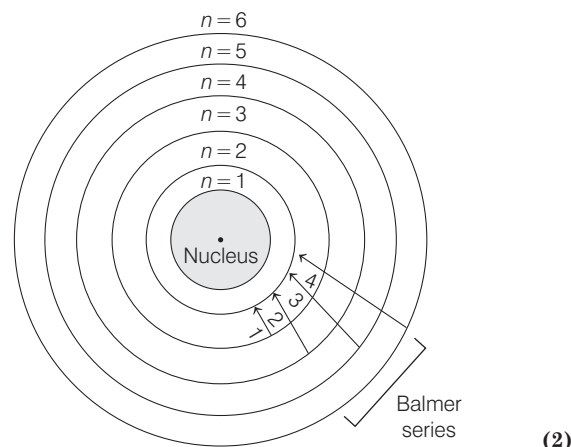
For first excited state, $n = 2$

$E_2 = \frac{-13.6}{2^2} = -3.4 \text{ eV}$

Therefore, required energy $= E_2 - E_1$
 $= -3.4 - (-13.6) = 10.2 \text{ eV}$ (1)

Q.68 Give orbital diagram to show transitions giving rise to Balmer series in hydrogen atom.

Sol When electrons jump from higher orbits to second orbit, Balmer series in hydrogen atom occurs.



Q.69 Calculate the frequency of radiation, if an electron in hydrogen atom jumps from $n_2 = 3$ to $n_1 = 2$.

[Take, Rydberg's constant, $R = 1.1 \times 10^7 \text{ m}^{-1}$]

Sol The frequency is given by

$$\nu = Rc \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$
 (1)

For transitions from $n_2 = 3$ to $n_1 = 2$,

$$\nu = (1.1 \times 10^7) (3 \times 10^8) \left[\frac{1}{2^2} - \frac{1}{3^2} \right]$$

$\Rightarrow \nu = (3.3 \times 10^{15}) \left[\frac{1}{4} - \frac{1}{9} \right] \Rightarrow \nu = 3.3 \times 10^{15} \times \frac{5}{36}$

$\therefore \nu = 4.6 \times 10^{14} \text{ Hz}$ (1)

Q.70 What is the shortest wavelength present in Paschen series of spectral lines?

Sol For Paschen series,

$$\frac{1}{\lambda} = R \left[\frac{1}{3^2} - \frac{1}{n^2} \right], \text{ where } n = 4, 5, 6, \dots, \infty$$

The shortest wavelength present in Paschen series of spectral lines is obtained for $n = \infty$. (1)

$$\therefore \frac{1}{\lambda} = R \left[\frac{1}{3^2} - \frac{1}{\infty^2} \right] \text{ or } \frac{1}{\lambda} = R \left(\frac{1}{9} \right)$$

$\Rightarrow \lambda = \frac{9}{R} = \frac{9}{1.097 \times 10^7} \Rightarrow \lambda = 820.4 \text{ nm}$ (1)

Q.71 State limitations of Bohr's atomic model.

Sol Limitations of Bohr's atomic model Refer to text on page 261. (2)

Q.72 Explain the terms

- excitation energy and
- ionisation energy. [Textbook]

Sol Ground state and excited state Refer to text on page 260. (2)

Q.73 Express velocity of an electron in terms of fundamental constants.

$$\text{Sol } \therefore v_n = \frac{Ze^2}{2h\epsilon_0} \frac{1}{n}, \text{ where } n = 1, 2, 3, \dots \quad (1)$$

For $n = 1, Z = 1$

$$\therefore v_1 = \frac{e^2}{2h\epsilon_0} = \frac{c}{137} \quad (1)$$

3 MARKS Questions

Exams' Question

Q.74 ν_{21} and ν_{31} are frequencies of the first and second lines of Lyman series. If ν_{32} is the frequency of the first line of Balmer series, then establish the relation among ν_{21} , ν_{31} and ν_{32} . [2019, 2013]

