Summary

Area Under Curve

1. Curve Tracing :

To find the approximate shape of a curve, the following procedure is adopted in order:

(i) Symmetry about x - axis, (ii) Symmetry about y-axis:

(iii) Symmetry about the line y = x, (iv) Symmetry in opposite quadrants:

(v) Find the points where the curve crosses the x-axis and also the y-axis.

(vi) Find $\frac{dy}{dx}$ and equate it to zero to find the points to the curve where you have horizontal tangents.

(vii) Examine if possible, the intervals when f(x) is increasing or decreasing

(viii) Examine what happens to y when x → ∞ or x → -∞

(ix) **Asymptotes :**

(a) *If* $\underset{x\to a}{Lt}\, f(x)=\infty$ *or* $\underset{x\to a}{Lt}\, f(x)=-\infty,$ then x = a is asymptote of y=f(x)

(b) *If* $\underset{x\to\infty}{Lt}\, f(x)=k$ *or* $\underset{x\to-\infty}{Lt}\, f(x)=k,$ then y = k is asymptote of y = f(x)

(c) *If* $\underset{x\to\infty}{Lt}\frac{f(x)}{x}=m_1,\ \underset{x\to\infty}{Lt}\left(f(x)-m_1x\right)=c$, then y = m_1x+c_1 is an asymptote

2. Quadrature :

(i) *If* $f(x)\geq 0$ *for* $x\in[a,b]$,then area bounded by curve y = f(x) , x-axis, x = a and x = b is

$$\int_a^b f(x)\,dx$$

(ii) If $f(x)\leq 0$ *for* $x\in[a.b]$, then area bounded by curve y = f(x), x-axis, x = a and x = b is

$$-\int_a^b f(x)\,dx$$

(iii) $f(x)\geq 0$ *for* $x\in[a,c]$ *and* $f(x)\leq 0$ *for* $x\in[c,b](a<c<b)$ then area bounded by curve y =f(x) and x-axis between x = a and x = b is $\int_a^c f(x)dx-\int_a^b f(x)\,dx$

(iv) If $f(x) \geq g(x)$ for $x \in [a,b]$ then area bounded by curves y = f(x) and y = g(x) between ordinates x = a and $x = b$ is $\int_a^b (f(x) - g(x))dx$

If $g(y) \geq 0$ for $y \in [c,d]$ then area bounded by curve x = g(y) and y-axis between abscissa y = c and $y = d$ is $\int_{y=c}^{d} g(y)dy$

Practice Questions

1. The area (in sq units) of the region $\{(x, y) : y^2 \geq 2x \text{ and } x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is (2016)

(a) $\pi - \frac{4}{3}$

(b) $\pi - \frac{8}{3}$

(c) $\pi - \frac{4\sqrt{2}}{3}$

(d) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$

2. The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then, the area (in sq units) of the quadrilateral PQRS is (2014)

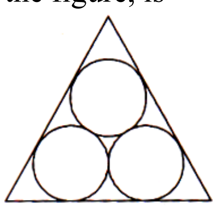
(a) 3

(b) 6

(c) 9

(d) 15

3. The area of the equilateral triangle, in which three coins of radius 1 cm are placed, as shown in the figure, is (2005)



(a) $(6 + 4\sqrt{3})$ sq cm

(b) $(4\sqrt{3} - 6)$ sq cm

(c) $(1 + 4\sqrt{3})$ sq cm

(d) $4\sqrt{3}$ sq cm

4. The area of the quadrilateral formed by the tangents at the end points of latusrectum to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is (2003)

(a) 27/4 sq units

(b) 9 sq units

(c) 27/2 sq units

(d) 27 sq units

5. The area (in sq units) bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is (2002)

(a) 1

(b) 2

(c) $2\sqrt{2}$

(d) 4

6. The triangle formed by the tangent to the curve $f(x) = x^2 + bx - b$ at the point (1, 1) and the coordinate axes, lies in the first quadrant. If its area is 2 sq units, then the value of b is (2001)

(a) – 1

(b) 3

(c) – 3

(d) 1

7. The area (in sq units) of the region $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y \text{ and } y \leq 1 + \sqrt{x}\}$ is (2017)

(a) $\frac{59}{12}$

(b) $\frac{3}{2}$

(c) $\frac{7}{3}$

(d) $\frac{5}{2}$

8. Area of the region $\{(x, y)\} \in R^2 : y \geq \sqrt{|x+3|}, 5y \leq (x + 9) \leq 15\}$ is equal to (2016)

(a) $\frac{1}{6}$

(b) $\frac{4}{3}$

(c) $\frac{3}{2}$

(d) $\frac{5}{3}$

9. The area (in sq units) of region described by (x, y) $y^2 \leq 2x$ and $y \geq 4x - 1$ is (2015)

(a) $\frac{7}{32}$

(b) $\frac{5}{64}$

(c) $\frac{15}{64}$

(d) $\frac{9}{32}$

10. The area (in sq units) of the region described by $A = \{(x, y) : x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\}$ is (2014)

