Chapter 15
FUNCTIONS & THEIR GRAPHS

A SINGLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

1. A certain polynomial $P(x),\ x \in R$ when divided by $x-a, x-b, x-c$ leaves remainders a, b, c respectively. The remainder when $P(x)$ is divided by $(x-a)(x-b)(x-c)$ is (a, b, c are disticnt)

(a) 0 (b) x

(c) $ax+b-c$ (d) ax^2+bx+c

2. If $f: R \to R, g: R \to R$ be two given functions then $h(x) = 2\min\{f(x)-g(x), 0\}$ equals

(a) $f(x)+g(x)-|g(x)-f(x)|$

(b) $f(x)+g(x)+|g(x)-f(x)|$

(c) $f(x)-g(x)+|g(x)-f(x)|$

(d) $f(x)-g(x)-|g(x)-f(x)|$

3. If $f(x) = \sqrt{3|x|-x-2}$ and $g(x) = \sin x$, then domain of definition of fog (x) is

(a) $\left\{2n\pi+\frac{\pi}{2}\right\}_{n\in I}$

(b) $\bigcup\limits_{n\in I}\left(2n\pi+\frac{7\pi}{6},\ 2n\pi+\frac{11\pi}{6}\right)$

(c) $\left\{2n\pi+\frac{7\pi}{6}\right\}_{n\in I}$

(d) $\{(4m+1)\frac{\pi}{2} : m \in I\} \bigcup\limits_{n\in I}\left[2n\pi+\frac{7\pi}{6},\ 2n\pi+\frac{11\pi}{6}\right]$

4. If $[x]$ denotes the integral part of x, then the domain of $f(x) = \cos^{-1}(x+[x])$ is

(a) $(0, 1)$ (b) $[0,1)$

(c) $[0, 1]$ (d) $[-1, 1]$

5. If $f(x) = \lim\limits_{t\to\infty} \frac{(1+\sin \pi x)^t - 1}{(1+\sin \pi x)^t + 1}$, then the range $f(x)$ is

(a) $\{-1, 1\}$ (b) $\{0, 1\}$

(c) $\{-1, 1\}$ (d) $\{-1, 0, 1\}$

6. If $f: (0, \pi) \to R$, is defined by $f(x) = \sum\limits_{k=1}^{n}\left[1+\sin\left(\frac{kx}{n}\right)\right]$, where $[x]$ denotes the integral part of x, then the range of $f(x)$ is

(a) $\{n-1, n+1\}$ (b) $\{n-1, n, n+1\}$

(c) $\{n, n+1\}$ (d) none of these

7. Let $f(x) = (x+1)^2 - 1,\ x \ge -1$, then the set $S = \{x : f(x) = f^{-1}(x)\}$ is

(a) $\left\{0, -1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}\right\}$

(b) $\{0, 1, -1\}$

(c) $\{0, -1\}$

(d) empty

8. Range of the function f defined by $f(x) = \left[\frac{1}{\sin\{x\}}\right]$ (where $[.]$ and $\{.\}$ respectively denote the greatest integer and the fractional part functions) is

(a) I, the set of integers

(b) N, the set of natural numbers

(c) W, the set of whole numbers

(d) $\{2, 3, 4, ...\}$

9. A function $F(x)$ satisfies the functional equation $x^2F(x) + F(1-x) = 2x - x^4$ for all real x. $F(x)$ must be

(a) x^2 (b) $1-x^2$

(c) $1+x^2$ (d) x^2+x+1

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	

10. If $2f(x-1)-f\left(\frac{1-x}{x}\right)=x,\ x\neq -1, 0$ then $f(x)$ is

(a) $\frac{1}{3}\left[2(1+x)+\frac{1}{1+x}\right]$ (b) $2\,(x-1)-\frac{1-x}{x}$

(c) $x^2+\frac{1}{x^2}+3$ (d) $\frac{1}{x}+\frac{1-x}{1+x}$

11. If x and y satisfy the equations $y=2\,[x]+3$ and $y=3\,[x-2]$ simultaneously, then $[x+y]$ is (where [ ] represents integral part function)

(a) 21 (b) 9
(c) 30 (d) 12

12. If a,b be two fixed positive integers such that

$f(a+x)=b+[b^3+1-3b^2f(x)+3b\{f(x)\}^2+\{f(x)\}^3]^{1/3}$

for all real x, then $f(x)$ is a periodic function with period

(a) a (b) $2a$
(c) b (d) $2b$

13. If $f:\boldsymbol{R}\to\boldsymbol{R}$ is a function satisfying the property $f(2x+3)+f(2x+7)=2,\ \forall\, x\in\mathbf{R}$, then the period of $f(x)$ is

(a) 2 (b) 4
(c) 8 (d) 12

14. Let $f:R\to\left[0,\ \frac{\pi}{2}\right)$ defined by $f(x)=\tan^{-1}(x^2+x+a)$, then the set of values of a for which f is onto is

(a) $[0,\infty)$ (b) $[1, 2]$

(c) $\left[\frac{1}{4},\infty\right)$ (d) none of these

15. If $f(x)=\sin^2 x+\sin^2\left(x+\frac{\pi}{3}\right)+\cos x\cos\left(x+\frac{\pi}{3}\right)$

and $g\left(\frac{5}{4}\right)=1$, then graph of $y=g\,[f(x)]$ is

(a) a circle (b) a straight line
(c) a parabola (d) none of these

16. Function $f:(-\infty,-1]\to(0,e^5]$ defined by

$f(x)=e^{x^3-3x+2}$ is

(a) many one and onto (b) Many one and into
(c) one one and onto (d) one one and into

17. If $f:R\to[0,\infty)$ is a function such that

$f(x-1)+f(x+1)=\sqrt{3}f(x)$, then period of $f(x)$ is

(a) 2 (b) 6
(c) 12 (d) none of these

18. If $x^2+y^2=1$, then minimum and maximum values of $x+y$ are respectively

(a) $-\sqrt{2},\sqrt{2}$ (b) $-1, 1$

(c) $-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}$ (d) $-\frac{1}{\sqrt{2}},\sqrt{2}$

19. The number of pairs $(x,\ y),\ x,\ y\in R,$ satisfying

$4x^2-4x+2=\sin^2 y$ and $x^2+y^2\leq 3$ are

(a) 0 (b) 4
(c) 2 (d) infinite

20. The maximum value of x^2y, subject to constraints

$x+y+\sqrt{2x^2+2xy+3y^2}=k$ (constant), $x,y\geq 0$ is

(a) $\frac{k^2}{(2+\sqrt{15})^2}$ (b) $\frac{4k^3}{(3+\sqrt{15})^3}$

(c) $\frac{4k^3+k^2}{(3+\sqrt{15})^3}$ (d) none of these

21. If $F(n+1)=\frac{2F(n)+1}{2};n=1,2,......$ and $F(1)=1$ then $F(2009)$ equals

(a) 1005 (b) 2009
(c) 2010 (d) 1006

22. Let f: $\mathbf{R}\to\mathbf{R}$ be a periodic function such that

$f(T+x)=1+[1-3f(x)+3f(x)+3(f(x))^2-(f(x))^3]^{1/3}$

where T is a fixed positive number, then period of $f(x)$ is

(a) T (b) $2T$
(c) $3T$ (d) none of these

23. If $f\left(2x+\frac{y}{8},2x-\frac{y}{8}\right)=xy$, then $f(m,n)+f(n,m)=0$

(a) only when $m=n$ (b) only when $m\neq n$
(c) only when $m=-n$ (d) for all m and n

MARK YOUR RESPONSE					
	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ
	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ
	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	

24. If $f(x+y)=f(x)+f(y)-xy-1\ \forall\ x,y\in R$ and $f(1)=1$, then the number of solutions of $f(n)=n, n\in N$ is

(a) 0 (b) 1
(b) 2 (d) more than 2

25. If $[x]$ denotes the integral part of x and $k=\sin^{-1}\frac{1+t^2}{2t}>0$, then the integral value of α for which the equation $(x-[k])(x+\alpha)-1=0$ has integral root is

(a) -1 (b) 1
(c) 2 (d) none of these

26. If $f(x)=\underset{n\to\infty}{\text{Lt}}\left[\frac{x}{x+1}+\frac{x}{(x+1)(2x+1)}+\frac{x}{(2x+1)(3x+1)}+.....+\text{ to } n \text{ terms}\right]$, then range of $f(x)$ is

(a) $\{0,1\}$ (b) $\{-1,0\}$
(c) $\{-1,1\}$ (d) $[-1,1]$

27. If for $x>0, f(x)=(a-x^n)^{1/n}$,

$g(x)=x^2+px+q; p,q\in R$

and the equation $g(x)-x=0$ has imaginary roots, then number of real roots of equation $g(g(x))-f(f(x))=0$ is

(a) 0 (b) 2
(c) 4 (d) n

28. Let $f(x)=ln\left(\frac{1-x}{1+x}\right)$. The set of values of '$\alpha$' for which

$f(\alpha)+f(\alpha^2)=f\left(\frac{\alpha}{\alpha^2-\alpha+1}\right)$ is satisfied are

(a) $(-\infty,-1)\cup(1,\infty)$ (b) $(-1,1)$
(c) $(0,1)$ (d) none of these

29. Period of the function $f(x)=\cos 2\pi\{2x\}+\sin 2\pi\{2x\}$ is (where $\{.\}$ denotes the fractional part of x)

(a) 1 (b) $\frac{\pi}{2}$
(c) $\frac{1}{2}$ (d) π

30. Let $f(x)=x^3+x^2+100x+7\sin x$, then the equation

$\frac{1}{y-f(1)}+\frac{2}{y-f(2)}+\frac{3}{y-f(3)}=0$ in variable y, has

(a) no real root (b) one real root
(c) two real roots (d) more than two real roots

31. Consider a real valued function $f(x)$ satisfying $2f(xy)=(f(x))^y+(f(y))^x\ \forall x,y\in R$ and $f(1)=a$ where $a\neq 1$, then $(a-1)\sum_{i=l}^{n}f(i)$ equals

(a) a^n (b) a^{n+1}
(b) $a^{n-1}+a$ (d) $a^{n+1}-a$

32. If $f(2x+3y, 2x-7y)=20x$, then $f(x,y)$ can be equal to

(a) $7x-3y$ (b) $7x+3y$
(c) $3x-7y$ (d) $x-y$

33. Let $f: R\to R$ and $g: R\to R$ be two one-one and onto functions such that they are the mirror images of each other about the line $y=a$. If h $(x)=f(x)+g(x)$, then $h(x)$ is

(a) one-one onto (b) one-one into
(c) many-one onto (d) many-one into

34. Let $f(x)$ be a polynomial of degree n, an odd positive integer, and has monotonic behaviour then number of real roots of the equation

$f(x)+f(2x)+f(3x)+......+f(nx)=\frac{1}{2}n(n+1)$ is eqaual to

(a) at least one (b) exactly one
(c) atmost one (d) none of these

35. If $f(x)$ is an even function and satisfies the relation $x^2f(x)-2f\left(\frac{1}{x}\right)=g(x)$ where $g(x)$ is an odd function, then $f(5)$ equals

(a) 0 (b) $\frac{2}{3}$
(c) $\frac{49}{75}$ (d) none of these

36. The function $f(x)$ is defined for all real x.

If $f(a+b)=f(ab)\ \forall\ a$ and b and $f\left(-\frac{1}{2}\right)=-\frac{1}{2}$,

then $f(2009)$ equals

(a) -2009 (b) 2009
(c) $-\frac{1}{2}$ (d) $-\frac{2009}{2}$

MARK YOUR RESPONSE					
	24. ⓐⓑⓒⓓ	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ
	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ	31. ⓐⓑⓒⓓ	32. ⓐⓑⓒⓓ	33. ⓐⓑⓒⓓ
	34. ⓐⓑⓒⓓ	35. ⓐⓑⓒⓓ	36. ⓐⓑⓒⓓ		

37. The range of the function

$f(x)=\sin^{-1}\left[x^2+\frac{1}{2}\right]+\cos^{-1}\left[x^2-\frac{1}{2}\right]$, where [.] is the greatest integer function, is

(a) $\left\{\frac{\pi}{2},\pi\right\}$ (b) $\left\{0,-\frac{1}{2}\right\}$

(c) $\{\pi\}$ (d) $\left(0,\frac{\pi}{2}\right)$

38. If $[2\sin x]+[\cos x]=-3$, then range of the function $f(x)=\sin x+\sqrt{3}\cos x$ in $[0,2\pi]$ is (where [.] denotes the greatest integer function)

(a) $[-2,-1)$ (b) $(-2,-1)$

(c) $\left(-1,-\frac{1}{2}\right)$ (d) none of these

39. The number of solutions of $\tan x-mx=0,\ m>1$ in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ is

(a) 1 (b) 2

(c) 3 (d) m

40. For $n=1,2,3....$ the value of

$\left[\frac{n+1}{2}\right]+\left[\frac{n+2}{4}\right]+\left[\frac{n+4}{8}\right]+\left[\frac{n+8}{16}\right]+.........=$

(where [.] denotes the greatest integer function)

(a) $n-1$ (b) n

(c) $n+2$ (d) $2n$

41. Let $f(x)$ be a function defined by

$f(x)=\int_1^x t(t^2-3t+2)\,dt,\ 1\le x\le 3$. Then the range of $f(x)$ is

(a) $[0,2]$ (b) $\left[-\frac{1}{4},4\right]$

(c) $\left[-\frac{1}{4},2\right]$ (d) none of these

42. Let $f(x)=\sqrt{[\sin 2x]-[\cos 2x]}$ (where [.] denotes the greatest integer function), then range of $f(x)$ will be

(a) $\{0\}$ (b) $\{1\}$

(c) $\{0,1\}$ (d) $\{0,1,\sqrt{2}\}$

43. The number of roots of the equation $x\sin x=1,\ x\in[-2\pi,0)\cup(0,2\pi]$, is

(a) 2 (b) 3

(c) 4 (d) 0

44. The number of points (x, y), where the curves $|y|=ln|x|$ and $(x-1)^2+y^2-4=0$ cut each other, is

(a) 2 (b) 3

(c) 1 (d) 6

45. Let α,β and γ be the roots of $f(x)=x^3+x^2-5x-1=0$. Then $[\alpha]+[\beta]+[\gamma]$, where [.] denotes the greatest integer function, is equal to

(a) 1 (b) -2

(c) -4 (d) -3

46. The value of 'a' for which $x^3-3x+a=0$ would have two distinct roots in (0, 1) is :

(a) 2 (b) -2

(c) 4 (d) none of these

47. The number of solutions of $2\cos x=|\sin x|, 0\le x\le 4\pi$ is

(a) 0 (b) 2

(c) 4 (d) infinite

48. The number of solutions of equation $2^{\cos x}=|\sin x|$, when $x\in[0,2\pi]$ is

(a) 4 (b) 3

(c) 0 (d) 2

49. The number of solutions of the equation $\sin\pi x=|\log|x||$ is

(a) infinite (b) 8

(c) 6 (d) 0

50. If x and y satisfy the equations max $(|x+y|,|x-y|)=1$ and $|y|=x-[x]$, then the number of ordered pairs (x, y) is

(a) 0 (b) 4

(c) 8 (d) infinite

51. The range of the function $f(x)$ $=\sin^{-1}(\log[x])+\log(\sin^{-1}[x])$; (where [.] denotes the greatest integer function) is

