CHAPTER

3

TRIGONOMETRY

TRIGONOMETRIC IDENTITY

An equation involving trigonometric functions which is true for all those angles for which the functions are defined is called a trigonometric identity.

Conditional Trigonometric Identities

If A + B + C = 180° (or π), or A, B, C are angles of a triangle.

Then,

(*a*) $\sin(\mathrm{A}+\mathrm{B}) = \sin(\pi-\mathrm{C}) = \sin \mathrm{C}$, etc.

(*b*) $\sin\left(\frac{\mathrm{A}}{2}+\frac{\mathrm{B}}{2}\right)=\sin\left(\frac{\pi}{2}-\frac{\mathrm{C}}{2}\right)=\cos\frac{\mathrm{C}}{2}$, etc

(*c*) $\sin \mathrm{A}+\sin \mathrm{B}+\sin \mathrm{C}=4\cos\frac{\mathrm{A}}{2}\cos\frac{\mathrm{B}}{2}\sin\frac{\mathrm{C}}{2}$

(*d*) $\cos \mathrm{A}+\cos \mathrm{B}+\cos \mathrm{C}=1+4\sin\frac{\mathrm{A}}{2}\sin\frac{\mathrm{B}}{2}\sin\frac{\mathrm{C}}{2}$

(*e*) $\tan \mathrm{A} + \tan \mathrm{B} + \tan \mathrm{C} = \tan \mathrm{A}\tan \mathrm{B}\tan \mathrm{C}$

(*f*) $\cot \mathrm{A}\cot \mathrm{B} + \cot \mathrm{B}\cot \mathrm{C} + \cot \mathrm{C}\cot \mathrm{A} = 1$

(*g*) $\tan\frac{\mathrm{A}}{2}\tan\frac{\mathrm{B}}{2}+\tan\frac{\mathrm{B}}{2}\tan\frac{\mathrm{C}}{2}+\tan\frac{\mathrm{C}}{2}\tan\frac{\mathrm{A}}{2}=1$

Periodic Properties of Trigonometric Functions

(*a*) sin *x*, cos *x*, sec *x* and cosec*x* are periodic functions with fundamental period 2π.

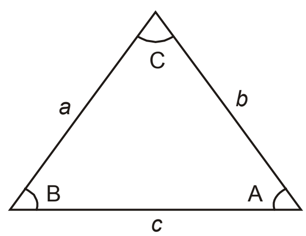
(*b*) tan *x* and cot *x* are periodic functions with fundamental period π.

(*c*) $|\sin x|$, $|\cos x|$, $|\tan x|$, $|\cot x|$, $|\text{sex } x|$, $|\text{cosec } x|$ are periodic functions with fundamental period π.

(*d*) $\sin^n x$, $\cos^n x$, $\sec^n x$, $\text{cosec}^n x$ are periodic functions with fundamental period 2π or π according as *n* is odd or even.

(*e*) $\tan^n x$ and $\cot^n x$ are periodic function with fundamental period π whether *n* is odd or even.

Properties of Triangle



(*a*) Sine Rule : $\frac{a}{\sin \mathrm{A}}=\frac{b}{\sin \mathrm{B}}=\frac{c}{\sin \mathrm{C}}$

(*b*) Consine Rule : $\cos \mathrm{A}=\frac{b^2+c^2-a^2}{2bc}$

(*c*) Projection formula

(*i*) $a = b\cos \mathrm{C} + c\cos \mathrm{B}$

(*ii*) $b = c\cos \mathrm{A} + a\cos \mathrm{C}$

(*iii*) $c = a\cos \mathrm{B} + b\cos \mathrm{A}$

(*d*) Tangent Rule:

$$\tan\left(\frac{\mathrm{B}-\mathrm{C}}{2}\right)=\frac{b-c}{b+c}\tan\left(\frac{\mathrm{B}+\mathrm{C}}{2}\right)=\frac{b-c}{b+c}\cot\frac{\mathrm{A}}{2}$$

(*e*) Half angle formula:

(*i*) $\sin\frac{\mathrm{A}}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$

(*ii*) $\cos\frac{\mathrm{A}}{2}=\sqrt{\frac{s(s-a)}{bc}}$

(*iii*) $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

(*f*) Area of a triangle:

$$\Delta = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B = \frac{1}{2}ab\sin C$$

Orthocentre of the Triangle and Pedal Triangle

(*a*) The distances of the orthocentre of the triangle from the vertices are 2R cos A, 2R cos B, 2R cos C and its distances from the sides are 2R cos B cos C, 2R cos C cos A, 2R cos A cos B.