(a) $\frac{\pi}{2}+\frac{4}{3}$

(b) $\frac{\pi}{2}-\frac{4}{3}$

(c) $\frac{\pi}{2}-\frac{2}{3}$

(d) $\frac{\pi}{2}+\frac{2}{3}$

11. The area enclosed by the curves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ over the interval $\left[0, \frac{\pi}{2}\right]$ is (2014)

(a) $4(\sqrt{2} - 1)$

(b) $2\sqrt{2}(\sqrt{2} - 1)$

(c) $2(\sqrt{2} + 1)$

(d) $2\sqrt{2}(\sqrt{2} + 1)$

12. The area (in sq units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, X-axis and lying in the first quadrant, is (2013)

(a) 9

(b) 6

(c) 18

(d) 27 4

13. Let f : [–1, 2] → [0, ∞) be a continuous function such that f (x) = f (1 – x), ∀ x ∈ [–1, 2]. If $R_1 = \int_{-1}^{2} xf(x)\,dx$ and R_2 are the area of the region bounded by y = f (x), x = – 1, x = 2 and the X-axis. Then, (2011)

(a) $R_1 = 2R_2$

(b) $R_1 = 3R_2$

(c) $2R_1 = R_2$

(d) $3R_1 = R_2$

14. If the straight line x = b divide the area enclosed by $y = (1 - x)^2$, y = 0 and x = 0 into two parts R_1 ($0 \le x \le b$) and R_2 ($b \le x \le 1$) such that $R_1 - R_2 = \frac{1}{4}$. Then, b equals (2011)

(a) $\frac{3}{4}$

(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) $\frac{1}{4}$

15. The area of the region between the curves $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$ and bounded by the lines x = 0 and x = $\frac{\pi}{4}$ is (2008)

(a) $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\,dt$

(b) $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$

(c) $\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$

(d) $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\,dt$

16. The area bounded by the curves $y = (x - 1)^2$, $y = (x + 1)^2$ and $y = \frac{1}{4}$ is (2005)

(a) $\frac{1}{3}$ sq unit

(b) $\frac{2}{3}$ sq unit

(c) $\frac{1}{4}$ sq unit

(d) $\frac{1}{5}$ sq unit

17. The area enclosed between the curves $y = ax^2$ and $x = ay^2$ $(a > 0)$ is 1 sq unit. Then, the value of a is (2004)

(a) $\frac{1}{\sqrt{3}}$

(b) $\frac{1}{2}$

(c) 1

(d) $\frac{1}{3}$

18. The area bounded by the curves $y = f(x)$, the X-axis and the ordinates $x = 1$ and $x = b$ is $(b - 1) \sin(3b + 4)$. Then, $f(x)$ is equal to (1982)

(a) $(x - 1) \cos(3x + 4)$

(b) $8\sin(3x + 4)$

(c) $\sin(3x + 4) + 3(x - 1)\cos(3x + 4)$

(d) None of the above

19. The slope of tangent to a curve $y = f(x)$ at $[x, f(x)]$ is $2x + 1$. If the curve passes through the point $(1, 2)$, then the area bounded by the curve, the X-axis and the line $x = 1$ is

(a) $\frac{3}{2}$

(b) $\frac{4}{3}$

(c) $\frac{5}{6}$

(d) $\frac{1}{12}$

20. If the line x = α divides the area of region R = {(x, y) ∈ R^2 : $x^3 \le y \le x, 0 \le x \le 1$} into two equal parts, then (2017)

(a) $2\alpha^4 - 4\alpha^2 + 1 = 0$

(b) $\alpha^4 + 4\alpha^2 - 1 = 0$

(c) $\frac{1}{2} < \alpha < 1$

(d) $0 < \alpha \le \frac{1}{2}$

21. If S be the area of the region enclosed by y = e^{-x^2}, y = 0, x = 0 and x = 1. Then, (2012)

(a) $S \ge \frac{1}{e}$

(b) $S \ge 1 - \frac{1}{e}$

(c) $S \le \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$

(d) $S \le \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

22. Area of the region bounded by the curve y = e^x and lines x = 0 and y = e is (2009)

(a) $e - 1$

(b) $\int_1^e \ln(e + 1 - y)\,dy$

(c) $e - \int_0^1 e^x\,dx$

(d) $\int_1^e \ln y\,dy$

23. For which of the following values of m, is the area of the region bounded by the curve y = x – x^2 and the line y = mx equals $\frac{9}{2}$? (1999)

(a) – 4

(b) – 2

(c) 2

(d) 4

24. If $\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix}\begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2+3a \\ 3b^2+3b \\ 3c^2+3c \end{bmatrix}$,

f(x) is a quadratic function and its maximum value occurs at a point V. A is a point of intersection of y = f (x) with X-axis and point B is such that chord AB subtends a right angle at V. Find the area enclosed by f (x) and chord AB. (2005)