(a) R (b) $[1,2)$

(c) $\left\{\log\frac{\pi}{2}\right\}$ (d) $\{-\sin 1\}$

MARK YOUR RESPONSE					
	37. ⓐⓑⓒⓓ	38. ⓐⓑⓒⓓ	39. ⓐⓑⓒⓓ	40. ⓐⓑⓒⓓ	41. ⓐⓑⓒⓓ
	42. ⓐⓑⓒⓓ	43. ⓐⓑⓒⓓ	44. ⓐⓑⓒⓓ	45. ⓐⓑⓒⓓ	46. ⓐⓑⓒⓓ
	47. ⓐⓑⓒⓓ	48. ⓐⓑⓒⓓ	49. ⓐⓑⓒⓓ	50. ⓐⓑⓒⓓ	51. ⓐⓑⓒⓓ

52. If $f(x+y)=f(x).f(y)$ for all real x,y and $f(0)\neq 0$ then the function $g(x)=\dfrac{f(x)}{1+\{f(x)\}^2}$ is

(a) even function (b) odd function
(c) odd if $f(x)>0$ (d) neither even nor odd

53. If the function f satisfies the relation
$f(x+y)+f(x-y)=2f(x)f(y)\ \forall\ x,y\in R$ and $f(0)\neq 0$,
then $f(x)$ is

(a) even function (b) odd function
(c) constant function (d) neither even nor odd

54. Let $g: R\to R$ be given by $g(x)=3+4x$.

If $g^n(x)=gogo.......og(x)$, then $g^{-n}(x)=$

(where $g^{-n}(x)$ denotes inverse of $g^n(x)$)

(a) $(4^n-1)+4^n x$ (b) $(x+1)4^{-n}-1$
(c) $(x+1)4^n-1$ (d) $(4^{-n}-1)x+4^n$

55. Let $f: R-\{2\}\to R$ be a function satisfying

$2f(x)+3f\left(\dfrac{2x+29}{x-2}\right)=100x+80\ \forall\ x\in R-\{2\}$,

then $f(x)=$

(a) $16-40x+\dfrac{60(2x+29)}{x-2}$

(b) $100x+80-\dfrac{3(2x+29)}{x-2}$

(c) $40-16x+\dfrac{30(2x+29)}{x-2}$

(d) none of these

56. If $\sum_{k=0}^{n} f(x+ka)=0$, where $a>0$, then the period of $f(x)$ is

(a) a (b) $(n+1)a$
(c) $\dfrac{a}{n+1}$ (d) $f(x)$ is non-periodic

57. Let $A=\{1,2,3,4,5\}$. If 'f' be a bijective function from A to A, then the number of such functions for which $f(k)\neq k$, $k=1,2,3,4,5$, is

(a) 5^5 (b) 120
(c) 44 (d) 5^5-120

58. If $f(x)=x^3+3x^2+4x+a\sin x+b\cos x\ \forall\ x\in R$ is a one-one function then the greatest value of a^2+b^2 is

(a) 1 (b) 2
(c) $\sqrt{2}$ (d) None

59. If $2<x^2<3$ then the number of positive roots of $\left\{\dfrac{1}{x}\right\}=\{x^2\}$, where $\{x\}$ denotes the fractional part of x, is

(a) 0 (b) 1
(c) 2 (d) 3

60. If $f(x)$ is a periodic function having period 7 and $g(x)$ is periodic having period 11 then the period of

$$D(x)=\begin{vmatrix} f(x) & f\left(\dfrac{x}{3}\right) \\ g(x) & g\left(\dfrac{x}{5}\right)\end{vmatrix}$$ is

(a) 77 (b) 231
(c) 385 (d) 1155

61. If $f(x+1)+f(x-1)=2f(x)\ \forall x\in R$ and $f(0)=0$ and $f(n)$, $n\in N$ is

(a) $nf(1)$ (b) $\{f(1)\}^n$
(c) 0 (d) n

62. Let $f(x)$ and $g(x)$ be bijective functions where f: $\{a,b,c,d\}\to\{1,2,3,4\}$ and $g:\{3,4,5,6\}\to\{w,x,y,z\}$ respectively. The number of elements in the range set of $g(f(x))$ are

(a) 1 (b) 2
(c) 3 (d) 4

63. If $g: D\to R$ be a function such that $g(x)=\underbrace{\ln\ln\ln\ln......\ln}_{\text{n times}}$ $[4(x^2+x+1)+\sin(\pi x)]$.
$(n\in N)$, then the least value of n for which g becomes onto, is

(a) 1 (b) 2
(c) 3 (d) 4

64. If $[f(x)]^2.f\left(\dfrac{1-x}{1+x}\right)=64x\quad\forall x\in R-\{-1,0,1\}$ then $f(x)$ is

(a) $4\left[x^2\left(\dfrac{1+x}{1-x}\right)\right]^{1/3}$ (b) $4\left[x^2\left(\dfrac{1-x}{1+x}\right)\right]^{1/3}$

(c) $4\left[x\left(\dfrac{1+x}{1-x}\right)\right]^{1/3}$ (d) $4\left[x\left(\dfrac{1-x}{1+x}\right)\right]^{1/3}$

MARK YOUR RESPONSE					
	52. ⓐⓑⓒⓓ	53. ⓐⓑⓒⓓ	54. ⓐⓑⓒⓓ	55. ⓐⓑⓒⓓ	56. ⓐⓑⓒⓓ
	57. ⓐⓑⓒⓓ	58. ⓐⓑⓒⓓ	59. ⓐⓑⓒⓓ	60. ⓐⓑⓒⓓ	61. ⓐⓑⓒⓓ
	62. ⓐⓑⓒⓓ	63. ⓐⓑⓒⓓ	64. ⓐⓑⓒⓓ		

65. If $f(x)$ is a polynomial of degree n such that $f(0) = 0, f(1) = \frac{1}{2}, \ldots\ldots, f(n) = \frac{n}{n+1}$, then $f(n+1)$ is

(a) 1 if n is odd (b) $\frac{n}{n+2}$ if n is even

(c) $\frac{n+1}{n+2}$ (d) none of these

It is odd and $\frac{n}{n+1}$, if n is even.

66. The domain of definition of the function

$$f(x) = \sqrt{\ln_{(|x|-1)}(x^2+4x+4)}$$

(a) $[-3,-1] \cup [1,2]$
(b) $(-2,-1) \cup [2, \infty)$
(c) $(-\infty,-3] \cup (-2,-1) \cup (2, \infty)$
(d) none of these

67. If $f: R \to R, g: R \to R$ be two given functions, then $|f(x)+g(x)|+|f(x)-g(x)|$ is equal to

(a) $2 \max \{f(x), g(x)\}$ (b) $2 \max \{|f(x)|, |g(x)|\}$
(c) $2 \min \{f(x), g(x)\}$ (d) $2 \min \{|f(x)|, |g(x)|\}$

68. If $f(x)$ is continuous such that $|f(x)| \le 1 \ \forall x \in R$ and $g(x) = \frac{e^{f(x)} - e^{|f(x)|}}{e^{f(x)} + e^{|f(x)|}}$, then range of g (x) is

(a) $[0, 1]$ (b) $\left[0, \frac{e^2+1}{e^2-1}\right]$

(c) $\left[0, \frac{e^2-1}{e^2+1}\right]$ (d) $\left[\frac{1-e^2}{1+e^2}, 0\right]$

69. If the graph of the functions $y = \ell n$ x and $y = ax$ intersect at exactly two points, then a must belong to

(a) $(0, e)$ (b) $\left(\frac{1}{e}, 0\right)$

(c) $\left(0, \frac{1}{e}\right)$ (d) none of these

MARK YOUR RESPONSE	65. ⓐⓑⓒⓓ	66. ⓐⓑⓒⓓ	67. ⓐⓑⓒⓓ	68. ⓐⓑⓒⓓ	69. ⓐⓑⓒⓓ

B COMPREHENSION TYPE

This section contains groups of questions. Each group is followed by some multiple choice questions based on a paragraph. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

PASSAGE-1

A functional equation is an equation, which relates the values assumed by a function at two or more points, which are themselves related in a particular manner. For example, we define an odd function by the relation $f(-x) = -f(x)$ for all x. The definition can be paraphrased to say that it is a function $f(x)$, which satisfies the functional relation $f(x) + f(y) = 0$, whenever $x + y = 0$. Of course, this does not identify the function uniquely, sometimes with some additional information, a function satisfying a given functional equation can be identified uniquely.
Suppose a funcional equation has a relation between $f(x)$ and $f\left(\frac{1}{x}\right)$, then due to the reason that reciprocal of a reciprocal gives back the original number, we can substitute $\frac{1}{x}$ for x. This will result into another equation and solving these two, we can find $f(x)$ uniquely. Similarly, we can solve an equation, which contains $f(x)$ and $f(-x)$. Such equations are of repetitive nature.

1. Suppose that for every $x \ne 0$, $af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5$, where $a \ne b$, then the value of the integral $\int_1^2 f(x)dx$ is

(a) $\frac{2a\ \ell n\ 2 - 10a + 7b}{2(a^2-b^2)}$ (b) $\frac{\pi}{2\sqrt{2}}$

(c) $\frac{7a + 10b - 2a\ \ell n\ 2}{2(a^2-b^2)}$ (d) none of these

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ				

2. If for every $x \in R$, the function $f(x)$ satisfies the relation $a f(x) + bf(-x) = g(x)$, then

(a) $f(x)$ can be uniquely determined if $ag(x) - bg(-x) \neq 0$ and $a = \pm b$

(b) $f(x)$ can have inifinitely many values if $ag(x) - bg(-x) = 0$ and $a = \pm b$

(c) $f(x)$ cannot be determined if $a \neq \pm b$

(d) all of the above

3. The function $f(x)$ is an even function and satisfies $x^2 f(x) - 2f\left(\frac{1}{x}\right) = g(x)$, where $g(x)$ is an odd function.

Then the value of $f(5)$ is

(a) -2 (b) 4

(c) 0 (d) depends on $g(x)$

4. Suppose $f(z)$ is a possibly complex valued function of a complex number z, which satisfies a functional equation of the form $af(z) + bf(w^2z) = g(z)$ for all $z \in C$, where a a and b are some fixed complex numbers and $g(z)$ is some function of z and w is cube root of unity ($w \neq 1$), then $f(z)$ can be determined uniquely if

(a) $a + b = 0$ (b) $a^2 + b^2 \neq 0$

(c) $a^3 + b^3 \neq 0$ (d) $a^3 + b^3 = 0$

PASSAGE-2

Let $f(x)$ and $g(x)$ are two distinct continuous functions such that $f(x)$ is an odd function and $g(x)$ is an even function $\forall x \in R$ and $f'(x) > g'(x)$ $\forall x \in R$. Let a function $h(x) = f(x) + g(x)$ is an odd function and $\phi(x) = f(g(x)) + g(f(x))$.

5. Function $g(x)$ is

(a) a trigonometrical function

(b) a polynomial function

(c) an absolute value function

(d) a constant function

6. The number of solutions of $f(x) = g(x)$ is

(a) 1 (b) 2

(c) 0 (d) 3

7. The number of solutions of $\phi(x) = h(x)$ is

(a) 2 (b) 1

(c) 0 (d) 3

PASSAGE-3

Let $f(x)$ be a real valued function satisfying the functional equation $f(x) + f(1-x) = k$ for all $x \in Q$, where k is a contant quantity. Let m be a positive integer. Put $x = \frac{r}{m+1}$ in the given equation, we get $f\left(\frac{r}{m+1}\right) + f\left(\frac{m+1-r}{m+1}\right) = k$

$$\Rightarrow \sum_{r=1}^{m} f\left(\frac{r}{m+1}\right) + \sum_{r=1}^{m} f\left(\frac{m+1-r}{m+1}\right) = mk$$

$$\Rightarrow \sum_{r=1}^{m} f\left(\frac{r}{m+1}\right) + \sum_{t=1}^{m} f\left(\frac{t}{m+1}\right) = mk \text{ (putting } m+1-r=t)$$

$$\therefore 2\sum_{r=1}^{m} f\left(\frac{r}{m+1}\right) = mk \Rightarrow \sum_{r=1}^{m} f\left(\frac{r}{m+1}\right) = \frac{mk}{2}$$

8. If $f(x) = \frac{4^x}{4^x + 2}$, where $x \in Q$, then $f\left(\frac{1}{2009}\right) + f\left(\frac{2}{2009}\right) + \ldots\ldots + f\left(\frac{2008}{2009}\right)$ equals to

(a) 1004 (b) 2008

(c) 2009 (d) none of these

9. If $f(x) = \frac{3^{x-3}}{3^{1-x} + 3^x}$ for all $x \in Q$, then the value of the sum $f\left(\frac{1}{55}\right) + f\left(\frac{2}{55}\right) + \ldots\ldots + f\left(\frac{54}{55}\right)$ is

(a) 1 (b) 27

(c) 54 (d) 55

10. If $f(x) = \frac{a^x}{a^x + \sqrt{a}} (a > 0)$, then $\sum_{r=1}^{2n-1} 2f\left(\frac{r}{2n}\right)$ is equal to

(a) 1 (b) $2n$

(c) $2n-1$ (d) $\frac{(2n-1)a}{2}$

PASSAGE-4

Let $x \in R$ be any real number such that x lies between any two consecutive integers say $n - 1$ and n, i.e., $n-1 < x \leq n$ then we can always find this unique integer n.

Let us call this n as super integral value of x.

We denote it symbolically as (x).

For example : if $x = 2.63$, then $(x) = 3$, if $x = -2.63$, then $(x) = -2$

MARK YOUR RESPONSE					
	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ
	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	

11. The range of function $y = \frac{(x)}{x}$ if $x \in (-\infty, 0)$, is

(a) $(-\infty, 0]$ (b) $[-1, 0]$

(c) $[0, 1]$ (d) $[-1, 1]$

12. If $-2 < x \le -1, -1 < y \le 0, 0 < z \le 1$ then the value of the determinant $\begin{vmatrix} (x)+1 & (y) & (z) \\ (x) & (y)+1 & (z) \\ (x) & (y) & (z)+1 \end{vmatrix}$ is

(a) 0 (b) 1

(c) 2 (d) –1

13. If $f(x) = \cos(\pi^2)x + \cos(-\pi^2)x$ then the value of $f\left(\frac{\pi}{4}\right) + f\left(\frac{\pi}{2}\right)$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $1 + \frac{1}{\sqrt{2}}$

(c) $1 - \frac{1}{\sqrt{2}}$ (d) $-1 + \frac{1}{\sqrt{2}}$

Let $y = f(x)$ is a parabola whose vertex is at $\left(\frac{3}{2}, -\frac{1}{4}\right)$, the length of latus rectum is 1 and axis is parallel to positive direction of y-axis

Let $g(x) = f(|x|), h(x) = |g(x)|$

14. The number of real roots of equation $g(x) = 0$ is

(a) 2 (b) 3

(c) 4 (d) 6

15. If $g(x) + a = 0$ has exactly two roots, then a belongs to

(a) $(0, 2)$ (b) $(-\infty, -2)$

(c) $(2, \infty)$ (d) $(-1, 2)$

16. The number of solutions of $h'(x) = 0$ is

(a) one (b) two

(c) three (d) four

Mark Your Response	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ
	16. ⓐⓑⓒⓓ				

C Reasoning Type

In the following questions two Statements (1 and 2) are provided. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct. Mark your responses from the following options:

(a) Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.

