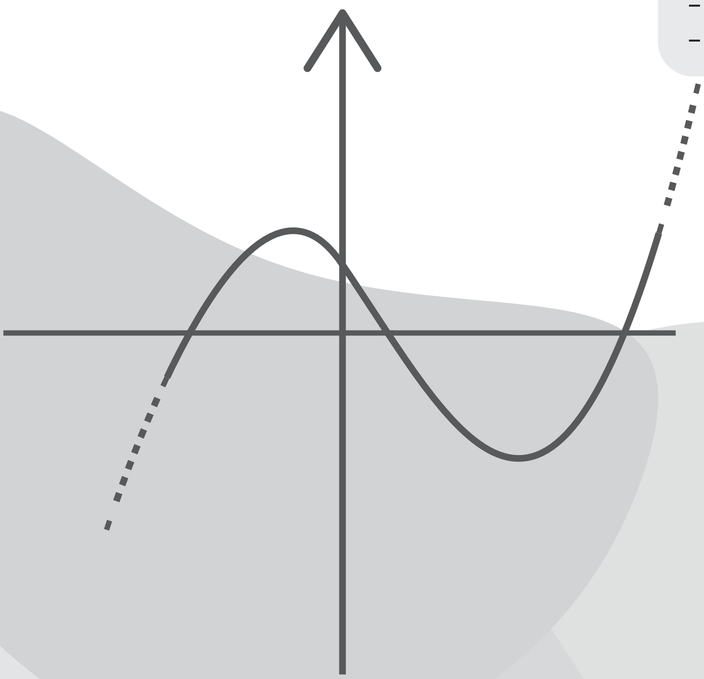
2

Polynomials



Objective Section MAP

POLYNOMIALS

Eg: $19x^5 + 8x + 1,\ 7x^4 + 3$

TYPES OF POLYNOMIAL

Based on number of terms

Monomial

- A polynomial with just one term.

Eg: $9x,\ 6x^2$

Binomial

- A polynomial with two terms.

Eg: $\left(\frac{4}{9}\right)x^2 + 6x,\ 7x + 9$

Trinomial

- A polynomial with three terms.

Eg: $6x^4 + x + 6$

Based on degree of Polynomial

Linear Polynomial

- Polynomial of degree 1.
- Has exactly one zero.
- Standard form- $p(x)$: $ax + b$, where $a \neq 0$

Eg: $6x + 17,\ -5x$

Quadratic Polynomial

- Polynomial of degree 2.
- Can have at most 2 zeroes.
- Standard form- $p(x)$: $ax^2 + bx + c,\ a \neq 0$

Eg: $9x^2 + 2x + 15$

Cubic Polynomial

- Polynomial of degree 3
- Can have at most 3 zeroes.
- Standard form- $p(x)$: $ax^3 + bx^2 + cx + d,\ a \neq 0$

Eg: $17x^3 - 9x^2 + 6$

RELATIONSHIP BETWEEN ZEROES AND COEFFICIENT OF A POLYNOMIAL

- If $p(x)$ is a polynomial in x, and if k is any real number, then the value obtained by replacing x by k in $p(x)$, is called the value of $p(x)$ at $x = k$, and is denoted by $p(k)$.
- A real number k is said to be a zero of a polynomial $p(x)$, if $p(k) = 0$

Zeroes of the Quadratic Polynomial

- If α and β are the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c,\ a \neq 0$ then

Sum of zeroes $= \alpha + \beta = -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of Zeroes $= \alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Zeroes of Cubic Polynomial

- If α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, then

$\alpha+\beta+\gamma = -\frac{b}{a} = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$\alpha\beta+\beta\gamma+\alpha\gamma = \frac{c}{a} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

$\alpha\beta\gamma = -\frac{d}{a} = -\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Zeroes of Linear Polynomial

- If k is a zero of $p(x) = ax + b$, then $p(k) = ak + b = 0$,

i.e., $k = -\frac{b}{a}$. So, the zero of the linear polynomial $ax + b$

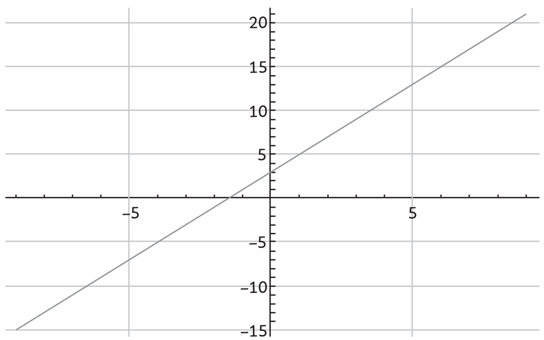
is $= -\frac{b}{a} = -\frac{\text{Constant term}}{\text{Coefficient of } x}$

GRAPH OF POLYNOMIAL

Geometrical representation of a linear Polynomial

- Graph of a linear polynomial $p(x) = ax + b$ is a straight line. It cuts the x-axis at exactly one point.
- In the $p(x) = ax + b$ for degree 1, a represent the slope of a line, the constant b represents y - intercept of a line.

Eg: $y = 2x + 3$
Here, $a = 2$ and $b = 3$

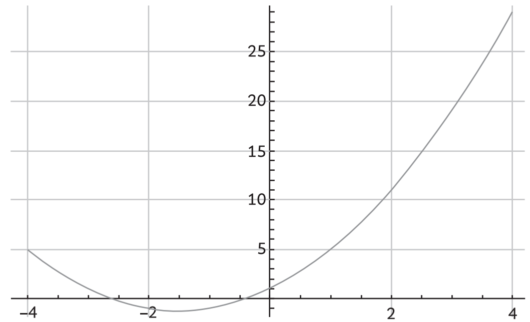


Note: *All constant function are linear function.*

Geometrical representation of a Quadratic Polynomial

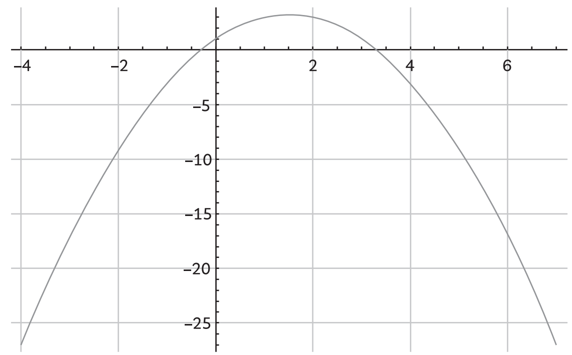
- Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open upwards , if $a > 0$.

Eg: $y = x^2 + 3x + 1$



- Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open downwards, if $a < 0$.

