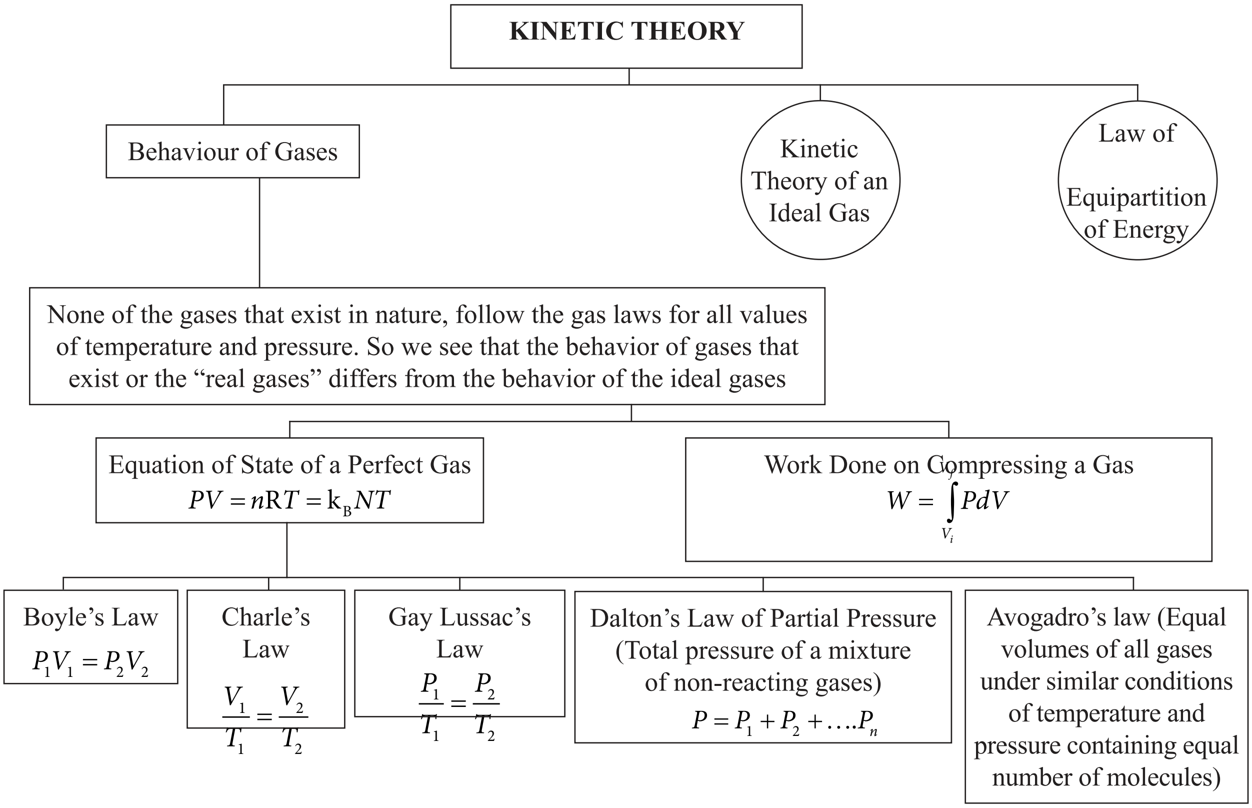
CHAPTER | 13 |

Kinetic Theory

Syllabus

- **Behaviour of gases:** Equation of state of a perfect gas; Work done on compressing a gas.
- **Kinetic theory of an ideal gas:** Assumptions; Concept of pressure; Kinetic energy and temperature, rms speed of gas molecule.
- **Law of equipartition of energy:** Degrees of freedom; Application of specific heat capacities of gases, concept of mean-free path.

MIND MAP



Mind Map 1: Kinetic Theory

KINETIC THEORY OF AN IDEAL GAS

(Gas consists of a very large number of molecules with perfect elastic spheres)

- **Assumptions of Kinetic Theory of Gases**
  - All the molecules of a gas are identical
  - Molecules of different gases are different
  - Molecules of gases are in a state of random motion
  - Collisions of gas molecules are perfectly elastic
- **Vander Waal's Gas Equations**
  - For 1 mole of gas
    $$\left(P+\frac{a}{V^2}\right)(V-b)=RT$$
  - For n moles of gas
    $$\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$$
- **Pressure of an Ideal Gas**
  $$P=\frac{1}{3}\rho v^2_{\text{rms}}$$
  $$=\frac{1}{3}\frac{M}{V}v^2_{\text{rms}}$$
- **Kinetic Energy and Temperature**
  Average kinetic energy per molecule of a gas $=\frac{3}{2}k_BT$
- **RMS Speed of a Gas**
  $$v_{\text{rms}}=\sqrt{\frac{3RT}{M}}$$
- **Maxwell Distribution Function**
  $$v+dv.dN(v)=4pNa^3e^{-bv^2}v^2dv=n_vdv$$

Mind Map 2: Kinetic Theory of an Ideal Gas

LAW OF EQUIPARTITION OF ENERGY

(For any system in thermal equilibrium, total energy is equally distributed among its various degrees of freedom)

- **Degree of Freedom**
  (Total number of independent modes in which a system can possess energy)
- **Application of Specific Heat Capacities of Gases**
  - For an ideal gas
    $$C_P-C_V=R$$
  - For monoatomic,
    $$\frac{C_P}{C_V}=\frac{5}{3}$$
  - For diatomic,
    $$\frac{C_P}{C_V}=\frac{7}{5}$$
  - For polyatomic gases,
    $$\frac{C_P}{C_V}=\frac{(4+f)}{(3+f)}$$
- **Mean Fee Path**
  $$\lambda=\frac{m}{\sqrt{2}\pi d^2\rho}$$

Mind Map 3: Law of Equipartition of Energy

Behaviour of Gases

- For ideal gas, the equation of states is
  $PV = nRT$
- It is a result of combination of Boyle's and Charle's laws. Where n is the number of moles and $R = N_A k_B$ is a universal constant. The temperature T is absolute temperature. Choosing kelvin scale for absolute temperature, $R = 8.314\ J\ mol^{-1}K^{-1}$.
  Here,
  $$n = \frac{M}{M_0} = \frac{N}{N_A}; P = \frac{\rho RT}{M_0}$$
- A gas that satisfies $PV=nRT$ exactly at all pressures and temperatures is defined to be an ideal gas. An ideal gas is a simple theoretical model of a gas. No real gas is truly ideal.

Boyle's Law

- At constant temperature, volume of a given mass of gas is inversely proportional to its pressure.
  $$V \propto \frac{1}{P}$$
  PV = constant (If T = constant) or, $P_1V_1=P_2V_2$

Charle's Law

- At constant pressure, volume of the given mass of a gas is directly proportional to its absolute temperature.
  $$V \propto T; \frac{V}{T} = \text{constant (If } P = \text{constant); or, } \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Gay Lussac's Law

- The volume remaining constant, the pressure of a given mass of a gas is directly proportional to its absolute temperature.
  $$P \propto T; \frac{P}{T} = \text{ constant (If } V = \text{constant); or, } \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Dalton's Law of Partial Pressure

- Consider a mixture of non-interacting ideal gases: n_1 moles of gas 1, n_2 moles of gas 2, etc. in a vessel of volume V at temperature T and pressure P. It is then found that the equation of state of the mixture is:
  $$PV = (n_1 + n_2 + \ldots)RT; \text{i.e., } P = n_1\frac{RT}{V} + n_2\frac{RT}{V} + \ldots$$
  Total pressure of a mixture of non-reacting gases
  $$P = P_1 + P_2 + \ldots .P_n$$
  Clearly $P_1 = n_1RT/V$ is the pressure that gas one would exert at the same conditions of volume and temperature if no other gases were present. This is called the partial pressure of the gas.

Avogadro's law

- Under the same condition of temperature and pressure equal volumes of all gases contain equal no. of molecules
  i.e., $N_1=N_2$
- The number of molecules per unit volume is the same for all gases at a fixed temperature and pressure. The number in 22.4 litres of any gas is 6.02×10^{23}.

Work done on compressing a gas

- It can be obtained from the graph of P-V, $W = \int_{V_i}^{V_f} PdV$

Kinetic Theory of an Ideal Gas

Assumptions of kinetic theory of gases

- All the molecules of a gas are identical.
- The molecules of different gases are different.
- The molecules of gases are in a state of random motion.
- The collisions of gas molecules are perfectly elastic.

Vander Waal's gas equations

- For 1 mole of gas

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT$$

For n moles of gas

$$\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$$

Here a, b are called vander woals constants

$\frac{an^{2}}{V^{2}}$ = Pressure correction for molecular atraction, nb = volume correction for finite size of molecular.

- **Critical temperature (T_c):** The maximum temperature below which a gas can be liquefied by pressure alone.
- **Critical pressure (P_c):** The minimum pressure necessary to liquefy a gas at critical temperature.
- **Critical volume (V_c):** The volume of one mole of gas at critical pressure and critical temperature.