(b) Both Statement-1 and Statement-2 are true and Statement-2 is not the correct explanation of Statement-1.

(c) Statement-1 is true but Statement-2 is false.

(d) Statement-1 is false but Statement-2 is true.

1. Statement-1 : The period of $f(x) = \sin 2x \cos[2x] - \cos 2x \sin[2x]$ is $\frac{1}{2}$

Statement-2 : The period of $x - [x]$ is 1

2. Statement-1 : If $f(x) = |x-1| + |x-2| + |x-3|$ where $2 < x < 3$ is an identity function.

Statement-2 : $f : A \longrightarrow A$ defined by $f(x) = x \ \forall\ x \in A$ is an identity function.

3. Statement-1 : $f : R \to R$ defined by $f(x) = \sin x$ is a bijection.

Statement-2 : If f is both one-one and onto then it is bijection.

4. Statement-1 : $f : R \to R$ is a function defined by $f(x) = \frac{2x+1}{3}$. Then $f^{-1}(x) = \frac{3x-1}{2}$

Statement-2 : $f(x)$ is not a bijection.

5. Statement-1 : If f is even function, g is odd function then $\frac{f}{g}$, $(g \neq 0)$ is an odd function

Statement-2 : If $f(-x) = -f(x)$ for every x of its domain, then $f(x)$ is called an odd function and if $f(-x) = f(x)$ for every x of its domain, then $f(x)$ is called an even function.

Mark Your Response	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ

6. **Statement-1** : $f : A \to B$ and $g : B \to C$ are two functions then $(gof)^{-1} = f^{-1} o g^{-1}$

Statement-2 : $f : A \to B$ and $g : B \to C$ are bijections then f^{-1}, g^{-1} are also bijections.

7. **Statement-1** : Let $A = \{x / -1 \le x \le 1\}$, then $f : A \to A$ is defined by $f(x) = \sin \pi x$, $\forall x \in A$ is onto but not one – one

Statement-2 : If range set is equal to the codomain set then the function is onto.

MARK YOUR RESPONSE	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ			

D MULTIPLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

1. Let $f : \left[-\frac{\pi}{3}, \frac{2\pi}{3}\right] \longrightarrow [0, 4]$ be a function defined as $f(x) = \sqrt{3}\sin x - \cos x + 2$. then $f^{-1}(x)$ is given by

(a) $\sin^{-1}\left(\frac{x-2}{2}\right) - \frac{\pi}{6}$ (b) $\sin^{-1}\left(\frac{x-2}{2}\right) + \frac{\pi}{6}$

(c) $\frac{2\pi}{3} - \cos^{-1}\left(\frac{x-2}{2}\right)$ (d) none of these

2. Let $R = \left\{(x, y) : x, y \in R, x^2 + y^2 \le 25\right\}$ and $R' = \left\{(x, y) : x, y \in R,\ y \ge \frac{4}{9}x^2\right\}$ then

(a) dom $R \cap R' = [-3, 3]$

(b) Range $R \cap R' \supset [0, 4]$

(c) Range $R \cap R' = [0, 5]$

(d) $R \cap R'$ defines a function

3. Let $f(x) = \max\{1 + \sin x, 1, 1 - \cos x\}$, $x \in [0, 2\pi]$ and $g(x) = \max\{1, |x - 1|\}$ $x \in R$, then

(a) $g(f(0)) = 1$ (b) $g(f(1)) = 1$

(c) $f(g(1)) = 1$ (d) $f(g(0)) = 1 + \sin 1$

4. Let n be a positive integer with $f(n) = 1! + 2! + 3! + ... + n!$ and $P(x)$ and $Q(x)$ be polynomials in x such that $f(n+2) = P(n) f(n+1) + Q(n) f(n)$ for all $n \ge 1$, then

(a) $P(x) = x + 3$ (b) $Q(x) = -x - 2$

(c) $P(x) = -x - 2$ (d) $Q(x) = x + 3$

5. Let $f(x) = \frac{e^{-x}}{1 + [x]}$, then

(a) Domain of definition of $f(x)$ is $R - [-1, 0)$

(b) Range of definition of $f(x)$ is $\left(-\frac{1}{e}, 1\right]$

(c) $f(x)$ is one-one in its deomain

(d) $f(x)$ is discontinuous at infinitely many points

6. A function $f : R \to R$ is defined by $f(x + y) - kxy = f(x) + 2y^2$ $\forall$ $x, y \in R$ and $f(1) = 2; f(2) = 8$, where k is some contant, then $f(x + y).f\left(\frac{1}{x + y}\right)$ is equal to (where $x + y \ne 0$).

(a) 1 (b) 4

(c) k (d) $f(1)$

7. If the range of the function $y = \frac{x - 1}{a - x^2 + 1}$ does not contain any values belonging to the interval $\left[-1, -\frac{1}{3}\right]$ then the integral value (s) of a can be

(a) –5 (b) –1

(c) 0 (d) 1

8. If $f(x) = 1 + x^2$ and $f[g(x)] = 1 + x^2 - 2x^3 + x^4$, $g(2) = 2$, then

(a) $g(0) = 0$ (b) $g(2) = 2$

(c) $g(x) = x$ (d) $g(x)$ is one-one

9. Let $f : D \to$ R be defined by $f(x) = ln(ln(ln x)))$ then

(a) $f(x)$ is into (b) $f(x)$ is one-one

(c) $f(x)$ is onto (d) $D = (e^e, \infty)$

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	

10. Let $f : X \to Y$ be defined as $f(x) = \sqrt{|x| - \{x\}}$, where $\{\bullet\}$ denotes the functional part of x, then

(a) $X = \left(-\infty, -\frac{1}{2}\right] \cup [0, \infty)$

(b) $Y = \left[\frac{1}{2}, \infty\right)$

(c) $f(x)$ is not one-one

(d) $Y \subseteq [0, \infty)$

11. π is the FUNDAMENTAL period of

(a) $\dfrac{1+\sin x}{\cos x(1+\text{cosec}x)}$ (b) $|\sin x| + |\cos x|$

(c) $\sin 2x + \cos 2x$ (d) $\cos(\sin x) + \cos(\cos x)$

12. The function $f(x) = \dfrac{|x-1|}{x^2}$ is

(a) one-one in $(2, \infty)$

(b) one-one in $(0, 1)$

(c) one-one in $(-\infty, 0)$

(d) one-one in $(1, 2)$

13. The value(s) of a, for which $\dfrac{x^2 - 6x^2 + 11x - 6}{x^3 + x^2 - 10x + 8} + \dfrac{a}{30} = 0$ does not have real solution is /are

(a) -10 (b) 12

(c) 5 (d) -30

MARK YOUR RESPONSE	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	

E MATRIX-MATCH TYPE

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labeled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column -I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A–p, s and t; B–q and r; C–p and q; and D–s and t; then the correct darkening of bubbles will look like the given.

	p	q	r	s	t
A	●	○	○	●	●
B	○	●	●	○	○
C	●	●	○	○	○
D	○	○	○	●	●

1. **Observe the following columns :**

Column-I	**Column-II**
(A) Domain of $\sin^{-1}(5x)$	p. 1
(B) Range of $\sqrt{1-25x^2}$	q. $[0, 1]$
(C) If $f(x) = \dfrac{x+1}{x-1}, x \neq 1$ then $(f \circ f)(x)$ can assume value(s)	r. $\left[-\frac{1}{5}, \frac{1}{5}\right]$
(D) Period of $x - [x]$ is	s. $\left[0, \frac{1}{5}\right]$

MARK YOUR RESPONSE

1.

	p	q	r	s
A	ⓟ	ⓠ	ⓡ	ⓢ
B	ⓟ	ⓠ	ⓡ	ⓢ
C	ⓟ	ⓠ	ⓡ	ⓢ
D	ⓟ	ⓠ	ⓡ	ⓢ

2. **Observe the following lists :**

Column-I		Column-II
(A) Range of $\sin x - \cos x$	p.	$\left(-\infty, \frac{1}{3}\right) \cup (3, \infty)$
(B) Range of $\frac{1}{2-\cos 3x}$	q.	$\left[\frac{-3\pi}{2}, \frac{\pi}{2}\right]$
(C) Range of $\tan^{-1} x - \cot^{-1} x$	r.	$\left[-\sqrt{2}, \sqrt{2}\right]$
(D) Range of $\frac{\sin x \cos 3x}{\sin 3x \cos x}$ is	s.	$\left[\frac{1}{3}, 1\right]$
	t.	$\left(\frac{-3\pi}{2}, \frac{\pi}{2}\right)$

3. **Observe the following columns :**

Column-I		Column-II
(A) The integral value of $x \in (-\pi, \pi)$ satisfying the equation $\left|x^2 - 1 + \cos x\right| = \left|x^2 - 1\right| + \left|\cos x\right|$ can be	p.	0
(B) The number of solutions of $[x]^2 = x + 2\{x\}$ is equal to	q.	1
(C) If $f(x) = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x$, then $[f(x)]$ can be equal to	r.	2
(D) An allowable value of $f(x) = \sqrt{\ln(\cos(\sin x))}$ can be	s.	4
	t.	–1

([.] and {.} represent integral and fractional parts respectively)

4. **Observe the following columns :**

Column-I		Column-II
(A) Let $f: R \to R$ satisfies $f(x+y) + f(x-y) = 2f(x)f(y)\ \forall\ x, y \in$ R and $f(0) \neq 0$, then $f(x)$ is	p.	Even
(B) Let $f: R \to R$ is defined by $f(x) = \frac{e^{\|x\|} - e^{-x}}{e^x + e^{-x}}$, then $f(x)$ is	q.	Odd
(C) Let $f: R \to R$ be a polynomial function satisfying $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ and $f(3) = 28$ then $f(x)$ is	r.	Into
(D) Let $f: R \to R$ is defined by $f(x) = 2x + \sin x$, then $f(x)$ is	s.	Many-one
	t.	Bijective

MARK YOUR RESPONSE

2.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

3.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

4.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

5. **Match the following columns :**

Column-I	Column-II
(A) Domain of $f(x) = \log_e\{(ax^3 + (a+b)x^2 + (b+c)x + c\}$ if $b^2 - 4ac < 0, a > 0$	p. $\left[-1, \frac{5}{4}\right]$
(B) Domain of $f(x) = \ln \tan^{-1}\{(x^3 - 6x^2 + 11x - 6)\, x\,(e^x - 1)\}$	q. $R - \left\{\frac{1}{5}, 1\right\}$
(C) Range of $f(x) = \dfrac{x^2 - 3x + 2}{x^2 + x - 6}$	r. $(-1, \infty)$
(D) Range of $f(x) = \sin^2 \dfrac{x}{4} + \cos \dfrac{x}{4}$	s. $(1,2) \cup (3, \infty)$

6. **Match the Column- 1 with Column- 1I**

Column-I	Column-II
(A) Range of the function $f(x) = \sin^{-1}(x^2 + 1) + \cos^{-1}(x^2 + 1) + [1 + x^2]^{1/x}$, where $[\bullet]$ is the greatest integer function, contains	p. 2
(B) Period of the function $f(x) = \cot \dfrac{\pi}{2}[x]$ is where $[\bullet]$ is the greatest integer function	q. $-\dfrac{1}{2}$
(C) If $f\left(\dfrac{2\tan x}{1 + \tan^2 x}\right) = \dfrac{(\cos 2x + 1)(\sec^2 + 2\tan x)}{2}$ then $f(4)$ is equal to	r. $\dfrac{\pi}{2} + 1$
(D) Let $f : R \to \left(-\dfrac{\pi}{2}, 0\right]$ given by $f(x) = \tan^{-1}(2a - 2x - x^2)$ is onto then a is equal to	s. π
	t. 5

MARK YOUR RESPONSE

5.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

6.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

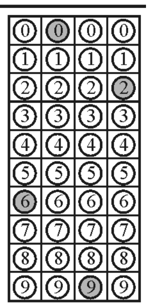
F NUMERIC/INTEGER ANSWER TYPE

The answer to each of the questions is either numeric (eg. 304, 40, 3010 etc.) or a single-digit integer, ranging from 0 to 9.

The appropriate bubbles below the respective question numbers in the response grid have to be darkened.

For example, if the correct answers to a question is 6092, then the correct darkening of bubbles will look like the given.

For single digit integer answer darken the extreme right bubble only.