Eg: $y = -x^2 + 3x + 1$



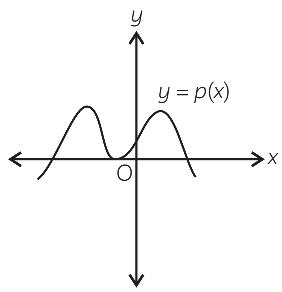
Note: *A polynomial $p(x)$ of degree n has at most n zeroes.*

OBJECTIVE Type Questions

[**1** mark]

Multiple Choice Question

1. **The number of zeroes for a polynomial $p(x)$ whose graph is given in figure-1, is:**



(a) 4 (b) 3

(c) 5 (d) 1 **[CBSE 2020]**

Ans. *(b) 3*

Explanation: Since the curve $y = p(x)$ cuts or meets the x-axis at three points, the function $p(x)$ has three zeroes.

2. **The zeroes of the polynomial $x^2 - 3x - m(m + 3)$ are:**

(a) $m, m + 3$ (b) $-m, m + 3$

(c) $m, -(m + 3)$ (d) $-m, -(m + 3)$

[CBSE 2020]

Ans. *(b) $-m, m + 3$*

Explanation:

Given, polynomial can be rewritten as

$x^2 - (m + 3)x + mx - m(m + 3)$

$= x[x - (m + 3)] + m[x - (m + 3)]$

$= [x - (m + 3)]\ [x + m]$

Hence, the two zeroes are $m + 3$ and $-m$.

3. **If one of the zeroes of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is**

(a) 10 (b) -10

(c) -7 (d) -2

Ans. *(b) -10*

Explanation:

Let, $p(x) = x^2 + 3x + k$

Since, 2 is one of the zero of p(x)

$\therefore \quad p(2) = 0$

$\Rightarrow 2^2 + 3(2) + k = 0$

$\Rightarrow \quad 4 + 6 + k = 0$

$\Rightarrow \quad k = -10$

4. **The quadratic polynomial, the sum of whose zeroes is -5 and their product is 6, is**

(a) $x^2 + 5x + 6$ (b) $x^2 - 5x + 6$

(c) $x^2 - 5x - 6$ (d) $-x^2 + 5x + 6$

[CBSE 2020]

Ans. *(a)* $x^2 + 5x + 6$

Explanation: A polynomial, in which sum of zeroes is -5 and product of zeroes is 6, is :

$x^2 + 5x + 6$

Since, the equadratic equation is:

$x^2 -$ (sum of roots) x + product of roots = 0

5. **If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and -3, then:**

(a) $a = -7, b = -1$ (b) $a = 5, b = -1$

(c) $a = 2, b = -6$ (d) $a = 0, b = -6$

[Diksha]

Ans. *(d) $a = 0, b = -6$*

Explanation: Let $p(x) = x^2 + (a + 1)x + b$.

We know that if α is one of the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, then $p(\alpha) = 0$.

It is given that 2 and -3 are the zeroes of the given quadratic polynomial.

Therefore, $p(2) = 0$ and $p(-3) = 0$

$p(2) = (2)^2 + (a + 1)(2) + b = 0$

$\Rightarrow \quad 4 + 2a + 2 + b = 0$

$\Rightarrow \quad 2a + b + 6 = 0 \quad$ ...(i)

Also, $p(-3) = (-3)^2 + (a + 1)(-3) + b = 0$

$\Rightarrow \quad 9 - 3a - 3 + b = 0$

$\Rightarrow \quad -3a + b + 6 = 0 \quad$ ...(ii)

From (i) and (ii), we get

$2a + b + 6 = -3a + b + 6$

$\Rightarrow \quad 5a = 0$

$\Rightarrow \quad a = 0$

Putting the value of 'a' in (i), we have

$2(0) + b + 6 = 0$

$\Rightarrow \quad b = -6$

Hence, if the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and -3, then the required values of a and b are $a = 0$ and $b = -6$.

Alternate Method:

It is given that 2 and -3 are the zeroes of the given quadratic polynomial.

Sum of the zeroes = $2 + (-3) = -1 \quad$ ...(i)

Product of the zeroes = $2(-3) = -6 \quad$ ...(ii)

The equation of a quadratic polynomial is given by

$p(x) = k\{x^2$ – (sum of the zeroes)x
+ (product of the zeroes)$\}$,

where, k is a constant.

Here, $p(x) = x^2 + (a + 1)x + b$.

Comparing the two equations we get:

Sum of the zeroes = – (coefficient of x) ÷ coefficient of x^2

$\Rightarrow$ sum of the zeroes = $-(a + 1)$

$\Rightarrow$ $-1 = -a - 1$ [Using (i)]

$\Rightarrow$ $-1 + 1 = -a$

$\Rightarrow$ $-a = 0$

$\Rightarrow$ $a = 0$

Product of the zeroes = constant term ÷ coefficient of x^2

$\Rightarrow$Product of the zeroes = b

$-6 = b$ [Using (ii)]

$\Rightarrow$ $b = -6$

Hence, if the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and –3, then the required values of a and b are $a = 0$ and $b = -6$.

6. **The number of polynomials having zeroes as –2 and 5 is:**

(a) 1 (b) 2

(c) 3 (d) more than 3

[NCERT]

Ans. *(d) more than 3*

Explanation: A quadratic polynomial is given by

$p(x) = k\{x^2$ – (sum of the zeroes)x
+ (product of the zeroes)$\}$,

where k is a constant.

Sum of the zeroes = – (coefficient of x) ÷ coefficient of x^2

and product of the zeroes = constant term ÷ coefficient of x^2

Sum of the zeroes = $-2 + 5 = 3$

and product of the zeroes = $(-2)5 = -10$

A quadratic polynomial is given by

$= k\{x^2$ – (sum of the zeroes)x
+ (product of the zeroes)$\}$

which becomes,

$= k\{x^2 - 3x - 10\}$

where k is any real number.

Thus, we can say that $kx^2 - 3kx - 10k$ will also have –2 and 5 as their zeroes.

As k can take any real value, there can be infinite polynomials having –2 and 5 as their zeroes.

Hence, the required number of polynomials are infinite i.e., more than 3.

7. **Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is:**

(a) $-\frac{c}{a}$ (b) $\frac{c}{a}$

(c) 0 (d) $-\frac{b}{a}$ **[CBSE 2012]**

Ans. *(b)* $\frac{c}{a}$

Explanation: Let $p(x) = ax^3 + bx^2 + cx + d$.

It is given that one of the zeroes of the cubic polynomial $p(x)$ is zero.

Let α, β and γ be the zeroes of the polynomial

$$p(x) = ax^3 + bx^2 + cx + d$$

And let $\alpha = 0$ [Given]

We know that:

Sum of the product of two zeroes at a time = coefficient of x ÷ coefficient of x^3 i.e.,

Sum of the product of two zeroes at a time = $\frac{c}{a}$

$\Rightarrow$ $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

$\Rightarrow$ $0 \times \beta + \beta\gamma + \gamma \times 0 = \frac{c}{a}$ [Using $\alpha = 0$]

$\Rightarrow$ $0 + \beta\gamma + 0 = \frac{c}{a}$

$\Rightarrow$ $\beta\gamma = \frac{c}{a}$

Hence, product of the other two zeroes is $\frac{c}{a}$.

