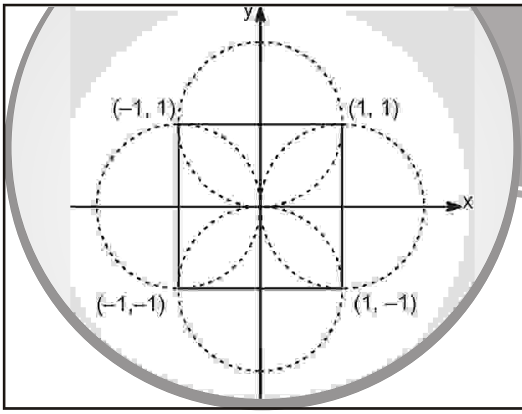


2

CIRCLE

KEY CONCEPTS

1. EQUATION OF A CIRCLE IN VARIOUS FORM

(a) The circle with centre (h,k) and radius 'r' has the equation;

$(x \quad h)^2 \quad (y \quad k)^2 \quad r^2$

(b) The general equation of a circle is $x^2 \quad y^2 \quad 2gx \quad 2fy \quad c \quad 0$ with centre as:

$(\quad g, \quad f)$ and radius $\sqrt{g^2 \quad f^2 \quad c}$.

Remember that every second degree equation in x and y in which coefficient of x^2 = coefficient of y^2 and there is no xy term always represents a circle.

If $g^2 \quad f^2 \quad c \quad 0$ real circle.

$g^2 \quad f^2 \quad c \quad 0$ point circle.

$g^2 \quad f^2 \quad c \quad 0$ imaginary circle.

Note that the general equation of a circle contains three arbitrary constants g, f and c which corresponds to the fact that a unique circle passes through three non collinear points.

(c) The equation of a circle with (x_1, y_1) and (x_2, y_2) as its diameter is:

$(x \quad x_1)(x \quad x_2) \quad (y \quad y_1)(y \quad y_2) \quad 0.$

Note that this will be the circle of least radius passing through (x_1, y_1) and (x_2, y_2).

2. INTERCEPTS MADE BY A CIRCLE ON THE AXES:

The intercepts made by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ on the co-ordinate axes are $2\sqrt{g^2 - c}$ and $2\sqrt{f^2 - c}$ respectively.

Note: *If* $g^2 - c > 0 \Rightarrow$ *circle cuts the x-axis at two distinct points.*

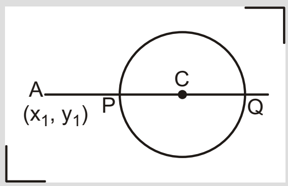
If $g^2 = c \Rightarrow$ *circle touches the x-axis.*

If $g^2 < c \Rightarrow$ *circle lies completely above or below the x-axis.*

3. POSITION OF A POINT w.r.t. A CIRCLE

The point (x_1, y_1) is inside, on or outside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ according as $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c \lessgtr 0$.

Note: *The greatest and the least distance of a point A from a circle with centre C and radius r is* $|AC + r|$ *and* $|AC - r|$ *respectively.*



4. LINE AND A CIRCLE

Let $L = 0$ be a line and $S = 0$ be a circle. If r is the radius of the circle and p is the length of the perpendicular from the centre on the line, then :

(i) $p > r \Leftrightarrow$ the line does not meet the circle *i.e.* passes out side the circle.

(ii) $p = r \Leftrightarrow$ the line touches the circle.

(iii) $p < r \Leftrightarrow$ the line is a secant of the circle.

(iv) $p = 0 \Rightarrow$ the line is a diameter of the circle.

5. PARAMETRIC EQUATION OF A CIRCLE

The parametric equation of $(x - h)^2 + (y - k)^2 = r^2$ are :

$x = h + r\cos\theta$; $y = k + r\sin\theta$; where (h, k) is the centre, r is the radius and θ is a parameter.

Note that equation of a straight line joining two point α and β on the circle $x^2 + y^2 = a^2$ is

$$x\cos\frac{\alpha+\beta}{2} + y\sin\frac{\alpha+\beta}{2} = a\cos\frac{\alpha-\beta}{2}.$$

6. TANGENT AND NORMAL

(a) The equation of the tangent to the circle $x^2+y^2=a^2$ at its point (x_1, y_1) is, $xx_1+yy_1=a^2$. Hence equation of a tangent at $(a\cos\alpha, a\sin\alpha)$ is ; $x\cos\alpha+y\sin\alpha=a$. The point of intersection of the tangents at the points $P(\alpha)$ and $Q(\beta)$ is $\left(\dfrac{a\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}, \dfrac{a\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}\right)$.

(b) The equation of the tangent to the circle $x^2+y^2+2gx+2fy+c=0$ at its point (x_1, y_1) is $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$.

(c) $y=mx+c$ is always a tangent to the circle $x^2+y^2=a^2$ if $c^2=a^2(1+m^2)$ and the point of contact is $\left(-\dfrac{a^2m}{c}, \dfrac{a}{c}\right)$.

(d) If a line is normal / orthogonal to a circle then it must pass through the centre of the circle. Using this fact normal to the circle $x^2+y^2+2gx+2fy+c=0$ at (x_1, y_1) is $y-y_1=\dfrac{y_1+f}{x_1+g}(x-x_1)$.

7. A FAMILY OF CIRCLE

(a) The equation of the family of circles passing through the points of intersection of two circles $s_1=0$ and $s_2=0$ is : $s_1+ks_2=0$ $(k\neq -1)$.

(b) The equation of the family of circles passing through the point of intersection of a circle $S=0$ and a line $L=0$ is given $S+kL=0$.

(c) The equation of a family of circle passing through two given points (x_1, y_1) and (x_2, y_2) can be written in the from :

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+K\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}=0 \text{ where } K \text{ is a parameter.}$$

(d) The equation of a family of circles touching a fixed line $y-y_1=m(x-x_1)$ at the fixed point (x_1, y_1) is $(x-x_1)^2+(y-y_1)^2+K[y-y_1-m(x-x_1)]=0$, where K is a parameter.

In case the line through (x_1, y_1) is parallel to y-axis the equation of the family of circles touching it at (x_1, y_1) becomes $(x-x_1)^2+(y-y_1)^2+K(x-x_1)=0$

Also if line is parallel to x-axis the equation of the family of circles touching it at (x_1, y_1) becomes $(x-x_1)^2+(y-y_1)^2+K(y-y_1)=0$.

(e) Equation of circle circumscribing a triangle whose sides are given by $L_1=0$; $L_2=0$ and $L_3=0$ is given by ; $L_1L_2+\lambda L_2L_3+\mu L_3L_1=0$ provided coefficient of $xy=0$ and coefficient of $x^2=$ coefficient of y^2.

(f) Equation of circle circumscribing a quadrilateral whose side in order are represented by the lines $L_1 = 0, L_2 = 0, L_3 = 0$ and $L_4 = 0$ is $L_1L_3 + \lambda L_2L_4 = 0$ provided coefficient of x^2 = coefficient of y^2 and coefficient of $xy = 0$.

8. LENGTH OF A TANGENT AND POWER OF A POINT

The length of a tangent from an external point (x_1, y_1) to the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ is given by $L = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2f_1y + c} = \sqrt{S_1}$.

Square of length of the tangent from the point P is also called **THE POWER OF POINT** w.r.t. a circle. Power of a point remains constant w.r.t. a circle.

Note that power of a point P is positive, negative or zero according as the point 'P' is outside, inside or on the circle respectively.

9. DIRECTOR CIRCLE:

The locus of the point of intersection of two perpendicular tangent is called the **DIRECTOR CIRCLES** of the given circle. The director circle of a circle is the concentric circle having radius equal to $\sqrt{2}$ times the original circle.

10. EQUATION OF THE CHORD WITH A GIVEN MIDDLE POINT

The equation of the chord of the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ in terms of its mid point $M(x_1, y_1)$ is $y - y_1 = -\frac{x_1 + g}{y_1 + f}(x - x_1)$. This on simplication can be put in the form $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ which is designated by $T = S_1$.

Note that the shortest chord of a circle passing through a point '***M***' **inside the circle, is one chord whose middle point is *M*.**

11. CHORD OF CONTACT :

If two tangents PT_1 and PT_2 are drawn from $p(x_1, y_1)$ to the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, then the equation of the chord of contact T_1T_2 is :

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0.$$

REMEMBER:

(a) Chord of contact exists only if the point 'P' is not inside.

(b) Length of chord of contact $T_1T_2 = \frac{2LR}{\sqrt{R^2 + L^2}}$.

(c) Area of the triangle formed by the pair of the tangent and its chord of contact $= \frac{RL^3}{R^2 + L^2}$ Where R is the radius of the circle and L is the tangent from (x_1, y_1) on $S = 0$.

(d) Angle between the pair of tangents from $(x_1, y_1) = \tan^{-1}\left(\frac{2RL}{L^2 - R^2}\right)$ where $R =$ radius ; $L =$ length of tangent.

(e) Equation of the circle circumscribing the triangle PT_1T_2 is :
$(x - x_1)(x + g) + (y - y_1)(y + f) = 0.$

(f) The joint equation of a pair of tangents drawn from the point $A(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is : $SS_1 = T^2$.

Where $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0;\ S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

$T \equiv xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c.$

12. POLE AND POLAR

(i) If through a point P in the plane of the circle, there be drawn any straight line to meet the circle in Q and R, the locus of the point of intersection of the tangents at Q and R *is* called the **Polar Of The Point *P*** ; also P is called the **Pole Of The Polar**.

(ii) The equation to the polar of a point $P(x_1, y_1)$ w.r.t. the circle $x^2 + y^2 = a^2$ is given by $\boldsymbol{xx_1 + yy_1 = a^2}$, and if the circle is general then the equation of the polar becomes $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$. Note that if the point (x_1, y_1) be on the circle then the chord of contact, tangent and polar will be represented by the same equation.

(iii) Pole of a given line $Ax + By + c = 0$ w.r.t. any circle $x^2 + y^2 = a^2$ is $\left(-\frac{Aa^2}{c}, -\frac{Ba^2}{c}\right)$.

(iv) If the polar of a point P pass through a point Q, then the polar of Q passes through P.

(v) Two lines L_1 and L_2 are conjugate of each other if pole of L_1 lies on L_2 and *vice versa*. Similarly two points P and Q are said to be conjugate of each other if the polar of P passes through Q and vice-versa.

13. COMMON TANGENTS TO TWO CIRCLE

(i) Where the two circle neither intersect nor touch each other , there are FOUR common tangents, two of them are transverse and the others are direct common tangents.

(ii) When they intersect there are two common tangents, both of them being direct.

(iii) When they touch each other :

(a) **EXTERNALLY:** There are three common tangents, two direct and one is the tangents at the point.

(b) **INTERNALLY:** Only one common tangents possible at their point of contact.

(iv) Length of an external common tangent and internal common tangent to the two circles is given by :$L_{ext} = \sqrt{d^2 - (r_1 - r_2)^2}$ and $L_{ext} = \sqrt{d^2 - (r_1 + r_2)^2}$.

Where $d =$ distance between the centres of the two circles. r_1 and r_2 are the radii of the two circle.

(v) The direct common tangents meet at a point which divides the line joining centre of circles externally in the ratio of their radii.

Transverse common tangents meet at a point which divides the line joining centre of circles internally in the ratio of their radii.

14. RADICAL AXIS AND RADICAL CENTRE

The radical axis of two circles is the locus of points whose powers w.r.t. the two circles are equal. The equation of radical axis of the two circles $S_1 = 0$ and $S_2 = 0$ is given ;

$S_1 - S_2 = 0$ *i.e.*, $2(g_1 - g_2)x + 2(f_1 - f_2)y + (c_1 - c_2) = 0$.

Note That :

(a) If two circles intersect, then the radical axis is the common chord of the two circles.

(b) If two circles touch each other then the radical axis is the common tangent of the two circles at the common point of contact.

(c) Radical axis is always perpendicular to the line joining the centres of the two circles.

(d) Radical axis need not always pass through the mid point of the line joining the centres of the two circles.

(e) Radical axis bisects a common tangent between the two circles.

(f) The common point of intersection of the radical axes of there circles taken two at a time is called the radical centre of three circles.

(g) A system of circles , every two which have the same radical axis, is called coaxal system.

(h) Pairs of circles which do not have radical axis are concentric.

15. ORTHOGONALITY OF TWO CIRCLE

Two circles $S_1 = 0$ and $S_2 = 0$ are said to be orthogonal or said to intersect orthogonally if the tangents at their point of intersection include a right angle. The condition for two circles to be orthogonal is :

$2g_1g_2 + 2f_1f_2 = c_1 + c_2$.

Note:

(a) *Locus of the centre of a variable circle orthogonal to two fixed circles is the radical axis between the two fixed circles.*

(b) *If two circles are orthogonal, then the polar of a point 'P' on first circle w.r.t. the second circle passes through the point Q which is the other end of the diameter through P . Hence locus of a point which moves such that its polars w.r.t. the circles $S_1 = 0, S_2 = 0$ and $S_3 = 0$ are concurrent in a circle which is orthogonal to all the three circles.*

EXERCISE 1

Only One Choice is Correct:

1. Circles of radii 36 and 9 touch externally. The radius of the circle which touches the two circles externally and also their common tangent is:

(a) 4 (b) 5

(c) $\sqrt{17}$ (d) $\sqrt{18}$

2. $A(0,a)$ and $B(0,b), a, b \quad 0$ are two vertices of a triangle ABC where the vertex $C(x,0)$ is variable. The value of x when $\quad ACB$ is maximum is:

(a) $\frac{a \quad b}{2}$ (b) $\sqrt{ab}$

(c) $\frac{2ab}{a \quad b}$ (d) $\frac{ab}{a \quad b}$

3. A circle passing through origin O cuts two straight lines $x \quad y \quad 0$ and $x \quad y \quad 0$ in points A and B respectively. If abscissae of A and B are roots of the equation $x^2 \quad ax \quad b \quad 0$, then the equation of the given circle is:

(a) $x^2 \quad y^2 \quad ax \quad by \quad 0$ (b) $x^2 \quad y^2 \quad x\sqrt{4b \quad a^2} \quad yb \quad 0$

(c) $x^2 \quad y^2 \quad ax \quad y\sqrt{a^2 \quad 4b} \quad 0$ (d) $x^2 \quad y^2 \quad ax \quad y\sqrt{a^2 \quad 4b} \quad 0$

4. The minimum distance between the circle $x^2 \quad y^2 \quad 9$ and the curve $2x^2 \quad 10y^2 \quad 6xy \quad 1$ is:

(a) $2\sqrt{2}$ (b) 2

(c) $3 \quad \sqrt{2}$ (d) $3 \quad \frac{1}{\sqrt{11}}$

5. A pair of tangents are drawn from a point P to the circle $x^2 \quad y^2 \quad 1$. If the tangents make an intercept of 2 on the line $x \quad 1$, then locus of P is:

(a) straight line (b) pair of lines

(c) circle (d) parabola

6. Let $ABCD$ be a quadrilateral in which $AB || CD$, $AB \quad AD$ and $AB \quad 3CD$. The area of quadrilateral $ABCD$ is 4. The radius of a circle touching all the sides of quadrilateral is:

(a) $\sin\frac{}{12}$ (b) $\sin\frac{}{6}$

(c) $\sin\frac{}{4}$ (d) $\sin\frac{}{3}$

7. A circle with center (2, 2) touches the coordinate axes and a straight line AB where A and B lie on positive direction of coordinate axes such that the circle lies between origin and the line AB. If O be the origin then the locus of circumcenter of $\quad OAB$ will be:

(a) $xy \quad x \quad y \quad \sqrt{x^2 \quad y^2}$

(b) $xy \quad x \quad y \quad \sqrt{x^2 \quad y^2}$

(c) $xy \quad x \quad y \quad \sqrt{x^2 \quad y^2}$

(d) $xy \quad x \quad y \quad \sqrt{x^2 \quad y^2} \quad 0$

8. The value of for which the point (, 2) is an interior point of smaller segment of the curve $x^2 \quad y^2 \quad 4 \quad 0$ made by the chord of the curve whose equation is $3x \quad 4y \quad 12 \quad 0$

(a) $\quad , \ \frac{20}{7}$

(b) $(\ 2,0)$

(c) $\quad , \frac{20}{7}$

(d)

9. The equation of circumcircle of an equilateral triangle is $x^2 \quad y^2 \quad 2gx \quad 2fy \quad c \quad 0$ and one vertex of triangle is $(1,1)$. The equation of incircle of triangle is:

(a) $4(x^2 \quad y^2) \quad g^2 \quad f^2$

(b) $4(x^2 \quad y^2) \quad 8gx \quad 8fy \quad (1 \quad g)(1 \quad 3g) \quad (1 \quad f)(1 \quad 3f)$

(c) $4(x^2 \quad y^2) \quad 8gx \quad 8fy \quad g^2 \quad f^2$

(d) $4(x^2 \quad y^2) \quad 8gx \quad 8fy \quad (2 \quad g)(1 \quad 3g) \quad (2 \quad f)(2 \quad 3f)$

10. The equation of the smallest circle passing through the points of intersection of the line $x \quad y \quad 1$ and the circle $x^2 \quad y^2 \quad 9$ is:

(a) $x^2 \quad y^2 \quad x \quad y \quad 8 \quad 0$

(b) $x^2 \quad y^2 \quad x \quad y \quad 8 \quad 0$

(c) $x^2 \quad y^2 \quad x \quad y \quad 8 \quad 0$

(d) $x^2 \quad y^2 \quad x \quad y \quad 8 \quad 0$

11. Let C be a circle $x^2 \quad y^2 \quad 1$. The line l intersects C at the point $(\ 1,0)$ and the point P. Suppose that the slope of the line l is a rational number m. Number of choices for m for which both the coordinates of P are rational, is:

(a) 3

(b) 4

(c) 5

(d) infinitely many

12. The line $2x \quad y \quad 1 \quad 0$ is tangent to the circle at the point (2, 5) and the centre of the circles lies on $x \quad 2y \quad 4$. The radius of the circle is:

(a) $3\sqrt{5}$

(b) $5\sqrt{3}$

(c) $2\sqrt{5}$

(d) $5\sqrt{2}$

13. One circle has a radius of 5 and its center at (0, 5). A second circle has a radius of 12 and its centre at (12, 0). The length of a radius of a third circle which passes through the center of the second circle and both points of intersection of the first 2 circles, is equal to:

(a) 13/2

(b) 15/2

(c) 17/2

(d) none

14. In the *xy*-plane, the length of the shortest path from (0, 0) to (12, 16) that does not go inside the circle $(x \quad 6)^2 \quad (y \quad 8)^2 \quad 25$ is:

(a) $10\sqrt{3}$ (b) $10\sqrt{5}$

(c) $10\sqrt{3} \quad \frac{5}{3}$ (d) $10 \quad 5$

15. Four unit circles pass through the origin and have their centres on the coordinate axes. The area of the quadrilateral whose vertices are the points of intersection (in pairs) of the circles, is:

(a) 1sq. unit

(b) $2\sqrt{2}$ sq. units

(c) 4 sq. units

(d) can not be uniquely determined, insufficient data

16. Consider 3 non-collinear points A, B, C with coordinates (0, 6), (5, 5) and (–1, 1) respectively. Equation of a line tangent to the circle circumscribing the triangle *ABC* and passing through the origin is:

(a) $2x \quad 3y \quad 0$ (b) $3x \quad 2y \quad 0$

(c) $3x \quad 2y \quad 0$ (d) $2x \quad 3y \quad 0$

17. A circle is inscribed in an equilateral triangle with side lengths 6 unit. Another circle is drawn inside the triangle (but outside the first circle), tangent to the first circle and two of the sides of the triangle. The radius of the smaller circle is:

(a) $1/\sqrt{3}$ (b) 2/3

(c) 1/2 (d) 1

18. A square *OABC* is formed by line pairs $xy \quad 0$ and $xy \quad 1 \quad x \quad y$ where '*O*'is the origin. A circle with centre C_1 inside the square is drawn to touch the line pair $xy \quad 0$ and another circle with centre C_2 and radius twice that of C_1, is drawn to touch the circle C_1 and the other line pair. The radius of the circle with centre C_1 is:

(a) $\frac{\sqrt{2}}{\sqrt{3}(\sqrt{2} \quad 1)}$ (b) $\frac{2\sqrt{2}}{3(\sqrt{2} \quad 1)}$

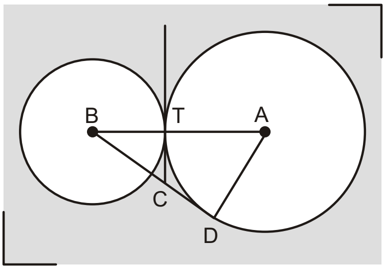
(c) $\frac{\sqrt{2}}{3(\sqrt{2} \quad 1)}$ (d) $\frac{\sqrt{2} \quad 1}{3\sqrt{2}}$

19. The shortest distance from the line $3x \quad 4y \quad 25$ to the circle $x^2 \quad y^2 \quad 6x \quad 8y$ is equal to:

(a) 7/5 (b) 9/5

(c) 11/5 (d) 32/5

20. Two circles with centres at *A* and *B*, touch at *T*. *BD* is the tangent at *D* and *TC* is a common tangent. *AT* has length 3 and *BT* has length 2. The length of *CD* is:



(a) 4/3 (b) 3/2

(c) 5/3 (d) 7/4

21. Triangle *ABC* is right angled at *A*. The circle with centre *A* and radius *AB* cuts *BC* and *AC* internally at *D* and *E* respectively. If *BD* 20 and *DC* 16 then the length *AC* equals:

(a) $6\sqrt{21}$ (b) $6\sqrt{26}$

(c) 30 (d) 32

22. From the point *A*(0, 3) on the circle $x^2 \quad 4x \quad (y \quad 3)^2 \quad 0$ a chord *AB* is drawn and extended to a point *M* such that *AM* 2*AB*. The equation of the locus of *M* is:

(a) $x^2 \quad 8x \quad y^2 \quad 0$ (b) $x^2 \quad 8x \quad (y \quad 3)^2 \quad 0$

(c) $(x \quad 3)^2 \quad 8x \quad y^2 \quad 0$ (d) $x^2 \quad 8x \quad 8y^2 \quad 0$

23. If $x \quad 3$ is the chord of contact of the circle $x^2 \quad y^2 \quad 81$, then the equation of the corresponding pair of tangents, is:

(a) $x^2 \quad 8y^2 \quad 54x \quad 729 \quad 0$ (b) $x^2 \quad 8y^2 \quad 54x \quad 729 \quad 0$

(c) $x^2 \quad 8y^2 \quad 54x \quad 729 \quad 0$ (d) $x^2 \quad 8y^2 \quad 729$

24. The locus of the mid points of the chords of the circle $x^2 \quad y^2 \quad ax \quad by \quad 0$ which subtend a right angle at $(a/2, b/2)$ is:

(a) $ax \quad by \quad 0$ (b) $ax \quad by \quad a^2 \quad b^2$

(c) $x^2 \quad y^2 \quad ax \quad by \quad \frac{a^2 \quad b^2}{8} \quad 0$ (d) $x^2 \quad y^2 \quad ax \quad by \quad \frac{a^2 \quad b^2}{8} \quad 0$

25. From (3, 4) chords are drawn to the circle $x^2 \quad y^2 \quad 4x \quad 0$. The locus of the mid points of the chords is:

(a) $x^2 \quad y^2 \quad 5x \quad 4y \quad 6 \quad 0$ (b) $x^2 \quad y^2 \quad 5x \quad 4y \quad 6 \quad 0$

(c) $x^2 \quad y^2 \quad 5x \quad 4y \quad 6 \quad 0$ (d) $x^2 \quad y^2 \quad 5x \quad 4y \quad 6 \quad 0$

26. The centre of the smallest circle touching the circles $x^2 \quad y^2 \quad 2y \quad 3 \quad 0$ and $x^2 \quad y^2 \quad 8x \quad 18y \quad 93 \quad 0$ is:

(a) (3, 2) (b) (4, 4)

(c) (2, 7) (d) (2, 5)

27. In a right triangle ABC, right angled at A, on the leg AC as diameter, a semicircle is described. The chord joining A with the point of intersection D of the hypotenuse and the semicircle, then the length AC equals to:

(a) $\dfrac{AB\ \ AD}{\sqrt{AB^2\ \ \ AD^2}}$ (b) $\dfrac{AB\ \ AD}{AB\ \ \ AD}$

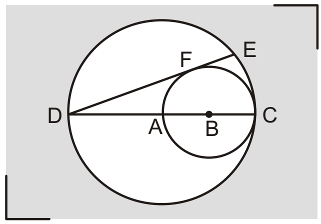
(c) $\sqrt{AB\ \ AD}$ (d) $\dfrac{AB\ \ AD}{\sqrt{AB^2\ \ \ AD^2}}$

28. If the circle $C_1: x^2\ \ \ y^2\ \ \ 16$ intersects another circle C_2 of radius 5 in such a manner that the common chord is of maximum length and has a slope equal to 3/4, then the coordinates of the centre of C_2 are:

(a) $\dfrac{9}{5}, \ \dfrac{12}{5}$ (b) $\dfrac{9}{5}, \mp\dfrac{12}{5}$

(c) $\dfrac{12}{5}, \ \dfrac{9}{5}$ (d) $\dfrac{12}{5}, \mp\dfrac{9}{5}$

29. In the diagram, DC is a diameter of the large circle centered at A, and AC is a diameter of the smaller circle centered at B. If DE is tangent to the smaller circle at F and $DC\ \ \ 12$ then the length of DE is:



(a) $8\sqrt{2}$ (b) 16

(c) $9\sqrt{2}$ (d) $10\sqrt{2}$

30. Let C be the circle of radius unity centred at the origin. If two positive numbers x_1 and x_2 are such that the line passing through $(x_1,\ \ 1)$ and $(x_2, 1)$ is tangent to C then:

(a) $x_1x_2\ \ \ 1$ (b) $x_1x_2\ \ \ \ 1$

(c) $x_1\ \ \ x_2\ \ \ 1$ (d) $4x_1x_2\ \ \ 1$

31. The distance between the chords of contact of tangents to the circle $x^2\ \ \ y^2\ \ \ 2gx\ \ \ 2fy\ \ \ c\ \ \ 0$ from the origin and the point (g, f) is:

(a) $\sqrt{g^2\ \ \ f^2}$ (b) $\dfrac{\sqrt{g^2\ \ \ f^2\ \ \ c}}{2}$

(c) $\dfrac{g^2\ \ \ f^2\ \ \ c}{2\sqrt{g^2\ \ \ f^2}}$ (d) $\dfrac{\sqrt{g^2\ \ \ f^2\ \ \ c}}{2\sqrt{g^2\ \ \ f^2}}$

32. The locus of the centers of the circles which cut the circles $x^2\ \ \ y^2\ \ \ 4x\ \ \ 6y\ \ \ 9\ \ \ 0$ and $x^2\ \ \ y^2\ \ \ 5x\ \ \ 4y\ \ \ 2\ \ \ 0$ orthogonally is:

(a) $9x\ \ \ 10y\ \ \ 7\ \ \ 0$ (b) $x\ \ \ y\ \ \ 2\ \ \ 0$

(c) $9x\ \ \ 10y\ \ \ 11\ \ \ 0$ (d) $9x\ \ \ 10y\ \ \ 7\ \ \ 0$

33. In a circle with centre 'O' PA and PB are two chords. PC is the chord that bisects the angle APB. The tangent to the circle at C is drawn meeting PA and PB extended at Q and R respectively. If $QC \quad 3, QA \quad 2$ and $RC \quad 4$, then length of RB equals:

(a) 2 (b) 8/3

(c) 10/3 (d) 11/3

34. Suppose that two circles C_1 and C_2 in a plane have no points in common. Then

(a) there is no line tangent to both C_1 and C_2.