1. If the function $f(x)=\dfrac{x-1}{c-x^2+1}$ does not take any value in the internal $\left[-1,-\dfrac{1}{3}\right]$, then the largest integral value that c can attain is equal to

2. The number of solutions of the equation $[\sin^{-1}x]=x-[x]$, where [.] denotes the greatest integer function is

3. The largest integral value of n, such that the function $f(x)=\cos(nx)\sin\left(\dfrac{5x}{n}\right)$ has period 3π, is equal to

4. Let $f\left(\dfrac{x+y}{2}\right)=\dfrac{f(x)+f(y)}{2}\ \forall\ x,y\in R$

If $f'(0)$ exists and equals -1 and $f(0)=1$, then the value of $f(-1)$ is equal to

5. Let $f:[0,1]\to[0,1]$ defined by $f(x)=\dfrac{1-x}{1+x}$, for $0\le x\le 1$ and let $g:[0,1]\to[0,1]$ defined by $g(x)=4x(1-x)$, $0\le x\le 1$. If range of $fog(x)$ is $[\alpha,\beta]$, then $\alpha+\beta=$

6. If $f(x)f(y)+2=f(x)+f(y)+f(xy)\ \forall\ x,y\in R$ and $f(1)=f'(1)=2$, then $f(2)$ is equal to

MARK YOUR RESPONSE

1.	2.	3.	4.	5.	6.
0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0
1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2
3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3
4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4
5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5
6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6
7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7
8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8
9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9

Answerkey

A SINGLE CORRECT CHOICE TYPE

1	(b)	**11**	(c)	**21**	(a)	**31**	(d)	**41**	(c)	**51**	(c)	**61**	(a)
2	(d)	**12**	(b)	**22**	(b)	**32**	(b)	**42**	(c)	**52**	(a)	**62**	(b)
3	(d)	**13**	(c)	**23**	(d)	**33**	(d)	**43**	(c)	**53**	(a)	**63**	(c)
4	(b)	**14**	(d)	**24**	(b)	**34**	(b)	**44**	(b)	**54**	(b)	**64**	(a)
5	(d)	**15**	(b)	**25**	(a)	**35**	(a)	**45**	(d)	**55**	(a)	**65**	(a)
6	(c)	**16**	(d)	**26**	(a)	**36**	(c)	**46**	(d)	**56**	(b)	**66**	(c)
7	(c)	**17**	(c)	**27**	(a)	**37**	(c)	**47**	(c)	**57**	(c)	**67**	(b)
8	(b)	**18**	(a)	**28**	(b)	**38**	(b)	**48**	(a)	**58**	(a)	**68**	(d)
9	(b)	**19**	(c)	**29**	(c)	**39**	(c)	**49**	(c)	**59**	(b)	**69**	(c)
10	(a)	**20**	(b)	**30**	(c)	**40**	(b)	**50**	(d)	**60**	(d)		

B COMPREHENSION TYPE

1	(a)	**5**	(d)	**9**	(a)	**13**	(d)
2	(b)	**6**	(a)	**10**	(c)	**14**	(c)
3	(c)	**7**	(b)	**11**	(c)	**15**	(b)
4	(c)	**8**	(a)	**12**	(b)	**16**	(b)

C REASONING TYPE

1	(a)	**3**	(d)	**5**	(a)	**7**	(a)
2	(a)	**4**	(c)	**6**	(d)		

D MULTIPLE CORRECT CHOICE TYPE

1	(b,c)	**6**	(b, c)	**11**	(a,c)
2	(a, b, c)	**7**	(a, b, c)	**12**	(a, b, c, d)
3	(a,b, d)	**8**	(a, b)	**13**	(b,c,d)
4	(a, b)	**9**	(b, c, d)		
5	(a, d)	**10**	(a, c, d)		

E MATRIX-MATCH TYPE

1. **A-r; B-q; C-r,s; D-p**
2. **A-r; B-s; C-t; D-p**
3. **A-q,t; B-s; C-p,q,r; D-p**
4. **A-p, r, s; B-r,s; C-t; D-q,t**
5. **A-r, B- s; C-q; D-p**
6. **A-r; B-p; C-t; D-q**

F NUMERIC/INTEGER ANSWER TYPE

1	0	**2**	1	**3**	15	**4**	2	**5**	1	**6**	5

Solutions

A SINGLE CORRECT CHOICE TYPE

1. **(b)** By remainder theorem, $P(a) = a$, $P(b) = b$ and $P(c) = c$. Let the required remainder be $R(x)$, then $P(x) = (x-a)(x-b)(x-c)Q(x) + R(x)$, where $R(x)$ is a polynomial of degree at most 2. We get $R(a) = a$, $R(b) = b$ and $R(c) = c$. So, the equation $R(x) - x = 0$ has three roots a, b and c. But its degree is at most 2, So, $R(x) - x$ must be zero polynomial (or identity). Hence, $R(x) = x$

2. **(d)** $h(x) = 2\min\{f(x) - g(x), 0\}$

$$\begin{cases} 0, \text{if } f(x) > g(x) \\ 2\{f(x) - g(x), \quad \text{if } f(x) \le g(x)\} \end{cases}$$

$$\begin{cases} f(x) - g(x) - |f(x) - g(x)|, \text{ if } f(x) \le g(x) \\ f(x) - g(x) - |f(x) - g(x)|, \text{ if } f(x) > g(x) \end{cases}$$

$\therefore\ h(x) = f(x) - g(x) - |f(x) - g(x)|$

3. **(d)** $f(x) = \sqrt{3|x| - x - 2}$ and $g(x) = \sin x$

for $\text{fog}(x) = \sqrt{3|\sin x| - \sin x - 2}$ which is defined if

$3|\sin x| - \sin x - 2 \ge 0$

If $\sin x > 0$ then $2\sin x - 2 \ge 0 \Rightarrow \sin x \ge 1$

$\Rightarrow \sin x = 1 \Rightarrow x = 2n\pi + \dfrac{\pi}{2}$

If $\sin x < 0$ then $-4\sin x - 2 \ge 0$

$\Rightarrow -1 \le \sin x \le -\dfrac{1}{2} \Rightarrow x \in \left[2n\pi + \dfrac{7\pi}{6}, 2n\pi + \dfrac{11\pi}{6}\right]$

$$x \in \left[2n\pi + \frac{7\pi}{6}, 2n\pi + \frac{11\pi}{6}\right] \cup \left\{2m\pi + \frac{\pi}{2}\right\}, n, m \in I$$

4. **(b)** For $f(x)$ to be defined, $-1 \le x + [x] \le 1$ ...(i)

When $x \in I$, Let $x = k$, then from (i), $-1 \le 2k \le 1$

$\Rightarrow -\dfrac{1}{2} \le k \le \dfrac{1}{2} \Rightarrow k = 0 \Rightarrow x = 0.$

When $x \notin I$, let $x = k + f$, where k is the integral part of x and $0 < f < 1$.

From (i), $-1 \le 2k + f \le 1$

$\Rightarrow \dfrac{-1-f}{2} \le k \le \dfrac{1-f}{2}. \Rightarrow k = 0 \Rightarrow 0 < x < 1$

$\left[\because \dfrac{-1-f}{2} > -1 \text{ and } \dfrac{1-f}{2} < \dfrac{1}{2}\right]$

So, all possible values of x are given by $0 \le x < 1$

$\therefore$ Domain of $f = [0, 1)$

5. **(d)** $f(x) = \lim\limits_{t \to \infty} \dfrac{(1 + \sin \pi x)^t - 1}{(1 + \sin \pi x)^t + 1}$

$$= \begin{cases} \dfrac{1 - \dfrac{1}{(1 + \sin \pi x)^t}}{1 + \left(\dfrac{1}{1 + \sin \pi x}\right)} & \sin \pi x > 0 \\ \dfrac{0 - 1}{0 + 1} & \sin \pi x < 0 \\ \dfrac{1 - 1}{1 + 1} & \sin \pi x = 0 \end{cases}$$

$$= \begin{cases} 1, & \sin \pi x > 0 \\ -1, & \sin \pi x < 0 \\ 0, & \sin \pi x = 0 \end{cases}$$

$\therefore$ Range $f = \{-1, 0, 1\}$

6. **(c)** $f(x) = \sum\limits_{k=1}^{n} \left(1 + \left[\sin\left(\dfrac{kx}{n}\right)\right]\right)$

$= n + \left[\sin \dfrac{x}{n}\right] + \left[\sin \dfrac{2x}{n}\right] + \ldots.. + [\sin x]$ ....(i)

Case I : When $x \ne \dfrac{\pi}{2}$

Since $0 < \dfrac{kx}{n} < \pi$ and $\dfrac{kx}{n} \ne \dfrac{\pi}{2}$ $\therefore\ 0 < \sin\left(\dfrac{kx}{n}\right) < 1$, for $k = 1, 2, \ldots., n$

$\therefore \left[\sin\left(\dfrac{kx}{n}\right)\right] = 0$, for $k = 1, 2, 3, \ldots., n$

$\therefore$ From (i), $f(x) = n$

Case II : When $x = \frac{\pi}{2}$

In this case $\sin x = 1$ and others lie between 0 and 1.

$\therefore$ From (i), $f(x) = n + 1$.

Hence range of $f = \{n, n+1\}$.

7. **(c)** Let $f(x) = y \Rightarrow (x+1)^2 - 1 = y \Rightarrow x = -1 + \sqrt{1+y}$

$[\because x \geq -1]$

$\therefore \ f^{-1}(x) = -1 + \sqrt{1+x}$

Now, $f(x) = f^{-1}(x) \Rightarrow (x+1)^2 - 1 = -1 + \sqrt{1+x}$

$\Rightarrow (x+1)^4 = (x+1)$

$\Rightarrow (x+1)[(x+1)^3 - 1] = 0 \Rightarrow x = -1$ or $x + 1 = (1)^{1/3}$

$\Rightarrow x = -1$ or 0 $[\because x \geq -1 \Rightarrow x \in R]$

8. **(b)** $\because \{x\} \in (0,1)$ as $f(x)$ is defined if $\sin\{x\} \neq 0$

$\therefore \ \frac{1}{\sin\{x\}} \in \left(\frac{1}{\sin 1}, \infty\right) \left[\frac{1}{\sin\{x\}}\right] \in \{1, 2, 3 \ldots\ldots\}$

Note that

$1 < \frac{\pi}{3} \Rightarrow \sin 1 < \sin\frac{\pi}{3} = 0.866 \Rightarrow \frac{1}{\sin 1} > 1.155$

9. **(b)** $x^2 F(x) + F(1-x) = 2x - x^4$ .....(1)

Replacing x by $1 - x$, gives

$(1-x)^2 F(1-x) + F(x) = 2(1-x) - (1-x)^4$ .....(2)

Multiplying (1) by $(1-x)^2$ and subtracting (2) from it gives

$\left[x^2(1-x)^2 - 1\right] F(x)$

$= 2x(1-x)^2 - x^4(1-x)^2 - 2(1-x) + (1-x)^4$

$\Rightarrow [x(1-x) - 1][x(1-x) + 1] F(x)$

$= 2(1-x)\,[x(1-x) - 1] - (1-x)^2\left[x^4 - (1-x)^2\right]$

$\Rightarrow [x - 1 - x^2][x + 1 - x^2]\ F(x)$

$= 2(1-x)[x - 1 - x^2] - (1-x)^2(x^2 - 1 + x)(x^2 + 1 - x)$

$\Rightarrow (x + 1 - x^2) F(x) = (1-x)\left[2 + (1-x)(x^2 + x - 1)\right]$

$= (1-x)\left[2 + 2x - x^3 - 1\right]$

$= (1-x)\left[2(1+x) - (x+1)(x^2 - x + 1)\right]$

$= (1 - x^2)\left[1 + x - x^2\right]$

$\therefore F(x) = 1 - x^2$

10. **(a)** $2f(x-1) - f\left(\frac{1-x}{x}\right) = x$ .... (1).

Replace x by $\frac{1}{x}$, we get

$2f\left(\frac{1}{x} - 1\right) - f(x-1) = \frac{1}{x} \Rightarrow 2f\left(\frac{1-x}{x}\right) - f(x-1) = \frac{1}{x}$

Eliminate $f\left(\frac{1-x}{x}\right)$ from (1) and (2), we get

$f(x-1) = \frac{1}{3}\left(2x + \frac{1}{x}\right)$. Replace $x - 1$ by x,

$f(x) = \frac{1}{3}\left[2(1+x) + \frac{1}{1+x}\right]$

11. **(c)** $y = 2[x] + 3 = 3[x-2]$'

$\Rightarrow 2[x] + 3 = 3[x] - 6 \Rightarrow [x] = 9$

$\therefore \ y = 2[x] + 3 = 21$

$\therefore \ [x+y] = [x+21] = [x] + 21 = 9 + 21 = 30$

12. **(b)** $f(a+x) = b + [1 + b^3 - 3b^2 f(x)$

$+ 3b\{f(x)\}^2 - \{f(x)\}^3]^{1/3}$

$= b + [1 + \{b - f(x)\}^3]^{1/3}$

$\Rightarrow f(a+x) - b = [1 - \{f(x) - b\}^3]^{1/3}$

$\Rightarrow \phi(a+x)] = [1 - \{\phi(x)\}^3]^{1/3}$ .....(1)

where $\phi(x) = f(x) - b$

$\Rightarrow \phi(2a + x) = [1 - \{\phi(x+a)\}^3]^{1/3} = \phi(x)$

[from (1)]

$\Rightarrow f(x + 2a) - b = f(x) - b \Rightarrow f(x + 2a) = f(x)$

$\therefore \ f(x)$ is periodic with period $2a$.

13. **(c)** $f(2x+3) + f(2x+7) = 2$ .....(1)

Replace x by $x + 1$, $f(2x+5) + f(2x+9) = 2$ .....(2)

Now replace x by $x + 2$, $f(2x+7) + f(2x+11) = 2$.....(3)

From (1) – (3) we get $f(2x+3) - f(2x+11) = 0$

$\Rightarrow f(2x+3) = f(2x+11) \Rightarrow f(x) = f(x+8)$

$\left[\text{Replacing } x \text{ by } \frac{x-3}{2}\right]$

14. **(d)** Since codomain $= \left[0, \frac{\pi}{2}\right)$

$\therefore$ for f to be onto, range $= \left[0, \frac{\pi}{2}\right)$

This is possible only when $x^2 + x + a \geq 0$

$\therefore \ l^2 - 4a \leq 0 \Rightarrow a \geq \frac{1}{4}$ $\because$ for $a > \frac{1}{4}$, range will not be entire $\left[0, \frac{\pi}{2}\right)$; So $a = \frac{1}{4}$

15. (b) $f(x)=\sin^2 x+\left(\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x\right)^2$

$$+\cos x\left(\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x\right)$$

$$=\sin^2 x+\frac{3}{4}\cos^2 x+\frac{1}{4}\sin^2 x+\frac{\sqrt{3}}{2}\cos x\sin x$$

$$+\frac{1}{2}\cos^2 x-\frac{\sqrt{3}}{2}\cos x\sin x$$

$$=\frac{5}{4}(\sin^2 x+\cos^2 x)=\frac{5}{4}$$

Now, $y=g(f(x))=g\left(\frac{5}{4}\right)=1.$

Clearly $y=1$ is a straight line.

16. (d) $f(x)=e^{x^3-3x+2}$

Let

$g(x)=x^3-3x+2;\ g'(x)=3x^2-3=3(x^2-1)$

$g'(x)\ge 0$ for $x\in(-\infty,-1]$

$\therefore$ $g(x)$ is increasing function $\therefore f(x)$ is one-one

$\lim\limits_{x\to-\infty} f(x)<f(x)\le f(-1)\Rightarrow 0<f(x)\le e^4$

So the range of $f(x)$ is $\left(0,e^4\right]$

But codomain is $(0,e^5]$ $\therefore f(x)$ is into function.

17. (c) $f(x-1)+f(x+1)=\sqrt{3}f(x)$.....(i)

Putting $x+2$ for x in relation (i) we get

$f(x+1)+f(x+3)=\sqrt{3}f(x+2)$ .....(ii)

from (i) and (ii) we get

$f(x-1)+2f(x+1)+f(x+3)$

$$=\sqrt{3}(f(x)+f(x+2))$$

$$=\sqrt{3}(\sqrt{3}f(x+1))\ \text{(From (i))}$$

$$=3f(x+1)$$

$\Rightarrow\ f(x-1)+f(x+3)=f(x+1)$ ......(iii)

Putting $x+2$ for x in relation (iii) we get;

$f(x+1)+f(x+5)=f(x+3)$ ......(iv)

Adding (iii) and (iv) results in $f(x-1)=-f(x+5)$

Now put $x+1$ for x, $f(x)=-f(x+6)$......(v)

Put $x+6$ in place of x in (v) we get

$f(x+6)=-f(x+12)$

$\therefore$ from (v) again $f(x)=-[-f(x+12)]=f(x+12)$

$\therefore$ Period of $f(x)$ is 12.