Relation between Vander Waal's constants and T_c, P_c, V_c

$$T_c=\frac{8a}{27Rb},\ P_c=\frac{a}{27b^2},\ V_c=3b$$

Root mean square speed

$$v_{\text{rms}}=\sqrt{\frac{3PV}{M}}=\sqrt{\frac{3P}{\rho}}=\sqrt{\frac{3RT}{M}}$$

Average speed,

$$v_{av}=\sqrt{\frac{8RT}{\pi M}}=0.92\ v_{\text{rms}}$$

Most probable speed

$$v_{mp}=\sqrt{\frac{2RT}{M}}=\left(\sqrt{\frac{2}{3}}\right)v_{\text{rms}}=0.816\ v_{\text{rms}};\ v_{\text{rms}}>v_{av}>v_{mp}$$

Pressure of an ideal gas

$$P=\frac{1}{3}\rho v^2_{\text{rms}}=\frac{1}{3}\frac{M}{V}v^2_{\text{rms}}$$

Kinetic energy and temperature

Average kinetic energy per molecule of a gas,

$$\frac{E}{N}=\frac{1}{2}mv^2=\frac{3}{2}k_BT$$

Maxwell distribution function

- In a given mass of gas, the velocities of all molecules are not the same, even when bulk parameters like pressure, volume and temperature are fixed. Collisions change the direction and the speed of molecules. However, in a state of equilibrium, the distribution of speeds is constant or fixed.

The molecular speed distribution gives the number of molecules between the speeds v and $v+dv.dN(v)=4pNa^3e^{-bv^2}v^2dv=n_v dv$. This is called Maxwell distribution.

Law of Equipartition of Energy

- In thermal equilibrium at absolute temperature T, for each translational mode of motion, the average energy is $\frac{1}{2}k_B T$. The most elegant principle of classical statistical mechanics states that this is so for each mode of energy: translational, rotational and vibrational. That is, in equilibrium, the total energy is equally distributed in all possible energy modes, with each mode having an average energy equal to $\frac{1}{2}k_B T$. This is known as the law of equipartition of energy. Accordingly, each translational and rotational degree of freedom of a molecule contributes $\frac{1}{2}k_B T$ to the energy, while each vibrational frequency contributes $2\times\frac{1}{2}k_B T = k_B T$, since a vibrational mode has both kinetic and potential energy modes.

Degree of freedom (*f*)

- Total number of independent modes in which a system can possess energy.
  - For monoatomic gas, $f = 3$
  - For diatomic gas:
    - at room temperature, $f = 5$; at high temperature, $f = 7$
  - For polyatomic gas :
    - at room temperature, $f = 6$; at high temperature, $f = 8$
- Law of equipartition of energy holds good for all degrees of freedom whether translational. rotational or vibrational.
- Each square term in the total energy expression of a molecule contributes towards one degree of freedom.
- A monoatomic gas molecule has only translational kinetic energy.
- $\varepsilon_t = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2$
- In addition to translational kinetic energy, a diatomic molecule has two rotational kinetic energies.
- $\varepsilon_t + \varepsilon_r = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2 + \frac{1}{2}I_y\omega_y^2 + \frac{1}{2}I_z\omega_z^2$
- Diatomic molecule like CO has a mode of vibration even at moderate temperatures. Its atoms vibrate along the interatomic axis and contribute a vibrational energy term ε, to the total energy.

$$\varepsilon = \varepsilon_t + \varepsilon_r + \varepsilon_r = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2 + \frac{1}{2}I_y\omega_y^2 + \frac{1}{2}I_z\omega_z^2 + \frac{1}{2}m\eta^2 + \frac{1}{2}k\eta^2$$

i.e., a diatomic molecule has 7 degree of freedom if it vibrates.

Mean free path

- Distance travelled by a gas molecule between two successive collisions.

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2} = \frac{k_B T}{\sqrt{2}\pi d^2 P}; \lambda = \frac{m}{\sqrt{2}\pi d^2 \rho}$$

- If volume of given mass of a gas is changed with P or T, $\lambda \propto T$ at constant pressure, and $\lambda \propto \frac{1}{P}$ at constant temperature.

Application to specific heat capacities of gases

For an ideal gas,

$$C_P - C_V = R$$

For monoatomic,

$$\frac{C_P}{C_V} = \gamma = \frac{5}{3}$$

For diatomic,

$$\frac{C_P}{C_V} = \gamma = \frac{7}{5}$$

For polyatomic gases,

$$\frac{C_P}{C_V} = \gamma = \frac{(4+f)}{(3+f)}$$

PRACTICE TIME

Behaviour of Gases

1. A real gas behaves like an ideal gas if its
(a) both pressure and temperature are low.
(b) both pressure and temperature are high.
(c) pressure is high and temperature is low.
(d) pressure is low and temperature is high.

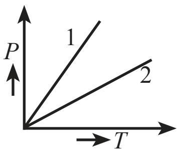
2. If the pressure and the volume of certain quantity of ideal gas are one-third of its initial value, then its temperature
(a) is doubled.
(b) becomes one-fourth.
(c) become one-ninth.
(d) remains constant.

3. An air bubble of volume 1.0 cm^3 rises from the bottom of a lake 30 m deep at a temperature of 12°C. To what volume does it grow when it reaches the surface, which is at a temperature of 35°C?
(a) 10.6×10^{-6} m^3 (b) 4.21×10^{-5} m^3
(c) 42.1×10^{-6} m^3 (d) 4.21×10^{-6} m^3

4. The diameter of an oxygen molecule is 2 Å. The ratio of molecular volume to the actual volume occupied by the oxygen gas at standard temperature and pressure (STP) is:
(a) 9×10^{-4} **(b)** 9.5×10^{-4}
(c) 9×10^{-3} **(d)** 9.5×10^{-3}

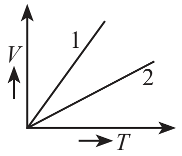
5. A balloon contains 1500 m^3 of helium at 27°C and 3 atmospheric pressures. The volume of helium at −3°C temperature and 2 atmospheric pressure will be:
(a) 2700 m^3 (b) 2520 m^3
(c) 2025 m^3 (d) 202.5 m^3

6. Pressure versus temperature graph of an ideal gas of equal number of moles of different volumes is plotted as shown in figure. Choose the correct alternative.



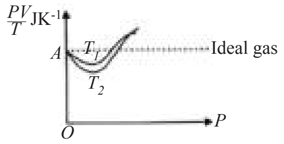
(a) $V_1 = V_2$ (b) $V_1 < V_2$
(c) $V_1 > V_2$ (d) $T_1 = T_2$

7. Volume versus temperature graph of an ideal gas of equal number of moles of different volumes is plotted as shown in figure. Choose the correct alternative.



(a) $V_1 > V_2$ (b) $P_1 > P_2$
(c) $P_1 < P_2$ (d) $P_1 = P_2$

8. Given is the graph between $\frac{PV}{T}$ and P for 1 g of oxygen gas at two different temperatures T_1 and T_2 as shown in figure. Given, density of oxygen = 1.427 kg m^3. The value of $\frac{PV}{T}$ at the point A and B:



(a) 0.259 JK^{-1} (b) 4.28 JK^{-1}
(c) 0.259 JK^{1} (d) 0.295 JK^{-1}

9. A vessel has 6 g of oxygen at pressure P and temperature 400 K. A small hole is made in it so that oxygen leaks out. How much oxygen leaks out if the final pressure is $\frac{P}{3}$ and temperature 300 K?
(a) 5 g (b) 3.33 g
(c) 2.67 g (d) 2 g

10. A vessel has 6 g of hydrogen at pressure P and temperature 500 K. A small hole is made in it so that hydrogen leaks out. How much hydrogen leaks out if the final pressure is $\frac{P}{3}$ and temperature falls to 300 K?
(a) 3.34 g (b) 3.45 g
(c) 2.66 g (d) 6 g

11. A vessel contains two non-reactive gases neon (monatomic) and oxygen (diatomic). The ratio of their partial pressures is 1:2. The ratio of number of molecules is:
(a) $\frac{3}{2}$ (b) $\frac{2}{3}$
(c) $\frac{1}{2}$ (d) $\frac{3}{5}$

12. A vessel contains two non-reactive gases A and B. The ratio of their partial pressures is 5:2. The ratio of number of molecules is
(a) $\frac{2}{5}$ (b) $\frac{3}{2}$
(c) $\frac{2}{3}$ (d) $\frac{5}{2}$

13. A cylinder contains 10 kg of gas at a pressure of 10^7 Nm^{-2}. The quantity of gas taken out of the cylinder, if final pressure is 3.5×10^6 Nm^{-2} is:

(a) 6.5 kg (b) 3.5 kg
(c) 4.5 kg (d) 5 kg

14. A gas at 200 K has pressure $4 \times 10^{-10} Nm^{-2}$. If $k_B = 1.38 \times 10^{-23}$ JK^{-1}, the number of molecules per cm^3 is of the order of:
(a) 10^5 (b) 10^4
(c) 10^3 (d) 10^2

15. The volume of vessel A is twice the volume of another vessel B, and both of them are filled with the same gas. If the gas in A is at twice the temperature and twice the pressure in comparison to the gas in B, then the ratio of the gas molecules in A to that of B
(a) $\frac{2}{1}$ (b) $\frac{1}{2}$
(c) $\frac{3}{2}$ (d) $\frac{2}{3}$

