Logarithms

INTRODUCTION

Logarithm, in mathematics, is the 'exponent' or 'power' to which a stated number called the *base* is raised to yield a specific number. For example, in the expression $10^2 = 100$, the logarithm of 100 to the **base** 10 is 2. This is written as $\log_{10} 100 = 2$. Logarithms were originally invented to help simplify the arithmetical processes of multiplication, division, expansion to a power and extraction of a 'root', but they are nowadays used for a variety of purposes in pure and applied mathematics.

Logarithm

If for a positive real number ($a \neq 1$), $a^m = b$, then the index m is called the logarithm of b to the base a. We write this as

$$\log_a b = m$$

'log' being the abbreviation of the word 'logarithm'. Thus,

$$a^m = b \Leftrightarrow \log_a b = m$$

where, $a^m = b$ is called the *exponential form* and $\log_a b = m$ is called the *logarithmic form*.

Illustration 1 Refer to the following table

Exponential form	Logarithmic form
$3^5 = 243$	$\log_3 243 = 5$
$2^4 = 16$	$\log_2 16 = 4$
$3^0 = 1$	$\log_3 1 = 0$
$8^{1/3} = 2$	$\log_8 2 = \frac{1}{3}$

LAWS OF LOGARITHMS

1. Product formula

The logarithm of the product of two numbers is equal to the sum of their logarithms.

i.e., $\log_a (mn) = \log_a m + \log_a n$

Generalisation: In general, we have

$\log_a (mnpq...) = \log_a m + \log_a n + \log_a p + \log_a q + ...$

2. Quotient formula

The logarithm of the quotient of two numbers is equal to the difference of their logarithms.

i.e., $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$

where a, m, n are positive and $a \neq 1$.

3. Power formula

The logarithm of a number raised to a power is equal to the power multiplied by logarithm of the number.

i.e., $\log_a (m^n) = n \log_a m$,

where a, m are positive and $a \neq 1$.

4. Base changing formula

$$\log_n{}^m = \frac{\log_a m}{\log_a n}. \text{ So, } \log_n m = \frac{\log m}{\log n}$$

where m, n, a are positived and $n \neq 1, a \neq 1$.

5. Reciprocal relation

$$\log_b a \times \log_a b = 1$$

where a and b are positive and not equal to 1.

6. $\log_b a = \dfrac{1}{\log_a b}$

7. ${}_a\log_a x = x$, where a and x are positive, $a \neq 1$.

8. If $a > 1$ and $x > 1$, then $\log_a x > 0$.

9. If $0 < a < 1$ and $0 < x < 1$, then $\log_a x > 0$.

10. If $0 < a < 1$ and $x > 1$, then $\log_a x > 0$.

11. If $a > 1$ and $0 < x < 1$, then $\log_a x < 0$.

Some useful Formulae

1. Logarithm of 1 to any base is equal to zero.
   i.e., $\log_a 1 = 0$, where $a > 0$, $a \neq 1$.
2. Logarithm of any number to the same base is 1.
   i.e., $\log_a a = 1$, where $a > 0$, $a \neq 1$.

Common Logarithms

There are two bases of logarithms that are extensively used these days. One is base e ($e = 2.71828$ approx.) and the other is base 10. The logarithms to base e are called natural logarithms. The logarithms to base 10 are called the common logarithms.

$\log_{10} 10 = 1$, since $10^1 = 10$.
$\log_{10} 100 = 2$, since $10^2 = 100$.
$\log_{10} 10000 = 4$, since $10^4 = 10000$.
$\log_{10} 0.01 = -2$, since $10^{-2} = 0.01$.
$\log_{10} 0.001 = -3$, since $10^{-3} = 0.001$
and, $\log_{10} 1 = 0$, since $10^{\circ} = 1$.

Practice Exercises

Difficulty Level-1 (Based on Memory)

1. The number of real solutions of the equation
   $\log(-x) = 2\log(x + 1)$ is:
   (*a*) One (*b*) Two
   (*c*) Three (*d*) Four
   [Based on MAT, 2005]

2. If $\log_5(x^2 + x) - \log_5(x + 1) = 2$, then the value of x is:
   (*a*) 5 (*b*) 32
   (*c*) 25 (*d*) 10
   [Based on FMS (Delhi), 2007]

3. If $\log_{12} 27 = a$, then $\log_6 16$ is:
   (*a*) $(3 - a)/4(3 + a)$ (*b*) $(3 + a)/4(3 - a)$
   (*c*) $4(3 + a)/(3 - a)$ (*d*) $4(3 - a)/(3 + a)$
   [Based on MAT (Dec), 2006]

4. If $\log_a b = \frac{1}{2}$, $\log_b c = \frac{1}{3}$ and $\log_c a = \frac{k}{5}$, then the value of k is:
   (*a*) 25 (*b*) 35
   (*c*) 30 (*d*) 20
   [Based on MAT (May), 2006]

5. Which of the following is true?
   (*a*) $\log_{17} 275 = \log_{19} 375$
   (*b*) $\log_{17} 275 < \log_{19} 375$
   (*c*) $\log_{17} 275 > \log_{19} 375$
   (*d*) None of the above
   [Based on MAT (May), 2005]

6. $\frac{1}{2}\log_{10} 25 - 2\log_{10} 3 + \log_{10} 18$ equals to:
   (*a*) 18 (*b*) 1
   (*c*) $\log_{10} 3$ (*d*) None of these
   [Based on MAT (Sept), 2003]

7. The difference between the logarithms of sum of the squares of two positive numbers A and B and the sum of logarithms of the individual numbers is a constant C. If $A = B$, then C is:
   (*a*) 2 (*b*) 1.3031
   (*c*) log 2 (*d*) exp (2)
   [Based on MAT (Sept), 2003]

8. If $\log_x a$, $a^{x/2}$, and $\log_b x$ are in GP, then x is:
   (*a*) $\log_a(\log_b a)$
   (*b*) $\log_a(\log_e a) + \log_a(\log_e b)$
   (*c*) $-\log_a(\log_a b)$
   (*d*) $\log_a(\log_e b) - \log_a(\log_e a)$
   [Based on MAT (Dec), 2002]

9. If $\log_a(ab) = x$, then $\log_b(ab)$ is:
   (*a*) $\frac{1}{x}$ (*b*) $\frac{x}{x+1}$
   (*c*) $\frac{x}{x-1}$ (*d*) $\frac{x}{1-x}$
   [Based on MAT (May), 2002]

10. If $\log_8 x + \log_8 \frac{1}{6} = \frac{1}{3}$, then the value of x is:
    (*a*) 18 (*b*) 24
    (*c*) 16 (*d*) 12
    [Based on MAT (May), 1998]

11. If $\log_8 x + \log_8 \frac{1}{6} = \frac{1}{3}$, then the value of x is:
    (*a*) 18 (*b*) 24
    (*c*) 16 (*d*) 12
    [Based on MAT, 1998]

12. If $\log_{10}(x^2 - 6x + 45) = 2$, then the value of x are:
(a) 6, 9 (b) 9, –5
(c) 10, 5 (d) 11, –5
[Based on FMS, 2005]

13. If $\log_{10} 125 + \log_{10} 8 = x$, then x is equal to:
(a) –3 (b) 3
(c) $\frac{1}{3}$ (d) 0.064
[Based on FMS, 2005]

14. $\log(a^2/bc) + \log(b^2/ac) + \log(c^2/ab)$ is:
(a) 1 (b) 0
(c) 39 (d) abc
[Based on FMS, 2006]

15. What is the value of the following expression?
$$\log(9/14) - \log(15/16) + \log(35/24)$$
(a) 0 (b) 1
(c) 2 (d) 3
[Based on IIFT, 2005]

16. The characteristics of the logarithm of 21735 is:
(a) 2 (b) 1
(c) 3 (d) 4
[Based on ATMA, 2005]

17. If $\log_a 3 = 2$ and $\log_b 8 = 3$, then $\log_a b$ is:
(a) $\log_3 2$ (b) $\log_2 3$
(c) $\log_3 4$ (d) $\log_4 3$

18. If $\log_7 2 = m$, then $\log_{49} 28$ is equal to:
(a) $2(1 + 2m)$ (b) $\frac{1+2m}{2}$
(c) $\frac{2}{1+2m}$ (d) $1 + m$

19. If $\log_{10} x = y$, then $\log_{1000} x^2$ is equal to:
(a) y^2 (b) $2y$
(c) $\frac{3y}{2}$ (d) $\frac{2y}{3}$

20. $\frac{1}{2}\log_{10} 25 - 2\log_{10} 3 + \log_{10} 18$ equals:
(a) 18 (b) 1
(c) $\log_{10} 3$ (d) None of these

21. If $\log_a(ab) = x$, then $\log_b(ab)$ is:
(a) $\frac{1}{x}$ (b) $\frac{x}{x+1}$
(c) $\frac{x}{x-1}$ (d) $\frac{x}{1-x}$

22. A certain type of bacteria reproduces itself at the rate of 10% every 5 minutes. In how many minutes will the number double itself?
(a) 20 minutes (b) 30 minutes
(c) 40 minutes (d) None of these

23. Find $\log_{3/2} 3.375$
(a) 2 (b) 3
(c) 5/2 (d) 17/2

24. $7\log\frac{16}{15} + 5\log\frac{25}{24} + 3\log\frac{81}{80} =$
(a) log 2 (b) log 3
(c) log 5 (d) None of these

25. $\frac{\log 49\sqrt{7} + \log 25\sqrt{5} - \log 4\sqrt{2}}{\log 17.5} =$
(a) 5 (b) 2
(c) 5/2 (d) 3/2

26. $\log_{10}\tan 40° \times \log_{10} 41° \ldots \log_{10}\tan 50° =$
(a) 1 (b) 0
(c) –1 (d) None of these

27. If $y = \frac{1}{a^{1-\log_a x}}$, $z = \frac{1}{a^{1-\log_a y}}$ and $x = a^k$, then $k =$
(a) $\frac{1}{a^{1-\log_a z}}$ (b) $\frac{1}{1-\log_a z}$
(c) $\frac{1}{1+\log_z a}$ (d) $\frac{1}{1-\log_z a}$

28. If $\log_e 2 \times \log_b 625 = \log_{10}{}^{16} \times \log_e 10$, then $b =$
(a) 4 (b) 5
(c) 1 (d) e

29. $5^{\sqrt{\log_5 7}} - 7^{\sqrt{\log_7 5}}$
(a) log 2 (b) 1
(c) 0 (d) None of these