18. (a) $\because (x-y)^2\ge 0\Rightarrow x^2+y^2\ge 2xy$

$\Rightarrow 2(x^2+y^2)\ge(x+y)^2$

$\Rightarrow(x+y)^2\le 2\ \Rightarrow -\sqrt{2}\le x+y\le\sqrt{2}$

Alternatively,

Let $x=\cos\alpha$ and $y=\sin\alpha$ $[\because x^2+y^2=1]$

Now, $x+y=\cos\alpha+\sin\alpha=\sqrt{2}\sin\left(\alpha+\frac{\pi}{4}\right)$

$\Rightarrow -\sqrt{2}\le x+y\le\sqrt{2}$

19. (c) $4x^2-4x+2=\sin^2 y$ and

$x^2+y^2\le 3\quad\Rightarrow(2x-1)^2+1=\sin^2 y$

and $x^2+y^2\le 3$

$\Rightarrow\ 2x-1=0$ and $\sin^2 y=1$ and $x^2+y^2\le 3$

$\Rightarrow\ x=\frac{1}{2},\sin^2 y=1$ and $y^2\le 3-\frac{1}{4}$

$\Rightarrow\ x=\frac{1}{2},y=n\pi\pm\frac{\pi}{2}$ and

$-\frac{\sqrt{11}}{2}\le y\le\frac{\sqrt{11}}{2}\Rightarrow x=\frac{1}{2},y=\pm\frac{\pi}{2}.$

Solution pairs are $\left(\frac{1}{2},\frac{\pi}{2}\right)$ and $\left(\frac{1}{2},-\frac{\pi}{2}\right)$

20. (b) Using A. M $\ge$ G. M. $x+y=\frac{x}{2}+\frac{x}{2}+y\ge 3\left(\frac{x^2y}{4}\right)^{1/3}$

Equality holds if and only if $\frac{x}{2}=y\Rightarrow x=2y$. Also.

$2x^2+2xy+3y^2$

$$=\underbrace{\frac{2x^2}{x}...+\frac{2x^2}{8}}_{8\text{ terms}}+\underbrace{\frac{2xy}{4}+....+\frac{2xy}{4}}_{4\text{ terms}}+y^2+y^2+y^2$$

$$\ge 15\left\{\left(\frac{2x^2}{8}\right)^8\left(\frac{2xy}{4}\right)^4(y^2)^3\right\}^{\frac{1}{15}}=15\left(\frac{x^2y}{4}\right)^{\frac{2}{3}}$$

Equality holds if and only if $\frac{2x^2}{8}=\frac{2xy}{4}=y^2$

or $x=2y$. Thus

$$k=x+y+(2x^2+2xy+y^2)^{1/2}\ge(3+\sqrt{15})\left(\frac{x^2y}{4}\right)^{1/3}$$

$\therefore$ Maximum value of $x^2y=\frac{4k^3}{(3+\sqrt{15})^3}$

21. (a) $F(n+1)=\dfrac{2F(n)+1}{2} \Rightarrow F(n+1)-F(n)=\dfrac{1}{2}$

Put $n = 1,2,3, ..., 2008$ and add,

$F(2009)-F(1) = 2008\times\dfrac{1}{2} \Rightarrow F(2009)=1005$

$[\because F(1) = 1]$

22. (b) Given :

$f(T+x)=1+\left[\{1-f(x)\}^3\right]^{1/3}=1+(1-f(x))$

$\Rightarrow f(T+x)+f(x)=2$ .....(1)

$\Rightarrow f(2T+x)+f(T+x)=2$ .....(2)

$(2)-(1) \Rightarrow f(2T+x)-f(x)=0 \Rightarrow f(2T+x)=f(x)$

Also T is positive and least therefore period of $f(x)=2T$

23. (d) Let $2x+\dfrac{y}{8}=\alpha$ and $2x-\dfrac{y}{8}=\beta$, then $x=\dfrac{\alpha+\beta}{4}$

and $y=4\ (\alpha-\beta)$

Given, $f\left(2x+\dfrac{y}{8}, 2x-\dfrac{y}{8}\right)=xy$

$\Rightarrow f(\alpha,\beta)=\alpha^2-\beta^2$

$\Rightarrow f(m,n)+f(n,m)=m^2-n^2+n^2-m^2=0$

for all m, n

24. (b) Given $f(x+y)=f(x)+f(y)-xy-1\ \forall\ x,y\in R$

$f(1)=1$

$f(2)=f(1+1)=f(1)+f(1)-1-1=0$

$f(3)=f(2+1)=f(2)+f(1)-2.1-1=-2$

$f(n+1)=f(n)+f(1)-n-1=f(n)-n<f(n)$

Thus $f(1)>f(2)>f(3)>......$

and $f(1)=1$ $\therefore f(1)=1$ and $f(n)<1$, for $n>1$

Hence $f(n) = n$, $n\in N$ has only one solution $n = 1$

25. (a) For $\sin^{-1}\dfrac{1+t^2}{2t}$ to be defined $\left|\dfrac{1+t^2}{2t}\right|\le 1$

$\Rightarrow 1+t^2\le 2|t| \Rightarrow (1-|t|)^2\le 0$

$\Rightarrow (1-|t|)^2=0 \Rightarrow t=\pm 1$

$\therefore k=\sin^{-1}\dfrac{1+t^2}{2t}>0 \Rightarrow k=\sin^{-1}1=\dfrac{\pi}{2}$

$\therefore [k]=\left[\dfrac{\pi}{2}\right]=1$. The given equation then becomes $(x-1)\ (x-\alpha)=1$. For integral values of α and x, we have

either $x-1=1$ and $x+\alpha=1 \Rightarrow x=2$ and $\alpha=-1$

or $x-1=-1$ and $x+\alpha=-1 \Rightarrow x=0$ and $\alpha=-1$

Thus, $\alpha=-1$.

26. (a) $S_n=\left(1-\dfrac{1}{1+x}\right)+\left(\dfrac{1}{1+x}-\dfrac{1}{1+2x}\right)+\left(\dfrac{1}{1+2x}-\dfrac{1}{1+3x}\right)$

$+......+\left(\dfrac{1}{1+(n-1)x}-\dfrac{1}{1+nx}\right)=1-\dfrac{1}{1+nx}$

But $\underset{n\to\infty}{\text{Lt}}\ nx=\begin{cases}\infty & \text{if } x>0\\ -\infty & \text{if } x<0\\ 0 & \text{if } x=0\end{cases}$

$\therefore f(x)=\underset{n\to\infty}{\text{Lt}}\ S_n=\begin{cases}1 \text{ when } x\ne 0\\ 0 \text{ when } x=0\end{cases}$

$\therefore$ Range of $f=\{0,1\}$

27. (a) $f(x)=(a-x^n)^{1-n}, x>0$

$\Rightarrow f(f(x))=[a-\{(a-x^n)^{1/n}\}^n]^{1/n}=x$

Also $g(x) = x^2+px+q$

$\therefore g(x)-x=0$ is quadratic equation

$x^2+(p-1)x+q=0$

Given that this equation has imaginary roots

$\therefore x^2+(p-1)x+q>0$ for all real x

$[\therefore$ coefficient of $x^2=1>0]$

$\therefore g(x)-x>0$ for all real x

$\Rightarrow g(g(x))-g(x)>0\ \forall\ x\in R$

Now $g(g(x))-f(f(x))=g(g(x))-x=$

$[g(g(x))-g(x)]+[g(x)-x]>0\ \forall\ x\in R$

$\therefore$ The equation $g(g(x))-f(f(x))=0$ has no real root.

28. (b) $f(\alpha)+f(\alpha^2)=ln\left[\left(\dfrac{1-\alpha}{1+\alpha}\right)\left(\dfrac{1-\alpha^2}{1+\alpha^2}\right)\right]=ln\left[\dfrac{(1-\alpha)^2}{1+\alpha^2}\right]$

$f\left(\dfrac{\alpha}{\alpha^2-\alpha+1}\right)=ln\left[\dfrac{1-\dfrac{\alpha}{\alpha^2-\alpha+1}}{1+\dfrac{\alpha}{\alpha^2-\alpha+1}}\right]=ln\left[\dfrac{(1-\alpha)^2}{1+\alpha^2}\right]$

$\therefore f(\alpha)+f(\alpha^2)=f\left(\dfrac{\alpha}{\alpha^2-\alpha+1}\right)$ for all values of α

for which the functions are defined, therefore

(i) $\dfrac{1-\alpha}{1+\alpha}>0 \Rightarrow -1<\alpha<1$ ....(1)

(ii) $\dfrac{1-\alpha^2}{1+\alpha^2}>0 \Rightarrow 1-\alpha^2>0 \Rightarrow -1<\alpha<1$.......(2)

From (1) and (2), we have $-1<\alpha<1$

$\therefore$ The set of values of $\alpha=(-1, 1)$.

29. **(c)** $f(x)=\cos 2\pi\{2x\}+\sin 2\pi\{2x\}$

$=\cos 2\pi(2x-[2x])+\sin 2\pi(2x-[2x])$

$=\cos 4\pi x\cos 2\pi[2x]+\sin 4\pi x\sin 2\pi[2x]$

$+\sin 4\pi\cos 22\pi[2x]-\cos 4\pi x\sin 2\pi[2x]$

Here cos $2\pi[2x]=1, \sin 2\pi[2x]=0$

$f(x)=\cos 4\pi x+\sin 4\pi x$. Hence period of $f(x)=\frac{1}{2}$.

30. **(c)** $f(x)=x^3+x^2+100x+7\sin x$

$\therefore\ f'(x)=3x^2+2x+100+7\cos x>0\quad\forall\ x\in R$

$\therefore f(x)$ is increasing function $\Rightarrow f(1)<f(2)<f(3)$

Let $\alpha=f(1),\beta=f(2),\gamma=f(3)$, then $\alpha<\beta<\gamma$ .....(1)

Now, the given equation is $\frac{1}{y-\alpha}+\frac{2}{y-\beta}+\frac{3}{y-\gamma}=0$

$\Rightarrow\ (y-\beta)(y-\gamma)+2(y-\alpha)(y-\gamma)+3(y-\alpha)(y-\beta)=0$ .....(2)

Let

$g(y)=(y-\beta)(y-\gamma)+2(y-\alpha)(y-\gamma)+3(y-\alpha)(y-\beta)$

Then

$g(\alpha)=(\alpha-\beta)(\alpha-\gamma)>0, g(\beta)=2(\beta-\alpha)(\beta-\gamma)<0$

and $g(\gamma)=3(\gamma-\alpha)(\gamma-\beta)>0$ [From (1)]

So, the given equation (2), i.e., $g(y)=0$ has one real root between α and β and other between β and γ.

31. **(d)** Putting $y=1$ in the given relation.

$2f(x)=f(x)+[f(1)]^x\ \Rightarrow\ f(x)=(f(1))^x=a^x$

Now, $\sum_{i=l}^{n}f(i)=a+a^2+a^3+.....+a^n=\frac{a(a^n-1)}{a-1}$

$\therefore\ (a-1)\sum_{i-l}^{n}f(i)=a^{n+1}-a$

32. **(b)** Notice that, $7(2x+3y)+3(2x-7y)=20x$

$\therefore\ f(2x+3y,2x-7y)=20x\Rightarrow f(x,y)=7x+3y$

33. **(d)** Since $f(x)$ and $g(x)$ are mirror images of each other about the line $y=a$, $f(x)$ and $g(x)$ are at equal distances from the line $y=a$. Let for some particular x_0

$f(x_0)=a+k$, then $g\ (x_0)=a-k$, then

$h(x_0)=f(x_0)+g(x_0)=2a$

$\therefore h(x)=2a\ \forall\ x\in R$. So, $h(x)$ must be a constant function, which is many -one into.

34. **(b)** Since $f(x)$ is monotonic so $f'(x)>0$ or $<0\ \forall\ x\in R$.

Then $\left(\frac{d}{dx}\right)f(rx)=rf'(rx)$ also >0 or <0 for all $x\in R$ and for positive integers r. Therefore, $f(x)$ and $f(rx)$ have the same monotonic behaviour. Since n is odd, so

$f(x)+f(2x).......+f(nx)$ is a polynomial of odd order and monotonic so attains all real values only once.

Hence the equation

$f(x)+f(2x).......+f(nx)=\frac{n(n+1)}{2}$ has exactly one solution.

35. **(a)** $x^2f(x)-2f\left(\frac{1}{x}\right)=g(x)$

and $2f\left(\frac{1}{x}\right)-4x^2f(x)=2x^2g\left(\frac{1}{x}\right)$

$\therefore\ -3x^2f(x)=g(x)+2x^2g\left(\frac{1}{x}\right)$

$\therefore\ f(x)=-\left(\frac{g(x)+2x^2g\left(\frac{1}{x}\right)}{3x^2}\right)$

$\because\ g(x)$ and x^2 are odd and even function respectively.

So, $f(x)$ is an odd function. But $f(x)$ is given even.

$\therefore\ f(x)=0\ \forall\ x$. Hence, $f(5)=0$

36. **(c)** Let $f(0)=k$. Let $a=0$

We get $f(b)=f(0)=k$ and again $b=0$ gives

$f(a)=k\Rightarrow f(a)=f(b)=k\forall a$,

$b\Rightarrow f(x)$ is a constant function.

$\therefore\ f(2009)=-\frac{1}{2}$

37. **(c)** $f(x)=\sin^{-1}\left[x^2+\frac{1}{2}\right]+\cos^{-1}\left[x^2-\frac{1}{2}\right]$

$=\sin^{-1}\left[x^2+\frac{1}{2}\right]+\cos^{-1}\left[x^2+\frac{1}{2}-1\right]$

$=\sin^{-1}\left[x^2+\frac{1}{2}\right]+\cos^{-1}\left(\left[x^2+\frac{1}{2}\right]-1\right)$

Since $x^2+\frac{1}{2}\geq\frac{1}{2}$, $\left[x^2+\frac{1}{2}\right]=0$ or 1 as

$\sin^{-1}\left[x^2+\frac{1}{2}\right]$ is defined only for these two values.

Hence $\left[x^2+\frac{1}{2}\right]=0$

$\Rightarrow f(x) = \sin^{-1} 0 + \cos^{-1}(-1) = \pi \left[x^2 + \frac{1}{2}\right] = 1$

$\Rightarrow f(x) = \sin^{-1}(1) + \cos^{-1} 0 = \pi$.

Therefore range of $f(x) = \{\pi\}$

38. **(b)** $[2\sin x] + [\cos x] = -3$ only if $[2\sin x] = -2$ and $[\cos x] = -1$

$\therefore\ -2 \le 2\sin x < -1$ and $-1 \le \cos x < 0$

$\Rightarrow\ -1 \le \sin x < -\frac{1}{2}$ and

$-1 \le \cos x < 0 \Rightarrow \frac{7\pi}{6} < x < \frac{11\pi}{6}$ and $\frac{\pi}{2} < x < \frac{3\pi}{2}$

Common values of x are given by $\frac{7\pi}{6} < x < \frac{3\pi}{2}$.

