Chapter 15

Introduction to Three Dimensional Geometry

Solutions

SECTION - A

Objective Type Questions (one option is correct)

1. If a parallelopiped is formed by planes drawn through the points (2, 5, 3) and (6, 7, 9) parallel to the coordinate planes, then the length of its diagonal is

(1) $\sqrt{48}$ units (2) $2\sqrt{14}$ units (3) $3\sqrt{14}$ units (4) $\sqrt{14}$ units

Sol. Answer (2)

Length of diagonal

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$= \sqrt{(6-2)^2 + (7-5)^2 + (9-3)^2}$

$= \sqrt{16+4+36}$

$= 2\sqrt{14}$ units

2. If the extremities of the diagonal of a square are (1, –2, 3) and (3, –4, 3), then the length of the side is

(1) $2\sqrt{2}$ units (2) 2 units (3) $\sqrt{2}$ units (4) 4 units

Sol. Answer (2)

Length of diagonal

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$= \sqrt{(3-1)^2 + (-4+2)^2 + (3-3)^2}$

$= 2\sqrt{2}$ units

Now, diagonal of square = $\sqrt{2} \times \text{side}$

$\therefore$ Side = 2 units

3. If origin is the centroid of the triangle with vertices $P(3a, 3, 6)$, $Q(-4, 2b, -8)$ and $R(8, 12, 2c)$, then the value of a, b and c are

(1) $\frac{4}{3}, 1, 2$ (2) $-\frac{4}{3}, -\frac{15}{2}, 1$ (3) $3, 2, -\frac{4}{5}$ (4) $\frac{4}{3}, \frac{15}{2}, 1$

Sol. Answer (2)

The coordinates of centroid are

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

$$= \left(\frac{3a-4+8}{3}, \frac{3+2b+12}{3}, \frac{6-8+2c}{3}\right)$$

$$= \left(\frac{3a+4}{3}, \frac{2b+15}{3}, \frac{2c-2}{3}\right)$$

But it is given that, origin is centroid

$$\Rightarrow \frac{3a+4}{3}=0, \frac{2b+15}{3}=0 \text{ and } \frac{2c-2}{3}=0$$

On solving these equations, we get

$$a=-\frac{4}{3}, b=-\frac{15}{2} \text{ and } c = 1$$

4. If the three vertices of a parallelogram *ABCD* are *A*(3, –1, 5), *B*(1, –2, –4) and *C*(0, 3, 0), then the coordinates of fourth vertex is

(1) (2, 4, 9) (2) (2, –4, –9) (3) (0, 4, 9) (4) (0, –4, –9)

Sol. Answer (1)

Let the fourth vertex be (*x*, *y*, *z*), then mid-point of *AC* = mid-point of *BD*

$$\Rightarrow \left(\frac{0+3}{2}, \frac{-1+3}{2}, \frac{5+0}{2}\right)=\left(\frac{x+1}{2}, \frac{y-2}{2}, \frac{z-4}{2}\right)$$

$$\Rightarrow \left(\frac{3}{2}, 1, \frac{5}{2}\right)=\left(\frac{x+1}{2}, \frac{y-2}{2}, \frac{z-4}{2}\right)$$

$$\Rightarrow \frac{3}{2}=\frac{x+1}{2}, 1=\frac{y-2}{2}, \frac{5}{2}=\frac{z-4}{2}$$

$\Rightarrow x = 2, y = 4, z = 9$

Hence, the fourth coordinates are (2, 4, 9).

5. The ratio in which the join of *A*(1, 2, 3) and *B*(3, 4, 6) is divided by *XY*-plane externally is

(1) 2 : 1 (2) 1 : 2 (3) 2 : 3 (4) 3 : 2

Sol. Answer (2)

Let the join of *A*(1, 2, 3) and *B*(3, 4, 6) is divided by *XY*-plane in *k* : 1.

Therefore, the coordinates are

$$\left(\frac{3k+1}{k+1}, \frac{4k+2}{k+1}, \frac{6k+3}{k+1}\right)$$

Now, *z*-coordinate on *XY*-plane is zero

$$\Rightarrow \frac{6k+3}{k+1}=0$$

$$\Rightarrow k=-\frac{1}{2}$$

Hence, the required ratio is 1 : 2.