(b) there are exactly four lines tangent to both C_1 and C_2.

(c) there are no lines tangent to both C_1 and C_2 or there are exactly two lines tangent to both C_1 and C_2.

(d) there are no lines tangent to both C_1 and C_2 or there are exactly four lines tangent to both C_1 and C_2.

35. If two chords of the circle $x^2 \quad y^2 \quad ax \quad by \quad 0$, drawn from the point (a, b) is divided by the x-axis in the ratio 2 : 1 then:

(a) $a^2 \quad 3b^2$ (b) $a^2 \quad 3b^2$

(c) $a^2 \quad 4b^2$ (d) $a^2 \quad 4b^2$

36. The angle at which the circles $(x \quad 1)^2 \quad y^2 \quad 10$ and $x^2 \quad (y \quad 2)^2 \quad 5$ intersect is:

(a) $\frac{\;}{6}$ (b) $\frac{\;}{4}$

(c) $\frac{\;}{3}$ (d) $\frac{\;}{2}$

37. The locus of the point of intersection of the tangent to the circle $x^2 \quad y^2 \quad a^2$, which include an angle of 45 is the curve $(x^2 \quad y^2)^2 \quad a^2(x^2 \quad y^2 \quad a^2)$. The value of is:

(a) 2 (b) 4

(c) 8 (d) 16

38. P is a point (a, b) in the first quadrant. If the two circles which pass through P and touch both the coordinate axes cut at right angles, then:

(a) $a^2 \quad 6ab \quad b^2 \quad 0$ (b) $a^2 \quad 2ab \quad b^2 \quad 0$

(c) $a^2 \quad 4ab \quad b^2 \quad 0$ (d) $a^2 \quad 8ab \quad b^2 \quad 0$

39. A circle of radius unity is centred at origin. Two particles start moving at the same time from the point (1, 0) and move around the circle in opposite direction. One of the particle moves counter clockwise with constant speed v and the other moves clockwise with constant speed $3v$. After leaving (1, 0), the two particles meet first at a point P, and continue until they meet next at point Q. The coordinates of the point Q are:

(a) (1, 0) (b) (0, 1)

(c) (0, 1) (d) (–1,0)

40. Three concentric circles of which the biggest is $x^2 \quad y^2 \quad 1$, have their radii in A.P. If the line $y \quad x \quad 1$ cuts all the circles in real and distinct points. The interval in which the common difference of the A.P. will lie is:

(a) $0, \frac{1}{4}$

(b) $0, \frac{1}{2\sqrt{2}}$

(c) $0, \frac{2 \quad \sqrt{2}}{4}$

(d) none

41. A circle is inscribed into a rhombus $ABCD$ with one angle 60 . The distance from the centre of the circle to the nearest vertex is equal to 1. If P is any point of the circle, then
$|PA|^2 \quad |PB|^2 \quad |PC|^2 \quad |PD|^2$ is equal to:

(a) 12

(b) 11

(c) 9

(d) none

42. The value of 'c' for which the set, $\{(x, y) | x^2 \quad y^2 \quad 2x \quad 1\} \quad \{(x, y) | x \quad y \quad c \quad 0\}$ contains only one point in common is:

(a) $(\quad , \quad 1] \quad [3, \quad)$

(b) $\{-1, 3\}$

(c) $\{ \quad 3\}$

(d) $\{ \quad 1\}$

43. A tangent at a point on the circle $x^2 \quad y^2 \quad a^2$ intersects a concentric circle C at two points P and Q. The tangents to the circle C at P and Q meet at a point on the circle $x^2 \quad y^2 \quad b^2$ then the equation of circle 'C' is:

(a) $x^2 \quad y^2 \quad ab$

(b) $x^2 \quad y^2 \quad (a \quad b)^2$

(c) $x^2 \quad y^2 \quad (a \quad b)^2$

(d) $x^2 \quad y^2 \quad a^2 \quad b^2$

44. Tangents are drawn to the circle $x^2 \quad y^2 \quad 1$ at the points where it is met by the circles, $x^2 \quad y^2 \quad (\quad 6)x \quad (8 \quad 2 \quad)y \quad 3 \quad 0.$ being the variable. The locus of the point of intersection of these tangents is:

(a) $2x \quad y \quad 10 \quad 0$

(b) $x \quad 2y \quad 10 \quad 0$

(c) $x \quad 2y \quad 10 \quad 0$

(d) $2x \quad y \quad 10 \quad 0$

45. $ABCD$ is a square of unit area. A circle is tangent to two sides of $ABCD$ and passes through exactly one of its vertices. The radius of the circle is:

(a) $2 \quad \sqrt{2}$

(b) $\sqrt{2} \quad 1$

(c) $\frac{1}{2}$

(d) $\frac{1}{\sqrt{2}}$

46. A pair of tangents are drawn to a unit circle with centre at the origin and these tangents intersect at A enclosing an angle of 60 . The area enclosed by these tangents and the arc of the circle is:

(a) $\frac{2}{\sqrt{3}} \quad \frac{}{6}$

(b) $\sqrt{3} \quad \frac{}{3}$

(c) $\frac{}{3} \quad \frac{\sqrt{3}}{6}$ (d) $\sqrt{3} \; 1 \quad \frac{}{6}$

47. A straight line with slope 2 and y-intercept 5 touches the circle, $x^2 \quad y^2 \quad 16x \quad 12y \quad c \quad 0$ at a point Q. Then the coordinates of Q are:

(a) $(\; 6, 11)$ (b) $(\; 9, \; 13)$

(c) $(\; 10, \; 15)$ (c) $(-6, -7)$

48. A variable circle is drawn to touch the x-axis at the origin. The locus of the pole of the straight line $lx \quad my \quad n \quad 0$ w.r.t the variable circle has the equation:

(a) $x(my \quad n) \quad ly^2 \quad 0$ (b) $x(my \quad n) \quad ly^2 \quad 0$

(c) $x(my \quad n) \quad ly^2 \quad 0$ (d) none of these

49. A foot of the normal from the point (4, 3) to a circle is (2, 1) and a diameter of the circle has the equation $2x \quad y \quad 2 \quad 0$. Then the equation of the circle is:

(a) $x^2 \quad y^2 \quad 4y \quad 2 \quad 0$ (b) $x^2 \quad y^2 \quad 4y \quad 1 \quad 0$

(c) $x^2 \quad y^2 \quad 2x \quad 1 \quad 0$ (d) $x^2 \quad y^2 \quad 2x \quad 1 \quad 0$

50. AB is a diameter of a circle. CD is a chord parallel to AB and $2CD \quad AB$. The tangent at B meets the line AC produced at E then AE is equal to:

(a) AB (b) $\sqrt{2}AB$

(c) $2\sqrt{2}AB$ (d) $2\,AB$

51. Points P and Q are 3 units apart. A circle centre at P with a radius of 3 units intersects a circle centre d at Q with a radius $\sqrt{3}$ units at point A and B. The area of the quadrilateral $APBQ$ is:

(a) $\sqrt{99}$ (b) $\frac{\sqrt{99}}{2}$

(c) $\sqrt{\frac{99}{2}}$ (d) $\sqrt{\frac{99}{16}}$

52. The equation of a line inclined at an angle $\frac{}{4}$ to the axis X, such that the two circles $x^2 \quad y^2 \quad 4, x^2 \quad y^2 \quad 10x \quad 14y \quad 65 \quad 0$ intercept equal lengths on it, is:

(a) $2x \quad 2y \quad 3 \quad 0$ (b) $2x \quad 2y \quad 3 \quad 0$

(c) $x \quad y \quad 6 \quad 0$ (d) $x \quad y \quad 6 \quad 0$

53. The equation of the circle symmetric to the circle $x^2 \quad y^2 \quad 2x \quad 4y \quad 4 \quad 0$ about the line $x \quad y \quad 3$ is:

(a) $x^2 \quad y^2 \quad 10x \quad 4y \quad 28 \quad 0$ (b) $x^2 \quad y^2 \quad 6x \quad 8 \quad 0$

(c) $x^2 \quad y^2 \quad 14x \quad 2y \quad 49 \quad 0$ (d) $x^2 \quad y^2 \quad 8x \quad 2y \quad 16 \quad 0$

54. Consider the circles, $x^2 \quad y^2 \quad 25$ and $x^2 \quad y^2 \quad 9$. From the point A (0, 5) two segments are drawn touching the inner circle at the points B and C while intersecting the outer circle at the points D and E. If 'O' is the centre of both the circles then the length of the segment OF that is perpendicular to DE, is:

(a) 7/5 (b) 7/2 (c) 5/2 (d) 3

55. The locus of the center of the circles such that the point (2, 3) is the mid point of the chord $5x \quad 2y \quad 16$ is:

(a) $2x \quad 5y \quad 11 \quad 0$ (b) $2x \quad 5y \quad 11 \quad 0$ (c) $2x \quad 5y \quad 11 \quad 0$ (d) none of these

56. If $a, \frac{1}{a}$, $b, \frac{1}{b}$, $c, \frac{1}{c}$ and $d, \frac{1}{d}$ are four distinct points on a circle of radius 4 units then, $abcd$ is equal to:

(a) 4 (b) 1/4 (c) 1 (d) 16

57. A circle of constant radius 'a' passes through origin 'O' and cuts the axes of coordinates in points P and Q, then the equation of the locus of the foot of perpendicular from O to PQ is:

(a) $(x^2 \quad y^2) \quad \frac{1}{x^2} \quad \frac{1}{y^2} \quad 4a^2$ (b) $(x^2 \quad y^2)^2 \quad \frac{1}{x^2} \quad \frac{1}{y^2} \quad a^2$

(c) $(x^2 \quad y^2)^2 \quad \frac{1}{x^2} \quad \frac{1}{y^2} \quad 4a^2$ (d) $(x^2 \quad y^2) \quad \frac{1}{x^2} \quad \frac{1}{y^2} \quad a^2$

58. If a circle of constant radius $3k$ passes through the origin 'O' and meets coordinate axes at A and B then the locus of the centroid of the triangle OAB is:

(a) $x^2 \quad y^2 \quad (2k)^2$ (b) $x^2 \quad y^2 \quad (3k)^2$

(c) $x^2 \quad y^2 \quad (4k)^2$ (d) $x^2 \quad y^2 \quad (6k)^2$

59. Tangents are drawn from (4, 4) to the circle $x^2 \quad y^2 \quad 2x \quad 2y \quad 7 \quad 0$ to meet the circle at A and B. The length of the chord AB is:

(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) $2\sqrt{6}$ (d) $6\sqrt{2}$

60. Tangents are drawn from any point on the circle $x^2 \quad y^2 \quad R^2$ to the circle $x^2 \quad y^2 \quad r^2$. If the line joining the points of intersection of these tangents with the first circle also touch the second, then R equals:

(a) $\sqrt{2}r$ (b) $2r$ (c) $\frac{2r}{2 \quad \sqrt{3}}$ (d) $\frac{4r}{3 \quad \sqrt{5}}$

61. The complete set of real values of a for which at least one tangent to the parabola $y^2 \quad 4ax$ becomes normal to the circle $x^2 \quad y^2 \quad 2ax \quad 4ay \quad 3a^2 \quad 0$ is:

(a) [1, 2] (b) $[\sqrt{2}, 3]$ (c) R (d) $R \quad \{0\}$

62. A ray of light incident at the point (2, 1) gets reflected from the tangent at (0, 1) to the circle $x^2 \quad y^2 \quad 1$. The reflected ray touches the circle. The equation of the line along which the incident ray moves, is:

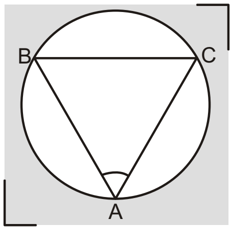
(a) $4x \quad 3y \quad 11 \quad 0$ (b) $4x \quad 3y \quad 11 \quad 0$ (c) $3x \quad 4y \quad 11 \quad 0$ (d) $4x \quad 3y \quad 7 \quad 0$

63. If from a moving point P on $x^2 \quad y^2 \quad 4$ tangents PA and PB are drawn to $x^2 \quad y^2 \quad a^2$, then the locus of the circumcentre of triangle PAB, is $(0 \quad |a| \quad 2)$:

(a) $x^2 \quad y^2 \quad a^2 \quad 4$ (b) $x^2 \quad y^2 \quad 4 \quad a^2$

(c) $x^2 \quad y^2 \quad 1$ (d) $x^2 \quad y^2 \quad 2$

64. Let B and C be two fixed points of a given circle and A a variable point lying on major arc of this circle as shown in figure. The locus of feet of perpendiculars dropped from the midpoint of AB on AC is:



(a) Circle such that BC subtends angle $\tan^{1}(\tan \quad)$ in its circumference

(b) Circle such that BC subtends angle $\tan^{1}(2\tan \quad)$ in its circumference

(c) Circle such that BC subtends angle $2\tan^{1}(\tan \quad)$ in its circumference

(d) Circle such that BC subtends angle $\cot^{1}(3\tan \quad)$ in its circumference

65. For any real k, the circle $x^2 \quad y^2 \quad 2kx \quad 2ky \quad 8 \quad 0$ passes through two fixed points A and B. Locus of the point of intersection of the tangents to the circle at A and B is:

(a) $x^2 \quad 4y$ (b) $y^2 \quad 4x$ (c) $x \quad y$ (d) $x \quad y \quad 0$

66. If a line having y-intercept 'c' makes a chord of length 'a' to the circle $x^2 \quad y^2 \quad a^2$, then:

(a) $4c^2 \quad 3a^2$ (b) $4c^2 \quad 3a^2$ (c) $4c^2 \quad 5a^2$ (d) $c^2 \quad 3a^2$

ANSWERS

1.	(a)	**2.**	(b)	**3.**	(c)	**4.**	(b)	**5.**	(d)	**6.**	(d)	**7.**	(a)	**8.**	(d)	**9.**	(b)	**10.**	(b)
11.	(d)	**12.**	(a)	**13.**	(a)	**14.**	(c)	**15.**	(c)	**16.**	(d)	**17.**	(a)	**18.**	(c)	**19.**	(a)	**20.**	(b)
21.	(b)	**22.**	(b)	**23.**	(b)	**24.**	(c)	**25.**	(a)	**26.**	(d)	**27.**	(d)	**28.**	(b)	**29.**	(a)	**30.**	(a)
31.	(c)	**32.**	(c)	**33.**	(b)	**34.**	(d)	**35.**	(a)	**36.**	(b)	**37.**	(c)	**38.**	(c)	**39.**	(d)	**40.**	(c)
41.	(b)	**42.**	(d)	**43.**	(a)	**44.**	(a)	**45.**	(a)	**46.**	(b)	**47.**	(d)	**48.**	(a)	**49.**	(c)	**50.**	(d)
51.	(b)	**52.**	(a)	**53.**	(a)	**54.**	(a)	**55.**	(a)	**56.**	(c)	**57.**	(c)	**58.**	(a)	**59.**	(b)	**60.**	(b)
61.	(d)	**62.**	(b)	**63.**	(c)	**64.**	(b)	**65.**	(c)	**66.**	(a)								

EXERCISE 2

One or More than One is/are Correct

1. If a circle passes through the point $3, \sqrt{\frac{7}{2}}$ and touches $x \quad y \quad 1$ and $x \quad y \quad 1$, then the centre of the circle is at:

(a) $(4, 0)$ (b) $(4, 2)$

(c) $(6, 0)$ (d) $(7, 0)$

2. Equations of four circles are $(x \quad a)^2 \quad (y \quad a)^2 \quad a^2$, then:

(a) The radius of the greatest circle touching all the four circles is $(\sqrt{2} \quad 1)\, a$

(b) The radius of the smallest circle touching all the four circles is $(\sqrt{2} \quad 1)\, a$

(c) Area of region enclosed by four given circles is $(4 \quad)\, a^2$ sq. units

(d) The centres of four circles are the vertices of a square

3. The equation of the circle which touches the axes of coordinates and the line $\frac{x}{3} \quad \frac{y}{4} \quad 1$ and whose centre lies in the first quadrant is $x^2 \quad y^2 \quad 2rx \quad 2ry \quad r^2 \quad 0$, then r can be equal to:

(a) 1 (b) 2 (c) 3 (d) 6

4. If $(a, 0)$ is a point on a diameter of the circle $x^2 \quad y^2 \quad 4$, then $x^2 \quad 4x \quad a^2 \quad 0$ must have:

(a) exactly one real root in $\frac{9}{10}, \frac{1}{10}$

(b) exactly one real root in $4, \frac{49}{10}$

(c) exactly one real root in $[0, 2]$

(d) two distinct real roots in $[\quad 1, 5]$

5. The locus of points of intersection of the tangents to $x^2 \quad y^2 \quad a^2$ at the extremeties of a chord of circle $x^2 \quad y^2 \quad a^2$ which touches the circle $x^2 \quad y^2 \quad 2ax \quad 0$ is/are:

(a) $y^2 \quad a(a \quad 2x)$ (b) $x^2 \quad a(a \quad 2y)$

(c) $x^2 \quad y^2 \quad (x \quad a)^2$ (d) $x^2 \quad y^2 \quad (y \quad a)^2$

6. The tangent drawn from the origin to the circle $x^2 \quad y^2 \quad 2gx \quad 2fy \quad f^2 \quad 0$ are perpendicular, if:

(a) $g \quad f$ (b) $g \quad f$

(c) $g \quad 2f$ (d) $2g \quad f$

7. Two chords are drawn from the point $P(h,k)$ on the circle $x^2 \quad y^2 \quad hx \quad ky$. If the y-axis divides both the chords in the ratio 2 : 3, then which of the following may be correct ?

(a) $k^2 \quad 15h^2$ (b) $15k^2 \quad h^2$

(c) $h^2 \quad 15k^2$ (d) $k^2 \quad 5h^2$

8. The equation(s) of the tangent at the point (0, 0) to the circle, making intercepts of lengths $2a$ and $2b$ units on the coordinates axes, is/are:

(a) $ax \quad by \quad 0$ (b) $ax \quad by \quad 0$

(c) $x \quad y$ (d) $bx \quad ay \quad 0$

9. Consider the circle $x^2 \quad y^2 \quad 10x \quad 6y \quad 30 \quad 0$. Let O be the centre of the circle and tangent at $A(7,3)$ and $B(5,1)$ meet at C. Let $S \quad 0$ represents family of circles passing through A and B, then:

(a) Area of quadrilateral $OACB \quad 4$

(b) the radical axis for the family of circles $S \quad 0$ is $x \quad y \quad 10$

(c) the smallest possible circle of the family $S \quad 0$ is $x^2 \quad y^2 \quad 12x \quad 4y \quad 38 \quad 0$

(d) the coordinates of point C are $(7,1)$

10. Let x, y be real variable satisfying the $x^2 \quad y^2 \quad 8x \quad 10y \quad 40 \quad 0$. Let $a \quad \max(\sqrt{(x \quad 2)^2 \quad (y \quad 3)^2})$ and $b \quad \min(\sqrt{(x \quad 2)^2 \quad (y \quad 3)^2})$, then:

(a) $a \quad b \quad 18$ (b) $a \quad b \quad 4\sqrt{2}$

(c) $a \quad b \quad 4\sqrt{2}$ (d) $ab \quad 73$

11. Coordinates of the centre of a circle, whose radius is 2 unit and which touches the line pair $x^2 \quad y^2 \quad 2x \quad 1 \quad 0$ are:

(a) (4, 0) (b) $(1 \quad 2\sqrt{2}, 0)$

(c) (4, 1) (d) $(1, 2\sqrt{2})$

12. Point M moved on the circle $(x \quad 4)^2 \quad (y \quad 8)^2 \quad 20$. Then it broke away from it and moving along a tangent to the circle, cuts the x-axis at the point $(\quad 2, 0)$. The coordinates of a point on the circle at which the moving point broke away is:

(a) $\frac{3}{5}, \frac{46}{5}$ (b) $\frac{2}{5}, \frac{44}{5}$

(c) (6, 4) (d) (3, 5)

13. If the area of the quadrilateral formed by the tangents from the origin to the circle $x^2 \quad y^2 \quad 6x \quad 10y \quad c \quad 0$ and the radii corresponding to the points of contact is 15, then a value of c is:

(a) 9 (b) 4

(c) 5 (d) 25

14. If $4a^2 \quad 5b^2 \quad 6a \quad 1 \quad 0$ and the line $ax \quad by \quad 1 \quad 0$ touches a fixed circle, then:

(a) centre of circle is at (3, 0)

(b) the radius of circle is $\sqrt{5}$

(c) the radius of circle is $\sqrt{3}$

(d) the circle passes through $(1, 1)$

15. Let $P(1, 2\sqrt{2})$ is a point on circle $x^2 \quad y^2 \quad 9$. Locate the points on the given circle, which are at 2 units distance from point P.

(a) $(\quad 1, 2\sqrt{2})$

(b) $(2\sqrt{2}, 1)$

(c) $\frac{23}{9}, \frac{10\sqrt{2}}{9}$

(d) $(3, 0)$

16. AC is diameter of circle. AB is a tangent. BC meets the circle again at D. $AC \quad 1$, $AB \quad a$, $CD \quad b$, then:

(a) $ab \quad 1$

(b) $ab \quad 1$

(c) $\frac{b}{a} \quad \frac{1}{a^2 \quad \frac{1}{2}}$

(d) $\frac{b}{a} \quad \frac{1}{a^2 \quad \frac{1}{2}}$

17. Equation of line that touches the curves $|y| \quad x^2$ and $x^2 \quad (y \quad 2)^2 \quad 4$ where $x \quad 0$ is:

(a) $y \quad 4\sqrt{5}x \quad 20$

(b) $y \quad 4\sqrt{3}x \quad 12$

(c) $y \quad\quad 4\sqrt{5}x \quad 20$

(d) $y \quad\quad 4\sqrt{5}x \quad 20$

18. Let l_1, l_2 and l_3 are the lengths of the tangents drawn from a variable point P to the circle $x^2 \quad y^2 \quad a^2$; $x^2 \quad y^2 \quad 2ax$ and $x^2 \quad y^2 \quad 2ay$ respectively. The lengths satisfy the relation $l_1^4 \quad l_2^2 l_3^2 \quad a^4$. Then the locus of P can be:

(a) Line

(b) Circle

(c) Parabola

(d) Hyperbola

ANSWERS

1.	(a, c)	**2.**	(a, b, c, d)	**3.**	(a, d)	**4.**	(a, b, d)	**5.**	(a, c)	**6.**	(a, b)
7.	(a, b, d)	**8.**	(a, b, d)	**9.**	(a, c, d)	**10.**	(a, c, d)	**11.**	(b, d)	**12.**	(b, c)
13.	(a, d)	**14.**	(a, b, d)	**15.**	(a, c)	**16.**	(b, c)	**17.**	(a, b, c)	**18.**	(a, b)

EXERCISE 3

Comprehension: (1)

An altitude *BD* and a bisector *BE* are drawn in the triangle *ABC* from the vertex *B*. It is known that the length of side *AC* 1, and the magnitudes of the angles *BEC*, *ABD*, *ABE*, *BAC* form an arithmetic progression.

1. The area of circle circumscribing *ABC* is:

(a) $\frac{\ }{8}$ (b) $\frac{\ }{4}$

(c) $\frac{\ }{2}$ (d)

2. Let '*O*' be the circumcentre of *ABC*, the radius of circle inscribed in *BOC* is :

(a) $\frac{1}{8\sqrt{3}}$ (b) $\frac{1}{4\sqrt{3}}$

(c) $\frac{1}{2\sqrt{3}}$ (d) $\frac{1}{2}$

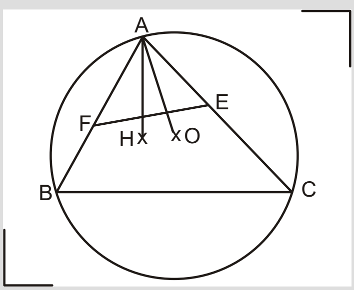
3. Let *B*' be the image of point *B* with respect to side *AC* of *ABC*, then the length *BB*' is equal to :

(a) $\frac{\sqrt{3}}{4}$ (b) $\frac{\sqrt{2}}{4}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

Comprehension: (2)

Triangle *ABC* has circumcentre '*O*' and orthocentre '*H*'. Points *E* and *F* are chosen on sides *AC* and *AB* respectively such that *AE* *AO* and *AF* *AH* as shown in figure. Let '*R*' be the radius of circle circumscribing the triangle *ABC*



1. The area of quadrilateral $BFEC$ is equal to:

(a) $\frac{R^2}{4}(\sin B \quad \sin C)$

(b) $\frac{R^2}{8}\sin A\cos(B \quad C)$

(c) $\frac{R^2}{2}\sin 2A$

(d) $\frac{R^2}{2}(\sin 2B \quad \sin 2C)$

2. The length of segment EF is equal to:

(a) $R\cos A$

(b) R

(c) $R\sin 2A$

(d) $2R\cos A$

3. Let $AFE \quad \frac{}{3}$, then the sum of squares of the 3 altitudes of triangle AFE is equal to:

(a) $\frac{9R^2}{4}$

(b) $3R^2$

(c) $\frac{3R^2}{2}$

(d) $\frac{8R^2}{3}$

Comprehension: (3)

Let S_1 and S_2 be two fixed externally tangent circles with radius 2 and 3 respectively. Let S_3 be a variable circle internally tangent to both S_1 and S_2 at points A and B respectively. The tangents to S_3 at A and B meet at T, and $TA \quad 4$.

