Coordinate Geometry 7

Objective Section (1 mark each)

Multiple Choice Questions

Q. 1. The point P on x-axis equidistant from the points A(– 1, 0) and B(5, 0) is :

[CBSE OD, Set 1, 2020]

(a) (2, 0) (b) (0, 2)

(c) (3, 0) (d) (2, 2)

Ans. Let P(h, 0) be the required point on x-axis.

So, according to the question

$$PA = PB$$

or $$(PA)^2 = (PB)^2$$

$$\Rightarrow (h + 1)^2 + (0 - 0)^2 = (h - 5)^2 + (0 - 0)^2$$

$$\Rightarrow (h + 1)^2 = (h - 5)^2$$

$$\Rightarrow h^2 + 2h + 1 = h^2 - 10h + 25$$

$$\Rightarrow 12h = 24$$

$$\Rightarrow h = 2$$

So, the required point is (2, 0)

$\therefore$ Option (a) is correct. **Ans.**

Q. 2. The co-ordinates of the point which is reflection of point (– 3, 5) in x-axis are

[CBSE OD, Set 1, 2020]

(a) (3, 5) (b) (3, – 5)

(c) (– 3, – 5) (d) (– 3, 5)

Ans. The co-ordinates of the point which is reflection of point (– 3, 5) in x-axis is (– 3, – 5).

$\therefore$ Option (c) is correct. **Ans.**

Q. 3. If the point P(6, 2) divides the line segment joining A(6, 5) and B(4, y) in the ratio 3 : 1, then the value of y is [CBSE OD, Set 1, 2020]

(a) 4 (b) 3

(c) 2 (d) 1

Ans. Using section formula,

$$y\text{-coordinate of P} = \frac{3 \times y + 1 \times 5}{3 + 1}$$

$$\Rightarrow 2 = \frac{3y + 5}{4}$$

$$\Rightarrow 8 = 3y + 5$$

$$\Rightarrow 3y = 3$$

$$\Rightarrow y = 1$$

$\therefore$ Option (d) is corrrect. **Ans.**

Q. 4. The distance between the points (a cos θ + b sin θ, 0) and (0, a sin θ – b cos θ), is

[CBSE Delhi, Set 1, 2020]

(a) $a^2 + b^2$ **(b)** $a^2 - b^2$

(c) $\sqrt{a^2 + b^2}$ **(d)** $\sqrt{a^2 - b^2}$

Ans. The distance formula,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here $x_1 = a\cos\theta + b\sin\theta,\ y_1 = 0$

$x_2 = 0,\ y_2 = a\sin\theta - b\cos\theta$

$$\therefore d = \sqrt{[0 - (a\cos\theta + b\sin\theta)]^2 + [(a\sin\theta - b\cos\theta) - 0]^2}$$

$$= \sqrt{(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2}$$

$$\sqrt{a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta}$$

$$= \sqrt{a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)}$$

$$= \sqrt{a^2 \times 1 + b^2 \times 1} \qquad \{\because \sin^2\theta + \cos^2\theta = 1\}$$

$$= \sqrt{a^2 + b^2}$$

$\therefore$ Option (c) is correct. **Ans.**

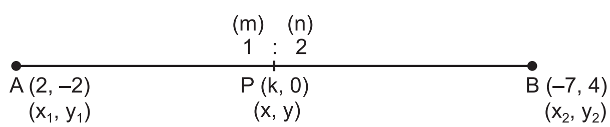
Q. 5. If the point P(k, 0) divides the line segment joining the points A(2, – 2) and B(– 7, 4) in the ratio 1 : 2, then the value of k is

[CBSE Delhi, Set 1, 2020]

(a) 1 (b) 2

(c) – 2 (d) – 1

Ans.



By section formula,

$$x = \frac{mx_2 + nx_1}{m + n},\ y = \frac{my_2 + ny_1}{m + n}$$

Here $x = k$, $m = 1$, $n = 2$, $x_1 = 2$, $x_2 = -7$

$\therefore$ we get,

$$k = \frac{(1)(-7)+(2)(2)}{1+2}$$

$$\Rightarrow \quad k = \frac{-7+4}{3}$$

$$\Rightarrow \quad k = \frac{-3}{3}$$

$$\Rightarrow \quad k = -1$$

$\therefore$ Option (d) is correct. **Ans.**

Q. 6. The value of p, for which the points A(3, 1), B(5, p) and C(7, – 5) are collinear, is

[CBSE Delhi, Set 1, 2020]

(a) – 2 **(b) 2**

(c) – 1 **(d) 1**

Ans. Given,

$$A(3, 1) \Rightarrow x_1 = 3, y_1 = 1$$

$$B(5, p) \Rightarrow x_2 = 5, y_2 = p$$

$$C(7, -5) \Rightarrow x_3 = 7, y_3 = -5.$$

If A, B and C are collinear points,

then, ar(ΔABC) = 0

$$\Rightarrow \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \frac{1}{2}[3(p - (-5)) + 5(-5 - 1) + 7(1 - p)] = 0$$

$$\Rightarrow \frac{1}{2}[3p + 15 - 25 - 5 + 7 - 7p] = 0$$

$$\Rightarrow \frac{1}{2}[-4p - 8] = 0$$

$$\Rightarrow -2p - 4 = 0$$

$$\Rightarrow p = \frac{4}{-2} = -2$$

$$\Rightarrow p = -2$$

$\therefore$ Option (a) is correct. **Ans.**

Very Short Answer Type Questions (1 mark each)

Q. 1. Write the coordinates of a point P on x-axis which is equidistant from the point $A(-2, 0)$ and $B(6, 0)$.

[CBSE OD, Set 1, 2019]

Ans. Let coordinates of P on x-axis is $(x, 0)$

Given, $A(-2, 0)$ and $B(6, 0)$

Here, $PA = PB$

$$\therefore \sqrt{(x+2)^2+(0-0)^2} = \sqrt{(x-6)^2+(0-0)^2}$$

$$\Rightarrow \sqrt{(x+2)^2} = \sqrt{(x-6)^2}$$

On squaring both sides, we get

$$(x+2)^2 = (x-6)^2$$

$$x^2 + 4 + 4x = x^2 + 36 - 12x$$

$$\Rightarrow 4 + 4x = 36 - 12x$$

$$\Rightarrow 16x = 32$$

$$\Rightarrow x = \frac{32}{16}$$

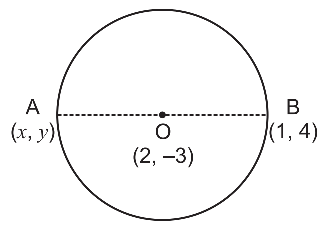
$$\Rightarrow x = 2$$

Co-ordinates of P are (2, 0).

Q. 2. Find the coordinates of a point A, where AB is diameter of a circle whose centre is (2, –3) and B is the point (1, 4).

[CBSE Delhi, Set 1, 2019]

Ans. Let the co-ordinates of point A be (x, y) and point O (2, –3) be the centre, then by mid-point formula,



$$\frac{x+1}{2} = 2 \quad \text{and} \quad \frac{y+4}{2} = -3$$

or $x = 4 - 1$ and $y = -6 - 4$

$x = 3$ $\quad y = -10$

$\therefore$ The co-ordinates of point A are (3, –10).

Q. 3. Find the coordinates of a point A, where AB is a diameter of the circle with centre (– 2, 2) and B is the point with coordinates (3, 4).

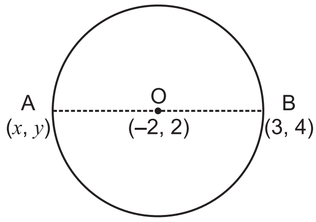
[CBSE Delhi, Set 2, 2019]

Ans. By mid-point formula

$$\frac{x+3}{2} = -2$$

$$x = -4 - 3$$

$$x = -7$$



and $\dfrac{y+4}{2} = 2$

$y = 0$

$\therefore$ Co-ordinates of point A are (–7, 0).

Q. 4. Find the distance of a point $P(x, y)$ from the origin. [CBSE, 2018]

Ans. The given point is $P\ (x, y)$.