6. A line is parallel to *YZ*-plane, if all the points on the line have equal

(1) *x*-coordinates (2) *y*-coordinates (3) *z*-coordinates (4) Distance from origin

Sol. Answer (1)

Any line parallel to *YZ*-plane is $x = C$

$\therefore$ *x*-coordinate is same.

7. The equation $x = C$ represents a plane parallel to

(1) *YZ*-plane (2) *XY*-plane (3) *XZ*-plane (4) *x*-axis

Sol. Answer (1)

$x = C$ is a line parallel to *YZ*-plane.

8. The plane uniquely determined by *x*-axis and *y*-axis is known as

(1) *XY*-plane (2) *YZ*-plane (3) *XZ*-plane (4) None of these

Sol. Answer (1)

XY-plane is uniquely determined by *x* and *y*-coordinates.

9. Three mutually perpendicular plane divide the space into

(1) 6 parts (2) 8 parts (3) 4 parts (4) None of these

Sol. Answer (2)

Three mutually perpendicular plane divide the space into 8 parts. Each known as octant.

10. A parallelopiped is formed by planes drawn through the points (1, 2, 3) and (6, 8, 18) parallel to the coordinate planes then which of the following is not the length of an edge of the rectangular parallelopiped?

(1) 5 units (2) 10 units (3) 15 units (4) 6 units

Sol. Answer (2)

Length of the edges are given by

$(x_2 - x_1)$, $(y_2 - y_1)$ and $(z_2 - z_1)$

$x_2 - x_1 = 6 - 1 = 5$

$y_2 - y_1 = 8 - 2 = 6$

$z_2 - z_1 = 18 - 3 = 15$

11. The coordinates of the point where the line joining (3, 4, 1) and (5, 1, 6) crosses the *XY*-plane are

(1) $\left(\frac{13}{5}, \frac{23}{5}, 0\right)$ (2) (0, 0, 0) (3) $\left(-\frac{13}{5}, -\frac{23}{5}, 0\right)$ (4) $\left(-\frac{13}{5}, \frac{23}{5}, 0\right)$

Sol. Answer (1)

Let the point on *XY*-plane divides the join of (3, 4, 1) and (5, 1, 6) in $k : 1$.

Therefore, the coordinates of the point are

$$\left(\frac{5k+3}{k+1}, \frac{k+4}{k+1}, \frac{6k+1}{k+1}\right) \quad ...(i)$$

z-coordinate on XY-plane is zero

$\Rightarrow \quad \frac{6k+1}{k+1} = 0$

$\Rightarrow \quad k = \frac{-1}{6}$

On putting the value of $k = \frac{-1}{6}$ in (i), we get the coordinates of the point as

$$\left(\frac{13}{5},\ \frac{23}{5},\ 0\right)$$

12. Two vertices of a triangle are (3, 4, 2) and (1, 3, 3). If the medians of the triangle intersect at (0, 3, –1), then the coordinates of the third vertex of the triangle are

(1) (3, 1, –8) (2) (4, 2, 8) (3) (0, 0, 0) (4) (–4, 2, –8)

Sol. Answer (4)

The point of intersection of median is known as centroid. Let the third vertex is (x, y, z), then

$$\left(\frac{3+1+x}{3},\ \frac{4+3+y}{3},\ \frac{2+3+z}{3}\right) = (0,\ 3,\ -1)$$

$$\Rightarrow \left(\frac{x+4}{3},\ \frac{y+7}{3},\ \frac{z+5}{3}\right) = (0,\ 3,\ -1)$$

$\Rightarrow \quad \frac{x+4}{3} = 0,\ \frac{y+7}{3} = 3$ and $\frac{z+5}{3} = -1$

$\Rightarrow \quad x = -4,\ y = 2$ and $z = -8$

Hence, the required point is (–4, 2, –8).

13. The ratio in which the plane $2x + 3y + 5z = 1$ divides the line segment joining the points (1, 0, 0) and (1, 3, –5) is

(1) 7 : 8 (2) 13 : 12 (3) 15 : 1 (4) 1 : 15

Sol. Answer (4)

Let the join of the given points (1, 0, 0) and (1, 3, –5) is divided by the plane $2x + 3y + 5z = 1$ in k : 1.