Sol For Lyman series,

$$\nu = Rc \left[\frac{1}{1^2} - \frac{1}{n^2} \right], \text{ where } n = 2, 3, 4, \dots$$

$$\nu_{21} = Rc \left[\frac{1}{1^2} - \frac{1}{2^2} \right] = Rc \left(\frac{3}{4} \right) \quad (1/2)$$

$$\nu_{31} = Rc \left[\frac{1}{1^2} - \frac{1}{3^2} \right] = Rc \left(\frac{8}{9} \right) \quad (1/2)$$

$$\text{and for Balmer series, } \nu_{32} = Rc \left[\frac{1}{2^2} - \frac{1}{3^2} \right]$$

$$\nu_{32} = Rc \left(\frac{5}{36} \right) \quad (1/2)$$

Therefore, relation among ν_{21} , ν_{31} and ν_{32} is given by

$$\nu_{32} : \nu_{21} : \nu_{31} :: 5 : 27 : 32 \quad (1\frac{1}{2})$$

Important Questions

Q.75 The series limit wavelength of Balmer series is emitted as electron of hydrogen atom falls from $n \rightarrow \infty$ state to $n = 2$ state. What is the wavelength of this line?

$$\text{Sol } \text{Since, } \Delta E = E_\infty - E_2 \Rightarrow \frac{hc}{\lambda} = 0 - \left(\frac{-13.6}{2^2} \right) \quad (1)$$

$$\Rightarrow \frac{hc}{\lambda} = \frac{13.6}{4} \text{ eV} \Rightarrow \frac{hc}{\lambda} = \frac{13.6 \times 1.6 \times 10^{-19}}{4} \text{ J} \quad (1)$$

$$\Rightarrow \lambda = \frac{h \times c \times 4}{13.6 \times 1.6 \times 10^{-19}}$$

$$\Rightarrow \lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8 \times 4}{13.6 \times 1.6 \times 10^{-19}}$$

$$\therefore \lambda = 3.64 \times 10^{-7} \text{ m} \quad (1)$$

Q.76 Using Bohr's formula for energy quantisation, calculate

- (i) the longest wavelength in Lyman series of hydrogen atom

- (ii) the excitation energy of $n = 3$ level of He^+ atom and
(iii) the ionisation potential of ground level of Li^{2+} atom

$$\text{Sol } \text{(i) Since, } \frac{1}{\lambda} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$\text{For Lyman series, } \frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{n^2} \right]$$

where, $n = 2, 3, 4, \dots$

For longest wavelength $n = 2$,

$$\frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{2^2} \right]$$

$$\Rightarrow \lambda = \frac{4}{3R} = \frac{4}{3 \times 1.097 \times 10^7} = 1216 \text{ \AA} \quad (1)$$

$$\text{(ii) Using } E_n = \frac{-13.6Z^2}{n^2} \text{ eV}$$

$$E_3 - E_1 = -13.6 \times (2)^2 \left[\frac{1}{3^2} - \frac{1}{1^2} \right]$$

$$\Rightarrow E_3 - E_1 = 48.35 \text{ eV} \quad (1)$$

$$\text{(iii) Again using } E_n = \frac{-13.6Z^2}{n^2} \text{ eV,}$$

$$E_\infty - E_1 = -13.6 \times (3)^2 \left[\frac{1}{\infty} - \frac{1}{1^2} \right]$$

$$\Rightarrow E_\infty - E_1 = 122.4 \text{ eV} \quad (1)$$

7 MARKS Questions

Exams' Questions

Q.77 Explain Bohr's model for hydrogen atom and derive an expression for energy of the electron in the n th stationary state. [2018]

Or State Bohr's postulates for the theory of atomic model of hydrogen. Hence, deduce an expression for the total energy of the electron in the n th orbit. [2016]

Or Using Bohr's theory of hydrogen atom, prove that the total energy of electron in n th stationary orbit varies inversely as n^2 . Hence, calculate the ground state energy.

[Take, $m_e = 9.11 \times 10^{-31} \text{ kg}$, $h = 6.63 \times 10^{-34} \text{ J-s}$ and $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$] [2014, 2006]

Or State the postulates of Bohr's theory of hydrogen atom. Derive an expression for energy of an electron in n th orbit. [2012 Instant, 2010 Instant]

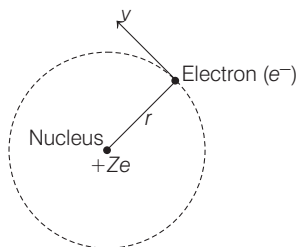
Or Write the postulates of Bohr's atomic model. Derive an expressions for radius of the n th stationary orbit and energy of an electron on this orbit according to Bohr's theory. [2010, 2008, 2002]

Or What are the postulates of Bohr's theory of hydrogen atom? Deduce an expression for the total energy of an allowed stationary state of hydrogen atom to show that it varies inversely as square of quantum number. What is the significance of the negative sign in the expression of total energy? [2006 Instant]

Or Explain the Bohr's model of hydrogen atom and obtain an expressions for its radius and energy of the n th state. [2012]

Sol Bohr's postulates Refer to text on pages 259 and 260. (3)

Expression for Total Energy A hydrogen atom consists of a tiny positively charged nucleus (with one proton) and an electron revolving in a stable circular orbit around the nucleus.