8. **If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is –1, then the product of the other two zeroes is:**

(a) $b - a + 1$ (b) $b - a - 1$

(c) $a - b + 1$ (d) $a - b - 1$

[CBSE 2012]

Ans. *(a) $b - a + 1$*

Explanation: Let $p(x) = x^3 + ax^2 + bx + c$

Let α, β and γ be the zeroes of the polynomial

$$p(x) = x^3 + ax^2 + bx + c$$

And $\alpha = -1$ [Given]

We know that if α is one of the zeroes of the quadratic polynomial p(x) = $ax^2 + bx + c$, then $p(\alpha) = 0$.

$\Rightarrow$ $p(\alpha) = p(-1) = 0$

$\Rightarrow$ $(-1)^3 + (-1)^2a + (-1)b + c = 0$

$\Rightarrow$ $-1 + a - b + c = 0$

$\Rightarrow$ $c = 1 - a + b$ ...(i)

We know that:

Product of the zeroes= – constant term ÷ coefficient of x^3

i.e.,Product of zeroes = $-\frac{c}{1}$

$\Rightarrow$ $\alpha\beta\gamma = -c$

$\Rightarrow$ $(-1)\beta\gamma = -c$ [Using $\alpha = -1$]

$\Rightarrow \qquad \beta\gamma = c$

$\Rightarrow \qquad \beta\gamma = 1 - a + b$

[From equation (i)]

Hence, product of the other two zeroes is $1 - a + b$ or $b - a + 1$.

9. If α, β are the zeros of the polynomial $5x^2 - 7x + 2$, then the sum of their reciprocal is:

(a) $\frac{7}{2}$ (b) $\frac{7}{5}$

(c) $\frac{2}{5}$ (d) $\frac{14}{25}$

Ans. *(a)* $\frac{7}{2}$

Here, $\alpha + \beta = \frac{-b}{a} = \frac{-(-7)}{5} = \frac{7}{5}$

and $\alpha\beta = \frac{c}{a} = \frac{2}{5}$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{\frac{7}{5}}{\frac{2}{5}} = \frac{7}{2}$$

10. The degree of the polynomial $(x + 1)(x^2 - x + x^4 - 1)$ is:

(a) 2 (b) 3

(c) 4 (d) 5

Ans. (d) *5*

Given polynomial can be written as,

$x^5 + x^4 + x^3 - 2x - 1$

This is a polynomial of degree 5.

11. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are:

(a) both positive

(b) both negative

(c) one positive and one negative

(d) both equal **[NCERT]**

Ans. *(b) both negative*

Explanation: Let $p(x) = x^2 + 99x + 127$

Then zeroes of the polynomial are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-99 \pm \sqrt{(99)^2 - 4(127)}}{2}$$

$$x = \frac{-99 \pm \sqrt{9801 - 508}}{2}$$

$$x = \frac{-99 \pm 96.4}{2}$$

$$x = \left(-\frac{2.6}{2}, -\frac{195.4}{2}\right)$$

$$x = (-1.3, -97.7)$$

Hence both the zeroes are negative.

12. The zeroes of the quadratic polynomial $x^2 + kx + k$, where $k \neq 0$,

(a) cannot both be positive

(b) cannot both be negative

(c) are always unequal

(d) are always equal **[NCERT]**

Ans. *(a) cannot both be positive*

Explanation: Let $p(x) = x^2 + kx + k$ where $k \neq 0$.

On comparing $p(x)$ with $ax^2 + bx + c$, we get

$a = 1$, $b = k$ and $c = k$.

Let α and β be the zeroes of the polynomial $p(x)$.

We know that:

Sum of the zeroes

$$\alpha + \beta = -\frac{b}{a}$$

$$\Rightarrow \qquad \alpha + \beta = -\frac{k}{1} = -k \qquad \text{...(i)}$$

And product of the zeroes

$$\alpha\beta = \frac{c}{a}$$

$$\Rightarrow \qquad \alpha\beta = \frac{k}{1} = k \qquad \text{...(ii)}$$

Case 1: k is negative

If k is negative,

$\alpha\beta$ [from equation (ii)] is negative.

It means α and β are of the opposite sign.

$\Rightarrow$ Both the zeroes are of the opposite signs.

Case 2: k is positive

If k is positive,

$\alpha\beta$ (from equation (ii)) is positive but $\alpha + \beta$ is negative.

If the product of the two numbers is positive, then either both are negative or both are positive. But the sum of these numbers is negative, so the numbers must be negative.

$\Rightarrow$ Both the zeroes are negative.

Hence, in both the cases, both the zeroes cannot be positive.

Alternate Method:

Let $\quad p(x) = x^2 + kx + k$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-k \pm \sqrt{k^2 - 4k}}{2}$$

$$x = \frac{-k \pm \sqrt{k(k-4)}}{2}$$

$\Rightarrow \qquad k(k - 4) > 0$

$\Rightarrow \qquad k \in (-\infty, 0) \cup (4, \infty)$

Here, k lies in two intervals; therefore, we need to consider both the intervals separately.

Case 1:

When $\quad k \in (-\infty, 0)$

i.e., $\quad k < 0$

We know that in a quadratic equation

$$p(x) = ax^2 + bx + c,$$

if either $a > 0$, $c < 0$ or $a < 0$, $c > 0$, then the zeroes of the polynomial are of the opposite signs.

Here, $a = 1 > 0$, $b = k < 0$ and $c = k < 0$.

$\Rightarrow$ Both the zeroes are of the opposite signs.

Case 2:

When $\quad k \in (4, \infty)$

i.e., $\quad k > 0$

We know, in a quadratic polynomial, if the coefficient of the terms are of the same sign, then the zeroes of the polynomial are negative.

i.e., if either $a > 0$, $b > 0$ and $c > 0$ or $a < 0$, $b < 0$ and $c < 0$, then both the zeroes are negative.

Here, $a = 1 > 0$, $b = k > 0$ and $c = k > 0$.

$\Rightarrow$ Both the zeroes are negative.

Hence, in both cases, both the zeroes cannot be positive.

13. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, where $c \neq 0$ are equal, then:

(a) c and a have opposite signs

(b) c and b have opposite signs

(c) c and a have the same sign

(d) c and b have the same sign [NCERT]

Ans. *(c) c and a have the same sign*

Explanation: Given that the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, where

$c \neq 0$, are equal.