Therefore, the coordinates of the given point are

$$\left(\frac{k+1}{k+1},\ \frac{3k}{k+1},\ \frac{-5k}{k+1}\right)$$

Since, the point lies on $2x + 3y + 5z = 1$

$\Rightarrow \quad 2 + \frac{9k}{k+1} - \frac{25k}{k+1} = 1$

$\Rightarrow \quad \frac{-16k}{k+1} = -1$

$\Rightarrow \quad -16k = -k - 1$

$\Rightarrow \quad k = \frac{1}{15}$

Hence, the required ratio is 1 : 15 internally.

SECTION - B

Objective Type Questions (More than One Options are correct)

1. If the distance between the points $P(a, -8, 4)$ and $Q(-3, -5, 4)$ is 5, then possible values of a is/are

(1) 1 (2) –2 (3) 3 (4) –7

Sol. Answer (1, 4)

$\sqrt{(a+3)^2+(-8+5)^2+(4-4)^2} = 5 \Rightarrow (a+3)^2 + 9 = 25$

$\Rightarrow (a+3)^2 = 16 \Rightarrow a + 3 = 4, -4$

$a = 1, -7$

2. If $A = (2, -3, 7)$, $B = (-1, 4, -5)$ and P is a point on the line AB such that $AP : BP = 3 : 2$, then P has the coordinates

(1) $\left(\frac{4}{5}, \frac{-1}{5}, \frac{11}{5}\right)$ (2) $\left(\frac{1}{5}, \frac{6}{5}, \frac{-1}{5}\right)$ (3) $\left(\frac{7}{5}, \frac{-18}{5}, \frac{29}{5}\right)$ (4) $(-7, 18, -29)$

Sol. Answer (2, 4)

Coordinates of P are $\left(\frac{3(-1)+2(2)}{3+2}, \frac{3(4)+2(-3)}{3+2}, \frac{3(-5)+2(7)}{3+2}\right) = \left(\frac{1}{5}, \frac{6}{5}, \frac{-1}{5}\right)$ for internal division, and

$\left(\frac{3(-1)-2(2)}{3-2}, \frac{3(4)-2(-3)}{3-2}, \frac{3(-5)-2(7)}{3-2}\right) = (-7, 18, -29)$ for external division.

3. If the direction ratios of a line are $1 + \lambda$, $1 - \lambda$, 2, and it makes an angle of 60° with the y-axis, then λ is

(1) $1+\sqrt{3}$ (2) $2+\sqrt{5}$ (3) $1-\sqrt{3}$ (4) $2-\sqrt{5}$

Sol. Answer (2, 4)

$a_1 = 1 + \lambda,\ b_1 = 1 - \lambda,\ c_1 = 2$

$a_2 = 0,\ b_2 = 1,\ c_2 = 0$

$$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{\sum a_1^2}\sqrt{\sum a_2^2}}$$

$$\Rightarrow \frac{1}{2} = \frac{0(1+\lambda)+1(1-\lambda)+0(2)}{\sqrt{(1+\lambda)^2+(1-\lambda)^2+2^2}\sqrt{0^2+1^2+0^2}} \Rightarrow \frac{1}{2} = \frac{1-\lambda}{\sqrt{2\lambda^2+6}}$$

$\Rightarrow 2\lambda^2 + 6 = 4(1-\lambda)^2 \Rightarrow 2\lambda^2 - 8\lambda - 2 = 0 \Rightarrow \lambda^2 - 4\lambda - 1 = 0 \Rightarrow \lambda = 2 \pm \sqrt{5}$

4. The direction cosines of a line whose direction ratios are 3, 4, 12 is/are

(1) $\frac{3}{13}, \frac{4}{13}, \frac{12}{13}$ (2) $\frac{-3}{13}, \frac{-4}{13}, \frac{-12}{13}$ (3) $\frac{1}{13}, \frac{2}{13}, \frac{3}{13}$ (4) $\frac{-1}{13}, \frac{-2}{13}, \frac{-3}{13}$

Sol. Answer (1, 2)

$a = 3,\ b = 4,\ c = 12$

$\sqrt{a^2+b^2+c^2} = \sqrt{3^2+4^2+12^2} = 13 \quad \therefore\ l, m, n$ are $\pm\frac{3}{13}, \pm\frac{4}{13}, \pm\frac{12}{13}$

