7 Areas Related to Circles

Topics Covered

1. Perimeter and Area of a Circle
2. Areas of Sector and Segment of a Circle

1. Perimeter and Area of a Circle

As we know that a **circle** is a closed curve consisting of a set of all those points of the plane which are at a constant distance from a fixed point in the plane. The fixed point is called its **centre**. The constant distance is called its **radius**. It is the line segment joining any point on the boundary (circumference) to centre. The boundary (or perimeter) of a circle is called its **circumference**.

- Circumference of a circle = $2\pi r$ (unit)
- Area of a circle = πr^2 unit^2
- Area of the circular path formed by two concentric circles of radii r_1 and r_2 $(r_1 > r_2)$ $= \pi r_1^{\,2} - \pi r^2_{\,2} = \pi\,(r_1^{\,2} - r_2^{\,2})$ (unit)^2,
- The distance travelled (covered) by a wheel in 1 round = its circumference = $2\pi r$ (unit)
- Total distance covered by a wheel = its circumference × number of rounds taken by it.
- Number of rounds made by a wheel = $\dfrac{\text{Total distance covered}}{\text{Its circumference}}$
- Speed of the wheel = $\dfrac{\text{Total distance covered}}{\text{Time taken}}$
- If speed = x km/hr, then speed = $x \times \dfrac{5}{18}$ m/s and if speed = x m/s, then speed = $x \times \dfrac{18}{5}$ km/hr.

Note: Unless stated otherwise, the value of π is to be taken as $\dfrac{22}{7}$.

Example 1. The area of the circle, the circumference of which is equal to the perimeter of a square of side 11 cm is

(*a*) 122 cm^2 (*b*) 144 cm^2 (*c*) 154 cm^2 (*d*) 180 cm^2

Solution. Since, side of the square = 11 cm

$\therefore$ Perimeter of the square = 4×11 cm = 44 cm

Given: Circumference of circle = perimeter of square

$\Rightarrow$ $2\pi r = 44 \Rightarrow 2 \times \dfrac{22}{7} \times r = 44 \Rightarrow r = 7$ cm

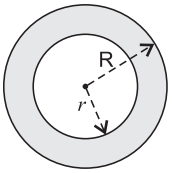
$\therefore$ Area of the circle = $\pi r^2 = \dfrac{22}{7} \times (7)^2 = 154$ cm^2

Hence, option (*c*) is the correct answer.

Example 2. The area of a ring shaped region enclosed between two concentric circles of radii 20 cm and 15 cm is

(*a*) 330 cm^2 (*b*) 415 cm^2 (*c*) 520 cm^2 (*d*) 550 cm^2

Solution. For the given ring shaped region; R = 20 cm and r = 15 cm



$\therefore$ Required area $= \pi R^2 - \pi r^2$

$= \pi (R + r)(R - r)$

$= \frac{22}{7}(20 + 15)(20 - 15)\ \text{cm}^2 = 550\ \text{cm}^2$

Hence, option (*d*) is the correct answer.

Example 3. If the perimeter and area of a circle are numerically equal; its radius will be

(*a*) 1 unit (*b*) 2 units (*c*) 4 units (*d*) None of these

Solution. Let the radius of the circle be r units; then given that: $2\pi r = \pi r^2$

$\Rightarrow$ $r = 2$ units

Hence, option (*b*) is the correct answer.

Example 4. A wheel has diameter 84 cm. Number of complete revolutions must it make to cover 792 metres will be

(*a*) 100 (*b*) 160 (*c*) 220 (*d*) 300

Solution. Since distance covered by the wheel in 1 round = $\pi \times$ diameter = $\frac{22}{7} \times 84$ cm = 264 cm

And, total distance covered = 792 m = 792×100 cm

$\therefore$ No. of complete revolutions made = $\frac{\text{Total distance covered}}{\text{Distance covered by wheel in 1 round}} = \frac{792 \times 100\ \text{cm}}{264\ \text{cm}} = 300$

Hence, option (*d*) is the correct answer.

Example 5. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

(*a*) 2275 (*b*) 2650 (*c*) 3815 (*d*) 4375

Solution. $\because$ Speed = 66 km per hour = $66 \times \frac{5}{18}$ m/s = $\frac{66 \times 5 \times 100}{18}$ cm/s

and, time taken = 10 minutes = 10×60 sec = 600 sec

$\therefore$ Distance covered in 10 min = Speed $\times$ time = $\frac{66 \times 5 \times 100}{18} \times 600$ cm = 11,00,000 cm

Given, diameter of each wheel = 80 cm

$\therefore$ Distance covered by each wheel in 1 revolution = its circumference.

$= \pi \times \text{diameter} = \frac{22}{7} \times 80\ \text{cm} = \frac{1760}{7}\ \text{cm}$

$\Rightarrow$ No. of revolutions made by each wheel in 10 min = $\frac{\text{Total distance covered}}{\text{Distance covered in 1 revolution}}$

$= \frac{11,00,000\ \text{cm}}{\frac{1760}{7}\ \text{cm}} = 4375$

Hence, option (*d*) is the correct answer.

Exercise 7.1

A. Multiple Choice Questions (MCQs)

. **Choose the correct answer from the given options:**

1. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is

(*a*) 22 : 7 (*b*) 14 : 11 (*c*) 7 : 22 (*d*) 11 : 14

2. The area of the square that can be inscribed in a circle of radius 8 cm is

(*a*) 256 cm^2 (*b*) 128 cm^2 (*c*) $64\sqrt{2}$ cm^2 (*d*) 64 cm^2

3. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is

(*a*) 31 cm (*b*) 25 cm (*c*) 62 cm (*d*) 50 cm

4. If the area of circle is numerically equal to twice its circumference, then the diameter of the circle is

(*a*) 4 units (*b*) 6 units (*c*) 8 units (*d*) 12 units

5. The perimeter (in cm) of a square circumscribing a circle of radius *a* cm is

(*a*) 2*a* (*b*) 4*a* (*c*) 6*a* (*d*) 8*a*

6. What is the diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm?

(*a*) 20 cm (*b*) 30 cm (*c*) 50 cm (*d*) 80 cm

7. What is the area of the circle that can be inscribed in a square of side 6 cm?

(*a*) 9 π cm^2 (*b*) 11 π cm^2 (*c*) 16 π cm^2 (*d*) 15 π cm^2

8. The area of a quadrant of a circle whose circumference is 25 cm is **[Imp.]**

(*a*) 24 cm^2 (*b*) 28 cm^2 (*c*) 32.5 cm^2 (*d*) 38.5 cm^2

9. If area of quadrant of a circle is 38.5 cm^2, then its diameter is $\left[\text{Use } \pi = \frac{22}{7}\right]$

(*a*) 10 cm (*b*) 14 cm

(*c*) 21 cm (*d*) None of these

10. The areas of two circles are in the ratio 9 : 4, then what is the ratio of their circumferences?

(*a*) 1 : 2 (*b*) 2 : 1 (*c*) 3 : 2 (*d*) 2 : 3

11. The cost of fencing a circular field at the rate of ₹ 24 per metre is ₹ 5280. The radius of the field is

(*a*) 15 m (*b*) 35 m (*c*) 25 m (*d*) 30 m

12. The radii of two circles are 8 cm and 6 cm respectively. The radius of the circle having area equal to the sum of the areas of the two circles is

(*a*) 5 cm (*b*) 10 cm (*c*) 12 cm (*d*) 15 cm

13. An athlete runs on a circular track of radius 49 m and covers a distance of 3080 m along its boundary. How many rounds has he taken to cover this distance? $\left[\text{Take } \pi = \frac{22}{7}\right]$

(*a*) 5 (*b*) 8 (*c*) 10 (*d*) 15

14. The area of the largest triangle that can be inscribed in a semi-circle of radius *r* units will be

(*a*) *r* sq. units (*b*) $\frac{r}{2}$ sq. units (*c*) r^2 sq. units (*d*) 2*r* sq. units