30. $2^{\log_3 7} - 7^{\log_3 2}$
(a) $\log_2 7$ (b) log 7
(c) log 2 (d) 0

31. If $\log_{30} 3 = a$, $\log_{30} 5 = b$, then $\log_{30} 8 =$
(a) $3(1 - a - b)$ (b) $a - b + 1$
(c) $1 - a - b$ (d) $1(a - b + 1)$

32. $\log_5 2$ is:
(a) an integer (b) a rational number
(c) an irrational number (d) a prime number

33. What is the value of $\frac{\log_{27} 9 \times \log_{16} 64}{\log_4 \sqrt{2}}$?
(a) $\frac{1}{6}$ (b) $\frac{1}{4}$
(c) 8 (d) 4
[Based on SNAP, 2013]

Difficulty Level-2
(Based on Memory)

1. If $\log_{10} x - \log_{10} \sqrt{x} = 2 \log_x 10$, then a possible value of x is given by:

(*a*) 10 (*b*) 1/100

(*c*) 1/1000 (*d*) None of these

[Based on CAT, 2004]

2. What is the sum of '*n*' terms in the series:

$$\log m + \log\left(\frac{m^2}{n}\right) + \log\left(\frac{m^3}{n^2}\right) + \log\left(\frac{m^4}{n^3}\right) + \ldots$$

(*a*) $\log\left[\frac{n^{(n-1)}}{m^{(n+1)}}\right]^{n/2}$ (*b*) $\log\left[\frac{n^m}{n^n}\right]^{n/2}$

(*c*) $\log\left[\frac{m^{(1-n)}}{n^{(1+n)}}\right]^{n/2}$ (*d*) $\log\left[\frac{m^{(n+1)}}{n^{(n-1)}}\right]^{n/2}$

[Based on CAT, 2004]

3. The length of the circumference of a circle equals the perimeter of a triangle of equal sides, and also the perimeter of a square. The areas covered by the circle, triangle, and square are c, t and s respectively. Then,

(*a*) $s > t > c$ (*b*) $c > t > s$

(*c*) $c > s > t$ (*d*) $s > c > t$

[Based on CAT, 2004]

4. If $\log_n 48 = a$ and $\log_n 108 = b$, what is the value of $\log_n 1296$ in terms of a and b?

(*a*) $\frac{2(2a+b)}{5}$ (*b*) $\frac{(a+3b)}{5}$

(*c*) $\frac{4(2a+b)}{5}$ (*d*) $\frac{2(a+3b)}{5}$

5. $\sum_{n=1}^{n} \frac{1}{\log_{2^n}(a)} =$

(*a*) $\frac{n(n+1)}{2}\log_a 2$ (*b*) $\frac{n(n-1)}{2}\log_2 a$

(*c*) $\frac{(n+1)^2 n^2}{4}\log_a 2$ (*d*) None of these

6. If $\log\frac{a}{b} + \log\frac{b}{a} = \log(a+b)$, then

(*a*) $a + b = 1$ (*b*) $a - b$

(*c*) $a^2 - b = 1$ (*d*) $a = b$

[Based on FMS (Delhi), 2007]

7. If $\log x\ (0.1) = -\frac{1}{3}$, then the value of x is:

(*a*) 10 (*b*) 100

(*c*) 1000 (*d*) $\frac{1}{1000}$

[Based on FMS (MS), 2006]

8. If $\log_7 \log_5 (\sqrt{x+5} + \sqrt{x}) = 0$, what is the value of x?

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 5

[Based on FMS (2009)]

9. What is the value of x, if

$$\frac{1}{\log_{\frac{442}{441}} x} + \frac{1}{\log_{\frac{443}{442}} x} + \frac{1}{\log_{\frac{444}{443}} x} + \ldots + \frac{1}{\log_{\frac{899}{898}} x} + \frac{1}{\log_{\frac{900}{899}} x} = 2?$$

(*a*) $\frac{2}{21}$ (*b*) 1

(*c*) $\frac{7}{10}$ (*d*) $\frac{10}{7}$

10. What is the value of $[\log_{10}(5 \log_{10} 100)]^2$?

(*a*) 25 (*b*) 10

(*c*) 2 (*d*) 1

[Based on FMS, 2010]

11. If $\log_{10} 2 = a$ and $\log_{10} 3 = b$, then $\log_5 12$ equals:

(*a*) $\frac{a+b}{1+a}$ (*b*) $\frac{2a+b}{1+a}$

(*c*) $\frac{a+2b}{1+a}$ (*d*) $\frac{2a+b}{1-a}$

[Based on FMS, 2010]

12. If $a = \log_8 225$ and $b = \log_2 15$, then a in terms of b, is:

(*a*) $b/2$ (*b*) $2b/3$

(*c*) b (*d*) $3b/2$

[Based on FMS, 2010]

13. If $\log x - 5 \log 3 = -2$, then x equals:

(*a*) 1.25 (*b*) 0.81

(*c*) 2.43 (*d*) 0.8

[Based on FMS, 2011]

14. The value of $\log_5 \frac{(125)(625)}{25}$ is equal to:

(*a*) 725 (*b*) 6

(*c*) 3125 (*d*) 5

[Based on FMS, 2011]

15. The values of a in the equation: $\log_{10}(a^2 - 15a) = 2$ are:

(*a*) $\frac{15 \pm \sqrt{233}}{2}$ (*b*) 20, – 5

(*c*) $\frac{15 \pm \sqrt{305}}{2}$ (*d*) ± 20

[Based on FMS, 2011]

16. If $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b}$, mark all the correct options.

(*a*) $xyz = 1$ (*b*) $x^a y^b z^c = 1$

(*c*) $x^{b+c} y^{c+a} z^{a+b} = 1$ (*d*) $x^{b+c} y^{c+a} z^{a+b} = 0$

[Based on ITFT, 2006]

17. If $2 \log x = 5 \log y + 3$, then the relation between x and y is:

(*a*) $x^2 = 100y^5$ (*b*) $x^{1/5} = 1000\, y^{1/2}$

(*c*) $x^2 = 1000y^5$ (*d*) $x^2 = y^5 + 1000$

18. What is the value of $\sqrt{\frac{a}{b}}$, if $\log_4 \log_4 4^{a-b} = 2 \log_4 (\sqrt{a} - \sqrt{b}) + 1$?

(*a*) $-\frac{5}{3}$ (*b*) 2

(*c*) $\frac{5}{3}$ (*d*) 1

[Based on IIFT, 2010]

19. The value of $\log_{10}\log_{10}\log_{10}\log_{10}\left(10^{10^{10^{10}}}\right)$ is:

(*a*) 1 (*b*) $\frac{1}{10}$

(*c*) 10 (*d*) Cannot be determined

20. $\log_{10}(\log_2 3) + \log_{10}(\log_3 4) + \ldots + \log_{10}(\log_{1023} 1024)$ equals:

(*a*) 10 (*b*) *e*

(*c*) 1 (*d*) 0

[Based on JMET, 2006]

21. $\log_4 2 - \log_8 2 + \log_{16} 2 - \ldots$ to ∞ is:

(*a*) e^2 (*b*) $ln\, 2 + 1$

(*c*) $ln\, 2 - 1$ (*d*) $1 - ln\, 2$

22. If $\log_2 x + \log_2 y \geq 6$, then the least value of xy is:

(*a*) 4 (*b*) 8

(*c*) 64 (*d*) 32

23. Solve for x, $\log_{10} x + \log_{\sqrt{10}} x + \log_{\sqrt[3]{100}} x = 27$

(*a*) 1 (*b*) 10^6

(*c*) 10^4 (*d*) 10

24. If $y = 2^{1/\log_x 4}$, then x is equal to:

(*a*) $\sqrt{y}$ (*b*) y

(*c*) y^2 (*d*) y^4

25. If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, then which of the following options holds true?

(*a*) $a^b \cdot b^c \cdot c^a = 1$ (*b*) $a^a b^b c^c = 1$

(*c*) $a^{2a} b^{2b} c^{2c} = 1$ (*d*) $a^{ab} b^{bc} c^{ac} = 1$

26. If a, b, c are distinct positive numbers different from 1 such that $(\log_b a \cdot \log_c a - \log_a a) + (\log_a b \cdot \log_c b - \log_b b) + (\log_a c \cdot \log_b c - \log_c c) = 0$ then $abc =$

(*a*) 0 (*b*) *e*

(*c*) 1 (*d*) None of these

27. The value of $3^{2\log 93}$ is:

(*a*) 4 (*b*) 3

(*c*) 47 (*d*) 8

28. If $\log_{10} x - \log_{10} \sqrt{x} = 2 \log_x 10$, then a possible value of x is given by:

(*a*) 10 (*b*) 1/100

(*c*) 1/1000 (*d*) None of these

29. The length of the circumference of a circle equals the perimeter of a triangle of equal sides, and also the perimeter of a square. The areas covered by the circle, triangle, and square are c, t and s respectively. Then,

(*a*) $s > t > c$ (*b*) $c > t > s$

(*c*) $c > s > t$ (*d*) $s > c > t$

30. The difference between the logarithms of sum of the squares of two positive numbers A and B and the sum of logarithms of the individual numbers is a constant C. If $A = B$, then C is

(*a*) 2 (*b*) 1.3031

(*c*) log 2 (*d*) exp (2)

31. The number of real solutions of the equation $\log(-x) = 2 \log(x + 1)$ is:

(*a*) One (*b*) Two

(*c*) Three (*d*) Four

32. If $\log_7 \log_5 \left((\sqrt{x} + 5 + \sqrt{x})\right) = 0$, find the value of x.

(*a*) 1 (*b*) 0

(*c*) 2 (*d*) None of these

33. $\log_2[\log_7 (x^2 - x + 37)] = 1$, then what could be the value of x?