The origin is $O\ (0, 0)$

Distance of point P from origin,

$$PO = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(x-0)^2 + (y-0)^2}$$

$$= \sqrt{x^2 + y^2} \text{ unit}$$

Q. 5. If the distance between the points (4, k) and (1, 0) is 5, then what can be the possible values of k?
[CBSE Delhi, Term 2, Set 1, 2017]

Ans. Distance between (4, k) and (1, 0) = 5

$$\sqrt{(1-4)^2 + (0-k)^2} = 5$$

On squaring both sides,

$$9 + k^2 = 25$$

$\Rightarrow$ $k^2 = 25 - 9 = 16$

So $k = \pm 4$

Short Answer Type Questions-I (2 marks each)

Q. 1. Find a relation between x and y if the points $A(x, y)$, $B(-4, 6)$ and $C(-2, 3)$ are collinear. [CBSE OD, Set 1, 2019]

Ans. Given, $A(x, y)$, $B(-4, 6)$, $C(-2, 3)$

$x_1 = x,\ y_1 = y,\ x_2 = -4,\ y_2 = 6,\ x_3 = -2,\ y_3 = 3$

If these points are collinear, then area of triangle made by these points is 0.

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\frac{1}{2}[x(6-3) + (-4)(3-y) + (-2)(y-6)] = 0$$

$$3x - 12 + 4y - 2y + 12 = 0$$

$$3x + 2y = 0$$

$$3x = -2y$$

$$x = \frac{-24}{3}$$

Q. 2. Find the area of a triangle whose vertices are given as (1, –1) (–4, 6) and (–3, –5). [CBSE OD, Set 1, 2019]

Ans. $x_1 = 1, y_1 = -1, x_2 = -4, y_2 = 6, x_3 = -3, y_3 = -5.$

Area of triangle

$$= \frac{1}{2}\,|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2}\,|1(6+5) + (-4)(-5+1) + (-3)(-1-6)|$$

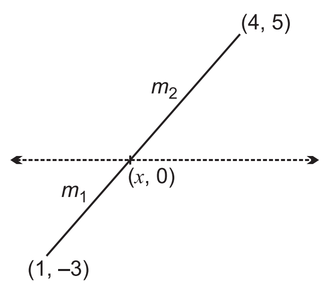
$$= \frac{1}{2}\,|11 + 16 + 21|$$

$$= \frac{1}{2} \times 48$$

$= 24$ square unit.

Q. 3. Find the ratio in which the segment joining the points (1, –3) and (4, 5) is divided by x-axis? Also find the coordinates of this point on x-axis.
[CBSE Delhi, Set 1, 2019]

Ans. Let the given points be A (1, –3) and B (4, –5) and the line-segment joining by these points is divided by x-axis, so the co-ordinates of the point of intersection will be P(x, 0)



Let the ratio be $m_1 : m_2$

So, by section formula

$$0 = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$0 = \frac{5m_1 - 3m_2}{m_1 + m_2}$$

or $\quad 5m_1 - 3m_2 = 0$

or $\quad \dfrac{m_1}{m_2} = \dfrac{3}{5}$

$\therefore$ Required ratio is 3 : 5

Now to find the co-ordinates of this point on x-axis

$$\therefore \quad x = \frac{3\times4+5\times1}{3+5}$$

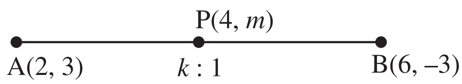
$$x = \frac{12+5}{8}$$

$$x = \frac{17}{8}$$

$\therefore$ The required point is $\left(\dfrac{17}{8},\ 0\right)$

Q. 4. Find the ratio in which $P(4, m)$ divides the line segment joining the points $A(2, 3)$ and $B(6, -3)$. Hence find m. [CBSE, 2018]

Ans. Let P divides line segment AB in the ratio $k : 1$.



$$P = \left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$(4, m) = \left(\frac{k\times6+1\times2}{k+1}, \frac{k\times(-3)+1\times3}{k+1}\right)$$

$$(4, m) = \left(\frac{6k+2}{k+1}, \frac{-3k+3}{k+1}\right) \quad \text{...(i)}$$

On comparing, we get

$$\left(\frac{6k+2}{k+1}\right) = 4$$

$$\Rightarrow \quad 6k + 2 = 4 + 4k$$

$$\Rightarrow \quad 6k - 4k = 4 - 2$$

$$\Rightarrow \quad 2k = 2$$

$$\Rightarrow \quad k = 1 \quad \text{...(ii)}$$

Hence, P divides AB in the ratio 1 : 1.

From (i) and (ii), $\dfrac{-3(1)+3}{1+1} = m$

$$\Rightarrow \quad \frac{-3+3}{2} = m$$

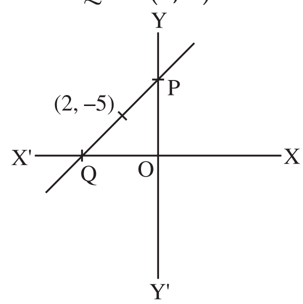
$$\Rightarrow \quad m = 0$$

Q. 5. A line intersects the y-axis and x-axis at the points P and Q respectively. If $(2, -5)$ is the mid-point of PQ, then find the co-ordinates of P and Q.

[CBSE OD, Term 2, Set 1, 2017]

Ans.

Let co-ordinates of P be $(0, y)$ and Co-ordinate of Q be $(x, 0)$.



Mid-point is $(2, -5)$

$$\therefore \quad \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (2, -5)$$

$$\Rightarrow \quad \left(\frac{x+0}{2}, \frac{0+y}{2}\right) = (2, -5)$$

$$\Rightarrow \quad \frac{x}{2} = 2\ ; \quad \frac{y}{2} = -5$$

$$\Rightarrow \quad x = 4\ ; \quad y = -10$$

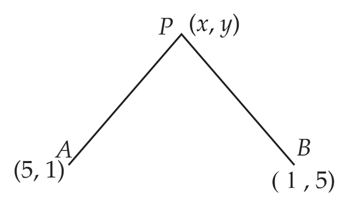
Co-ordinates of $P = (0, -10)$

Co-ordinates of $Q = (4, 0)$

Q. 6. If the distances of $P(x, y)$, from $A(5, 1)$ and $B(-1, 5)$ are equal, then prove that $3x = 2y$. [CBSE OD, Term 2, Set 1, 2017]

Ans. Given, PA = PB

$$\Rightarrow \sqrt{(x-5)^2+(y-1)^2} = \sqrt{(x+1)^2+(y-5)^2}$$



Squaring both sides,

$$(x-5)^2 + (y-1)^2 = (x+1)^2 + (y-5)^2$$

$$\Rightarrow x^2 + 25 - 10x + y^2 + 1 - 2y = x^2 + 1 + 2x + y^2 + 25 - 10y$$

$$\Rightarrow \quad -10x - 2y = 2x - 10y$$

$$\Rightarrow \quad -10x - 2x = -10y + 2y$$

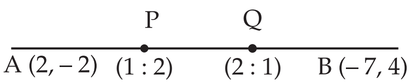
$$\Rightarrow \quad 12x = 8y$$

$$\Rightarrow \quad 3x = 2y$$

Hence Proved.