15. The cost of fencing a circular field at the rate of ₹ 24 per metre is ₹ 5280. The field is to be ploughed at the rate of ₹ 0.50 per m^2. The cost of ploughing the field is $\left[\text{Take } \pi = \frac{22}{7}\right]$

(*a*) ₹ 1925 (*b*) ₹ 1650

(*c*) ₹ 2010 (*d*) ₹ 2525

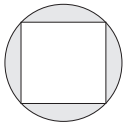
16. The area of circle whose circumference is 22 cm is

(*a*) $\frac{32}{2}$ cm^2 (*b*) $\frac{45}{2}$ cm^2 (*c*) $\frac{55}{2}$ cm^2 (*d*) $\frac{77}{2}$ cm^2

17. A road which is 7 m wide surrounds a circular park whose circumference is 88 m. The area of the road is

(*a*) 220 m^2 (*b*) 340 m^2 (*c*) 550 m^2 (*d*) 770 m^2

18. In the given figure, a square of diagonal 8 cm is inscribed in a circle. The area of the shaded region is



(*a*) $12\frac{1}{2}$ cm (*b*) $18\frac{2}{7}$ cm^2 (*c*) $24\frac{3}{4}$ cm^2 (*d*) $11\frac{2}{9}$ cm^2

19. The diameter of a circle whose circumference is equal to the sum of the circumference of the two circles of diameters 36 cm and 20 cm is
(*a*) 22 cm (*b*) 32 cm (*c*) 56 cm (*d*) 84 cm

20. It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park is
(*a*) 10 m (*b*) 12 m (*c*) 15 m (*d*) 18 m

B. Assertion-Reason Type Questions

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct choice as:

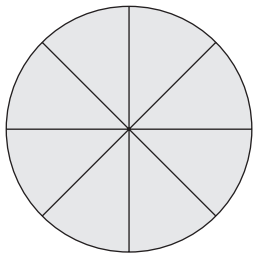
(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(*c*) Assertion (A) is true but reason (R) is false.
(*d*) Assertion (A) is false but reason (R) is true.

1. **Assertion (A):** If the circumference of a circle is 176 cm, then its radius is 28 cm.
Reason (R): Circumference = $2\pi \times$ radius.

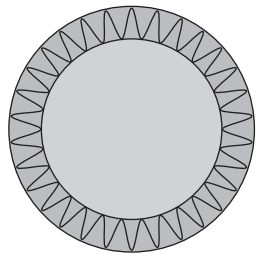
2. **Assertion (A):** If the outer and inner diameter of a circular path is 10 m and 6 m respectively, then area of the path is $16\ \pi\ m^2$.
Reason (R): If R and r be the radius of outer and inner circular path respectively, then area of circular path = $\pi\ (R^2 - r^2)$.

Case Study Based Questions

I. A brooch is a small piece of jewellery which has a pin at the back so it can be fastened on a dress, blouse or coat.
Designs of some brooch are shown below. Observe them carefully.



A



B

Design A: Brooch A is made with silver wire in the form of a circle with diameter 28 mm. The wire used for making 4 diameters which divide the circle into 8 equal sectors.
Design B: Brooch B is made with two colours — Gold and Silver. Outer part is made with gold. The circumference of silver part is 44 mm and the gold part is 3 mm wide everywhere.

Refer to Design A

1. The circumference of the brooch is
(*a*) 28 mm (*b*) 44 mm (*c*) 56 mm (*d*) 88 mm

2. The total length of the silver wire required is
(*a*) 180 mm (*b*) 200 mm (*c*) 250 mm (*d*) 280 mm

Refer to Design B

3. The circumference of outer part (golden) is
(*a*) 48.49 mm (*b*) 82.2 mm (*c*) 72.50 mm (*d*) 62.86 mm

4. The difference in areas of golden and silver parts is
(*a*) $18\pi\ mm^2$ (*b*) $44\pi\ mm^2$ (*c*) $51\pi\ mm^2$ (*d*) $64\pi\ mm^2$

5. A boy is playing with the brooch B. He makes revolution with it along its edge. How many complete revolutions must it take to cover 80π mm?

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

Answers and Hints

A. Multiple Choice Questions (MCQs)

1. (*b*) 14 : 11 **2.** (*b*) 128 cm^2

3. (*d*) 50 cm **4.** (*c*) 8 units

5. (*d*) $8a$

6. (*c*) 50 cm

According to the question,

$$\pi R^2 = \pi r_1^2 + \pi r_2^2$$

$$\Rightarrow \quad \pi(R^2) = \pi(r_1^2 + r_2^2) \Rightarrow R^2 = r_1^2 + r_2^2$$

$$= (24)^2 + (7)^2 \quad [r_1 = 24 \text{ cm}, \ r_2 = 7 \text{ cm}]$$

$$= 576 + 49 = 625$$

$$\Rightarrow \quad R = \sqrt{625} \quad \Rightarrow R = 25$$

$\therefore$ Diameter $= 2R = 2 \times 25 = 50$ cm.

7. (*a*) 9π cm^2

Diameter of the circle inscribed in a square = side of square

$$\therefore \quad 2r = a$$

$$\Rightarrow \quad r = \frac{a}{2} = \frac{6}{2} = 3 \text{ cm}$$

$\therefore$ Area of circle $= \pi r^2 = \pi(3)^2 = 9\pi$ cm^2

8. (*d*) 38.5 cm^2

$$\frac{2\pi r}{4} + 2r = 25 \text{ cm}$$

$$\Rightarrow \quad \pi r + 4r = 50 \Rightarrow r\left(\frac{22}{7} + 4\right) = 50$$

$$\Rightarrow \quad r = 50 \times \frac{7}{50} = 7 \text{ cm}$$

$$\therefore \text{Area of quadrant} = \frac{\pi r^2}{4} = \frac{22 \times 7 \times 7}{7 \times 4}$$

$$= 38.5 \text{ cm}^2$$

9. (*b*) 14 cm

$$\text{Area of quadrant} = \frac{1}{4} \times \frac{22}{7} \times r^2$$

$$= 38.5 = \frac{1}{4} \times \frac{22}{7} \times r^2$$

$$\Rightarrow \quad r = 7 \text{ cm}$$

$$\therefore \quad \text{Diameter} = 14 \text{ cm}$$

10. (*c*) 3 : 2

Let r_1 and r_2 be the radii of two circles, A_1 and A_2 be areas of two circles and C_1 and C_2 be circumferences of two circles.

Then, $A_1 : A_2 = 9 : 4 \Rightarrow \frac{A_1}{A_2} = \frac{9}{4}$

$$\Rightarrow \quad \frac{\pi r_1^2}{\pi r_2^2} = \frac{9}{4} \Rightarrow \frac{r_1}{r_2} = \frac{3}{2}$$

Now, $C_1 : C_2 = 2\pi r_1 : 2.\pi r_2$

$$= \frac{2\pi r_1}{2\pi r_2} = \frac{3}{2}$$

So, $C_1 : C_2 = 3 : 2$

11. (*b*) 35 m

$$\text{Length of the fence} = \frac{\text{Total cost}}{\text{Rate}}$$

$$= \frac{₹\ 5280}{₹\ 24/\text{metre}} = 220 \text{ m}$$

So, length of fence = Circumference of the field

$$\therefore \quad 220 \text{ m} = 2\pi r = 2 \times \frac{22}{7} \times r$$

$$\text{So,} \quad r = \frac{220 \times 7}{2 \times 22} \text{ m} = 35 \text{ m}$$

12. (*b*) 10 cm

$$\because \quad \pi r^2 = 64\pi + 36\pi$$

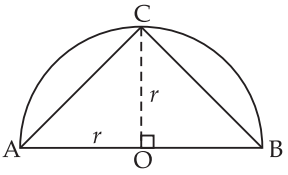
$$\Rightarrow \quad \pi r^2 = 100\pi \quad \Rightarrow \quad r^2 = 100$$

$$\Rightarrow \quad r = 10 \text{ cm}$$

$\therefore$ Radius of the required circle is 10 cm.