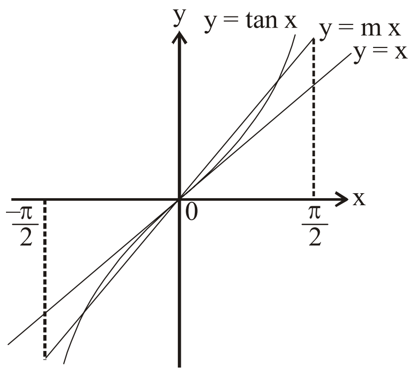
For these value of x, $\sin x + \sqrt{3}\cos x = 2\sin\left(\frac{\pi}{3} + x\right)$

lies between -2 and -1

$\therefore$ Range of $f(x)$ is $(-2, -1)$.

39. **(c)** In $\left(-\frac{\pi}{2}, 0\right)$ the graph of $y = \tan x$ lies below the line $y = x$ which is the tangent at $x = 0$ and in $\left(0, \frac{\pi}{2}\right)$ it lies above the line $y = x$.

For $m > 1$, the line $y = mx$ lies below $y = x$ in $\left(-\frac{\pi}{2}, 0\right)$ and



above $y = x$ in $\left(0, \frac{\pi}{2}\right)$. As $|\tan x|$ mono-tonically increases form O at $x = 0$ to ∞ for $|x| = \frac{\pi}{2}$, the graphs of $y = \tan x$ and $y = mx$, $m > 1$, meet three points including $x = 0$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ independent of m.

40. **(b)** We have $n = \left[\frac{n}{2}\right] + \left[\frac{n+1}{2}\right]$ or

$n = \left[\frac{n+1}{2}\right] + \left[\frac{n}{4}\right] + \left[\frac{n+2}{4}\right]$ (using (1) again and again)

$= \left[\frac{n+1}{2}\right] + \left[\frac{n+2}{4}\right] + \left[\frac{n+4}{8}\right] + ...$

41. **(c)** $f'(x) = x(x^2 - 3x + 2) = x(x-1)(x-2)$. The sign scheme for $f'(x)$ is as shown in figure.

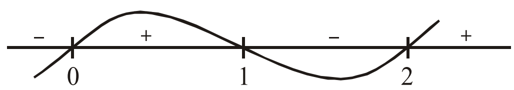
$\therefore\ f'(x) \le 0$ in $1 \le x \le 2$ and $f'(x) \ge 0$ and $2 \le x \le 3$

$\therefore f(x)$ is decreasing in [1, 2] and increasing in [2, 3]

$\therefore$ min $f(x) = f(2) = \int_1^2 x(x^2 - 3x + 2)dx$

$= \left[\frac{x^4}{4} - x^3 + x^2\right]_1^2 = -\frac{1}{4}$

max. $f(x)$ = the greatest among $[f(1), f(3)]$



$f(1) = \int_1^1 x(x^2 - 3x + 2)dx = 0$

$f(3) = \int_1^3 x(x^2 - 3x + 2)dx = 2$

$\therefore$ max $f(x) = 2$, so the range $= \left[-\frac{1}{4}, 2\right]$

42. **(c)** We should have $[\sin 2x] \ge [\cos 2x]$ $\Rightarrow$ we can have

$[\sin 2x] = 1, [\cos 2x] = 1, 0 - 1$

$[\sin 2x] = 0$, $[\cos 2x] = 0, -1$

$[\sin 2x] = -1$, $[\cos 2x] = -1$

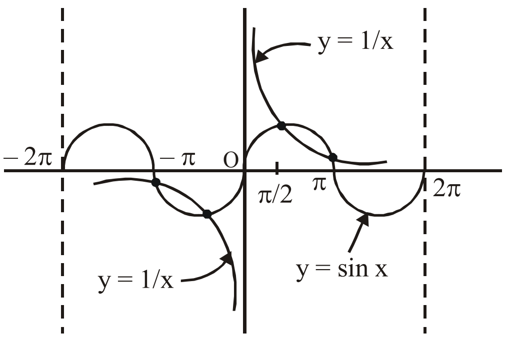
but $[\sin 2x] = 1$, $[\cos 2x] = 1$ and

$[\sin 2x] = 1$, $[\cos 2x] = -1$ are not possible

So range = {0, 1}

43. **(c)** $x \sin x = 1$ ..... (1)

$\Rightarrow\ y = \sin x = \frac{1}{x}$



Root of equation (1) will be given by the point (s) of intersection of the graphs $y = \sin x$ and $y = \frac{1}{x}$.

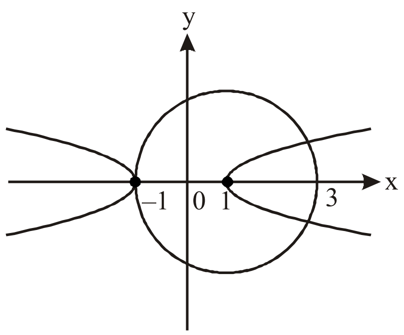
By graph, it is clear that, we get four roots.

44. **(b)** $(x-1)^2 + y^2 - 4 = 0$ is circle of radius 2 and centre $(1, 0)$

For, $|y| = ln|x|$, $ln|x| \geq 0 \Rightarrow |x| \geq 1$

$\Rightarrow x \leq -1$ or $x \geq 1$

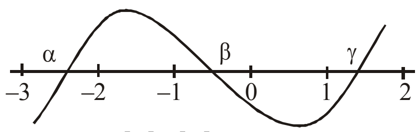
Graph of the curves are drawn, we notice that there are three points of intersection.



45. **(d)** $f(0) = -1, f(1) = -4, f(2) = 1,$

$f(-1) = 4, f(-2) = 5, f(-3) = -4$

So graph of $f(x)$ is as shown in the figure



Clearly $[\alpha]+[\beta]+[\gamma] = -3-1+1 = -3$

46. **(d)** Let $f(x) = x^3 - 3x + a$

$f'(x) = 3x^2 - 3 = 0 \Rightarrow x = \pm 1$

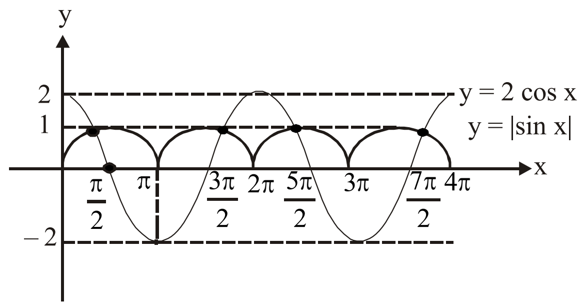
$f''(x) = 6x$

$f''(x) > 0$ at $x = 1$ and $f''(x) < 0$ at $x = -1$

Thus $x = 1$ is point of minima and $x = -1$ is point of maxima.

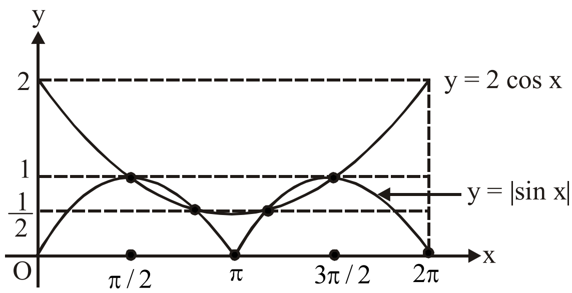
Hence either one root lies in [–1, 1] or no root lies in[–1, 1]. So no such value exist for which the given condition is possible.

47. **(c)** See the graph of $y = 2\cos x$ and $y = |\sin x|$. Their points of intersection represent the solution of the given equation.



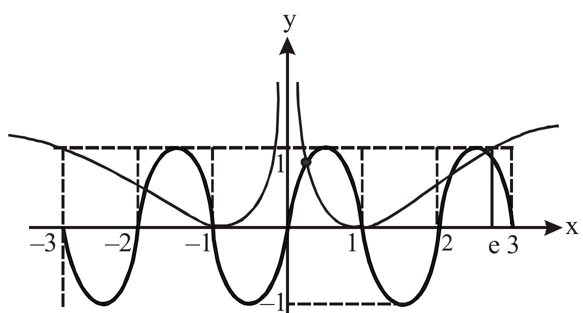
We find that the graphs intersect at 4 points. Hence the equation has 4 solutions.

48. **(a)** See the graph $y = 2^{\cos x}$ and $y = |\sin x|$. Two curves meet at four points for $x \in [0, 2\pi]$



So, the equation $2^{\cos x} = |\sin x|$ has four solutions.

49. **(c)** The graph of $y = \sin \pi x$ and $y = |\log|x||$ intersect the 6 poins (four positive and two negative roots) as clear from the follwing figures :



50. **(d)** First consider max $(|x+y|, |x-y|) = 1$

If $|x+y| \geq |x-y|$ then $|x+y| = 1$

i..e if $4xy \geq 0$ then $x + y = \pm 1$,

$\Rightarrow$ In first and third quadrants $x + y = \pm 1$

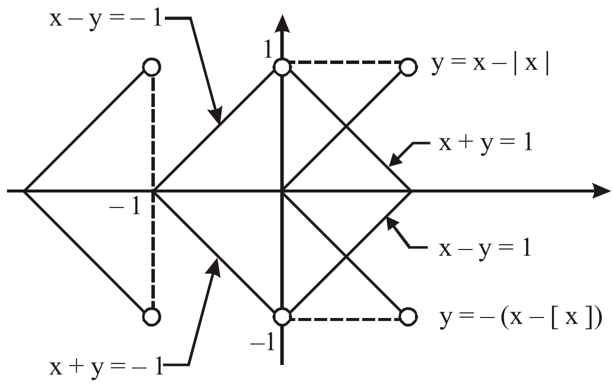
If $|x+y| \leq |x-y|$ then $|x-y| = 1$

i.e., if $4xy \leq 0$, then $x - y = \pm 1$

$\Rightarrow$ In second and fourth quadrants $x - y = \pm 1$

Also $|y| = x - |x| \Rightarrow y = \pm(x - [x])$ as $x - [x] \geq 0$

The graph of the curves is shown :



We note that for $-1 \le x < 0$ two curves concide. So, there are infinitely many solutions.

51. **(c)** $\sin^{-1}(\log[x])$ is defined if $-1 \le \log[x] \le 1$ and $[x] > 0$

$$\Rightarrow \frac{1}{e} \le [x] \le e \Rightarrow [x] = 1, 2 \Rightarrow x \in [1, 3)$$

Again, $\log(\sin^{-1}[x])$ is defined if $\sin^{-1}[x] > 0$ and $-1 \le [x] \le 1$

$\Rightarrow [x] > 0$ and $-1 \le [x] \le 1 \Rightarrow 0 < [x] \le 1$

$\Rightarrow x \in [1, 2)$

$\therefore$ Domain of $f(x) = [1, 2)$

For $1 \le x < 2$, $[x] = 1$

$$\therefore \ f(x) = \sin^{-1} 0 + \log\frac{\pi}{2} = \log\frac{\pi}{2}, \forall x \in [1, 2)$$

$$\therefore \text{ Range of } f(x) = \left\{\log\frac{\pi}{2}\right\}.$$

52. **(a)** Given $f(x+y) = f(x)f(y)$ Put $x = y = 0$, then $f(0) = 1$

Put $y = -x$ then $f(0) = f(x)f(-x)$

$$\Rightarrow f(-x) = \frac{1}{f(x)}$$

Now $g(x) = \dfrac{f(x)}{1 + \{f(x)\}^2}$

$$g(-x) = \frac{f(-x)}{1+\{f(-x)\}^2} = \frac{\frac{1}{f(x)}}{1 + \frac{1}{\{f(x)\}^2}} = \frac{f(x)}{1+\{f(x)\}^2} = g(x)$$

53. **(a)** Given, $f(x+y) + f(x-y) = 2f(x)f(y)$ ..... (1)

Interchange x and y in (1), we get

$f(y+x) + f(y-x) = 2f(y)f(x)$ ...... (2)

From (1) and (2) $f(x-y) = f(y-x)$

Putting $y = 2x$, we get $f(x) = f(-x)$

54. **(b)** Here $g^2(x) = gog(x) = g\{g(x)\} = g(3+4x)$

$$= 15 + 4^2 x = (4^2 - 1) + 4^2 x$$

$$g^3(x) = gogog(x) = g(15 + 4^2 x) = 3 + 4(15 + 4^2 x)$$

$$= 63 + 4^3 x = (4^3 - 1) + 4^3 x$$

Generalizing, we get $g^n(x) = (4^n - 1) + 4^n x = y$ (say)

then $x = (y + 1 - 4^n)4^{-n}$

$\Rightarrow g^{-n}(y) = (y+1)4^{-n} - 1$

$\therefore g^{-n}(x) = (x+1)4^{-n} - 1$

55. **(a)** Given $f(x) = -\dfrac{3}{2} f\left(\dfrac{2x+29}{x-2}\right) + 50x + 40$.

Replace x by $\dfrac{2x+29}{x-2}$, we get

$$f\left(\frac{2x+29}{x-2}\right) = -\frac{3}{2} f\left[\frac{2\left(\frac{2x+29}{x-2}\right)+29}{\left(\frac{2x+29}{x-2}\right)-2}\right] + 50\left(\frac{2x+29}{x-2}\right) + 40$$

$$\Rightarrow f\left(\frac{2x+29}{x-2}\right) = -\frac{3}{2} f(x) + 50\left(\frac{2x+29}{x-2}\right) + 40$$

$$\therefore f(x) = \frac{9}{4} f(x) - 75\left(\frac{2x+29}{x-2}\right) - 60 + 50x + 40$$

$$\Rightarrow \quad f(x) = 60\left(\frac{2x+29}{x-2}\right) + 16 - 40x$$

56. **(b)** Given $f(x) + f(x+a) + \ldots\ldots\ldots + f(x+an) = 0$.....(1)

Replace x by $x + a$,

$f(x+a) + f(x+2a) + \ldots\ldots. + f\{x + a(n+1)\} = 0$.....(2)

Subtracting (2) from (1), we get

$f(x) - f\{x + a\,(n+1)\} = 0$

$\Rightarrow f(x)$ is periodic with period a $(n + 1)$.