1. The radius of circle S_3 is equal to:

(a) 2

(b) 4

(c) 6

(d) 8

2. The area of circle circumscribing TAB is:

(a) 10

(b) 20

(c) 40

(d) 80

3. Let C_1, C_2, C_3 be centres of circles S_1, S_2, S_3 respectively then which of the following must be true:

(a) $C_3C_1 \quad C_3C_2 \quad 5$

(b) $C_3C_1 \quad C_3C_2 \quad 3$

(c) $C_3C_1 \quad C_3C_2 \quad 3$

(d) $C_3C_1 \quad C_3C_2 \quad 1$

Comprehension: (4)

Let A, B, C, D lie on a line such that $AB \quad BC \quad CD \quad 1$. The points A and C are also joined by a semicircle with AC as diameter and P is a variable point on this semicircle such that $PBD \quad , 0 \quad$. Let R is the region bounded by arc AP, the straight line PD and line AD.

1. The maximum possible area of region R is:

(a) $\dfrac{2 \quad \sqrt{3}}{12}$ (b) $\dfrac{2 \quad 3\sqrt{3}}{6}$

(c) $\dfrac{(2 \quad 6\sqrt{3})}{12}$ (d) $\dfrac{2 \quad 3\sqrt{3}}{12}$

2. Let 'L' is the perimeter of region 'R', then L is equal to:

(a) $3 \quad \sqrt{5 \quad 4\cos}$ (b) $3 \quad \sqrt{5 \quad 4\cos}$

(c) $3 \quad \sqrt{5 \quad 4\cos}$ (d) $3 \quad \sqrt{5 \quad 4\cos}$

3. The non negative difference of greatest and least values of L is:

(a) $3 \quad \sqrt{3} \quad \dfrac{}{3}$ (b) $\sqrt{3} \quad 3 \quad \dfrac{}{3}$

(c) $\sqrt{3} \quad 3 \quad \dfrac{2}{3}$ (d) 2

Comprehension: (5)

Let $P(a,b)$ be a variable point satisfying $4 \quad a^2 \quad b^2 \quad 9$ and $b^2 \quad 4ab \quad a^2 \quad 0$. Let R be the complete equation represented in x-y plane in which P can lie.

1. Area of region R is:

(a) $\dfrac{2}{3}$ (b)

(c) $\dfrac{4}{3}$ (d) $\dfrac{5}{3}$

2. Minimum value of $|a \quad b|$ for all positions of P lying in region R is:

(a $\sqrt{3}$ (b) $2\sqrt{3}$

(c) $\sqrt{6}$ (d) $2\sqrt{6}$

3. Let A and B be two points in first quadrant lying in region R, then maximum possible distance between them is:

(a) 2 (b) $\sqrt{5}$

(c) $\sqrt{6}$ (d) $\sqrt{7}$

Comprehension: (6)

Let $f(x,y) \quad 0$ be the equation of a circle such that $f(0,y) \quad 0$ has equal real roots and $f(x,0) \quad 0$ has two distinct real roots. Let $g(x,y) \quad 0$ be the locus of points 'P' from where tangents to circle $f(x,y) \quad 0$ make angle $/3$ between them and $g(x,y) \quad x^2 \quad y^2 \quad 5x \quad 4y \quad c, c \quad R$.

1. Let Q be a point from where tangents drawn to circle $g(x, y)$ 0 are mutually perpendicular.If A, B are the points of contact of tangent drawn from Q to circle $g(x, y)$ 0, then area of triangle QAB is

(a) $\frac{25}{12}$ (b) $\frac{25}{8}$

(c) $\frac{25}{4}$ (d) $\frac{25}{2}$

2. The area of region bounded by circle $f(x, y)$ 0 with x-axis in the first quadrant is

(a) $3 \quad \frac{25}{8} \quad \tan^{1}\frac{1}{2}$ (b) $3 \quad \frac{25}{4}\tan^{1}\frac{24}{11}$

(c) $3 \quad \frac{25}{8} \quad 2 \quad \tan^{1}\frac{3}{4}$ (d) $3 \quad \frac{25}{8} \quad 2 \quad \tan^{1}\frac{24}{7}$

3. The number of points with positive integral coordinates satisfying $f(x, y)$ 0, $g(x, y)$ 0; y 3 and x 6 is

(a) 7 (b) 8

(c) 10 (d) 11

Comprehension: (7)

Consider two circles S_1 and S_2 (externally touching) having centres at points A and B whose radii are 1 and 2 respectively. A tangent to circle S_1 from point B intersects the circle S_1 at point C.D is chosen on circle S_2 so that AC is parallel to BD and the two segments BC and AD do not intersect. Segment AD intersect the circle S_1 at E. The line through B and E intersects the circle S_1 at another point F.

1. The length of segment EF is:

(a) $2\sqrt{2}$ (b) $\frac{2\sqrt{2}}{3}$

(c) $\frac{2\sqrt{3}}{3}$ (d) $\sqrt{3}$

2. The area of triangle ABD is:

(a) 2 (b) $\sqrt{3}$

(c) $2\sqrt{2}$ (d) $\sqrt{5}$

3. The length of the segment DE is:

(a) $\sqrt{3}$ (b) $2\sqrt{2}$

(c) 2 (d) 3

Comprehension: (8)

In an acute triangle *ABC*, point *H* is the intersection point of altitude *CE* to *AB* and altitude *BD* to *AC*. A circles with *DE* as its diameter intersects *AB* and *AC* at points *F* and *G* respectively. If *BC* 25, *BD* 20 and *BE* 7.

1. The sum of the length of all the sides of *ABC* is:

(a) 65 (b) 70

(c) 75 (d) 80

2. Area of the circle *S* is:

(a) 100 (b) 196

(c) $\frac{225}{4}$ (d) 400

3. Let *FG* and *AH* intersect at point *K*, then the length of *AK*

(a) $\frac{192}{25}$ (b) $\frac{216}{25}$

(c) $\frac{225}{24}$ (d) 9

Comprehension: (9)

The circle '*S*' touches the sides *AB* and *AD* of the rectangle *ABCD* and cuts the side *DC* at single point *F* and the side *BC* at a single point *E*. If $|AB|$ 32, $|AD|$ 40 and $|BE|$ 1.

1. The angle between pair of tangents drawn from the point *D* to the circle '*S*' is:

(a) $\tan^{1}\frac{25}{8}$ (b) $\tan^{1}\frac{15}{7}$

(c) $\tan^{1}\frac{15}{8}$ (d) $\frac{}{2}\ \tan^{1}\frac{5}{3}$

2. The area of trapezoid *AFCB* is:

(a) 960 (b) 1020

(c) 1140 (d) 1180

3. The radius of circle is:

(a) 22 (b) 23

(c) 25 (d) 27

Comprehension: (10)

A point $P(\quad,\quad)$ is called rational point if both and are rational numbers and if both and are integers, then point $P(\quad,\quad)$ is called lattice point.

1. Line $x \quad y \quad n(n \quad N)$ cuts coordinates axes at A and B. If O is origin then number of lattice points within the triangle AOB is:

(a) $\frac{n^2 \quad 4n \quad 3}{2}$ (b) $\frac{n^2 \quad n}{2}$

(c) $\frac{n^2 \quad n}{2}$ (d) $\frac{n^2 \quad 3n \quad 2}{2}$

2. The number of rational points that could be on a circle, whose centre is $(2, \sqrt{3})$ and radius is $2\sqrt{3}$ units, is :

(a) 1 (b) 2 (c) 3 (d) Infinite

3. Number of lattice points on the circumference of circle $x^2 \quad y^2 \quad 25$ is:

(a) 14 (b) 8 (c) 12 (d) 10

Comprehension: (11)

Consider a right triangle *ABC* right angled at vertex *B* with *AB* 3 and *BC* 4. A circle '*S*' touching the side *BC* is drawn intersecting the sides *AB* at points *D* and *E* and the side *AC* at points *F* and *G* respectively. Also *D* and *F* are the midpoints of sides *AB* and *AC* respectively, then:

1. Radius of circle S is:

(a) 1 (b) $\frac{5}{4}$

(c) $\frac{7}{6}$ (d) $\frac{13}{12}$

2. The length of portion FG of hypotenuse AC is:

(a) $\frac{5}{2}$ (b) $\frac{13}{10}$

(c) $\frac{11}{10}$ (d) 1

3. The length of chord EF of circle S is:

(a) $\frac{17}{6}$ (b) $\frac{5}{2}$

(c) $\frac{13}{6}$ (d) $\frac{11}{6}$

Comprehension: (12)

Given a line segment $AB, A \quad (0,0)$ and $B(a,0)$. Three circles S_1, S_2, S_3, of radius R are centred at the end points and the midpoint of the line segment AB. A fourth circles S_4 is drawn touching the 3 given circles.

1. If $0 \quad R \quad \frac{a}{4}$, then sum of all possible distinct values of radius of S_4 is:

(a) $\frac{a^2}{16R}$ (b) $\frac{a^2}{7R}$ (c) $\frac{3a^2}{16R}$ (d) $\frac{a^2}{4R}$

2. If $0 \quad R \quad \frac{a}{4}$, then number of possible circles S_4 is:

(a) 2 (b) 4 (c) 6 (d) 8

3. If $\frac{a}{4} \quad R \quad \frac{a}{2}$, then radius of circle S_4 is:

(a) $\frac{a^2}{16R}$ (b) $\frac{a^2}{8R}$

(c) $\frac{3a^2}{16R}$ (d) $\frac{a^2}{4R}$

Comprehension: (13)

Let $A \quad (0,0), B \quad (4,0)$ and on segment AB is given a point M. On the same side of AB squares $AMCD$ and $BMFE$ are constructed above AB. The circumcircles S_1 and S_2 of two squares $AMCD$ and $BMFE$ respectively whose centres are P and Q, intersect in M and another point N.

1. The point of intersection of the lines FA and BC is:

(a) N (b) outside S_1 but inside S_2

(c) outside S_2 but inside S_1 (d) inside S_1 and S_2 both

2. For all positions of M varying along the segment AB, the lines MN passes through the fixed point $R(a,b)$, then $a \quad b$

(a) 0 (b) 1 (c) $\sqrt{3}$ (d) 2

3. The locus of midpoints of all segments PQ as M varies along the segment AB is:

(a) line segment $(x,1), x \quad [1,3]$ (b) line segment $(x,1), x \quad [2,4]$

(c) line segment $(x,1), x \quad [0,3]$ (d) line segment $(x,1), x \quad [0,4]$

Comprehension: (14)

Let C be a circle of radius r with centre at O, let P be a point outside C and D be a point on C. A line through P intersects C at Q and R, S is the midpoint of QR.

1. For different choices of lines through P, what is the curve on which S lies:
(a) a straight line
(b) an arc of circle with P as centre
(c) an arc of circle with PS as diameter
(d) an arc of circle with OP diameter

2. Let P is situated at a distance 'd' from centre O, then which of the following does not equal the product $(PQ)(PR)$.
(where T is a point on C and PT is tangent to C)
(a) $d^2 \quad r^2$
(b) $(PT)^2$
(c) $(PS)^2 \quad (QS)(RS)$
(d) $(PS)^2$

3. Let ABC be an equilateral triangle inscribed in C. If , , denote the distances of D from vertices A, B, C respectively, what is the value of product ( )( )( ):
(a) 0
(b) $\frac{\quad}{8}$
(c) $\frac{\quad^3 \quad^3 \quad^3 \quad 3}{6}$
(d)

Comprehension: (15)

Given a line segment AB, $A \quad (0,0)$ and $B(a,0)$. Three circles S_1, S_2, S_3 of radius R are centred at the end points and the midpoint of the line segment AB. A fourth circle S_4 is drawn touching the 3 given circles.

1. If $0 \quad R \quad \frac{a}{4}$, then sum of all possible distinct values of radius of S_4 is:
(a) $\frac{a^2}{16R}$
(b) $\frac{a^2}{7R}$
(c) $\frac{3a^2}{16R}$
(d) $\frac{a^2}{4R}$

2. If $0 \quad R \quad \frac{a}{4}$, then number of possible circle S_4 is:
(a) 2
(b) 4
(c) 6
(d) 8

3. If $\frac{a}{4} \quad R \quad \frac{a}{2}$, then radius of circle S_4 is:
(a) $\frac{a^2}{16R}$
(b) $\frac{a^2}{8R}$
(c) $\frac{3a^2}{16R}$
(d) $\frac{a^2}{4R}$

Comprehension: (16)

The line $y \quad ax \quad b$ intersects the curve $C: x^2 \quad y^2 \quad 6x \quad 10y \quad 1 \quad 0$ at the points A and B. If the lines segment AB subtends a right angle at origin then the locus of the point (a, b) is the curve $g(x, y) \quad 0$.

1. The equation of curve $g(x, y) \quad 0$ is:

(a) $x^2 \quad 2y^2 \quad 6xy \quad 10y \quad 1 \quad 0$
(b) $x^2 \quad 2y^2 \quad 6xy \quad 10y \quad 1 \quad 0$
(c) $x^2 \quad 2y^2 \quad 6xy \quad 10y \quad 1 \quad 0$
(d) $x^2 \quad 2y^2 \quad 6xy \quad 10y \quad 1 \quad 0$

2. The slope of tangent to the curve $g(x, y) \quad 0$ at the point where the line $y \quad 1$ intersects it in first quadrant is:

(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/6

3. The equation of chord of the curve C whose middle point is (1, 2) is:

(a) $4x \quad 3y \quad 2 \quad 0$ (b) $4x \quad 3y \quad 10 \quad 0$ (c) $3x \quad 4y \quad 11 \quad 0$ (d) $3x \quad 4y \quad 5 \quad 0$

ANSWERS

Comprehension-1:	1.	(b)	2.	(b)	3.	(d)
Comprehension-2:	1.	(d)	2.	(b)	3.	(a)
Comprehension-3:	1.	(d)	2.	(b)	3.	(d)
Comprehension-4:	1.	(b)	2.	(c)	3.	(d)
Comprehension-5:	1.	(d)	2.	(c)	3.	(d)
Comprehension-6:	1.	(d)	2.	(d)	3.	(d)
Comprehension-7:	1.	(c)	2.	(c)	3.	(c)
Comprehension-8:	1.	(d)	2.	(c)	3.	(b)
Comprehension-9:	1.	(c)	2.	(d)	3.	(c)
Comprehension-10:	1.	(d)	2.	(b)	3.	(c)
Comprehension-11:	1.	(d)	2.	(c)	3.	(c)
Comprehension-12:	1.	(c)	2.	(c)	3.	(a)
Comprehension-13:	1.	(a)	2.	(a)	3.	(a)
Comprehension-14:	1.	(d)	2.	(d)	3.	(a)
Comprehension-15:	1.	(c)	2.	(c)	3.	(a)
Comprehension-16:	1.	(b)	2.	(d)	3.	(a)

EXERCISE 4

Assertion and Reason

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1

(B) Statement-1 is true, Statement-2 is true and Statement-2 is not correct explanation for Statement-1

(C) Statement-1 is true, Statement-2 is false

(D) Statement-1 is false, Statement-2 is true

1. Consider 3 equal circles of radius r_1 within a circle of radius r_2 each to touch the other two and the given circle.

Statement-1: $\frac{r_1}{r_2} \quad \frac{\sqrt{3}}{\sqrt{3} \quad 1}$

because

Statement-2: Incentre of triangle formed by joining centres of 3 equal circles is same as centre of given circle.

2. Let a circle be inscribed in the quadrant of a circle of diameter 4, then

Statement-1: The radius of inscribed circle is the positive root of the equation $r^2 \quad 4r \quad 4 \quad 0$

because

Statement-2: Distance between their centres = 2 (radius of circle inscribed).

3. Let A and B are two points both lying within a given circle S, and 'P' be a point on circumference of 'S' at which AB subtends the greatest angle.

Statement-1: If $A \quad (1, 1), B \quad (1, \quad 1)$ and equation of S is $x^2 \quad y^2 \quad 4$ then P will be (2,0)

because

Statement-2: 'P' will be the point where a circle passing through A and B touches the circle S.

4. Let PQ be fixed chord in a circle 'S' and AB is any diameter, then

Statement-1: If PQ is represented by equation $y \quad 1$, S represented by $x^2 \quad y^2 \quad 4$ and A, B lie on same side of PQ then the sum of the perpendiculars let fall from A and B on PQ is equal to 4.

because

Statement-2: The sum of the perpendiculars let fall from A and B on PQ is same for all position of AB.

5. If two equal chords AB and CD of a circle intersect at point P,

Statement-1: if ACB and CBD are minor segments of AB and CD. $AP \quad 2$ and $PB \quad 7$, then $PC \quad 7$ and $PD \quad 2$.

because

Statement-2: $AP \quad PD$ and $PB \quad PC$.

6. AB is a fixed chord of a circle and XY is any chord having its middle point Z on AB, then

Statement-1: XY is greatest if XY coincides with AB

because

Statement-2: XY is greatest if Z becomes the middle point of AB.

7. Let the bisector of $\quad A$ of $\quad ABC$ meets BC in D and the circumcircle of $\quad ABC$ in E, then

Statement-1: AD is less than the G..M. of AB and AC

because

Statement-2: $\quad ABD$ is similar to $\quad AEC$.

8. Let $ABCD$ is a rectangle, the diagonal AC and BD intersect at O. A straight line through B intersects DC produced at E and DA produced at F such that $OE \quad OF$, then

Statement-1: $(CE)(DE) \quad (AF)(DF)$

because

Statement-2: If a line through any point P intersects the circle with centre at 'O' and radius 'r' at Q and R then $(PQ)(PR) \quad (OP)^2 \quad r^2$.

9. Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle ABC.

Statement-1: If angle C is obtuse then the quantity $(x_3 \quad x_1)(x_3 \quad x_2) \quad (y_3 \quad y_1)(y_3 \quad y_2)$ is negative.

because

Statement-2: Diameter of a circle subtends obtuse angle at any point lying inside the semicircle.

10. Let C be a circle with centre 'O' and HK is the chord of contact of tangents drawn from a point A. OA intersects the circle 'C' at P and Q and B is the midpoint of HK, then

Statement-1: AB is the Harmonic mean of AP and AQ

because

Statement-2: AK is the Geometric mean of AB and AO and OA is the arithmetic mean of AP and AQ.

11. Let the diagonals of a convex quadrilateral $ABCD$ intersect at point P and a, b, c, d denote the length of sides AB, BC, CD and DA respectively, then

Statement-1: Diagonals of quadrilateral $ABCD$ are perpendicular if $a^2 \quad c^2 \quad b^2 \quad d^2$

because

Statement-2: $a^2 \quad c^2 \quad b^2 \quad d^2 \quad (AP)^2 \quad (BP)^2 \quad (CP)^2 \quad (DP)^2$

or $\quad b^2 \quad d^2 \quad a^2 \quad c^2 \quad (AP)^2 \quad (BP)^2 \quad (CP)^2 \quad (DP)^2$

ANSWERS

1.	(d)	**2.**	(c)	**3.**	(a)	**4.**	(c)	**5.**	(c)	**6.**	(d)	**7.**	(a)	**8.**	(c)	**9.**	(a)	**10.**	(a)
11.	(c)																		

EXERCISE 5

Match the Columns:

1. Let E, F, G, H be 4 distinct points inside square $ABCD$ whose area is 1 square units such that $EDC \quad ECD \quad HDA \quad HAD = \quad GAB \quad GBA \quad FBC \quad FCB \quad 15$

	Column-I		Column-II
(a)	If area of quadrilateral $EFGH$ is equal to $a \quad \sqrt{b}$ where $a, b \quad N$, then $a \quad b$	**(p)**	1
(b)	Let $AEB \quad \frac{}{k}$; then k	**(q)**	3
(c)	The radius of circle circumscribing the AHD is equal to	**(r)**	5
(d)	Let the lengths of perpendiculars from vertices G, A, H to opposite sides of triangle AHG be h_1, h_2, h_3 respectively. Let $\frac{1}{h_1^2} \quad \frac{1}{h_2^2} \quad \frac{1}{h_3^2} \quad a \quad \sqrt{b}$ where $a, b \quad N$, then $\frac{b}{a}$	**(s)**	6

2. Let $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ be 3 distinct points lying on circle $S: x^2 \quad y^2 \quad 1$, such that $x_1x_2 \quad y_1y_2 \quad x_2x_3 \quad y_2y_3 \quad x_3x_1 \quad y_3y_1 \quad \frac{3}{2}$

	Column-I		Column-II
(a)	Let P be any arbitrary point lying on S, then $(PA)^2 \quad (PB)^2 \quad (PC)^2$	**(p)**	3
(b)	Let the perpendicular dropped from point 'A' to BC meets S at Q and $OBQ \quad \frac{}{k}$, where 'O' is origin, then k	**(q)**	4
(c)	Let R be the point lying on line $x \quad y \quad 2$, at the minimum distance from S and the square of maximum distance of R from S is $a \quad b\sqrt{b}$, then $a \quad b$	**(r)**	5
(d)	Let I and G represent incenter and centroid of ABC respectively, then $IA \quad IB \quad IC \quad GA \quad GB \quad GC$	**(s)**	6

3. In the triangle ABC, the angle bisector AK is perpendicular to the median BM and $\quad ABC \quad 120 \quad$,

	Column-I		Column-II
(a)	The value of ratio $\frac{BC}{AB}$ is equal to	**(p)**	$\frac{3\sqrt{3}}{32}(\sqrt{13} \quad 1)$
(b)	The value of ratio of radius of the circle circumscribing the triangle ABC to the side length AB is equal to	**(q)**	$\frac{\sqrt{13} \quad 1}{2}$
(c)	The ratio of the area of $\quad ABC$ to the area of the circle circumscribing $\quad ABC$ is equal to	**(r)**	$\frac{1}{2}$
(d)	The value of ratio of the sides AB to AC is equal to	**(s)**	$\frac{2}{\sqrt{3}}$

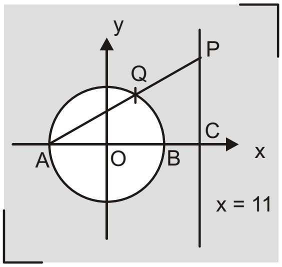
4. There are two circles in a parallelogram. One of them of radius 3, is inscribed in the parallelogram, and the other touches two sides of the parallelogram and the first circle. The distance between the points of tangency which lie on the same side of the parallelogram is equal to 3.

	Column-I		Column-II
(a)	The radius of the other circle is	**(p)**	$\frac{75}{2}$
(b)	Area of the parallelogram is equal to	**(q)**	75
(c)	Let d_1, d_2 denote the lengths of the diagonals of parallelogram, then the product $d_1 d_2$ is equal to	**(r)**	$5\sqrt{31}$
(d)	Let d_1, d_2 be the diagonals of the parallelogram then the value of $d_1 \quad d_2$ is equal to	**(s)**	$\frac{3}{4}$

5. In the parallelogram $ABCD$ with angle $A \quad 60$, the bisector of angle B is drawn which cuts the side CD at a point E. A circle S_1 of radius 'r' is inscribed in the $\quad ECB$. Another circle 'S_2' is inscribed in the trapezoid $ABED$.

	Column-I		Column-II
(a)	The value of radius of S_2 is	**(p)**	$2\sqrt{3}\,r$
(b)	The value of distance between the centres of S_1 and S_2 is	**(q)**	$\frac{\sqrt{3}}{2}r$
(c)	The value of the length of internal common tangent of S_1 and S_2 is	**(r)**	$\sqrt{7}\,r$
(d)	The value of the length CE is	**(s)**	$\frac{3}{2}r$

6. In the given figure, the circle $x^2 \quad y^2 \quad 25$ intersects x-axis at points A and B. The line $x \quad 11$ intersects x-axis, at point C. Point P moves along the line $x \quad 11$ above the x-axis and AP intersects the circle at Q.



	Column-I		Column-II
(a)	The coordinates of point P if the $\quad AOB$ has the maximum area is	**(p)**	(11, 0)
(b)	The coordinates of point P if Q is the middle point of AP is	**(q)**	(11, 8)
(c)	The co-ordinates of P if the area of $\quad AQB$ is $\frac{1}{4}^{th}$ of the area of $\quad APC$ is	**(r)**	(11, 12)
(d)	The co-ordinates of P if $\lvert AP \quad BP \rvert$ is maximum	**(s)**	(11, 16)

7.

	Column-I		Column-II
(a)	Two intersecting circles	**(p)**	have a common tangent
(b)	Two circles touching each other	**(q)**	have a common normal
(c)	Two non concentric circles, one strictly inside the other	**(r)**	do not have a common normal
(d)	Two concentric circles of different radii	**(s)**	do not have a radical axis

8. Match the following : Let C and C_1 be circles of radii 1 and $r \quad 1$ respectively touching the coordinate axes, Column-II gives values of r for the conditions in Column-I .

	Column-I		Column-II
(a)	C passes through the centre of C_1	**(p)**	3
(b)	C and C_1 touch each other	**(q)**	$\frac{2 \quad \sqrt{2}}{2}$
(c)	C and C_1 are orthogonal	**(r)**	$2 \quad \sqrt{3}$
(d)	C and C_1 have longest common chord	**(s)**	$3 \quad 2\sqrt{2}$

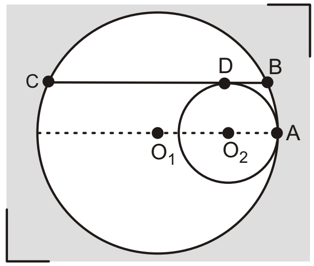
9. Triangles *ABC* are described on a given base *BC* and of a given vertical angle .