13. (*c*) 10

14. (*c*) r^2 square units.



Base AB of triangle ABC in semicircle is constant, *i.e.*, equal to $2r$, and maximum altitude may be equal to r.

$$\therefore \quad \text{Area of triangle} = \frac{1}{2} \text{ base} \times \text{altitude}$$

$$= \frac{1}{2} \text{ AB} \times \text{OC} = \frac{1}{2}(2r) \times r = r^2$$

∴ Area of triangle in semicircle
$= r^2$ square units.

15. (*a*) ₹ 1925

Circumference of field $= \frac{5280}{24} = 220$ m

$\therefore \quad 2\pi r = 220 \Rightarrow r = \frac{220 \times 7}{2 \times 22} = 35$ m

Now,

area of the field $= \frac{22}{7} \times (35)^2 = 22 \times 5 \times 35$ m^2

The cost of ploughing the field
$= ₹ (22 \times 5 \times 35 \times 0.50)$
$= ₹ 1925$

16. (*d*) $\frac{77}{2}$ cm^2

$2\pi r = 22$ cm

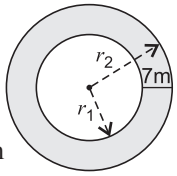
$r = \frac{22}{2\pi} = \frac{22 \times 7}{2 \times 22} \Rightarrow r = \frac{7}{2}$

Area of circle $= \pi r^2 = \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = \frac{77}{2}$ cm^2

17. (*d*) 770 m^2

Circumference of circular park = 88 m

$2\pi r_1 = 88$ m



$\Rightarrow r_1 = \frac{88}{2 \times \pi} = \frac{88 \times 7}{2 \times 22} = 14$ m

Width of road = 7 m

So, $r_2 = r_1 + 7$ m $= 14 + 7 = 21$ m

So, area of the road $= \pi r_2^2 - \pi r_1^2 = \frac{22}{7}(21^2 - 14^2)$

$= \frac{22}{7}(21 + 14)(21 - 14)$

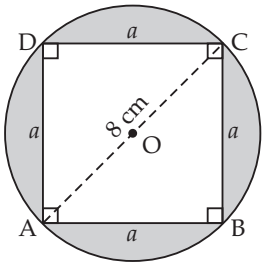
$= \frac{22}{7} \times 35 \times 7 = 770$ m^2

18. (*b*) $18\frac{2}{7}$ cm^2

Let the side of the square be *a* cm.

So, radius of the circle,

$r = OA = \frac{AC}{2}$



$\Rightarrow r = \frac{8}{2} = 4$ cm

So, in right angled ΔABC,

$AB^2 + BC^2 = AC^2$

$\Rightarrow \quad a^2 + a^2 = 8^2 \quad \Rightarrow \quad 2a^2 = 64$

$\Rightarrow \quad a^2 = \frac{64}{2} \quad \Rightarrow \quad a^2 = 32$

Area of shaded part
= Area of circle – Area of square

$= \pi r^2 - a^2 = \frac{22}{7} \times 4 \times 4 - 32$

$= 16\left[\frac{22}{7} - \frac{2}{1}\right] = 16\left[\frac{22 - 14}{7}\right]$

$= \frac{16 \times 8}{7} = \frac{128}{7}$

Area of shaded region $= 18\frac{2}{7}$ cm^2

19. (*c*) 56 cm

$2\pi r = 2\pi \times \frac{36}{2} + 2\pi \times \frac{20}{2}$

$= 36\pi + 20\pi = 56\pi$

$\Rightarrow \quad r = 28$ cm

$\therefore$ Diameter of the required circle = 28 cm × 2
= 56 cm

20. (*a*) 10 m

Let the radius of the new park be R.

$\therefore$ Area of new park
= Area of old park I + Area of park II

$\Rightarrow \quad \pi R^2 = \pi r_1^2 + \pi r_2^2$

$\Rightarrow \quad \pi(R^2) = \pi(r_1^2 + r_2^2)$

$\Rightarrow \quad \pi(R^2) = \pi[8^2 + 6^2] \quad \begin{bmatrix} r_1 = 28 \text{ cm} \\ r_2 = 6 \text{ cm} \end{bmatrix}$

$\Rightarrow \quad R^2 = 64 + 36$

$\Rightarrow \quad R = \sqrt{100} = 10$ m

B. Assertion-Reason Type Questions

1. (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

2. (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Case Study Based Questions

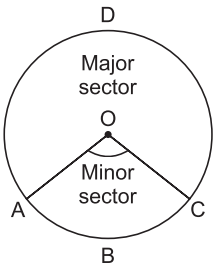
I. 1. (*d*) 88 mm **2.** (*b*) 200 mm

3. (*d*) 62.86 mm **4.** (*c*) 51π

5. (*c*) 4

2. Areas of Sector and Segment of a Circle

A sector is a part of the circular region which is enclosed by two radii and the corresponding arc. Hence, OABC is a minor sector and OCDA is a major sector. ∠AOC is called the angle of sector.

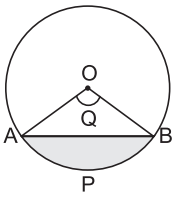


- Area of the sector of a circle of radius r with central angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$, where θ is measured in degrees.

OR

$$\text{Area of the sector} = \frac{1}{2} \times \text{length of arc} \times \text{radius} = \frac{1}{2} lr$$

- Length of the arc of the sector of a circle of radius r with central angle $\theta = \frac{\theta}{360^\circ} \times 2\pi r$, where θ is measured in degrees.



- Area of the minor segment APB of the circle in the given figure
  = area of sector OAPB – area of ΔOAB
  $= \frac{\theta}{360^\circ} \times \pi r^2 - \frac{1}{2} r^2 \times \sin\theta$
- Area of the major sector of a circle of radius r
  $= \pi r^2$ – area of the corresponding minor sector.

Example 1. The area of a sector of a circle with radius 6 cm if angle of the sector is 60° is

(*a*) $15\frac{2}{3}$ cm^2 (*b*) $16\frac{1}{2}$ cm^2 (*c*) $18\frac{6}{7}$ cm^2 (*d*) $19\frac{3}{8}$ cm^2

Solution. Given: $r = 6$ cm and $\theta = 60^\circ$

$$\therefore \quad \text{Area of sector} = \pi r^2 \times \frac{\theta}{360^\circ} = \frac{22}{7} \times 6 \times 6 \times \frac{60^\circ}{360^\circ} \text{ cm}^2 = \frac{132}{7} \text{ cm}^2 = 18\frac{6}{7} \text{ cm}^2$$

Hence, option (*c*) is the correct answer.

Example 2. A chord AB of a circle of radius 10 cm subtends a right angle at the centre. The area of the minor-sector is [Take $\pi = 3.14$] **[Imp.]**

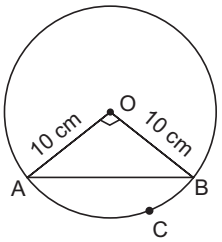
(*a*) 38.5 cm^2 (*b*) 42 cm^2 (*c*) 78.5 cm^2 (*d*) 82 cm^2

Solution: ∵ $\theta = 90^\circ$ and $r = 10$ cm

$$\therefore \quad \text{Area of the minor-sector OACB} = \pi r^2 \times \frac{\theta}{360^\circ}$$

$$= 3.14 \times 10 \times 10 \times \frac{90^\circ}{360^\circ} \text{ cm}^2$$

$$= 78.5 \text{ cm}^2$$



Hence, option (*c*) is the correct answer.

Areas of Combinations of Plane Figures

Example 3. A square ABCD is inscribed in a circle of radius 10 units. The area of the circle, not included in the square is (Take $\pi = 3.14$)

(*a*) 84 cm^2 (*b*) 108 cm^2 (*c*) 114 cm^2 (*d*) 122 cm^2

Solution. When a square or a rectangle is inscribed in a circle then diagonal of the square is the diameter of the circle.