(a) $\frac{3}{125}$ sq. units

(b) $-\frac{3}{125}$ sq. units

(c) $\frac{125}{3}$ sq. units

(d) $-\frac{125}{3}$ sq. units

25. Find the area bounded by the curves $x^2 = y$, $x^2 = -y$ and $y^2 = 4x - 3$. (2005)

(a) $\frac{1}{3}$ sq. units

(b) – 3 sq. units

(c) $-\frac{1}{3}$ sq. units

(d) 3 sq. units

26. A curve passes through (2, 0) and the slope of tangent at point P(x, y) equals $\frac{(x+1)^2+y-3}{(x+1)}$.

Find the equation of the curve and area enclosed by the curve and the X-axis in the fourth quadrant. (2004)

(a) $y = x^2 - 2x, \frac{4}{3}$ sq. units

(b) $y = x^2 + 2x, -\frac{4}{3}$ sq. units

(c) $y = x^2 + 2x, \frac{4}{3}$ sq. units

(d) $y = x^2 - 2x, -\frac{4}{3}$ sq. units

27. Find the area of the region bounded by the curves $y = x^2$, $y = |2 - x^2|$ and $y = 2$, which lies to the right of the line $x = 1$. (2002)

(a) $\left(\dfrac{20+12\sqrt{2}}{3}\right)$ sq. units

(b) $\left(\dfrac{20-12\sqrt{2}}{3}\right)$ sq. units

(c) $\left(\dfrac{10+12\sqrt{2}}{-3}\right)$ sq. units

(d) $\left(\dfrac{-10-12\sqrt{2}}{-3}\right)$ sq. units

28. Let $b \neq 0$ and for $j = 0,1,2..., n$. If S_j is the area of the region bounded by the Y-axis and the curve $xe^{ay} = \sin by$, $\dfrac{j\pi}{b} \leq y \leq \dfrac{(j+1)\pi}{b}$. Then, show that S_0, S_1, S_2, ..., S_n are in geometric progression. Also, find their sum for $a = -1$ and $b = \pi$. (2001)

(a) $\left[\dfrac{\pi(1+e)}{(1+\pi^2)}+\left(\dfrac{e^{n+1}-1}{e-1}\right)\right]$

(b) $\left[\dfrac{\pi(1+e)}{(1+\pi^2)}\cdot\left(\dfrac{e^{n+1}-1}{e-1}\right)\right]$

(c) $\left[\dfrac{\pi(1+e)}{(1+\pi^2)}-\left(\dfrac{e^{n+1}-1}{e-1}\right)\right]$

(d) None of these

29. If f (x) is a continuous function given by $f(x)=\begin{cases} 2x, & |x|\le 1 \\ x^2+ax+b, & |x|>1 \end{cases}$. Then, find the area of the region in the third quadrant bounded by the curves x = – 2y^2 and y = f (x) lying on the left on the line 8x + 1 = 0. (1999)

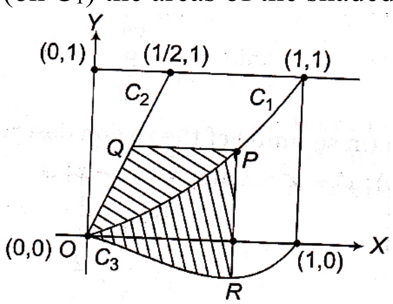
(a) $\left(-\frac{761}{192}\right)$ sq. units

(b) $\left(\frac{192}{761}\right)$ sq. units

(c) $\left(\frac{761}{192}\right)$ sq. units

(d) $\left(-\frac{192}{761}\right)$ sq. units

30. Let C_1 and C_2 be the graphs of functions y = x^2 and y = 2x, 0 ≤ x ≤ 1, respectively. Let C_3 be the graph of a function y = f (x), 0 ≤ x ≤ 1, f (0) = 0. For a point P on C_1, let the lines through P, parallel to the axes, meet C_2 and C_3 at Q and R respectively (see figure). If for every position of P (on C_1) the areas of the shaded regions OPQ and ORP are equal, then determine f (x). (1998)



(a) $f(x)=3x^3-2x^2, 0\le x\le 1$

(b) $f(-x)=x^3-x^2, 0\le x\le 1$

(c) $f(x)=3x^3-x^2, 0\le x\le 1$

(d) $f(x)=x^3-x^2, 0\le x\le 1$

31. Let f (x) = max $\{x^2, (1-x)^2, 2x(1-x)\}$, where $0 \leq x \leq 1$. Determine the area of the region bounded by the curves y = f (x), X-axis, x = 0 and x = 1. (1997)

(a) $\frac{17}{27}$ sq unit

(b) $-\frac{15}{25}$ sq unit

(c) $-\frac{17}{27}$ sq unit

(d) $\frac{15}{25}$ sq unit

32. Find all the possible values of b > 0, so that the area of the bounded region enclosed between the parabolas $y = x - bx^2$ and $y = \frac{x^2}{b}$ is maximum. (1997)

(a) 0
(b) 1
(c) 2
(d) 3

33. Consider a square with vertices at (1, 1), (–1, 1), (–1, –1) and (1, –1). If S is the region consisting of all points inside the square which are nearer to the origin than to any edge. Then, sketch the region S and find its area. (1995)