16. When the temperature of a gas filled in a closed vessel is increased by 1°C, its pressure increases by 0.5%. The initial temperature of gas was:
(a) 300 K (b) 250 K
(c) 200 K (d) 150 K

17. The equation of state for 3 g of oxygen at a pressure P and temperature T, when occupying a volume V, will be:
(a) $PV = \frac{5}{32}RT$ (b) $PV = \frac{3}{32}RT$
(c) $PV = \frac{32}{3}RT$ (d) $PV = \frac{31}{32}RT$

18. A vessel containing one mole of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing one mole of He gas (molar mass 4) at temperature $3T$ has a pressure of:
(a) 6 P (b) 5 P
(c) P (d) 3 P

19. In a certain region of space there are only four gaseous molecules per cm^3 on an average. The temperature there is 3 K. The pressure of this gas is: $(k_B = 1.38 \times 10^{-23}\ mol^{-1}K^{-1})$
(a) $20.7 \times 10^{-16}\ Nm^{-2}$
(b) $10.7 \times 10^{-16}\ Nm^{-2}$
(c) $16.6 \times 10^{-16}\ Nm^{-2}$
(d) $26.6 \times 10^{-16}\ Nm^{-2}$

20. A sample of an ideal gas occupies a volume V at pressure P and absolute temperature T. The mass of each molecule is m, then the density of the gas is:
(a) mkT (b) $\frac{P}{kT}$
(c) $\frac{Pm}{kT}$ (d) P

21. One third mole each of nitrogen, oxygen and carbon dioxide are mixed in enclosure of volume 5 litres and temperature 27°C. The pressure exerted by mixture is: $(R = 8.31\ mol^{-1}K^{-1})$
(a) $7.48 \times 10^5\ Nm^{-2}$ (b) $4.99 \times 10^5\ Nm^{-2}$
(c) $5.99 \times 10^5\ Nm^{-2}$ (d) $9.99 \times 10^5\ Nm^{-2}$

22. From the volume of water molecule is: (Take, density of water is $10^3\ kgm^{-3}$ and Avogadro's number $= 6 \times 10^{23}\ mole^{-1}$)
(a) $3.5 \times 10^{-28}\ m^3$ (b) $3 \times 10^{-29}\ m^3$
(c) $1.5 \times 10^{-28}\ m^3$ (d) $2.5 \times 10^{-29}\ m^3$

23. Critical temperature of CO_2 is 31.2°C. In summer, the room temperature is 40°C.
(a) CO_2 cannot be liquefied
(b) Can be liquefied with increase of pressure
(c) Can be liquefied with decrease of pressure
(d) Can be liquefied, if temperature of CO_2 is decreased below 31.2°C

Kinetic Theory of an Ideal Gas

24. Which one of the following is not an assumption of kinetic theory of gases?
(a) The volume occupied by the molecules of the gas is negligible.
(b) The force of attraction between the molecules is negligible.
(c) The collision between the molecules is elastic.
(d) All molecules have same speed.

25. Pressure of a gas at constant volume is proportional to
(a) temperature of the gas.
(b) average kinetic energy of the molecules.
(c) average potential energy of the molecules.
(d) total internal energy of the gas.

26. Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will the mass of the gas present in the volume V:
(a) Increase
(b) Decrease
(c) Remain the same
(d) Decreases for some, while increase for others

27. When an ideal gas is compressed adiabatically, its temperature rises the molecules on the average have more kinetic energy than before. The kinetic energy increases
(a) because of collisions with moving parts of the wall only.

(b) because of collisions with the entire wall.
(c) because the molecules gets accelerated in their motion inside the volume.
(d) because the redistribution of energy amongst the molecules.

28. A gas is filled in a container at pressure P_0. If the mass of molecules is one-third of initial mass and their rms speed is doubled, then the resultant pressure would be:
(a) $\frac{1}{3}P_0$ (b) $\frac{4}{3}P_0$
(c) $2P_0$ (d) $4P_0$

29. A vessel is filled with a gas at a pressure of 76 cm of Hg at a certain temperature. The mass of the gas is increased by 50% by introducing more gas in the vessel at the same temperature. The resultant pressure of the gas is:
(a) 114 cm of Hg (b) 112 cm of Hg
(c) 108 cm of Hg (d) 76 cm of Hg

30. In a mixture of gases at a fixed temperature
(a) lighter molecule has higher average speed.
(b) lighter molecule has lower average speed.
(c) heavier molecule has higher average speed.
(d) None of these

31. The average kinetic energy of gas molecules depends upon which of the following factor?
(a) Volume of the gas
(b) Temperature of the gas
(c) Nature of the gas
(d) Both (b) and (c)

32. What is the average velocity of the molecules of an ideal gas?
(a) Infinity (b) Constant
(c) Unstable (d) Zero

33. The ratio of the speed of sound in nitrogen gas to that in helium gas, at 300 K is:
(a) $\frac{\sqrt{3}}{5}$ (b) $\frac{\sqrt{7}}{5}$
(c) $\frac{\sqrt{3}}{7}$ (d) $\frac{\sqrt{7}}{3}$

34. 0.028 kg of nitrogen is enclosed in a vessel at a temperature of 37°C. At which temperature the rms velocity of nitrogen gas is twice the rms velocity at 37°C?
(a) 1200 K (b) 1240 K
(c) 1480 K (d) 148 K

35. At what temperature is the rms velocity of hydrogen molecule equal to that of an oxygen molecule at 27°C?
(a) 10 K (b) 20 K
(c) 18.75 K (d) 19.75 K

36. If three molecules have velocities 0.5 ms^{-1}, 1 ms^{-1}, and 2 ms^{-1} the ratio of the rms speed and average speed and rms speed is:
(a) $\frac{39}{44}$ (b) $\frac{44}{39}$
(c) $2\frac{39}{44}$ (d) $\frac{49}{44}$

37. The kinetic energy of 1 g molecule of a gas, at normal temperature and pressure, is:
(a) 0.56×10^4 J (b) 3.4×10^3 J
(c) 3.4×10^4 J (d) 3.6×10^4 J

38. An insulated container containing monatomic gas of molar mass m moving with a velocity $\frac{4mv_0^2}{3R}v_0$. If the container is suddenly stopped. The change in temperature is:
(a) $\frac{2mv_0^2}{3R}$ (b) $\frac{3mv_0^2}{2R}$
(c) $\frac{4mv_0^2}{3R}$ (d) $\frac{3mv_0^2}{4R}$

39. One moles of a gas A at 27°C mixed with a two moles of gas at 37°C. If both are monatomic ideal gases, what will be the temperature of the mixture?
(a) 33.67°C (b) 34.27°C
(c) 27°C (d) 37°C

40. The average kinetic energy, of O_2 at a particular temperature is 0.768 eV. The average kinetic energy of N_2 molecules in eV at the same temperature is:
(a) 0.0015 (b) 0.0030
(c) 0.048 (d) 0.768

41. The molecules of a given mass of a gas have root mean square speeds of 100 ms^{-1} at 27°C and one atmospheric pressure. The root mean square speeds of the molecules of the gas at 127°C and two atmospheric pressure is:
(a) $\frac{200}{\sqrt{3}}$ ms^{-1} (b) $\frac{100}{\sqrt{3}}$ ms^{-1}
(c) $\frac{400}{\sqrt{3}}$ ms^{-1} (d) $\frac{100}{3}$ ms^{-1}

42. The temperature of an ideal gas is increased from 37°C to 137°C, then percentage increase in v_{rms} is:
(a) 14% (b) 14.9%
(c) 15.9% (d) 15%

43. At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at −10°C? (Atomic mass of Ar = 39.9 u and He = 4 u)

(a) 2.5×10^3 K (b) 3.5×10^3 K
(c) 4.5×10^3 K (d) 2.9×10^3 K

44. The temperature of an ideal gas is increased from 130 K to 520 K. If at 130 K, the rms velocity of the gas molecules is v_{rms}, then at 520 K, it becomes:

(a) v_{rms} (b) $4v_{rms}$
(c) $3v_{rms}$ (d) $2v_{rms}$

45. A gas is compressed at a constant pressure from a volume of 10 m^3 to a volume of 4 m^3, then work done on the system is:

(a) $nRT\ln\frac{2}{5}$ (b) $nRT\ln\frac{5}{2}$
(c) $nRT\ln 6$ (d) $nRT\ln\frac{1}{6}$

46. Match Column 1 (Physical variables) with Column II (Expressions). (n = number of gas molecules present per unit volume, k_B = Boltzmann constant, T = absolute temperature, m= mass of the particle):

	Column I		Column II
A	Most probable velocity	(i)	nk_BT
B	Energy per degree of freedom	(ii)	$\sqrt{(3k_BT/m)}$
C	Pressure	(iii)	$\sqrt{(2k_BT/m)}$
D	v_{rms}	(iv)	$k_BT/2$

(a) A→(iii), B→(ii), C→(i), D→(ii)
(b) A→(ii), B→(iii), C→(i), D→(ii)
(c) A→(iii), B→(ii), C→(ii), D→(i)
(d) A→(iii), B→(i), C→(iv), D→(ii)

Law of Equipartition of Energy

47. According to equipartition law of energy each particle in a system of particles have thermal energy equal to:

(a) $\frac{3}{2}k_BT$ (b) $\frac{1}{2}k_BT$
(c) k_BT (d) $2k_BT$

48. If a gas has 'n' degrees of freedom, the ratio of the specific heats γ of the gas is:

(a) $1+\frac{n}{2}$ (b) $1+\frac{1}{n}$
(c) $\frac{n+1}{2}$ (d) $1+\frac{2}{n}$

49. If one mole of monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with one mole of diatomic gas $\left(\gamma = \frac{7}{5}\right)$ the value of γ for the mixture is:

(a) 1.60 (b) 1.53
(c) 1.50 (d) 1.40

50. How is the mean free path (λ) in a gas related to the interatomic distance?

(a) λ is 10 times the interatomic distance.
(b) λ is 100 times the interatomic distance.
(c) λ is 1000 times the interatomic distance.
(d) λ is 0.1 times the interatomic distance.