(*a*) 3 (*b*) 5

(*c*) 4 (*d*) None of these

34. If $\frac{1}{3} \log_3 M + 3 \log_3 N = 1 + \log 5_{0.008}$, then:

(*a*) $M^9 = \frac{9}{N}$ (*b*) $N^9 = \frac{9}{M}$

(*c*) $M^3 = \frac{3}{N}$ (*d*) $N^9 = \frac{3}{N}$

35. If $\log_{10} x - \log_{10}\sqrt{x} = 2\log_x 10$, then a possible value of x is given by:

(a) 10 (b) $\frac{1}{100}$

(c) $\frac{1}{1000}$ (d) None of these

36. Let, $u = (\log_2 x)^2 - 6\log_2 x + 12$, where x is a real number. Then, the equation $x^u = 256$, has:

(a) no solution for x

(b) exactly one solution for x

(c) exactly two distinct solutions for x

(d) exactly three distinct solutions for x

37. If $\log_8 x + \log_8 \frac{1}{6} = \frac{1}{3}$, then the value of x is:

(a) 18 (b) 24 (c) 16 (d) 12

38. Which of the following is true?

(a) $\log_{17}275 = \log_{19}375$ (b) $\log_{17}275 < \log_{19}375$

(c) $\log_{17}275 > \log_{19}375$ (d) None of these

39. If $x = \log_{2a} a, y = \log_{3a} 2a$ and $z = \log_{4a} 3a$, find $yz(2 - x)$.

(a) 1 (b) –1

(c) 2 (d) –2

40. $\frac{\log x}{l + m - 2m} = \frac{\log y}{m + n - 2l} = \frac{\log z}{n + l - 2m}$, find $x^2y^2z^2$.

(a) 2 (b) –1

(c) 4 (d) 1

41. If $\log \frac{x+y}{5} = \frac{1}{2}(\log x + \log y)$, then $\frac{x}{y} + \frac{y}{x} =$

(a) 20 (b) 23

(c) 22 (d) 21

42. If $\log(x + y) = \log\left(\frac{3x - 3y}{2}\right)$, then $\log x - \log y =$

(a) log 2 (b) log 3

(c) log 5 (d) log 6

43. If $\log_2 x + \log_4 x + \log_{16} x = 21/4$, then $x =$

(a) 8 (b) 4 (c) 2 (d) 16

44. If $0 < a\, x$, the minimum value of $\log_a x + \log_x a$ is:

(a) 1 (b) 2

(c) 3 (d) 5

45. If $\frac{\log x}{b - c} = \frac{\log y}{c - a} = \frac{\log}{a \quad b}$, then $xyz = x^a \times y^b \times z^c$ $= x^{b+c} \times y^{c+a} \times z^{a+b} =$

(a) 1 (b) 0

(c) 2 (d) None of these

46. $x^{\log y - \log z} \times y^{\log z - \log x} \times z^{\log x - \log y} =$

(a) 0 (b) 2

(c) 1 (d) None of these

47. If $\log_{10}[98 + \sqrt{x^2 - 12x + 36}] = 2$, then $x =$

(a) 4 (b) 8 (c) 12 (d) 4, 8

48. If $x = \log_a bc, y = \log_b ca, z = \log_c ab$, then

(a) $xyz = x + y + z + 2$ (b) $xyz = x + y + z + 1$

(c) $x + y + z = 1$ (d) $xyz = 1$

49. If $a^x = b^y = c^z = d^w$, then $\log_a(bcd) =$

(a) $\frac{1}{x}\left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right)$ (b) $x\left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right)$

(c) $\frac{y + z + w}{x}$ (d) None of these

50. If $\log_{10} 2 = 0.3010$, then $\log_{10}(1/2) =$

(a) –0.3010 (b) 0.6990

(c) $\bar{1}.6990$ (d) $\bar{1}.3010$

51. If $\log_2(3^{2x-2} + 7) = 2 + \log_2(3^{x-1} + 1)$, then $x =$

(a) 0 (b) 1 (c) 2 (d) 1 or 2

52. If $\log_a b = \log_b c = \log_c a$, then:

(a) $a > b\, c$ (b) $a < b < c$

(c) $a = b = c$ (b) $a < b \leq c$

53. If $\frac{1}{\log_x 10} = \frac{2}{\log_a 10} - 2$, then $x =$

(a) $a/2$ (b) $a/100$ (c) $a^2/10$ (d) $a^2/100$

54. If $a^2 + b^2 = c^2$, then $\frac{1}{\log_{c+a} b} + \frac{1}{\log_{c-a} b} =$

(a) 1 (b) 2 (c) –1 (d) –2

55. If $\log_{10} 87.5 = 1.9421$, then the number of digits in $(875)^{10}$ is:

(a) 30 (b) 29 (c) 20 (d) 19

56. If $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$, then the number of zeros between the decimal point and the first significant figure in $(0.0432)^{10}$ is:

(a) 10 (b) 13 (c) 14 (d) 15

57. If $(4.2)^x = (0.42)^y = 100$, then $\frac{1}{x} - \frac{1}{y} =$

(a) 1 (b) 2 (c) 1/2 (d) –1

58. $\frac{\log_9 11}{\log_5 13} - \frac{\log_3 11}{\log_{\sqrt{5}} 13} =$

(a) 1 (b) –1

(c) 0 (d) None of these

59. If $\frac{\log x}{2} = \frac{\log y}{3} = \frac{\log z}{5}$, then yz in terms of x is:

(a) x (b) x^2 (c) x^3 (d) x^4

60. If $4^x + 2^{2x-1} = 3^{x+\frac{1}{2}} + 3^{x-\frac{1}{2}}$, then $x =$

(a) 1/2 (b) 3/2 (c) 5/2 (d) 1

61. If $\log_8 p = 2.5$, $\log_2 q = 5$, then p in terms of q is:

(a) $\sqrt[3]{q}$ (b) $2q$

(c) q (d) $q/2$

62. If $0 < a < 1$, $0 < x < 1$ and $x < a$, then $\log_a x$

(a) < 1 (b) > 1 (c) < 0 (d) ≤ 1

63. $\log_5\left(1+\frac{1}{5}\right)+\log_5\left(1+\frac{1}{6}\right)+\log_5\left(1+\frac{1}{7}\right)+ \ldots + \log_5\left(1+\frac{1}{624}\right)$

(a) 5 (b) 4 (c) 3 (d) 2

64. If $\log_{10} 2986 = 3.4751$, then $\log_{10} 0.02986 =$

(a) $\bar{1}.2986$ (b) $\bar{2}.4751$

(c) 0.34751 (d) None of these

65. If $\log(2a - 3b) = \log a - \log b$, then $a =$

(a) $\frac{3b^2}{2b-1}$ (b) $\frac{3b}{2b-1}$

(c) $\frac{b^2}{2b+1}$ (d) $\frac{3b^2}{2b+1}$

66. If $\log(x-y) - \log 5 - \frac{1}{2}\log x - \frac{1}{2}\log y = 0$, then $\frac{x}{y}+\frac{y}{x} =$

(a) 25 (b) 26 (c) 27 (d) 28

67. If $\log x{:}3 = \log y{:}4 = \log z{:}5$, then $zx =$

(a) $2y$ (b) y^2 (c) $8y$ (d) $4y$

68. If $3 + \log_5 x = 2\log_{25} y$, then $x =$

(a) $y/125$ (b) $y/25$

(c) $y^2/625$ (d) $3 - y^2/25$

69. If $\frac{\log_2 a}{3} = \frac{\log_3 b}{3} = \frac{\log_4 c}{4}$ and $a^{1/2} \times b^{1/3} \times c^{1/4} = 24$, then:

(a) $a = 24$ (b) $b = 81$

(c) $c = 64$ (d) $c = 256$

70. If $\frac{\log_2 x}{3} = \frac{\log_2 y}{4} = \frac{\log_2 z}{5k}$ and $\frac{z}{x^3 y^4} = 1$, then $k =$

(a) 3 (b) 4 (c) 5 (d) –5

71. $\frac{3+\log_{10} 343}{2+\frac{1}{2}\log\left(\frac{49}{4}\right)+\frac{1}{3}\log\left(\frac{1}{125}\right)} =$

(a) 3 (b) 3/2

(c) 2 (d) 1

72. If $\frac{\log x}{a^2+ab+b^2} = \frac{\log y}{b^2+bc+c^2} = \frac{\log z}{c^2+ca+a^2}$, then $x^{a-b} \times y^{b-c} \times z^{c-a} =$

(a) 0 (b) –1 (c) 1 (d) 2

73. If $3^{x-2} = 5$ and $\log_{10} 2 = 0.20103$, $\log_{10} 3 = 0.4771$, then $x =$

(a) $1\frac{22187}{47710}$ (b) $2\frac{22187}{47710}$

(c) $3\frac{22187}{47710}$ (d) None of these

74. If $\log_2 = 0.30103$ and $\log_3 = 0.4771$, then number of digits in $(648)^5$ is:

(a) 12 (b) 13 (c) 14 (d) 15

75. If $\log x = \frac{\log y}{2} = \frac{\log z}{5}$, then $x^4 \times y^3 \times z^{-2} =$

(a) 2 (b) 10 (c) 1 (d) 0

76. $\frac{\log\sqrt{27}+\log\sqrt{1000}+\log 8}{\log 120}$

(a) 1/2 (b) 1 (c) 3/2 (d) 2

77. For $x > 0$, if $y = \frac{10^{\log_{10} x}}{x^2}$ and $x = y^a$, then $a =$

(a) 1 (b) –1 (c) 0 (d) 2

78. If $x = 100_{4/3}(1/2)$, $y = \log_{1/2}(1/3)$, then:

(a) $x > y$ (b) $x < y$ (c) $x = y$ (d) $x \ge y$

79. Let $u = (\log_2 x)\ 2 - 6\log_2 x + 12$ where, x is a real number. Then the equation $x^u = 256$, has

(a) no solution for x.

(b) exactly one solution for x.

(c) exactly two distinct solutions for x.

(d) exactly three distinct solutions for x.

[Based on CAT, 2010]

80. If $\log_x(a-b) - \log_x(b/a)$, find $\frac{a^2}{b^2}+\frac{b^2}{a^2}$.

(a) 4 (b) 2 (c) 3 (d) 6

[Based on CAT, 2012]

81. $\log_2[\log_7(x^2 - x + 37)] = 1$, then what could be the value of 'x'?