(a) $\left[\frac{1}{3}\left(16\sqrt{2}-20\right)\right]$ sq unit

(b) $\left[-\frac{1}{3}\left(16\sqrt{2}-20\right)\right]$ sq unit

(c) $\left[-\frac{5}{7}\left(16\sqrt{2}-20\right)\right]$ sq unit

(d) $\left[-\frac{5}{7}\left(16\sqrt{2}+20\right)\right]$ sq unit

34. In what ratio, does the X-axis divide the area of the region bounded by the parabolas $y = 4x - x^2$ and $y = x^2 - x$? (1994)

(a) $121 : \frac{1}{2}$

(b) 4 : 121
(c) 121 : 4
(d) none of these

35. Sketch the region bounded by the curves y = x^2 and y = $\frac{2}{\left(1+x^2\right)}$. Find its area. (1992)

(a) $\left(\pi-\frac{2}{3}\right)$ sq. units

(b) $\left(\pi+\frac{2}{3}\right)$ sq. units

(c) $\left(2\pi+\frac{2}{3}\right)$ sq. units

(d) none of these

36. Sketch the curves and identify the region bounded by x = 1/2, x = 2, y = log x and y = 2^x. Find the area of this region. (1991)

(a) $\left(\frac{4-\sqrt{2}}{4e}-\frac{5}{2}\log 2-\frac{3}{2}\right)$ sq. units

(b) $\left(\frac{4-\sqrt{2}}{4e}+\frac{5}{2}\log 2-\frac{3}{2}\right)$ sq. units

(c) $\left(\frac{4-\sqrt{2}}{4e}+\frac{5}{2}\log 2+\frac{3}{2}\right)$ sq. units

(d) $\left(\frac{4-\sqrt{2}}{4e}-\frac{5}{2}\log 2+\frac{3}{2}\right)$ sq. units

37. Compute the area of the region bounded by the curves y = ex log x and y = $\frac{\log x}{ex}$, where log e = 1. (1990)

(a) $\left(\frac{e^2-5}{4e}\right)$ sq. units

(b) $\left(\frac{4e}{e^2+5}\right)$ sq. units

(c) $\left(\frac{4e}{e^2-5}\right)$ sq. units

(d) $\left(\frac{e^2+5}{4e}\right)$ sq. units

38. Find all maxima and minima of the function $y = x(x-1)^2$, $0 \le x \le 2$. Also, determine the area bounded by the curve $y = x(x-1)^2$, the Y-axis and the line $x = 2$. (1989)

(a) $\left(y_{\max} = \dfrac{4}{27}, y_{\min} = 0, \dfrac{10}{3}\right)$ sq. units

(b) $\left(y_{\max} = -\dfrac{4}{27}, y_{\min} = -0, -\dfrac{10}{3}\right)$ sq. units

(c) $\left(y_{\max} = \dfrac{4}{27}, y_{\min} = 0, -\dfrac{10}{3}\right)$ sq. units

(d) none

39. Find the area of the region bounded by the curve C: $y = \tan x$, tangent drawn to C at $x = \pi/4$ and the X-axis. (1988)

(a) $\left[\left(\log\dfrac{\sqrt{2}}{4} - \dfrac{1}{4}\right)\right]$ sq. units

(b) $\left[\left(\log\sqrt{2} + \dfrac{1}{4}\right)\right]$ sq. units

(c) $\left[\left(\log\sqrt{2} - \dfrac{1}{4}\right)\right]$ sq. units

(d) None of these

40. Find the area bounded by the curves $x^2 + y^2 = 25$, $4y = |4 - x^2|$ and $x = 0$ above the X-axis. (1987)

(a) $\left[4 + 25\sin^{-1}\left(\dfrac{4}{5}\right)\right]$ sq. units

(b) $\left[4 - 25\sin^{-1}\left(\dfrac{4}{5}\right)\right]$ sq. units

(c) $\left[25 - 4\sin^{-1}\left(\dfrac{4}{5}\right)\right]$ sq. units

(d) none

41. Find the area bounded by the curves $x^2 + y^2 = 4$, $x^2 = -\sqrt{2}\,y$ and $x = y$. (1986)

(a) $\left(\frac{1}{3} - \pi\right)$ sq. units

(b) $\left(\frac{1}{3} + \pi\right)$ sq. units

(c) π

(d) $\left(\frac{1}{3} + \frac{\pi}{6}\right)$ sq. units

42. Sketch the region bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ and find its area. (1985)

(a) $\left(\frac{5\pi}{4} - \frac{1}{2}\right)$ sq. units

(b) $\left(\frac{5\pi}{4} + \frac{2}{5}\right)$ sq. units

(c) $\left(\frac{1}{2} + \frac{5\pi}{6}\right)$ sq. units

(d) sq. units

43. Find the area of the region bounded by the X-axis and the curves defined by $y = \tan x, -\frac{\pi}{3} \le x \le \frac{\pi}{3}$ and $y = \cot x, \frac{\pi}{6} \le x \le \frac{\pi}{3}$ (1984)