Q. 7. Let P and Q be the points of trisection of the line segment joining the points $A(2, -2)$ and $B(-7, 4)$ such that P is nearer to A. Find the coordinates of P and Q.
[CBSE OD, Term 2, Set 1, 2016]

Ans. Since, P and Q are the points of trisection of AB then, P divides AB in 1 : 2.



$$\therefore \text{Coordinates of } P = \left(\frac{1(-7)+2(2)}{1+2}, \frac{1(4)+2(-2)}{1+2}\right)$$

$$= \left(\frac{-3}{3}, \frac{0}{3}\right) = (-1, 0)$$

And, Q is the mid-point of PB

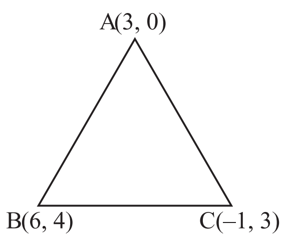
$$\therefore \text{Coordinates of } Q = \left(\frac{-1+(-7)}{2}, \frac{0+4}{2}\right)$$

$$= (-4, 2)$$

So, $\quad P \equiv (-1, 0), Q \equiv (-4, 2)$

Q. 8. Prove that the points (3, 0), (6, 4) and (–1, 3) are the vertices of a right angled isosceles triangle.
[CBSE OD, Term 2, Set 1, 2016]

Ans. Let $A(3, 0)$, $B(6, 4)$ and $C(-1, 3)$ be the vertices of a triangle ABC.



$$\text{Length of } AB = \sqrt{(6-3)^2+(4-0)^2}$$

$$= \sqrt{(3)^2+(4)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$

$$\text{Length of } BC = \sqrt{(-1-6)^2+(3-4)^2}$$

$$= \sqrt{(7)^2+(-1)^2}$$

$$= \sqrt{49+1} = \sqrt{50} = 5\sqrt{2} \text{ units.}$$

$$\text{And, Length of } AC = \sqrt{(-1-3)^2+(3-0)^2}$$

$$= \sqrt{(-4)^2+(3)^2}$$

$$= \sqrt{16+9} = \sqrt{25}$$

$$= 5 \text{ units}$$

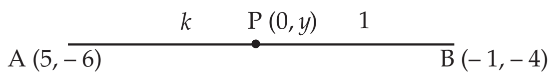
$\therefore \quad AB = AC$

And, $(AB)^2 + (AC)^2 = (BC)^2$

Hence, $\Delta\, ABC$ is a isosceles, right angled triangle. **Hence Proved.**

Q. 9. Find the ratio in which y-axis divides the line segment joining the points $A(5, -6)$ and $B(-1, -4)$. Also find the coordinates of the point of division.
[CBSE Delhi, Term 2, Set 1, 2016]

Ans. Let the required ratio be $k : 1$ and point on y-axis be $(0, y)$



$\therefore \quad AP : PB = k : 1$

Then, by section formula

$$\frac{5-k}{k+1} = 0$$

$\Rightarrow \quad 5 - k = 0$

$\Rightarrow \quad k = 5$

Hence, required ratio is 5 : 1

$$\therefore \quad y = \frac{(-4)(5) + (-6)(1)}{5+1}$$

$$\therefore \quad y = \frac{-26}{6} = -\frac{13}{3}$$

Hence, point on y-axis is $\left(0, -\frac{13}{3}\right)$

Q. 10. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from $Q(2, -5)$ and $R(-3, 6)$, find the coordinates of P. [CBSE Delhi, Term 2, Set 1, 2016]

Ans. Let the coordinates of point P be $(2y, y)$

Since, P is equidistant from Q and R

$\therefore \quad PQ = PR$

$$\Rightarrow \sqrt{(2y-2)^2+(y+5)^2} = \sqrt{(2y+3)^2+(y-6)^2}$$

$\Rightarrow (2y - 2)^2 + (y + 5)^2 = (2y + 3)^2 + (y - 6)^2$

$\Rightarrow 4y^2 + 4 - 8y + y^2 + 25 + 10y$
$= 4y^2 + 9 + 12y + y^2 + 36 - 12y$

$\Rightarrow \quad 2y + 29 = 45$

$\Rightarrow \quad 2y = 45 - 29$

$\Rightarrow \quad y = \frac{16}{2} = 8$

Hence, the co-ordinates of point P are (16, 8).

Q. 11. The points $A(4, 7)$, $B(p, 3)$ and $C(7, 3)$ are the vertices of a right triangle, right-angled at B. Find the value of p.

[CBSE OD, Term 2, Set 1, 2015]

Ans. The given points are $A(4, 7)$, $B(p, 3)$ and $C(7, 3)$.

Since A, B and C are the vertices of a right angled triangle

then, $(AB)^2 + (BC)^2 = (AC)^2$

[By Pythagoras theorem]

$\Rightarrow [(p-4)^2 + (3-7)^2] + [(7-p)^2 + (3-3)^2] = [(7-4)^2 + (3-7)^2]$

$\Rightarrow (p-4)^2 + (-4)^2 + (7-p)^2 = (3)^2 + (-4)^2$

$\Rightarrow p^2 + 16 - 8p + 16 + 49 + p^2 - 14p = 9 + 16$

$\Rightarrow \quad 2p^2 - 22p + 56 = 0$

$\Rightarrow \quad p^2 - 11p + 28 = 0$

$\Rightarrow \quad p^2 - 7p - 4p + 28 = 0$

$\Rightarrow \quad p(p-7) - 4(p-7) = 0$

$p = 4$ or 7

$p \neq 7$

(As B and C will coincide)

So, $\quad p = 4.$

Q. 12. Find the relation between x and y if the points A(x, y), B(–5, 7) and C(–4, 5) are collinear.

[CBSE OD, Term 2, Set 1, 2015]

Ans. Given that the points $A(x, y)$, $B(-5, 7)$ and $C(-4, 5)$ are collinear.

So, the area formed by the vertices is 0.

Therefore,

$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow \quad \frac{1}{2}[x(7-5) - 5(5-y) - 4(y-7)] = 0$

$\Rightarrow \quad \frac{1}{2}[x(2) - 5(5-y) - 4(y-7)] = 0$

$\Rightarrow \quad 2x - 25 + 5y - 4y + 28 = 0$

$\Rightarrow \quad 2x + y + 3 = 0$

$\Rightarrow \quad -2x - 3 = y$

which is the required relation between x and y.

Q. 13. If $A(4, 3)$, $B(-1, y)$ and $C(3, 4)$ are the vertices of a right triangle ABC, right-angled at A, then find the value of y.

[CBSE OD, Set 2, 2015]

Ans. Given the triangle ABC, right angled at A.

Now, $\quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(-1-4)^2 + (y-3)^2}$

$AB = \sqrt{(-5)^2 + (y-3)^2}$

$AB = \sqrt{25 + y^2 + 9 - 6y}$

$\therefore \quad AB = \sqrt{34 + y^2 - 6y}$

$BC = \sqrt{(3-(-1))^2 + (4-y)^2}$

$BC = \sqrt{(4)^2 + (4-y)^2}$

$BC = \sqrt{16 + 16 + y^2 - 8y}$

$\therefore \quad BC = \sqrt{32 + y^2 - 8y}$

and $\quad AC = \sqrt{(3-4)^2 + (4-3)^2}$

$AC = \sqrt{(-1)^2 + (1)^2}$

$AC = \sqrt{1+1}$

$\therefore \quad AC = \sqrt{2}$ units

Given, $\Delta\, ABC$ is a right angled triangle

So, by pythagoras theorem

$$BC^2 = AC^2 + AB^2$$

$$(\sqrt{32 + y^2 - 8y})^2 = (\sqrt{2})^2 + (\sqrt{34 + y^2 - 6y})^2$$

$$32 + y^2 - 8y = 2 + 34 + y^2 - 6y$$

$$-2y = 4$$

$$y = -2$$

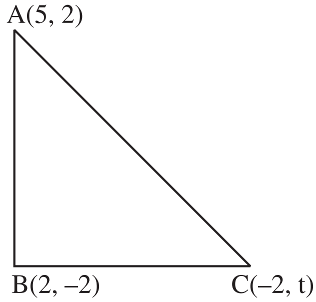
Hence, the value of y is -2.

Q. 14. If A (5, 2), B(2, – 2) and C(– 2, t) are the vertices of a right angled triangle with $\angle B = 90°$, then find the value of t.

[CBSE Delhi, Term 2, Set 1, 2015]

Ans. Given, ABC are the vertices of a right angled triangle, then,

By Pythagoras theorem,



$$(AC)^2 = (BC)^2 + (AB)^2 \quad ...(i)$$

Now, $(AC)^2 = (5 + 2)^2 + (2 - t)^2$

$= 49 + (2 - t)^2$

$(BC)^2 = (2 + 2)^2 + (-2 - t)^2$

$= 16 + (t + 2)^2$

And $(AB)^2 = (5 - 2)^2 + (2 + 2)^2$

$= 9 + 16 = 25$

Putting these values in (i)

$49 + (2 - t)^2 = 16 + (t + 2)^2 + 25$

$\Rightarrow \quad 49 + (2 - t)^2 = 41 + (t + 2)^2$

$\Rightarrow \quad 8 = (t + 2)^2 - (2 - t)^2$

$\Rightarrow \quad 8 = t^2 + 4 + 4t - 4 - t^2 + 4t$

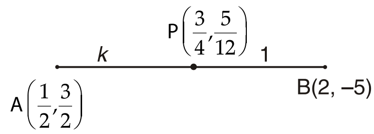
$\Rightarrow \quad 8 = 8t$

$\therefore \quad t = 1$

Q. 15. Find the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and $B(2, -5)$.

[CBSE Term 1, Set 1, 2015]

Ans. Let point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line AB in ratio $k : 1$.