5. A line segment has length 63 and direction ratios are 3, –2, 6. The components of the line vector are

(1) –27, 18, 54 (2) 27, –18, –54 (3) 27, –18, 54 (4) –27, 18, –54

Sol. Answer (3, 4)

Direction Ratio's are 3, –2, 6

$\therefore$ Direction cosine's are $\frac{3}{7}, \frac{-2}{7}, \frac{6}{7}$

$\therefore$ Components are $\pm\left(\frac{3}{7}\times 63, \frac{-2}{7}\times 63, \frac{6}{7}\times 63\right)$ = ± (27, –18, 54)

6. A point Q at a distance 3 from the point $P(1, 1, 1)$ lying on the line joining the points $A(0, -1, 3)$ and P, has the coordinates

(1) (2, 3, –1) (2) (4, 7, –5) (3) (0, –1, 3) (4) (–2, –5, 7)

Sol. Answer (1, 3)

Direction ratio's of AP are 1, 2, –2

$\therefore$ Equation of AP is

$$\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-1}{-2}=r\,(say)$$

To put r as the distance between points P and Q, direction ratios must be converted to direction cosines.

$$\therefore \quad \frac{x-1}{\left(\frac{1}{3}\right)}=\frac{y-1}{\left(\frac{2}{3}\right)}=\frac{z-1}{\left(\frac{-2}{3}\right)}=r$$

Put r = 3, and –3, the coordinates of Q are (2, 3, –1) and (0, –1, 3).

7. The direction cosines of two lines are given by the equations $3m + n + 5l = 0$, $6nl - 2lm + 5mn = 0$, then the direction cosines are

(1) $\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}$ (2) $\frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}$ (3) $\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$ (4) $\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$

Sol. Answer (1, 2, 3, 4)

Eliminating n between the given equations

$(6l + 5m)(3m + 5l) + 2lm = 0$

$\Rightarrow$ $30l^2 + 15m^2 + 45\,lm = 0$ $\Rightarrow$ $2\left(\frac{l}{m}\right)^2+3\left(\frac{l}{m}\right)+1=0$

$$\frac{l_1}{m_1}=-1,\ \frac{l_2}{m_2}=-\frac{1}{2}$$

Similarly, eliminating m between the given equations $6nl-\left(\frac{n+5l}{3}\right)(5n-2l)=0$

$\Rightarrow$ $10l^2 - 5nl - 5n^2 = 0$ $\Rightarrow$ $2\left(\frac{l}{n}\right)^2-\left(\frac{l}{n}\right)-1=0$

$\Rightarrow$ $\frac{l_1}{n_1}=\frac{-1}{2},\ \frac{l_2}{n_2}=+1$ $\quad\therefore$ $\frac{l_1}{-1}=\frac{m_1}{1}=\frac{n_1}{2}=\pm\frac{\sqrt{l_1^2+m_1^2+n_1^2}}{\sqrt{(-1)^2+1^2+2^2}}=\pm\frac{1}{\sqrt{16}}$

$$\frac{l_2}{-1}=\frac{m_2}{2}=\frac{n_2}{-1}=\frac{\pm\sqrt{l_2^2+m_2^2+n_2^2}}{\sqrt{(-1)^2+2^2+(-1)^2}}=\pm\frac{1}{\sqrt{6}}$$

∴ The direction-cosines of the lines are

$$\frac{1}{\sqrt{6}},\frac{-1}{\sqrt{6}},\frac{-2}{\sqrt{6}};\frac{-1}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{2}{\sqrt{6}};\frac{-1}{\sqrt{6}},\frac{2}{\sqrt{6}},\frac{-1}{\sqrt{6}};\frac{1}{\sqrt{6}},\frac{-2}{\sqrt{6}},\frac{1}{\sqrt{6}}$$

8. The ratio in which the sphere $x^2 + y^2 + z^2 = 504$ divides the line joining the points (12, –4, 8) and (27, –9, 18) is

(1) 2 : 3 internally (2) 2 : 3 externally (3) 3 : 4 internally (4) 3 : 4 externally

Sol. Answer (1, 2)