(*b*) Circumradius of the pedal triangle = $\frac{R}{2}$

(*c*) Area of the pedal triangle = $2\Delta\cos A\cos B\cos C$.

(*d*) Circumcentre O, centroid G and orthocentre O′ are collinear and G divides OO′ in the ratio 1 : 2.

(*e*) Distance between the circumcentre O and the incentre I is

$$OI = R\sqrt{1-8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}$$

Heights and Distances

(*a*) The angle of elevation or depression is the angle between the line of observation and the horizontal line according as the object is at a higher or lower level than the observer.

(*b*) The angle of elevation or depression is always measured from horizontal line through the point of observation.

Trigonometric Equations

The general solution of some trigonometrical equations

	Trigonometric equation	**General solution**
(*i*)	$\sin\theta = \sin\alpha$	$\theta = n\pi + (-1)^n\alpha$, $n \in Z$
(*ii*)	$\cos\theta = \cos\alpha$	$\theta = 2n\pi \pm \alpha$ $n \in Z$
(*iii*)	$\tan\theta = \tan\alpha$	$\theta = n\pi + \alpha$ $n \in Z$
(*iv*)	$\sin^2\theta = \sin^2\alpha$	$\theta = n\pi \pm \alpha$ $n \in Z$
(*v*)	$\cos^2\theta = \cos^2\alpha$	$\theta = n\pi \pm \alpha$ $n \in Z$
(*vi*)	$\tan^2\theta = \tan^2\alpha$	$\theta = n\pi \pm \alpha$ $n \in Z$

Since all the trigonometrical ratios are periodic, the equations of the form $\sin\theta = k$, $\cos\theta = k$, or $\tan\theta = k$ etc., can have infinite number of angles satisfying it.

MULTIPLE CHOICE QUESTIONS

1. If (1 + sin A) (1 + sin B) (1 + sin C) = (1 – sin A) (1 – sin B) (1 – sin C), then each side is equal to:
A. ± sin A sin B sin C B. tan A tan B tan C
C. ± cot A cot B cot C D. ± cos A cos B cos C

2. If $T_n = \sin^2 5n° - \cos^2 5n°$, the value of $T_1 + T_2 + ... + T_{18}$ is:
A. 18 B. 1
C. 0 D. None

3. If $m\tan(\theta - 30°) = n\tan(\theta + 120°)$, then $\frac{m+n}{m-n}$ equals:
A. $2\cos 2\theta$ B. $\frac{1}{2}\cos 2\theta$
C. $\sin 2\theta$ D. $\frac{1}{2}\sin 2\theta$

4. If S = tan 9° – tan 27° – tan 63° + tan 81°, T = tan 9° + cot 9° – tan 27° – cot 27°, R = cosec 18° – cosec 54°, then
A. S = R = T B. S = R = 2T
C. 2S = R = T D. S = 2R = T

5. If $H = \frac{\cot 3x}{\cot x}$, then
A. $H \in \left[1, \frac{1}{2}\right]$ B. $H \notin \left[\frac{1}{3}, 3\right]$
C. $H \in \left[\frac{1}{3}, \frac{1}{2}\right]$ D. None

6. The greatest value of $\cos 2x + 3\sin x$ is:
A. $\sqrt{10}$ B. 9
C. $\frac{17}{8}$ D. $\frac{15}{4}$

7. The period of which of the functions is not $\frac{\pi}{2}$:
A. $|\cos 2x|$ B. $\sin^6 x + \cos^6 x$
C. $\sin 4x$ D. None

8. $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$ has how many solutions in the interval $(0, 2\pi)$:
A. 2 B. 4
C. 6 D. 8

9. The general solution of $\sqrt{2}\sec\theta + \tan\theta = 1$ is:
A. $\theta = n\pi + \frac{\pi}{4}$ B. $\theta = 2n\pi - \frac{\pi}{4}$
C. $\theta = n\pi \pm \frac{\pi}{4}$ D. $\theta = n\pi + (-1)^n\frac{\pi}{4}$