51. Mean free path of a gas molecule is:

(a) Inversely proportional to number of molecules per unit volume.
(b) inversely proportional to diameter of the molecule.
(c) directly proportional to the square root of die absolute temperature.
(d) directly proportional to the molecular mass.

52. The specific heat of a gas:

(a) has only two values C_p and C_V
(b) has a unique value at a given temperature
(c) can have any value between zero and ∝
(d) depends upon the mass of the gas

53. The degree of freedom of a triatomic gas is:

(a) 2 (b) 3
(c) 6 (d) 8

54. The specific heats at constant pressure is greater than that of the same gas at constant volume because:

(a) at constant pressure work is done in expanding the gas.
(b) at constant volume work is done in expanding the gas.
(c) the molecular attraction increases more at constant pressure.
(d) the molecular vibration increases more at constant pressure

55. The internal energy of one gram of helium at 100 K and one atmospheric pressure is:

(a) 1200 J (b) 30 J
(c) 3000 J (d) 300 J

56. If for a gas $\frac{R}{C_V} = 0.67$, this gas is made up of molecules which are:

(a) Monatomic
(b) Diatomic
(c) Polyatomic
(d) Mixture of diatomic and polyatomic molecules

57. The heat capacity per mole of water is: (R is universal gas constant)
(a) 5 R (b) 4 R
(c) 3 R (d) 9 R

58. A molecule of a gas has six degrees of freedom. Then the molar specific heat of the gas at constant volume is:
(a) R (b) 3 R
(c) 1.5 R (d) 0.5 R

HIGH-ORDER THINKING SKILL

Behaviour of Gases

1. Two chamber containing *m* grams of a gas at pressures and respectively are put in communication with each other, temperature remaining constant. The common pressure reached will be:

(a) $\frac{m_1 m_2 (P_1 + P_2)}{P_2 m_1 + P_1 m_2}$ **(b)** $\frac{P_1 P_2 (m_1 + m_2)}{P_2 m_1 + P_1 m_2}$

(c) $\frac{P_1 P_2 m_2}{P_2 m_1 + P_1 m_2}$ **(d)** $\frac{P_1 P_2 m_1}{P_2 m_1 + P_1 m_2}$

2. Two vessels separately contain two ideal gases A and B at the same temperature. The pressure of A being twice that of B. Under such conditions, the density of A is found to be 1.5 times the density of B. The ratio of molecular weight of A and B is:

(a) 2 **(b)** $\frac{3}{4}$

(c) $\frac{1}{2}$ **(d)** $\frac{4}{3}$

Kinetic Theory of an Ideal Gas

3. The molecules of a given mass of a gas have rms velocity of 200 ms^{-1} at 27°C and 1.0 × 10^5 Nm^{-2} pressure. When the temperature and pressure of gas are respectively, 127°C and 0.05 × 10^5 Nm^{-2} the rms velocity of its molecules in ms^{-1} is:

(a) $\frac{400}{\sqrt{3}}$ ms^{-1} **(b)** $\frac{800}{\sqrt{3}}$ ms^{-1}

(c) $\frac{300}{\sqrt{3}}$ ms^{-1} **(d)** $\frac{500}{\sqrt{3}}$ ms^{-1}

4. A thermally insulated vessel contains an ideal gas of molecular mass *M* and ratio of specific heats γ. It is moving with speed *v* and its suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by:

(a) $\frac{1}{2\gamma R} Mv^2$ K

(b) $\frac{(\gamma - 1)}{2R} Mv^2$ K

(c) $\frac{1}{2(\gamma + 1)R} Mv^2$ K

(d) $\frac{1}{2(\gamma - 1)R} Mv^2$ K

5. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If N_i and N_f are the number of molecules in the room before and after heating, then $N_f – N_i$ will be:
(a) 2.5 × 10^{25}
(b) -2.5 × 10^{25}
(c) -1.61 × 10^{23}
(d) 1.38 × 10^{23}

6. At what temperature will the rms speed of oxygen molecules become just sufficient for escaping from the Earth's atmosphere? [Given: Mass of oxygen molecule (*m*)= 2.76 × 10^{-26} kg, Boltzmann's constant k_B = 1.38 × 10^{-23} JK^{-1}]
(a) 2.508 × 10^4 K
(b) 8.360 × 10^4 K
(c) 1.254 × 10^4 K
(d) 5.016 × 10^4 K

Law of Equipartition of Energy

7. 4.0 g of a gas occupies 22.4 litres at STP. The specific heat capacity of the gas at constant volume is 5.0 JK^{-1}. If the speed of sound in this gas at STP is 952 ms^{-1}, then the heat capacity at constant pressure is: (Take gas constant R = 8.3 $JK^{-1}mol^{-1}$)
(a) 7.5 $JK^{-1}mol^{-1}$
(b) 8.5 $JK^{-1}mol^{-1}$
(c) 7.0 $JK^{-1}mol^{-1}$
(d) 8.0 $JK^{-1}mol^{-1}$

8. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as where V^q is the volume of the gas. The value of q is:

(a) $\frac{\gamma + 1}{2}$ **(b)** $\frac{\gamma - 1}{2}$

(c) $\frac{3\gamma + 5}{6}$ **(d)** $\frac{3\gamma - 5}{6}$

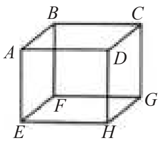
NCERT EXEMPLAR PROBLEMS

Behaviour of Gases

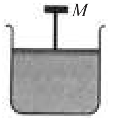
1. Boyle's law is applicable for an:
   (a) a diabatic process
   (b) is othermal process
   (c) is obaric process
   (d) is ochoric process

Kinetic Theory of an Ideal Gas

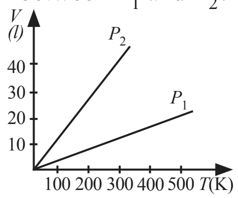
2. A cubic vessel (with faces horizontal + vertical) contains an ideal gas at normal temperature and pressure. The vessel is being carried by a rocket which is moving at a speed of 500 ms^{-1} in vertical direction. The pressure of the gas inside the vessel is observed by us on the ground:
   (a) remains the same because 500 ms^{-1} is very much smaller than v_{rms} of the gas.
   (b) remains the same because motion of the vessel as a whole does not affect the relative motion of the gas molecules and the walls.
   (c) will increase by a factor equal to $\left[v_{rms}^2 + (500)^2\right] / v_{rms}^2$ where v_{rms} was the original mean square velocity of the gas.
   (d) will be different on the top wall and bottom wall of the vessel.
3. One mole of an ideal gas is contained in a cubical volume *V*, *ABCDEFGH* at 300 K as shown in figure. One face of the cube (*EFGH*) is made up of a material which totally absorbs any gas molecule incident on it. At any given time:



   (a) the pressure on *EFGH* would be zero.
   (b) the pressure on all the faces will be equal.
   (c) the pressure of *EFGH* would be double the pressure on *ABCD*.
   (d) the pressure on *EFGH* would be half that on *ABCD*.
4. An inflated rubber balloon contains one mole of an ideal gas, has a pressure *P*, volume *V* and temperature *T*. If the temperature rises to 1.1 *T*, and the volume is increased to 1.05 *V*, the final pressure will be:
   (a) 1.1 *P*
   (b) *P*
   (c) Less than *P*
   (d) between *P* and 1.1*P*
5. A cylinder containing an ideal gas is in vertical position and has a piston of mass *M* that is able to move up or down without friction. If the temperature is increased:



   (a) both *P* and *V* of the gas will change.
   (b) only *P* will increase according to Charle's law.
   (c) *V* will change but not *P*.
   (d) *P* will change but not *V*.
6. Volume versus temperature graphs for a given mass of an ideal gas is shown in figure at two different values of constant pressure. What can be inferred about relation between P_1 and P_2?