(a) 3 (b) 5

(c) 4 (d) None of these

[Based on CAT, 1997]

82. $\log_6 216\sqrt{6}$ is:

(a) 3 (b) $\frac{3}{2}$

(c) $\frac{7}{2}$ (d) None of these

[Based on CAT, 1994]

83. If $\log_7 \log_5 (\sqrt{x+5} + \sqrt{x}) = 0$, find the value of x.

(*a*) 1 (*b*) 0

(*c*) 2 (*d*) None of these

[Based on CAT, 1994]

84. The value of the expression:

$\sum_{i=2}^{100} \frac{1}{\log_i 100!}$ is:

(*a*) 0.01 (*b*) 0.1

(*c*) 1 (*d*) 10

(*e*) 100

[Based on XAT, 2014]

Answer Keys

Difficulty Level-1

1. (*a*)	**2.** (*c*)	**3.** (*d*)	**4.** (*c*)	**5.** (*b*)	**6.** (*b*)	**7.** (*c*)	**8.** (*a*)	**9.** (*c*)	**10.** (*d*)	**11.** (*d*)	**12.** (*d*)	**13.** (*b*)
14. (*b*)	**15.** (*a*)	**16.** (*d*)	**17.** (*c*)	**18.** (*b*)	**19.** (*d*)	**20.** (*b*)	**21.** (*c*)	**22.** (*d*)	**23.** (*b*)	**24.** (*a*)	**25.** (*c*)	**26.** (*b*)
27. (*b*)	**28.** (*b*)	**29.** (*c*)	**30.** (*d*)	**31.** (*a*)	**32.** (*c*)	**33.** (*d*)						

Difficulty Level-2

1. (*b*)	**2.** (*d*)	**3.** (*c*)	**4.** (*d*)	**5.** (*a*)	**6.** (*a*)	**7.** (*c*)	**8.** (*c*)	**9.** (*d*)	**10.** (*d*)	**11.** (*d*)	**12.** (*b*)	**13.** (*c*)
14. (*d*)	**15.** (*b*)	**16.** (*a*, *b*, *c*)	**17.** (*c*)	**18.** (*c*)	**19.** (*c*)	**20.** (*c*)	**21.** (*d*)	**22.** (*c*)	**23.** (*b*)	**24.** (*c*)	**25.** (*b*)	**26.** (*c*)
27. (*b*)	**28.** (*b*)	**29.** (*c*)	**30.** (*c*)	**31.** (*a*)	**32.** (*b*)	**33.** (*c*)	**34.** (*b*)	**35.** (*b*)	**36.** (*b*)	**37.** (*d*)	**38.** (*b*)	**39.** (*a*)
40. (*d*)	**41.** (*b*)	**42.** (*c*)	**43.** (*a*)	**44.** (*b*)	**45.** (*a*)	**46.** (*c*)	**47.** (*d*)	**48.** (*a*)	**49.** (*b*)	**50.** (*c*)	**51.** (*d*)	**52.** (*c*)
53. (*d*)	**54.** (*b*)	**55.** (*a*)	**56.** (*b*)	**57.** (*c*)	**58.** (*c*)	**59.** (*d*)	**60.** (*b*)	**61.** (*a*)	**62.** (*b*)	**63.** (*b*)	**64.** (*b*)	**65.** (*a*)
66. (*c*)	**67.** (*b*)	**68.** (*a*)	**69.** (*d*)	**70.** (*c*)	**71.** (*a*)	**72.** (*c*)	**73.** (*c*)	**74.** (*d*)	**75.** (*c*)	**76.** (*c*)	**77.** (*b*)	**78.** (*b*)
79. (*b*)	**80.** (*d*)	**81.** (*a*)	**82.** (*c*)	**83.** (*b*)	**84.** (*c*)							

Explanatory Answers

Difficulty Level-1

1. (*a*) $-x = (x+1)^2$

$\Rightarrow x^2 + 3x + 1 = 0$

$\therefore \quad x = \frac{-3+\sqrt{5}}{2}, \frac{-3-\sqrt{5}}{2}$

But only $\frac{-3+\sqrt{5}}{2}$ satisfies the other condition of $x + 1 > 0$.

2. (*c*) $\frac{\log(x^2+x)}{\log 5} - \frac{\log(x+1)}{\log 5} = 2$

$\log(x^2+x) - \log(x+1) = 2 \log 5$

$\Rightarrow \quad \log\left(\frac{x^2+x}{x+1}\right) = \log 5^2$

$\Rightarrow \quad \frac{x^2+x}{x+1} = 25$

$\Rightarrow \quad x^2 - 24x - 25 = 0$

$\Rightarrow \quad (x-25)(x+1) = 0$

$\Rightarrow \quad x = 25$

3. (*d*) $\log_{12} 27 = a$

$\Rightarrow \quad \log_{12} 3^3 = a$

$\Rightarrow \quad \log_{12} 3 = \frac{a}{3}$

$\Rightarrow \quad \log_3 12 = \frac{3}{a}$

$\Rightarrow \log_3 4 + \log_3 3 = \frac{3}{a}$

$\Rightarrow \quad 2 \log_3 2 = \frac{3-a}{a}$

$\Rightarrow \quad \log_3 2 = \frac{3-a}{2a} \qquad (1)$

Now, $\log_6 16 = 4 \log_6 2 = A$ (let)

$$\Rightarrow \quad \log_6 2 = \frac{A}{4}$$

$$\Rightarrow \quad \log_2 6 = \frac{4}{A}$$

$$\Rightarrow \quad \log_2 3 = \frac{4-A}{A}$$

$$\Rightarrow \quad \log_3 2 = \frac{A}{4-A} \qquad (2)$$

From Eqs. (1) and (2),

$$\frac{3-a}{2a} = \frac{A}{4-A} \Rightarrow \frac{2a}{3-a} = \frac{4-A}{A}$$

$$\Rightarrow \frac{2a}{3-a}+1 = \frac{4-A}{A}+1$$

$$\Rightarrow \frac{2a+3-a}{3-a} = \frac{4-A+A}{A}$$

$$\therefore \quad A = \frac{4(3-a)}{(3+a)}$$

4. (*c*) $\log_a b = \frac{1}{2}, \log_b c = \frac{1}{3}, \log_c a = \frac{k}{5}$

$$\Rightarrow \frac{\log b}{\log a} = \frac{1}{2}, \frac{\log c}{\log b} = \frac{1}{3}, \frac{\log a}{\log c} = \frac{k}{5}$$

$$\Rightarrow \frac{1}{2} \times \frac{1}{3} \times \frac{k}{5} = 1 \Rightarrow k = 30$$

5. (*b*) Let, $\log_{17} 275 = \log_{19} 375$

$$\Rightarrow \frac{\log 275}{\log 17} = \frac{\log 375}{\log 19}$$

$$\Rightarrow \frac{275}{17} = \frac{375}{19}$$

$\therefore \quad 16.18 < 19.74$

Hence, $\log_{17} 275 < \log_{19} 375$

6. (*b*) $\frac{1}{2}\log_{10} 25 - 2\log_{10} 3 + \log_{10} 18$

$= \log_{10}(25)^{1/2} - \log_{10}(3)^2 + \log_{10} 18$

$= \log_{10} 5 - \log_{10} 9 + \log_{10} 18$

$= \log_{10}\frac{5}{9} + \log_{10} 18$

$= \log_{10}\frac{5 \times 18}{9} = \log_{10} 10 = 1$

7. (*c*) Given $\log(A^2 + B^2) - (\log A + \log B) = C$

If $A = B$, then

$$\Rightarrow \log(2A^2) - 2\log A = C$$

$$\Rightarrow \log(2A^2) - \log A^2 = C$$

$$\Rightarrow \log\left(\frac{2A^2}{A^2}\right) = C$$

$$\Rightarrow \log 2 = C$$

8. (*a*) $\log_x a, a^{x/2}$ and $\log_b x$ are in GP, then

$$(a^{x/2})^2 = (\log_x a) \times (\log_b x)$$

$$\Rightarrow a^x = \log_b a$$

$$\Rightarrow x \log a = \log_a (\log_b a)$$

9. (*c*) $\log_a (ab) = x$

$$\Rightarrow \frac{\log ab}{\log a} = x$$

$$\Rightarrow \frac{\log b}{\log a} = x - 1$$

$$\Rightarrow \frac{\log a}{\log b} = \frac{1}{x-1}$$

Now, $\log_b (ab) = \frac{\log ab}{\log b} = \frac{\log a + \log b}{\log b}$

$$= \frac{\log a}{\log b} + 1 = \frac{1}{x-1} + 1$$

$$= \frac{1+x-1}{x-1} = \frac{x}{x-1}$$

10. (*d*) $\log_8 x + \log_8 \frac{1}{6} = \frac{1}{3}$

$$\Rightarrow \log_8\left(x \times \frac{1}{6}\right) = \frac{1}{3}$$

$$\Rightarrow \log_8\left(\frac{x}{6}\right) = \frac{1}{3}$$

$$\Rightarrow (8)^{1/3} = \frac{x}{6}$$

[$\because$ if $\log_a y = x$, than $(a)^x = y$]

$$\Rightarrow (2^3)^{1/3} = \frac{x}{6}$$

$$\Rightarrow x = 12$$

11. (*d*) $\log_8 x + \log_8 \frac{1}{6} = \frac{1}{3}$

$$\Rightarrow \frac{\log x}{\log 8} + \frac{\log \frac{1}{6}}{\log 8} = \frac{1}{3}$$

$$\Rightarrow \frac{\log x + \log 1 - \log 6}{\log 2^3} = \frac{1}{3}$$

$\Rightarrow \quad \log x + \log 1 - \log 6 = \log 2$

$\log x + \log 1 = \log 6 + \log 2$

$\log x = \log 12$

$\therefore \quad x = 12$

12. (*d*) $x^2 - 6x + 45 = 100$

$x = -5, 11$

13. (*b*) $\log_{10} 125 + \log_{10} 8 = x$

$\Rightarrow \log_{10}(125 \times 8) = x$

$\Rightarrow \quad 10^x = 1000$

$\therefore \quad x = 3$

14. (*b*) $\log\left(\frac{a^2}{b^2} \times \frac{b^2}{c^2} \times \frac{c^2}{a^2}\right) = \log(1) = 0$

15. (*a*) $\log\left(\frac{9}{14}\right) - \log\left(\frac{15}{16}\right) + \log\left(\frac{35}{24}\right)$

$= \log \frac{9}{14} \times \frac{16}{15} \times \frac{35}{24} = \log 1 = 0$

16. (*d*) Characteristics = number of digits – 1 = 5 – 1 = 4

17. (*c*) $\log_b 8 = 3 \Rightarrow 3\log_b 2 = 3 \Rightarrow \log_b 2 = 1$

$\log_a b = \log_2 b \times \log_a 2 = \log_2 b \times \log_3 2 \times \log_a 3$

$= 1 \times \log_3 2 \times 2 = 2\log_3 2 = \log_3 4$

18. (*b*) $\log_{49} 28 = \frac{1}{2}\log_7(7 \times 4) = \frac{1}{2}(1 + \log_7 4)$