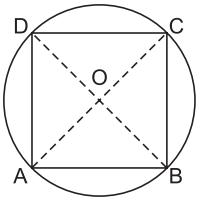
$$\therefore \quad \text{The diagonal AC of the square ABCD} = \text{Diameter of the circle} = 2 \times 10 \text{ cm}$$

$$= 20 \text{ cm}$$

Since, the diagonals of a square are equal and bisect each other at 90°; therefore; AC = BD = 20 cm

and area of the square $= \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 20 \times 20 \text{ cm}^2 = 200 \text{ cm}^2$



Also, area of the circle $= \pi r^2 = 3.14 \times (10)^2 \text{ cm}^2 = 314 \text{ cm}^2$

$\therefore$ The required area = Area of circle – Area of the square

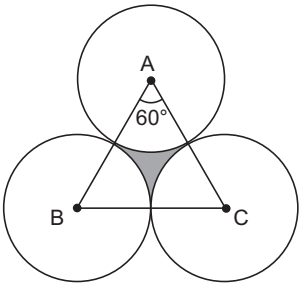
$= 314 \text{ cm}^2 - 200 \text{ cm}^2 = 114 \text{ cm}^2$.

Hence, option (*c*) is the correct answer.

Example 4. The area of an equilateral triangle is 17320.5 cm^2. With each vertex as centre, a circle is described with radius equal to half the length of the side of the triangle. The area of the triangle not included in the circles is (Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$).

(*a*) 1620.51 cm^2 (*b*) 1810.25 cm^2 (*c*) 2430.60 cm^2 (*d*) None of these

Solution. We know that area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{side})^2$



$\therefore \quad \frac{\sqrt{3}}{4} \times (\text{side})^2 = 17320.5$

$\Rightarrow \quad (\text{side})^2 = 17320.5 \times \frac{4}{\sqrt{3}} = 17320.5 \times \frac{4}{1.73205}$

$\Rightarrow$ Side = 200 cm.

$\Rightarrow$ Radius of each circle drawn $= \frac{200}{2}$ cm = 100 cm

For each minor sector; $\theta = 60°$

$\therefore$ Area of each sector inside the triangle $= \pi r^2 \times \frac{\theta}{360°}$

$= 3.14 \times (100)^2 \times \frac{60°}{360°} \text{ cm}^2 = 5233.33 \text{ cm}^2$

$\therefore$ Area of the triangle not included in the circles

= Area of triangle – 3 × area of each sector inside the triangle

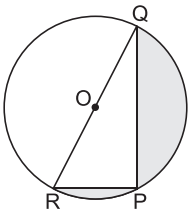
$= (17320.5 - 3 \times 5233.33) \text{ cm}^2 = 1620.51 \text{ cm}^2$.

Hence, option (*a*) is the correct answer.

Example 5. In the given figure, PQ = 24 cm, PR = 7 cm and O is the centre of the circle. The area of the shaded portion is

(*a*) 132.58 cm^2 (*b*) 148.20 cm^2

(*c*) 154.36 cm^2 (*d*) 161.54 cm^2



Solution. We know that the angle subtended by semi-circle is 90°

$\Rightarrow \quad \angle RPQ = 90°$,

i.e., ΔRPQ is a right-angled triangle with hypotenuse = RQ

$\therefore \quad RQ^2 = PR^2 + PQ^2$

$\Rightarrow \quad RQ^2 = (7)^2 + (24)^2 = 49 + 576 = 625$

$\Rightarrow \quad RQ = 25$ cm = diameter of the circle.

$\therefore$ Radius of the circle $= \frac{RQ}{2} = \frac{25}{2}$ cm = 12.5 cm

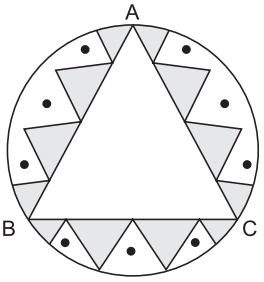
Area of given shaded portion = Area of semicircle – Area of right ΔRPQ

$= \frac{1}{2}\pi r^2 - \frac{1}{2} \times RP \times PQ = \frac{1}{2} \times \frac{22}{7} \times (12.5)^2 - \frac{1}{2} \times 7 \times 24 = \frac{1718.75}{7} - 84$

$$= \frac{1718.75 - 588}{7} = \frac{1130.75}{7} \text{ cm}^2 = 161.54 \text{ cm}^2$$

Hence, option (*d*) is the correct answer.

Example 6. In a circular table cover of radius 32 cm, a design is formed having an equilateral triangle ABC in the middle, as shown below. The area of the design is **[Imp.]**



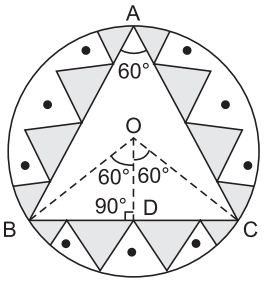
(*a*) 777.36 cm^2 (*b*) 1888.11 cm^2

(*c*) 2010.54 cm^2 (*d*) None of these

Solution. Let O be the centre of the circle. Join OB and OC, also draw OD $\perp$ BC.

Since, ABC is an equilateral triangle, $\angle BAC = 60°$.

Also, angle subtended by an arc at the centre of the circle is twice the angle at remaining circumference.



$\therefore \quad \angle BOC = 2\angle BAC = 2 \times 60° = 120°$

Now, by R.H.S., $\Delta OBD \cong \Delta OCD \Rightarrow \angle BOD = \angle COD = 60°$

In ΔOBD, OB = radius of the circle = 32 cm

$$\Rightarrow \quad \sin 60° = \frac{BD}{OB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{BD}{32} \Rightarrow BD = 16\sqrt{3} = CD$$

$$\therefore \quad BC = 2\,BD = 2 \times 16\sqrt{3} \text{ cm} = 32\sqrt{3} \text{ cm}$$

Area of the design = Area of the circle – Area of equilateral ΔABC

$$= \pi r^2 - \frac{\sqrt{3}}{4} \times (\text{side})^2 = \left[\frac{22}{7} \times (32)^2 - \frac{\sqrt{3}}{4} \times (32\sqrt{3})^2\right] \text{cm}^2$$

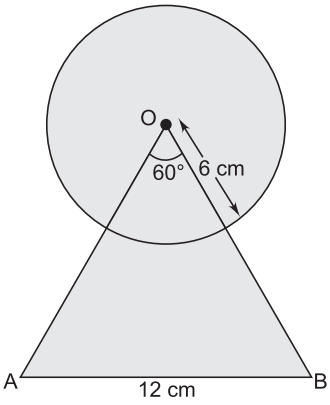
$$= \left(\frac{22528}{7} - 768\sqrt{3}\right) \text{cm}^2 = 1888.11 \text{ cm}^2$$

Hence, option (*b*) is the correct answer.

Example 7. The area of the shaded portion in the figure, given below, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre is **[Imp.]**

(*a*) 156.64 cm^2 (*b*) 188.46 cm^2 (*c*) 256.64 cm^2 (*d*) 310.25 cm^2

Solution. Since, ΔOAB is equilateral, $\angle AOB = 60°$



Required area = Area of circle + Area of ΔOAB – Area of sector with angle 60°

$$= \pi r^2 + \frac{\sqrt{3}}{4} \times (\text{side})^2 - \pi r^2 \times \frac{\theta}{360°}$$

$$= \frac{22}{7} \times 6^2 + \frac{\sqrt{3}}{4} \times (12)^2 - \frac{22}{7} \times 6^2 \times \frac{60}{360} \text{ cm}^2$$

$$= \frac{22}{7}(36 - 6) + \frac{\sqrt{3}}{4} \times 144 \text{ cm}^2$$

$$= \left(\frac{660}{7} + 36\sqrt{3}\right) = 156.64 \text{ cm}^2$$

Hence, option (*a*) is the correct answer.