	Column-I		Column-II
(a)	The locus of orthocentre of *ABC* is	**(p)**	Part of circle such that *BC* subtend angle t its circumference
(b)	The locus of incentre of *ABC* is	**(q)**	Part of circle such that *BC* subtend angle $\frac{}{2} \quad \frac{}{2}$ at its circumference
(c)	The locus of the excentre corresponding to vertex opposite to base of *ABC* is	**(r)**	Part of circle such that *BC* subtend angle $\frac{}{2} \quad \frac{}{2}$ at its circumference
(d)	The locus of centroid of *ABC* is:	**(s)**	Part of circle such that *PQ* subtend angle at its circumference where *P*, *Q* are points of trisection of segment *BC*.

10. Match the column:

	Column-I		Column-II
(a)	Line *L* is the radical axis of the circles $S_1 \quad x^2 \quad y^2 \quad 2x \quad 2y \quad 7 \quad 0$ and $S_2 \quad x^2 \quad y^2 \quad 6x \quad 8y \quad 0$. If (x_1, y_1) and (x_2, y_2) denote the coordinates of the extremeties of the diameter of S_2 which is perpendicular to *L*, then $\frac{1}{5}(x_1^2 \quad x_2^2 \quad y_1^2 \quad y_2^2)$ is equal to	**(p)**	13
(b)	The pair of lines represented by $x^2 \quad y^2 \quad 3xy \quad 4x \quad y \quad 1 \quad 0$ intersect at *P*. If *Q* and *R* are the point of intersection of the pair of lines with the *x*-axis and the area of the *PQR* is , then ^{2}	**(q)**	20
(c)	If the coordinates of radical centre of circles $x^2 \quad y^2 \quad 25 \quad 0$; $2x^2 \quad 2y^2 \quad 4x \quad 6y \quad 7 \quad 0$; $x^2 \quad y^2 \quad x \quad 2y \quad 9 \quad 0$ is (,) then, 2( ) is equal to	**(r)**	25
(d)	Let m_1 and m_2 are the slopes of the tangents drawn to circle $x^2 \quad y^2 \quad 4x \quad 8y \quad 5 \quad 0$ from the point $P(\ 1, \ 2)$, and $\lvert m_1 \quad m_2 \rvert \quad p/q$ where *p* and *q* are relatively prime natural numbers, then $p \quad q$ is equal to	**(s)**	27
		(t)	29

11. Let two circles S_1 and S_2 having centres O_1 and O_2 have radius R and r respectively ($r \quad R$) touching each other internally at a point A. A tangent to smaller circle at point D intersects the larger one at points B and C as shown. Let AB and AC intersects the circles S_2 at points L and K respectively.



	Column-I		Column-II
(a)	$\dfrac{AK}{AC}$	**(p)**	$\dfrac{\sqrt{R}}{\sqrt{R} \quad \sqrt{R \quad r}}$
(b)	$\dfrac{AC}{CD}$	**(q)**	$\dfrac{R}{r}$
(c)	$\dfrac{AL}{AB}$	**(r)**	$\dfrac{r}{R \quad r}$
(d)	$\dfrac{AO}{AD}$	**(s)**	$\dfrac{r}{R}$
	(where O is incentre of $\quad ABC$)	**(t)**	$\sqrt{\dfrac{R}{R \quad r}}$

12. Let $S \quad \{(x, y) : x^2 \quad y^2 \quad 6x \quad 8y \quad 21 \quad 0\}$

Match the Column-I with Column-II

	Column-I		Column-II
(a)	$\max \dfrac{12x}{7} \quad \dfrac{5y}{7}; (x, y) \quad S$	**(p)**	3
(b)	$\min \dfrac{1}{2}(x^2 \quad y^2 \quad 1) \quad (x \quad y); (x, y) \quad S$	**(q)**	4
(c)	$\max \dfrac{3x}{7} \quad \dfrac{4y}{7}; (x, y) \quad S$	**(r)**	5
(d)	$\min \dfrac{\sqrt{3}y \quad \lvert x \quad 3\rvert}{\lvert x \quad 3\rvert}; (x, y) \quad S$	**(s)**	6
		(t)	7

ANSWERS

1. a r; b q; c p; d s

2. a s; b p; c r; d s

3. a q; b s; c p; d r

4. a s; b p; c q; d r

5. a s; b r; c q; d p

6. a s; b q; c r; d p

7. a p,q; b p,q; c q; d q,s

8. a q; b s; c r; d p

9. a p; b q; c r; d s

10. a q; b q; c t; d p

11. a s; b t; c s; d p

12. a s; b q; c r; d q

EXERCISE 6

Subjective Problems

1. If the circle C_1 touches x-axis and the line $y \quad x \tan \quad , \qquad 0, \frac{}{2}$ in first quadrant and circle C_2 touches the line $y \quad x \tan \quad$, y-axis and circle C_1 in such a way that ratio of radius of C_1 to radius of C_2 is 2 : 1, then value of $\tan \frac{}{2} \quad \frac{\sqrt{a} \quad b}{c}$ where a, b, c are relatively prime natural numbers find $a \quad b \quad c$.

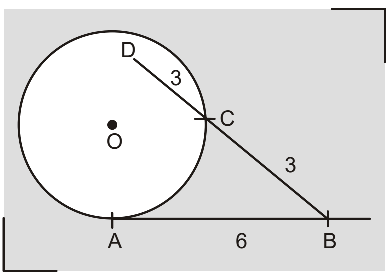
2. A point D is taken on the side AC of an acute triangle ABC, such that $AD \quad 1, DC \quad 2$ and BD is an altitude of $\quad ABC$. A circle of radius 2, which passes through points A and D, touches at point D a circle circumscribed about the $\quad BDC$. The area of $\quad ABC$ is A, then $\frac{A^2}{15} \quad$.

3. How many ordered pair of integers (a, b) satisfy all the following inequalities $a^2 \quad b^2 \quad 16, a^2 \quad b^2 \quad 8a, a^2 \quad b^2 \quad 8b$?

4. Circle S_1 is centered at $(0, 3)$ with radius 1. Circle S_2 is externally tangent to circle S_1 and also tangent to x-axis. If the locus of the centre of the variable circle S_2 can be expressed as $y \quad 1 \quad \frac{x^2}{}$. Find $\quad$.

5. Let two parallel lines L_1 and L_2 with positive slope are tangent to the circle $C_1 : x^2 \quad y^2 \quad 2x \quad 16y \quad 64 \quad 0$. If L_1 is also tangent to the circle $C_2 : x^2 \quad y^2 \quad 2x \quad 2y \quad 2 \quad 0$ and equation of L_2 is $a\sqrt{a}x \quad by \quad c \quad a\sqrt{a} \quad 0$ where $a, b, c \quad N$, then find the value of $\frac{a \quad b \quad c}{2}$.

6. In the figure AB is tangent at A to circle with centre O; point D is interior to circle and DB intersects the circle at C. If $BC \quad DC \quad 3, OD \quad 2$ and $AB \quad 6$, then find the radius of the circle.



7. The lines $3x \quad 4y \quad 4 \quad 0$ and $6x \quad 8y \quad 7 \quad 0$ are tangents to the same circle whose radius is r, then $4r$ is equal to.

8. There exists two circles passing through $(\quad 1,1)$ and touching the lines $x \quad y \quad 2$ and $x \quad y \quad 2$ whose radii are r_1 and r_2, and $r_1 \quad r_2 \quad a\sqrt{2}$, then a is equal to.

9. Let A be the centre of the circle $x^2 \quad y^2 \quad 2x \quad 4y \quad 20 \quad 0$. Suppose the tangents at the points. $B\,(1,7)$ and $D\,(4, \; 2)$ on the circle meet at C. The area of the quadrilateral $ABCD$ is N, then $\frac{N}{15}$.

10. If radii of the smallest and the largest circle passing through $(\sqrt{3}, \sqrt{2})$ and touching the circle $x^2 \quad y^2 \quad 2\sqrt{2}y \quad 2 \quad 0$ are r_1 and r_2 respectively, then find the mean of r_1 and r_2.

11. Given a convex quadrilateral $ABCD$ circumscribed about a circle of diameter 1. Inside $ABCD$, there is a point M such that $|MA|^2 \quad |MB|^2 \quad |MC|^2 \quad |MD|^2 \quad 2$. Find the area of $ABCD$.

12. Tow circle of radii 8 and 4 touch each other externally at a point A. Through a point B taken on the larger circle a straight line is drawn touching the smaller circle at C. Find BC if $AB \quad \sqrt{6}$.

13. Given a rectangle $ABCD$ where $|AB| \quad 2a, |BC| \quad a\sqrt{2}$. On the side AB, as on diameter, a semicircle is constructed externally. Let M be an arbitrary point on the semicircle, the line MD intersect AB at N, and MC at L. If $|AL|^2 \quad |BN|^2 \quad\quad a^2$, then

14. The distance between centres of two circles is equal to 1. Let x be the side of a rhombus two opposite vertices of which lie on one circle, and the other two on the other if the radii of the circles are 6 and 3, then $[x]$ (where [.] denote greates integer function)

15. Circle of radii 4 and 2 touch each other internally. The side of equilateral triangle one vertex of which coincides with the point of tengency and the other two lying on the two given circle is:

16. Two circles of radius 8 are placed inside a semi-circle of radius 25. The two circles are each tangent to the diameter and to the semicircle. If the distance between the centers of the two circles is , then the value of $\frac{60}{}$.

ANSWERS

1.	22	**2.**	9	**3.**	6	**4.**	8	**5.**	7	**6.**	4	**7.**	3	**8.**	6	**9.**	5	**10.**	1
11.	1	**12.**	3	**13.**	4	**14.**	6	**15.**	4	**16.**	2								

EXERCISE 7

1. (A) If two distinct chords, drawn from the point (p, q) on the circle $x^2 \quad y^2 \quad px \quad qy$ (where $p, q \quad 0$) are bisected by the x-axis, then:

(a) $p^2 \quad q^2$ (b) $p^2 \quad 8q^2$ (c) $p^2 \quad 8q^2$ (d) $p^2 \quad 8q^2$

(B) Let L_1 be a straight line through the origin and L_2 be the straight line $x \quad y \quad 1$. If the intercepts made by the circle $x^2 \quad y^2 \quad x \quad 3y \quad 0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1 ? **[IIT-JEE 1999]**

(a) $x \quad y \quad 0$ (b) $x \quad y \quad 0$ (c) $x \quad 7y \quad 0$ (d) $x \quad 7y \quad 0$

2. (A) The triangle PQR is inscribed in the circle, $x^2 \quad y^2 \quad 25$. If Q and R have co-ordinates $(3, 4)$ and $(\quad 4, 3)$ respectively, then $\quad QPR$ is equal to:

(a) $\frac{\ }{2}$ (b) $\frac{\ }{3}$ (c) $\frac{\ }{4}$ (d) $\frac{\ }{6}$

(B) If the circles, $x^2 \quad y^2 \quad 2x \quad 2ky \quad 6 \quad 0$ and $x^2 \quad y^2 \quad 2ky \quad k \quad 0$ intersect orthogonally, then 'k' is: **[IIT-JEE (Screening) 2000]**

(a) 2 or $\frac{3}{2}$ (b) 2 or $\frac{3}{2}$ (c) 2 or $\frac{3}{2}$ (d) 2 or $\frac{3}{2}$

3. (A) Extremities of a diagonal of a rectangle are $(0, 0)$ and $(4, 3)$. Find the equation of the tangents to the circumcircle of a rectangle which are parallel to this diagonal.

(B) Find the point on the straight line, $y \quad 2x \quad 11$ which is nearest to the circle,

$$16(x^2 \quad y^2) \quad 32x \quad 8y \quad 50 \quad 0$$

(C) A circle of radius 2 units rolls on the outerside of the circle, $x^2 \quad y^2 \quad 4x \quad 0$, touching it externally. Find the locus of the centre of this outer circle. Also find the equations of the common tangents of the two circles when the line joining the centres of the two circles is inclined at an angle of 60 with x-axis. **[REE (Mains) 2000]**

4. (A) Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle then $2r$ equals:

[IIT-JEE (Screening) 2001]

(a) $\sqrt{PQ \quad RS}$ (b) $\frac{PQ \quad RS}{2}$

(c) $\frac{2PQ \quad RS}{PQ \quad RS}$ (d) $\sqrt{\frac{(PQ)^2 \quad (RS)^2}{2}}$

(B) Let $2x^2 \quad y^2 \quad 3xy \quad 0$ be the equation of a pair of tangents drawn from the origin 'O' to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA. **[IIT-JEE (Mains) 2001]**

5. (A) Find the equation of the circle which passes through the points of intersection of circles $x^2 \quad y^2 \quad 2x \quad 6y \quad 6 \quad 0$ and $x^2 \quad y^2 \quad 2x \quad 6y \quad 6 \quad 0$ and intersects the circle $x^2 \quad y^2 \quad 4x \quad 6y \quad 4 \quad 0$ orthogonally. **[REE (Mains) 2001]**

(B) Tangents TP and TQ are drawn from a point T to the circle $x^2 \quad y^2 \quad a^2$. If the point T lies on the line $px \quad qy \quad r$, find the locus of centre of the circumcircle of triangle TPQ.

[REE (Mains) 2001]

6. (A) If the tangent at the point P on the circle $x^2 \quad y^2 \quad 6x \quad 6y \quad 2$ meets the straight line $5x \quad 2y \quad 6 \quad 0$ at a point Q on the y-axis, then the length of PQ is:

(a) 4 (b) $2\sqrt{5}$ (c) 5 (d) $3\sqrt{5}$

(B) If $a \quad 2b \quad 0$ then the positive value of m for which $y \quad mx \quad b\sqrt{1 \quad m^2}$ is a common tangent to $x^2 \quad y^2 \quad b^2$ and $(x \quad a)^2 \quad y^2 \quad b^2$ is: **[IIT-JEE (Screening) 2002]**

(a) $\dfrac{2b}{\sqrt{a^2 \quad 4b^2}}$ (b) $\dfrac{\sqrt{a^2 \quad 4b^2}}{2b}$ (c) $\dfrac{2b}{a \quad 2b}$ (d) $\dfrac{b}{a \quad 2b}$

7. The radius of the circle, having centre at $(2, 1)$, whose one of the chord is a diameter of the circle $x^2 \quad y^2 \quad 2x \quad 6y \quad 6 \quad 0$ **[IIT-JEE (Screening) 2004]**

(a) 1 (b) 2 (c) 3 (d) $\sqrt{3}$

8. Line $2x \quad 3y \quad 1 \quad 0$ is a tangent to a circle at $(1, \quad 1)$. This circle is orthogonal to a circle which is drawn having diameter as a line segment with end points $(0, \quad 1)$ and $(\quad 2, 3)$. Find equation of circle. **[IIT-JEE 2004]**

9. A circle is given by $x^2 \quad (y \quad 1)^2 \quad 1$, another circle C touches it externally and also the x-axis, then the locus of its centre is: **[IIT-JEE (Screening) 2005]**

(a) $\{(x, y) : x^2 \quad 4y\} \quad \{(x, y) : y \quad 0\}$

(b) $\{(x, y) : x^2 \quad (y \quad 1)^2 \quad 4\} \quad \{(x, y) : y \quad 0\}$

(c) $\{(x, y) : x^2 \quad y\} \quad \{(0, y) : y \quad 0\}$

(d) $\{(x, y) : x^2 \quad 4y\} \quad \{(0, y) : y \quad 0\}$

10. (A) Let $ABCD$ be a quadrilateral with area 18, with side AB parallel to the side CD and $AB \quad 2CD$. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral $ABCD$ touching all the sides, then its radius is:

(a) 3 (b) 2 (c) 3/2 (d) 1

(B) Tangents are drawn from the point $(17, 7)$ to the circle $x^2 \quad y^2 \quad 169$.

Statement-1: The tangents are mutually perpendicular.

because

Statement-2: The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 \quad y^2 \quad 338$. **[IIT-JEE 2007]**

(a) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true.

11. (A) Consider the two curves

$C_1 : y^2 \quad 4x; \;\; C_2 : x^2 \quad y^2 \quad 6x \quad 1 \quad 0.$ Then:

(a) C_1 and C_2 touch each other only at one point

(b) C_1 and C_2 touch each other exactly at two points

(c) C_1 and C_2 intersect (but do not touch) at exactly two points

(d) C_1 and C_2 neither intersect nor touch each other

(B) Consider, $L_1 : 2x \quad 3y \quad P \quad 3 \quad 0; \;\; L_2 : 2x \quad 3y \quad P \quad 3 \quad 0,$

where P is a real number and $C : x^2 \quad y^2 \quad 6x \quad 10y \quad 30 \quad 0.$

Statement-1: If line L_1 is a diameter of circle C, then line L_2 is not always a diameter of circle C.

because

Statement-2: If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C.

(a) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true.

(C) Comprehension

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F respectively. The line PQ is given by the equation $\sqrt{3}x \quad y \quad 6 \quad 0$ and the point D is $\frac{3\sqrt{3}}{2}, \frac{3}{2}$.

Further, it is given that the origin and the centre of C are on the same side of the line PQ. [IIT-JEE 2008]

(i) The equation of circle C is:

(a) $(x \quad 2\sqrt{3})^2 \quad (y \quad 1)^2 \quad 1$

(b) $(x \quad 2\sqrt{3})^2 \quad y \quad \frac{1}{2}^{\,2} \quad 1$

(c) $(x \quad \sqrt{3})^2 \quad (y \quad 1)^2 \quad 1$

(d) $(x \quad \sqrt{3})^2 \quad (y \quad 1)^2 \quad 1$

(ii) Points E and F are given by:

(a) $\frac{\sqrt{3}}{2}, \frac{3}{2}, (\sqrt{3}, 0)$ (b) $\frac{\sqrt{3}}{2}, \frac{1}{2}, (\sqrt{3}, 0)$

(c) $\frac{\sqrt{3}}{2}, \frac{3}{2}, \frac{\sqrt{3}}{2}, \frac{1}{2}$ (d) $\frac{3}{2}, \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, \frac{1}{2}$

(iii) Equations of the sides RP, RQ are:

(a) $y \quad \frac{2}{\sqrt{3}}x \quad 1, y \quad \frac{2}{\sqrt{3}}x \quad 1$ (b) $y \quad \frac{1}{\sqrt{3}}x, y \quad 0$

(c) $y \quad \frac{\sqrt{3}}{2}x \quad 1, y \quad \frac{\sqrt{3}}{2}x \quad 1$ (d) $y \quad \sqrt{3}x, y \quad 0$

12. Tangents drawn from the point $P(1, 8)$ to the circle $x^2 \quad y^2 \quad 6x \quad 4y \quad 11 \quad 0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is: **[IIT 2009]**

(a) $x^2 \quad y^2 \quad 4x \quad 6x \quad 19 \quad 0$ (b) $x^2 \quad y^2 \quad 10y \quad 19 \quad 0$

(c) $x^2 \quad y^2 \quad 2x \quad 6y \quad 29 \quad 0$ (d) $x^2 \quad y^2 \quad 6x \quad 4y \quad 19 \quad 0$

13. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid-point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is: **[IIT 2009]**

14. The circle passing through the point $(\quad 1, 0)$ and touching the y-axis at $(0, 2)$ also passes through the point: **[IIT 2011]**

(a) $\frac{3}{2}, 0$ (b) $\frac{5}{2}, 2$ (c) $\frac{3}{2}, \frac{5}{2}$ (d) $(\quad 4, 0)$

15. The straight line $2x \quad 3y \quad 1$ divides the circular region $x^2 \quad y^2 \quad 6$ into two parts. If

$$S \quad 2, \frac{3}{4}, \quad \frac{5}{4}, \frac{3}{4}, \quad \frac{1}{4}, \quad \frac{1}{4}, \quad \frac{1}{8}, \frac{1}{4},$$

then the number of point(s) in S lying inside the smaller part is: **[IIT 2011]**

16. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x \quad 5y \quad 20$ to the circle $x^2 \quad y^2 \quad 9$ is: **[IIT-JEE 2012]**

(a) $20(x^2 \quad y^2) \quad 36x \quad 45y \quad 0$ (b) $20(x^2 \quad y^2) \quad 36x \quad 45y \quad 0$

(c) $36(x^2 \quad y^2) \quad 20x \quad 45y \quad 0$ (d) $36(x^2 \quad y^2) \quad 20x \quad 45y \quad 0$

Paragraph for question Nos. 17 to 18

A tangent PT is drawn to the circle $x^2 \quad y^2 \quad 4$ at the point $P(\sqrt{3}, 1)$. A straight line L, perpendicular to PT is a tangent to the circle $(x \quad 3)^2 \quad y^2 \quad 1$.

[IIT-JEE 2012]

17. A common tangent of the two circles is:

(a) $x \quad 4$ (b) $y \quad 2$ (c) $x \quad \sqrt{3}y \quad 1$ (d) $x \quad 2\sqrt{2}y \quad 6$

18. A possible equation of L is:

(a) $x \quad \sqrt{3}y \quad 1$ (b) $x \quad \sqrt{3}y \quad 1$

(c) $x \quad \sqrt{3}y \quad 1$ (d) $x \quad \sqrt{3}y \quad 5$

19. The circle passing through $(1, \quad 2)$ and touching the axis of x at $(3, 0)$ also passes through the point : **[IIT-JEE (Mains) 2013]**

(a) $(\quad 2, 5)$ (b) $(\quad 5, 2)$

(c) $(2, \quad 5)$ (d) $(5, \quad 2)$

20. Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y-axis is(are) : **[IIT-JEE (Advance) 2013]**

(a) $x^2 \quad y^2 \quad 6x \quad 8y \quad 9 \quad 0$ (b) $x^2 \quad y^2 \quad 6x \quad 7y \quad 9 \quad 0$

(c) $x^2 \quad y^2 \quad 6x \quad 8y \quad 9 \quad 0$ (d) $x^2 \quad y^2 \quad 6x \quad 7y \quad 9 \quad 0$

ANSWERS

1. **(A)** d; **(B)** b, c **2.** **(A)** c; **(B)** a

3. **(A)** $6x \quad 8y \quad 25 \quad 0$ and $6x \quad 8y \quad 25 \quad 0$; **(B)** $(\quad 9/2, 2)$

(C) $x^2 \quad y^2 \quad 4x \quad 12 \quad 0, T_1 : \sqrt{3}x \quad y \quad 2\sqrt{3} \quad 4 \quad 0, T_2 : \sqrt{3}x \quad y \quad 2\sqrt{3} \quad 4 \quad 0$ (D.C.T.)

$T_3 : x \quad \sqrt{3}y \quad 2 \quad 0, T_4 : x \quad \sqrt{3}y \quad 6 \quad 0$ (T.C.T.)

4. **(A)** a; **(B)** $OA \quad 3(3 \quad \sqrt{10})$

5. **(A)** $x^2 \quad y^2 \quad 14x \quad 6y \quad 6 \quad 0$; **(B)** $2px \quad 2qy \quad r$

6. **(A)** c; **(B)** a

7. c **8.** $2x^2 \quad 2y^2 \quad 10x \quad 5y \quad 1 \quad 0$ **9.** d

10. **(A)** b; **(B)** a **11.** **(A)** b; **(B)** c; **(C)** **(i)** d; **(ii)** a; **(iii)** d

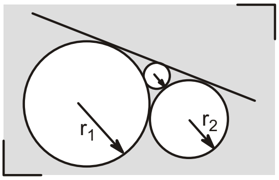
12. b **13.** 8 **14.** d **15.** 2

16. a **17.** d **18.** a **19.** d

20. a, c

Only One Choice is Correct:

1. (a)



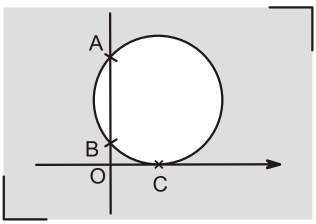
$2\sqrt{rr_1} \quad 2\sqrt{rr_2} \quad 2\sqrt{r_1 r_2}$

$\sqrt{r}(6 \quad 3) \quad 6 \quad 3$

$r \quad 4$

2. (b) For $\quad ACB$ to be maximum,

circle passing through A, B will touch x-axis at C.

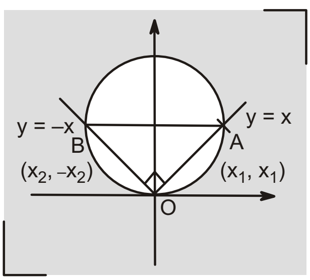


$OC^2 \quad (OA)(OB)$

$x^2 \quad ab$

$x \quad \sqrt{ab}$

3. (c)



$(x \quad x_1)(x \quad x_2) \quad (y \quad x_1)(y \quad x_2) \quad 0$

$x^2 \quad y^2 \quad (x_1 \quad x_2)x \quad (x_2 \quad x_1)y \quad 0$

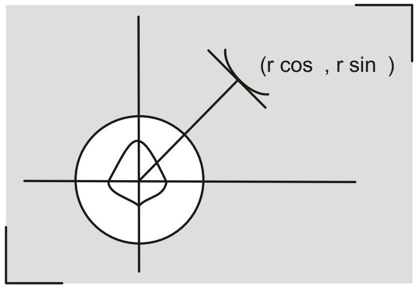
$x^2 \quad y^2 \quad ax \quad (\sqrt{a^2 \quad 4b})y \quad 0$

4. (b)

$r^2(2\cos^2 \quad 10\sin^2 \quad 6\sin \quad \cos \quad) \quad 1$

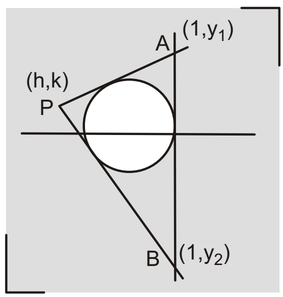
$r^2 \quad \dfrac{1}{3\sin 2 \quad 4\cos 2 \quad 6} \quad \dfrac{1}{6 \quad 5} \quad 1$

$r \quad 1$



Minimum distance between curves $3 \quad 1 \quad 2$.