Then, by section formula, coordinates of P are

$$\frac{2k + \frac{1}{2}}{k+1} = \frac{3}{4}$$

$\Rightarrow \quad 8k + 2 = 3k + 3$

$\Rightarrow \quad 8k - 3k = 3 - 2$

$\Rightarrow \quad 5k = 1$

$\Rightarrow \quad k = \frac{1}{5}$

and $\quad \dfrac{-5k + \frac{3}{2}}{k+1} = \dfrac{5}{12}$

$\Rightarrow \quad -60k + 18 = 5k + 5$

$\Rightarrow \quad -60k - 5k = 5 - 18$

$\Rightarrow \quad -65 - k = -15$

$\Rightarrow \quad k = \frac{-15}{-65} = \frac{1}{5}$

$\Rightarrow \quad k = \frac{1}{5}$ in each case

Hence the required ratio is $\frac{1}{5} : 1$ *i.e.* $1 : 5$.

Short Answer Type Questions-II (3 marks each)

Q. 1. If the mid-point of the line segment joining the points A(3, 4) and B(k, 6) is P(x, y) and $x + y - 10 = 0$. Find the value of k.

[CBSE OD, Set 1, 2020]

Ans. Coordinates of the mid-point of A(3, 4) and

B(k, 6) $= \left\{\frac{3+k}{2}, \frac{4+6}{2}\right\}$

$= \left(\frac{3+k}{2}, 5\right)$

$\because$ P(x, y) is the mid-point of AB

$\therefore \quad P(x, y) = \left(\frac{3+k}{2}, 5\right)$

On comparing, we get

$$x = \frac{3+k}{2} \text{ and } y = 5$$

Now, $\quad x + y - 10 = 0$

$\Rightarrow \quad \frac{3+k}{2} + 5 - 10 = 0$

$\Rightarrow \quad \frac{3+k}{2} - 5 = 0$

$\Rightarrow \quad 3 + k - 10 = 0$

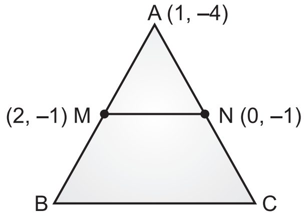
$\Rightarrow \quad k - 7 = 0$

$\Rightarrow \quad k = 7$ **Ans.**

Q. 2. Find the area of triangle ABC with A(1, – 4) and the mid-points of sides through A being (2, – 1) and (0, – 1).

[CBSE OD, Set 1, 2020]

Ans. Let the points M and N be the mid-points of AB and AC, respectively.



Now, $\text{ar}(\Delta AMN) = \frac{1}{2}[1(-1+1)+2(-1+4)+0(-4+1)]$

$= \frac{1}{2}[1(0)+2(3)+0]$

$= \frac{1}{2}[6] = 3$ sq. units

$\because \quad \Delta AMN \sim \Delta ABC$

$\therefore \quad \frac{\text{ar}(\Delta AMN)}{\text{ar}(\Delta ABC)} = \frac{AM^2}{AB^2}$

$= \left(\frac{AM}{AB}\right)^2$

$= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\Rightarrow \quad \frac{3}{\text{ar}(\Delta ABC)} = \frac{1}{4}$

$\Rightarrow \quad \text{ar}(\Delta ABC) = 12$ sq. units. **Ans.**

Q. 3. Find the area of triangle PQR formed by the points P(– 5, 7), Q(– 4, – 5) and R(4, 5).

[CBSE Delhi, Set 1, 2020]

Ans. Given P(– 5, 7), Q(– 4, – 5), R(4, 5)

$\therefore \quad x_1 = -5, x_2 = -4, x_3 = 4$

$y_1 = 7, y_2 = -5, y_3 = 5$

Area of ΔPQR

$= \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$= \frac{1}{2}[(-5)(-5-5)+(-4)(5-7)+4(7-(-5))]$

$= \frac{1}{2}[(-5)(-10)-4(-2)+4(7+5)]$

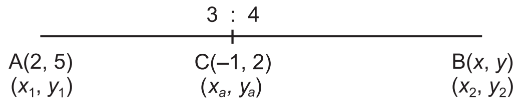
$= \frac{1}{2}[50+8+48]$

$= \frac{1}{2}\times 106 = 53$ sq. units. **Ans.**

Q. 4. If the point C(– 1, 2) divides internally the line segment joining A(2, 5) and B(x, y) in the ratio 3 : 4, find the coordinates of B.

[CBSE Delhi, Set 1, 2020]

Ans.



By section formula,

$x = \frac{mx_2+nx_1}{m+n}, \quad y = \frac{my_2+ny_1}{m+n}$

$\Rightarrow -1 = \frac{3(x)+4(2)}{3+4}, 2 = \frac{3(y)+4(5)}{3+4}$

$\Rightarrow -1 = \frac{3x+8}{7}, 2 = \frac{3y+20}{7}$

$\Rightarrow -7 = 3x+8, \; 14 = 3y+20$

$\Rightarrow -7-8 = 3x, -6 = 3y$

$\Rightarrow 3x = -15, \quad y = \frac{-6}{3}$

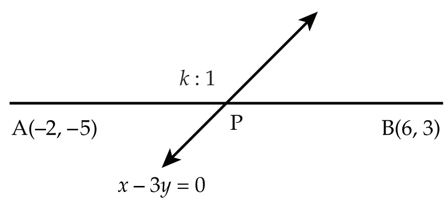
$\Rightarrow x = -5, \quad y = -2$

$\therefore$ (– 5, – 2) are the coordinates of the point B. **Ans.**

Q. 5. Find the ratio in which the line $x - 3y = 0$ divides the line segment joining the points (– 2, – 5) and (6, 3). Find the coordinates of the point of intersection.

[CBSE OD, Set 1, 2019]

Ans. Let required ratio be $k : 1$



By section formula, we have

$$x = \frac{mx_2+nx_1}{m+n}, \; y = \frac{my_2+ny_1}{m+n}$$

Here, $x_1 = -2, x_2 = 6, y_1 = -5, y_2 = 3$

$m = k, n = 1$

$\Rightarrow \quad x = \frac{k(6)+(-2)}{k+1} = \frac{6k-2}{k-1}$

$\Rightarrow \quad y = \frac{k(3)+(-5)}{k+1} = \frac{3k-5}{k+1}$

$\left(\dfrac{6k-2}{k+1}, \dfrac{3k-5}{k+1}\right)$ points lie on the line $x-3y=0$

$$\therefore \quad \left(\frac{6k-2}{k+1}\right)-3\left(\frac{3k-5}{k+1}\right)=0$$

$$\frac{6k-2}{k+1}-\frac{(9k-15)}{k+1}=0$$

$$6k-2-9k+15=0$$

$$-3k+13=0$$

$$k=\frac{13}{3}$$

Hence required ratio is $\left(\dfrac{13}{3}:1\right)$ *i.e.*, (13 : 3)

Here, intersection point is,

$$x=\frac{6k-2}{k+1}=\frac{\frac{6\times 13}{3}-2}{\frac{13}{3}+1}$$

$$=\frac{(26-2)\times 3}{16}$$

$$=\frac{72}{16}=\frac{9}{2}$$

$$y=\frac{3k-5}{k+1}=\frac{3\times\frac{13}{3}-5}{\frac{13}{3}+1}$$

$$=\frac{(13-5)\times 3}{13+3}=\frac{24}{16}$$

$$=\frac{3}{2}$$

$\therefore$ intersection point is $\left(\dfrac{9}{2}, \dfrac{3}{2}\right)$

Q. 6. Point A lies on the line segment XY joining $X(6, -6)$ and $Y(-4, -1)$ in such a way that $\dfrac{XA}{XY}=\dfrac{2}{5}$. If point A also lies on the line $3x+k(y+1)=0$, find the value of k.
[CBSE OD, Set 2, 2019]

Ans. Given,

$$\frac{XA}{XY}=\frac{2}{5}$$

$$\frac{XA}{XA+AY}=\frac{2}{5}$$

$$5XA=2XA+2AY$$

$$3XA=2AY$$

$$\frac{XA}{AY}=\frac{2}{3}$$

$$XA:AY=2:3$$

So A divides XY in ratio 2 : 3

Here, $m=2$, $n=3$, $x_1=6$, $y_1=-6$, $x_2=-4$ and $y_2=-1$

Coordinates of point A are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$\Rightarrow \quad \left(\frac{2\times(-4)+3(6)}{2+3}, \frac{2(-1)+3(-6)}{2+3}\right)$$

$$\Rightarrow \quad \left(\frac{-8+18}{5}, \frac{-2-18}{5}\right)=(2,-4)$$

Since, point $A(2, -4)$ lies on line $3x+k(y+1)=0$.