Example 8. ABCD is a square of side 4 cm. At each corner of the square, a quarter circle of radius 1 cm, and at the centre, a circle of radius 1 cm, are drawn, as shown in the given figure. The area of the shaded region is

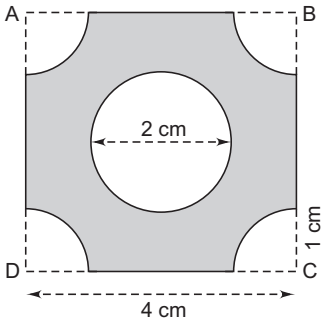
(*a*) 8.46 cm^2 (*b*) 7.25 cm^2 (*c*) 9.71 cm^2 (*d*) 10.43 cm^2

Solution. Area of the shaded portion

= Area of given square ABCD
– 4 × area of each quarter circle
– area of the circle at the centre.

$$= 4\times4-4\left[\frac{\pi(1)^2}{4}\right]-\pi(1)^2 \text{ sq. cm}$$

$$= \left[16-\frac{22}{7}-\frac{22}{7}\right] cm^2 = 9\frac{5}{7} cm^2 = 9.71 cm^2$$



Hence, option (*c*) is the correct answer.

Exercise 7.2

A. Multiple Choice Questions (MCQs)

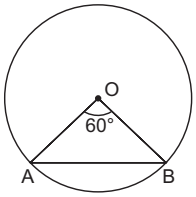
Choose the correct answer from the given options.

1. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. The length of the arc is
(*a*) 11 cm (*b*) 22 cm (*c*) 27 cm (*d*) 44 cm

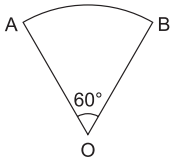
2. An arc of length 15.7 cm subtends a right angle at the centre of the circle. Then the radius of the circle is
(*a*) 20 cm (*b*) 10 cm (*c*) 15 cm (*d*) 12 cm

3. The angle described by a minute hand in 5 minutes is
(*a*) 30° (*b*) 60° (*c*) 90° (*d*) None of these

4. The area of the sector in the following figure showing a chord AB of a circle of radius 18 cm subtending an angle of 60° at the centre O is **[Imp.]**
[Take π = 3.14]
(*a*) 151.31 cm^2 (*b*) 169.56 cm^2
(*c*) 173.33 cm^2 (*d*) None of these



5. The given figure is a sector of circle of radius 10.5 cm. The perimeter of the sector is **[Imp.]**
[Take π = $\frac{22}{7}$]
(*a*) 32 cm (*b*) 44 cm
(*c*) 54 cm (*d*) None of these

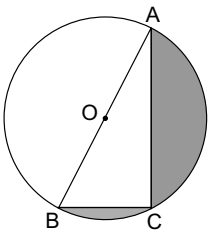


6. In a circle of diameter 42 cm,if an arc subtends an angle of 60° at the centre where π = $\frac{22}{7}$, then what will be the length of arc?
(*a*) 11 cm (*b*) 20 cm (*c*) 22 cm (*d*) 28 cm

7. A horse is tied to a pole with 28 m long rope. The perimeter of the field where the horse can graze is (Take π = 22/7)
(*a*) 60 cm (*b*) 85 cm (*c*) 124 cm (*d*) 176 cm

8. A car has two wipers which do not overlap. Each wiper has a blade of length 21 cm sweeping through an angle 120°. The total area cleaned at each sweep of the blades is [Take π = $\frac{22}{7}$]
(*a*) 360 cm^2 (*b*) 448 cm^2 (*c*) 556 cm^2 (*d*) 924 cm^2

9. The area of the shaded region in the given figure, if AC = 24 cm, BC = 10 cm and O is the centre of the circle is [Take $\pi = 3.14$]



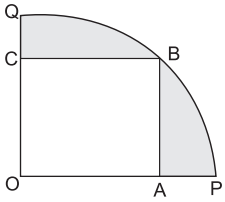
(*a*) 128.56 cm^2

(*b*) 145.33 cm^2

(*c*) 248.16 cm^2

(*d*) None of these

10. In the given figure, a square OABC is inscribed in a quadrant OPBQ of a circle. If OA = 20 cm, the area of the shaded region is [Use $\pi = 3.14$]



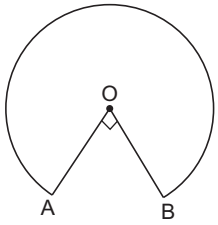
(*a*) 126 cm^2

(*b*) 200 cm^2

(*c*) 228 cm^2

(*d*) None of these

11. In the given figure, the shape of the top of a table is that of a sector of a circle with centre O and $\angle AOB = 90°$. If AO = OB = 42 cm, then the perimeter of the top of the table is [Take $\pi = \frac{22}{7}$]



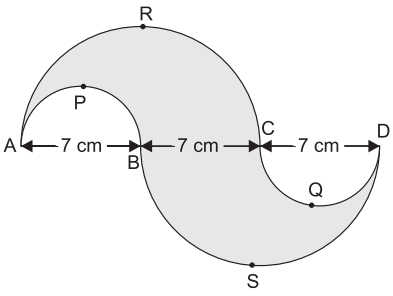
(*a*) 282 cm

(*b*) 266 cm

(*c*) 320 cm

(*d*) 480 cm

12. In the given figure, APB and CQD are semicircles of diameter 7 cm each, while ARC and BSD are semicircles of diameter 14 cm each. The perimeter of the shaded region is [Take $\pi = \frac{22}{7}$]



(*a*) 22 cm

(*b*) 33 cm

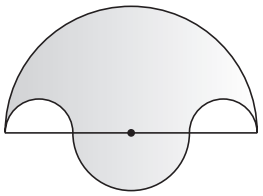
(*c*) 44 cm

(*d*) 66 cm

13. A piece of wire 22 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. The radius of the circle is [Take $\pi = \frac{22}{7}$]

(*a*) 7 cm (*b*) 14 cm (*c*) 21 cm (*d*) 28 cm

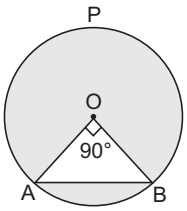
14. In the given figure, the boundary of shaded region consists of four semicircular arcs, two smallest being equal. If diameter of the largest is 14 cm and that of the smallest is 3.5 cm, the area of the shaded region is [Take $\pi = \frac{22}{7}$]



(*a*) 72.225 cm^2 (*b*) 86.625 cm^2

(*c*) 92.330 cm^2 (*d*) None of these

15. The area of the major segment APB in figure of a circle of radius 35 cm and $\angle AOB = 90°$ is [Take $\pi = \frac{22}{7}$]



(*a*) 1200 cm^2

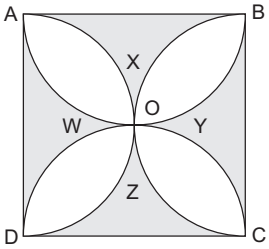
(*b*) 2400 cm^2

(*c*) 3500 cm^2

(*d*) None of these

16. In the given figure, ABCD is a square of side 14 cm. Semicircles are drawn with each side of square as diameter. The area of the shaded region is

[Take $\pi = \frac{22}{7}$]



(*a*) 84 cm^2
(*b*) 96 cm^2
(*c*) 110 cm^2
(*d*) 220 cm^2

17. The short and long hands of a clock are 4 cm and 6 cm long respectively. The sum of distances travelled by their tips in 2 days is

(*a*) 1148 cm (*b*) 1426.35 cm (*c*) 1910.85 cm (*d*) None of these

18. The area of the sector of a circle of radius 5 cm, if the corresponding arc length is 3.5 cm is

(*a*) 3.25 cm^2 (*b*) 8.75 cm^2 (*c*) 4.60 cm^2 (*d*) 5.50 cm^2

19. The area of the largest circle that can be drawn inside the given rectangle of length '*a*' cm and breadth '*b*' cm ($a > b$) is