Let a point on sphere divides joining of two points in the ratio $\lambda : 1$

∴ Its co-ordianates are $\frac{27\lambda+12}{\lambda+1},\frac{-9\lambda-4}{\lambda+1},\frac{18\lambda+8}{\lambda+1}$

It lies on the sphere, therefore

$(27\lambda + 12)^2 + (-9\lambda - 4)^2 + (18\lambda + 8)^2 = 504(\lambda + 1)^2$

$729\lambda^2 + 144 + 648\lambda + 81\lambda^2 + 16 + 72\lambda + 324\lambda^2 + 64 + 288\lambda = 504\lambda^2 + 1008\lambda + 504$

$\Rightarrow 630\lambda^2 = 280$

$$\lambda^2=\frac{4}{9}\Rightarrow\lambda=\pm\frac{2}{3}$$

∴ (1) & (2) are correct

SECTION - C

Linked Comprehension Type Questions

Comprehension-I

If the direction cosines of two lines are (l_1, m_1, n_1) and (l_2, m_2, n_2) and the angle between them is θ, then

$$l_1^2+m_1^2+n_1^2=1=l_2^2+m_2^2+n_2^2 \text{ and } \cos\theta=l_1l_2+m_1m_2+n_1n_2$$

Now give the answers of the following questions.

1. If $l_1=\frac{1}{\sqrt{3}}, m_1=\frac{1}{\sqrt{3}}$, then the value of n_1 is equal to

(1) $\pm\frac{1}{\sqrt{3}}$ (2) $+\frac{1}{\sqrt{3}}$ (3) $-\frac{1}{\sqrt{3}}$ (4) 0

Sol. Answer (1)

$$l_1^2+m_1^2+n_1^2=1$$

$$\Rightarrow \frac{1}{3}+\frac{1}{3}+n_1^2=1$$

$$\Rightarrow n_1=\pm\frac{1}{\sqrt{3}}$$

2. If the angle between the lines is 60° then the value of $l_1(l_1 + l_2) + m_1(m_1 + m_2) + n_1(n_1 + n_2)$ is

(1) 0 (2) $\frac{3}{2}$ (3) $\frac{1}{2}$ (4) 1

Sol. Answer (2)

$$\cos 60° = l_1l_2 + m_1m_2 + n_1n_2$$

$$\Rightarrow l_1l_2 + m_1m_2 + n_1n_2 = \frac{1}{2}$$

$$l_1(l_1 + l_2) + m_1(m_1 + m_2) + n_1(n_1 + n_2)$$

$$= (l_1^2 + m_1^2 + n_1^2) + (l_1l_2 + m_1m_2 + n_1n_2)$$

$$= 1 + \frac{1}{2} = \frac{3}{2}$$

3. The angle between the lines whose direction cosines are $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ is

(1) 0° (2) 60° (3) 90° (4) 120°

Sol. Answer (3)

$$\cos\theta = \frac{1}{2} \times \left(-\frac{1}{2}\right) + \frac{1}{2}\left(-\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$= -\frac{1}{4} - \frac{1}{4} + \frac{1}{2} = 0$$

$\theta = 90°$

Comprehension-II

Let A (1, 1, 1), B (3, 7, 4) and C (–1, 3, 0) be three points in a plane.

1. The co-ordinates of the foot of perpendicular drawn from point C to the line segment AB is

(1) $\left(\frac{59}{49}, \frac{79}{49}, \frac{64}{49}\right)$ (2) $\left(\frac{-59}{49}, \frac{79}{49}, \frac{64}{49}\right)$ (3) $\left(\frac{59}{49}, \frac{-79}{49}, \frac{64}{49}\right)$ (4) $\left(\frac{59}{49}, \frac{79}{49}, \frac{-64}{49}\right)$

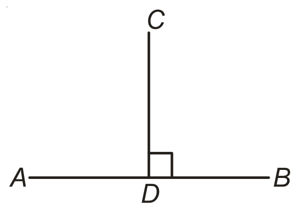
Sol. Answer (1)

Let $D \equiv (x, y, z)$

Then

$$\frac{x-1}{3-1} = \frac{y-1}{7-1} = \frac{z-1}{4-1}$$

i.e. $\frac{x-1}{2} = \frac{y-1}{6} = \frac{z-1}{3}$ …(1)