10. If $e^{\cos x} - e^{-\cos x} - 4 = 0$, then which of the following is true?

A. has two real solutions in $(0, \pi)$
B. has two real solutions in $\{0, 2\pi)$
C. has no real solution
D. has exactly one real solution

11. The solution of the equation $\cos^{58}x + \sin^{40}x = 1$

A. $x = n\pi$ B. $n = \frac{n\pi}{2}$

C. $x = \frac{n\pi}{2} \pm \frac{\pi}{4}$ D. None

12. If $32\tan^8\theta = 2\cos^2\alpha - 3\cos\alpha$, and $3\cos 2\theta = 1$ the general value of θ is:

A. $\alpha = n\pi \pm \frac{\pi}{3}$ B. $\alpha = 2n\pi \pm \frac{2\pi}{3}$

C. $\alpha = \frac{n\pi}{2} \pm \pi$ D. None

13. $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 =$

A. $\tan^{-1}3$ B. $\cot^{-1}\frac{1}{3}$

C. $\sin^{-1}\left(\frac{1}{\sqrt{10}}\right)$ D. None

14. $\tan^{-1}\left(\frac{a-b}{1+ab}\right) + \tan^{-1}\left(\frac{b-c}{1+bc}\right) + \tan^{-1}\left(\frac{c-a}{1+ca}\right) =$

A. 0 B. $\tan^{-1}\left(\frac{a+b+c}{1-ab-bc-ca}\right)$

C. $\tan^{-1}(a + b + c)$ D. None

15. $\tan^{-1}\left(\frac{c_1x-y}{c_1y+x}\right) + \tan^{-1}\left(\frac{c_2-c_1}{1+c_2c_1}\right) + \tan^{-1}\left(\frac{c_3-c_2}{1+c_3c_2}\right) + \ldots$ n terms $= \tan^{-1}\frac{Y}{x} - \tan^{-1}(k)$ then k is equal to:

A. $\frac{1}{c_n}$ B. c_n

C. c_{n-1} D. $\frac{1}{c_{n-1}}$

16. The number of real solutions of the equation $\tan^{-1}(x - 1) + \tan^{-1}x = \tan^{-1}(x + 1) = \tan^{-1}(3x)$ is:
A. 0 B. 1
C. 2 D. 3

17. $\underset{n\to\infty}{\text{Lt}} \sum_{1}^{\infty} \tan^{-1}\left(\frac{1}{n^2+n+1}\right) =$
A. 0 B. 1
C. $\frac{\pi}{4}$ D. $\frac{\pi}{2}$

18. If $\cos^{-1}\left(\frac{x}{2}\right) + \cos^{-1}\left(\frac{y}{3}\right) = \theta$, the value of $9x^2 - 12xy \cos\theta + 4y^2$ is:
A. $9\sin^2\theta$ B. $36\sin^2\theta$
C. $18\cos^2\theta$ D. $\cos^2\theta$

19. If the angles of a triangle are in the ratio 3 : 4 : 5, the least and the greatest sides are in the ratio:
A. $\sqrt{3}-1:1$ B. $\sqrt{3}:2+\sqrt{3}$
C. $\sqrt{2}:1+\sqrt{3}$ D. $1:\sqrt{3}+1$

20. The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 4.6 m away from the wall. The length of the ladder is:
A. 2.3 m B. 4.6 m
C. 7.8 m D. 9.2 m

ANSWERS

1	2	3	4	5	6	7	8	9	10
D	B	A	D	B	C	D	C	B	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	B	C	A	A	D	C	B	A	D

EXPLANATORY ANSWERS

1. Multiplying both sides by $(1 - \sin A)(1 - \sin B)(1 - \sin C)$, we have $(1 - \sin^2 A)(1 - \sin^2 B)(1 - \sin^2 C) = (1 - \sin A)^2 (1 - \sin B)^2 (1 - \sin C)^2$:
i.e., $\cos^2 A \cos^2 B \cos^2 C$
$= (1 - \sin A)^2 (1 - \sin B)^2 (1 - \sin C)^2$
$\Rightarrow (1 - \sin A)(1 - \sin B)(1 - \sin C)$
$= \pm \cos A \cos B \cos C$
$\therefore$ each side is equal to $\pm \cos A \cos B \cos C$