Therefore it will satisfy the equation.

On putting $x=2$ and $y=-4$ in the equation, we get

$$3\times 2+k(-4+1)=0$$

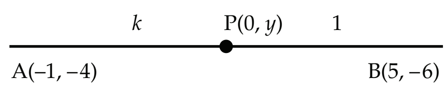
$$6-3k=0$$

$$3k=6$$

$$k=2$$

Q. 7. Find the ratio in which the y-axis divides the line segment joining the points $(-1, -4)$ and $(5, -6)$. Also find the coordinates of the point of intersection.
[CBSE OD, Set 3, 2019]

Ans. Let the y-axis cut the line joining point $A(-1, -4)$ and point $B(5, -6)$ in the ratio $k:1$ at the point $P(0, y)$



Then, by section fromula, we have

$$x=\frac{mx_2+nx_1}{m+n}$$

$$0=\frac{k(5)+(-1)}{k+1}$$

$$0=\frac{5k-1}{k+1}$$

$$\Rightarrow \quad 5k-1=0$$

$$k = \frac{1}{5}$$

Then the required ratio is $\left(\frac{1}{5} : 1\right)$ *i.e.*, (1 : 5)

Again, by section formula, we have

$$y = \frac{my_2 + ny_1}{m+n}$$

$$= \frac{1(-6)+5(-4)}{1+5}$$

$$= \frac{-6-20}{6}$$

$$= \frac{-26}{6} = \frac{-13}{3}$$

Hence, co-ordinates of the intersection point is $\left(0, -\frac{13}{3}\right)$

Q. 8. Find the point on *y*-axis which is equidistant from the points (5, –2) and (–3, 2). [CBSE Delhi, Set 1, 2019]

Ans. We know that a point on the *y*-axis is of the form (0, *y*). So, let the point *P*(0, *y*) be equidistant from *A* (5, –2) and *B* (–3, 2)

Then $AP = BP$

or $AP^2 = BP^2$

or $(5-0)^2 + (-2-y)^2 = (-3-0)^2 + (2-y)^2$

or $25 + 4 + y^2 + 4y = 9 + 4 + y^2 - 4y$

$$8y = -16$$

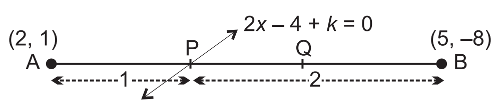
$$y = -2$$

So, the required point is (0, – 2)

Q. 9. The line segment joining the points *A*(2, 1) and *B*(5, – 8) is trisected at the points *P* and *Q* such that *P* is nearer to *A*. If *P* also lies on the line given by $2x - y + k = 0$, find the value of *k*. [CBSE Delhi, Set 1, 2019]

Ans. The line segment *AB* is trisected at the points *P* and *Q* and *P* is nearest to *A*

So, *P* divides *AB* in the ratio 1 : 2



Then co-ordinates of P, by section formula

$$= P\left[\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}\right]$$

$$= P\left[\frac{1(5)+2(2)}{1+2}, \frac{1(-8)+2(1)}{1+2}\right]$$

$$= P\left[\frac{5+4}{3}, \frac{-8+2}{3}\right] = P\,(3, -2)$$

$\because$ P lies on the line $2x - y + k = 0$

$\therefore$ It will satisfy the equation.

On putting $x = 3$ and $y = -2$ in the given equation, we get

$$2(3) - (-2) + k = 0$$

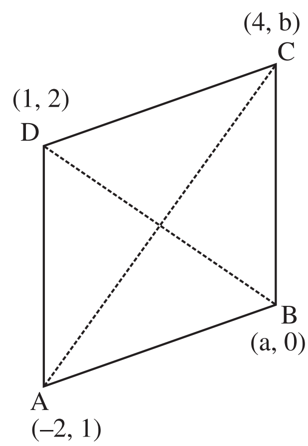
$$6 + 2 + k = 0$$

$\Rightarrow$ $k = -8$

Hence, $k = -8$

Q. 10. If *A*(–2, 1) and B(*a*, 0), *C*(4, b) and *D*(1, 2) are the vertices of a parallelogram *ABCD*, find the values of *a* and *b*. Hence find the lengths of its sides. [CBSE Term 1, 2018]

Ans. Given *ABCD* is a parallelogram.



$$\text{Mid point of AC} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-2+4}{2}, \frac{1+b}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{1+b}{2}\right) = \left(1, \frac{1+b}{2}\right)$$

$$\text{Midpoint of BD} = \left(\frac{x'_1+x'_2}{2}, \frac{y'_1+y'_2}{2}\right)$$

$$= \left(\frac{a+1}{2}, \frac{0+2}{2}\right)$$

$$= \left(\frac{a+1}{2}, \frac{2}{2}\right) = \left(\frac{a+1}{2}, 1\right)$$

Since, diagonals of a parallelogram bisect each other,

$$\therefore \quad \left(1, \frac{1+b}{2}\right) = \left(\frac{a+1}{2}, 1\right)$$

On comparing, we get

$$\therefore \quad \frac{a+1}{2} = 1 \qquad \Big| \qquad \frac{1+b}{2} = 1$$

$$\Rightarrow \quad a+1 = 2 \qquad \Big| \qquad \Rightarrow \quad 1+b = 2$$

$$\Rightarrow \quad a = 1 \qquad \Big| \qquad \Rightarrow \quad b = 1$$

Therefore, the coordinates of vertices of parallelogram $ABCD$ are $A(-2, 1)$, $B(1, 0)$, $C(4, 1)$ and $D(1, 2)$

Length of side $AB = DC = \sqrt{(1+2)^2 + (0-1)^2}$

$$= \sqrt{9+1} = \sqrt{10} \text{ units}$$

And, $\quad AD = BC = \sqrt{(1+2)^2 + (2-1)^2}$

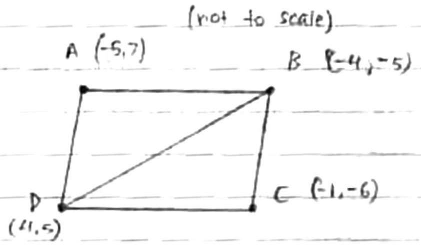
$$= \sqrt{9+1} = \sqrt{10} \text{ units}$$

Q. 11. If $A(-5, 7)$, $B(-4, -5)$, $C(-1, -6)$ and $D(4, 5)$ are the vertices of a quadrilateral, find the area of the quadrilateral $ABCD$. [CBSE, 2018]

Ans.

Topper's Answers

15) Vertices of quadrilateral ABCD:
(choice 2) A (-5,7), B(-4,-5), C(-1,-6), D(4,5)



Area of quad ABCD.

= area △ABD + area △BCD.

area △ABD →

$= \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$ sq. units.

$= \frac{1}{2}[-5(-5-5) + (-4)(5-7) + 4(7+5)]$

$= \frac{1}{2}[50 + 8 + 48]$

$= \frac{1}{2}[58+48]$.

$= \frac{1}{2} \times 106 = 53$ sq. units².

area △BCD $= \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$.

$= \frac{1}{2}[-4(-6-5) + (-1)(5+5) + (4)(-5+6)]$.