$= \frac{1}{2} + \frac{1}{2}.2\log_7 2 = \frac{1}{2} + \log_7 2$

$= \frac{1}{2} + m = \frac{1+2m}{2}$

19. (*d*) $\log_{1000}(x^2) = \frac{2}{3}\log_{10} x = \frac{2}{3}y$

20. (*b*) $\frac{1}{2}\log 25 - 2\log 3 + \log 18$

$= \frac{1}{2}\log 5^2 - 2\log 3 + \log(2 \times 3 \times 3)$

$= \log 5 - 2\log 3 + \log 2 + \log 3^2$

$= \log 5 + \log 2$

$= \log(5 \times 2) = \log 10 = 1$

21. (*c*) $\log_a ab = x$

$\Rightarrow \quad \frac{\log ab}{\log a} = x$

$\Rightarrow \quad \frac{\log a + \log b}{\log a} = x$

$\Rightarrow \quad \frac{\log b}{\log a} + 1 = x$

$\Rightarrow \quad \frac{\log b}{\log a} = x - 1$

$\Rightarrow \quad \frac{\log a}{\log b} = \frac{1}{x-1}$

$\therefore \quad \log_b ab = \frac{\log ab}{\log b} = 1 + \frac{\log a}{\log b}$

$= 1 + \frac{1}{x-1} = \frac{x}{x-1}$

22. (*d*) Let, the number of bacteria in the beginning be *P*, which doubles after time *T*.

$\therefore \quad P\left(1 + \frac{10}{100}\right)^T = 2P$

[Here 1 unit of time = 5 minutes]

$\Rightarrow \quad (1.1)^T = 2$

$\Rightarrow T\log(1.1) = \log 2$

$\Rightarrow \quad T = \frac{\log 2}{\log 1.1}$

$\Rightarrow \quad T = \frac{0.3010}{0.0414} = \frac{3010}{414}$

= 7.27 units

= 36.35 minutes

23. (*b*) $\log_{3/2} 3.375 = x$

$\Rightarrow \quad \left(\frac{3}{2}\right)^x = 3.375$

$\Rightarrow \quad (1.5)^x = (1.5)^3 \Rightarrow x = 3$

24. (*a*) $7\log\left(\frac{2^4}{5 \times 3}\right) + 5\log\left(\frac{5^2}{2^3 \times 3}\right) + 3\log\left(\frac{3^4}{2^4 \times 5}\right)$

$= 28\log 2 - 7\log 5 - 7\log 3 + 10\log 5 - 15\log 2$

$-5\log 3 + 12\log 3 - 12\log 2 - 3\log 5 = \log 2$

25. (*c*) $\frac{\log 7^{5/2} + \log 5^{5/2} - \log 2^{5/2}}{\log 17.5}$

$= \frac{5(\log 7 + \log 5 - \log 2)}{2\log\left(\frac{35}{2}\right)} = \frac{5}{2}$

26. (*b*) $\log_{10}\tan 40° \times \log_{10}\tan 41° \ldots \log_{10}\tan 50°$

$= 0$, since $\log_{10}\tan 45° = 0$

27. (*b*) $\log_a y = \dfrac{1}{1-\log_a x}$, $\log_a z = \dfrac{1}{1-\log_a y}$

$\therefore\quad \log_a z = \dfrac{1}{1-\left(\dfrac{1}{1-\log_a x}\right)} = \dfrac{1-\log_a x}{-\log_a x}$

$\Rightarrow\quad -\log_a z = -1 + \dfrac{1}{\log_a x}$

$\Rightarrow\quad \dfrac{1}{\log_a x} = 1 - \log_a z$

$\therefore\quad \log_a x = \dfrac{1}{1-\log_a z}$

$\Rightarrow\quad x = \dfrac{1}{a^{1-\log_a^z}} = a^k$ (given)

$\therefore\quad k = \dfrac{1}{1-\log_a z}$

28. (*b*) $\log_e 2 \times 4 \log_b 5 \;= 4 \times \log_{10} 2 \times \log_e 10 = 4 \log_e 2$

$\Rightarrow\quad \log_b 5 = 1 \Rightarrow b = 5$

29. (*c*) $5^{\sqrt{\log_5 7}} - (7^{\log_7 5})^{\frac{1}{\sqrt{\log_7 5}}}$

$= 5^{\sqrt{\log_5 7}} - \dfrac{1}{5^{\sqrt{\log_7 5}}}$

$\Rightarrow\quad 5^{\sqrt{\log_5 7}} - 5^{\sqrt{\log_5 7}} \;= 0$

30. (*d*) $2^{\log 37} - 7^{\log 32} = 2^{\log 27 \times \log 32} - 7^{\log 32}$

$= 7^{\log 32} - 7^{\log 32} = 0$

31. (*a*) $a + b = \log_{30} 15 = \log_{30}\left(\dfrac{30}{2}\right) = 1 - \log_{30} 2$

$\Rightarrow\quad \log_{30} 2 = 1 - a - b.$

$\therefore\quad \log_{30} 8 = 3(1 - a - b)$

32. (*c*) $\log_5 2 = \dfrac{p}{q} \Rightarrow 2 = 5^{p/q} = 2^q = 5^p$

$\Rightarrow$ even number = odd number,

which is a contradiction.

$\therefore$ $\log_5 2$ is an irrational number.

33. (*d*) We have

$\dfrac{\log_{27} 9 \log_{16} 64}{\log_4 \sqrt{2}} = \dfrac{\log 9}{\log 27} \times \dfrac{\log 64}{\log 16} \times \dfrac{\log 4}{\log\sqrt{2}}$

$= \dfrac{2\log 3}{3\log 3} \times \dfrac{6\log 2}{4\log 2} \times \dfrac{2\log 2}{\frac{1}{2}\log 2}$

$= \dfrac{1}{3} \times \dfrac{6}{4} \times 4 = 4.$

Difficulty Level-2

1. (*b*) $\log_{10} x - \log_{10} \sqrt{x} \;= 2 \log_x 10$

$\Rightarrow\quad \log_{10} x - \dfrac{1}{2}\log_{10} x = \dfrac{2}{\log_{10} x}$

$\Rightarrow\quad \dfrac{\log_{10} x}{2} = \dfrac{2}{\log_{10} x}$

$(\log_{10} x)^2 = 4$

$\Rightarrow\quad \log_{10} x = \pm 2$

$\Rightarrow\quad x = 10^{-2}, 10^2 = \dfrac{1}{100}, 100.$

2. (*d*) $\log m + \log\left(\dfrac{m^2}{n}\right) + \log\left(\dfrac{m^3}{n^2}\right)$

$+ \log\left(\dfrac{m^4}{n^3}\right) + \ldots + n\text{th term}$

$= \log\left[\dfrac{m \cdot m^2 \cdot m^2 \ldots m^n}{n \cdot n^2 \cdot n^3 \ldots n^{n-1}}\right] = \log\left[\dfrac{m^{(1+2+3+\ldots+n)}}{n^{(1+2+3+\ldots+n-1)}}\right]$

$= \log\left[\dfrac{m^{\frac{n(n+1)}{2}}}{n^{\frac{(n-1)n}{2}}}\right] = \log\left[\dfrac{m^{(n+1)}}{n^{(n-1)}}\right]^{n/2}.$

3. (*c*) If the length of the circumference of a circle equals the perimeter of a regular polygon then

Area of circle > Area of regular polygon

Also, if two regular polygons have the same perimeter, then the regular polygon having larger no. of sides will have area greater than that of the regular polygon having less number of sides.

$\therefore\quad c > s > t.$

4. (*d*) Given $\log_n 48 = 4 \log_n 2 + \log_n 3 = a$ and,

$\log_n 108 = 2.\log_n 2 + 3 \log_n 3 = b$

Let, $\log_n 2 = x$ and, $\log_n 3 = y$

$\Rightarrow\quad 4x + y = a \qquad (1)$

$2x + 3y = b \qquad (2)$

$2 \times (2) - (1)$ gives

$$6y - y = 2b - a$$

$$\Rightarrow \quad y = \frac{2b - a}{5}$$

Similarly, $x = \frac{3a - b}{10}$

$$\log_n 1296 = 4(\log_n 2 + \log_n 3)$$

$$= 4\left[\frac{2b - a}{5} + \frac{3a - b}{10}\right]$$

$$= 4\left[\frac{a + 3b}{10}\right] = \frac{2(a + 3b)}{5}$$

5. (*a*) $\sum_{n=1}^{n} \frac{1}{\log_{2^n}(a)} = \sum_{n=1}^{n} \log_a(2^n)$

$$= \sum_{n=1}^{n} n \log_a 2$$

$$= \log_a 2 \sum_{n=1}^{n} n = \frac{n(n+1)}{2} \log_a 2$$

6. (*a*) $\log \frac{a}{b} + \log \frac{b}{a} = \log (a + b)$

$$\Rightarrow \quad \log\left(\frac{a}{b} \times \frac{b}{a}\right) = \log (a + b)$$

$$\therefore \quad a + b = 1$$

7. (*c*) $\log_x (0.1) = -\frac{1}{3}$

$$\Rightarrow \quad x^{-\frac{1}{3}} = 0.1 \Rightarrow x = 1000$$

8. (*c*) $\log_7 \log_5 (\sqrt{x + 5} + \sqrt{x}) = 0$

$$\Rightarrow \quad 7^0 = \log_5(\sqrt{x + 5} + \sqrt{x})$$

$$\Rightarrow \quad \log_5(\sqrt{x + 5} + \sqrt{x}) = 1$$

$$\Rightarrow \quad 5^1 = \sqrt{x + 5} + \sqrt{x}$$

$$\Rightarrow \quad x = 4$$

9. (*d*) The given equation

$$= \frac{1}{\log_{\frac{442}{441}} x} = \frac{\log \frac{442}{441}}{\log x} = \log_x \frac{442}{441}$$

$$\log_x \frac{442}{441} + \log_x \frac{443}{442} + \log_x \frac{444}{443} + \ldots$$

$$+ \log_x \frac{899}{898} + \log_x \frac{900}{899} = 2$$

$$\Rightarrow \log_x \left(\frac{442}{441} \times \frac{443}{442} \times \frac{444}{443} \times \ldots \times \frac{899}{898} \times \frac{900}{899}\right) = 2$$

$$\Rightarrow \quad \log_x \frac{900}{441} = 2 \Rightarrow x^2 = \frac{900}{441} \Rightarrow x = \frac{30}{21} = \frac{10}{7}$$