2. Here $T_n = -(\cos^2 5n° - \sin^2 5n°) = -\cos 10n°$
$T_1 + T_2 + ... + T_{18}$
$= -[\cos 10° + \cos 20° + ... + \cos 180°]$
$= -[\cos 10° + \cos 20° + ... + \cos 160° + \cos 170° + \cos 180°]$
$= [\cos 180°]$ $(\because \cos\theta + \cos(180 - \theta) = 0)$
$= -(-1) = 1.$

3. $\frac{m}{n} = \frac{\tan(\theta+120°)}{\tan(\theta-30°)} = \frac{-\cot(\theta+30°)}{\tan(\theta-30°)}$

$\Rightarrow \quad \frac{m}{n} = \frac{\cos(\theta+30°)\cos(\theta-30°)}{-\sin(\theta+30°)\sin(\theta-30°)}$

$\Rightarrow \quad \frac{m}{n} = \frac{\cos^2\theta - \sin^2 30°}{\sin^2 30° - \sin^2\theta}$

Applying componendo and dividendo, we have

$$\frac{m+n}{m-n} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta - 2\sin^2 30°} = 2\cos 2\theta.$$

4. S = tan 9° – tan27° – tan63° + tan81°

= tan9° – tan27° – tan(90° – 27°) + tan (90° – 9°)

= tan9° – tan27° – cot27° + cot9°

= (tan9° + cot9°) – (tan27° + cot27°) (= T)

$= \left(\frac{\sin 9°}{\cos 9°} + \frac{\cos 9°}{\sin 9°}\right) - \left(\frac{\sin 27°}{\cos 27°} + \frac{\cos 27°}{\sin 27°}\right)$

$= \frac{\sin^2 9° + \cos^2 9°}{\sin 9° \cos 9°} - \left(\frac{\sin^2 27° + \cos^2 27°}{\sin 27° \cos 27°}\right)$

$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°}$ (= 2R) $= \frac{2(\sin 54° - \sin 18°)}{\sin 18° \sin 54°}$

$= \frac{2.2\cos 36° \cdot \sin 18°}{\sin 18° \sin(90° - 36°)} = \frac{4\cos 36°}{\cos 36°} = 4$

5. Let $y = \frac{\cot 3x}{\cot x} = \frac{\tan x}{\tan 3x} = \frac{\tan x(1 - 3\tan^2 x)}{3\tan x - \tan^3 x}$

$\Rightarrow (3 - \tan^2 x)y = 1 - 3\tan^2 x$

$\Rightarrow \quad \tan^2 x = \frac{1-3y}{3-y}$

but $\tan^2 x > 0 \ \therefore \ \frac{1-3y}{3-y} > 0$

$\Rightarrow (3 - y), (1 - 3y) > 0$

(multiplying by $(3 - y)^2$ on both sides)

$(y - 3)(3y - 1) > 0$

$\Rightarrow \quad y < \frac{1}{3}$ or $y > 3 \ \Rightarrow y \notin \left[\frac{1}{3}, 3\right]$

6. Let $\cos 2x + 3\sin x = y$, then

$y = 1 - 2\sin^2 x + 3\sin x$

$= 1 - 2\left\{\left(\sin x - \frac{3}{4}\right)^2 - \frac{9}{16}\right\}$

$= \frac{17}{8} - 2\left(\sin x - \frac{3}{4}\right)^2$

y is greatest, when $\left(\sin x - \frac{3}{4}\right)^2$ is least.

$\therefore$ for $\sin x = \frac{3}{4}$, we get the maximum value of y.

7. Note: (*i*) The period of cos *ax* or sin *ax* is $\frac{2\pi}{a}$

(*ii*) The period of $|\cos ax|$ or $|\sin ax|$ is $\frac{\pi}{a}$

Here the period of $|\cos 2x|$ is $\frac{\pi}{2}$ and period of $\sin 4x$ is $\frac{\pi}{2}$.

Also $\sin^6 x + \cos^6 x = (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x\cos^2 x(\sin^2 x + \cos^2 x)$

$= 1 - \frac{3}{4}(2\sin x\cos x)^2 = 1 - \frac{3}{4}\sin^2 2x$

$= 1 - \frac{3}{4}(1 - \cos 4x) = \frac{1}{4} + \frac{3}{4}\cos 4x,$

which is a periodic function of period $\frac{\pi}{2}$.