(*a*) $\frac{1}{2}\pi b^2$ cm^2 (*b*) $\frac{1}{3}\pi b^2$ cm^2 (*c*) $\frac{1}{4}\pi b^2$ cm^2 (*d*) πb^2 cm^2

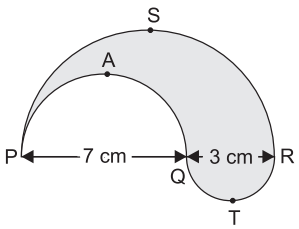
20. All the vertices of a rhombus lie on a circle. The area of the rhombus, if the area of the circle is 1256 cm^2 is [Use $\pi = 3.14$]

(*a*) 300 cm^2 (*b*) 600 cm^2 (*c*) 800 cm^2 (*d*) 900 cm^2

21. The difference of the areas of two segments of a circle formed by a chord of radius 5 cm subtending an angle of 90° at the centre is **[Imp.]**

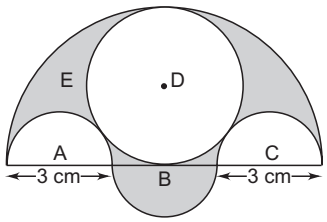
(*a*) $\left(\frac{25\pi}{4} - \frac{25}{2}\right)$ cm^2 (*b*) $\left(\frac{15\pi}{4} - \frac{7}{2}\right)$ cm^2 (*c*) $\left(\frac{7\pi}{4} - \frac{3}{2}\right)$ cm^2 (*d*) None of these

22. In the given figure, PSR, RTQ and PAQ are three semicircles of diameters 10 cm, 3 cm and 7 cm respectively. The perimeter of the shaded region is [Use $\pi = 3.14$]



(*a*) 22.5 cm
(*b*) 31.4 cm
(*c*) 36.6 cm
(*d*) None of these

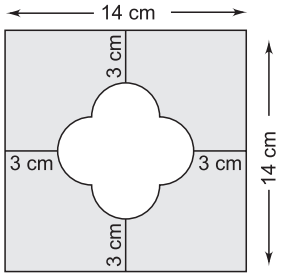
23. There are three semi-circles A, B and C having diameter 3 cm each and another semicircle E having a circle D with diameter 4.5 cm as shown in the figure. The area of the shaded region is



(*a*) $\frac{15}{16}\pi$ cm^2 (*b*) $\frac{18}{19}\pi$ cm^2

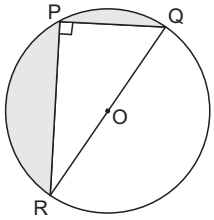
(*c*) $\frac{37}{54}\pi$ cm^2 (*d*) $\frac{63}{16}\pi$ cm^2

24. The area of the shaded region given in the figure is



(*a*) $(180 - 2\pi)$ cm^2
(*b*) $(90 - 8\pi)$ cm^2
(*c*) $(180 - 8\pi)$ cm^2
(*d*) $(90 - 2\pi)$ cm^2

25. The area of the shaded region in the given figure, if PR = 12 cm, PQ = 5 cm and O is the centre of the circle is [Take $\pi = \frac{22}{7}$]



(*a*) 36.39 cm^2

(*b*) 48.24 cm^2

(*c*) 28.76 cm^2

(*d*) 62.62 cm^2

B. Assertion-Reason Type Questions

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct choice as:

(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(*c*) Assertion (A) is true but reason (R) is false.

(*d*) Assertion (A) is false but reason (R) is true.

1. Assertion (A): In a circle of radius 6 cm, the angle of a sector is 60°. Then the area of the sector is $18\frac{6}{7}$ cm^2.

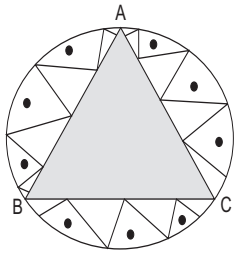
Reason (R): Area of the circle with radius r is πr^2.

2. Assertion (A): The length of the minute hand of a clock is 7 cm. Then the area swept by the minute hand in 5 minute is $12\frac{5}{6}$ cm^2.

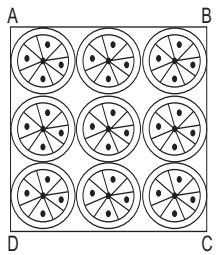
Reason (R): The length of an arc of a sector of angle θ and radius r is given by $l = \frac{\theta}{360°} \times 2\pi r$.

Case Study Based Questions

I. Pookalam is the flower bed or flower pattern designed during Onam in Kerala. It is similar as Rangoli in North India and Kolam in Tamil Nadu.
During the festival of Onam, your school is planning to conduct a Pookalam competition. Your friend who is a partner in competition, suggests two designs given below.
Observe these carefully.



I



II

Design I: This design is made with a circle of radius 32 cm leaving equilateral triangle ABC in the middle as shown in the given figure.

Design II: This Pookalam is made with 9 circular design each of radius 7 cm.

Refer Design I:

1. The side of equilateral triangle is

(*a*) $12\sqrt{3}$ cm (*b*) $32\sqrt{3}$ cm (*c*) 48 cm (*d*) 64 cm

2. The altitude of the equilateral triangle is

(*a*) 8 cm (*b*) 12 cm (*c*) 48 cm (*d*) 52 cm

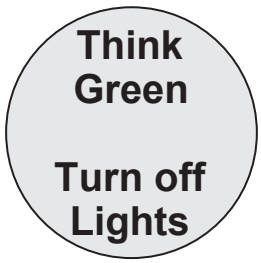
Refer Design II:

3. The area of square is
   (*a*) 1264 cm^2 (*b*) 1764 cm^2 (*c*) 1830 cm^2 (*d*) 1944 cm^2
4. Area of each circular design is
   (*a*) 124 cm^2 (*b*) 132 cm^2 (*c*) 144 cm^2 (*d*) 154 cm^2
5. Area of the remaining portion of the square ABCD is
   (*a*) 378 cm^2 (*b*) 260 cm^2 (*c*) 340 cm^2 (*d*) 278 cm^2

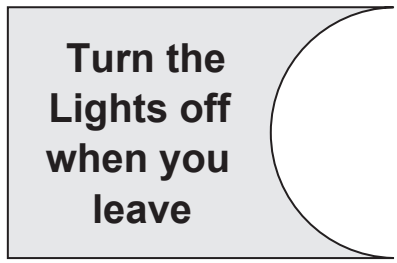
II. Energy conservation is the effect made to reduce the consumption of energy by using less of an energy service.

Some children of a school prepared posters on 'Save Energy' which are shown below.

Poster I



Poster II



Poster III

Refer to Poster I

1. If radius of the circle is 21 cm and angle subtended by arc at centre is 60°, the length of the arc is
   (*a*) 22 cm (*b*) 28 cm (*c*) 33 cm (*d*) None of these.
2. The area of sector formed by the arc (radius = 21 cm and angle of sector = 60°) is
   (*a*) 124 cm^2 (*b*) 231 cm^2 (*c*) 285 cm^2 (*d*) 310 cm^2

Refer to Poster II

3. What is the radius of circular region if length of poster is 18 cm and breadth is 14 cm?
   (*a*) 14 cm (*b*) 18 cm (*c*) 7 cm (*d*) 9 cm
4. What is the area of circular region if length of poster is 18 cm and breadth is 14 cm?
   (*a*) 154 cm^2 (*b*) 172 cm^2 (*c*) 196 cm^2 (*d*) 216 cm^2

Refer to Poster III

5. The area of poster if the length and breadth of it are 18 cm and 14 cm respectively, is
   (*a*) 155 cm^2 (*b*) 175 cm^2 (*c*) 195 cm^2 (*d*) 205 cm^2

Answers and Hints

A. Multiple Choice Questions (MCQs)

1. (*b*) 22 cm **2.** (*b*) 10 cm **3.** (*a*) 30°

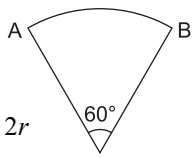
4. (*b*) 169.56 cm^2

$$\text{Area of sector} = \frac{60°}{360°} \times 3.14 \times 18 \times 18$$
$$= 169.56 \text{ cm}^2$$

5. (*a*) 32 cm

Radius (r) = 10.5 cm

Angle (θ) = 60°



$$\text{Perimeter} = \frac{\theta}{360°} \times 2\pi r + 2r$$
$$= \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 10.5 + 2 \times 10.5$$
$$= 11 + 21 = 32 \text{ cm}$$

6. (*c*) 22 cm

$$\text{Length of arc} = \frac{\theta}{360°}(2\pi r)$$
$$= \frac{60°}{360°}\left(2 \times \frac{22}{7} \times 21\right)$$
$$= 22 \text{ cm}$$

7. (*d*) 176 cm

Horse can graze in the field which is a circle of radius 28 cm.