The zeroes of a quadratic polynomial are equal when the discriminant is equal to 0

i.e., $\quad D = 0$

$$b^2 - 4ac = 0$$

$$\Rightarrow \quad 4ac = b^2$$

$$\Rightarrow \quad ac = \frac{b^2}{4} > 0$$

[square of any positive or negative number is positive]

$$\Rightarrow \quad ac > 0$$

Therefore, for $ac > 0$, a and c must have the same sign

i.e., either $a > 0$ and $c > 0$ or $a < 0$ and $c < 0$.

Alternate Method:

Given that the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, where $c \neq 0$, are equal.

Let α and β be the zeroes of the polynomial $p(x)$.

If α and β are equal, these must have the same sign (both positive or both negative).

$$\Rightarrow \quad \alpha\beta > 0$$

Product of zeroes

$$\alpha\beta = \frac{c}{a}$$

$$\Rightarrow \quad \frac{c}{a} > 0 \quad \text{[Using } \alpha\beta > 0]$$

As $\frac{c}{a} > 0$, which is only possible when a and c have the same signs, so α and β have the same sign.

14. If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it:

(a) has no linear term and the constant term is negative.

(b) has no linear term and the constant term is positive.

(c) can have a linear term but the constant term is negative.

(d) can have a linear term but the constant term is positive. [NCERT]

Ans. *(a) has no linear term and the constant term is negative.*

Explanation: Let

$$p(x) = x^2 + ax + b.$$

And let α be one of the zeroes, and

$-\alpha$ is the other zero of the polynomial $p(x)$.

[Given]

Product of the zeroes= constant term $\div$ coefficient of x^2

Product of the zeroes $= \frac{b}{1}$

$$\alpha(-\alpha) = b$$

$$-\alpha^2 = b \quad \text{i.e.,} \quad b < 0$$

i.e., the constant term is negative.

Sum of the zeroes= – (coefficient of x) $\div$ coefficient of x^2

$$\alpha - \alpha = -\frac{a}{1}$$

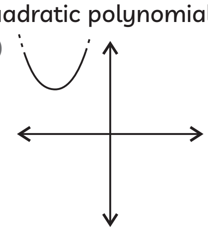
$$0 = -a$$

$$\Rightarrow \quad a = 0$$

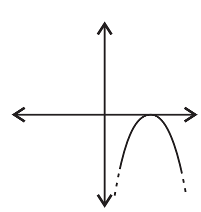
Hence, it has no linear term and the constant term is negative.

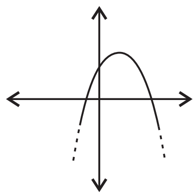
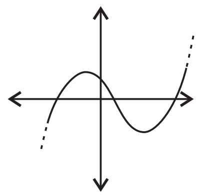
15. Which of the following is not the graph of a quadratic polynomial?

(a)



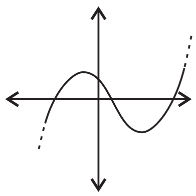
(b)



(c)  (d) 

[NCERT]

Ans. *(d)*



Explanation: From the given options only option D has more than two roots so it cannot be graph of quadratic polynomial.

For any quadratic polynomial $ax^2 + bx + c$, $a \neq 0$, the graph of the corresponding polynomial $ax^2 + bx + c$, has one of the two shapes: either open upwards like $\cup$ (parabolic shape) or open downwards like $\cap$ (parabolic shape), depending on whether $a > 0$ or $a < 0$. These curves are called parabolas. So, option (d) cannot be possible.

Alternate Method:

Also, the curve of a quadratic polynomial crosses the x-axis at most two points but in option (d), the curve crosses the x–axis at three points, so it does not represent a quadratic polynomial.

Hence, (d) is not the graph of a quadratic polynomial.

Fill in the Blanks

Fill in the blanks/tables with suitable information:

16. **If one root of the equation $(k - 1)x^2 - 10x + 3 = 0$ is the reciprocal of the other, then the value of k is ....**

Ans. *4*

17. **The sum and product of the zeroes of a quadratic polynomial are 3 and – 10 respectively. The quadratic polynomial is ....**

Ans. $x^2 - 3x - 10$

Explanation: Sum of zeroes = 3

Product of zeroes = – 10

Quadratic polynomial

x^2 – (sum of zeroes) x + product of zeroes

$= x^2 - 3x - 10$

18. **If two of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ are 0, then the third zero is ....**

Ans. $\dfrac{-b}{a}$

Explanation: Two zeroes of the cubic polynomial are zero.

$$\text{sum of zeroes} = \frac{-b}{a}$$

$$\Rightarrow \quad (0 + 0 + x) = \frac{-b}{a}$$

(where, x is the third zero)

$$x = \frac{-b}{a}$$

19. **Zeroes of $p(x) = x^2 - 2x - 3$ are ....**

Ans. *3 and – 1*

Explanation: We have, $x^2 - 2x - 3$

$x^2 - 3x + x - 3$

$\Rightarrow x(x - 3) + 1(x - 3)$

$\Rightarrow (x - 3)(x + 1)$

zeroes of $p(x)$ are 3 and – 1.

20. **If $x - 2$ is a factor of the polynomial $x^3 - 6x^2 + ax - 8$, then the value of a is equal to ....**

Ans. *12*

Explanation: $(x - 2)$ is factor of polynomial $p(x)$

$p(x) = x^3 - 6x^2 + ax - 8$

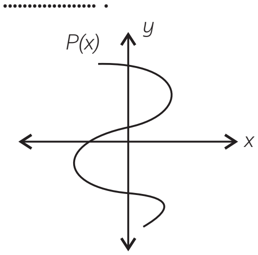
Therefore, $x = 2$ is a zero of polynomial

$$p(2) = 0$$

$\Rightarrow 2^3 - 6(2)^2 + 2a - 8 = 0$

$\Rightarrow 8 - 24 + 2a - 8 = 0 \Rightarrow 2a = 24 \Rightarrow a = 12$

21. **The number of zeroes of $p(x)$ in the given figure is ....**



Ans. *1*

Explanation: The graph $p(x)$ intersects the x-axis at only one point.

So, number of zero is 1.

22. **If the sum of the zeroes of the quadratic polynomial $kx^2 + 2x + 3k$ is equal to the product of its zeroes then k = ...**

Ans. $\dfrac{-2}{3}$

Explanation: Given, polynomial $P(x) = kx^2 + 2x + 3k$

$$\text{sum of zeroes} = \frac{-2}{k}$$

$$\text{Product of zeroes} = \frac{3k}{k} = 3$$

According to question,

$$\frac{-2}{k} = 3 \Rightarrow k = \frac{-2}{3}$$

23. **If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - x - 4$, find the value of $\frac{1}{\alpha}+\frac{1}{\beta} \quad \alpha\beta$ = ...**

Ans. $\frac{15}{4}$

Explanation: $f(x) = x^2 - x - 4$

Let α and β are the zeroes of $f(x)$

$$\therefore \quad \alpha + \beta = \frac{(-1)}{1} = 1$$

$$\alpha\beta = \frac{-4}{1} = -4$$

$$\text{So, } \frac{1}{\alpha}+\frac{1}{\beta} - \alpha\beta = \frac{\alpha+\beta}{\alpha\beta} - \alpha\beta$$

$$= \frac{1}{-4} - (-4) = -\frac{1}{4} + 4 = \frac{15}{4}$$

24. **A monomial has .... term/terms.**

Ans. *One*

Explanation: A monomial is number, variable or a product of a number and variable where all exponents are whole numbers.