10. (*d*) $[\log_{10} (5 \log_{10} 100)]^2 = [\log_{10} (5 \times 2)]^2$

$$= [\log_{10} 10]^2$$

$$= 1^2 = 1$$

11. (*d*) $\log_5 12 = \log_5 (3 \times 4)$

$$= \log_5 3 + \log_5 4$$

$$= \log_5 3 + 2 \log_5 2$$

$$= \frac{\log_{10} 3}{\log_{10} 5} + \frac{2 \log_{10} 2}{\log_{10} 5}$$

$$= \frac{\log_{10} 3}{\log_{10} 10 - \log_{10} 2} + \frac{2 \log_{10} 2}{\log_{10} 10 - \log_{10} 2}$$

$$= \frac{b}{1 - a} + \frac{2a}{1 - a} = \frac{2a + b}{1 - a}$$

12. (*b*) $a = \log_8 225, b = \log_2 15$

$$\therefore \; a = \log_{2^3} 15^2 = s\frac{2}{3} \log_2 15 = \frac{2}{3} \times b$$

13. (*c*) $\log x - 5 \log 3 = -2$

$$\Rightarrow \quad \log_{10} \frac{3^5}{x} = -2$$

$$\frac{x}{243} = 10^{-2}$$

$$\Rightarrow \quad x = \frac{243}{100} \Rightarrow x = 2.43$$

14. (*d*) $\log_5 \frac{(125)(625)}{25} = \log_5 \frac{(5^3)(5^4)}{5^2}$

$$= \log_5 5^{7-2} = \log_5 5^5$$

$$= 5 \log 5^5 = 5$$

15. (*b*) $\log_{10} (a^2 - 15a) = 2$

$$a^2 - 15a = 10^2$$

$$\Rightarrow \; a^2 - 15a - 100 = 0$$

$$(a - 20)(a + 5) = 0$$

$$\Rightarrow \quad a = 20, -5$$

16. (*a*, *b*, *c*) Option (*d*) is wrong as the expression evaluate to 1 as in (*c*) and not zero. In all, options (*a*), (*b*) and (*c*) are correct.

17. (*c*) $2 \log x = 5 \log y + 3$

$\log x^2 = \log y^5 + 3 \log_{10}10$

$\log x^2 = \log y^5 + 3 \log_{10}10^3$

$= \log (y^5 \times 10^3)$

$\Rightarrow \quad x^2 = 1000y^5$

18. (*c*) Given, $\log_4 \log_4 4^{a-b} = 2 \log_4 (\sqrt{a} - \sqrt{b}) + 1$

$\log_4 (a-b) \log_4 4 = \log_4(\sqrt{a} - \sqrt{b})^2 + \log_4 4$

$\log_4(a-b) = \log_4 4(\sqrt{a} - \sqrt{b})^2$

$(a-b) = 4(\sqrt{a} - \sqrt{b})^2$

$\Rightarrow \quad \sqrt{a} + \sqrt{b} = 4\sqrt{a} - 4\sqrt{b}$

$3\sqrt{a} = 5\sqrt{b}$

$\Rightarrow \quad \dfrac{\sqrt{a}}{\sqrt{b}} = \dfrac{5}{3}.$

19. (*c*) $\log_a a \quad = 1$

$= \log_{10} 10^{10^{10^{10^{10}}}}$

$= 10^{10^{10^{10}}} \log_{10}10 = 10^{10^{10^{10}}}$

i.e., with each $\log_{10}$ one 10^{10} is removed there are $5 - 10s$ (including the ones in powers) and $4 - \log_{10}$ therefore, last will be $\log_{10}10^{10} = 10\log_{10}10 = 10$.

20. (*c*) $\log_{10} (\log_2 3) + \log_{10} (\log_3 4) + \ldots + \log_{10} \log_{1023} 1024)$

$= \log_{10} [(\log_2 3)(\log_3 4)(\log_4 5) \ldots (\log_{1023} 1024)]$

$= \log_{10}\left(\dfrac{\log 3}{\log 2} \times \dfrac{\log 4}{\log 3} \times \dfrac{\log 5}{\log 4} \times \ldots \times \dfrac{\log 1024}{\log 1023}\right)$

$\left[\because \log_b a = \dfrac{\log a}{\log b}\right]$

$= \log_{10} (\log_2 1024) = \log_{10} \log_2 2^{10} = \log_{10} 10 = 1$

21. (*d*) Required sum $= \dfrac{1}{2} - \dfrac{1}{3} + \dfrac{1}{4} - \ldots$ to ∞

Now, $\log_e(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \ldots$

$\therefore \quad \log_e 2 = 1 - \dfrac{1}{2} + \dfrac{1}{3} - \ldots$ to ∞

$\Rightarrow \dfrac{1}{2} - \dfrac{1}{3} + \dfrac{1}{4}$ .. to $\infty = 1 - \log_e 2 = 1 - \ln 2$

22. (*c*) Given: $\log_2 x + \log_2 y \geq 6$

$\Rightarrow \quad \log_2(xy) \geq 6$

$\Rightarrow \quad xy \geq 64$

23. (*b*) Changing the base to 10, we get

$\log_{\sqrt{10}} x = \dfrac{\log_{10} x}{\log_{10} \sqrt{10}} = 2 \log_{10} x$

$\log_{\sqrt[3]{100}} x = \dfrac{\log_{10} x}{\log_{10} \sqrt[3]{100}} = \dfrac{3\log_{10} x}{2}$

$\therefore \quad \log_{10} x + 2\log_{10} x + \dfrac{3}{2}\log_{10} x = 27$

$\Rightarrow \quad \dfrac{9}{2}\log_{10} x = 27$

$\Rightarrow \quad \log_{10} x = 6$

$\therefore \quad x = 10^6$

24. (*c*) $y = 2^{\frac{1}{\log_x 4}} = 2^{\log_4 x} = 2^{\frac{1}{2}\log_2 x}$

$= 2^{\log_2 \sqrt{x}} = \sqrt{x}$

$\therefore \quad x = y^2$

25. (*b*) $\dfrac{\log a}{b-c} = \dfrac{\log b}{c-a} = \dfrac{\log c}{a-b} = r$

$\therefore \; a \log a + b \log b + c \log c$

$= r[a(b-c) + b(c-a) + c(a-b)]$

$\Rightarrow \quad \log a^a + \log b^b + \log c^c = 0$

$\Rightarrow \quad \log(a^a \, b^b \, c^c) = \log 1$

$\Rightarrow \quad a^a \, b^b \, c^c = 1$

26. (*c*) $(\log_b a \log_c a - 1) + (\log_a b \,.\, \log_c b - 1)$

$+ (\log_a c \log_b c - 1) = 0$

$\Rightarrow \dfrac{\log a}{\log b} \cdot \dfrac{\log a}{\log c} + \dfrac{\log b}{\log a} \cdot \dfrac{\log b}{\log c} + \dfrac{\log c}{\log a} \cdot \dfrac{\log c}{\log b} = 3$

$\Rightarrow (\log a)^3 + (\log b)^3 + (\log c)^3 = 3 \log a \log b \log c$

$\Rightarrow (\log a + \log b + \log c) = 0$

$[\because$ If $a^3 + b^3 + c^3 - 3abc = 0$ then $a + b + c = 0$ if $a \neq b \neq c]$

$\Rightarrow \quad \log abc = \log 1 \Rightarrow abc = 1$

27. (*b*) $3^{2\log_9 3} = 3^{2 \cdot \frac{1}{2}\log_3 3} = 3$

28. (*b*) $\log_{10} x - \log_{10} \sqrt{x} = 2 \log_x 10$

$\Rightarrow \quad \log_{10} x - \dfrac{1}{2}\log_{10} x = \dfrac{2}{\log_{10} x}$

$\Rightarrow \quad \dfrac{\log_{10} x}{2} = \dfrac{2}{\log_{10} x}$

$\Rightarrow \quad (\log_{10} x)^2 = 4 \Rightarrow \log_{10} x = \pm 2$

$\Rightarrow \quad x = 10^{-2}, 10^2 = \dfrac{1}{100}, 100$

29. (*c*) If the length of the circumference of a circle equals the perimeter of a regular polygon, then,

Area of circle > Area of regular polygon

Also, if two regular polygons have the same perimeter, then the regular polygon having larger number of sides will have area greater than that of the regular polygon having less number of sides.

$\therefore \quad c > s > t$

30. (*c*) $\log (A^2 + B^2) - [\log (A) + \log (B)] = C$

$\therefore \quad A = B \Rightarrow \log (2A^2) - 2 \log A = C$

$\Rightarrow \log 2A^2 - \log A^2 = C$

$\Rightarrow \quad \log \dfrac{2A^2}{A^2} = C \Rightarrow C = \log 2$

31. (*a*) $\quad -x = (x + 1)^2$

$\Rightarrow x^2 + 3x + 1 = 0$

$\therefore \quad x = \dfrac{-3+\sqrt{5}}{2}, \dfrac{-3-\sqrt{5}}{2}$

But only $= \dfrac{-3+\sqrt{5}}{2}$ satisfies the other condition of $x + 1 > 0$.