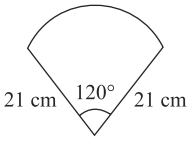
So, required perimeter

$= 2\pi r = 2\pi\,(28)$ cm

$= 2 \times \frac{22}{7} \times 28$ cm $= 176$ cm

8. (*d*) 924 cm^2

Here, $r = 21$ cm, $\theta = 120°$



Area of a sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{120°}{360°} \times \frac{22}{7} \times 21 \times 21$

$= 462$ cm^2

$\therefore$ Total area cleaned by two wipers

$= 2 \times 462 = 924$ cm^2

9. (*b*) 145.33 cm^2

Here, AB is diameter,

AC = 24 cm, BC = 10 cm and $\angle ACB = 90°$

[Angle in a semicircle is 90°]

$\therefore$ $AB^2 = AC^2 + BC^2$

[By Pythagoras theorem]

$\Rightarrow$ $AB = \sqrt{(24)^2 + (10)^2}$ cm

$= \sqrt{576 + 100}$ cm

$= \sqrt{676}$ cm $= 26$ cm

$\Rightarrow$ $OB = OA = \frac{AB}{2} = 13$ cm

$\therefore$ Area of shaded region = Area of semicircle – Area of ΔACB

$= \left[\frac{1}{2}\pi(13)^2 - \frac{1}{2} \times 24 \times 10\right]$cm^2

$= \left[\frac{1}{2} \times 3.14 \times 169 - 120\right]$cm^2

$= [265.33 - 120]$ cm$^2 = 145.33$ cm^2

10. (*c*) 228 cm^2

$OA = 20$ cm

$OB^2 = OA^2 + AB^2 = 400 + 400 = 2 \times 400$

$\Rightarrow$ $OB = 20\sqrt{2}$ $\Rightarrow$ $OB = r = 20\sqrt{2}$

$\therefore$ Shaded area = Area of quadrant – Area of square

$= \frac{1}{4}\pi r^2 - (20)^2$

$= \frac{1}{4} \times 3.14 \times 20\sqrt{2} \times 20\sqrt{2} - 400$

$= 400\left(\frac{3.14}{2} - 1\right) = 400 \times (1.57 - 1)$

$= 400 \times 0.57 = 228$ cm^2

11. (*a*) 282 cm

Perimeter = length of major arc + $2r$

$= \frac{270°}{360°} \times 2 \times \pi r + 2r$

$= \frac{3}{2} \times \frac{22}{7} \times 42 + 2 \times 42$

$= 198 + 84 = 282$ cm

12. (*d*) 66 cm

Perimeter of shaded region

= Perimeter of semicircles (ARC + APB + BSD + CQD)

$= \pi[r_1 + r_2 + r_3 + r_4]$

$= \frac{22}{7}\left[7 + \frac{7}{2} + 7 + \frac{7}{2}\right] = \frac{22}{7} \times 21$

$= 66$ cm

13. (*c*) 21 cm

Length of wire = Length of Arc

$\Rightarrow$ 22 cm $= \frac{\theta}{360°} \times 2 \times \frac{22}{7} \times r$

$\Rightarrow$ $22 = \frac{60°}{360°} \times 2 \times \frac{22}{7} \times r$

$\Rightarrow$ $r = \frac{22 \times 360 \times 7}{60 \times 2 \times 22} = 21$ cm

Thus, the radius of the circle = 21 cm

14. (*b*) 86.625 cm^2

Area of shaded region

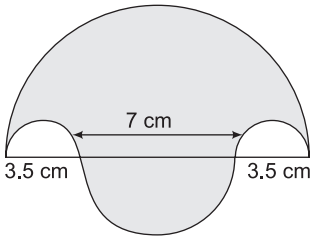
= area of big semicircle – area of 2 small semicircles + area of middle semicircle

$= \frac{1}{2}\pi \times (7)^2 - 2 \times \frac{1}{2} \times \pi \times \left(\frac{3.5}{2}\right)^2 + \frac{1}{2}\pi \times \left(\frac{7}{2}\right)^2$

$= \frac{49}{2}\pi - \left(\frac{7}{4}\right)^2 \pi + \frac{1}{2}\left(\frac{49}{4}\right)\pi = \left(\frac{49}{2} - \frac{49}{16} + \frac{49}{8}\right)\pi$

$= \left(\frac{1}{2} - \frac{1}{16} + \frac{1}{8}\right) \times 49 \times \frac{22}{7} = \left(\frac{8 - 1 + 2}{16}\right) \times 7 \times 22$

$= \frac{9}{16} \times 7 \times 22 = 86.625$ cm^2.



15. (*c*) 3500 cm^2

Radius of circle = 35 cm

$\angle AOB = 90°$

Area of sector OAB = $\frac{\pi r^2\theta}{360°}$

$= \frac{22}{7}\times 35\times 35\times\frac{90°}{360°} = \frac{1925}{2}\ cm^2$

Area of minor segment

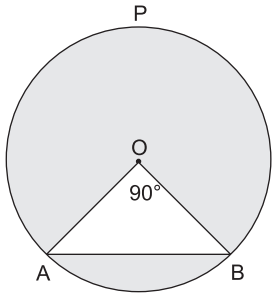
= area of sector OAB – area of ΔOAB

$= \frac{1925}{2} - \frac{1}{2}\times 35\times 35 = \frac{1925}{2} - \frac{1225}{2}$

$= \frac{700}{2} = 350\ cm^2$

Area of major segment APB = area of circle – area of minor segment

$= \frac{22}{7}\times 35\times 35 - 350 = 3500\ cm^2$



16. (*a*) 84 cm^2

Area of the square ABCD = $14\times 14 = 196\ cm^2$

Area of semicircle AOB = $\frac{1}{2}\times\pi r^2$

$= \frac{1}{2}\times\frac{22}{7}\times 7\times 7 = 77\ cm^2 \quad (\because r = \frac{14}{2})$

Similarly area of semicircle DOC = 77 cm^2

Hence, the area of shaded region (Part W and Part Y)

= Area of square – Area of two semicircles AOB and COD

= 196 – 154 = 42 cm^2

Therefore, area of four shaded parts (*i.e.* X, Y, W, Z) = $(2\times 42)\ cm^2 = 84\ cm^2$

17. (*c*) 1910.85 cm

18. (*b*) 8.75 cm^2

Here, $l = 3.5$ cm, $r = 5$ cm

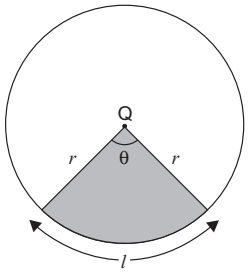
Length of arc

$l = \frac{2\pi r\theta}{360°}$

$\Rightarrow \quad 3.5 = \frac{2\times\pi\times 5\times\theta}{360°}$

$\Rightarrow \quad \frac{\pi\theta}{36} = 3.5$

$\Rightarrow \quad \theta = \frac{3.5\times 36}{\pi}$



Now, Area of sector

$= \frac{\pi r^2\theta}{360°} = \frac{\pi\times 5\times 5\times 35\times 36}{360°\times\pi\times 10}$

$= \frac{25\times 35}{100} = \frac{875}{100}$

$= 8.75\ cm^2$

$\therefore$ Area of sector = 8.75 cm^2

19. (*c*) $\frac{1}{4}\pi b^2\ cm^2$

The diameter of circle that can be drawn inside the rectangle is equal to the breadth of rectangle.