Let e , m and v be respectively, the charge, mass and velocity of the electron and r_n the radius of the orbit.

The positive charge on the nucleus is Ze , where Z is the atomic number (in case of hydrogen atom, $Z = 1$). As, the centripetal force is provided by the electrostatic force of attraction.

We have,

$$\frac{mv_n^2}{r_n} = \frac{1}{4\pi\epsilon_0} \cdot \frac{(Ze) \times e}{r_n^2} \text{ or } mv_n^2 = \frac{Ze^2}{4\pi\epsilon_0 r_n} \quad \dots(i) \quad (1)$$

From the second postulate, the angular momentum of the electron is

$$mv_n r_n = n \frac{h}{2\pi} \quad \dots(ii)$$

where, n ($= 1, 2, 3, \dots$) is principal quantum number.

From Eqs. (i) and (ii), we get

$$\text{Radius of } n\text{th orbit, } r_n = n^2 \frac{h^2 \epsilon_0}{\pi m Z e^2} \quad \dots(iii)$$

The energy E of an electron in an orbit is the sum of kinetic and potential energies.

Using Eq. (i), kinetic energy of electron is

$$\text{KE} = \frac{1}{2} mv_n^2 = \frac{Ze^2}{8\pi\epsilon_0 r_n} \quad \dots(iv) \quad (1)$$

The potential energy of the electron in an orbit of radius r_n due to the electrostatic attraction by the nucleus is given by

$$\begin{aligned} \text{PE} &= \frac{1}{4\pi\epsilon_0} \frac{(Ze)(-e)}{r_n} \\ &= -\frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r_n} \quad \dots(v) \end{aligned}$$

The total energy of the electron is therefore

$$E = \text{KE} + \text{PE} = \frac{Ze^2}{8\pi\epsilon_0 r_n} - \frac{Ze^2}{4\pi\epsilon_0 r_n} = -\frac{Ze^2}{8\pi\epsilon_0 r_n}$$

Substituting for r_n from Eq. (iii), we get

$$E_n = -\frac{mZ^2 e^4}{8\epsilon_0^2 h^2} \left(\frac{1}{n^2} \right)$$

where, $n = 1, 2, 3, \dots$. This is the expression for the energy of the electron in the n th orbit. (1)

For hydrogen atom $Z = 1$, substituting the standard values, we get $E_n = \frac{-13.6}{n^2}$ eV, negative energy of the

electron shows that the electron is bound to the nucleus and is not free to leave it.

In ground state, $n = 1$

$$E_1 = -13.6 \text{ eV} \quad (1)$$

Important Questions

Q.78 Explain Rutherford's α -particle scattering experiment. What information did it convey about the structure of an atom? Mention the limitations of Rutherford's atomic model.

Sol Rutherford's α -particle scattering experiment

Refer to text on pages 258 and 259. (4 + 1)

Limitations of Rutherford's Atomic Model

Refer to text on page 259. (2)

Q.79 Explain the meaning of the terms: Energy level, excitation potential, ground state. Write down the names of the series obtained in hydrogen spectrum. Drawing the excited energy diagram of hydrogen atom, show these series on an energy level diagram. [Textbook]

Sol Energy Level Refer to text on page 261. (1)

Excitation Potential The minimum energy required to excite an atom and is corresponding potential is called excitation potential,

$$V_{\text{excitation}} = \frac{E_{\text{exe}}}{e} \quad (1)$$

Ground State Refer to text on page 260. (1)

Series and Energy Level Diagram Refer to text and figures on page 261. (4)