32. (*b*) $\quad \log_7 \log_5 (\sqrt{x} + 5 + \sqrt{x}) = 0$

$\Rightarrow \quad \log_5 (\sqrt{x} + 5 + \sqrt{x}) = 7^0 = 1$

$\sqrt{x} + 5 + \sqrt{x} = 5^1 = 5$

$\Rightarrow \quad 2\sqrt{x} = 0 \therefore x = 0$

33. (*c*) $\quad \log_2 [\log 7(x^2 - x + 37)] = 1$

$\Rightarrow \quad 2 = \log_7 (x^2 - x + 37)$

$\Rightarrow \quad 49 = x^2 - x + 37$

or, $\quad x^2 - x - 12 = 0$

$\Rightarrow \quad (x - 4)(x + 3) = 0. \therefore x = 4$

34. (*b*) $\dfrac{1}{3} \log_3 M + 3 \log_3 N = 1 + \log_{0.008} 5$

$\Rightarrow \quad \log_3 M^{1/3} + \log_3 N^3 = 1 + \dfrac{\log_e 5}{\log_e 0.008}$

$\Rightarrow \quad \log_3 (M. N^9)^{1/3} = 1 + \dfrac{\log_e 5}{\log_e \dfrac{8}{1000}}$

$= 1 + \dfrac{\log_e 10 - \log_e 2}{\log_e 8 - \log_e 1000}$

$= 1 + \dfrac{\log_e 10 - \log_e 2}{3 \log_e 2 - 3 \log_e 10}$

$= 1 + \dfrac{\log_e 10 - \log_e 2}{-3 (\log_e 10 - \log_e 2)}$

$\therefore \log_3 (MN^9)^{1/3} = 1 - \dfrac{1}{3} = \dfrac{2}{3}$

$\Rightarrow \quad (MN^9)^{\frac{1}{3}} = 3^{\frac{2}{3}} = (9)^{\frac{1}{3}}$

$\Rightarrow \quad MN^9 = 9$

$\therefore \quad N^9 = \dfrac{9}{M}$

35. (*b*) $\log_{10} x - \log_{10} \sqrt{x} = 2 \log_x 10$

$\Rightarrow \log_{10} x - \dfrac{1}{2} \log_{10} x = 2 \log_x 10$

$\Rightarrow \dfrac{1}{2} \log_{10} x = 2. \log_x 10$

$\Rightarrow \quad \log_{10} x = 4. \log_x 10$

$\Rightarrow \quad \log_{10} x = \log_x 10^4$

or, $\quad \log_{10} x = \log_x 10000$

Now putting the value of $x = 10$

$1 = 4$, which is not possible

Putting the value of $x = \dfrac{1}{100}$, we get $-2 = -2$.

Thus answer is (*b*).

36. (*b*) $\quad u = (\log_2 x)^2 - 6(\log_2 x) + 12$

Let, $\quad \log_2 x = p \quad (1)$

$\Rightarrow \quad u = p^2 - 6p + 12$

$x^u = 256 = (2^8)$

Applying log to base 2 on both sides, we get $u \log_2 x = \log_2 2^8$.

$\Rightarrow \quad u \log_2 x = 8 \quad (2)$

Dividing (2) by (1), we get

$u = 8/p$

$\Rightarrow \quad 8/p = p^2 - 6p + 12$

$\Rightarrow 8 - p^3 - 6p^2 + 12p = 0$

or, $\quad p^3 - 6p^2 + 12p - 8 = 0$

$\Rightarrow \quad (p - 2)^3 = 0$ or, $p = 2$

$\therefore \quad \log_2 x = 2 \Rightarrow 2 \Rightarrow x = 2^2 = 4$

Thus, we have exactly one solution.

37. (*d*) $\log_8 x + \log_8 \frac{1}{6} = \frac{1}{3}$

$\Rightarrow \log_8\left(x \times \frac{1}{6}\right) = \frac{1}{3}$ or, $\log_8\left(\frac{x}{6}\right) = \frac{1}{3}$

$\Rightarrow (8)^{1/3} = \frac{x}{6} \Rightarrow (2^3)^{1/3} = \frac{x}{6}$

$\therefore x = 12$

38. (*b*) $\log_{17}275 = \log_{19}375$

$\Rightarrow \frac{\log 275}{\log 17} = \frac{\log 375}{\log 19}$

$\Rightarrow \frac{275}{17} = \frac{375}{19}$

$\because 16.18 < 19.74$

$\therefore \log_{17}275 < \log_{19}375$

39. (*a*) $yz(2 - x) = 2yz - xyz = 2\log_{4a} 2a - \log_{4a} a$

$= \log_{4a}\left(\frac{4a^2}{a}\right) = 1$

40. (*d*) Each is equal to k

$\Rightarrow \log x = k(l + m - 2n)$,

$\log y = k(m + n - 2l)$,

$\log z = k(n + l - 2m)$.

$\Rightarrow \log xyz = k(0)$

$\Rightarrow xyz = e^0 = 1 \Rightarrow x^2y^2z^2 = 1$

41. (*b*) $\log\left(\frac{x+y}{5}\right) = \frac{1}{2}[\log x + \log y]$

$\Rightarrow x + y = 5\sqrt{xy} \Rightarrow x^2 + y^2 = 23xy$

$\Rightarrow \frac{x}{y} + \frac{y}{x} = 23$

42. (*c*) $x + y = \frac{3x - 3y}{2}$

$\Rightarrow x = 5y \Rightarrow \frac{x}{y} = 5$

$\Rightarrow \log x - \log y = \log 5$

43. (*a*) $\log_2 x + \frac{1}{2}\log_2 x + \frac{1}{4}\log_2 xs = \frac{21}{4}$

$\Rightarrow \log_2{}^x\left(1 + \frac{1}{2} + \frac{1}{4}\right) = \frac{21}{4}$

$\Rightarrow \log_2 x = 3 \Rightarrow x = 8$

44. (*b*) $0 < a \le x$; Min. value of $\log_a x + \log_x a$ is 2 when we put $x = a$

45. (*a*) $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b} = k$ (say)

$\Rightarrow \log x = k(b - c), \log y = k(c - a)$,

$\log z = k(a - b)$

$\Rightarrow \log x + \log y + \log z = 0 \Rightarrow xyz = 1.$

Also, $a\log x + b\log y + c\log z = 0$

$\Rightarrow x^a . y^b . z^c = 1.$

Again $(b + c)\log x + (c + a)\log y + (a + b)\log z = 0.$

$\Rightarrow x^{b+c} \times y^{c+a} \times z^{a+b} = 1.$

$\therefore xyz = x^a \times y^b \times z^c = x^{b+c} \times y^{c+a} \times z^{a+b} = 1$

46. (*c*) $x^{\log y - \log z} \times y^{\log z - \log x} \times z^{\log x - \log y} = k$ (say)

$\Rightarrow (\log y - \log z)\log x + (\log z - \log x)\log y$

$+ (\log x - \log y)\log z = \log k = 0$

$\Rightarrow k = 1$

47. (*d*) $98 + \sqrt{x^2 - 12x + 36} = 100$

$\Rightarrow \sqrt{x^2 - 12x + 36} = 2$

$\Rightarrow x^2 - 12x + 32 = 0$

$\Rightarrow x = 8, 4$

48. (*a*) $x = \log_a bc$

$\Rightarrow a^x = bc$

$\Rightarrow a^{x+1} = abc$

$\Rightarrow a = (abc)^{1/x+1}.$

Similarly, $b = (abc)^{1/y+1}$ and $c = (abc)^{1/z+1}$

$\therefore abc = (abc)^{\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}}$

$\Rightarrow 1 = \frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$

$\Rightarrow (x + 1)(y + 1)(z + 1) = (y + 1)(z + 1)$

$+ (x + 1)(z + 1) + (x + 1)(y + 1)$

$\Rightarrow xyz = x + y + z + 2$

49. (*b*) $b^y = a^x \Rightarrow b = a^{\frac{x}{y}}, c = a^{\frac{x}{z}}, d = a^{\frac{x}{w}}$

$\log_a(bcd) = \log_a\left(a^{\frac{x}{y}}.a^{\frac{x}{z}}.a^{\frac{x}{w}}\right) = \frac{x}{y} + \frac{x}{z} + \frac{x}{w}$

$= x\left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right)$

50. (*c*) $\log_{10}\left(\frac{1}{2}\right) = -\log_{10} 2 = -0.3010$

$= 1 - 0.3010 - 1 = \bar{1}.6990$

51. (*d*) $\log_2(3^{2x-2}+7) = \log_2{}^4 + \log_2(3^{x-1}+1)$

$[\because 2 = 2\log_2{}^2 = \log_2 2^2]$

$\Rightarrow \quad 3^{2x-2}+7 = 4(3^{x-1}+1)$

$\Rightarrow \quad t^2+7 = 4(t+1)$, where, $3^{x-1} = t$

$\Rightarrow t^2 - 4t + 3 = 0 \Rightarrow t = 1, 3$

When $t = 1 \Rightarrow 3^{x-1} = 1 \Rightarrow x = 1$

When $t = 3 \Rightarrow 3^{x-1} = 3^1 \Rightarrow x = 2$

52. (*c*) $\log_a b = \log_b c = \log_c a = k$ (say)

$\Rightarrow \quad b = a^k, c = b^k, a = c^k$

$\Rightarrow \quad c = (a^k)^k = a^{k2} = (c^{k2})^k = c^{k3}$

$\Rightarrow \quad k^3 = 1 \Rightarrow k = 1. \ \therefore \ a = b = c$

53. (*d*) $\log_{10} x = 2\log_{10} a - 2$

$\Rightarrow \quad \log_{10} x = 2(\log_{10} a - 1)$

$\Rightarrow \quad \log_{10} x = 2\log_{10}\left(\frac{a}{10}\right) \Rightarrow x = \frac{a^2}{100}$

54. (*b*) $\log_b(c+a) + \log_b(c-a)$

$= \log_b(c^2 - a^2) = \log_b b^2 = 2$

55. (*a*) $x = (875)^{10} = (87.5 \times 10)^{10}$

$\therefore \quad \log_{10} x = 10(\log_{10} 87.5 + 1)$

$= 10(1.9421 + 1)$

$= 10(2.9421) = 29.421.$

$\therefore \quad x = \text{Antilog}(29.421).$

$\therefore$ Number of digits in $x = 30$.

56. (*b*) $x = (0.0432)^{10} = \left(\frac{432}{10000}\right)^{10}$

$= \left(\frac{3^3.2^4}{10^4}\right)^{10}$

$\therefore \quad \log_{10} x = 10(3\log_{10} 3 + 4\log_{10} 2 - 4)$

$= 10(1.4313 + 1.2040 - 4)$

$= 10(-1.3647) = -13.647$

$= \overline{14}.353$

$\therefore \quad x = \text{Antilog}(.053)$

$\therefore$ Number of zeros between the decimal and the first significant figure = 13.