The length of the rectangle = a cm

The breadth of the rectangle = b cm

$\therefore$ Diameter of circle = b cm

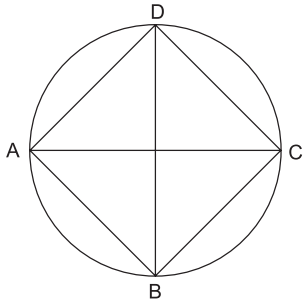
$\Rightarrow \quad r = \frac{b}{2}$ cm

$\therefore$ Area of circle A = πr^2

$= \pi\left(\frac{b}{2}\right)^2 = \frac{1}{4}\pi b^2\ cm^2$

20. (*c*) 800 cm^2

Diagonal of a rhombus are perpendicular bisector of each other.



$\therefore$ Each diagonal is diameter of the circle.

Now,

area of circle = 1256 cm^2

$\Rightarrow \quad \pi r^2 = 1256 \quad \Rightarrow \quad r^2 = \dfrac{1256}{\pi}$

$\Rightarrow \quad r^2 = \dfrac{1256}{3.14} = 400$

$\Rightarrow \quad r = 20$ cm

$\therefore$ Diameter of the circle = 40 cm

= Each diagonal of the rhombus

Area of rhombus

$= \dfrac{1}{2}(d_1 \times d_2) = \dfrac{1}{2} \times 40 \times 40$

$= 800\ cm^2$

21. (*a*) $\left(\dfrac{25\pi}{4} - \dfrac{25}{2}\right) cm^2$

22. (*b*) 31.4 cm

Perimeter of shaded region = length of arc PSR + length of arc PAQ + length of arc QTR

$= 5\pi + 3.5\pi + 1.5\pi = 10\pi = 10 \times 3.14 = 31.4$ cm

23. (*d*) $\dfrac{63}{16}\pi\ cm^2$

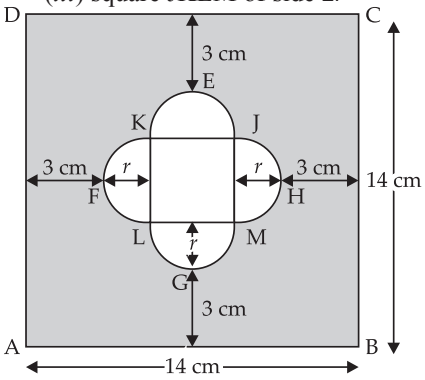
24. (*c*) $(180 - 8\pi)\ cm^2$.

Identification of shapes of figures:

(*i*) 4 semi circles of radius *r*

(*ii*) square ABCD of side 14 cm

(*iii*) square JKLM of side 2*r*



From figure,

AB = 3 + 3 + *r* + 2*r* + *r*

$\Rightarrow \quad 14 = 6 + 4r \quad \Rightarrow \quad 4r = 14 - 6$

$\Rightarrow \quad 4r = 8 \quad \Rightarrow \quad r = \dfrac{8}{4} = 2$ cm

So, the area of shaded region

= Area of square – Area of 4 semi circles – Area of square (JKLM)

One square ABCD	One square JKLM	Four semicircles
$a_1 = 14$ cm	$a_2 = 2r$	$r = 2$ cm
	$\Rightarrow a_2 = 2 \times 2$	
	$\Rightarrow a_2 = 4$ cm	

$\therefore$ Required area

$= a_1^2 - 4 \times \dfrac{\pi r^2}{2} - a_2^2$

$= 14 \times 14 - \dfrac{4 \times \pi \times 2 \times 2}{2} - 4 \times 4$

$= 196 - 16 - 8\pi = (180 - 8\pi)\ cm^2$

Hence, the shaded area = $(180 - 8\pi)\ cm^2$.

25. 36.39 cm^2

B. Assertion-Reason Type Questions

1. (*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

2. (*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Case Study Based Questions

I. 1. (*b*) $32\sqrt{3}$ cm **2.** (*c*) 48 cm

3. (*b*) 1764 cm^2 **4.** (*d*) 154 cm^2

5. (*a*) 378 cm^2

II. 1. (*a*) 22 cm **2.** (*b*) 231 cm^2

3. (*c*) 7 cm **4.** (*a*) 154 cm^2

5. (*b*) 175 cm^2

Experts' Opinion

Questions based on following types are very important for Exams. So, students are advised to revise them thoroughly.

1. To find the area of sector of a circle.
2. To find the area of segment.
3. To find area of combination of plane figures.

IMPORTANT FORMULAE

- Circumference of a circle = $2\pi r$
- Area of a circle = πr^2
- Circumference of a semicircle = $\pi r + 2r$
- Area of semicircle = $\frac{1}{2}\pi r^2$
- Area of quadrant = $\frac{1}{4}\pi r^2$
- Area of the ring = $\pi(R^2 - r^2)$, R = outer radius and r = inner radius.
- Length of an arc of a sector of a circle with radius r and angle with degree measure $\theta = \frac{\theta}{360^\circ} \times 2\pi r$
- Area of a sector of circle with radius r and angle with degrees measure $\theta = \frac{\theta}{360^\circ} \times \pi r^2$
- Area of the major sector = πr^2 – Area of minor sector
- Area of segment of a circle = Area of the corresponding sector – Area of the corresponding triangle
- Area of major segment = πr^2 – Area of minor segment

COMMON ERRORS

Errors	Corrections
(*i*) Interpreting the concept of sector and segment incorrectly.	(*i*) Make concept of both term clear by understanding them.
(*ii*) Finding perimeter of sector, without taking the radius into consideration.	(*ii*) Draw the figure and note the data given to get the answer correctly.
(*iii*) Using formula for area of sector and length of arc of sector incorrectly.	(*iii*) Remember all the formulae and use them properly.
(*iv*) While finding the area of shaded region students forget to subtract the common area overlapped.	(*iv*) Be careful in such problems. Do enough practice to avoid the errors.

QUICK REVISION NOTES

- A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point always remains the same. The fixed point is called as centre and constant distance is known as radius.
- Area and perimeter of a circle with radius r
- Circumference (perimeter) = $2\pi r$ unit or πd unit, where d (diameter) = $2r$ unit
  - Area = πr^2 or $\frac{\pi d^2}{4}$ $(\text{unit})^2$
  - Area of semicircle = $\frac{\pi r^2}{2}$ or $\frac{\pi d^2}{8}$ $(\text{unit})^2$
  - Area of a of a quadrant = $\frac{\pi r^2}{4}$ $(\text{unit})^2$
- Area of a sector of circle with radius r and an angle θ at the centre

$$= \pi r^2 \times \frac{\theta}{360^\circ} \ (\text{unit})^2$$

OR

$$= \frac{1}{2} \times \text{length of arc} \times \text{radius} = \frac{1}{2} lr$$

- Area of segment of a circle with radius r and angle with degree measure θ

$$= \text{Area of the sector} - \text{Area of the triangle}$$

$$= \frac{\theta}{360^\circ} \times \pi r^2 - \frac{1}{2} r^2 \times \sin\theta \ (\text{unit})^2$$

- Area enclosed by two concentric circles:

  If R and r are radii of two concentric circles, then area enclosed by the two circles

$$= \pi R^2 - \pi r^2 = \pi (R^2 - r^2) = \pi (R - r)(R + r) \ (\text{unit})^2$$