Example : 42, $5x$, $2xy - 2$.

Very Short Answer Type Questions

25. **Form a quadratic polynomial, the sum and product of whose zeroes are (–3) and 2 respectively. [CBSE 2020]**

Ans. A general form of a quadratic polynomial is

$$ax^2 + bx + c$$

Here, $\alpha + \beta = -\frac{b}{a} = -3$ and $\alpha.\beta = -\frac{b}{a} = 2$

where, α and β are the roots of given polynomial.

So, the required polynomial is $x^2 + 3x + 2$.

26. **Find the value of k for which the roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other. [CBSE 2019]**

Ans. Given, equation is $3x^2 - 10x + k = 0$, where roots are reciprocals of each other.

Let the roots be α and $\frac{1}{\alpha}$

$$\therefore \text{ Product of roots} = \frac{c}{a}$$

$$\Rightarrow \qquad \alpha . \frac{1}{\alpha} = \frac{k}{3} \qquad [\because a = 3, b = -10, c = k]$$

$$\Rightarrow \qquad 1 = \frac{k}{3}$$

$$\Rightarrow \qquad k = 3$$

27. **Determine the degree of the polynomial $(x + 1)(x^2 - x - x^4 + 1)$.**

Ans. Given polynomial in standard form is :

$-x^5 - x^4 + x^3 + 1$

So, its degree is 5

28. **If the product of two zeros of the polynomial $p(x) = 2x^3 + 6x^2 - 4x + 9$ is 3, find the third zero of the polynomial.**

Ans. If α, β, and γ be the three zeros of $p(x)$. Then,

$$\alpha\beta\gamma = -\frac{9}{2}$$

Since, $\qquad \alpha\beta = 3,$

we get $\qquad \gamma = -\frac{9}{2} \times \frac{1}{3} = -\frac{3}{2}$

Thus, the third zero of $p(x)$ is $-\frac{3}{2}$.

29. **If α and β are the zeros of the polynomial $p(x) = 4x^2 - 2x - 3$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.**

Ans. Here, $\qquad \alpha + \beta = \frac{2}{4}$ or $\frac{1}{2}$ and $\alpha.\beta = \frac{-3}{4}$.

$$\text{So, } \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha.\beta} = \frac{\frac{1}{2}}{\frac{-3}{4}} = -\frac{2}{3}$$

30. **If one of the zeros of polynomial $p(x) = (k - 1)x^2 - kx + 1$ is –3, find the value of k.**

Ans. Since, (–3) is a zero of $p(x)$, we have,

$$(k - 1)(-3)^2 - k(-3) + 1 = 0$$

$$\Rightarrow \qquad 9k - 9 + 3k + 1 = 0,$$

$$\Rightarrow \qquad 12k = 8 \Rightarrow k = \frac{2}{3}$$

31. **If α and β be the roots of the equation $x^2 - 1 = 0$, then show that: $\alpha + \beta = \frac{1}{\alpha} + \frac{1}{\beta}$**

Ans. Here, $\qquad \alpha + \beta = \frac{0}{1} = 0$

$$\left[\because \text{ Sum of roots} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}\right]$$

Also, $\qquad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{0}{1} = 0$

Thus, $\qquad \alpha + \beta = \frac{1}{\alpha} + \frac{1}{\beta}$

SHORT ANSWER (SA-I) Type Questions

[**2** marks]

32. A teacher asked 10 of his students to write a polynomial in one variable on a paper and then to handover the paper. The following were the answer given by the students:

$2x + 3$, $3x^2 + 7x + 2$, $4x^3 + 3x^2 + 2$, $x^3 + \sqrt{3x} + 7$, $7x + \sqrt{7}$, $5x^3 - 7x + 2$, $2x^3 + 3 - \frac{5}{x}$, $5x - \frac{1}{2}$, $ax^3 + bx^2 + cx + d$, $x + \frac{1}{x}$.

Answer the following questions :

(A) How many of the above ten are not polynomials?

(B) How many of the above ten are quadratic polynomials? [CBSE 2020]

Ans. (A) Three, namely: $x^3 + \sqrt{3x} + 7$,

$$2x^2 + 3 - \frac{5}{x}, x + \frac{1}{x}$$

(As they contain square roots of the variable and negative power of x)

(B) One, namely $3x^2 + 7x + 2$

33. If one of the zeroes of the quadratic polynomial $f(x) = 4x^2 - 8kx - 9$ is equal in magnitude but opposite in sign of the other, then find the value of k. [Diksha]

Ans. $f(x) = 4x^2 - 8kx - 9$

Let one of the zeroes of the polynomial be α and the other zeroes be $-\alpha$

$$\text{Sum of zeroes} = \left(-\frac{b}{a}\right) = \frac{8k}{4}$$

$$\alpha + (-\alpha) = 0$$

So, $\frac{8k}{4} = 0 \Rightarrow k = 0$

34. Can $(x - 5)$ be the remainder on division of a polynomial $p(x)$ by $(x + 8)$? [CBSE 2014]

Ans. *No.*

We know that we cannot divide the polynomials which have same degree.

As we can see that degree of $(x - 5)$ = degree of $(x + 8)$

So, they are not divisible.

35. If the zeros of the polynomial $x^3 - 2x^2 + x + 1$ are $a - b$, a and $a + b$, then find the values of a and b.

Ans. As $(a - b)$, a and $(a + b)$ are zeros of $x^3 - 3x^2 + x + 1$, we have:

$$a - b + a + a + b = 3$$

$\Rightarrow$ $3a = 3$, or $a = 1$ ...(i)

$$a(a - b) + a(a + b) + (a - b)(a + b) = 1$$

$\Rightarrow$ $3a^2 - b^2 = 1$ ...(ii)

and $(a - b)\,a\,(a + b) = -1$

$\Rightarrow$ $a(a^2 - b^2) = -1$ ...(iii)

From (i) and (ii), we have $b = \pm\sqrt{2}$

Thus, $a = 1, b = \pm\sqrt{2}$

36. What number should be added to the polynomial $x^2 - 5x + 4$ so that 3 is the zero of the polynomial?

Ans. Let k be the number to be added to the given polynomial.. Then the polynomial becomes $x^2 - 5x + (4 + k)$

As 3 is the zero of the polynomial, we get:

$$(3)^2 - 5(3) + (4 + k) = 0$$

$\Rightarrow$ $(4 + k) = 15 - 9$

$\Rightarrow$ $4 + k = 6$

$\Rightarrow$ $k = 2$

Thus, 2 is to be added to the polynomial.