   (a) $P_1 > P_2$
   (b) $P_1 = P_2$
   (c) $P_1 < P_2$
   (d) Data is insufficient
7. 1 mole of H_2 gas is contained in a box of volume $V = 1.00$ m^3 at $T = 300$ K. The gas is heated to a temperature of $T = 3000$ K and the gas gets converted to a gas of hydrogen atoms. The final pressure would be: (considering all gases to be ideal)
   (a) same as the pressure initially
   (b) Two times the pressure initially
   (c) 10 times the pressure initially
   (d) 20 times the pressure initially
8. A vessel of volume *V* contains a mixture of one mole of hydrogen and 1 mole of oxygen (both considered as ideal). Let $f_1(v)dv$ denote the fraction of molecules with speed between v and $(v + dv)$ with $f_2(v)dv$, similarly for oxygen. Then,
   (a) $f_1(v) + f_2(v) = f(v)$, obeys the Maxwell's distribution law.
   (b) $f_1(v), f_2(v)$, obeys the Maxwell's distribution law separately.
   (c) Neither $f_1(v)$ nor $f_2(v)$ will obey the Maxwell's distribution law.
   (d) $f_2(v)$ and $f_1(v)$ will be the same.

ASSERTION AND REASONS

Directions: In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as:
(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

Behaviour of Gases

1. **Assertion:** Air pressure in a car tyre increases during driving.
   Reason: Absolute zero temperature is not zero energy temperature.
2. **Assertion:** For a mixture of non-reactive ideal gases, the total pressure gets contribution from each gas in the mixture.
   Reason: In equilibrium, the average kinetic energy of the molecules of different gases will be equal.

Kinetic Theory of an Ideal Gas

3. **Assertion:** In case of collision of gas molecules in a given amount of gas, total kinetic energy is conserved.
   Reason: All collisions of the gas molecules in a given amount of gas are elastic.
4. **Assertion:** The root mean square and most probable speeds of the molecules in a gas are symmetrical.
   Reason: The Maxwell distribution for the speed to molecules in a gas is symmetrical.
5. **Assertion:** The ratio of rms speed and average speed of a gas molecules at a given temperature is $\sqrt{3} : \sqrt{8/\neq}$
   Reason: $v_{\text{rms}} < v_{av}$

Law of Equipartition of Energy

6. **Assertion:** Mean free path of a gas molecule varies inversely as density of the gas.
   Reason: Mean free path varies inversely as pressure of the gas.
7. **Assertion:** The ratio of specific heat of a gas at constant pressure and specific heat at constant volume for a diatomic gas is more than that for a mono atomic gas.
   Reason: The molecules of a mono atomic gas have more degree of freedom than those of a diatomic gas.
8. **Assertion:** One mole of any substance at any temperature or volume always contains 6.02×10^{23} molecules.
   Reason: One mole of a substance always refers to STP conditions.
9. **Assertion:** Each vibrational mode gives two degrees of freedom.
   Reason: By law of equipartition of energy, the energy for each degree of freedom in thermal equilibrium is $2\ k_B T$.
10. **Assertion:** Specific heat of a gas at constant pressure is greater than its specific heat at constant volume.
    Reason: At constant pressure, some heat is spent in expansion of the gas.
11. **Assertion:** The total translation kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and volume.
    Reason: The molecules of gas collide with each other and the velocities of the molecules change due to the collision.

ANSWER KEYS

Practice Time

1	(d)	2	(c)	3	(d)	4	(a)	5	(c)	6	(b)	7	(c)	8	(a)	9	(b)	10	(c)
11	(c)	12	(d)	13	(a)	14	(a)	15	(a)	16	(c)	17	(b)	18	(d)	19	(c)	20	(c)
21	(b)	22	(b)	23	(a)	24	(d)	25	(d)	26	(c)	27	(a)	28	(b)	29	(a)	30	(a)
31	(b)	32	(d)	33	(a)	34	(b)	35	(c)	36	(a)	37	(b)	38	(c)	39	(a)	40	(d)
41	(a)	42	(c)	43	(b)	44	(d)	45	(a)	46	(a)	47	(b)	48	(d)	49	(c)	50	(b)
51	(a)	52	(c)	53	(c)	54	(a)	55	(d)	56	(a)	57	(d)	58	(b)				

High-Order Thinking Skill

1	(a)	2	(d)	3	(a)	4	(b)	5	(b)	6	(b)	7	(d)	8	(a)

NCERT Exemplar Problems

1	(b)	2	(d)	3	(d)	4	(d)	5	(c)	6	(a)	7	(d)	8	(b)

Assertion and Reasons

1	(b)	2	(a)	3	(a)	4	(d)	5	(c)	6	(a)	7	(d)	8	(c)	9	(c)	10	(a)
11	(b)																		

HINTS AND EXPLANATIONS

Practice Time

1 (d) At high temperature and low pressure the real gas behaves as an ideal gas.

2 (c) According to ideal gas equation

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$$

$$\text{or, } T_2=T_1\frac{P_2V_2}{P_1V_1}$$

$$T_2=T_1\frac{\frac{P_1}{3}\times\frac{V_1}{3}}{P_1V_1}$$

$$=\frac{T_1}{9}$$

3 (d) By using,

$$P_1=P_2+\rho gh$$

After putting values

$$\therefore \quad P_1=1.013\times10^5+10^3\times9.8\times30$$

$$=395300 \text{ Pa}$$

$$\text{Now, } \frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$$

Here, $T_1=285$ K

$T_2=308$ K

$V_1=1\times10^{-6}\ \text{m}^3$

V_2 is the volume of the air bubble when it reaches the surface

$$\therefore \quad V_2=\frac{P_1V_1T_2}{T_1P_2}$$

$$=\frac{(395300\times1\times10^{-6})}{285\times1.013\times10^5}\times308$$

$$=4.21\times10^{-6}\ \text{m}^3$$

4 (a) As we know that

$$\because\ r=\frac{d}{2}$$

$$=\frac{4}{2}\times10^{-10}\ \text{m}$$

$$=\frac{4}{2}\times10^{-8}\ \text{cm}$$

$$=2\times10^{-8}\ \text{cm}$$

Molecular volume of oxygen gas,

$$V=\frac{4}{3}\pi r^3\times N_A$$

Actual volume occupied by 1 mole of oxygen gas at STP

$$=22400\ \text{cm}^3$$

$$\therefore V=\frac{4}{3}\times3.14\times\left(2\times10^{-8}\right)^3\times6.023\times10^{23}$$

$$=20.17\ \text{cm}^3$$

Therefore, ratio of molecular volume to actual volume of oxygen,

$$\frac{V}{V_1}=\frac{20.17}{22400}$$

$$=9\times10^{-4}$$

5 (c) Here, V_1 = 1500 m^3, T_1 = 27°C = 300 K

P_1 = 3 atm, T_2 = –3°C = 270K, P_2 = 2 atm

According to ideal gas equation

$$V_2=\frac{P_1V_1}{T_1}\times\frac{T_2}{P_2}$$

$$=\frac{3\times1500\times270}{300\times2}$$

$$=2025\ \text{m}^3$$

6 (b) As P-T graphs are straight lines, passing through origin, therefore, $P \propto T$ and $V \propto T$, so $V_1 < V_2$.

7 (c) As V-T graphs are straight lines, passing through origin, therefore, $V \propto T$ and $P \propto T$, so $P_1 < P_2$.

8 (a) According to ideal gas equation

$$\frac{PV}{T} = n\text{R}$$

$$= \frac{m}{M}\text{R} \quad \left(\frac{m}{M} = n = \text{number of moles}\right)$$

$$= \text{ constant for all values of } P$$

That is why, ideally it is a straight line

$$\therefore \frac{PV}{T} = \frac{1}{32} \times 8.31$$

$$= 0.259 \text{ JK}^{-1}$$

9 (b) According to ideal gas equation

$$PV = \frac{m}{M}RT \text{ (for } m \text{ grams of gas)}$$

$$\therefore \quad m' = \frac{P'}{P} \times \frac{T}{T'} \times m$$

$$= \left(\frac{P/3}{P}\right) \times \frac{400}{300} \times 6$$

$$= 2.67 \text{ g}$$

$\therefore$ Mass of oxygen leaked,

$$\Delta m = m - m'$$

$$= 6 - 2.67$$

$$= 3.33 \text{ g}$$

10 (c) According to ideal gas equation

$$\because \quad PV = \frac{m}{M}RT \text{ (for } m \text{ grams of gas)}$$

$$\therefore \quad m' = \frac{P'}{P} \times \frac{T}{T'} \times m$$

$$= \left(\frac{P/3}{P}\right) \times \frac{500}{300} \times 6$$

$$= 3.34 \text{ g}$$

$\therefore$ Mass of oxygen leaked,

$$\Delta m = m - m'$$

$$= 6 - 3.34$$

$$= 2.66 \text{ g}$$

11 (c) As partial pressure of a gas in a mixture is the pressure it would exert for the same volume and temperature, if it alone occupied the vessel. Therefore, for common V and T, P_1 and P_2 are partial pressures.