Direction ratio's of CD are $x + 1$, $y - 3$, $z - 0$

Since $CD \perp AB$

$\therefore$ $(3 - 1)(x + 1) + (7 - 1)(y - 3) + (4 - 1)(z - 0) = 0$

$\Rightarrow$ $2(x + 1) + 6(y - 3) + 3z = 0$ …(2)

By equation (1)

$x = 2k + 1,\ y = 6k + 1,\ z = 3k + 1$

Substituting in (2)

$2(2k + 2) + 6(6k - 2) + 3(3k + 1) = 0$

$\Rightarrow\ 49\ k = 5 \quad \Rightarrow\ k = \dfrac{5}{49}$

$\therefore\ x = \dfrac{59}{49},\ y = \dfrac{79}{49},\ z = \dfrac{64}{49}$

2. The perpendicular distance from C to AB is

(1) $\dfrac{\sqrt{25312}}{49}$ (2) $\dfrac{\sqrt{23571}}{49}$ (3) $\dfrac{(14321)^{1/2}}{49}$ (4) $\dfrac{\sqrt{20384}}{49}$

Sol. Answer (4)

Required distance is

$$\sqrt{\left(\frac{59}{49}+1\right)^2+\left(\frac{79}{49}-3\right)^2+\left(\frac{64}{49}-0\right)^2} = \frac{\sqrt{20384}}{49}$$

3. Area of the triangle ABC is (in sq. units)

(1) $\dfrac{\sqrt{23571}}{14}$ (2) $\dfrac{\sqrt{20384}}{14}$ (3) $\dfrac{\sqrt{25312}}{14}$ (4) $\dfrac{\sqrt{14321}}{14}$

Sol. Answer (2)

Distance $AB = \sqrt{(3-1)^2+(7-1)^2+(4-1)^2} = 7$

$\therefore$ Area of $\Delta ABC = \dfrac{1}{2}(AB)(CD) = \dfrac{\sqrt{20384}}{14}$

SECTION - D

Matrix-Match Type Questions

1. Match the following :

Column-I	**Column-II**
(A) Centroid of the triangle with vertices A (2, 3, 7), B (6, 7, 5), C (1, 2, 3)	(p) (1, 6, 5)
(B) Mid-point of the line joining the points A (7, 9, 11) and B (–5, 3, –1)	(q) (3, 4, 5)
(C) A point on the line $\dfrac{x}{2}=\dfrac{y}{3}=\dfrac{z}{5}$, at a distance 2 from the origin	(r) (3, 3, 2)
(D) Coordinates of the point dividing the join of (5, 5, 0) and (0, 0, 5) in the ratio 2 : 3	(s) $\left(\dfrac{4}{\sqrt{38}}, \dfrac{6}{\sqrt{38}}, \dfrac{10}{\sqrt{38}}\right)$

Sol. Answer : A(q), B(p), C(s), D(r)

(A) $\left(\frac{2+6+1}{3}, \frac{3+7+2}{3}, \frac{7+5+3}{3}\right) = (3, 4, 5)$

(B) $\left(\frac{7-5}{2}, \frac{9+3}{2}, \frac{11-1}{2}\right) = (1, 6, 5)$

(C) Any point on $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ can be taken as $(2k, 3k, 5k)$.

$\therefore \sqrt{(2k-0)^2 + (3k-0)^2 + (5k-0)^2} = 2$

$\Rightarrow k = \pm\frac{2}{\sqrt{38}}$

$\therefore$ Required point is $\left(\frac{4}{\sqrt{38}}, \frac{6}{\sqrt{38}}, \frac{10}{\sqrt{38}}\right)$

(D) $\left(\frac{2\times0+3\times5}{2+3}, \frac{2\times0+3\times5}{2+3}, \frac{2\times5+0\times3}{2+3}\right)$ = (3, 3, 2)

2. If α, β, γ are the angles which a line makes with positive direction of the axes, then match the quantities in column-I to their values in column-II.