(a) $\left(-\frac{1}{2} + 2\log_e 3\right)$

(b) $\left(\frac{1}{2}\log_e 3\right)$

(c) $\left(-\frac{1}{2}\log_e 3\right)$

(d) none

44. Find the area bounded by the X-axis, part of the curve $y = \left(1 + \frac{8}{x^2}\right)$ and the ordinates at $x = 2$ and $x = 4$. If the ordinate at $x = a$ divides the area into two equal parts, then find a. (1983)

(a) $5\sqrt{2}$

(b) $2\sqrt{2}$

(c) $\frac{5\sqrt{2}}{7}$

(d) none

45.Statement-1: Area formed by curve y = cos x with y = 0, x = 0 and x = $\frac{3\pi}{4}$ is $2-\frac{1}{\sqrt{2}}$

Statement-2: Area of curve y = f(x) with x-axis between ordinates x = a and x = b is $\int_a^b f(x)\,dx$

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement -1 is False, Statement-2 is True

46. The area of the region described by $A=\{(x,y): x^2+y^2\le 1 \text{ and } y^2\le 1-x\}$ is (2014)

(a) $\frac{\pi}{2}+\frac{4}{3}$

(b) $\frac{\pi}{2}-\frac{4}{3}$

(c) $\frac{\pi}{2}-\frac{2}{3}$

(d) $\frac{\pi}{2}+\frac{2}{3}$

47. The area (in sq units) bounded by the curves $y=\sqrt{x}, 2y-x+3=0$, *X*-axis and lying in the first quadrant is (2013)

(a) 9
(b) 36
(c) 18
(d) $\frac{27}{4}$

48. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$, is

(2012)

(a) $20\sqrt{2}$

(b) $\frac{10\sqrt{2}}{3}$

(c) $\frac{20\sqrt{2}}{3}$

(d) $10\sqrt{2}$

49. The area of the region enclosed by the curves $y = x$, $x = e$, $y = \frac{1}{x}$ and the positive X-axis is

(2011)

(a) 1 sq unit

(b) $\frac{3}{2}$ sq units

(c) $\frac{5}{2}$ sq units

(d) $\frac{1}{2}$ sq units

50. The area bounded by the curves $y^2 = 4x$ and $x^2 = 4y$ is (2011)

(a) 0

(b) $\frac{32}{3}$

(c) $\frac{16}{3}$

(d) $\frac{8}{3}$

51. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$, is (2009)

(a) $\left(4\sqrt{2} - 2\right)$ sq units

(b) $\left(4\sqrt{2} + 2\right)$ sq units

(c) $\left(4\sqrt{2} - 1\right)$ sq units

(d) $\left(4\sqrt{2}+1\right)$ sq units

52. The area of the region bounded by the parabola $(y-2)^2 = x-1$, the tangent to the parabola at the point (2, 3) and the X-axis is (2009)

(a) 6 sq units

(b) 9 sq units

(c) 12 sq units

(d) 3 sq units

53. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to (2008)

(a) $\frac{4}{3}$ sq units

(b) $\frac{5}{3}$ sq units

(c) $\frac{1}{3}$ sq units

(d) $\frac{2}{3}$ sq units

54. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is (2007)

(a) $\frac{2}{3}$ sq unit

(b) 1 sq unit

(c) $\frac{1}{6}$ sq unit

(d) $\frac{1}{3}$ sq unit

55. If $f(x)$ is a non-negative continuous function such that the area bounded by the curve $y = f(x)$, X-axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is equal to (2005)

(a) $\left(1+\frac{\pi}{4}+\sqrt{2}\right)$

(b) $\left(1-\frac{\pi}{4}-\sqrt{2}\right)$

(c) $\left(\frac{\pi}{4}-\sqrt{2}+1\right)$

(d) $\left(\frac{\pi}{4}+\sqrt{2}-1\right)$

56. The area enclosed between the curve $y = \log_e (x + e)$ and the coordinate axes is (2005)
(a) 4 sq units
(b) 3 sq units
(c) 2 sq units
(d) 1 sq units

57. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2 and S_3 are respectively the area of these parts numbered from top to bottom, then $S_1 : S_2 : S_3$ is equal to (2005)
(a) 1 : 1 : 1
(b) 2 : 1 : 2
(c) 1 : 2 : 3
(d) 1 : 2 : 1

58. The area of the region bounded by the curves $y = |x - 2|$, $x = 1$, $x = 3$ and the X-axis is (2004)
(a) 1 sq unit
(b) 2 sq units
(c) 3 sq units
(d) 4 sq units

59. The area of the region bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$ is (2003)
(a) 2 sq units
(b) 3 sq units
(c) 4 sq units
(d) 6 sq units

60. The area bounded by the curve $y = 2x - x^2$ and the straight line $y = - x$ is given by
(a) $\frac{9}{2}$ sq units
(b) $\frac{43}{6}$ sq units
(c) $\frac{35}{6}$ sq units
(d) None of these

61. Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of ΔOPQ is $3\sqrt{2}$, then which of the following is/are the coordinates of P? (2015)

(a) $\left(-4, \sqrt{2}\right)$

(b) $\left(9, 3\sqrt{2}\right)$

(c) $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$

(d) $\left(1, \sqrt{2}\right)$

62. The area of the triangle formed by the positive X-axis and the normal and the tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is …. (1989)

(a) 2 sq. units

(b) $2\sqrt{3}$ sq. units

(c) 1/2 sq. units

(d) none

63. The area enclosed within the curve $|x| + |y| = 1$ is …… (1981)

(a) 1 sq. units

(b) 2 sq. units

(c) 3 sq. units

(d) 4 sq. units

64. Let O (0, 0), A (2, 0) and B $\left(1, \frac{1}{\sqrt{3}}\right)$ be the vertices of a triangle. Let R be the region consisting of all those points P inside Δ OAB which satisfy d(P, OA) $\geq$ min {d (P, OB), (P, AB)}, where d denotes the distance from the point to the corresponding line. Sketch the region R and find its area. (1997)

(a) $\left(2+\sqrt{3}\right)$ sq. units

(b) $\left(3+\sqrt{3}\right)$ sq. units

(c) $\left(2-\sqrt{3}\right)$ sq. units

(d) $\left(3+2\sqrt{3}\right)$ sq. units

65. Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real-valued differentiable function $y = f(x)$. If $x \in (-2, 2)$, the equation implicitly defines a unique real-valued differentiable function $y = g(x)$, satisfying $g(0) = 0$.

If $f\left(-10\sqrt{2}\right) = 2\sqrt{2}$, then $f''\left(-10\sqrt{2}\right)$ is equal to (2008)

(a) $\frac{4\sqrt{2}}{7^3 3^2}$

(b) $-\frac{4\sqrt{2}}{7^3 3^2}$

(c) $\frac{4\sqrt{2}}{7^3 3}$

(d) $-\frac{4\sqrt{2}}{7^3 3}$

66. Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real-valued differentiable function $y = f(x)$. If $x \in (-2, 2)$, the equation implicitly defines a unique real-valued differentiable function $y = g(x)$, satisfying $g(0) = 0$.

The area of the region bounded by the curve $y = f(x)$, the X-axis and the lines $x = a$ and $x = b$, where $-\infty < a < b < -2$, is (2008)

(a) $\int_a^b \frac{x}{3\left[\{f(x)\}^2 - 1\right]} dx + bf(b) - af(a)$

(b) $-\int_a^b \frac{x}{3\left[\{f(x)\}^2 - 1\right]} dx + bf(b) - af(a)$

(c) $\int_a^b \frac{x}{3\left[\{f(x)\}^2 - 1\right]} dx - bf(b) + af(a)$

(d) $-\int_a^b \frac{x}{3\left[\{f(x)\}^2 - 1\right]} dx - bf(b) + af(a)$

67. Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real-valued differentiable function $y = f(x)$. If $x \in (-2, 2)$, the equation implicitly defines a unique real-valued differentiable function $y = g(x)$, satisfying $g(0) = 0$.

$\int_{-1}^{1} g'(x)\,dx$ is equal to (2008)

(a) $2g(-1)$
(b) 0
(c) $-2g(1)$
(d) $2g(1)$

68. The area bounded by the curve $2x^2 + y^2 = 2$ is
(a) π
(b) $\sqrt{2}\pi$
(c) $\frac{\pi}{2}$
(d) 2π

69. The area bounded by the curve $y^2(1-x) = x^2(1+x)$ between x = 0, x = 1 is
(a) $\frac{\pi}{2} + 2$
(b) $2 - \frac{\pi}{2}$
(c) $\pi + 2$
(d) $\frac{\pi}{2} + 1$

70. The area bounded by the curve $y = xe^{-x^2}$, $y = 0$ and the maximum ordinate is
(a) $\frac{1}{2}$
(b) $\frac{1}{2\sqrt{e}}$
(c) $\frac{1}{2}\left(1 - \frac{1}{\sqrt{e}}\right)$
(d) $\frac{1}{2}\left(1 - \frac{1}{e}\right)$

71. The area enclosed between the curve $y = \sin^2x$ and $y = \cos^2x$, $0 \leq x \leq \pi$ is
(a) 1 sq unit
(b) ½ sq unit
(c) 2 sq units
(d) none of these

72. The area bounded by $y = \log_e x$ x, x-axis and the ordinate $x = e$ is given by
(a) 4 sq. units
(b) ½ sq. units
(c) 1 sq unit
(d) none of these