57. **(c)** The problem is equivalent to deragement. The required number of functions

$$= 5!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right] = 44.$$

58. **(a)** $f(x)$ is one-one if $f'(x) \ge 0 \ \forall\ x \in R$

$\therefore 3x^2 + 6x + 4 + a\cos x - b\sin x \ge 0 \ \forall\ x \in R$

$\Rightarrow 3x^2 + 6x + 4 \ge b\sin x - a\cos x \ \forall x \in R$

$\Rightarrow 3x^2 + 6x + 4 \ge \sqrt{a^2 + b^2} \ \forall x \in R$

or $3(x+1)^2 + 1 \ge \sqrt{a^2 + b^2} \ \forall\ x \in R$

$\therefore \sqrt{a^2 + b^2} \le 1 \cdot$

59. **(b)** $2 < x^2 < 3 \Rightarrow \sqrt{2} < x < \sqrt{3}$ $(\because x > 0)$

$\therefore \left\{\frac{1}{x}\right\} = \frac{1}{x}$ and $\{x^2\} = x^2 - 2$

So, the equation becomes

$\frac{1}{x} = x^2 - 2 \Rightarrow x^3 - 2x - 1 = 0$

or $(x+1)(x^2 - x - 1) = 0 \Rightarrow x = -1, \frac{1 \pm \sqrt{5}}{2}$

$\therefore x = \frac{1+\sqrt{5}}{2}$ $(\because x > 0)$

60. **(d)** We have, $D(x) = f(x)g\left(\frac{x}{5}\right) - g(x)f\left(\frac{x}{3}\right)$

Now period of $f(x)g\left(\frac{x}{5}\right)$ is $7 \times 55 = 385$

Period of $g(x)f\left(\frac{x}{3}\right)$ is $11 \times 21 = 231$

$\therefore$ Period of $D(x) =$ LCM of $(385, 231) = 1155$

61. **(a)** $f(x+1) + f(x-1) = 2f(x)$

$\Rightarrow f(2) + f(0) = 2f(1)$

$\Rightarrow f(2) = 2f(1)$

$f(3) + f(1) = 2f(2)$

$\Rightarrow f(3) = 4f(1) - f(1) = 3f(1)$

Similarly, $f(4) + f(2) = 2f(3)$

$\Rightarrow f(4) = 6f(1) - 2f(1) = 4f(1)$

$\Rightarrow f(4) = 4f(1)$

Hence $f(n) = nf(1)$

62. **(b)** Range of $f(x)$ for which $g(f(x))$ is defined is $\{3, 4\}$

Hence domain of g $(f(x))$ has two elements.

$\therefore$ Range of g $(f(x))$ also has two elements.

63. **(c)** $4x^2 + 4x + 4 + \sin(\pi x) = (2x+1)^2 + \sin(\pi x) + 3 \geq 2.$

Now as $1 < 2 < e$, the required value of n is 3.

64. **(a)** $[f(x)]^2 . f\left(\frac{1-x}{1+x}\right) = 64x$ ...(1)

Replace x by $\frac{1-x}{1+x}$

$\Rightarrow \left(f\left(\frac{1-x}{1+x}\right)\right)^2 . f(x) = 64\left(\frac{1-x}{1+x}\right)$ ...(2)

[Putting values of $f\left(\frac{1-x}{1+x}\right)$ from (1)]

$\Rightarrow f(x)^3 = 64x^2\left(\frac{1+x}{1-x}\right)$

65. **(a)** $(x+1)f(x) - x$ is a polynomial of degree $n+1$

$\Rightarrow (x+1)f(x) - x = k(x)[x-1][x-2] .... [x-n]$

$\Rightarrow [n+2]f(n+1) - (n+1) = k[(n+1)!]$

Also, $1 = k(-1)(-2) .... ((-n+1))$

$1 = k(-1)^{n+1}(n+1)!$

$\Rightarrow (n+2)f(n+1) - (n+1) = (-1)^{n+1}$

$\Rightarrow f(n+1) = 1$

If n is odd and $\frac{n}{n+1}$, if n is even.

66. **(c)** **Case I :**

$0 < |x| - 1 <$ i.e., $1 < |x| < 2$, then

$x^2 + 4x + 4 \leq 1$

$\Rightarrow x^2 + 4x + 3 \leq 0$

$\Rightarrow -3 \leq x \leq -1$

So, $x \in (-2, -1)$ ....(1)

Case II :

$|x| - 1 > 1$ i.e., $|x| > 2$, then

$x^2 + 4x + 4 \geq 1$

$\Rightarrow x^2 + 4x + 3 \geq 0$

$\Rightarrow x \geq -1$ or $x \leq -3$

So, $x \in (-\infty, -3] \cup (2, \infty)$

67. **(b)** Let $y = |f(x) + g(x)| + |f(x) - g(x)|$

$y^2 = f^2(x) + g^2(x) + 2f(x)g(x)$

$+ f^2(x) + g^2(x) - 2f(x)g(x) + 2|f^2(x) - g^2(x)|$

$= 2f^2(x) + 2g^2(x) + 2|f^2(x) - g^2(x)|$

Case I : If $f^2(x) \geq g^2(x)$, then $y^2 = 4f^2(x)$

$y = 2|f(x)|$

Case I : If $g^2(x) \geq f^2(x)$, then $y^2 = 4g^2(x)$

$y = 2|g(x)|$

So, $y = 2\max\{|f(x)|, |g(x)|\}$

68. **(d)** $g(x) = \frac{e^{f(x)} - e^{|f(x)|}}{e^{f(x)} + e^{|f(x)|}}$ $-1 \leq f(x) \leq 1$

Let $0 \leq f(x) \leq 1$ then $g(x) = 0$

Let $-1 \leq f(x) < 0$ then

$g(x) = \frac{e^{f(x)} - e^{-f(x)}}{e^{f(x)} + e^{-f(x)}} = \frac{e^{2f(x)} - 1}{e^{2f(x)} + 1} = 1 - \frac{2}{e^{2f(x)} + 1}$

Thus for $-1 \leq f(x) \leq 0$

$g(x) \in \left[\frac{1-e^2}{1+e^2}, 0\right)$

So, for $-1 \leq f(x) \leq 1$

$g(x) \in \left[\frac{1-e^2}{1+e^2}, 0\right]$

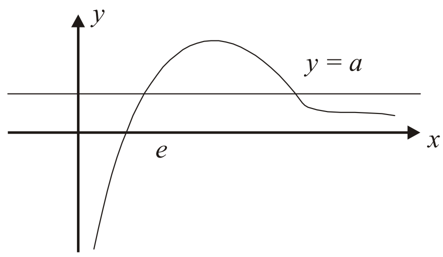
69. **(c)** Given curves $y = \ln x$ and $y = ax$

$\Rightarrow \ln x = ax$ has exactly two solutions

$\Rightarrow \frac{\ln x}{x} = a$ has exactly two solutions.

To find the range of $\frac{\ln x}{x}$

Let $y = \frac{\ln x}{x}$, $x > 0$; $\frac{dy}{dx} = \frac{x.\frac{1}{x} - \ln x}{x^2} = \frac{1-\ln x}{x^2}$



y is increasing if $1 - \ln x > 0 \Rightarrow \ln x < 1 \Rightarrow 0 < x < e$

Range of $y \in \left(-\infty, \frac{1}{e}\right]$; Graph of $y = \frac{\ln x}{x}$

For exactly two solution of $\frac{\ln x}{x} = a$

$a \in \left(0, \frac{1}{e}\right)$

B COMPREHENSION TYPE

1. **(a)** We have $af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5$ ...(1)

Replace x by $\frac{1}{x} \Rightarrow af\left(\frac{1}{x}\right) + bf(x) = x - 5$ ...(2)

Eliminating $f\left(\frac{1}{x}\right)$, we get

$(a^2 - b^2)f(x) = \frac{a}{x} - 5a - bx + 5b$ ...(3)

LHS = 0, where $a = b$ or $a + b = 0$. We are given that $a \neq b$ and if $a + b = 0$, then $b = -a \neq 0$, then (3) becomes quadratic in x, which can have at most two solutions. But that contradicts the fact that (1) and (2) are valid for all $x \neq 0$. So $a^2 - b^2 \neq 0$

$\therefore f(x) = \frac{a}{(a^2 - b^2)x} - \frac{bx}{a^2 - b^2} - \frac{5}{a+b}$ for all $x \neq 0$

Now $\int_1^2 f(x)dx = \int_1^2 \left[\frac{a}{(a^2-b^2)x} - \frac{bx}{a^2-b^2} - \frac{5}{a+b}\right]dx$

$= \frac{1}{a^2 - b^2}\left[a\ln x - \frac{bx^2}{2} - 5(a-b)x\right]_1^2$

$\therefore \int_1^2 f(x)dx = \frac{2a\ln 2 - 10a + 7b}{2(a^2 - b^2)}$

2. **(b)** We have $af(x) + bf(-x) = g(x)$ ...(1)

Replacing x by $-x \Rightarrow af(-x) + bf(x) = g(-x)$ ...(2)

Eliminating $f(-x)$,

we get $(a^2 - b^2) f(x) = ag(x) - bg(-x)$ ...(3)

$f(x)$ can be determined uniquely if $a^2 - b^2 \neq 0$.

In the degenerate case (i.e., when $a^2 - b^2 = 0$), equation (3) will have no solution if $ag(x) - bg(-x) \neq 0$ and has infinitely many solution if $ag(x) - bg(-x) = 0$

3. **(c)** Replace x by $(-x)$ and with the property that $f(x)$ is even, we get

$x^2 f(x) - 2f\left(\frac{1}{x}\right) = g(-x) = -g(x)$

$\Rightarrow x^2 f(x) - 2f\left(\frac{1}{x}\right) = 0$

Now replace x by $\frac{1}{x}$, we get $\frac{1}{x^2} f\left(\frac{1}{x}\right) - 2f(x) = 0$

On solving, we get $f(x) = 0$

4. **(c)** Given $af(z) + bf(w^2z) = g(z)$ ...(1)

Replace z by wz and by w^2z, we get

$af(wz) + bf(z) = g(wz)$ ...(2)

and $af(w^2z) + bf(wz) = g(w^2z)$ ...(3)

Equations (1), (2) and (3) together form a linear system of three equations in three unknowns $f(z)$, $f(wz)$ and $f(w^2z)$ which can be expressed in a matrix form as

$$\begin{bmatrix} a & 0 & b \\ b & a & 0 \\ 0 & b & a \end{bmatrix}\begin{bmatrix} f(z) \\ f(wz) \\ f(w^2z) \end{bmatrix} = \begin{bmatrix} g(z) \\ g(wz) \\ g(w^2z) \end{bmatrix}$$

The determinant of the coefficient matrix is $a^3 + b^3$. So, when $a^3 + b^3 \neq 0$, the functional equation (1) has a unique solution. Futher $a^3 + b^3 \neq 0 \Rightarrow a + b \neq 0$.

5. **(d)** Since $h(-x) = -h(x)$

$f(-x) + g(-x) = -f(x) - g(x)$ ($\because f$ is odd and g is even)

$\therefore 2g(x) = 0$

$\Rightarrow g(x) = 0 \ \forall x \in R$

$\Rightarrow$ g (x) is zero function which is constant function.

6. **(a)** $g(x) = 0 \Rightarrow g'(x) = 0 \ \forall \ x$

So, $f'(x) > 0 \ \forall x \in R$

$\Rightarrow f$ is increasing function and f is odd.

Let $f(x) > 0$ for some x (without loss of generality)

$f(-x) = -f(x) < 0$ for that x

Since $f(x)$ and $f(-x)$ take opposite signs and f is increasing

$\Rightarrow f$ is zero at only one point

$\therefore f(x) = g(x) \Rightarrow f(x) = 0 \Rightarrow x = 0$

such that $f(0) = 0$ ($\because f(-x) = -f(x)$ takes $x = 0$ then $f(0) = f(0) \Rightarrow f(0) = 0$)

$\therefore f(x) = g(x)$ has only one solution.

7. **(b)** The number of solutions of $\phi(x) = h(x)$ is

$\phi(x) = f(g(x)) + g(f(x))$

$= f(0) + 0$ ($\because$ g (x) $= 0 \ \forall x \in R$)

$\Rightarrow \phi(x) = 0$ and $h(x) = f(x)$

$\therefore \phi(x) = h(x) \Rightarrow 0 = f(x)$

$\Rightarrow f$ takes 0 at $x = 0$

$\therefore$ equation has only one solution.

8. **(a)** If $f(x) = \dfrac{4^x}{4^x + 2}$ where $x \in Q$

then $f(x) + f(1-x) = \dfrac{4^x}{4^x+2} + \dfrac{4^{1-x}}{4^{1-x}+2}$

$$= \frac{4^x}{4^x+2} + \frac{4}{4+2.4^x} = \frac{4^x}{4^x+2} + \frac{4}{2+4^x} = 1 = k$$

Now, $f\left(\dfrac{1}{2009}\right) + f\left(\dfrac{2}{2009}\right) + ... + f\left(\dfrac{2008}{2009}\right)$

$$= \sum_{r=1}^{2008} f\left(\frac{r}{2009+1}\right) = \frac{2008 \times 1}{2} = 1004$$

9. **(a)** $f(x) + f(1-x) = \dfrac{3^{x-3}}{3^{1-x}+3^x} + \dfrac{3^{-x-2}}{3^x + 3^{1-x}}$

$$= 3^{-3}\left[\frac{3^x + 3^{1-x}}{3^{1-x}+3^x}\right] = 3^{-3}$$

Thus, $f\left(\dfrac{1}{55}\right) + f\left(\dfrac{2}{55}\right) + ... + f\left(\dfrac{54}{55}\right)$

$$= \sum_{r=1}^{54}\left(\frac{r}{54+1}\right) = \frac{54.3^{-3}}{2} = 1$$

10. **(c)** $f(x) = \dfrac{a^x}{a^x + \sqrt{a}}$

$$f(x) + f(1-x) = \frac{a^x}{a^x+\sqrt{a}} + \frac{a^{1-x}}{a^{1-x}+\sqrt{a}}$$

$$= \frac{a^x}{a^x+\sqrt{a}} + \frac{a}{a + a^x\sqrt{a}}$$

$$= \frac{a^x}{a^x+\sqrt{a}} + \frac{\sqrt{a}}{\sqrt{a}+a^x} = 1 = k$$

So, $\displaystyle\sum_{r=1}^{2n-1} 2f\left(\frac{r}{2n}\right) = \sum_{r=1}^{2n-1} 2f\left(\frac{r}{2n-1+1}\right)$

$= (2n-1).\,1 = 2n-1$

11. **(c)** If $x \in R^-$, then $x \le (x) \ge 0$

$\therefore 0 \le \dfrac{(x)}{x} \le 1 \qquad \therefore y \in [0,1]$

12. **(b)** Since $-2 < x \le -1 \Rightarrow (x) = -1$

Similarly, $(y) = 0$, $(z) = 1$

So, determinant $= \begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & 0 & 2 \end{vmatrix}$

Expanding along R_1, we get the values of determinant as $1(0+1) = 1$.

13. **(d)** $f(x) = \cos 10x + \cos(-9x)$

$f(x) = \cos 10x + \cos 9x$

$$f\left(\frac{\pi}{4}\right) = \cos\frac{9\pi}{4} + \cos\frac{10\pi}{4} = \frac{1}{\sqrt{2}} + 0 = \frac{1}{\sqrt{2}}$$

$$\Rightarrow f\left(\frac{\pi}{2}\right) = \cos\frac{9\pi}{2} + \cos 5\pi = 0 + (-1) = -1$$

$$\Rightarrow f\left(\frac{\pi}{4}\right) + f\left(\frac{\pi}{2}\right) = \left(-1 + \frac{1}{\sqrt{2}}\right)$$

14. **(c)** Let the equation of parabola be $y = ax^2 + bx + c$

then $\dfrac{1}{a} = 1 \Rightarrow a = 1$. Also, its vertex is

$$\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$$

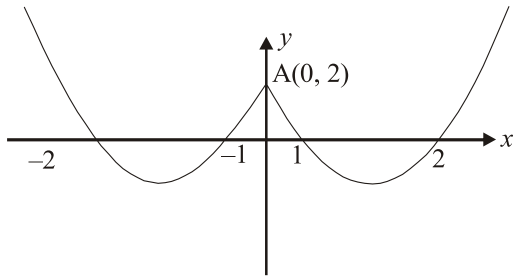
$\therefore -\dfrac{b}{2a} = \dfrac{3}{2}$ and $\dfrac{4ac-b^2}{4a} = -\dfrac{1}{4}$

We get b = –3 and c = 2. Thus $y = x^2 - 3x + 2$ is the required parabola.