8. $\tan\theta + \tan 2\theta = \sqrt{3}\,(1 - \tan\theta\tan 2\theta)$

$\Rightarrow \quad \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta} = \sqrt{3}$

$\Rightarrow \quad \tan 3\theta = \tan\frac{\pi}{3}.$

$\Rightarrow \quad 3\theta = n\pi + \frac{\pi}{3}$

$\Rightarrow \quad \theta = \frac{(3n+1)\pi}{9}$ for all $n \in$ Z

for $n = 0, 1, 2, 3, 4, 5$, $\theta \in (0, 2\pi)$

9. We have $\frac{\sqrt{2}}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \quad \cos\theta - \sin\theta = \sqrt{2}$

$\Rightarrow \quad \frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta = 1$

$\Rightarrow \quad \cos\left(\theta + \frac{\pi}{4}\right) = 1 = \cos\theta$

$\Rightarrow \quad \theta + \frac{\pi}{4} = 2n\pi$

$\Rightarrow \quad \theta = 2n\pi - \frac{\pi}{4}$ for all $n \in$ Z.

10. We have $(e^{\cos x})^2 - 4e^{\cos x} - 1 = 0$

$\Rightarrow \quad e^{\cos x} = \frac{4 \pm \sqrt{20}}{2} = 2 \pm \sqrt{5}$

Since $e^{\cos x} > 0$, we have $e^{\cos x} = 2 + \sqrt{5}$

$\therefore \ \cos x = \log_e\left(2 + \sqrt{5}\right) > 1$ and $2 + \sqrt{5} > e$

Hence the given equation has no real solution.

11. **Case I:** If $\cos^2 x < 1$, $\sin^2 x < 1$,

We have $(\cos^2 x)^{29} + (\sin^2 x)^{20} < \cos^2 x + \sin^2 x = 1$ not possible.

Case II: If $\cos^2 x = 1$, $\sin^2 x = 0$, then $x = n\pi$.

Case III: If $\sin^2 x = 1$, $\cos^2 x = 0$, then $x = n\pi \pm \frac{\pi}{2}$.

Thus combining Case (II) and Case (III),

We get the solution $x = \frac{n\pi}{2}, n \in Z$.

12. $\cos 2\theta = \frac{1}{3} \Rightarrow \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1}{3}$

$\tan^2\theta = \frac{1}{2} \Rightarrow \tan^8\theta = \frac{1}{16}$

$\therefore \quad 2\cos^2\alpha - 3\cos\alpha = 32\tan^8\theta = 2$

$\Rightarrow \quad 2\cos^2\alpha - 3\cos\alpha - 2 = 0$

$\Rightarrow (2\cos\alpha + 1)(\cos\alpha - 2) = 0$

$\Rightarrow \quad \cos\alpha = -\frac{1}{2} \quad (\because \cos\alpha \neq 2)$

$\Rightarrow \quad \alpha = 2n\pi \pm \frac{2\pi}{3}, n \in Z$

14. $\tan^{-1}a - \tan^{-1}b + \tan^{-1}(b) - \tan^{-1}c + \tan^{-1}c - \tan^{-1}a = 0$

15. $\tan^{-1}\left(\frac{c_1 x - y}{c_1 y - x}\right) + \tan^{-1}\left(\frac{c_2 - c_1}{1 + c_2 c_1}\right) + \ldots + \tan^{-1}\left(\frac{c_n - c_{n-1}}{1 + c_n c_{n-1}}\right)$

$= \tan^{-1}\left(\frac{\frac{x}{y} - \frac{1}{c_1}}{1 + \frac{x}{y}\cdot\frac{1}{c_1}}\right) + \tan^{-1}\left(\frac{\frac{1}{c_1} - \frac{1}{c_2}}{1 + \frac{1}{c_1}\cdot\frac{1}{c_2}}\right) + \ldots + \tan^{-1}\left(\frac{\frac{1}{c_{n-1}} - \frac{1}{c_n}}{1 + \frac{1}{c_{n-1}}\cdot\frac{1}{c_n}}\right)$

$= \tan^{-1}\frac{x}{y} - \tan^{-1}\frac{1}{c_1} + \tan^{-1}\frac{1}{c_1} - \tan^{-1}\frac{1}{c_2} +$

$\ldots. + \tan^{-1}\frac{1}{c_{n-1}} \tan^{-1}\left(\frac{1}{c_n}\right)$

$= \tan^{-1}\frac{x}{y} - \tan^{-1}\left(\frac{1}{c_n}\right)$.