57. (*c*) $(4.2)^x = 100$

$\Rightarrow \quad (42)^x = 10^{2+x}$

$\Rightarrow \quad 42 = 10^{\frac{2}{x}+1}$ (1)

$\left(\frac{42}{100}\right)^y = 100$

$\Rightarrow \quad (42)^y = 10^{2+2y}$

$\Rightarrow \quad 42 = 10^{\frac{2}{y}+2}$ (2)

From (1) and (2), $\frac{2}{x} - \frac{2}{y} = 1 \Rightarrow \frac{1}{x} - \frac{1}{y} = \frac{1}{2}$

58. (*c*) $\frac{\log_9 11}{\log_5 13} - \frac{\log_3 11}{\log_{\sqrt{5}} 13} = \frac{\log_3 11}{2.\log_5 13} - \frac{\log_3 11}{2.\log_5 13} = 0$

59. (*d*) $\frac{\log x}{2} = \frac{\log y}{3} = \frac{\log z}{5} = k$ (say)

$\Rightarrow \quad \log x = 2k, \log y = 3k, \log z = 5k$

$\Rightarrow \quad \log yz = 3k + 5k = 8k; \log x^4 = 8k$

$\therefore \quad \log yz = \log x^4 \Rightarrow yz = x^4$

60. (*b*) $4^x + \frac{4^x}{2} = \frac{3^x}{\sqrt{3}} + 3^x.\sqrt{3}$

$\Rightarrow \quad 4^x \times \frac{3}{2} = 3^x \times \frac{4}{\sqrt{3}} \Rightarrow \left(\frac{4}{3}\right)^x = \frac{8}{3\sqrt{3}}$

$\Rightarrow \quad \left(\frac{4}{3}\right)^x = \left(\frac{4}{3}\right)^{3/2} \Rightarrow x = \frac{3}{2}$

61. (*a*) $\log_8 p = \frac{5}{2} \Rightarrow p = (8)^{5/2} = 2^{\frac{15}{2}} = (2^5)^{3/2}$

$\log_2 q = 5 \Rightarrow q = 2^5. \therefore p = q^{3/2}$

62. (*b*) $0 < a < 1, 0 < x < 1$ and $x < a$

$\Rightarrow \log_a x > \log_a a \Rightarrow \log_a x > 1$

63. (*b*) $\log_5 \frac{6}{5} + \log_5 \frac{7}{6} + \log_5 \frac{8}{7} + \ldots + \log_5 \frac{625}{624}$

$= \log_5\left(\frac{6}{5}\frac{7}{6}\frac{8}{7}\ldots\frac{625}{624}\right) = \log_5\left(\frac{625}{5}\right) = 4$

64. (*b*) $\log_{10}(0.02986) = \log_{10}\left(\frac{2986}{100000}\right)$

$= 3.4751 - 5 = -1.5249$

$= \overline{2}.4751$

65. (*a*) $2a - 3b = \frac{a}{b}$

$\Rightarrow 2ab - 3b^2 = a$

$\Rightarrow \quad 3b^2 = a(2b-1)$

$\Rightarrow \quad a = \dfrac{3b^2}{2b-1}$

66. (*c*) $(x-y)^2 = 25xy$

$\Rightarrow \quad x^2 + y^2 = 27xy$

$\Rightarrow \quad \dfrac{x}{y} + \dfrac{y}{x} = 27$

67. (*b*) $\dfrac{\log x}{3} = \dfrac{\log y}{4} = \dfrac{\log z}{5} = k$

$\Rightarrow \quad \log x = 3k; \log y = 4k; \log z = 5k.$

$\Rightarrow \quad \log (zx) = \log z + \log x = 8k = 2 \log y$

$\therefore \quad zx = y^2$

68. (*a*) $3 + \log_5 x = \log_5 y$

$\Rightarrow \log_5(125x) = \log_5 y$

$\Rightarrow \quad x = \dfrac{y}{125}$

69. (*d*) $\dfrac{\log_2 a}{3} = \dfrac{\log_3 b}{3} = \dfrac{\log_4 c}{4} = k$

$\Rightarrow \quad a = 2^{2k}, b = 3^{3k}, c = 4^{4k}$ and,

$a^{1/2} \times b^{1/3} \times c^{1/4} = 2^k \times 3^k \times 4^k = 24$

$\Rightarrow \quad 24^k = 24^1$

$\Rightarrow \quad k = 1.$

$\therefore \quad a = 4, b = 27, c = 256$

70. (*c*) $\dfrac{z}{x^3 y^4} = 1$

$\Rightarrow \quad \log_2 z - 3 \log_2 x - 4 \log_2 y = 0$

$\Rightarrow \log_2 z - \dfrac{3.3}{5k} \cdot \log_2 z - 4 \cdot \dfrac{4}{5k} \cdot \log_2 z = 0$

$\Rightarrow \quad 1 - \dfrac{9}{5k} - \dfrac{16}{5k} = 0$

$\Rightarrow \quad 5k - 25 = 0$

$\Rightarrow \quad k = 5$

71. (*a*) $\dfrac{3(1+\log_{10} 7)}{2 + \log\dfrac{7}{2} + \log\dfrac{1}{5}} = \dfrac{3(1+\log_{10} 7)}{2 + \log\left(\dfrac{7}{10}\right)}$

$= \dfrac{3(1+\log_{10} 7)}{1 + \log_{10} 7} = 3$

72. (*c*) Each ratio $= k \Rightarrow \log x = k(a^2 + ab + b^2)$

$\Rightarrow (a-b)\log x = k(a^3 - b^3)$

$\Rightarrow \quad \log x^{a-b} = k(a^3 - b^3) \Rightarrow x^{a-b} = e^{k(a3-b3)}$

Similarly, $y^{b-c} = e^{k(b3-c3)}$, $z^{c-a} = e^{k(c3-a3)}$.

$\therefore \; x^{a-b} \cdot y^{b-c} \cdot z^{c-a} = e^0 = 1$

73. (*c*) $3^{x-2} = 5$

$\Rightarrow \quad 3^x = 45 = \left(\dfrac{90}{2}\right)$

$\Rightarrow \quad x \log_{10}3 = \log_{10}90 - \log_{10}2$

$= 2 \log_{10}3 + 1 - \log_{10}2$

$\Rightarrow \quad x(0.4771) = 1.65317$

$\Rightarrow \; x = \dfrac{165317}{47710} = 3\dfrac{22187}{47710}$

74. (*d*) $\log (648)^5 = 5 \log (81 \times 8) = 20 \log 3 + 15 \log 2$

$= 20(0.4771) + 15(0.30103)$

$= 14.05745.$

$\therefore$ Number of digits in $(648)^5$ is 15

75. (*c*) $\dfrac{\log x}{1} = \dfrac{\log y}{2} = \dfrac{\log z}{5} = k$

$\Rightarrow \quad \log x = k, \log y = 2k, \log z = 5k.$

$\therefore \quad \log(x^4 \cdot y^3 \cdot z^{-2}) = 4 \log x + 3 \log y - 2 \log z = 0$

$\Rightarrow \quad x^4 \cdot y^3 \cdot z^{-2} = 1$

76. (*c*) $\dfrac{\log\sqrt{27} + \log\sqrt{1000} + \log 8}{\log 120}$

$= \dfrac{\frac{3}{2}(\log 3 + \log 10 + \log 4)}{\log 3 + \log 10 + \log 4} = \dfrac{3}{2}$

77. (*b*) $y = \dfrac{10^{\log_{10} x}}{x^2} = \dfrac{1}{x} = \dfrac{1}{y^a} = y^{-a} \Rightarrow a = -1$

78. (*b*) $x = \log_{4/3}(1/2) = -\log_{4/3} 2 < 0$

and, $\quad y = \log_{1/2}(1/3) = \log_2 3 > 0$

$\Rightarrow \quad y > x$

79. (*b*) $x^u = 256$

Take log on base 2, $u \log_2 x = 8$

Let, $\log_2 x = p$

Then,

$\dfrac{8}{p} = p^2 - 6p + 12$

$p^3 - 6p^2 + 12p - 8 = 0$

$p^2(p-2) - 4p(p-2) + 4(p-2)$

$(p^2 - 4p + 4)(p-2) = 0$

$(p-2)^3 = 0$

$p = 2$

80. (*d*) We have

$$\log_x(a-b)-\log_x(a+b)=\log_x\frac{b}{a}$$

$$\Rightarrow \log_x\frac{a-b}{a+b}=\log_x\frac{b}{a}$$

$$\Rightarrow a(a-b)=b(a+b)$$

$$\Rightarrow a^2-ab=ab+b^2$$

$$\Rightarrow a^2-b^2=2ab$$

$$\Rightarrow a^2-2ab-b^2=0$$

$$\Rightarrow \left(\frac{a}{b}\right)^2-2\left(\frac{a}{b}\right)-1=0$$

This is quadratic in $\frac{a}{b}$. The product of the root is -1, i.e., if $\frac{a}{b}$ is a root, then $-\frac{1}{\frac{a}{b}}$ will also be a root.

Therefore,

$$\left(\frac{a}{b}\right)^2+\left(\frac{b}{a}\right)^2=\left(\frac{a}{b}\right)^2+\left(\frac{-1}{\frac{a}{b}}\right)^2$$

$$=\left(\frac{a}{b}+\left(\frac{-1}{\frac{a}{b}}\right)\right)^2+2=2^2+2=6$$

(As the sum of the roots is 2).

81. (*a*) We have,

$$\log_2[\log_7(x^2-x+37)]=1$$

$$\Rightarrow \log_7(x^2-x+37)=(2)^1$$

$$\Rightarrow (x^2-x+37)=(7)^2$$

$$\Rightarrow x^2-x+37-49=0$$

$$\Rightarrow x^2-x-12=0$$

$$\Rightarrow x=4.$$

82. (*c*) Let $\log_6 216\sqrt{6}=x$, then $216\sqrt{6}=(6)^x$

$$\Rightarrow (6)^3\times(6)^{\frac{1}{2}}=(6)^x \Rightarrow (6)^{\frac{7}{2}}=(6)^x \Rightarrow x=\frac{7}{2}$$

83. (*b*) $\log_7\log_5(\sqrt{x}+5+\sqrt{x})=0$

$$\Rightarrow \log_5(\sqrt{x}+5+\sqrt{x})=(7)^0=1$$

$$\Rightarrow (\sqrt{x}+5+\sqrt{x})=(5)^1=5$$

$$2\sqrt{x}=0$$

$$\therefore x=0$$

84. (*c*) $\sum_{i=2}^{100}\frac{1}{\log_i 100!}=\sum_{i=2}^{100}\log_{100!}i$

$$=\log_{100!}2+\log_{100!}3+\log_{100!}4+....,+\log_{100!}100$$

$$=\log_{100!}100!=1$$