5. (d) Equation of pair of tangents PA and PB is



$(xh \quad yk \quad 1)^2 \quad (x^2 \quad y^2 \quad 1)$
$(h^2 \quad k^2 \quad 1)$

Put $x \quad 1, (h \quad 1)^2 \quad 2ky(h \quad 1)$
$y^2(h^2 \quad 1)$

$y^2(h \quad 1) \quad 2ky \quad (h \quad 1) = 0 \begin{smallmatrix} y_1 \\ y_2 \end{smallmatrix}$

$AB \quad |y_1 \quad y_2| \quad 2$

$$4 \quad \frac{4k^2}{(h \quad 1)^2} \quad \frac{4(h \quad 1)}{(h \quad 1)}$$

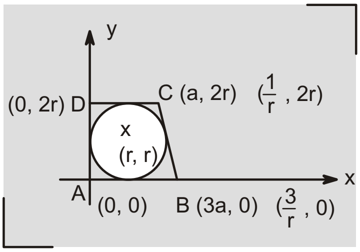
$$(h \quad 1) \quad k^2 \quad (h^2 \quad 1) \quad k^2 \quad 2(h \quad 1)$$

$$y^2 \quad 2(x \quad 1)$$

6. (d) Area of trapezium

$$ABCD \quad \frac{1}{2}(a \quad 3a)(2r) \quad 4$$

$$ar \quad 1$$



Equation of BC is $y \quad r^2 \quad x \quad \frac{3}{r}$

$$y \quad r^2 x \quad 3r \quad 0$$

∵ BC is tangent to circle

$$\frac{|r \quad r^3 \quad 3r|}{\sqrt{1 \quad r^4}} \quad r$$

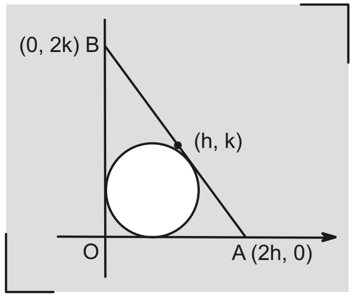
$$r^4 \quad 4 \quad 4r^2 \quad 1 \quad r^4$$

$$r \quad \frac{\sqrt{3}}{2}$$

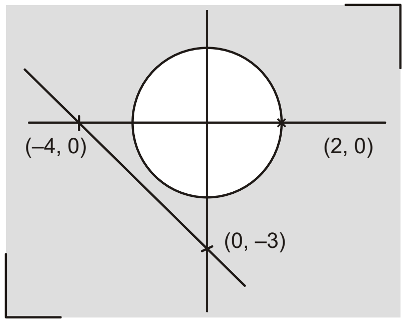
7. (a) $r \quad \frac{}{s} \quad \frac{\frac{1}{2} \quad (2h)(2k)}{\frac{1}{2}(2h \quad 2k \quad 2\sqrt{h^2 \quad k^2})}$

$$2 \quad \frac{2hk}{h \quad k \quad \sqrt{h^2 \quad k^2}}$$

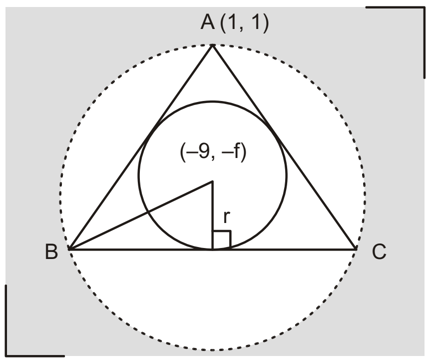
Locus of $xy \quad x \quad y \quad \sqrt{x^2 \quad y^2}$



8. (d) Line does not intersect the circle



9. (b) $r \quad R\cos 60 \quad \frac{R}{2}$



Equation of incircle is

$$x^2 \quad y^2 \quad 2gx \quad 2fy \quad g^2 \quad f^2$$

$$\frac{g^2 \quad f^2 \quad c}{4} \quad \frac{(g \quad 1)^2}{4} \quad \frac{(f \quad 1)^2}{4}$$

$$R^2 \quad (g \quad 1)^2 \quad (f \quad 1)^2$$

$g^2 \quad f^2 \quad 2g \quad 2f \quad 2 \quad g^2 \quad f^2 \quad c$

Equation of incircle becomes

$4(x^2 \quad y^2) \quad 8gx \quad 8fy$

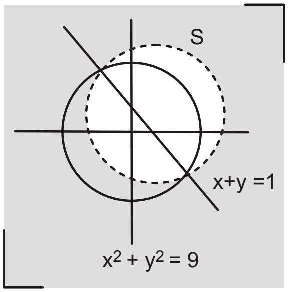
$2(g \quad f) \quad 3(g^2 \quad f^2) \quad 2$

$(2g \quad 3g^2 \quad 1) \quad (2f \quad 3f^2 \quad 1)$

$(1 \quad g)(1 \quad 3g) \quad (1 \quad f)(1 \quad 3f)$

10. (b) $S \quad x^2 \quad y^2 \quad 9 \quad (x \quad y \quad 1)$

Centre $\frac{}{2}, \frac{}{2}$



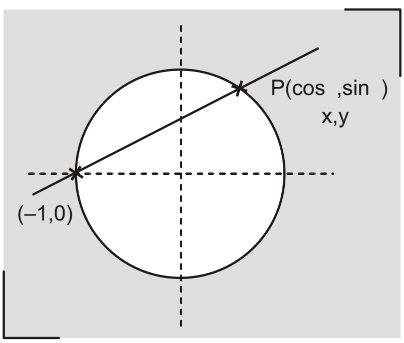
Centre lies on $x \quad y \quad 1$

1

$S \quad x^2 \quad y^2 \quad x \quad y \quad 8 \quad 0$

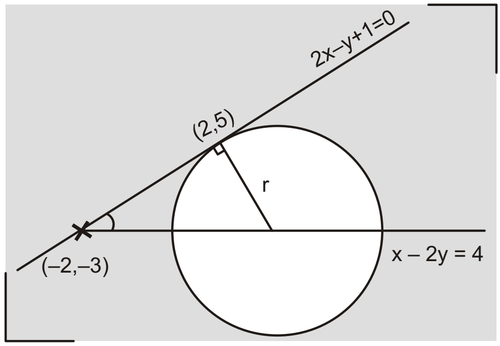
11. (d) $m \quad \frac{\sin}{\cos \quad 1} \quad \tan \frac{}{2}$

$P(x, y) \quad \frac{1 \quad \tan^2 \frac{}{2}}{1 \quad \tan^2 \frac{}{2}}, \frac{2\tan \frac{}{2}}{1 \quad \tan^2 \frac{}{2}}$



$x, y \quad Q$, if $m \quad Q$ Infinite points are possible.

12. (a)

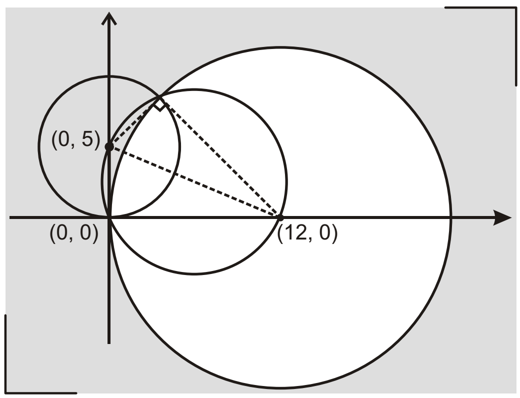


$\tan \quad \frac{2 \quad \frac{1}{2}}{1 \quad 2 \quad \frac{1}{2}} \quad \frac{3}{4}$

$\frac{r}{\sqrt{8^2 \quad 4^2}} \quad \tan \quad \frac{3}{4} \quad \frac{r}{4\sqrt{5}}$

$r \quad 3\sqrt{5}$

13. (a)



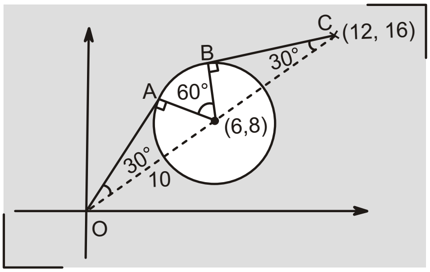
From figure it is clear that circle has AB as diameter

$r \quad \frac{13}{2}$

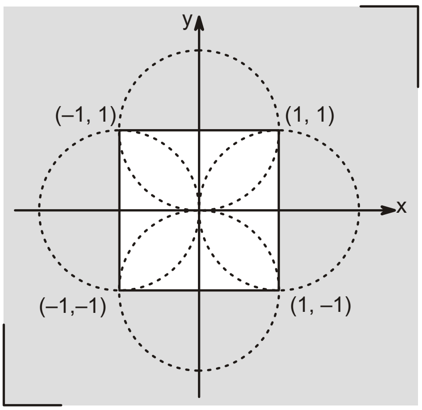
14. (c) *OABC* in the shortest path

$$OA \quad \overline{AB} \quad BC \quad 10\cos 30 \quad \frac{5}{3} \quad 10\cos 30$$

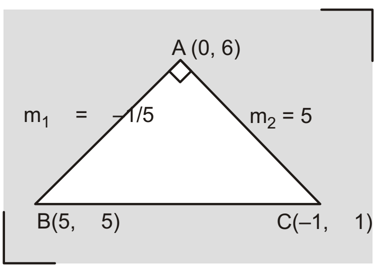
$$10\sqrt{3} \quad \frac{5}{3}$$



15. (c) Ar. (square of sides 2) = 4 sq. units



16. (d) Note that the *ABC* is right angled at *A*



equation of circle

$$(x \quad 1)(x \quad 5) \quad (y \quad 1)(y \quad 5) \quad 0$$

$$x^2 \quad y^2 \quad 4x \quad 6y \quad 5 \quad 5 \quad 0$$

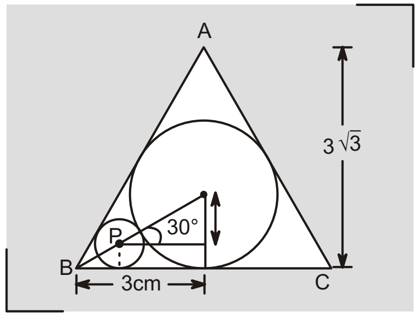
$$x^2 \quad y^2 \quad 4x \quad 6y \quad 0$$

Hence the circle passes through the origin

tangent at (0, 0) is $4x \quad 6y \quad 0$

$$2x \quad 3y \quad 0$$

17. (a)



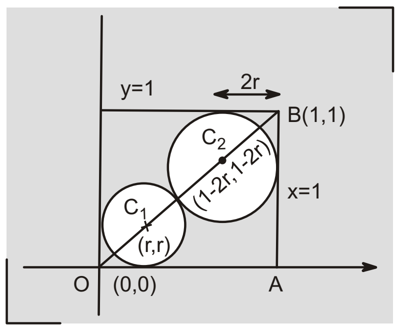
$$x \quad \sqrt{3} \quad r \quad PB$$

now $\frac{\sqrt{3} \quad r}{\sqrt{3} \quad r} \quad \sin 30 \quad \frac{1}{2}$

$$\frac{r}{\sqrt{3}} \quad \frac{2 \quad 1}{2 \quad 1} \quad \frac{1}{3}$$

$$r \quad \frac{1}{\sqrt{3}}$$

18. (c) C_1, C_2 lie on $y \quad x$ line



$$OB \quad OC_1 \quad C_1C_2 \quad C_2B$$

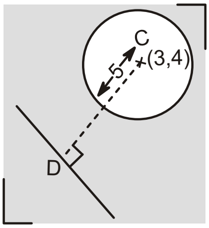
$$\sqrt{2} \quad r\sqrt{2} \quad 3r \quad 2r\sqrt{2}$$

$$r \quad \frac{\sqrt{2}}{3(\sqrt{2} \quad 1)}$$

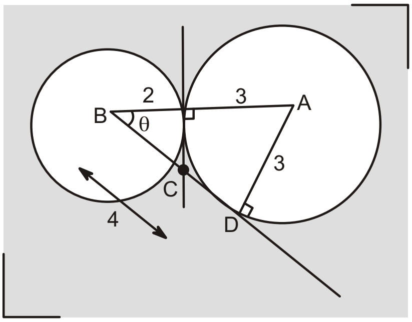
19. (a) Shortest distance $CD \quad r$

$$\frac{|9 \quad 16 \quad 25|}{5} \quad 5$$

$$\frac{32}{5} \quad 5 \quad \frac{7}{5}$$



20. (b)



$$BC \quad \frac{2}{\cos} \quad \frac{2}{\frac{4}{5}} \quad \frac{5}{2}$$

$$CD \quad 4 \quad \frac{5}{2} \quad \frac{3}{2}$$

21. (b) $(AC)^2 \quad r^2 \quad (36)^2$ ...(1)

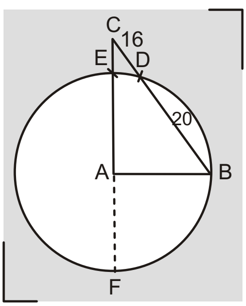
$(CE)(CF) \quad (CD)(BC)$

$(AC \quad r)(AC \quad r) \quad 16 \quad 36$ ...(2)

On adding (1) and (2), we get

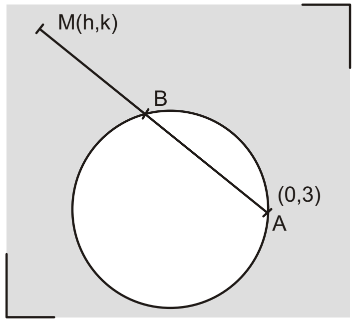
$2(AC)^2 \quad 36 \quad 52$

$AC \quad 6\sqrt{26}$



22. (b) $B \quad \frac{h}{2}, \frac{k \quad 3}{2}$

Put B to equation of circle



$$\frac{h}{2}^2 \quad 4 \quad \frac{h}{2} \quad \frac{k \quad 3}{2} \quad 3^2 \quad 0$$

Required locus is

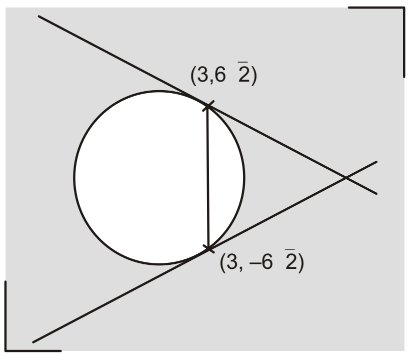
$x^2 \quad 8x \quad (y \quad 3)^2 \quad 0$

23. (b) Equation of pair of tangents is

$(3x \quad 6\sqrt{2}y \quad 81)(3x \quad 6\sqrt{2}y \quad 81) \quad 0$

$(x \quad 27)^2 \quad 8y^2 \quad 0$

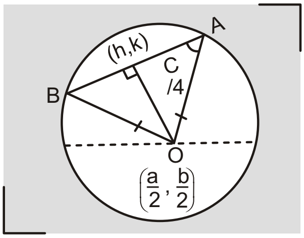
$x^2 \quad 54x \quad 8y^2 \quad 729 \quad 0$



24. (c) $OA^2 \quad AC^2 \quad OC^2$

$$\frac{a^2}{4} \quad \frac{b^2}{4} \quad 2 \quad h \quad \frac{a}{2}^2 \quad k \quad \frac{b}{2}^2$$

$$h^2 \quad k^2 \quad ah \quad bk \quad \frac{a^2}{8} \quad \frac{b^2}{8} \quad 0$$

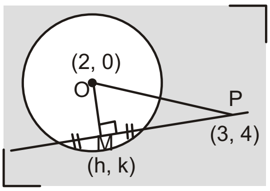


Required Locus is

$$x^2 \quad y^2 \quad ax \quad by \quad \frac{a^2 \quad b^2}{8} \quad 0$$

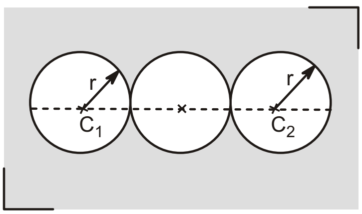
25. (a) Locus is Arc of the circle with OP as diameter intercepted by the given circle. *i.e.*,

$$(x \quad 2)(x \quad 3) \quad y(y \quad 4) \quad 0$$



26. (d) Centre will be mid point of C_1 and C_2

$$\frac{0 \quad 4}{2}, \frac{1 \quad 9}{2} \quad (2,5)$$

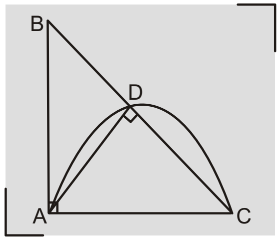


27. (d) $(AC^2) \quad (AD)^2 \quad (CD)^2$

$$(AB)^2 \quad (BD)(BC) \quad BD(BD \quad DC)$$

$$(BD)(DC) \quad (AB)^2 \quad (BD)^2 \quad (AD)^2$$

$$DC \quad \frac{AD^2}{BD} \quad \frac{(AD)^2}{\sqrt{(AB)^2 \quad (AD)^2}}$$



$$(AC)^2 \quad (AD)^2 \quad \frac{(AD)^4}{(AB^2) \quad (AD)^2}$$

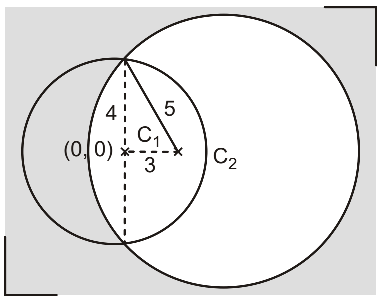
$$\frac{(AB)^2(AD)^2}{(AB)^2 \quad (AD)^2}$$

$$AC \quad \frac{(AB)(AD)}{\sqrt{(AB)^2 \quad (AD)^2}}$$

28. (b) Slope of $C_1C_2 \quad \frac{4}{3} \quad \tan$

$$C_2 \quad 0 \quad 3 \quad \frac{3}{5}, 0 \quad 3 \quad \frac{4}{5}$$

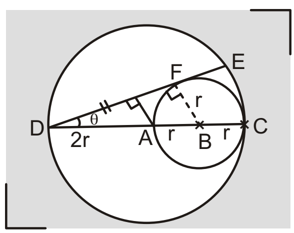
$$\text{or} \quad 0 \quad 3 \quad \frac{3}{5}, 0 \quad 3 \quad \frac{4}{5}$$



$$C_2 \quad \frac{9}{5}, \quad \frac{12}{5} \quad \text{or} \quad \frac{9}{5}, \frac{12}{5}$$

$$4r \quad 12 \quad r \quad 3$$

29. (a)

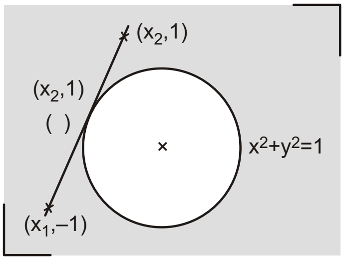


$\sin \quad \frac{r}{3r} \quad \frac{1}{3}$

$DE \quad 2(2r\cos\ ) \quad 4 \quad 3 \quad \frac{2\sqrt{2}}{3}$

$DE \quad 8\sqrt{2}$

30. (a) Let the equation of tangent is



$x\cos \quad y\sin \quad 1$

$(x_1, -1)$ lies on tangent

$x_1 \cos \quad \sin \quad 1$

$x_1 \cos \quad 1 \quad \sin$ ...(1)

now $(x_2, 1)$ lies on tangent

$x_2 \cos \quad 1 \quad \sin$ ...(2)

(1) (2), $x_1 x_2 \cos^2 \quad 1 \quad \sin^2 \quad \cos^2$

$x_1 x_2 \quad 1$

31. (c) Equation of chord of contact of the pair of tangent from (0, 0) and (g, f) are

$gx \quad fy \quad c \quad 0$ ...(1)

and $gx \quad fy \quad \frac{g^2 \quad f^2 \quad c}{2} \quad 0$ ...(2)

These lines are parallel

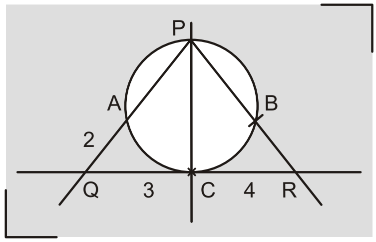
Hence distance $\left| \frac{c \quad \frac{g^2 \quad f^2 \quad c}{2}}{\sqrt{g^2 \quad f^2}} \right|$

32. (c) Locus of centres will be the radical axis of two given circles given by

$9x \quad 10y \quad 11 \quad 0$

33. (b) $(PQ)(AQ) \quad (QC)^2$

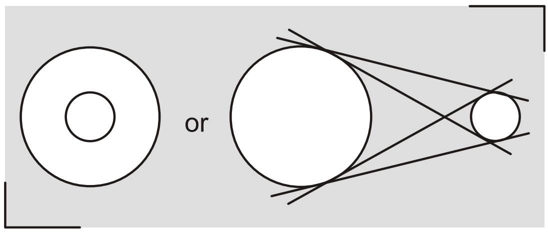
$PQ \quad \frac{9}{2}$



$\frac{QC}{RC} \quad \frac{PQ}{PR} \quad PR \quad 6$

$(RC)^2 \quad (RB)(RP) \quad RB \quad \frac{8}{3}$

34. (d)



35. (a) Put $h, \frac{b}{2}$ to equation of circle

$h^2 \quad \frac{b^2}{4} \quad ah \quad \frac{b^2}{2} \quad 0$

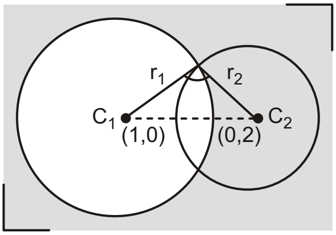
$h^2 \quad ah \quad \frac{3b^2}{4} \quad 0$ must get two distinct real roots

$D \quad 0 \quad a^2 \quad 3b^2 \quad 0$

36. (b) $\cos \quad \frac{r_1^2 \quad r_2^2 \quad (C_1C_2)^2}{2r_1r_2}$

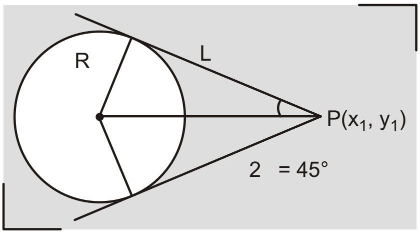
$\frac{10 \quad 5 \quad 5}{2\ \sqrt{10}.\sqrt{5}} \quad \frac{1}{\sqrt{2}}$

$\frac{}{4}$



37. (c) $\tan \quad \frac{R}{L}$ where $2 \quad 45$

$$\tan 2 \quad \frac{2\tan}{1 \quad \tan^2}$$



$$1 \quad \frac{2(R/L)}{1 \quad (R^2/L^2)} \quad \frac{2RL}{L^2 \quad R^2}$$

$$\frac{2a\sqrt{x_1^2 \quad y_1^2 \quad a^2}}{x_1^2 \quad y_1^2 \quad a^2 \quad a^2}$$

$$(x^2 \quad y^2 \quad 2a^2)^2 \quad 4a^2(x^2 \quad y^2 \quad a^2)$$

$$(x^2 \quad y^2)^2 \quad 4a^4 \quad 4a^2(x^2 \quad y^2)$$

$$4a^2(x^2 \quad y^2 \quad a^2)$$

$$(x^2 \quad y^2)^2 \quad 8a^4 \quad 8a^2(x^2 \quad y^2)$$

$$(x^2 \quad y^2)^2 \quad 8a^2(x^2 \quad y^2 \quad a^2)$$

$$8$$

38. (c) Let the circle be

$$(x \quad r)^2 \quad (y \quad r)^2 \quad r^2$$

$$x^2 \quad y^2 \quad 2rx \quad 2ry \quad r^2 \quad 0 \langle \begin{matrix} r_1 \\ r_2 \end{matrix}$$

Orthogonality

$$2r_1r_2 \quad 2r_1r_2 \quad r_1^2 \quad r_2^2$$

$$6r_1r_2 \quad (r_1 \quad r_2)^2 \qquad ...(1)$$

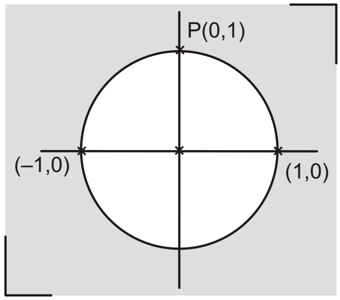
Circle passes through (a, b)

$$r^2 \quad 2(a \quad b)r \quad (a^2 \quad b^2) \quad 0$$

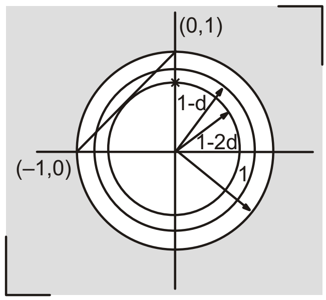
From (1), $6(a^2 \quad b^2) \quad 4(a \quad b)^2$

$$a^2 \quad b^2 \quad 4ab \quad 0$$

39. (d)



40. (c) For max common difference, the smallest circle just touches the line



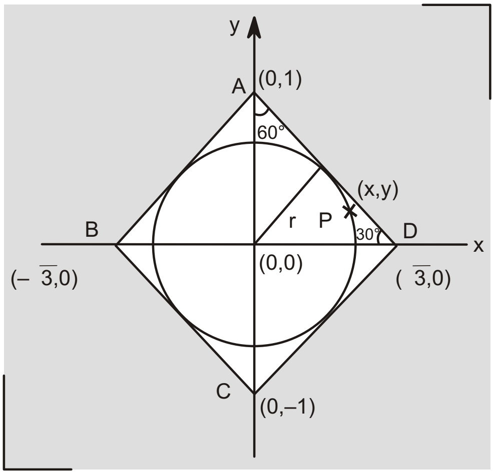
$$1 \quad 2d \quad \frac{1}{\sqrt{2}}$$

$$d \quad \frac{\sqrt{2} \quad 1}{2\sqrt{2}}$$

$$d \quad 0, \frac{\sqrt{2} \quad 1}{2\sqrt{2}}$$

i.e., $\quad 0, \frac{2 \quad \sqrt{2}}{4}$

41. (b) $r \quad \sqrt{3}\sin 30 \quad \frac{\sqrt{3}}{2}$



$(PA)^2 \quad (PB)^2 \quad (PC)^2 \quad (PD)^2$

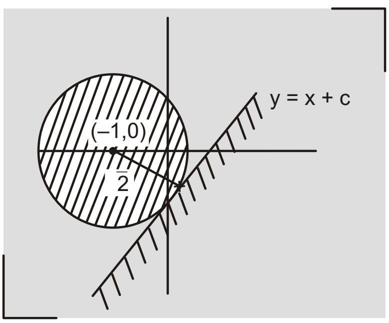
$(x \quad \sqrt{3})^2 \quad y^2 \quad x^2 \quad (y \quad 1)^2$