37. If the zeroes of a polynomial $x^2 - 8x + k = 0$, is the HCF of (6, 12), then find the value of k. [Diksha]

Ans. HCF of (6, 12) = 6

So, 6 is one of the roots of the polynomial.

$$f(x) = x^2 - 8x + k = 0$$

$$f(6) = (6)^2 - 8(6) + k = 0$$

$$36 - 48 + k = 0$$

$$-12 + k = 0 \Rightarrow k = 12.$$

SHORT ANSWER (SA-II) Type Questions

[**3** marks]

38. Find the quadratic polynomial sum and product of whose zeroes are – 1 and – 20 respectively. Also, find the zeroes of the polynomial so obtained. [CBSE 2019]

Ans. Let α and β the zeroes of the polynomial.

Given : sum of zeroes, $\alpha + \beta = -1$

product of zeroes, $\alpha\beta = -20$

Equation of polynomial :

x^2 – (sum of zeroes) x + product of zeroes = 0

$\therefore \quad x^2 - (-1)x + (-20) = 0$

$\Rightarrow \quad x^2 + x - 20 = 0$

On spliting the middle term,

$x^2 + 5x - 4x - 20 = 0$

$\Rightarrow \quad x(x + 5) - 4(x + 5) = 0$

$\Rightarrow \quad (x - 4)(x + 5) = 0$

$\Rightarrow \quad x = 4, -5$

Hence, the zeroes of the polynomial are 4 and –5.

39. Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c, a \neq 0, c \neq 0$. [CBSE 2020]

Ans. Let α, β be the zeroes of $f(x) = ax^2 + bx + c$. Thus

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

Now, $$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-b/a}{c/a} = -\frac{b}{c}$$

$$\frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{c/a} = \frac{a}{c}$$

$\therefore$ Polynomial is: x^2 – (sum of roots) x – product roots

$$x^2 - \left(\frac{-b}{c}\right)x + \frac{a}{c} = 0$$

$\Rightarrow \quad cx^2 + bx + a = 0$

So, the required polynomial is $cx^2 + bx + a$.

40. If the zeroes of the polynomial $x^2 + px + q$ are double the value to the zeroes of $2x^2 - 5x - 3$, find the value of p and q. [Diksha]

Ans. Let α and β are zeroes of the $2x^2 - 5x - 3$

$$\alpha + \beta = -\frac{b}{a} = \frac{5}{2} \quad \text{...(i)}$$

$$\alpha\beta = \frac{c}{a} = -\frac{3}{2} \quad \text{...(ii)}$$

According to the question,

2α and 2β are zeroes of $x^2 + px + q$

$$2\alpha + 2\beta = -p \Rightarrow 2(\alpha + \beta) = -p$$

$$2 \times \left(\frac{5}{2}\right) = -p \quad \text{[from eqn. (i)]}$$

$$p = -5$$

$$2\alpha \times 2\beta = q \Rightarrow 4\alpha\beta = q$$

$$4 \times \left(-\frac{3}{2}\right) = q \quad \text{[from eqn. (ii)]}$$

$$q = -6$$

Hence, $p = -5$ and $q = -6$.

41. Find the value of k such that the polynomial $x^2 - (k + 6)x + 2(2k - 1)$ has the sum of its zeros equal to half of their product. [CBSE 2019]

Ans. Given polynomial is :

$$p(x) = x^2 - (k + 6)x + 2(2k - 1)$$

In the given quadratic equation:

$$a = 1$$
$$b = -(k + 6)$$
$$c = 2(2k - 1)$$

$$\text{Sum of zeroes} = -\frac{b}{a} = k + 6 \quad \text{...(i)}$$

$$\text{Product of zeroes} = \frac{c}{a} = 2(2k - 1) \quad \text{...(ii)}$$

According to the given condition:

$$\text{Sum of the zeroes} = \frac{1}{2} \times \text{Product of zeroes}$$

$\Rightarrow \quad k + 6 = \frac{1}{2} \times 2(2k - 1)$

$\Rightarrow \quad k + 6 = 2k - 1$

$\Rightarrow \quad 2k - k = 6 + 1$

$\Rightarrow \quad k = 7$

Hence, the value of k is 7.

42. Find the zeroes of following polynomials by factorisation method and verify relation between the zeroes and coeffcients of polynomials.

(A) $2x2 + \frac{7}{2}x + \frac{3}{4}$ [NCERT]

(B) $2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$ [NCERT]

(C) $7y^2 - \frac{11}{3}y - \frac{2}{3}$ [NCERT]

Ans.

(A) Let $\quad f(x) = 2x^2 + \frac{7}{2}x + \frac{3}{4}$

$= 8x^2 + 14x + 3$

(Multiplying the given equation by 4)

$= 8x^2 + (12x + 2x) + 3$

$= 8x^2 + 12x + 2x + 3$

$= 4x(2x + 3) + 1(2x + 3)$

$= (2x + 3)(4x + 1)$

The zeroes of $f(x)$ are given by $f(x) = 0$.

So, the value of $2x^2 + \frac{7}{2}x + \frac{3}{4}$ is zero when $x = -\frac{3}{2}$ or $x = -\frac{1}{4}$

$\Rightarrow \quad x = -\frac{3}{2}, -\frac{1}{4}$

Verification:

Sum of the zeroes = – (coefficient of x) ÷ coefficient of x^2

$$\alpha + \beta = -\frac{b}{a}$$

$$\left(-\frac{3}{2}\right)+\left(-\frac{1}{4}\right) = -\frac{7}{4}$$

$$-\frac{7}{4} = -\frac{7}{4}$$

Product of the zeroes = constant term ÷ coefficient of x^2

$$\alpha\beta = \frac{c}{a}$$

$$\left(-\frac{3}{2}\right)\left(-\frac{1}{4}\right) = \frac{3}{4} \div \frac{2}{1}$$

$$\frac{3}{8} = \frac{3}{8}$$

Hence, verified.

(B) Let $f(s) = 2s^2 - (1 + 2\sqrt{2})\,s + \sqrt{2}$

$$= 2s^2 - s - 2\sqrt{2}s + \sqrt{2}$$

$$= s(2s - 1) - \sqrt{2}(2s - 1)$$

$$= (2s - 1)(s - \sqrt{2})$$

The zeroes of $f(s)$ are given by $f(s) = 0$

So, the value is zero when $2s^2 - (1 + 2\sqrt{2})s + \sqrt{2} = 0$

i.e., when $s = \frac{1}{2}$ or $\sqrt{2}$

$$\Rightarrow \quad s = \frac{1}{2}, \sqrt{2}$$

Verification:

Sum of the zeroes = – (coefficient of s) ÷ coefficient of s^2

$$\alpha + \beta = -\frac{b}{a}$$

$$\frac{1}{2} + \sqrt{2} = -\frac{-(1 + 2\sqrt{2})}{2}$$

$$+\frac{(1 + 2\sqrt{2})}{2} = +\frac{(1 + 2\sqrt{2})}{2}$$

Product of the zeroes = constant term ÷ coefficient of s^2

$$\alpha\beta = \frac{c}{a}$$

$$\frac{1}{2} \times \sqrt{2} = \frac{\sqrt{2}}{2}$$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Hence, verified.