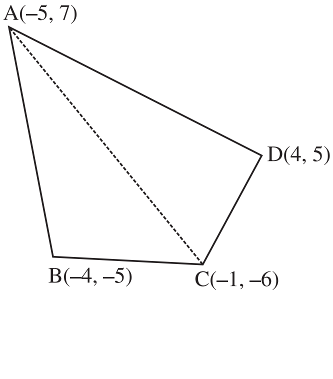
$= \frac{1}{2}[44 - 10 + 4]$

$= \frac{1}{2} \times 38 = 19$ sq. units².

⇒ Area of quadrilateral = Area of two triangles = 53+19 = 72 sq. units.

$\boxed{\text{Area of quadrilateral ABCD is 72 sq. units}}$

Given $ABCD$ is quadrilateral.



By joining points A and C, the quadrilateral is divided into two triangles.

Now, Area of quad. $ABCD$ = Area of ΔABC + Area of ΔACD

Area of ΔABC

$$= \frac{1}{2} \,|\,[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]\,|$$

$$= \frac{1}{2} \,|\,[-5(-5+6) - 4(-6-7) - 1(7+5)]\,|$$

$$= \frac{1}{2} \mid [-5(1) - 4(-13) - 1(12)] \mid$$

$$= \frac{1}{2} \mid (-5 + 52 - 12) \mid$$

$$= \frac{1}{2} \mid (35) \mid = \frac{35}{2} \text{ sq. units.}$$

Area of ΔADC

$$= \frac{1}{2} \mid [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \mid$$

$$= \frac{1}{2} \mid [-5(5+6) + 4(-6-7) + (-1)(7-5)] \mid$$

$$= \frac{1}{2} \mid [-5(11) + 4(-13) - 1(2)] \mid$$

$$= \frac{1}{2} \mid -55 - 52 - 2 \mid$$

$$= \frac{1}{2} \mid -109 \mid = \frac{109}{2} \text{ sq. units.}$$

Area of quadrilateral $ABCD$

$$= \frac{35}{2} + \frac{109}{2} = \frac{144}{2}$$

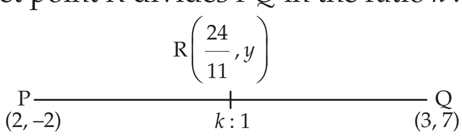
$$= 72 \text{ sq. units.}$$

Q. 12. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the points $P(2, -2)$ and $Q(3, 7)$? Also find the value of y.

[CBSE OD, Term 2, Set 1, 2017]

Ans.

Let point R divides PQ in the ratio $k : 1$



$$R = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$\Rightarrow \left(\frac{24}{11}, y\right) = \left(\frac{k(3) + 1(2)}{k+1}, \frac{k(7) + 1(-2)}{k+1}\right) \quad ...(i)$$

$$= \left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$$

$$\Rightarrow \quad \frac{3k+2}{k+1} = \frac{24}{11}$$

$$\Rightarrow \quad 33k + 22 = 24k + 24$$

$$\Rightarrow \quad 9k = 2 \Rightarrow k = 2/9$$

$$\therefore \quad k : 1 = 2 : 9$$

Now, from equation (i)

$$y = \frac{7k-2}{k+1} = \frac{7\left(\frac{2}{9}\right) - 2}{\frac{2}{9} + 1}$$

$$= \frac{\frac{14}{9} - 2}{\frac{2}{9} + 1} = \frac{\frac{14-18}{9}}{\frac{2+9}{2}} = \frac{-4}{11}$$

Line PQ divides in the ratio $2 : 9$ and value of $y = \frac{-4}{11}$

Q. 13. Show that $\Delta\, ABC$, where $A(-2, 0)$, $B(2, 0)$, $C(0, 2)$ and $\Delta\, PQR$ where $P(-4, 0)$, $Q(4, 0)$, $R(0, 4)$ are similar triangles.

[CBSE Delhi, Term 2, Set 1, 2017]

Ans. Coordinates of vertices are

$A(-2, 0)$, $B(2, 0)$, $C(0, 2)$

$P(-4, 0)$, $Q(4, 0)$, $R(0, 4)$

$$AB = \sqrt{(2+2)^2 + (0-0)^2} = 4 \text{ units}$$

$$BC = \sqrt{(0-2)^2 + (2-0)^2}$$

$$= \sqrt{4+4} = 2\sqrt{2} \text{ units}$$

$$CA = \sqrt{(-2-0)^2 + (0-2)^2}$$

$$= \sqrt{8} = 2\sqrt{2} \text{ units}$$

$$PR = \sqrt{(0+4)^2 + (4-0)^2}$$

$$= \sqrt{4^2 + (4)^2} = 4\sqrt{2} \text{ units}$$

$$QR = \sqrt{(0-4)^2 + (4-0)^2}$$

$$= \sqrt{4^2 + (4)^2} = 4\sqrt{2} \text{ units}$$

$$PQ = \sqrt{(4+4)^2 + (0-0)^2}$$

$$= \sqrt{(8)^2} = 8 \text{ units}$$

We see that sides of $\Delta\ PQR$ are twice the sides of $\Delta\ ABC$.
Hence, both triangles are similar.

Hence Proved.

Q. 14. The area of a triangle is 5 sq units. Two of its vertices are (2, 1) and (3, –2). If the third vertex is $\left(\frac{7}{2}, y\right)$, find the value of y.

[CBSE Delhi, Term 2, Set 1, 2017]

Ans. Given,
$A(2, 1)$, $B(3, -2)$ and $C\left(\frac{7}{2}, y\right)$

Now, Area $(\Delta ABC) = \frac{1}{2}\,|x_1(y_2 - y_3)$
$+ x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$$5 = \frac{1}{2}\,|2(-2-y) + 3(y-1) + \frac{7}{2}(1+2)|$$

$$\Rightarrow \quad 10 = |-4 - 2y + 3y - 3 + \frac{7}{2} + 7|$$

$$\Rightarrow \quad 10 = \left|y + \frac{7}{2}\right|$$

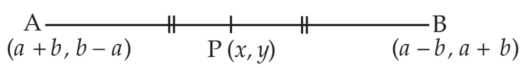
$$\Rightarrow \quad 10 = y + \frac{7}{2} \text{ or } -10 = \left(y + \frac{7}{2}\right)$$

$$\Rightarrow \quad y = \frac{13}{2} \quad \text{or} \quad y = \frac{-27}{2}$$

Q. 15. If the point $P(x, y)$ is equidistant from the points $A(a + b, b - a)$ and $B(a - b, a + b)$, prove that $bx = ay$.

[CBSE OD, Term 2, Set 1, 2016]

Ans. Given, P is equidistant from points A and B.



$$\therefore \quad PA = PB$$

$$\text{or} \quad (PA)^2 = (PB)^2$$

$$\Rightarrow (a + b - x)^2 + (b - a - y)^2 = (a - b - x)^2 + (a + b - y)^2$$

$$\Rightarrow (a + b)^2 + x^2 - 2ax - 2bx + (b - a)^2 + y^2 - 2by + 2ay = (a - b)^2 + x^2 - 2ax + 2bx + (a + b)^2 + y^2 - 2ay - 2by$$

$$\Rightarrow \quad -2bx + 2ay = 2bx - 2ay$$

$$\Rightarrow \quad 4ay = 4bx$$

$$\Rightarrow \quad ay = bx$$

or $\quad bx = ay$ **Hence Proved.**

Q. 16. In Fig. 6, ABC is a triangle coordinates of whose vertex A are (0, – 1). D and E, respectively are the mid-points of the sides AB and AC and their coordinates are (1, 0) and (0, 1) respectively. If F is the mid-point of BC, find the areas of $\Delta\ ABC$ and $\Delta\ DEF$. [CBSE Delhi, Term 2, Set 1, 2016]

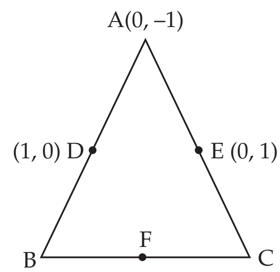


Fig. 6

Ans. Given, the coordinates of vertex A $(0, -1)$ and, mid points D $(1, 0)$ and $E(0, 1)$ respectively.
Since, D is the mid-point of AB
Let, coordinates of B are (x, y)

then, $\left(\frac{x+0}{2}, \frac{y-1}{2}\right) = (1, 0)$

which gives B (2, 1)
Similarly, E is the mid-point of AC

Let, coordinates of C are (x', y')

then, $\left(\frac{x'+0}{2}, \frac{y'-1}{2}\right) = (0, 1)$

which gives C (0, 3)
Now, Area of $\Delta\ ABC$

$$= \frac{1}{2}\,|[0(1-3) + 2(3+1) + 0(-1-1)]|$$

$= 4$ sq. units.