Here, 1 and 2 refer to neon gas and oxygen gas respectively.

$$\text{Now, } \frac{P_1}{P_2} = \frac{n_1}{n_2}$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{1}{2} \quad \left(\because \frac{P_1}{P_2} = \frac{1}{2}\right)$$

If N_1 and N_2 are number of molecules of two gases, then

$$\therefore \quad \frac{N_1}{N_2} = \frac{n_1}{n_2} \quad \left(\because n = \frac{N}{N_A}\right)$$

$$= \frac{1}{2}$$

12 (d) As partial pressure of a gas in a mixture is the pressure it would exert for the same volume and temperature, if it alone occupied the vessel. Therefore, for common V and T, P_1 and P_2 are partial pressures.

$$P_A V = n_A RT \text{ and } P_B V = n_B RT$$

Here, A and B refer to neon gas and oxygen gas respectively.

$$\text{Now, } \frac{P_A}{P_B} = \frac{n_A}{n_B}$$

$$\Rightarrow \quad \frac{n_A}{n_B} = \frac{5}{2} \quad \left(\because \frac{P_A}{P_B} = \frac{5}{2}\right)$$

If N_A and N_B are number of molecules of two gases, then

$$\therefore \quad \frac{N_A}{N_B} = \frac{n_A}{n_B} \quad \left(\because n = \frac{N}{N_{\text{Avogadro}}}\right)$$

$$= \frac{5}{2}$$

13 (a) By using:

$$\because \quad \frac{P_1}{P_2} = \frac{\rho_1}{\rho_2}$$

$$= \frac{m_1}{m_2}$$

$$\therefore \quad m_2 = \frac{P_2}{P_1} \times m_1$$

$$= \frac{3.5 \times 10^6}{10^7} \times 10$$

$$= 3.5 \text{ kg}$$

Mass of gas taken out,

$$\Delta m = m_1 - m_2$$

$$= 10 - 3.5$$

$$= 6.5 \text{ kg}$$

14 (a) By using,

$$PV = nRT$$

$$= n\text{N}_\text{A}\left(\frac{\text{R}}{\text{N}_\text{A}}\right)T$$

$$= nN_A k_B T$$

$$\frac{nN_A}{V} = \frac{P}{k_B T}$$

$$= \text{No. of molecules per m}^3$$

Therefore, no. of molecules per cm^3,

$$= \frac{P}{k_B T} \times 10^{-6}$$

$$= \frac{4 \times 10^{-10} \times 10^{-6}}{1.38 \times 10^{-23} \times 200}$$

$$= 1.44 \times 10^5$$

15 (a) According to ideal gas equation

$$\therefore \quad n_B = \frac{PV}{RT} \text{ and}$$

$$n_A = \frac{2P \times 2V}{R \times 2T}$$

or $$n_A = \frac{2PV}{RT}$$

$$\therefore \quad \frac{n_A}{n_B} = \frac{2}{1}$$

16 (c) By using,

$$PV = nRT$$

Since volume is constant

$$\therefore \quad \frac{P_2}{P_1} = \frac{T_2}{T_1}$$

or $$T_1 = T_2 \times \frac{P_1}{P_2}$$

or $$T_1 = (T_1 + 1)\left(\frac{100}{100.5}\right)$$

$$\Rightarrow \quad T_1 = 200 \text{ K}$$

17 (b) By using,

$$PV = nRT$$

$$n = \frac{m}{\text{molecular mass}}$$

$$= \frac{3}{32}$$

$$\Rightarrow PV = \left(\frac{3}{32}\right)RT$$

18 (d) Using ideal gas equation,

$$\because \quad P_1 V = n_1 R T_1 \text{ and}$$

$$P_2 V = n_2 R T_2$$

$$\therefore \quad \frac{P_2}{P_1} = \frac{n_2}{n_1} \times \frac{T_2}{T_1}$$

$$= \frac{1}{1} \times \frac{3T}{T}$$

$$= 3$$

or $$P_2 = P_1 \times 3$$

$$= P \times 3 \quad (\because P_1 = P)$$

$$= 3P$$

19 (c) Let n be the number of molecules in the gas

$$PV = nk_B T$$

or $$P = \frac{nk_B T}{V}$$

Here, $$\frac{n}{V} = 4 \text{ cm}^{-3}$$

$$= 4 \times 10^6 \text{ m}^{-3}$$

$$k_B = 1.38 \times 10^{-23} \text{ Jmol}^{-1}\text{K}^{-1}$$

$$\therefore \quad P = 4 \times 10^6 \times 1.38 \times 10^{-23} \times 3$$

$$= 16.6 \times 10^{-17} \text{ Nm}^{-2}$$

20 (c) According to ideal gas equation

$PV = k_B TN$, where N is the number of molecules

$$P\left(\frac{Nm}{\rho}\right) = k_B TN \quad \left(\because V = \frac{m}{\rho}\right)$$

Density of gas,

$$\rho = \frac{Pm}{k_B T}$$

21 (b) Using, Dalton's law of partial pressures

$$P = P_1 + P_2 + P_3$$

$$= \frac{nRT}{V} + \frac{nRT}{V} + \frac{nRT}{V}$$

$$= \frac{3nRT}{V}$$

After putting values,

$$\therefore \quad P = \frac{3 \times 8.31 \times 300}{3 \times 5 \times 10^{-3}}$$

$$P = 4.99 \times 10^5 \text{ Nm}^{-2}$$

22 (b) Molecular mass of water = 18

Number of molecules in 18 g or 0.018 kg = 6 × 10^{23}

Mass of molecule of water

$$= \frac{0.018}{6 \times 10^{23}}$$

$$= 3 \times 10^{-26}$$

Volume of a water molecule

$$= \frac{\text{Mass}}{\text{Density}}$$

$$= \frac{3 \times 10^{-26}}{1000}$$

$$= 3 \times 10^{-29} \text{ m}^3$$

23 (a) In summer, at room temperature gases cannot be liquefied above critical temperature.

24 (d) Molecules of an ideal gas move randomly with different speeds.

25 (d) As we know that

Pressure, $P = \frac{1}{3}\frac{M}{V}v_{rms}^2$

$= \frac{2}{3}E$

Here, $E = \frac{1}{2}Mv_{rms}^2$ is total internal energy of the gas.

26 (c) The centre of mass of the gas molecules moves with uniform speed along with the lorry. As there is no change in relative motion, the translational kinetic energy and hence the temperature of the gas molecules will remain the same.

27 (a) If an ideal gas is compressed adiabatically, its temperature rises, because heat produced cannot be lost to the surroundings. Each molecule has more k_B than before because of collisions of molecules with moving parts of the wall (i.e. piston compressing the gas).

28 (b) By using,

$$P_0 = \frac{1}{3}\rho v_{rms}^2$$

$$= \frac{1}{3}\frac{mN}{V}v_{rms}^2$$

$\therefore\ P_0 \propto mv_{rms}^2$

As m is one third and v_{rms} is doubled then

P becomes $\frac{4}{3}$.

or $P = \frac{4}{3}P_0$

29 (a) Pressure exerted by a gas,

$$P = \frac{1}{2}\frac{M}{V}v_{rms}^2$$

Since temperature T is kept constant,

v_{rms}^2 and V are also constant.

$\therefore\ P \propto M$

or $\frac{P_2}{P_1} = \frac{M_2}{M_1}$

According to the question,

$$\therefore\ \frac{P_2}{76} = \frac{\left[M_1 + \left(\frac{50}{100}\right)M_1\right]}{M_1}$$

$$= \frac{3}{2}$$

$$\Rightarrow\ P_2 = \frac{3}{2}\times 76$$

$= 114$ cm of mercury

30 (a) Lighter the molecule, higher the average speed.

31 (b) Average kinetic energy of gas molecules depends on the temperature of the gas as $E = \frac{3}{2}k_BT$

32 (d) The average velocity of the molecules of an ideal gas is zero, because the molecules possess all sorts of velocities in all possible directions so their vector sum and hence the average is zero.

33 (a) As we know that,

$$\because\ v = \sqrt{\frac{\gamma RT}{M}}$$

$$\therefore\ \frac{v_{N_2}}{v_{He}} = \sqrt{\frac{\gamma_{N_2}M_{He}}{\gamma_{He}M_{N_2}}}$$

$$= \sqrt{\frac{7/5}{5/3}\times\frac{4}{28}}$$

$$= \frac{\sqrt{3}}{5}$$

34 (b) By using,

$$v_{rms} = \sqrt{\frac{3RT}{m}}$$

$$\therefore\ \frac{(v_{rms})_1}{(v_{rms})_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\because\ (v_{rms})_2 = 2(v_{rms})_1$$

$$\therefore\ \frac{(v_{rms})_1}{2(v_{rms})_1} = \sqrt{\frac{310}{T_2}}$$

$$\Rightarrow\ \frac{1}{4} = \frac{310}{T_2}$$

$$T_2 = 310\times 4$$

$$= 1240\ \text{K}$$

35 (c) By using,

$$v_{rms} = \sqrt{\frac{3RT}{m}}$$

Now, rms velocity of H_2,

Molecule = rms velocity of O_2

$$\sqrt{\frac{3R\times T}{2}} = \sqrt{\frac{3R\times(27+273)}{32}}$$

$$T = \frac{2\times 300}{32}$$

$$= 18.75\ \text{K}$$

36 (a) As we know that,

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2}{3}}$$

$$= \sqrt{\frac{(0.5)^2 + (1)^2 + (2)^2}{3}}$$

$$= 1.32\ \text{ms}^{-1}$$

Average speed,

$$v_{av} = \frac{v_1 + v_2 + v_3}{3}$$

$$= \frac{0.5+1+2}{3}$$

$$= 1.17 \text{ ms}^{-1}$$

$$\therefore \frac{v_{av}}{v_{\text{rms}}} = \frac{1.17}{1.32} = \frac{39}{44}$$

37 (b) Kinetic energy of 1 g molecule of a gas at temperature T,

$$= \frac{3}{2}\text{R}T$$

$$= \frac{3}{2} \times 831 \times 273$$

$$= 3.4 \times 10^3 \text{ J}$$

38 (c) If the container is suddenly stopped loss in kinetic energy of gas $= \frac{1}{2}(mn)(2v_0)^2$ where n is number of moles of gas.