$(x \quad \sqrt{3})^2 \quad y^2 \quad x^2 \quad (y \quad 1)^2$

$4(x^2 \quad y^2 \quad 2)$

$4 \quad \frac{3}{4} \quad 2 \qquad 11$

42. (d) Line is tangent to circle as shown



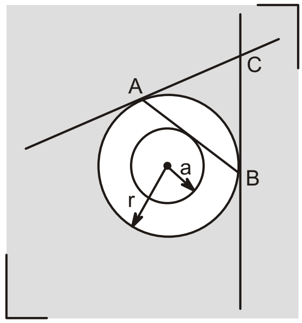
$\frac{1 \quad c}{\sqrt{2}} \quad \sqrt{2}$

[$\because$ (1,0) lies above the line $y \quad x \quad c \quad 0$]

$c \quad 1$

43. (a) Equation of AB is $xh \quad yk \quad r^2$

AB is tangent to $x^2 \quad y^2 \quad a^2$



$\frac{r^2}{\sqrt{h^2 \quad k^2}} \quad a$

$r^2 \quad ab \qquad [\because h^2 \quad k^2 \quad b^2]$

Equation of circle is $x^2 \quad y^2 \quad ab$

44. (a) Equation of AB is

$xh \quad yk \quad 1$ ...(1)

$AB \quad S_1 \quad S_2 \quad 0$

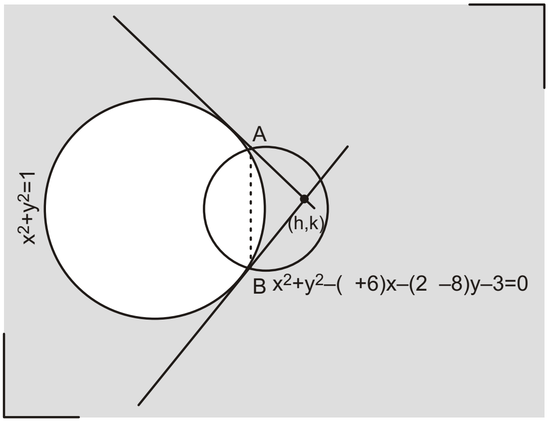
$(\quad 6)x \quad (2 \quad 8)y \quad 2 \quad 0$ ...(2)

(1) & (2) are identical

$\frac{h}{\quad 6} \quad \frac{k}{2 \quad 8} \quad \frac{1}{2}$

$\frac{2h}{2 \quad 12} \quad \frac{k}{2 \quad 8} \quad \frac{2h \quad k}{20}$

$\frac{1}{2}$ as $\frac{a}{b} \quad \frac{c}{b} \quad \frac{a \quad c}{b \quad d}$

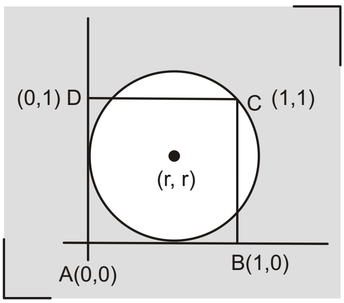


$2x \quad y \qquad 10$

$2x \quad y \quad 10 \quad 0$

45. (a) Equation of circle is

$x^2 \quad y^2 \quad 2rx \quad 2ry \quad r^2 \quad 0$



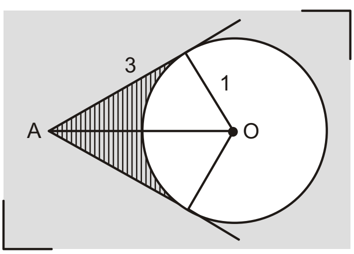
Put $(1,1)$

$r^2 \quad 4r \quad 2 \quad 0, r \quad 2 \quad \sqrt{2}$

$r \quad 2 \quad \sqrt{2} \qquad\qquad (\because r \quad 1)$

46. (b) $r \quad 1; \ L \quad \sqrt{3}$

area of quadrilateral $\quad \sqrt{3}$

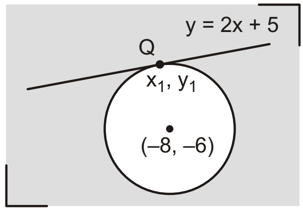


area of sector $\quad \frac{1}{2} \ 1 \ \frac{2}{3} \quad \frac{}{3}$

shaded region $\quad \sqrt{3} \quad \frac{}{2}$

47. (d) $y_1 \quad 2x_1 \quad 5 \quad]$

and $\quad \frac{(y_1 \quad 6)}{x_1 \quad 8} \quad 2 \quad 1$

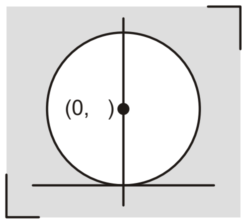


$x_1 \qquad 6$ and $y_1 \qquad 7$

48. (a) Equation of ⊙ is

$x^2 \quad (y \quad)^2 \qquad ^2$

$x^2 \quad y^2 \quad 2 \ y \quad 0$ ...(1)



Let the pole be (h,k). It's polar w.r.t. (1) is

$xh \quad ky \qquad (k \quad y) \quad 0$

$xh \quad (k \qquad)y \qquad k \quad 0$ ...(2)

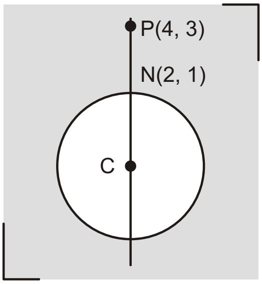
compare it with $lx \quad my \quad n \quad 0$ and eliminate .

49. (c) Diameter of the circle is given as

$2x \quad y \quad 2 \quad 0$ ...(1)

slope of PN: $\frac{3 \quad 1}{4 \quad 2} \quad 1$

equation of normal through PN is



$$y \quad 1 \quad (x \quad 2)$$

$x \quad y \quad 1 \quad 0$ ...(2)

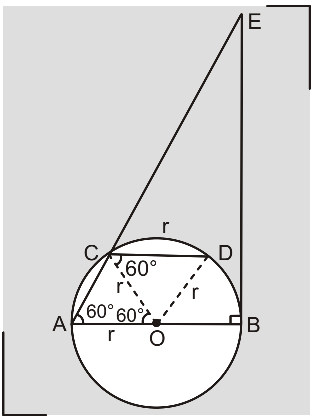
solving (1) and (2), centre is (1, 0)

Hence equation of the circle is

$$(x \quad 1)^2 \quad y^2 \quad (2 \quad 1)^2 \quad 1$$

$$x^2 \quad y^2 \quad 2x \quad 1 \quad 0$$

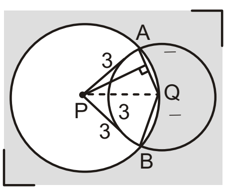
50. (d)



From figure it is clear that

$$AE \quad \frac{AB}{\cos 60} \quad 2(AB)$$

51. (b) Area of $APBQ$ 2 (area APQ)



$$2 \quad \frac{1}{2} \quad \sqrt{3} \quad \sqrt{3^2 \quad \frac{\sqrt{3}}{2}^{2}}$$

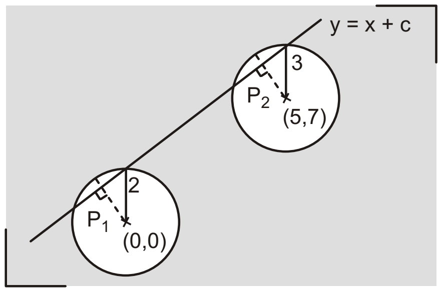
$$\sqrt{3} \quad \frac{\sqrt{33}}{2} \quad \frac{\sqrt{99}}{2}$$

52. (a) $r_1^2 \quad P_1^2 \quad r_2^2 \quad P_2^2$

$$4 \quad \frac{C^2}{2} \quad 9 \quad \frac{(2 \quad C)^2}{2}$$

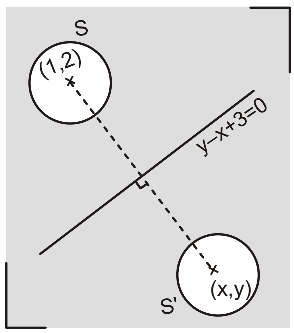
$$C \quad \frac{3}{2}$$

equation of line $y \quad x \quad \frac{3}{2}$



53. (a) $\dfrac{x \quad 1}{1} \quad \dfrac{y \quad 2}{1} \quad 2\dfrac{(2 \quad 1 \quad 3)}{2}$

$$(x, y) \quad (5, \quad 2)$$



Eqn. of S is $(x \quad 5)^2 \quad (4 \quad 2)^2 \quad 1$

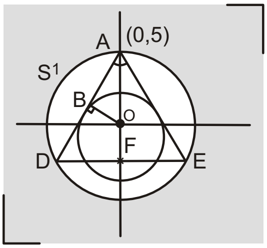
$$x^2 \quad y^2 \quad 10x \quad 4y \quad 28 \quad 0$$

54. (a) sin $\frac{3}{5}$

Slope of AB, AE are

tan $\frac{}{2}$,tan $\frac{}{2}$ $\frac{4}{3}, \frac{4}{3}$

Equation of OB is $y \quad \frac{3}{4}x$



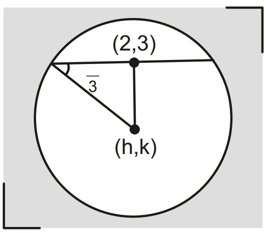
$4y \quad 3x \quad 0$

D is image of A w.r.t. OB

$\frac{x \quad 0}{3} \quad \frac{y \quad 5}{4} \quad 2\frac{4(5)}{25} \quad \frac{8}{5}$

$D \quad \frac{24}{5}, \frac{7}{5} \qquad OF \quad \frac{7}{5}$

55. (a)



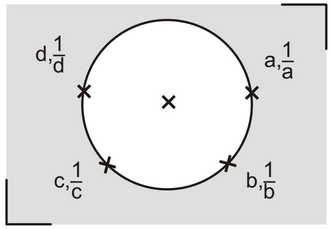
$\frac{k \quad 3}{h \quad 2} \quad \frac{2}{5}$

$5k \quad 2h \quad 11$

$2x \quad 5y \quad 11 \quad 0$

56. (c) Let circle is

$x^2 \quad y^2 \quad 2gx \quad 2fy \quad c \quad 0$



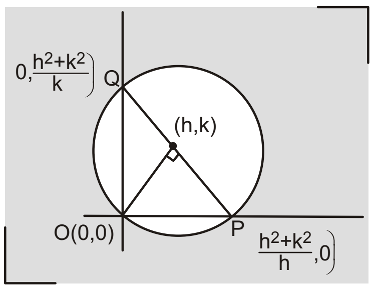
Put $t, \frac{1}{t}$

$t^4 \quad 2gt^3 \quad ct^2 \quad 2ft \quad 1 \quad 0$ — a, b, c, d

$abcd \quad 1$

57. (c) Equation of PQ is

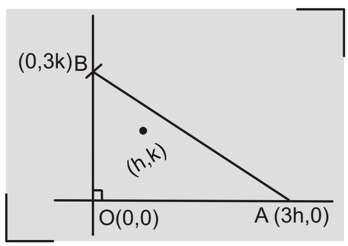
$hx \quad ky \quad h^2 \quad k^2 \quad 0, \frac{h^2 \quad k^2}{k}$



$PQ^2 \quad 4a^2$

$(h^2 \quad k^2)^2 \quad \frac{1}{h^2} \quad \frac{1}{k^2} \quad 4a^2$

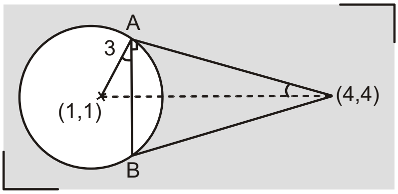
58. (a)



$(3h)^2 \quad (3k)^2 \quad (6k)^2$

$h^2 \quad k^2 \quad 4k^2$

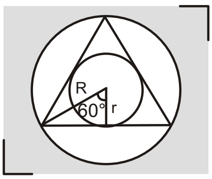
59. (b) sin $\frac{3}{3\sqrt{2}}$ $\frac{1}{\sqrt{2}}$



$\overline{}$
4

$AB \quad 2r\cos \quad 2(3)\frac{1}{\sqrt{2}} \quad 3\sqrt{2}$

60. (b) $r \quad R\cos 60 \quad \frac{R}{2}$



61. (d) Let equation of tangent to $y^2 \quad 4ax$ be

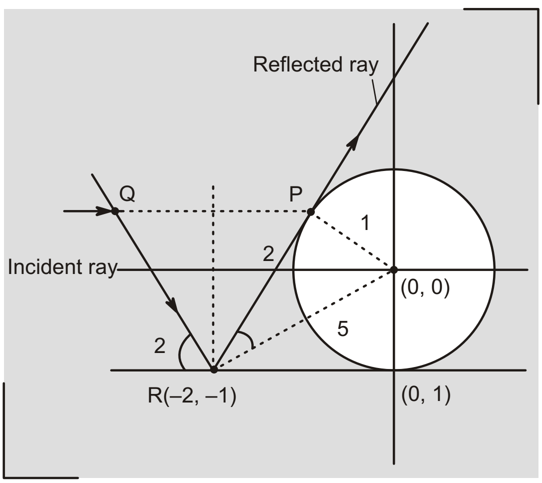
$ty \quad x \quad at^2$

If passes through $(a, 2a)$

$2at \quad a \quad at^2 \qquad a(t \quad 1)^2 \quad 0$

$a \quad R \quad \{0\}$

62. (b)



$\tan 2 \quad \frac{2\ \frac{1}{2}}{1\ \frac{1}{4}} \quad \frac{4}{3}$

Slope of QR $\frac{4}{3}$

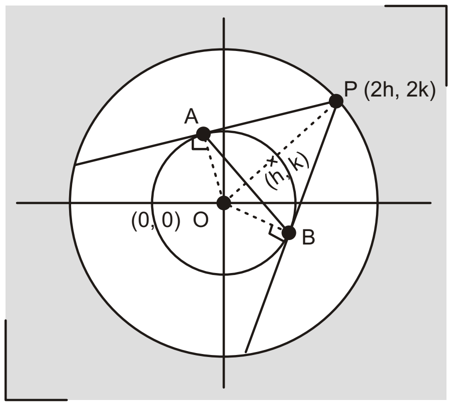
Equation of incident ray QR is

$y \quad 1 \quad \frac{4}{30}(x$

$3y \quad 4x \quad 11 \quad 0$

63. (c) Circumcircle of PAB has OP as diameter

If circumcentre (h, k)



$P \quad (2h, 2k)$

P lies on $x^2 \quad y^2 \quad 4$

$(2h)^2 \quad (2k)^2 \quad 4$

Locus of circumcentre is $x^2 \quad y^2 \quad 1.$

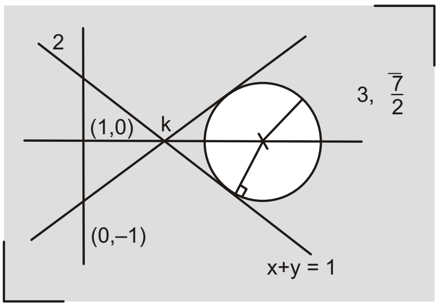
SOLUTIONS 2

One or More Than One Correct

1. (a, c)

$$(x \quad 3)^2 \quad y \quad \sqrt{\frac{7}{2}}^{\,2} \quad \left(\frac{x \quad y \quad 1}{\sqrt{2}}\right)^2$$

$$\left(\frac{x \quad y \quad 1}{\sqrt{2}}\right)^2$$



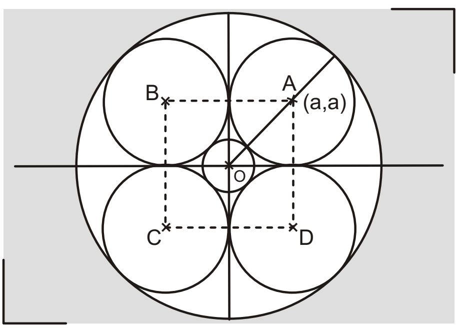
From $x \quad y \quad 1 \quad (x \quad y \quad 1)$

$y \quad 0$ or $x \quad 1$ (rejected)

$$(x \quad 3)^2 \quad \frac{7}{2} \quad \frac{(x \quad 1)^2}{2}$$

$x \quad 4,6$

2. (a, b, c, d)



$OA \quad \sqrt{2}a$

Largest radius $\sqrt{2}a \quad a$

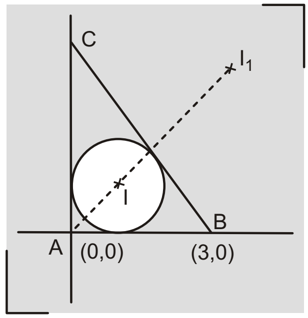
Smallest radius $\sqrt{2}a \quad a$

Area enclosed by circles

$$(2a)^2 \quad 4 \quad \frac{}{4}a^2 \quad (4 \quad)a^2$$

3. (a, d)

Circles will be in circle and excentral circle of ABC as shown

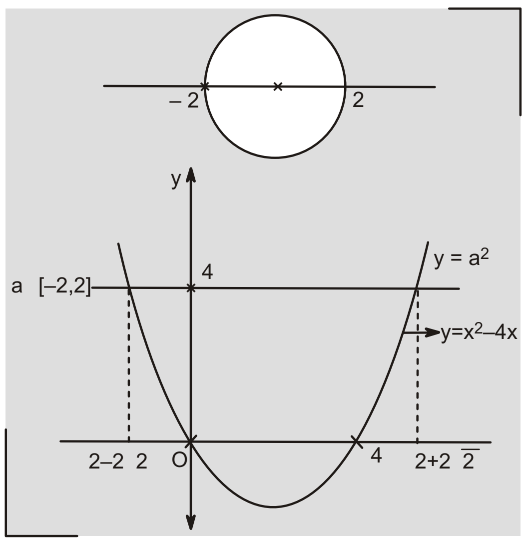


$$I \quad \frac{4(3)}{4 \quad 3 \quad 5}, \frac{3(4)}{4 \quad 3 \quad 5} \quad (1,1)$$

$$I_1 \quad \frac{4(3)}{5 \quad 4 \quad 3}, \frac{4(3)}{5 \quad 4 \quad 3} \quad (6,6)$$

$r \quad 1,6$

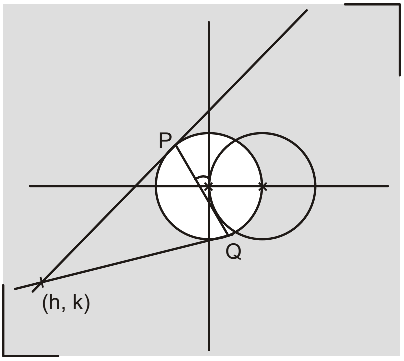
4. (a, b, d)



Exactly one root in $[2 \quad 2\sqrt{2}, 0] \quad [4, 2 \quad 2\sqrt{2}]$

5. (a, c)

PQ is chord of contact of (h, k) w.r.t. $x^2 \quad y^2 \quad a^2$



Equation of PQ is $xh \quad yk \quad a^2$

$\because$ PQ is tangent to $(x \quad a)^2 \quad y^2 \quad a^2$

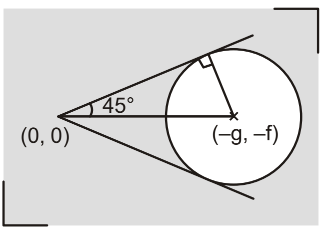
$$\frac{|ah \quad a^2|}{\sqrt{h^2 \quad k^2}} \quad a$$

$(h \quad a)^2 \quad h^2 \quad k^2$

Locus is $(x \quad a)^2 \quad x^2 \quad y^2$

$i.e., y^2 \quad a(a \quad 2x)$

6. (a, b)

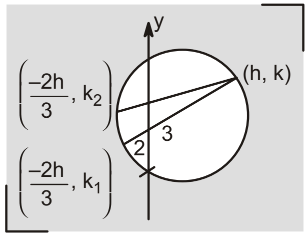


$$\sqrt{g^2 \quad f^2} \quad \frac{\sqrt{g^2 \quad f^2 \quad f^2}}{\sin 45}$$

$g^2 \quad f^2 \quad 2(g)^2$

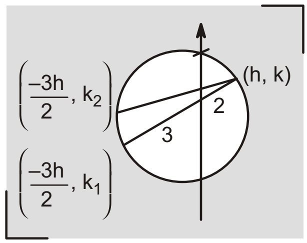
$g^2 \quad f^2 \qquad g \qquad f$

7. (a, b, d)



Put $x \quad \frac{2h}{3}, y^2 \quad ky \quad \frac{10h^2}{9} \quad 0$

$D \quad 0 \qquad k^2 \quad \frac{40}{9}h^2 \quad 0$



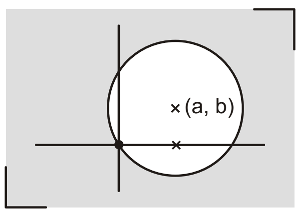
put $x \quad \frac{3h}{2}$

$y^2 \quad ky \quad \frac{15h^2}{4} \quad 0$

$D \quad 0 \qquad k^2 \quad 15h^2 \quad 0$

$k^2 \quad 15h^2$

8. (a, b, d)



Eqn. of circle is

$(x \quad a^2) \quad (y \quad b)^2 \quad a^2 \quad b^2$

or $(x \quad b)^2 \quad (y \quad a)^2 \quad a^2 \quad b^2$

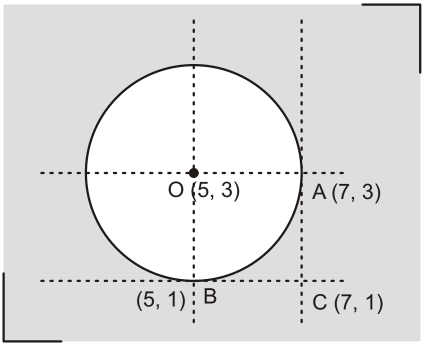
$x^2 \quad y^2 \quad 2ax \quad 2by \quad 0$

or $x^2 \quad y^2 \quad 2bx \quad 2ay \quad 0$

Eqn. of tangent origin is

$ax \quad by \quad 0$ or $bx \quad ay \quad 0$

9. (a, c, d)



Area of square $OACB \quad 2 \quad 2 \quad 4$

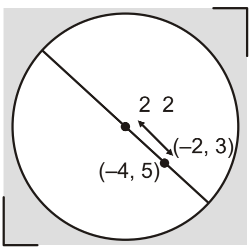
Radical axis of the family $S \quad 0$ is line AB whose equation $x \quad y \quad 4$

Smallest possible circle of family $S \quad 0$ has AB as diameter given by

$(x \quad 5)(x \quad 7) \quad (y \quad 1)(y \quad 3) \quad 0$

$x^2 \quad y^2 \quad 12x \quad 4y \quad 38 \quad 0$

10. (a, c, d)



$a \quad 9 \quad 2\sqrt{2}$

$b \quad 9 \quad 2\sqrt{2}$

$ab \quad 81 \quad 8 \quad 73$

$a \quad b \quad 18$

$a \quad b \quad 4\sqrt{2}$

11. (b, d)

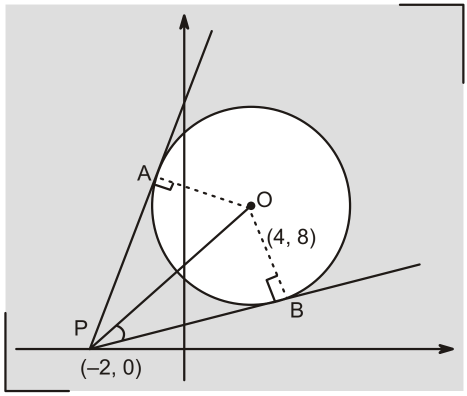
Centre of circles which touches the lines

$x \quad y \quad 1$ and $x \quad y \quad 1$

are $(1 \quad 2\sqrt{2}, 0)$ and

$(1, \quad 2\sqrt{2})$

12. (b, c)



$\sin \quad \frac{2\sqrt{5}}{10} \quad \frac{1}{\sqrt{5}}$

Slopes of PA and PB are $\tan(\qquad)$

where $\tan \quad \frac{8}{6} \quad \frac{4}{3}$

$$\frac{\frac{4}{3} \quad \frac{1}{2}}{1 \quad \frac{4}{3} \quad \frac{1}{2}}, \frac{\frac{4}{3} \quad \frac{1}{2}}{1 \quad \frac{4}{3} \quad \frac{1}{2}}$$

$$\frac{11}{2}, \frac{5}{10}$$

$$A, B \quad 4 \quad 2\sqrt{5} \quad \frac{11}{5\sqrt{5}}, 8 \quad 2\sqrt{5} \quad \frac{2}{5\sqrt{5}},$$

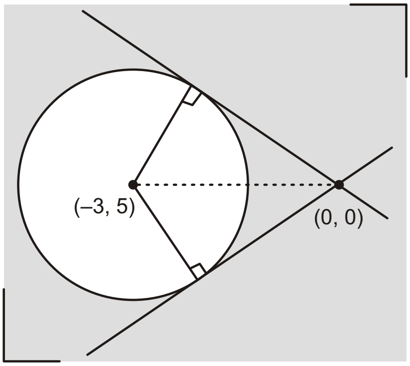
$$4 \quad 2\sqrt{5} \quad \frac{1}{\sqrt{5}}, 8 \quad 2\sqrt{5} \quad \frac{2}{\sqrt{5}} \qquad \frac{2}{5}, \frac{44}{5}$$

$$(6, 4)$$

13. (a, d)