Now, F is the mid-point of BC.

$\Rightarrow$ Coordinates of F are $\left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$

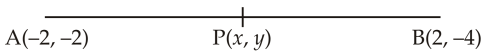
$$\therefore \quad \text{Area of } \Delta\ DEF = \frac{1}{2}\,|[1(1-2) + 0(2-0) + 1(0-1)]|$$

$$= \frac{|-2|}{2} = 1 \text{ sq. unit}$$

Q. 17. If the coordinates of points A and B are $(-2, -2)$ and $(2, -4)$ respectively find the coordinates of P such that $AP = \frac{3}{7}AB$, where P lies on the line segment AB.

[CBSE OD, Term 2, Set 1, 2015]

Ans. Here $P(x, y)$ divides line segment AB such that $AP = \frac{3}{7}AB$



$$\Rightarrow \quad \frac{AP}{AB} = \frac{3}{7}$$

$$\Rightarrow \quad \frac{AB}{AP} = \frac{7}{3}$$

$$\Rightarrow \quad \frac{AB}{AP} - 1 = \frac{7}{3} - 1$$

$$\Rightarrow \quad \frac{AB - AP}{AP} = \frac{4}{3}$$

$$\Rightarrow \quad \frac{BP}{AP} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{AP}{BP} = \frac{3}{4}$$

$\therefore$ P divides AB in the ratio $3 : 4$ $(m : n)$

Let the coordinates of P are (x, y)

Therefore,

$$x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}$$

$$x = \frac{3 \times 2 + 4(-2)}{3+4}, y = \frac{3(-4) + 4(-2)}{3+4}$$

$[\because m : n = 3 : 4]$

$$\Rightarrow \quad x = \frac{6-8}{7}, y = \frac{-12-8}{7}$$

$$\Rightarrow \quad x = \frac{-2}{7}, \quad y = \frac{-20}{7}$$

Therefore, co-ordinates of P(x, y) are

$$\left(\frac{-2}{7}, \frac{-20}{7}\right)$$

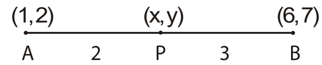
Q. 18. Find the coordinates of a point P on the line segment joining $A(1, 2)$ and $B(6, 7)$ such that $AP = \frac{2}{5}AB$.

[CBSE OD, Set 3, 2015]

Ans. Given, $A(1, 2)$ and $B(6, 7)$ are the points of a line segment AB with a point P on it.

Let the co-ordinates of point P be (x, y).

Also, $\quad AP = \frac{2}{5}AB \quad$ (Given)



$$AB = AP + PB$$

$$\Rightarrow \quad \frac{AP}{AB} = \frac{2}{5}$$

$$\Rightarrow \quad \frac{AP}{AP+PB} = \frac{2}{5} \Rightarrow 5AP = 2AP + 2PB$$

$$\Rightarrow \quad 3AP = 2PB$$

$$\Rightarrow \quad \frac{AP}{PB} = \frac{2}{3}$$

$$\therefore \quad m = 2, n = 3$$

Then, by section formula, we have

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$

$$x = \frac{2 \times 6 + 3 \times 1}{2+3} \text{ and } y = \frac{2 \times 7 + 3 \times 2}{2+3}$$

$$x = \frac{15}{5} \text{ and } y = \frac{20}{5}$$

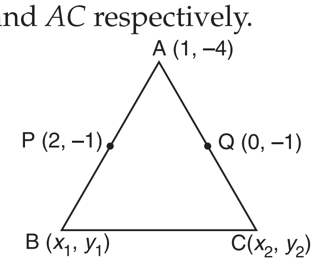
$\therefore$ $x = 3$ and $y = 4$

Hence, the required point is $P(3, 4)$.

Q. 19. Find the area of the triangle ABC with $A(1, -4)$ and mid-points of sides through A being $(2, -1)$, and $(0, -1)$.

[CBSE Delhi, Term 2, Set 1, 2015]

Ans. Let $A(1, -4)$, $B(x_1, y_1)$ and $C(x_2, y_2)$ be the vertices of a triangle ABC and let $P(2, -1)$ and $Q(0, -1)$ be the mid-points of AB and AC respectively.



$\because$ P is the mid-point of AB.

$$\therefore \quad \frac{1 + x_1}{2} = \mathbf{2}, \frac{-4 + y_1}{2} = \mathbf{-1}$$

$$x_1 = 3, y_1 = 2$$

So, $B(x_1, y_1) \equiv B(3, 2)$

Similarly, Q is the mid-point of AC.

$$\frac{1+x_2}{2} = \mathbf{0}, \ \frac{-4+y_2}{2} = -1$$

$\Rightarrow \qquad x_2 = -1, y_2 = 2$

So, $C(x_2, y_2) \equiv C(-1, 2)$

Thus, Area of ΔABC

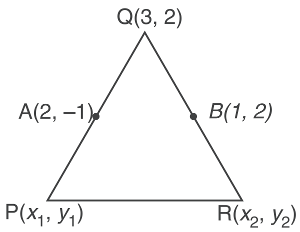
$$= \frac{1}{2}|(2-2)+3(2+4)-1(-4-2)|$$

$$= \frac{1}{2}\times 24 = 12 \text{ sq. units.}$$

Q. 20. Find the area of the triangle PQR with $Q(3, 2)$ and the mid-points of the sides through Q being (2, – 1) and (1, 2).

[CBSE Delhi, Term 2, Set 3, 2015]

Ans. Let $P(x_1, y_1)$, $Q(3, 2)$ an $R(x_2, y_2)$ be the vertices of a triangle PQR and let $A(2, -1)$ and $B(1, 2)$ be the mid-points of PQ and QR respectively.



$\because A$ is the mid-point of PQ

$\therefore \qquad \frac{3+x_1}{2} = \mathbf{2}, \ \frac{2+y_1}{2} = -1$

$\Rightarrow \qquad x_1 = 1, y_1 = -4$

So, $P(1, -4)$

$\because B$ is the mid-point of QR

$\therefore \qquad \frac{3+x_2}{2} = \mathbf{1}, \ \frac{2+y_2}{2} = 2$

$\Rightarrow \qquad x_2 = -1, y_2 = 2$

So, $R(-1, 2)$

Thus, Area of ΔPQR

$$= \frac{1}{2}\ |[1(2-2) - 1(2+4) + 3(-4-2)]|$$

$$= \frac{1}{2}|[1(0)-1(6)+3(-6)]|$$

$$= \frac{1}{2}|[-6-18]|$$

$$= \frac{24}{2} = 12 \text{ sq. units.}$$

Long Answer Type Questions

(4 marks each)

Q. 1. If $a \neq b \neq c$, prove that the points (a, a^2), (b, b^2) (c, c^2) will not be collinear.

[CBSE Delhi, Term 2, Set 1, 2017]

Ans. Area $= \frac{1}{2}\ |a(b^2 - c) + b(c - a^2) + c(a^2 - b^2)|$

$$= \frac{1}{2}\ |(b-c)(ab + ac - a^2 - bc)|$$

$$= \frac{1}{2}\ |(b-c)(a-b)(c-a)|$$

The area can never be zero as $a \neq b \neq c$

Hence, these points can never be collinear.

Hence Proved.

Q. 2. In fig. 8, the vertices of ΔABC are $A(4, 6)$, $B(1, 5)$ and $C(7, 2)$. A line-segment DE is drawn to intersect the sides AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. Calculate the area of ΔADE and compare it with area of ΔABC.

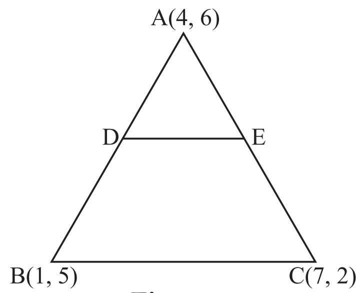
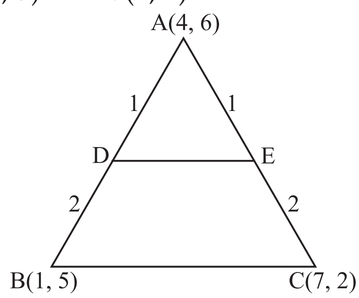


Figure 8

[CBSE OD, Term 2, Set 1, 2016]

Ans. We have, the vertices of ΔABC as $A(4, 6)$, $B(1, 5)$ and $C(7, 2)$.