16. $\tan^{-1}(x - 1) + \tan^{-1}(x + 1) = \tan^{-1}3x - \tan^{-1}x$

$\Rightarrow \tan^{-1}\left(\frac{(x-1)+(x+1)}{1-(x-1)(x+1)}\right) = \tan^{-1}\left(\frac{3x - x}{1 + 3x\cdot x}\right)$

$\Rightarrow \quad x(1 + 3x^2) = (2 - x^2)x$

$\Rightarrow \quad x(4x^2 - 1) = 0$

$\Rightarrow \quad x = 0,\ x = \pm\frac{1}{2}$

17. $S_n = \sum_1^\infty \tan^{-1}\left(\frac{1}{n^2 + n + 1}\right)$

$= \sum_1^\infty \tan^{-1}\left(\frac{(n+1) - n}{1 + (n+1)n}\right)$

$= \sum_1^\infty \left[\tan^{-1}(n+1) - \tan^{-1} n\right]$

$= [\tan^{-1} 2 - \tan^{-1} 1] + [\tan^{-1} 3 - \tan^{-1} 2] + \ldots + [\tan^{-1}(n + 1) - \tan^{-1} n]$

$= \tan^{-1}(n + 1) - \tan^{-1}(1)$

$= \tan^{-1}(n+1) - \frac{\pi}{4}$

$\therefore \operatorname{Lt}_{n\to\infty} S_n = \operatorname{Lt}_{n\to\infty}\left\{\tan^{-1}(n+1) - \frac{\pi}{4}\right\} = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

18. Let $\cos^{-1}\left(\frac{x}{2}\right) = A$, $\cos^{-1}\left(\frac{y}{3}\right) = B$,

Thus $A + B = \theta$.

$\Rightarrow \quad \cos(A + B) = \cos\theta$

$\Rightarrow \cos A \cos B - \sin A \sin B = \cos\theta$

$\Rightarrow \quad \frac{x}{2} \times \frac{y}{3} - \cos\theta = \sqrt{1-\cos^2 A}\sqrt{1-\cos^2 B}$

$\Rightarrow \quad xy - 6\cos\theta = \sqrt{4-x^2}\sqrt{9-y^2}$

squaring both sides, we have

$x^2y^2 - 12xy\cos\theta + 36\cos^2\theta = (4-x^2)(9 - y^2)$

$\Rightarrow \quad 9x^2 - 12xy\cos\theta + 4y^2 = 36(1 - \cos^2\theta)$

$\Rightarrow \quad 9x^2 - 12xy\cos\theta + 4y^2 = 36\sin^2\theta$

19. Let $A = 3\theta$, $B = 4\theta$, $c = 5\theta$

$A + B + C = \pi \Rightarrow 12\theta = 180° \Rightarrow \theta = 15°$

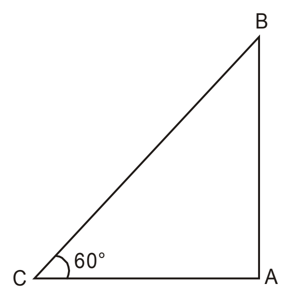
thus the angles are $A = 45°$, $B = 60°$, $C = 75°$

$\therefore$ a and c are the least and greatest sides opposite to the smallest and greatest angle A and C, respectively. By sine rule,

$\frac{a}{c} = \frac{\sin A}{\sin C} = \frac{\sin 45°}{\sin 75°} = \frac{\sin 45°}{\sin(30° + 45°)}$

$= \frac{2}{\sqrt{3}+1} = \frac{2(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{\sqrt{3}-1}{1}$

20. Let AB be the wall and BC be the ladder.



Then, $\angle ACB = 60°$ and $AC = 4.6$ m.

$\frac{AC}{BC} = \cos 60° = \frac{1}{2}$

$\Rightarrow \quad BC = 2 \times AC = (2 \times 4.6)\ m = 9.2\ m$