(C) Let $f(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$

$$= 21y^2 - 11y - 2$$

$$= 21y^2 + (3y - 14y) - 2$$

$$= 21y^2 + 3y - 14y - 2$$

$$= 3y(7y + 1) - 2(7y + 1)$$

$$= (7y + 1)(3y - 2)$$

The zeroes of $f(y)$ are given by $f(y) = 0$

So, the value of $7y^2 - \frac{11}{3}y - \frac{2}{3}$ is zero when $y = -\frac{1}{7}$ or $y = \frac{2}{3}$

$$\Rightarrow y = -\frac{1}{7}, \frac{2}{3}$$

Verification:

Sum of the zeroes = – (coefficient of y) ÷ coefficient of y^2

$$\alpha + \beta = -\frac{b}{a}$$

$$-\frac{1}{7} + \frac{2}{3} = -\frac{-11}{21}$$

$$\frac{11}{21} = \frac{11}{21}$$

Product of the zeroes = constant term ÷ coefficient of y^2

$$\alpha\beta = \frac{c}{a}$$

$$\left(-\frac{1}{7}\right)\left(\frac{2}{3}\right) = \frac{-\frac{2}{3}}{7}$$

$$-\frac{2}{21} = -\frac{2}{21}$$

Hence, verified.

43. Find a quadratic polynomial whose zeroes are 1 and –3. Verify the relation between the coeffcients and zeroes of polynomial. **[Diksha]**

Ans. Sum of zeroes,

$$S = 1 + (-3) = -2 \quad \text{...(i)}$$

Product of zeroes, $P = 1 \times (-3) = -3$ ...(ii)

Quadratic polynomial

$$p(x) = x^2 - Sx + P$$

$$= x^2 - (-2)x - 3 = x^2 + 2x - 3$$

Here, $a = 1, b = 2, c = -3$

$$-\frac{b}{a} = -\frac{2}{1} = -2$$

Sum of zeroes = $-\frac{b}{a} = 2$ [using eqn. (i)]

Also, $\frac{c}{a} = -\frac{3}{1} = -3$

Product of zeroes = $\frac{c}{a} = -3$ [using eqn. (ii)]

Hence, verified.

44. If one root of the equation $3x^2 - 8x + 2k + 1 = 0$ is seven times the other, find the two roots and the value of k.

Ans. Let α and 7α be the two roots of the equation:

$$3x^2 - 8x + (2k + 1) = 0$$

Then, $\alpha + 7\alpha = 8\alpha = \frac{8}{3}$ .....(i)

and $\alpha.(7\alpha) = 7\alpha^2 = \frac{2k + 1}{3}$ .....(ii)

From (i) $\alpha = \frac{1}{3}$. So, the two roots are $\frac{1}{3}$ and $\frac{7}{3}$

Using $\alpha = \frac{1}{3}$ in (ii), we have:

$$7\left(\frac{1}{3}\right)^2 = \frac{2k+1}{3}$$

$$\Rightarrow \quad 2k + 1 = \frac{7}{3}$$

$$\Rightarrow \quad 2k = \frac{4}{3}$$

$$\Rightarrow \quad k = \frac{2}{3}$$

45. **Without actually calculating the zeroes, form a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $5x^2 + 2x - 3$. [CBSE 2020]**

Ans. Let α and β be the zeroes of $5x^2 + 2x - 3$

Then, $\alpha + \beta = \left(-\frac{b}{a}\right) = -\frac{2}{5}$ and $\alpha\beta = \frac{c}{a} = -\frac{3}{5}$

Now $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-2/5}{-3/5} = \frac{2}{3}$

and $\frac{1}{\alpha}\cdot\frac{1}{\beta} = -\frac{5}{3}$

Thus, a quadratic polynomial where zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is x^2 – (sum of roots) x + product of roots = 0

$$\Rightarrow x^2 - \frac{2}{3}x - \frac{5}{3} = 0$$

i.e., $3x^2 - 2x - 5$

LONG ANSWER Type Questions

[**4** marks]

46. **Obtain other zeroes of the polynomial $f(x) = 2x^4 + 3x^3 - 5x^2 - 9x - 3$ if two of its zeroes are $-\sqrt{3}$ and $-\sqrt{3}$. [CBSE 2018]**

Ans. Since $\sqrt{3}$ and $-\sqrt{3}$ are zeroes of $f(x)$, $(x-\sqrt{3})(x+\sqrt{3})$ *i.e.*, $(x^2 - 3)$ is a factor of $f(x)$ to obtain other two zeroes, we shall determine the quotient, by dividing $f(x)$ with $(x^2 - 3)$

$$\begin{array}{r|l}
 & 2x^2+3x+1 \\
\hline
x^2-3 & 2x^4+3x^3-5x^2-9x-3 \\
 & \underline{2x^4 \qquad -6x^2} \\
 & \quad \;\; - \qquad\quad + \\
 & 3x^3+x^2-9x-3 \\
 & \underline{3x^3 \qquad -9x} \\
 & \quad \;\; - \qquad\quad + \\
 & x^2-3 \\
 & \underline{x^2-3} \\
 & \;\; - \quad + \\
 & 0
\end{array}$$

Here, quotient $= 2x^2 + 3x + 1$

$= (2x + 1)(x + 1)$

So, the two zeroes are -1 and $-\frac{1}{2}$.

47. **Given that the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form a, $a + b$, $a + 2b$ for some real numbers a and b, find the values of a and b as well as the zeroes of the given polynomial. [NCERT]**

Ans. Let $p(x) = x^3 - 6x^2 + 3x + 10$

and (a), $(a + b)$ and $(a + 2b)$ are the zeroes of $p(x)$.

We know:

Sum of the zeroes = –(coefficient of x^2) ÷ coefficient of x^3

$\Rightarrow a + (a + b) + (a + 2b) = -(-6)$

$\Rightarrow \quad 3a + 3b = 6$

$\Rightarrow \quad a + b = 2$

$\Rightarrow \quad a = 2 - b \quad$ ...(i)

Product of all the zeroes = –(constant term) ÷ coefficient of x^3

$a(a + b)(a + 2b) = -10$

$(2 - b)(2)(2 + b) = -10 \quad$ [Using eqn. (i)]

$(2 - b)(2 + b) = -5$

$4 - b^2 = -5$

$\Rightarrow \quad b^2 = 9$

$\Rightarrow \quad b = \pm 3$

When $b = 3, a = 2 - 3 = -1 \quad$ [Using equation (i)]

$\Rightarrow a = -1$ when $b = 3$.