Column-I	Column-II
(A) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$	(p) –2
(B) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma$	(q) 1
(C) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$	(r) –1
(D) $2\cos(\alpha-\beta)\cos(\alpha+\beta) + 2\cos(\beta-\gamma)\cos(\beta+\gamma) + 2\cos(\gamma-\alpha)\cos(\gamma+\alpha)$	(s) 2

Sol. Answer : A(s), B(q), C(r), D(p)

Since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$(1-\sin^2\alpha) + (1-\sin^2\beta) + (1-\sin^2\gamma) = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) - 3$

$= 2(1) - 3 = -1$

Since, $2\cos(\alpha+\beta)\cos(\alpha-\beta) = \cos2\alpha + \cos2\beta$

$\therefore \sum 2\cos(\alpha+\beta)\cos(\alpha-\beta) = 2\left(\cos^2\alpha + \cos^2\beta + \cos^2\gamma\right) = -2$

3. Match column I to column II according to the given condition.

In column I the direction ratios of lines are given. In column II angle between them is given.

Column I	Column II
(A) (1, 2, 3), (1, 1, –1)	(p) 90°
(B) (1, 2, 3), (2, 4, 6) then acute angle is less than	(q) 60°
(C) (1, 1, 1), (2, 1, –1) the acute angle is greater than	(r) 0°
(D) (1, 0, 0), (0, 1, 0) the angle is greater than	(s) 30°
	(t) 70°

Sol. Answer A(p), B(p, q, s, t), C(q, r, s), D(q, r, s, t)

(A) $1 \times 1 + 2 \times 1 + 3 \times (-1) = 0$

hence $\theta = 90°$

(B) $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$, hence angle between the lines is zero.

(C) $\cos\theta = \frac{1\times 2 + 1\times 1 + 1\times(-1)}{\sqrt{1+1+1}\,\sqrt{4+1+1}} = \frac{2}{\sqrt{3}\,\sqrt{6}} = \frac{2}{3\sqrt{2}} = \frac{\sqrt{2}}{3}$

$\cos\theta \approx \frac{1.411}{3} \approx 0.47 < \frac{1}{2} \Rightarrow \theta > 60^{\circ}$

(D) $1 \times 0 + 0 \times 1 + 0 \times 0 = 0 \Rightarrow \theta = 90°$

SECTION - E

Assertion-Reason Type Questions

1. STATEMENT-1 : The triangle with vertices (1, 3, 5), (2, 4, 6) and (0, 5, 7) must be a right angle triangle.

and

STATEMENT-2 : If the dot product of two non-zero vectors is zero then they must be perpendicular.

Sol. Answer (1)

Let *A*, *B*, *C* be the points (1, 3, 5), (2, 4, 6) and (0, 5, 7) respectively.

Direction Ratio's of *AB*, *BC*, and *CA* are 1, 1, 1 ; –2, 1, 1 ; 1, –2, –2 respectively.

Clearly, –2(1) + 1(1) + 1(1) = 0

$\therefore$ $AB \perp BC$.

In vector from $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \hat{i} + \hat{j} + \hat{k}$ $\quad (\overrightarrow{AB} \neq 0)$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = -2\hat{i} + \hat{j} + \hat{k}$ $\quad (\overrightarrow{BC} \neq 0)$

$\overrightarrow{AB} \,.\, \overrightarrow{BC} = -2(1) + 1(1) + 1(1) = 0$

2. STATEMENT-1 : The points (2, 3, 5), (7, 5, 7) and (–3, 1, 3) are vertices of an equilateral triangle.

and

STATEMENT-2 : The triangle with equal sides is called an equilateral triangle.

Sol. Answer (4)

Let points (2, 3, 5), (7, 5, 7) and (–3, 1, 3) be represented by *A*, *B* and *C* respectively.

$AB = \sqrt{(7-2)^2 + (5-3)^2 + (7-5)^2} = \sqrt{33}$

$BC = \sqrt{(-3-7)^2 + (1-5)^2 + (3-7)^2} \neq \sqrt{33}$

$\therefore$ $\Delta\, ABC$ is not equilateral

3. STATEMENT-1 : If a line making an angle $\pi/4$ with x-axis, $\pi/4$ with y-axis then it must be perpendicular to z-axis.

and

STATEMENT-2 : If direction ratios of two lines are l_1, m_1, n_1 and l_2, m_2, n_2 then the angle between them is given by $\theta = \cos^{-1}(l_1l_2 + m_1m_2 + n_1n_2)$.