Area of quadrilateral

$15 \quad \sqrt{34 \quad C}\sqrt{C}$



$C^2 \quad 34C \quad 225 \quad 0$

$(C \quad 25)(C \quad 9) \quad 0$

$C \quad 9, 25$

14. (a, b, d)

$9a^2 \quad 6a \quad 1 \quad 5(a^2 \quad b^2)$

$\dfrac{3a \quad 1}{\sqrt{a^2 \quad b^2}} \quad \sqrt{5}$

ar distance of (3, 0)

from $ax \quad by \quad 1 \quad 0$ is $\sqrt{5}$

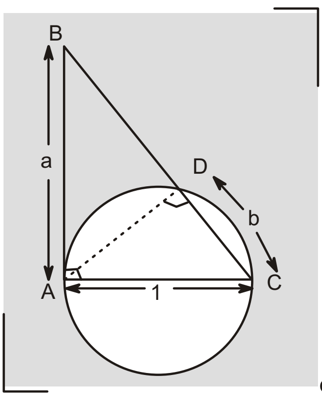
Centre of circle $(3, 0)$

radius $\sqrt{5}$

16. (b, c)

$\tan C \quad \dfrac{AD}{b} \quad \dfrac{a}{1}$

$AD \quad ab \quad AC \quad 1 \qquad ab \quad 1$



c

$(AC)^2 \quad (AD)^2 \quad (CD)^2 \quad a^2b^2 \quad b^2 \quad 1$

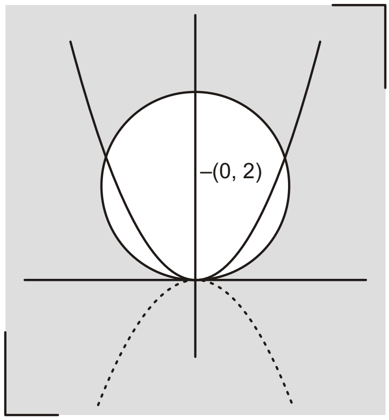
$b^2 \quad \dfrac{1}{a^2 \quad 1}$

$\dfrac{b^2}{a^2} \quad \dfrac{1}{a^4 \quad a^2} \quad \dfrac{1}{a^4 \quad a^2 \quad \frac{1}{4}}$

$\dfrac{1}{a^2 \quad \frac{1}{2}}^{2}$

$\dfrac{b}{a} \quad \dfrac{1}{a^2 \quad \frac{1}{2}}$

17. (a, b, c)



Eqn. of tangent to $y \quad x^2$ is

$tx \quad y \quad \dfrac{1}{4}t^2$

ar from centre $(0, 2) \quad 2$

$\dfrac{\left|\frac{1}{4}t^2 \quad 2\right|}{\sqrt{1 \quad t^2}} \quad 2$

$(t^2 \quad 8)^2 \quad 64(1 \quad t^2)$

$t^4 \quad 48t^2 \quad 0$

$t \quad 0, \quad 4\sqrt{3}$

Eqn. of tangent is $4\sqrt{3}x \quad y \quad 12$

Eqn. of tangent to $y \quad x^2$ is

$tx \quad y \quad \frac{1}{4}t^2$

ar from $(0,2) \quad 2$

$$\frac{\frac{1}{4}t^2 \quad 2}{\sqrt{1 \quad t^2}} \quad 2$$

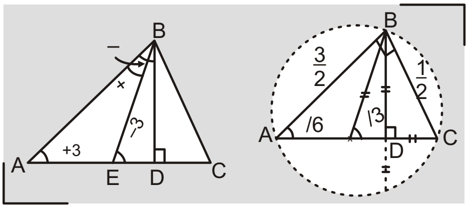
$t^4 \quad 80t^2 \quad 0$

$t \quad 4\sqrt{5}$

Equation of tangents are

$4\sqrt{5}x \quad y \quad 20$

Comprehension: (1)



Also, 3 (3) ( )

[using exterior angle theorem]

a 7

From ABD, 3 $\frac{}{2}$

$\frac{}{24}$, $\frac{7}{24}$

B 2( ) $\frac{}{2}$,

A $\frac{}{6}$, C $\frac{}{3}$

1. (b) Area of circle circumscribing

ABC $\frac{1}{2}$ ^{2} $\frac{}{4}$

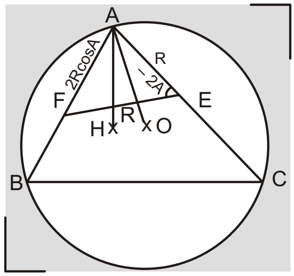
2. (b) BOC is equilateral

$$r \quad \frac{\frac{\sqrt{3}}{4} \quad \frac{1}{2}^{\;2}}{\frac{1}{2} \quad \frac{3}{2}} \quad \frac{1}{4\sqrt{3}}$$

3. (d) BD $\frac{\sqrt{3}}{2}\sin\frac{}{6}$ $\frac{\sqrt{3}}{4}$

BB' $2BD$ $\frac{\sqrt{3}}{2}$

Comprehension: (2)



1. (d) Area of quadrilateral

$BFEC$ ABC AFE

AOB BOC COA AFE

$\frac{1}{2}R^2(\sin 2C$ $\sin 2A$ $\sin 2B$

$\sin($ $2A))$

$\frac{1}{2}R^2(\sin 2B$ $\sin 2C)$

2. (b) AE R; AF $2R\cos A$

EF R

3. (a) If AFE $\frac{}{3}$

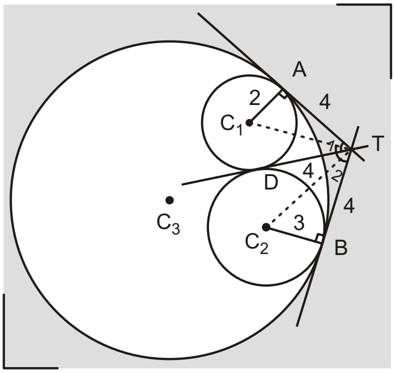
AFE is equilateral

sum of squares of altitudes

3 $\frac{R\sqrt{3}}{2}^{\;2}$ $\frac{9}{4}R^2$

Comprehension: (3)

AT and BT are radical axis to C_3 and C_1 and C_3 & C_2 respectively.



T is radical centre radical axis of C_1 and C_2 *i.e.*, common tangent passes through *T*.

$TA \quad TB \quad TD \quad 4$

$\tan\frac{1}{2} \quad \frac{2}{4} \quad \frac{1}{2}, \tan\frac{2}{2} \quad \frac{3}{4},$

$(\quad ATD \quad _1, \quad BTD \quad _2)$

1. (d) $r_3 \quad TA\tan\frac{1 \quad 2}{2} \quad 4\,\frac{\frac{1}{2} \quad \frac{3}{4}}{1 \quad \frac{1}{2}\,\frac{3}{4}} \quad 8$

2. (b) Circumcircle of TAB will pass through C_3 has TC_3 as diameter

Area $\quad \frac{TC_3}{2}^{2},$

$TC_3 \quad \frac{TA}{\cos\frac{1 \quad 2}{2}} \quad \frac{4}{1/\sqrt{5}} \quad 4\sqrt{5}$

Area $\quad \sqrt{} \quad 20$

3. (d) $C_3C_1 \quad r_3 \quad r_1$

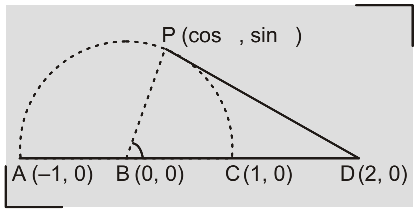
$C_3C_2 \quad r_3 \quad r_2$

$C_3C_1 \quad C_3C_2 \quad r_2 \quad r_1 \quad 1$

Comprehension: (4)

1. (b) Area $\frac{1}{2}(1)^2(\quad) \quad \frac{1}{2}(2)\sin$

$\frac{}{2} \quad \sin \quad \frac{}{2}$

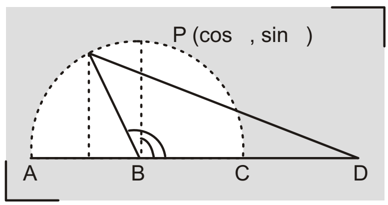


$A \quad \frac{}{2} \quad \sin \quad -$

$\frac{dA}{d} \quad \cos \quad \frac{1}{2}, \quad /3$

A is max. for $\frac{}{3}$

$A_{max} \quad \frac{}{3} \quad \frac{\sqrt{3}}{2} \quad \frac{2 \quad 3\sqrt{3}}{6}$



$A \quad \frac{1}{2}\sin \ (2) \quad \frac{1}{2}(\quad)$

2. (c) $L \quad 3 \quad (\quad) \quad \sqrt{(2 \quad \cos \ )^2 \quad \sin^2}$

$3 \quad (\quad) \quad \sqrt{5 \quad 4\cos}$

3. (c) $\frac{dL}{d} \quad 1 \quad \frac{4\sin}{2\sqrt{5 \quad 4\cos}} \quad 0$

$4\sin^2 \quad 5 \quad 4\cos$

$4\cos^2 \quad 4\cos \quad 1 \quad 0$

$\cos \quad \frac{1}{2} \quad \frac{}{3}$

$L(0) \quad 3 \quad \sqrt{1} \quad 4$

$L(\ )\ \ 3\ \ \sqrt{5\ \ 4}\ \ 6$

$L(\ /3)\ \ 3\ \ \frac{2}{3}\ \ \sqrt{5\ \ \frac{4}{2}}\ \ 3\ \ \sqrt{3}\ \ \frac{2}{3}$

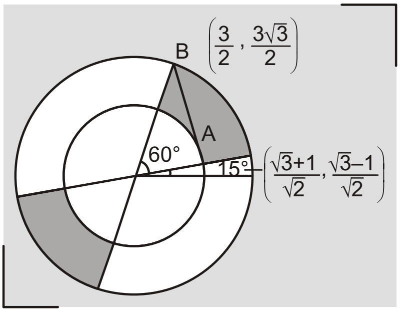
$L_{max}\ \ 4$

$L_{min}\ \ 6$

$L_{max}\ \ L_{min}\ \ \ \ 2$

Comprehension: (5)

Let $\frac{b}{a}\ \ \tan$



$\frac{b}{a}^{\ 2}\ \ \frac{4b}{a}\ \ 1\ \ 0$

$\tan\ \ \ [2\ \ \sqrt{3}, 2\ \ \sqrt{3}]$

$[15\ \ , 75\ \ ]$

1. (d) Area $\ \frac{}{3}(9\ \ 4)\ \ \frac{5}{3}$

2. (c) $|r(\cos\ \ \ \sin\ \ )|\ \ r\sqrt{2}|\sin(\ \ \ \ 45\ \ )|$

$2\sqrt{2}\sin(15\ \ \ 45\ \ )$

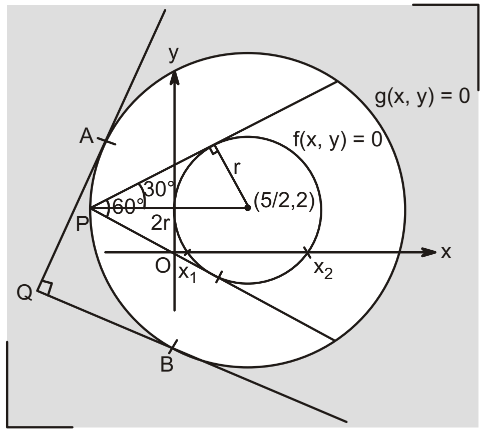
minimum $\ 2\sqrt{2}\ \frac{\sqrt{3}}{2}\ \ \sqrt{6}$

3. (d) $(AB)\ \ \sqrt{9\ \ \ 4\ \ \ 2(3)(2)\cos 60}$

$\sqrt{7}$

Comprehension: (6)

$g(x, y)\ \ 0$ represent a circle concentric with circle $f(x, y)\ \ 0$ with radius twice the radius of $f(x, y)\ \ 0$, centre $\ \frac{5}{2}, 2$



$f(x, y)\ \ \ x^2\ \ \ y^2\ \ \ 5x\ \ \ 4y\ \ \ 4\ \ 0$

$g(x, y)\ \ \ x^2\ \ \ y^2\ \ \ 5x\ \ \ 4y\ \ \ \frac{59}{4}\ \ \ 0$

1. (d) Area of $\ \ QAB\ \ \frac{1}{2}\ \ 5\ \ 5\ \ \frac{25}{2}$

2. (d) $\ \ \ \tan^{\ 1}\frac{3}{4}$

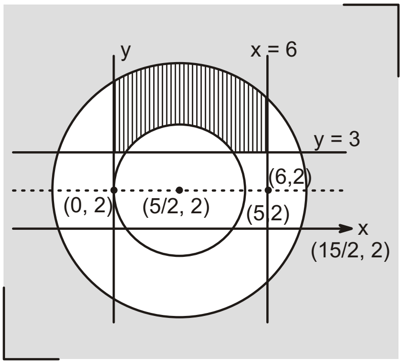
$2\ \ \ \tan^{\ 1}\frac{2\ \ \frac{3}{4}}{1\ \ \ \frac{9}{16}}\ \ \ \tan^{\ 1}\frac{24}{7}$

Area of region inside $f(x, y)\ \ 0$ above x-axis

$\frac{1}{2}\ \frac{5}{2}^{\ 2}\ \ 2\ \ \ \ \tan^{\ 1}\frac{24}{7}\ \ \ \ \frac{1}{2}\ \ 3\ \ 2$

$3\ \ \frac{25}{8}\ \ 2\ \ \ \ \tan^{\ 1}\frac{24}{7}$

3. (d) Points satisfying the conditions are



(1, 5) (1, 6), (2, 5), (2, 6) (3, 5), (3, 6)
(4, 5), (4, 6), (5, 4), (5, 5), (5, 6).

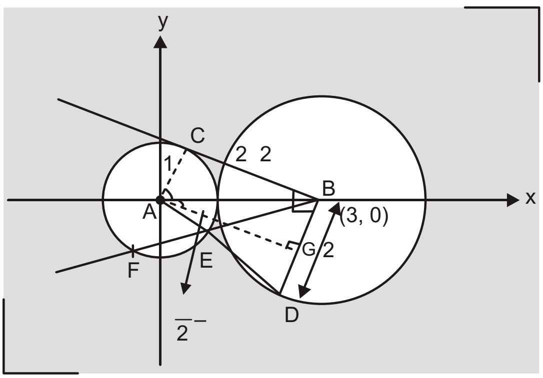
Comprehension: (7)

1. (c) 2. (c) 3. (c)

$\tan \quad 2\sqrt{2}$

$AG \quad 2\sqrt{2}$

$m_{BD} \quad 2\sqrt{2}$



$$D \quad 3 \quad 2 \quad \frac{1}{3}, 0 \quad 2 \quad \frac{2\sqrt{2}}{3}$$

$$\frac{7}{3}, \frac{4\sqrt{2}}{3}$$

$$m_{AE} \quad \tan(\quad 2 \ ) \quad \tan 2$$

$$\frac{2(2\sqrt{2})}{1 \quad 8} \quad \frac{4\sqrt{2}}{7}$$

$$E \quad 0 \quad 1 \quad \frac{7}{9}, 0 \quad 1 \quad \frac{4\sqrt{2}}{9}$$

$$\frac{7}{9}, \frac{4\sqrt{2}}{9}$$

Eqn. of BE is $y \quad \frac{\sqrt{2}}{5}(x \quad 3)$, Solving intersection of line BE with circle S_1,

$$x^2 \quad \frac{2}{25}(x \quad 3)^2 \quad 1$$

$$27x^2 \quad 12x \quad 7 \quad 0$$

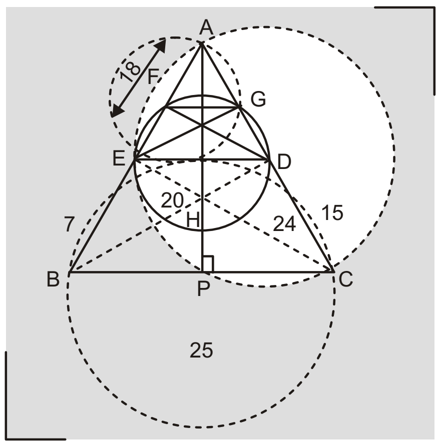
$$(9x \quad 7)(3x \quad 1) \quad 0$$

$$F \quad \frac{1}{3}, \frac{2\sqrt{2}}{3}$$

Comprehension: (8)

1. (d) 2. (c) 3. (b)

$$\sin C \quad \frac{4}{5}, \sin B \quad \frac{24}{25}$$



$$\tan A \quad \tan(B \quad C) \quad \frac{\frac{24}{7} \quad \frac{4}{3}}{1 \quad \frac{24}{7} \quad \frac{4}{3}}$$

$\frac{4}{3}$ $\tan C$

A C

BC AB 25

AC $2(CD)$ 30

AB BC CA 80

AP CE 24

DE AD DC 15

Area of S $\frac{(15)^2}{4}$ $\frac{225}{4}$

($AEPC$ is cyclic quadrilateral with AC as diameter)

$\because$ $EFGD$ is cyclic

AFG GDE

$\because$ $BEDC$ is cyclic GDE ABC

AFG ABC $FG||BC$

$AFG \sim ABC$

$\frac{AK}{AP}$ $\frac{AF}{AB}$

AF $\frac{1}{2}AE$ 9 $\because AD$ DE

AK $\frac{9 \quad 24}{25}$ $\frac{216}{25}$

Comprehension: (9)

1. (c) 2. (d) 3. (c)

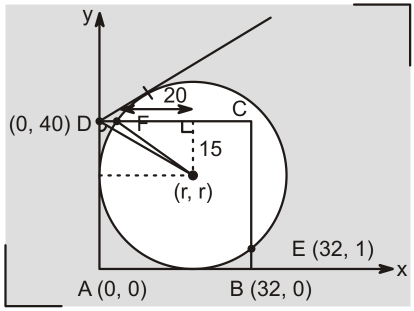
Equation of circle is

$x^2 \quad y^2 \quad 2rx \quad 2ry \quad r^2 \quad 0$

Put (32, 1)

$r^2 \quad 66r \quad 1025 \quad 0$

$r \quad 25,\ r \quad 41,\ r \quad 41$, is rejected



$\tan^{1} \frac{25}{15}$

$2 \quad 2\tan^{1}\frac{5}{3} \quad \tan^{1}\frac{15}{8}$

Area of

$AFCB$ $\frac{1}{2}(40)(32 \quad 27) \quad 1180$

Comprehension: (10)

1. (d) n $n \quad 1$

$1 \quad n \quad 1, 1 \quad n \quad 1, \quad , \quad 1$

No. of points (,)

${}^{n\ 1\ 2\ 2}C_2 \quad {}^{n\ 1}C_2 \quad \frac{(n \quad 1)(n \quad 2)}{2}$

$\frac{n^2 \quad 3n \quad 2}{2}$

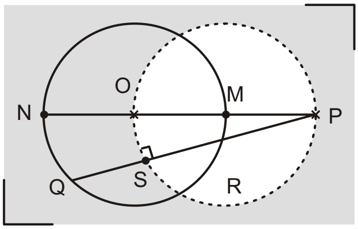
2. (b) Number of rational points is 2 given by (5,0) and (1,0).

3. (c) Lattice points are (5,0),(0, 5), (3, 4),(4, 3)

$2 \quad 2 \quad 4 \quad 4 \quad 12$

Comprehension: (14)

1. (d) Locus of S is a part of circle with OP as diameter passing inside the circle C.



2. (d) $(PR)(PQ) \quad (NP)(MP) \quad (d \quad r)(d \quad r)$

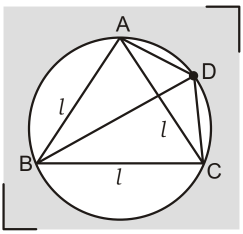
$$d^2 \quad r^2$$

$$(PS \quad SR)(PS \quad SQ) \quad PS^2 \quad SQ^2$$

$$(\because SQ \quad SR)$$

$$(PS)^2 \quad (SQ)(SR)$$

3. (a) Using Ptolemy's theorem



$$(BD)(AC) \quad (AB)(CD) \quad (BC)(AD)$$

$$l \quad l \qquad l$$

Comprehension: (16)

1. (b) By homogenisation,

$$x^2 \quad y^2 \quad 2(3x \quad 5y)$$

$$\frac{y \quad ax}{b} \qquad \frac{y \quad ax}{b}^{2} \quad 0$$

As the angle subtended at origin $\quad /2$

coefficient of x^2 $\quad$ coefficient of $y^2 \quad 0$

$$1 \quad 1 \quad \frac{6a}{b} \quad \frac{10}{b} \quad \frac{1}{b^2} \quad \frac{a^2}{b^2} \quad 0$$

$$a^2 \quad 2b^2 \quad 6ab \quad 10b \quad 1 \quad 0$$

locus of (a, b) will be

$$g(x,y) \quad x^2 \quad 2y^2 \quad 6xy \quad 10y \quad 1 \quad 0$$

2. (d) $y \quad 1$ cuts the curve $g(x, y)$ at $x^2 \quad 6x \quad 7 \quad 0$

$$x \quad 7, \quad 1$$

in first quadrant at $(7, 1)$

$$\frac{dy}{dx} \qquad \frac{x \quad 3y}{2y \quad 3x \quad 5}\bigg|_{(7,1)} \qquad \frac{4}{24} \quad \frac{1}{6}$$

3. (a) Equation of the chord with middle point (1, 2) will be $T \quad S_1$

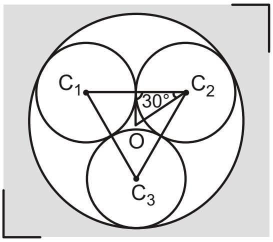
$$x \;\; 1 \quad y \;\; 2 \quad 3(x \quad 1) \quad 5(y \quad 2) \quad 1$$

$$1^2 \quad 2^2 \quad 6 \;\; 1 \quad 10 \;\; 2 \quad 1$$

$$4x \quad 3y \quad 2 \quad 0$$

Assertion and Reason

1. (D) $C_1C_2C_3$ is equilateral



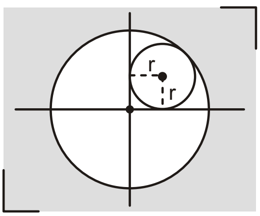
$$OC_2 \quad r_2 \quad r_1$$

$$r_1 \quad (r_2 \quad r_1)\cos 30$$

$$\frac{r_2}{r_1} \quad \frac{2}{\sqrt{3}} \quad 1 \quad \frac{2 \quad \sqrt{3}}{\sqrt{3}}$$

$$\frac{r_1}{r_2} \quad \frac{\sqrt{3}}{2 \quad \sqrt{3}}$$

2. (C)

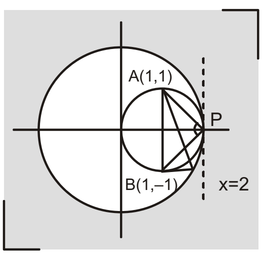


$$C_1C_2 \quad r\sqrt{2}$$

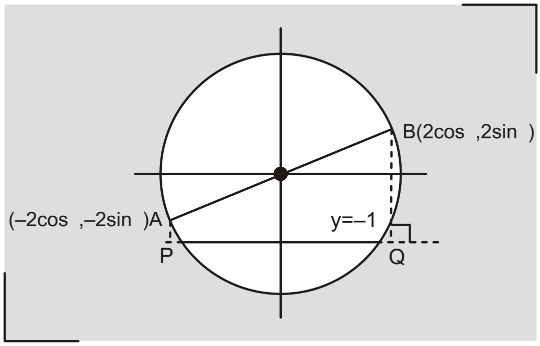
$$2 \quad r \quad r\sqrt{2}$$

$$r^2 \quad 4r \quad 4 \quad 0$$

3. (A)



4. (C)



$$P_1 \quad P_2 \quad |2\sin \quad 1| \ | \ 2\sin \quad 1| \quad 2$$

If A, B lie on opposite sides of PQ, then

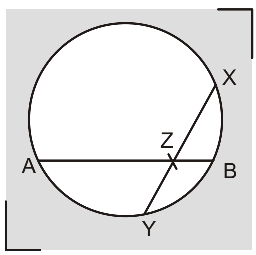
$P_1 \quad P_2$ = constant

5. (C) If ACB and CBD are minor segments of AB and CD then $AP \quad PD, PB \quad PC$

If ACB is minor and CBD is major segment of AB and CD respectively then, $PB \quad PD$ and $AP \quad PC$.

6. (D) $(XZ)(YZ) \quad (XZ)^2$

$$(AZ)(ZB) \quad \frac{AZ \quad ZB}{2}^{2}$$

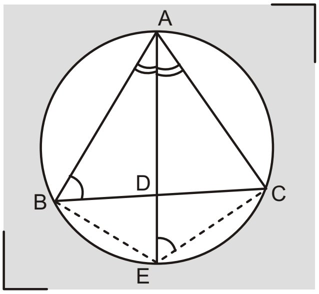


$$XZ \quad \frac{AB}{2}$$

$$XY \quad AB$$

7. (A) ∵ $ABD \sim AEC$

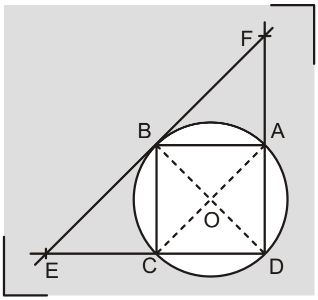
$$\frac{AB}{AE} \quad \frac{AD}{AC}$$



$$(AD)(AE) \quad (AB)(AC) \quad (AD)^2$$

$$AD \quad \sqrt{(AB)(AC)}$$

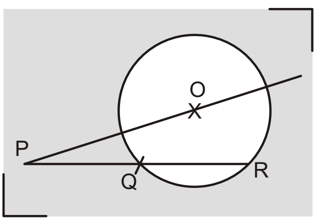
8. (C)



$$(CE)(DE) \quad (OE)^2 \quad r^2$$

$$(FA)(FD) \quad (OF)^2 \quad r^2$$

∵ $OE \quad OF$



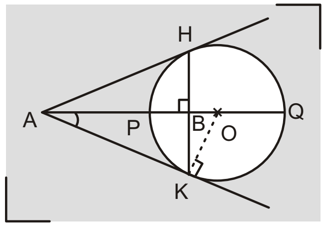
$$(CE)(DE) \quad (FA)(FD)$$

∵ $(PQ)(PR) \quad (OP \quad r)(OP \quad r)$

$$(OP)^2 \quad r^2$$

9. (A) Use diametrical form of circle with $A(x_1, y_1)$ and $B(x_2, y_2)$ as diameter.