Chapter Test

1 MARK Questions

- 1 Name one series of hydrogen spectrum which consists of lines of minimum frequencies.
- 2 Define Rydberg's constant. What is its value?
- 3 If the frequency of the incident light is doubled on a metallic plate, will kinetic energy of electrons be also doubled? [Textbook]
- 4 Derive an expression for the speed of electron in the n th orbit. [Textbook]
- 5 What is the radius of inner most orbit of hydrogen atom. [Textbook]
- 6 In Rutherford's experiment of scattering of α -particles, transfer of maximum energy is possible only when the scattering angle is ".....". [Textbook]
- 7 Emission spectra are due to transition of electrons from lower orbit to higher orbit. (True/False) [Textbook]
- 8 Write an expression for Bohr radius in hydrogen atom. [Textbook]

2 MARKS Questions

- 9 A 10 kg satellite circles, the earth once in two hours in an orbit of radius 800 km. If Bohr's theory applies to the satellite earth system, calculate the quantum n . [Textbook]
- 10 The second member of the Balmer series has a wavelength of 486.1 nm. Calculate the wavelength of the first member. [Textbook]
- 11 A doubly ionised lithium atom is hydrogen like with atomic number 3.
 - (i) Find the wavelength of radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit. The ionisation energy of the hydrogen atom is 13.6 eV.
 - (ii) How many spectral lines are observed in the emission of spectra of the above excited system. [Textbook]
- 12 Use Bohr's postulate for permitted orbits to prove that the circumference of the n th permitted orbit for the electron can contain exactly n wavelengths of the de-Broglie wavelength associated with the electron in that orbit. [Textbook]

3 MARKS Questions

- 13 What is the ratio of radii or orbits corresponding to first excited state and ground state in a hydrogen atom?
- 14 In hydrogen atom, if electron is replaced by a particle which is 200 times heavier but has same charge, how would its radius change?
- 15 The ground state energy of hydrogen atom is -13.6 eV. Calculate its kinetic and potential energies in this state.
- 16 Show that the tangential speed of an electron in its orbit is $\frac{Ze^2}{2\epsilon_0 n h}$ and its angular speed is $\frac{\pi m Z^2 e^2}{2\epsilon_0 n^3 h^2}$.
- 17 In Bohr's theory of hydrogen atom, calculate the energy of the photon emitted during a transition of the electron from the first excited state to the ground state. Write in which region the spectrum lies. [Textbook]

7 MARKS Questions

- 18** (i) Using Bohr's model, calculate the speed of electron in a hydrogen atom in $n = 1, 2$ and 3 levels. [Textbook]
 (ii) Calculate the orbital period in each of these levels.
- 19** Discuss different atomic models and compare their merits and demerits. [Textbook]

HINTS and ANSWERS

- 1.** Pfund series.
- 3.** Yes, because kinetic energy = |Energy| = $h\nu$,
 i.e. $\text{KE} \propto \nu$.
- 5.** 0.53 \AA .
- 6.** small.
- 7.** False, **higher** to **lower**.
- 9.** Hint $mvr = \frac{nh}{2\pi} \Rightarrow n = \frac{2\pi mvr}{h}$
 Here, $n = \frac{2\pi r}{T} = \frac{2\pi (80 \times 1000)}{2 \times 60 \times 60}$ [Ans. 380×10^{45}]
- 10.** Hint $\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$ [Ans. 3280 nm]
- 11.** Hint (i) $\frac{1}{\lambda} = Z^2 R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$
 (ii) 3 spectral lines are observed
 [Ans. $\lambda = 911.6 \text{ \AA}$]
- 13.** Hint $r_n = \frac{n^2 h^2 \epsilon_0}{\pi m Z e^2}$, $\frac{r_2}{r_1} = \frac{4}{1}$ [Ans. 4 : 1]
- 14.** $r_n = \frac{n^2 h^2 \epsilon_0}{\pi m e^2}$
 $\Rightarrow r \propto \frac{1}{m}$
 So, radius will become $\frac{1}{200}$ times, i.e. will reduce.
- 15.** Hint $\text{KE} = \frac{mZ^2 e^4}{8\epsilon_0 n^2 h^2}$
 and $\text{PE} = \frac{mZ^2 e^4}{4\epsilon_0 n^3 h^2}$ [Ans. $13.6 \text{ eV}, -27.2 \text{ eV}$]
- 16.** Hint Angular speed $\omega = \frac{v}{r}$
- 18.** Hint $v_n = \frac{Ze^2}{2n\epsilon_0 n}$