73. The area bounded the curve $y = \sin^{-1} x$ and the lines $x = 0$, $|y| = \frac{\pi}{2}$

(a) 2 sq units
(b) 4 sq units
(c) 8 sq units
(d) 16 sq units

74. The area of the figure bounded by the straight line $x = 0$, $x = 2$ and the curves $y = 2^x$, $y = 2x - x^2$ is

(a) $\left(\frac{4}{\ell n2} - \frac{8}{3}\right)$ sq. units

(b) $\left(\frac{4}{\ell n2} + \frac{8}{3}\right)$ sq. units

(c) $\left(\frac{8}{\ell n3} - \frac{4}{3}\right)$ sq. units

(d) $\left(\frac{3}{\ell n2} - \frac{4}{3}\right)$ sq. units

75. The area bounded by the curve $y = |x - 1|$ and $y = 3 - |x|$

(a) 4 sq. units

(b) 2 sq. units

(c) 6 sq. units

(d) 8 sq. units

76. The area of the region R = $\{(x, y) : |x| \le |y| \text{ and } x^2 + y^2 \le 1\}$ is

(a) $\frac{3\pi}{8}$ sq. units

(b) $\frac{5\pi}{8}$ sq. units

(c) $\frac{\pi}{2}$ sq. units

(d) $\frac{\pi}{8}$ sq. units

77. If y = f(x) make positive intercepts of 2 and 1 units on x and y coordinate axes and encloses an area of $\frac{3}{4}$ square units with the axes, then $\int_0^2 xf'(x)\,dx$ is 0

(a) $\frac{3}{2}$

(b) 1

(c) $\frac{5}{4}$

(d) $-\frac{3}{4}$

78. The area of one curvilinear triangle formed by the curves y = sin x, y = cos x and x-axis

(a) $(2 + \sqrt{2})$ sq. units

(b) $2 - \sqrt{2}$ sq. units

(c) $(\sqrt{2} - 2)$ sq. units

(d) none of these

79. The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = 1$, $x = -1$

(a) $\frac{4}{3}$ sq. units

(b) $\frac{2}{3}$ sq. units

(c) $\frac{1}{6}$ sq. units

(d) $\frac{4}{5}$ sq. units

80. The area of the region lying between the line $x - y + 2 = 0$ and the curve $x = \sqrt{y}$

(a) $\frac{4}{3}$ sq. units

(b) $\frac{2}{3}$ sq. units

(c) $\frac{5}{3}$ sq. units

(d) $\frac{10}{3}$ sq. units

81. The area enclosed between the curve $y^2 = x$ and $y = |x|$

(a) $\frac{1}{6}$ sq. units

(b) $\frac{2}{3}$ sq. units

(c) $\frac{5}{3}$ sq. units

(d) $\frac{10}{3}$ sq. units

82. The area bounded by the parabola $x = y^2$ and the straight line $y = 4$ and y-axis

(a) $\frac{1}{6}$ sq. units

(b) $\frac{64}{3}$ sq. units

(c) $\frac{5}{3}$ sq. units

(d) $\frac{32}{3}$ sq. units

83. The area inside the parabola $5x^2 - y = 0$ but outside the parabola $2x^2 - y + 9 = 0$ is

(a) $12\sqrt{3}$

(b) $6\sqrt{3}$

(c) $8\sqrt{3}$

(d) $-8\sqrt{3}$

84. The area of the region on plane bounded by max $(|x|, |y|) \leq 1$ and $xy \leq \frac{1}{2}$ is

(a) $1/2 + \ell n\ 2$

(b) $3 + \ell n\ 2$

(c) $31/4$

(d) $1 + 2\ \ell n\ 2$

85. The area bounded by $y = x^2$, $y = [x + 1]$, $x \leq 1$ and the y-axis is

(a) 1/3
(b) 2/3

(c) 1

(d) 7/3

86. The area contained between the curve $xy = a^2$, the vertical line x = a, x = 4a (a > 0) and x-axis is

(a) $a^2 \ell n\, 2$

(b) $2a^2 \ell n\, 2$

(c) $a \ell n\, 2$

(d) $2a \ell n\, 2$

87. The area of the closed figure bounded by the curves y = $\sqrt{x}$, y = $\sqrt{4-3x}$ and y = 0 is:

(a) $\frac{4}{9}$

(b) $\frac{8}{9}$

(c) $\frac{16}{9}$

(d) none

88. The area bounded by the curve $x^2 = 4y$, x-axis and the line x = 2 is

(a) 1

(b) $\frac{2}{3}$

(c) $\frac{3}{2}$

(d) 2

89. The area bounded by the parabola $y = 4x^2$, $x = 0$ and $y = 1$, $y = 4$ is

(a) 7

(b) $\frac{7}{2}$

(c) $\frac{7}{3}$

(d) $\frac{7}{4}$

90. The area bounded by the curve $y = \frac{1}{x^2}$ and its asymptote from $x = 1$ to $x = 3$ is

(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) $\frac{1}{2}$

(d) $\frac{1}{6}$

91. The area bounded by the curve $y^2 = 4x$ and the line $2x - 3y + 4 = 0$ is

(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) $\frac{4}{3}$

(d) $\frac{5}{3}$

92. If the curve $y = ax^{1/2} + bx$ passes through the point (1, 2) and lies above x-axis for $0 \le x \le 9$ and the area enclosed by the curve, the x-axis and the line x = 4 is 8 sq. units, then