Let ΔT is the fall in temperature of gas,

$$\therefore \quad n\left(\frac{3}{2}R\Delta T\right) = \frac{1}{2}mn4v_0^2$$

$$\Delta T = \frac{4mv_0^2}{3\text{R}}$$

39 (a) Since there is no loss of energy in the process.

So, Temperature of the mixture,

$$T = \frac{n_1T_1 + n_2T_2}{n_1 + n_2}$$

$$= \frac{1(27+273) + 2(37+273)}{1+2}$$

$$= \frac{920}{3}$$

$$\therefore \quad T = 306.67 \text{ K}$$

$$= 33.67°\text{C}$$

40 (d) Average kinetic energy per molecular of a gas = $\frac{3}{2}\text{k}_\text{B}T$ = a constant at a given temperature.

41 (a) Here, $v_{\text{rms}_1} = 100 \text{ ms}^{-1}$, $T_1 = 27°\text{C} = (27+273)\, K$ $= 300\text{K}$, $P_1 = 1$ atm

and, $v_{\text{rms}_2} = ?$, $T_2 = 127°\text{C} = (127+273)\, K$ $= 400$ K, $P_2 = 3$ atm

From, $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{V_1}{V_2} = \frac{P_2}{P_1} \cdot \frac{T_1}{T_2}$$

$$= \frac{3}{1} \times \frac{300}{400}$$

$$= \frac{9}{4}$$

Again, $$P_1 = \frac{1}{3}\frac{M}{V_1}v_{\text{rms}_1}^2$$

$$P_2 = \frac{1}{3}\frac{M}{V_2}v_{\text{rms}_2}^2$$

$$\therefore \quad \frac{v_{\text{rms}_2}^2}{v_{\text{rms}_1}^2} \times \frac{V_1}{V_2} = \frac{P_2}{P_1}$$

$$v_{\text{rms}_2}^2 = v_{\text{rms}_1}^2 \times \frac{P_2}{P_1} \times \frac{V_2}{V_1}$$

$$= (100)^2 \times 3 \times \frac{4}{9}$$

$$\Rightarrow \quad v_{\text{rms}_2} = \frac{200}{\sqrt{3}} \text{ ms}^{-1}$$

42 (c) As we know that,

$$\because \quad v_{\text{rms}} = \sqrt{\frac{3\text{R}T}{M}}$$

% increase in v_{rms}

$$= \frac{\sqrt{\frac{3\text{R}T_2}{M}} - \sqrt{\frac{3\text{R}T_1}{M}}}{\sqrt{\frac{3\text{R}T_1}{M}}} \times 100$$

$$= \frac{\sqrt{T_2} - \sqrt{T_1}}{\sqrt{T_1}} \times 100$$

$$= \frac{\sqrt{410} - \sqrt{310}}{\sqrt{310}} \times 100$$

$$= \frac{20.24 - 17.61}{17.61} \times 100$$

$$= 14.9\%$$

43 (b) By using,

$$\because \quad v_{\text{rms}} = \sqrt{\frac{3\text{R}T}{M}}$$

$$v_{\text{rms}_1} = \sqrt{\frac{3\text{R}T_1}{M_1}}$$

$$v_{\text{rms}_2} = \sqrt{\frac{3\text{R}T_2}{M_2}}$$

According to question,

$$v_{\text{rms}_1} = v_{\text{rms}_2}$$

$$\therefore \quad \sqrt{\frac{3\text{R}T_1}{M_1}} = \sqrt{\frac{3\text{R}T_2}{M_2}}$$

$$\Rightarrow \quad \frac{T_1}{M_1} = \frac{T_2}{M_2}$$

$$\therefore \quad T_1 = \frac{T_2}{M_2} \times M_1$$

$$= \frac{263}{4} \times 39.9$$

$$= 2.6 \times 10^3 \text{ K}$$

44 (d) By using,

$$\because \quad v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$\because \quad \frac{v_{\text{rms}_2}}{v_{\text{rms}_1}} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{520}{130}}$$

$$\therefore \quad v_{\text{rms}_2} = 2v_{\text{rms}_1}$$

45 (a) Since,

$$W = \int_{V_i}^{V_f} PdV = nRT\ln\frac{V_f}{V_i} = nRT\ln\frac{4}{10} = nRT\ln\frac{2}{5}$$

46 (a) By using,

$$v_{\text{rms}} = \sqrt{3k_B T/m}$$

$$v_{\text{mp}} = \sqrt{\frac{2}{3}}v_{\text{rms}}$$

47 (b) Each particle in a system of particles have thermal energy $\frac{1}{2}k_B T$, where k_B is Boltzmann constant.

48 (d) By using,

$$\frac{C_P}{C_V} = \gamma = 1 + \frac{2}{n}$$

49 (c) By using,

$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

or $$\frac{2}{\gamma - 1} = \frac{1}{\frac{5}{3} - 1} + \frac{1}{\frac{7}{5} - 1}$$

$$\therefore \quad \gamma = \frac{3}{2}$$

50 (b) Mean free path in a gas is 100 times the interatomic distance.

51 (a) Mean free path,

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$$

52 (c) By using,

$$C = \frac{Q}{m\Delta T}$$

If $\Delta T = 0$, $C = \infty$ and if $Q = 0$, then $C = 0$

53 (c) A triatomic gas molecule has 3 degrees of freedom due to translator motion and 3 degrees of freedom due to rotator motion.

54 (a) Woris to be done in expanding the gas at constant pressure.

55 (d) The helium molecule is monoatomic and hence its internal energy per molecule is $\frac{3}{2}k_B T$ (where k_B is the Boltzmann, constant). The internal energy per mole is $\frac{3}{2}RT$. One gram of helium is one fourth mole and hence its internal energy is $\frac{1}{4} \times \frac{3}{2} R \times 100 = 300$ J, taking the value of R to be approximately 8 J mol^{-1}K^{-1}.

56 (a) For a gas, we know

$$\frac{R}{C_V} = \gamma - 1$$

or, $0.67 = \gamma - 1$

or, $\gamma = 1.67$

Hence the gas is monoatomic.

57 (d) We treat water like a solid. For each atom average energy is 3 $k_B T$. Water molecule has three atoms, two hydrogen and one oxygen. The total energy of one mole of water is

$$U = 3 \times 3k_B T \times N_A = 9RT$$

Therefore heat capacity per molecule of water is,

$$C = \frac{\Delta Q}{\Delta T} = \frac{\Delta U}{\Delta T} = 9\,R$$

58 (b) Here, number of degrees of freedom, $f = 6$. Molar specific heat of the gas at constant volume is

$$C_V = \frac{f}{2}R = \frac{6}{2}R = 3\,R$$

High-Order Thinking Skill

1 (a) According to Boyle's law, $PV = k_B T$ (a constant)

$$P\frac{m}{\rho} = k_B T \quad \Rightarrow \quad \rho = \frac{Pm}{k_B T}$$

or $\rho = \frac{P}{K}$ (where $\frac{k_B T}{m} = K$ a constant)

So, $\rho_1 = \frac{P_1}{K}$

and $V_1 = \frac{m_1}{\rho_1}$

$= \frac{m_1}{P_1 / K}$

$= \frac{Km_1}{P_1}$

Similarly, $V_2 = \frac{Km_2}{P_2}$

Total volume $= V_1 + V_2$

$= K\left(\frac{m_1}{P_1} + \frac{m_2}{P_2}\right)$

Let P be the common pressure and ρ be the common density of mixture. Then,

$\rho = \frac{m_1 + m_2}{V_1 + V_2}$

$= \frac{m_1 + m_2}{P_1} + \frac{m_2}{P_2}$

$\therefore \quad P = K\rho$

$= \frac{m_1 + m_2}{\frac{m_1}{P_1} + \frac{m_2}{P_2}}$

$= \frac{P_1 P_2 (m_1 + m_2)}{(m_1 P_2 + m_2 P_1)}$

2 (d) As we know that,

From $PV = nRT$

$P_A = \frac{\rho_A M_A}{RT}$

and $P_B = \frac{\rho_B M_B}{RT}$

From question,

$\frac{P_A}{P_B} = \frac{\rho_A}{\rho_B}\frac{M_A}{M_B} = 2$

$\therefore \quad \frac{M_A}{M_B} = 2 \times \frac{\rho_B}{\rho_A}$

$= 2 \times \frac{1}{1.5} \qquad \left(\because \frac{\rho_A}{\rho_B} = 1.5\right)$

$\frac{M_A}{M_B} = \frac{4}{3}$

3 (a) Here $v_1 = 200$ m/s;

Temperature $T_1 = 27°C = 27+273 = 300$ K

Temperature $T_2 = 127°C = 127+273 = 400$ K, v_2=?

rms velocity,

$v_2 \propto \sqrt{T}$

$\frac{v_2}{200} = \sqrt{\frac{400}{300}}$

$\Rightarrow v_2 = \frac{200 \times 2}{\sqrt{3}}$ ms^{-1}

$= \frac{400}{\sqrt{3}}$ ms^{-1}

4 (b) As no heat is lost,

Loss of kinetic energy = Gain of internal energy of gas

$\frac{1}{2}mv^2 = nC_V \Delta T$

$\Rightarrow \frac{1}{2}mv^2 = \frac{m}{M}\cdot\frac{R}{\gamma - 1}\Delta T$

$\Rightarrow \Delta T = \frac{Mv^2(\gamma - 1)}{2R}K$

5 (b) Given: Temperature, $T_i = 17+273 = 290$ K

Temperature, $T_f = 27+273 = 300$ K

Atmospheric pressure, $P_0 = 1 \times 10^5$ Pa

Volume of room, $V_0 = 30$ m^3

Difference in number of molecules, $N_f - N_i$ =?