And $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$

Then, coordinates of D are

$$\left(\frac{1(1)+2(4)}{1+2}, \frac{1(5)+2(6)}{1+2}\right)$$

$\left(\frac{1+8}{3}, \frac{5+12}{3}\right)$ *i.e*, $D\left(3, \frac{17}{3}\right)$

and coordinates of E are

$$\left(\frac{1(7)+2(4)}{1+2}, \frac{1(2)+2(6)}{1+2}\right)$$

$\left(\frac{7+8}{3}, \frac{2+12}{3}\right)$ *i.e*, $E\left(5, \frac{14}{3}\right)$

Now, Area of ΔADE

$$= \frac{1}{2}\left[4\left(\frac{17}{3}-\frac{14}{3}\right)+3\left(\frac{14}{3}-6\right)+5\left(6-\frac{17}{3}\right)\right]$$

$$= \frac{1}{2}\left[4(1)+3\left(-\frac{4}{3}\right)+5\left(\frac{1}{3}\right)\right]$$

$$= \frac{5}{6} \text{ units}$$

and Area of ΔABC

$$= \frac{1}{2}\ [4(5-2) + 1\ (2-6) + 7\ (6-5)]$$

$$= \frac{1}{2}\ [4(3) + 1(-4) + 7(1)]$$

$$= \frac{15}{2} \text{ units.}$$

$$\therefore\ \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta ABC)} = \frac{5/6}{15/2} = \frac{1}{9}$$

i.e, ar (ΔADE) : ar (ΔABC) = 1 : 9

Q. 3. Prove that the area of a triangle with vertices (t, t – 2), (t + 2, t + 2) and (t + 3, t) is independent of t.

[CBSE Delhi, Term 2, Set 1, 2016]

Ans. Given, the vertices of a triangle (t, t – 2), (t + 2, t + 2) and (t + 3, t)

$\therefore$ Area of the triangle

$$= \frac{1}{2}\ |\,[t\ (t+2-t) + (t+2)\ (t-t+2) + (t+3)\ (t-2-t-2)]\,|$$

$$= \frac{1}{2}\ |\,(2t + 2t + 4 - 4t - 12)\,|$$

$$= \frac{1}{2}\ |\ -8\ |\ = 4 \text{ sq. units}$$

which is independent of t

Hence Proved.

Q. 4. If the points $A(k + 1, 2k)$, $B(3k, 2k + 3)$ and $C(5k - 1, 5k)$ are collinear, then find the value of k. [CBSE OD, Term 2, Set 3, 2015]

Ans. Since $A(k + 1, 2k)$, $B(3k, 2k + 3)$ and $C(5k - 1, 5k)$ are collinear points, so area of triangle = 0.

$$\Delta = \frac{1}{2}\ |\,(k+1)\ (2k + 3 - 5k) + 3k\ (5k - 2k) + (5k - 1)\ \{2k - (2k + 3)\}\,|$$

$$\Rightarrow\quad 0 = |\,(k+1)(3-3k) + 3k(3k) + (5k-1)(-3)\,|$$

$$0 = |\,3k - 3k^2 + 3 - 3k + 9k^2 - 15k + 3\,|$$

$$0 = \frac{1}{2}\ |\,6k^2 - 15k + 6\,|$$

$$\Rightarrow\quad 6k^2 - 15k + 6 = 0$$

$$\Rightarrow\quad 6k^2 - 12k - 3k + 6 = 0$$

$$\Rightarrow\quad 6k\ (k-2) - 3\ (k-2) = 0$$

$$\Rightarrow\quad (k-2)\ (6k-3) = 0$$

$$\therefore\quad k = 2 \text{ or } k = \frac{1}{2}$$

Q. 5. Find the values of k so that the area of the triangle with vertices (1, – 1), (– 4, 2k) and (– k, – 5) is 24 sq. units.

[CBSE OD, Term 2, Set 1, 2015]

Ans. The vertices of the given ΔABC are $A(1, -1)$, $B(-4, 2k)$ and $C(-k, -5)$

$$\therefore \text{ Area of } \Delta ABC = \frac{1}{2}\ [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}\,[1(2k+5) + (-4)(-5+1) + (-k)\ (-1-2k)]$$

$$= \frac{1}{2}\,[2k + 5 + 16 + k + 2k^2]$$

$$= \frac{1}{2}\,[2k^2 + 3k + 21]$$

Area of ΔABC = 24 sq. units (Given)

$$\therefore\quad \frac{1}{2}[2k^2 + 3k + 21] = 24$$

$$\Rightarrow\quad [2k^2 + 3k + 21] = 48$$

$$\Rightarrow\quad 2k^2 + 3k + 21 = 48$$

$$\Rightarrow\quad 2k^2 + 3k - 27 = 0$$

$\Rightarrow \quad 2k^2 + 9k - 6k - 27 = 0$

$\Rightarrow \quad k(2k + 9) - 3(2k + 9) = 0$

$\Rightarrow \quad (k - 3)(2k + 9) = 0$

$$k = 3 \text{ or } -\frac{9}{2}$$

Hence, $k = \mathbf{3}$ or $k = -\frac{9}{2}$

Q. 6. Find the values of k so that the area of the triangle with vertices $(k + 1, 1)$, $(4, -3)$ and $(7, -k)$ is 6 sq. units.

[CBSE OD, Term 2, Set 2, 2015]

Ans. Given, the vertices are $(k + 1, 1)$, $(4, -3)$ and $(7, -k)$ and the area of the triangle is 6 square units.

Therefore,

$$\text{Area} = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$\Rightarrow \quad 6 = \frac{1}{2}[(k+1)(-3+k) + 4(-k-1) + 7(1+3)]$

$\Rightarrow \quad 12 = (k + 1)(k - 3) + 4(-k - 1) + 28$

$\Rightarrow \quad 12 = k^2 - 3k + k - 3 - 4k - 4 + 28$

$\Rightarrow \quad k^2 - 6k + 9 = 0$

$\Rightarrow \quad k^2 - 3k - 3k + 9 = 0$

$\Rightarrow \quad k(k - 3) - 3(k - 3) = 0$

$\Rightarrow \quad (k - 3)(k - 3) = 0$

$\therefore \quad \text{k} = 3, 3$

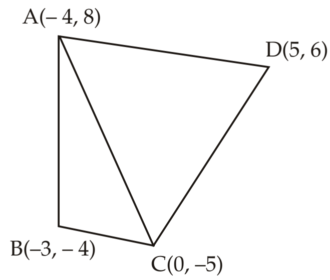
Hence, value of k is 3.

Q. 7. If $A(-4, 8)$, $B(-3, -4)$, $C(0, -5)$ and $D(5, 6)$ are the vertices of a quadrilateral *ABCD*, find its area.

[CBSE Delhi, Term 2, Set 1, 2015]

Ans. We have, $A(-4, 8)$, $B(-3, -4)$, $C(0, -5)$ and $D(5, 6)$ are the vertices of a quadrilateral.

Join A and C. Then, area of quad. $ABCD$



$= (\text{area of } \Delta ABC) + (\text{area of } \Delta ACD)$

Area of ΔABC

$= \frac{1}{2}[-4(-4 + 5) - 3(-5 - 8) + 0(8 + 4)]$

$= \frac{1}{2}[-4 + 39]$

$= \frac{35}{2}$ sq. units

And, area of ΔACD

$= \frac{1}{2}[-4(-5 - 6) + 0(6 - 8) + 5(8 + 5)]$

$= \frac{1}{2}[44 + 65]$

$= \frac{109}{2}$ sq. units.

$\because$ Area of quadilateral $ABCD$

$= \text{Area of } \Delta ABC + \text{Area of } \Delta ACD$

$= \frac{35}{2} + \frac{109}{2}$

$= \frac{144}{2}$ sq. units $= 72$ sq. units.