(a) a = 1, b= 1

(b) a = 3 b = - 1

(c) a = 3, b = 1

(d) a = 1, b = - 1

93. The area enclosed by the curve $x = a \sin^3 t$ and $y = a \cos^3 t$ is given by

(a) $12a^2 \int_0^{\pi/2} \cos^4 t \sin^2 t \, dt$

(b) $12a^2 \int_0^{\pi/2} \cos^2 t \sin^4 t \, dt$

(c) $2\int_{-a}^{a} \left(a^{2/3} - x^{2/3}\right)^{3/2} dx$

(d) all of these

94. The parabola $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines x = 4, y = 4 and the coordinates axes. If S_1, S_2, S_3 are the areas of these parts numbered from top to bottom, respectively, then -

(a) $S_1 : S_2 : S_3 = 1 : 1 : 1$

(b) $S_1 : S_2 : S_3 = 1 : 2 : 3$

(c) $S_1 : S_2 : S_3 = 3 : 2 : 1$

(d) $S_1 : S_2 : S_3 = 1 : 2 : 4$

95. If A_i is the area bounded by $|x - a| + |y| = b_i$ $i \in N$, where $a_{i+1} = a_i + \frac{3}{2}b_i$ and $b_{i+1} = \frac{b_i}{2}$, $a_1 = 0$ and $b_1 = 32$, then

(a) $A_3 = 64$

(b) $A_3 = 256$

(c) $\lim_{n\to\infty}\sum_{i=1}^{n} A_i = \frac{8}{3}(32)^2$

(d) $\lim_{n\to\infty}\sum_{i=1}^{n} A_i = \frac{4}{3}(16)^2$

96. Find the area enclosed between the curve $y^2(2a - x) = x^3$ and the line $x = 2a$ above x-axis

(a) $\frac{5\pi a^2}{2}$

(b) $\frac{3\pi a^2}{2}$

(c) $\frac{\pi a^2}{2}$

(d) $\frac{3\pi a}{2}$

97. The area enclosed by the curve $y = \sqrt{4 - x^2}, y \geq \sqrt{2}\sin\frac{\pi x}{2\sqrt{2}}$ and x-axis is divided by the y-axis in the ratio

(a) $\frac{2\pi^2}{2\pi^2 + \pi - 12}$

(b) $\frac{2\pi^2}{2\pi^2 + \pi - 8}$

(c) $\frac{\pi}{2\pi + \pi^2 - 8}$

(d) $\frac{2\pi^2}{\pi + \pi^2 - 8}$

98. The area cut off a parabola by any double ordinate is k times the corresponding rectangle contained by that double ordinate and its distance from the vertex, then find the value of k.

(a) 2/5

(b) 4/3

(c) 2/3

(d) 5/2

99. The slope of the tangent to the curve y = f(x) at (x, f(x)) is 2x + 1. If the curve passes through the point (1, 2). Then find the area of the region bounded by the curve, the x-axis and the line x = 1.

(a) $\frac{5}{7}$

(b) $\frac{5}{6}$

(c) $\frac{1}{6}$

(d) $\frac{2}{5}$

100. Statement-1: The area bounded by parabola $y = x^2 - 4x + 3$ and y = 0 is $\frac{4}{3}$ sq. units

Statement-2: The area bounded by the curve $y = f(x) \geq 0$ and y = 0 between the ordinates x = a and x = b, where $b \geq a$) is $\int_a^b f(x)\,dx$

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement -1 is False, Statement-2 is True

Answer Keys

1. (b) 2. (d) 3. (a) 4. (d) 5. (b) 6. (c) 7. (d) 8. (c) 9. (d) 10. (a) 11. (b) 12. (a)
13. (c) 14. (b) 15. (b) 16. (a) 17. (a) 18. (c) 19. (c) 20. (a) 21. (d) 22. (c) 23. (b) 24. (c)
25. (a) 26. (a) 27. (b) 28. (b) 29. (c) 30. (b) 31. (a) 32. (b) 33. (a) 34. (c) 35. (a) 36. (d)
37. (a) 38. (a) 39. (c) 40. (a) 41. (a) 42. (a) 43. (b) 44. (b) 45. (c) 46. (a) 47. (a) 48. (c)
49. (b) 50. (c) 51. (a) 52. (b) 53. (a) 54. (c) 55. (a) 56. (d) 57. (a) 58. (a) 59. (c) 60. (a)
61. (d) 62. (b) 63. (b) 64. (c) 65. (b) 66. (a) 67. (d) 68. (b) 69. (a) 70. (c) 71. (a) 72. (c)
73. (a) 74. (d) 75. (a) 76. (c) 77. (d) 78. (b) 79. (b) 80. (d) 81. (a) 82. (b) 83. (a) 84. (b)
85. (b) 86. (b) 87. (b) 88. (b) 89. (c) 90. (b) 91. (a) 92. (b) 93. (d) 94. (a) 95. (c) 96. (b)
97. (b) 98. (c) 99. (b) 100. (b)

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