Sol. Answer (3)

The angle given as $\theta = \cos^{-1}(l_1 l_2 + m_1 m_2 + n_1 n_2)$ is the angle between the lines whose direction-cosines are l_1, m_1, n_1 and l_2, m_2, n_2 (not direction ratio's)

4. STATEMENT-1 : The centroid of a tetrahedron with vertices (0, 0, 0), (4, 0, 0), (0, –8, 0), (0, 0, 12) is (1, –2, 3).

and

STATEMENT-2 : The centroid of a triangle with vertices (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

Sol. Answer (2)

Centroid of a tetrahedron with vertices (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), (x_4, y_4, z_4) is

$$\left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right)$$

$\therefore$ The centroid of the tetrahedron with given vertices is

$$\left(\frac{0+4+0+0}{4}, \frac{0+0-8+0}{4}, \frac{0+0+0+12}{4}\right) = (1, -2, 3)$$

Statement 2 is true, but does not explain statement 1.

5. Consider three planes

$P_1 : x - y + z = 1$

$P_2 : x + y - z = -1$

$P_3 : x - 3y + 3z = 2$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, P_1 and P_2, respectively

STATEMENT-1 : At least two of the lines L_1, L_2 and L_3 are non-parallel.

and

STATEMENT-2 : The three planes do not nave a common point.

Sol. Answer (4)

The given equations are

$x - y + z = 1$

$x + y - z = -1$

$x - 3y + 3z = 2$

The system of equations can be put in matrix form as $Ax = B$

$$i.e.\quad \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -3 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$\sim\begin{bmatrix}1 & -1 & 1\\0 & 2 & -2\\0 & -2 & 2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\-2\\1\end{bmatrix}\quad \begin{matrix}R_2\to R_2-R_1\\R_3\to R_3-R_1\end{matrix}$$

$$\sim\begin{bmatrix}1 & -1 & 1\\0 & 2 & -2\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\-2\\-1\end{bmatrix}$$

which is inconsistent as

$\Rightarrow$ The three planes do not have a common point.

$\Rightarrow$ Statement–2 is true.

Since planes P_1, P_2, P_3 are pairwise intersecting, their lines of intersection are parallel.

Statement–1, is false.

SECTION - F

Integer Answer Type Questions

1. If the direction ratios of two lines are (1, λ, 2) and (λ, λ + 1, λ) and the angle between the lines is 90° then the modulus of sum of all values of λ is _____.

Sol. Answer (4)

$(1 \times \lambda) + (\lambda(\lambda + 1)) + 2\lambda = 0$

$\Rightarrow \lambda + \lambda^2 + \lambda + 2\lambda = 0$

$\lambda^2 + 4\lambda = 0$

$\lambda = 0, -4$

sum = 0 – 4 = –4

Modulus of sum = 4

2. If $A \equiv (2, 3, 4)$, $B \equiv (3, 4, 5)$. The direction cosine of a line are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$. Now integral value of the projection of AB on the given line is _____.

Sol. Answer (1)

Projection $= \frac{1}{\sqrt{3}}(3-2)+\frac{1}{\sqrt{3}}(4-3)+\frac{1}{\sqrt{3}}(5-4)=\sqrt{3}$

$= \sqrt{3}$

Integral value = 1

3. The direction ratios of a line are (–2, 3, 6). If the line makes an acute angle with positive direction of x-axis then the modulus of integral value of sum of all direction cosines, is _____.

Sol. Answer (1)

Direction cosine = $\left(\pm\left(-\frac{2}{7}\right), \pm\frac{3}{7}, \pm\frac{6}{7}\right)$

$= (\cos\alpha, \cos\beta, \cos\gamma)$

But $\cos\alpha > 0$

Hence directions cosines are

$$\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$$

sum = $\frac{2-3-6}{7} = -1$

Modulus of sum = 1

4. If the centroid of triangles formed by the vertices (1, 2, 3), (2, 1, 0) and (3, 1, 4) is (α, β, γ) then the value of [α] + [β] + [γ], where [ ] represents the greatest integer function, is _____.

Sol. Answer (5)

$$\alpha = \frac{1+2+3}{3} = 2$$

$$\beta = \frac{2+1+1}{3} = \frac{4}{3}$$

$$\gamma = \frac{3+0+4}{3} = \frac{7}{3}$$

[α] = 2 , [β] = 1, [γ] = 2

[α] + [β] + [γ] = 5

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