The number of molecules,

$\Rightarrow N = \frac{PV}{RT}(N_0)$

$\therefore N_f - N_i = \frac{P_0 V_0}{R}\left(\frac{1}{T_f} - \frac{1}{T_i}\right)N_0$

$= \frac{1 \times 10^5 \times 30}{8.314} \times 6.023 \times 10^{23}\left(\frac{1}{300} - \frac{1}{290}\right)$

$= -2.5 \times 10^{25}$

6 (b) Let at temperature T, rms speed of oxygen molecules become just sufficient for escaping from the Earth's atmosphere

$v_{escape} = 11200$ ms^{-1}

$\because v_{rms} = v_{escape}$

$= \sqrt{\frac{3k_B T}{m_{O_2}}}$

$= 11200$ ms^{-1}

Putting value of k_B and m_{O_2} we get,

$T = 8.360 \times 10^4$ K

7 (d) Molar mass of the gas = 4 g/mol

Speed of sound

$v = \sqrt{\frac{\gamma RT}{m}}$

$\Rightarrow 952 = \sqrt{\frac{\gamma \times 8.3 \times 273}{4 \times 10^{-3}}}$

$\Rightarrow \gamma = 1.6$

$= \frac{8}{5}$

Also, $= \frac{C_P}{C_V}$

$= \frac{8}{5}$

So, $C_P = \gamma \times C_V$

$= \frac{8}{5} \times 5 \quad (\because C_V = 5.0\ \text{JK}^{-1})$

$= 8\ \text{JK}^{-1}\text{mol}^{-1}$

8 (a) Time of collision between molecules,

$$\tau = \frac{1}{\sqrt{2}\pi d^2 \left(\frac{N_A}{V}\right)\sqrt{\frac{3RT}{M}}}$$

$\Rightarrow \quad \tau^2 \propto \frac{V^2}{T}$

$\Rightarrow \quad T \propto \frac{V^2}{\tau^2}$ ...(i)

For an adiabatic process.

$\because \ TV^{\gamma-1} = K$

$\therefore \quad \tau \propto V^{+1/2} \quad \left(\text{From(i)}, T \propto \frac{V^2}{\tau^2}\right)$

$\Rightarrow \quad q = \frac{\gamma+1}{2}$

NCERT Exemplar Problems

1 (b) Boyle's law is applicable for an isothermal process where temperature remains constant.

2 (d) As $P = \frac{nRT}{V}$, it remains unaffected by n, R, T, and V.

3 (d) The gas molecules keep on colliding among themselves as well as with the walls of containing vessel in an ideal gas. These collisions are perfectly elastic. So, their kinetic energy and momentum remains conserved. So, the momentum transferred to the face ABCD = $2mv$. And the gas molecule is absorbed by the face EFGH. Hence it does not rebound. So, momentum transferred to the face EFGH = mv. And the pressure on the faces is due to the total momentum to the faces. So, pressure on EFGH would be half that on ABCD.

4 (d) By using,

$PV = nRT$

$\therefore \quad P_2 = \frac{P_1 V_1 T_2}{V_2 T_1}$

After putting values,

$= \frac{P \times V \times 1.1T}{1.05V \times T}$

$= 1.05\ P$

5 (c) As we know that

$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$

$= \frac{Mg}{\text{Area of piston}}$

$= \text{Constant}$

If the temperature is increased, only the volume increases as the piston moves up without friction. Recall $V \propto T$ at constant pressure.

6 (a) According to Charle's law

$V \propto T$

or $\frac{V}{T} = \text{constant}$

$= \frac{1}{P}$

As in graph, slope at P_2 is more than slope at P_1

$\therefore \quad P_1 > P_2$

7 (d) According to gas equation,

$PV = nRT$

or $\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1}$

or $\frac{P_2}{P_1} = \frac{V_1}{V_2} \cdot \frac{T_2}{T_1}$

Here, $T_2 = 3000$ K, $T_1 = 300$ K

Volume becomes half since, H_2 splits into hydrogen atoms, i.e.,

$V_2 = \frac{1}{2} V_1$

$\therefore \quad \frac{P_2}{P_1} = \frac{V_1}{\frac{1}{2}V_1} \times \frac{3000}{300}$ or $\frac{P_2}{P_1} = 20$

8 (b) The Maxwell-Boltzmann speed distribution function $\left(N_v = \frac{dN}{dv}\right)$ depends on the mass of the gas molecule. [Here, dN is the number of molecules with speeds between v and $(v + dv)$]. The masses of hydrogen and oxygen molecules are different.

Assertion and Reasons

1 (b) When a person is driving a car then the temperature of air inside the tyre is increased because of motion. From the Gay Lussac's law, $P \propto T$.
Hence, when temperature increases the pressure also increase.

2 (a) Dalton's law of partial pressures states that the total pressure of a mixture of gases is equal to the sum of the partial pressures of the component gases. The average kinetic energy of gas particles is proportional to the absolute temperature of the gas, and all gases at the same temperature have the same average kinetic energy.

3 (a) All collisions between molecules among themselves or between molecules and the wall are elastic. This implies that total kinetic energy is conserved.

4 (d) The two speeds are different from each other. Also, the Maxwell distribution for the speed to molecules in gas is symmetrical.

5 (c) By using:

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}}$$

and $$v_{av} = \sqrt{\frac{8k_B T}{\pi m}}$$

$$\therefore \quad \frac{v_{\text{rms}}}{v_{av}} = \frac{\sqrt{3}}{(\sqrt{8/\pi})} > 1$$

Thus, assertion is true. Here, reason is false because $\frac{v_{\text{rms}}}{v_{av}} = \frac{\sqrt{3}}{(\sqrt{8/\pi})} > 1$, so, $v_{\text{rms}} > v_{av}$

6 (a) The mean free path of a gas molecule is the average distance between two successive collisions. It is represented by λ

$$\lambda = \frac{1}{\sqrt{2}} \frac{k_B T}{\pi \sigma^2 P}$$

and $$\lambda = \frac{m}{\sqrt{2} \cdot \pi \sigma^2 d}$$

Here, $\sigma = 0$ diameter of molecule and

$k_B =$ Boltzmann's constant.

$\Rightarrow \quad \lambda \propto 1/d,\ \lambda \propto T$ and $\lambda \propto 1/P$

7 (d) For a monatomic gas, number of degree of freedom, $n = 3$, and for a diatomic gas, $n = 5$.

$$\text{As, } \frac{C_P}{C_V} = \gamma$$

$$= 1 + \frac{2}{n}$$

For monatomic gas,

$$\frac{C_P}{C_V} = 1.67 \text{ and}$$

For diatomic gas,

$$\frac{C_P}{C_V} = 1.4$$

$$\left(\frac{C_P}{C_V}\right)_{\text{monatomic}} > \left(\frac{C_P}{C_V}\right)_{\text{diatomic}}$$

Hence, when temperature increases the pressure also increase.

8 (c) The number 6.02×10^{23} is Avogadro's number and one mole of a substance contains Avogadro's number of molecules.

9 (c) By law of equipartition of energy, the energy for each degree of freedom in thermal equilibrium is $\frac{1}{2} k_B T$

Each quadratic term form in the total energy expression of a molecules is to be counted as a degree of freedom. Thus, each vibrational mode gives 2 degree of freedom, i.e., kinetic and potential energy modes, corresponding to the energy $2\left(\frac{1}{2} k_B T\right) = k_B T$

10 (a) In case of C_V, volume of the gas is kept constant and heat is required only for raising the temperature of one gram mole of the gas through 1°C or 1 K. No heat, what so ever, is spent in expansion of the gas. In case of C_P, as pressure of the gas is kept constant, the gas would expand on heating. Therefore, some heat is spent in expansion of the gas against external pressure. This is in addition to the amount of heat energy required for raising the temperature of one gram mole of the gas through 1°C or 1 K. Hence specific heat at constant pressure is greater than the specific heat at constant volume.

11 (b) The total translational kinetic energy of gas molecules is given

$$E = \frac{3}{2} nRT$$

$$= \frac{3}{2} PV$$

$$= 1.5 PV$$