When $b = -3, a = 2 - (-3) = 5$ [Using equation (i)]

$\Rightarrow a = 5$ when $b = -3$.

Case 1: when $a = -1$ and $b = 3$

The zeroes of the polynomial are:

$a = -1$

$$a + b = -1 + 3 = 2$$
$$a + 2b = -1 + 2(3) = 5$$

$\Rightarrow$ −1, 2 and 5 are the zeroes.

Case 2: when $a = 5$ and $b = -3$

The zeroes of the polynomial are:

$$a = 5$$
$$a + b = 5 - 3 = 2$$
$$a + 2b = 5 - 2(3) = -1$$

$\Rightarrow$ −1, 2 and 5 are the zeroes.

In both the cases, the zeroes of the polynomial are −1, 2, 5.

48. Given that $\sqrt{2}$ is a zero of the cubic polynomial $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$, find its other two zeroes. [CBSE 2010]

Ans. Let $\quad p(x) = 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$

As $\sqrt{2}$ is one of the zeroes of $p(x)$.

$\Rightarrow g(x) = (x - \sqrt{2})$ is one of the factors of $p(x)$.

$$\begin{array}{r|l}
(x-\sqrt{2}) & 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} \;\big(\; 6x^2 + 7\sqrt{2}x + 4 \\
 & 6x^3 - 6\sqrt{2}x^2 \\
 & - \quad + \\
\hline
 & 7\sqrt{2}x^2 - 10x - 4\sqrt{2} \\
 & 7\sqrt{2}x^2 - 14x \\
 & - \quad + \\
\hline
 & 4x - 4\sqrt{2} \\
 & 4x - 4\sqrt{2} \\
 & - \quad + \\
\hline
 & 0
\end{array}$$

Then,

$$\Rightarrow 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$$
$$= (x - \sqrt{2})\,(6x^2 + 7\sqrt{2}x + 4)$$
$$= (x - \sqrt{2})\,\{6x^2 + (3\sqrt{2}x + 4\sqrt{2}x) + 4\}$$
(by splitting the middle term)
$$= (x - \sqrt{2})\,\{6x^2 + 3\sqrt{2}x + 4\sqrt{2}x + 4\}$$
$$= (x - \sqrt{2})\,\{3\sqrt{2}x\,(\sqrt{2}x + 1) + 4(\sqrt{2}x + 1)\}$$
$$= (x - \sqrt{2})\,(\sqrt{2}x + 1)\,(3\sqrt{2}x + 4)$$
$$\Rightarrow \qquad x = \sqrt{2}, -\frac{1}{\sqrt{2}} \text{ or } -\frac{4}{3\sqrt{2}}$$

Thus, the other two zeroes are $-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$ and $-\frac{4}{3\sqrt{2}}$ or $-\frac{2\sqrt{2}}{3}$.

49. Given that $x - \sqrt{5}$ is a factor of the cubic polynomial $x^3 - 3\sqrt{5}x + 13x - 3\sqrt{5}$, find all the zeroes of the polynomial. [CBSE 2013]

Ans. Let $\quad p(x) = x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$

As $\sqrt{5}$ is one of the zeroes of $p(x)$.

$\Rightarrow (x - \sqrt{5})$ is one of the factors of $p(x)$.

$$\begin{array}{r|l}
 & \qquad x^2 - 2\sqrt{5}x + 3 \\
(x-\sqrt{5}) & x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5} \;\big(\\
 & x^3 - \sqrt{5}x^2 \\
 & - \quad + \\
\hline
 & -2\sqrt{5}x^2 + 13x - 3\sqrt{5} \\
 & -2\sqrt{5}x^2 + 10x \\
 & + \quad - \\
\hline
 & 3x - 3\sqrt{5} \\
 & 3x - 3\sqrt{5} \\
 & - \quad + \\
\hline
 & 0
\end{array}$$

Now $p(x) = (x - \sqrt{5})(x^2 - 2\sqrt{5}x + 3)$
$$= (x - \sqrt{5})(x^2 - 2\sqrt{5}x + 3)$$
$$= (x - \sqrt{5})[x^2 - \{(\sqrt{5} + \sqrt{2})x + (\sqrt{5} - \sqrt{2})x\} + 3)$$
$$= (x - \sqrt{5})[x\{x - (\sqrt{5} + \sqrt{2})\} - (\sqrt{5} - \sqrt{2})\{x - (\sqrt{5} + \sqrt{2})\} + 3]$$
$$= (x - \sqrt{5})\{x - (\sqrt{5} + \sqrt{2})\}\,\{x - (\sqrt{5} - \sqrt{2})\}$$

So, all the zeroes of the given polynomial are $(\sqrt{5} + \sqrt{2})$, $(\sqrt{5} - \sqrt{2})$ and $\sqrt{5}$.

50. For which values of a and b are the zeroes of $q(x) = x^3 + 2x^2 + a$ also the zeroes of the polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$? [NCERT]

Ans. Let $\quad p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$

and $\quad q(x) = x^3 + 2x^2 + a.$

Since, the zeroes of the polynomial $q(x)$ are also zeroes of $p(x)$, we can say that $q(x)$ is a factor of $p(x)$.

Then, on dividing

$$\begin{array}{r|l}
 & x^2 - 3x + 2 \\
x^3 + 2x^2 + a & x^5 - x^4 - 4x^3 + 3x^2 + 3x + b \;\big(\\
 & x^5 + 2x^4 \qquad + ax^2 \\
 & - \quad - \qquad\quad - \\
\hline
 & -3x^4 - 4x^3 + (3-a)x^2 + 3x + b \\
 & -3x^4 - 6x^3 \qquad\qquad - 3ax \\
 & + \quad + \qquad\qquad + \\
\hline
 & +2x^3 + (3-a)x^2 + (3+3a)x + b \\
 & +2x^3 + 4x^2 \qquad\qquad + 2a \\
 & - \quad - \qquad\qquad - \\
\hline
 & -(a+1)x^2 + 3(1+a)x + b - 2a
\end{array}$$

But remainder,

$$r(x) = -(a + 1)x^2 + 3(1 + a)x + b - 2a = 0$$

[since, $q(x)$ is factor of $p(x)$

$$\Rightarrow -(a + 1)x^2 + 3(1 + a)x + b - 2a$$
$$= 0.x^2 + 0.x + 0$$

On comparing the coefficients of x^2 and constant term, we get

$$-(a + 1) = 0 \quad \Rightarrow \quad a = -1$$

and $b - 2a = 0 \quad \Rightarrow \quad b = 2a = 2(-1) = -2$