10. (A)



$$\frac{AP \quad AQ}{2} \quad \frac{(OA \quad r) \quad (OA \quad r)}{2} \quad OA$$

$$\cos \quad \frac{AK}{OA} \quad \frac{AB}{AK}$$

$$(AK)^2 \quad (OA)(AB)$$

Also $(AK)^2 \quad (AP)(AQ)$

$$(OA)(AB) \quad (AP)(AQ)$$

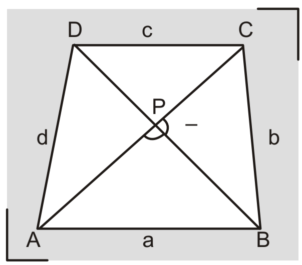
$$\frac{AP \quad AQ}{2} \; AB \quad (AP)(AQ)$$

$$AB \quad \frac{2(AP)(AQ)}{(AP \quad AQ)}$$

11. (C) S-1 is true, S-2 is false

Case-I If $[\;/2,\;)$

$$a^2 \quad (PA)^2 \quad (PB)^2 \quad 2(PA)(PB)\cos$$
$$(PA)^2 \quad (PB)^2$$



$$b^2 \quad (PB)^2 \quad (PC)^2$$

$$c^2 \quad (PC^2) \quad (PD)^2$$

$$d^2 \quad (PD)^2 \quad (PA)^2$$

$$a^2 \quad c^2 \quad (PA)^2 \quad (PB)^2 \quad (PC)^2$$
$$(PD)^2 \quad b^2 \quad d^2$$

Case-II If $0, \overline{}/2$

$$a^2 \quad (PA)^2 \quad (PB)^2,$$

$$b^2 \quad (PB)^2 \quad (PC)^2,$$

$$c^2 \quad (PC)^2 \quad (PD)^2,$$

$$d^2 \quad (PD)^2 \quad (PA)^2$$

$$a^2 \quad c^2 \quad (PA)^2 \quad (PB)^2 \quad (PC)^2$$

$$(PD)^2 \quad b^2 \quad d^2$$

If $a^2 \quad c^2 \quad b^2 \quad d^2$

$$a^2 \quad (PA)^2 \quad (PB)^2,$$

$$b^2 \quad (PB)^2 \quad (PC)^2,$$

$$c^2 \quad (PC)^2 \quad (PD)^2,$$

$$d^2 \quad (PD)^2 \quad (PA)^2$$

$$\overline{}/2$$

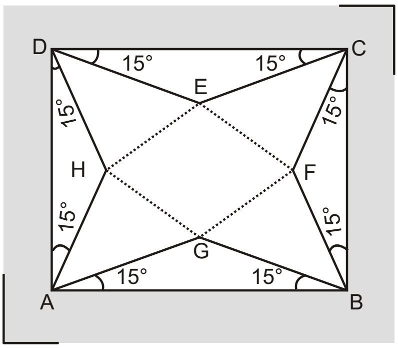
Match the Columns:

1. a r; b q; c p; d s

(a) $FH \quad 1 \quad 2 \quad \frac{1}{2}\tan 15 \quad \sqrt{3} \quad 1$

Area, $A \quad \frac{1}{2}d_1 d_2 \quad \frac{1}{2}(\sqrt{3} \quad 1)^2 \quad 2 \quad \sqrt{3}.$

$a \quad b \quad 5$



(b) $AEB \quad \frac{}{3}$

(c) $2R \quad \frac{AD}{\sin(150 \ )} \quad \frac{1}{(1/2)} \quad 2$

$R \quad 1$

(d) $h_1 \quad a\sin 60 \quad \frac{1/2}{\cos 15} \quad \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{\sqrt{2}(\sqrt{3} \quad 1)}$

$\frac{1}{h_1^2} \quad \frac{1}{h_2^2} \quad \frac{1}{h_3^2} \quad 3 \quad \frac{2(4 \quad 2\sqrt{3})}{3}$

$8 \quad 4\sqrt{3} \quad 8 \quad \sqrt{48} \quad \frac{b}{a} \quad 6$

2. a s; b p; c r; d s

$(2x_1x_2 \quad 2x_2x_3 \quad 2x_3x_1 \quad x_1^2 \quad x_2^2 \quad x_3^2)$

$(y_1^2 \quad y_2^2 \quad y_3^2 \quad 2y_1y_2 \quad 2y_2y_3 \quad 2y_3y_1) \quad 0$

$(x_1 \quad x_2 \quad x_3)^2 \quad (y_1 \quad y_2 \quad y_3)^2 \quad 0$

$x_1 \quad x_2 \quad x_3 \quad 0$

and $y_1 \quad y_2 \quad y_3 \quad 0$

Circumcenter and centroid of ABC coincide

ABC is equilateral

(a)

$(PA)^2 \quad (PB)^2 \quad (PC)^2$

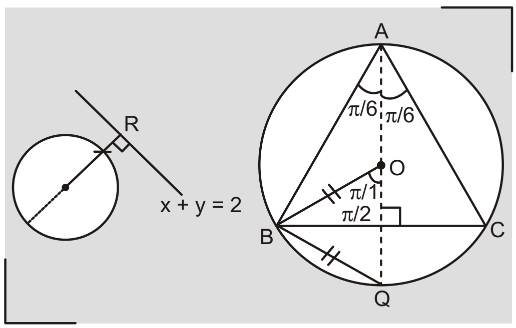
$(x \quad x_1)^2 \quad (y \quad y_1)^2 \quad (x \quad x_2)^2$

$(y \quad y_2)^2$

$(x \quad x_3)^2 \quad (y \quad y_3)^2$

$3(x^2 \quad y^2 \quad 1) \quad 6$

(b) $OBQ \quad \frac{}{3}, k \quad 3$



(c) Max. distance of R from S,

$d \quad \frac{2}{\sqrt{2}} \quad 1 \quad \sqrt{2} \quad 1$

$d^2 \quad 3 \quad 2\sqrt{2}$

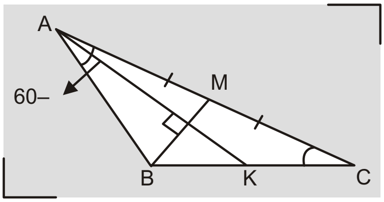
$a \quad b \quad 3 \quad 2 \quad 5.$

(d) I and G coincide with origin

$IA \quad IB \quad IC \quad GA \quad GB \quad GC \quad 1$

$IA \quad IB \quad IC \quad GA \quad GB \quad GC \quad 6.$

3. a q; b s; c p; d r



AMB is isosceles

∵ angle bisector of A is to MB

$$\frac{c}{\sin} \quad \frac{2c}{\sin 60} \qquad \sin \quad \frac{\sqrt{3}}{4}$$

[∵ AB c, AC $2c$]

$$a^2 \quad c^2 \quad 4c^2 \quad 4c^2 \cos(60 \quad)$$

$$5c^2 \quad 4c^2 \quad \frac{\sqrt{13}}{4}\frac{1}{2} \quad \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{4}$$

$$a \quad \frac{\sqrt{7 \quad \sqrt{13}}}{\sqrt{2}} c$$

$$\frac{\sqrt{14 \quad 2\sqrt{13}}}{2} c \quad \frac{\sqrt{13} \quad 1}{2} c$$

(a) $\frac{a}{c} \quad \frac{\sqrt{13} \quad 1}{2}$

(b) $R \quad \frac{abc}{4} \quad \frac{ac(2c)}{2ac\frac{\sqrt{3}}{2}} \quad \frac{2c}{\sqrt{3}}$

Alt. $2R \quad \frac{c}{\sin} \qquad \frac{R}{c} \quad \frac{1}{\frac{2\sqrt{3}}{4}} \quad \frac{2}{\sqrt{3}}$

(c) $\frac{}{R^2} \quad \frac{\frac{1}{2} ac\frac{\sqrt{3}}{2}}{\frac{4}{3}c^2}$

$$\frac{3\sqrt{3}}{16}\frac{a}{c} \quad \frac{3\sqrt{3}}{32}(\sqrt{13} \quad 1)$$

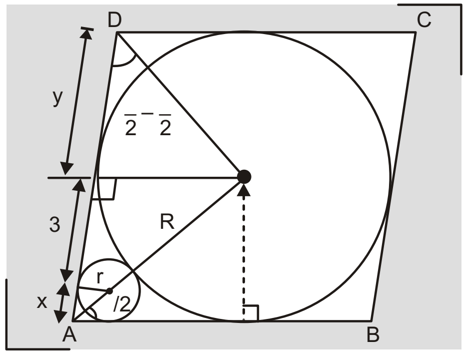
(d) $AB \quad c, AC \quad 2c, \frac{AB}{AC} \quad \frac{1}{2}.$

4. a s; b p; c q; d r

$$2\sqrt{Rr} \quad 3 \qquad r \quad \frac{9}{4 \quad 3} \quad \frac{3}{4}$$

$$\frac{r}{R} \quad \frac{x}{x \quad 3} \quad \frac{1}{4} \qquad x \quad 1$$

$$\tan\frac{}{2} \quad \frac{3}{4}.$$



$$y \quad R\cot\frac{}{2} \quad \frac{}{2}$$

$$3\tan\frac{}{2} \quad \frac{9}{4}$$

$ABCD$ is a rhombus with side

$$4 \quad \frac{9}{4} \quad \frac{25}{4}.$$

Area of $ABCD$

$$\frac{25}{4} \quad \frac{25}{4}\sin \qquad \frac{25}{4} \quad \frac{25}{4}\frac{2\frac{3}{4}}{1 \quad \frac{9}{16}}$$

$$A \quad \frac{75}{2} \quad \frac{1}{2}d_1 d_2$$

$$d_1 d_2 \quad 75$$

$$\frac{d_1^2}{4} \quad \frac{d_2^2}{4} \quad \frac{625}{16} \quad \frac{(d_1 \quad d_2)^2 \quad 2d_1 d_2}{16}$$

$$d_1 \quad d_2 \quad \sqrt{775} \quad 5\sqrt{31}$$

5. a s; b r; c q; d p

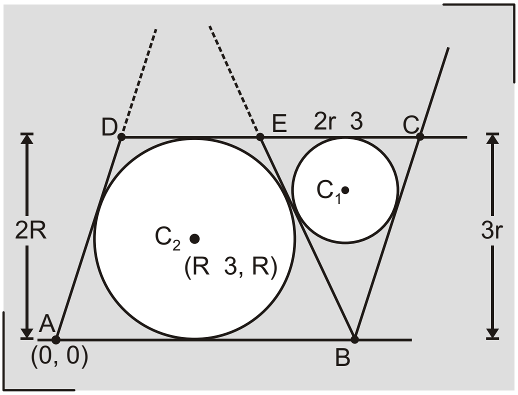
ar distance between AB and CD

$2R \quad 3r \qquad R \quad \frac{3r}{2}$

$C_1 \quad (2R\sqrt{3}, 2r) \quad (3\sqrt{3}r, 2r)$

$C_2 \quad (R\sqrt{3}, R) \quad \frac{3\sqrt{3}r}{2}, \frac{3r}{2}$

$C_1C_2 \quad \sqrt{\frac{r^2}{4} \quad \frac{27r^2}{4}} \quad r\sqrt{7}$



Length of common tangent

$\sqrt{7r^2 \quad (R \quad r)^2}$

$\sqrt{7r^2 \quad \frac{25r^2}{4}} \quad \frac{\sqrt{3}}{2}r.$

6. a s; b q; c r; d p

(a) Q lies on y-axis *i.e.*, OQ = radius

$\frac{y_P \quad 0}{11 \quad 5} \quad 1$

$y_P \quad 16$

(b) $x_Q \quad \frac{11 \quad 5}{2} \quad 3, y_Q$

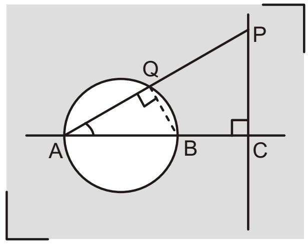
$\sqrt{25 \quad 9} \quad 4$

$\frac{y_P \quad Q}{2} \quad 4$

$y_P \quad 8$

(c) Let QAB

$\frac{1}{4}\frac{1}{2}(16)(16\tan \quad) \quad \frac{1}{2} \quad (10\cos \quad)(10\sin \quad)$



$\cos^2 \quad \frac{16 \quad 16}{4 \quad 10 \quad 10} \quad \frac{16}{25}$

$\cos \quad \frac{4}{5}$

$y_P \quad 16 \quad \frac{3}{4} \quad 12.$

(d) $y_P \quad 0.$

7. a p, q; b p, q; c q; d q, s

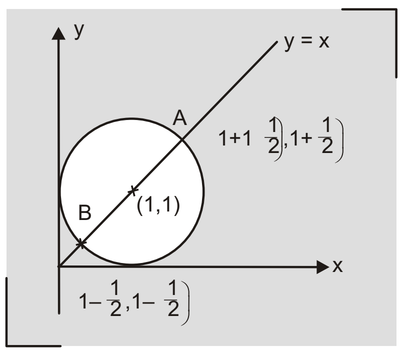
8. a q; b s; c r; d p

(a) Using parametric form,

$A \quad 1 \quad \frac{1}{\sqrt{2}}, 1 \quad \frac{1}{\sqrt{2}} \quad ,$

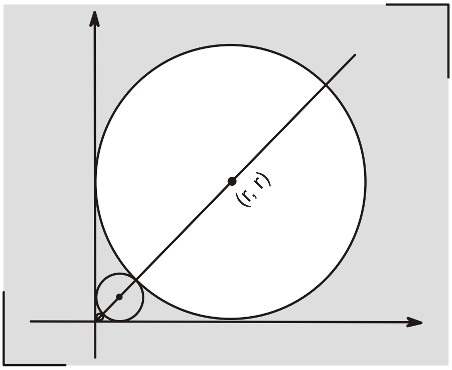
$B \quad 1 \quad \frac{1}{\sqrt{2}}, 1 \quad \frac{1}{\sqrt{2}} \quad ,$

$r \quad 1 \quad \frac{1}{\sqrt{2}} \quad \frac{\sqrt{2} \quad 1}{\sqrt{2}} \quad \frac{2 \quad \sqrt{2}}{2}$



(b) $C_1C_2 \quad r_1 \quad r_2$

$2(r \quad 1)^2 \quad (r \quad 1)^2$



$r^2 \quad 6r \quad 1 \quad 0$

$r \quad 3 \quad 2\sqrt{2}$

(c) $C \quad x^2 \quad y^2 \quad 2x \quad 2y \quad 1 \quad 0$

$C_1 \quad x^2 \quad y^2 \quad 2rx \quad 2ry \quad r^2 \quad 0$

orthogonality

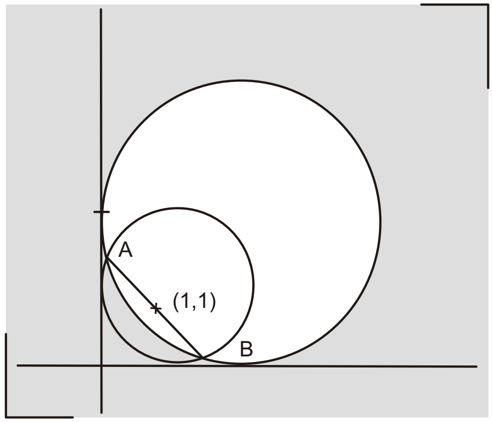
$2r(1) \quad 2r(1) \quad r^2 \quad 1$

$r^2 \quad 4r \quad 1 \quad 0$

$r \quad 2 \quad \sqrt{3}$

$r \quad 2 \quad \sqrt{3}$

(d) Equation of common chord *AB* is



$(x^2 \quad y^2 \quad 2x \quad 2y \quad 1) \quad (x^2 \quad y^2 \quad 2rx \quad 2ry \quad r^2) \quad 0$

$(2r \quad 2)x \quad (2r \quad 2)y \quad (1 \quad r^2) \quad 0$

Put $(1,1)$

$4r \quad 4 \quad 1 \quad r^2 \quad 0$

$r^2 \quad 4r \quad 3 \quad 0 \qquad (r \quad 1)(r \quad 3) \quad 0$

$r \quad 3$

9. a p; b q; c r; d s

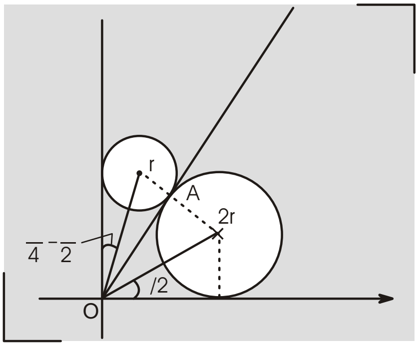
10. a q; b q; c t; d p

Subjective Problems

1. (22)

$OA \quad r\cot\frac{}{4} \quad \frac{}{2} \qquad 2r\cot\frac{}{2}$

Let, $\tan\frac{}{2} \quad t$



$\frac{1 \quad t}{1 \quad t} \quad \frac{2}{t}$

$t \quad \frac{3 \quad \sqrt{17}}{2}$

$\tan\frac{}{2} \quad \frac{\sqrt{17} \quad 3}{2}$

$a \quad b \quad c \quad 17 \quad 3 \quad 2 \quad 22$

2. (9)

$\cos \quad \frac{1/2}{2} \quad \frac{1}{4}$ [from OED]

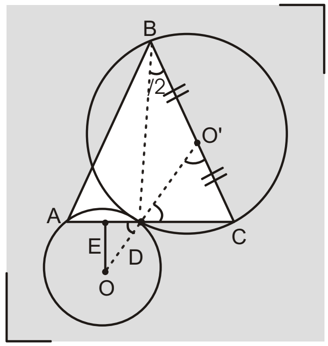
2

$BD \quad 2\cot\frac{}{2} \quad 2\cot\frac{}{2} \qquad 2\tan$

[from BDC]

$2\sqrt{15}$

Area of $ABC \quad \frac{1}{2} \quad (AC)(BD)$



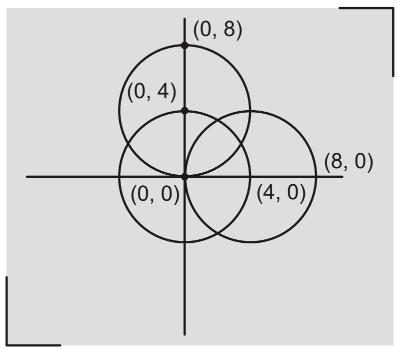
$\frac{1}{2} \quad 3 \quad 2\sqrt{15} \quad 3\sqrt{15} \quad \sqrt{135}$

$\frac{A^2}{15} \quad 9$

3. (6)

Points satisfying the inequalities are common to all 3 circles given by (1, 1), (1, 2); (2, 1), (2, 2), (2, 3), (3, 2)

number of ordered pairs $(a, b) \quad 6$



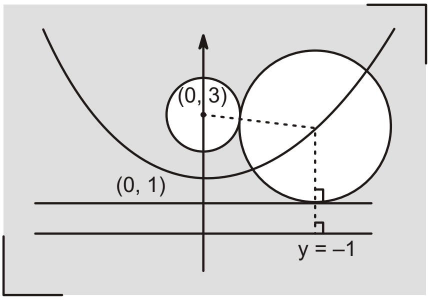
4. (8) Locus is parabola with directrix

$y \quad 1$ and focus (0,3) given by

$x^2 \quad 4(2)(y \quad 1)$

$\frac{x^2}{8} \quad 1 \quad y$

8

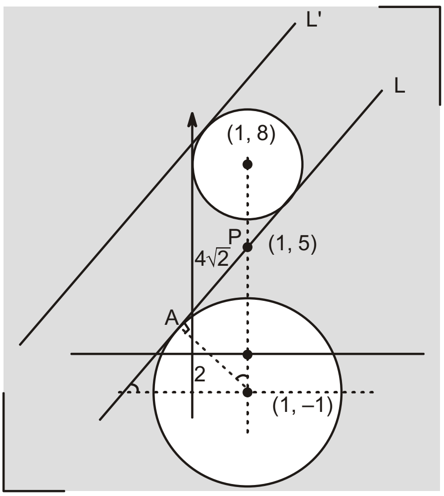


5. (7)

Equation of transverse common tangent L with positive slope is $y \quad 5 \quad 2\sqrt{2}(x \quad 1)$

$$[\because \text{slope} \quad \tan \quad \frac{4\sqrt{2}}{2} \quad 2\sqrt{2}]$$

$L \quad 2\sqrt{2}x \quad y \quad (5 \quad 2\sqrt{2}) \quad 0$



Let equation of tangent L' which is parallel to L is $2\sqrt{2}x \quad y \quad C$

$$\left|\frac{2\sqrt{2}(1) \quad 8 \quad C}{\sqrt{8 \quad 1}}\right| \quad 1$$

$2\sqrt{2} \quad 8 \quad C \quad 3$

$C \quad 2\sqrt{2} \quad 11, 2\sqrt{2} \quad 5$

equation of L is

$2\sqrt{2}x \quad y \quad 11 \quad 2\sqrt{2} \quad 0$

$$\frac{a \quad b \quad c}{2} \quad \frac{2 \quad 1 \quad 11}{2} \quad 7$$

6. (4)

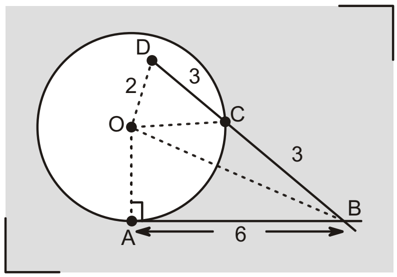
$$OC^2 \quad r^2 \quad \frac{2(OD^2 \quad OB^2) \quad BD^2}{4}$$

[Length of median dropped from 'O' to BD in OBD]

$$r^2 \quad \frac{2(4 \quad (OB)^2) \quad 6^2}{4} \qquad \text{...(1)}$$

Also,

$$(OB)^2 \quad (OA)^2 \quad (AB)^2 \quad r^2 \quad 36 \qquad \text{...(2)}$$



From (1) and (2), we get

$4r^2 \quad 2(r^2 \quad 36) \quad 28$

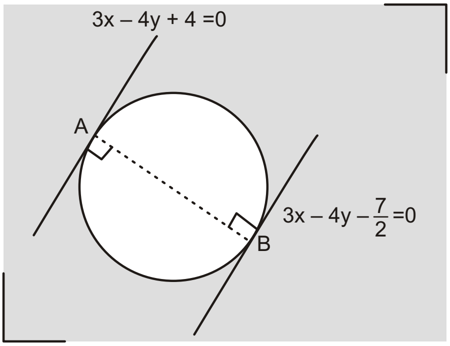
$r^2 \quad 22 \quad r \quad \sqrt{22}$

$[r] \quad 4$

7. (3)

$2r$ perpendicular distance between two parallel tangents

$$2r \quad \left|\frac{4 \quad \frac{7}{2}}{5}\right| \quad \frac{15}{2 \quad 5} \quad \frac{3}{2}$$



$r \quad \frac{3}{4} \qquad 4r \quad 3$

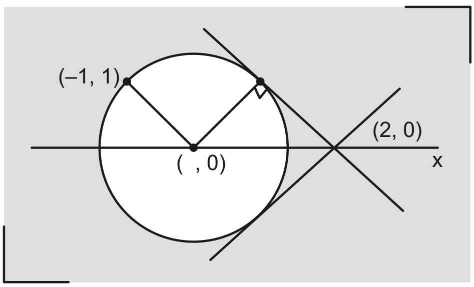
8. (6)

$r^2 \quad (\quad 1)^2 \quad 1 \quad \frac{| \quad 2|}{\sqrt{2}}^2$

$\frac{(\quad 2)^2}{2}$

$^2 \quad 8 \quad 0$

$0, \quad 8$



Equation of circle are

$x^2 \quad y^2 \quad 2$

and $(x \quad 8)^2 \quad y^2 \quad 50$

$i.e., x^2 \quad y^2 \quad 2 \quad 0$

and $x^2 \quad y^2 \quad 16x \quad 14 \quad 0$

$r_1 \quad r_2 \quad \sqrt{2} \quad \sqrt{50} \quad 6\sqrt{2}$

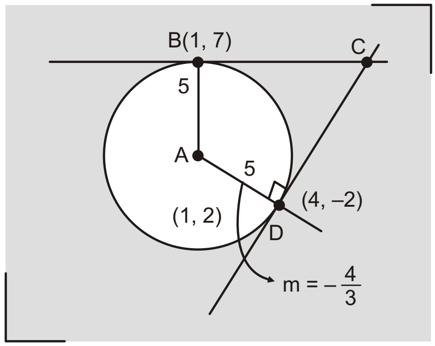
9. (5)

Equation of CD is

$y \quad 2 \quad \frac{3}{4}(x \quad 4)$

$4y \quad 3x \quad 20 \quad 0$

Put $y \quad 7$



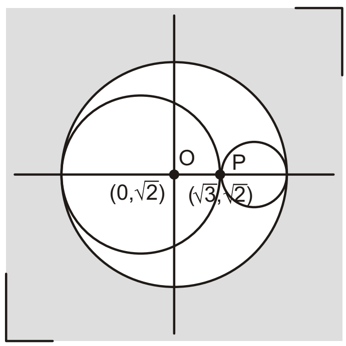
$C \quad (16,7)$

Area of quadrilateral $ABCD$,

$N \quad 2 \quad \frac{1}{2} \quad 5 \quad 15 \quad 75$

$\frac{N}{15} \quad 5$

10. (1) Maximum and minimum distance of point $P(\sqrt{3}, \sqrt{2})$ from circle is $2 \quad \sqrt{3}$ and $2 \quad \sqrt{3}$



Largest and smallest circles passing through P must have $\sqrt{3} \quad 2$ and $2 \quad \sqrt{3}$ as diameters.

$2(r_1 \quad r_2) \quad \sqrt{3} \quad 2 \quad 2 \quad \sqrt{3} \quad 4$

$\frac{r_1 \quad r_2}{2} \quad 1$