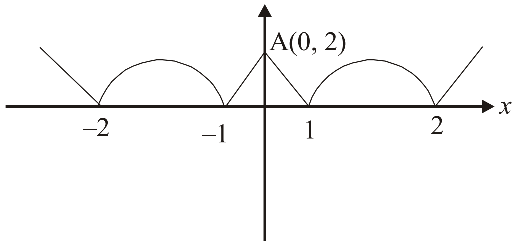
Hence $f(x) = x^2 - 3x + 2$

$g(x) = f(|x|)$

$h(x) = |g(x)|$



Graph of $g(x)$



Graph of $h(x)$

Number of real solutions of $g(x) = 0$ is 4.

15. **(b)** $g(x) + a = 0 \Rightarrow g(x) = -a$

A line parallel to x-axis cuts the graph of y = g(x) exactly twice if it is above the point A(0, 2), so

$-a > 2 \Rightarrow a < -2$

16. **(b)** From the graph of h(x) we see that $h'(x) = 0$ at exactly two points, one of which lies in (–2, –1) and other is $(1, 2)$

C REASONING TYPE

1. **(a)** $f(x) = x - [x]$

$f(x+1) = x + 1 - ([x]+1) = x - [x]$

period of $x - [x]$ is 1

$f(x) = \sin(2x - [2x])$

$$f\left(x+\frac{1}{2}\right) = \sin\left(2\left(x+\frac{1}{2}\right) - \left[2\left(x+\frac{1}{2}\right)\right]\right)$$

$= \sin(2x + 1 - [2x] - 1) \quad = \sin(2x - [2x])$

period is $\frac{1}{2}$

2. **(a)** $2 < x < 3 \Rightarrow \ x - 1 > 0$

$x - 2 > 0$

$x - 3 < 0$

$\Rightarrow f(x) = x - 1 + x - 2 + 3 - x = x$

$\Rightarrow f$ is an identity function

3. **(d)** Range of $\sin x$ is $[-1, 1]$

$\Rightarrow f : R \to R$ defined by $f(x) = \sin x$ is not onto

$\Rightarrow$ it is not bijection.

If f is both one and onto then f is bijection.

4. **(c)** $f: R \to R, f(x) = \frac{2x+1}{3}$ is a bijection.

$\Rightarrow f^{-1} = \frac{3x-1}{2}$.

5. **(a)** Let $h(x) = \frac{f(x)}{g(x)}$ then

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -h(x)$$

$\therefore \ h(x) = \frac{f}{g}$ is an odd function

6. **(d)** $f : A \to B$, $g : B \to C$ are function $(g\,of)^{-1} \neq f^{-1}og^{-1}$ (since functions need not possess inverses) Bijective functions are invertible.

7. **(a)** A : sin$x$: – [–1 1]$\to$ [– 1 1] Also,

$f\left(\frac{1}{2}\right) = f(5/6) \Rightarrow f$ is not one-one

D MULTIPLE CORRECT CHOICE TYPE

1. **(b,c)** $f(x) = \sqrt{3}\sin x - \cos x + 2 = 2\sin\left(x - \frac{\pi}{6}\right) + 2$

Since $f(x)$ is one-one and onto, f is invertible

Now $\left(f\,o\,f^{-1}\right)$ (x) = x

$\Rightarrow 2\sin\left(f^{-1}(x) - \frac{\pi}{6}\right) + 2 = x$

$\Rightarrow \sin\left(f^{-1}(x) - \frac{\pi}{6}\right) = \frac{x}{2} - 1$

$$\Rightarrow\ f^{-1}(x)=\sin^{-1}\left(\frac{x}{2}-1\right)+\frac{\pi}{6}$$

Because $\left|\frac{x}{2}-1\right|\le 1$ for all $x\in[0,4]$.

Also using $\sin^{-1}\alpha+\cos^{-1}\alpha=\frac{\pi}{2}$

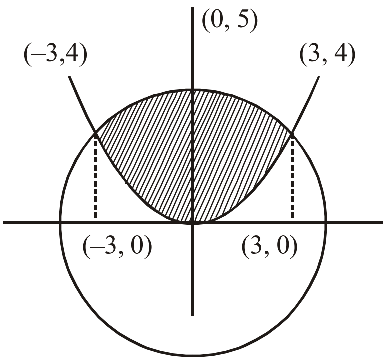
$$f^{-1}x=\frac{\pi}{2}-\cos^{-1}\left(\frac{x-2}{2}\right)-\frac{\pi}{6}=\frac{2\pi}{3}-\cos^{-1}\left(\frac{x-2}{2}\right)$$

.

2. **(a,b,c)** The equation $x^2+y^2=25$ represents a circle with center (0, 0) and

radius 5 and the equation $y=\frac{4}{9}x^2$ represent a parabola with

vertex (0, 0) and focus (0, 5). Hence $R\cap R'$ is the set of points



indicated in the figure = $\{(x,y):-3\le x\le 3,$ $0\le y\le 5\}$.

Thus dom $R\cap R'=[-3,3]$ and range $R\cap R'=[0,5]\supset[0,4]$

Since $(0,0)\in R\cap R'$ and $(0,5)\in R\cap R'$

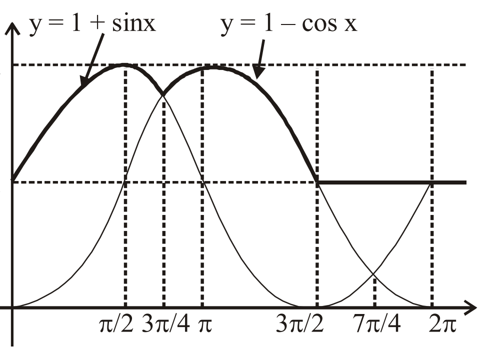
$\therefore$ 0 is related to 0 as well as 5.

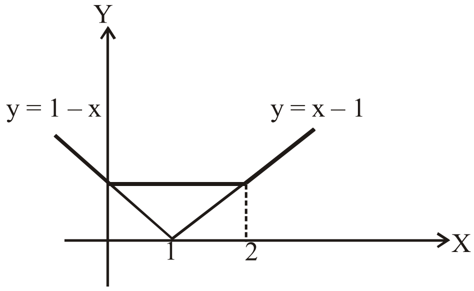
Hence, $R\cap R'$ does not define a function. Also any line parallel to y-axis contains infinite points in the region $-3<x<3$

So, there are infinitely many images for each value of $x\in(-3,3)$, thus $R\cap R'$ cannot be a function.

3. **(a,b,d)**

$$f(x)=\max\{1+\sin x,\ 1,\ 1-\cos x\}=\begin{cases}1+\sin x, & 0\le x\le\frac{3\pi}{4}\\ 1-\cos x, & \frac{3\pi}{4}\le x\le\frac{3\pi}{2}\\ 1, & \frac{3\pi}{2}\le x\le 2\pi\end{cases}$$





$$g(x)=\max\{1,|x-1|\}=\begin{cases}1-x, & x\le 0\\ 1, & 0\le x\le 2\\ x-1, & x\ge 2\end{cases}$$

$\therefore f(0)=1\Rightarrow g(f(0))=1$ and $f(1)=1+\sin 1$

$\left(\because 0<1<\frac{3\pi}{4}\right)$

$\Rightarrow g(f(1)=1$

$(\because 1<1+\sin 1<2)$

Again $g(1)=1\Rightarrow f(g(1))=1+\sin 1$ and

$g(0)=1\Rightarrow f(g(0))=1+\sin 1$

4. **(a,b)** We have $f(n+2)-f(n+1)=(n+2)!$

$=(n+2)(n+1)!=(n+2)[f(n+1)-f(n)]$

$\Rightarrow f(n+2)=(n+3)f(n+1)-(n+2)f(n)$

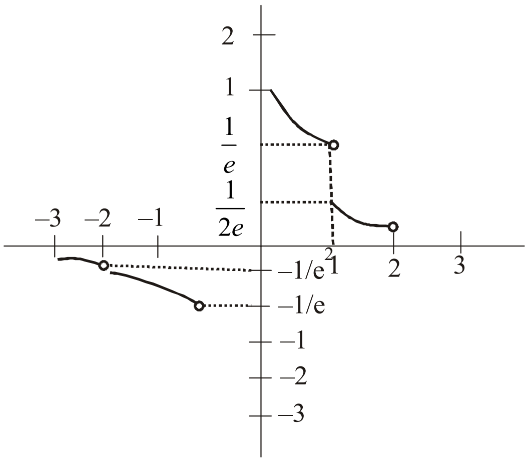
$\therefore P(x)=x+3$ and $Q(x)=-x-2$

5. **(a,d)** $f(x)$ is defined if $1+[x]\ne 0\Rightarrow x\notin[-1,0)$

Now,

$$f(x)=\begin{cases}\ldots\ldots\ldots\\ \frac{e^{-x}}{3}, & \text{if}\quad 2\le x<3\\ \frac{e^{-x}}{2}, & \text{if}\quad 1\le x<2\\ e^{-x} & \text{if}\quad 0\le x<1\\ -e^{-x} & \text{if}\quad -2\le x<-1\\ -\frac{e^{-x}}{2} & \text{if}\quad -3\le x-2\\ \ldots\ldots\ldots\end{cases}$$

The graph of the function is shown in the figure.



From the graph it is clear that $f(x)$ is discontinuous at all integral points. Further $f(x)$ does not attain many values in the interval $\left(-\frac{1}{e}, 1\right]$

6. **(b,c)** Given $f(x+y) - kxy = f(x) + 2y^2$.
Replace y by $-x$, then

$f(0) + kx^2 = f(x) + 2x^2$

$\Rightarrow\ f(x) = f(0) + kx^2 - 2x^2$ ...... (1)

Now $f(1) = f(0) + k - 2 = 2 \ \Rightarrow\ f(0) = -k + 4$

and $f(2) = f(0) + 4k - 8 = 8 \ \Rightarrow\ f(0) = -4k + 16$

Which give $k = 4$ and $f(0) = 0$

Thus, from (1) $f(x) = 2x^2$

$\therefore\ f(x+y) f\left(\frac{1}{x+y}\right) = 4 = k$

7. **(a,b,c)** $y = \frac{x-1}{a - x^2 + 1}$ also $y < -1$ or $y > -\frac{1}{3}$

$\Rightarrow (y+1)\left(y + \frac{1}{3}\right) > 0$

$\Rightarrow \left(\frac{x-1}{a-x^2+1} + 1\right)\left(\frac{x-1}{a-x^2+1} + \frac{1}{3}\right) > 0$

$\Rightarrow (x - 1 + a - x^2 + 1)\,(3x - 3 + a - x^2 + 1) > 0$

$(-x^2 + x + a)\,(-x^2 + 3x + a - 2) > 0$

$(x^2 - x - a)\,(x^2 - 3x + 2 - a) > 0$

$\Rightarrow x^2 - x - a > 0 \quad \forall x \in R$

and $x^2 - 3x + 2 - a > 0 \ \forall x \in R$

$\Rightarrow 1 + 4a < 0 \Rightarrow a < -\frac{1}{4}$

8. **(a,b)** $f(x) = 1 + x^2$ ....(1)
$f(g(x)) = 1 + x^2 - 2x^3 + x^4$
From (1) $f(g(x)) = 1 + g^2(x)$
So, $1 + g^2(x) = 1 + x^2 - 2x^3 + x^4$
$g^2(x) = x^2 - 2x^3 + x^4 = x^2(1-x)^2 = x^2(x-1)^2$
$g(x) = \pm\ x(x-1)$
as $g(2) = 2 \Rightarrow g(x) = x(x-1)$.

9. **(b,c,d)** For $f(x)$ to be real
$x > 0$, $\ln x > 0$, $\ln(\ln x) > 0$ and $\ln(\ln(\ln x)) > 0$
$\Rightarrow x > 0, x > 1, x > e$ and $x > e^e \Rightarrow D = (e^e, \infty)$
Clearly Range of $f(x) = R \Rightarrow f(x)$ is onto

Also, $f'(x) = \frac{1}{x\ln(x)\ln(\ln x)} > 0$ if $x > e^e$

$\therefore f(x)$ is one -one in its domain.

10. **(a,c,d)** $f(x) = \sqrt{|x| - \{x\}}$

If $x \ge 0$ then $f(x) = \sqrt{x - (x - [x])} = \sqrt{[x]}$

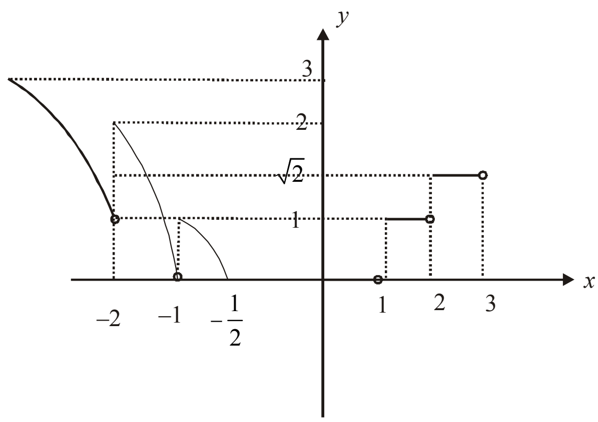
and $[x] \ge 0$, so $f(x)$ is defined

If $x < 0$ then $f(x) = \sqrt{-x - (x - [x])} = \sqrt{-2x + [x]}$

Now $-2x + [x] \le -2x - 1$ ($\because [x] \le -1$ if $x < 0$)

$\therefore f(x)$ is defined if $-2x - 1 \ge 0 \Rightarrow x \le -\frac{1}{2}$

Clearly for $x \le -\frac{1}{2}$ $f(x) \ge 0$ (see the graph)



So the range $Y = [0, \infty)$

11. **(a,c)** Let $f(x) = \frac{1 + \sin x}{\cos x(1 + \operatorname{cosec} x)}$

$= \frac{(1 + \sin x)\sin x}{\cos x(1 + \sin x)} = \tan x,\ x \ne \frac{n\pi}{2}$

Clearly, $f(x)$ has π as fundamental period
Let $f(x) = |\sin x| + |\cos x|$
$f(\pi + x) = |\sin x| + |\cos x| = f(x)$

$f\left(\frac{\pi}{2} + x\right) = |\cos x| + |\sin x| = f(x) \Rightarrow \frac{\pi}{2}$ is the fundamental period

Let $f(x) = \sin 2x + \cos 2x$
$f(\pi + x) = \sin 2x + \cos 2x = f(x)$

$f\left(\frac{\pi}{2} + x\right) = -\sin 2x - \cos 2x \ne f(x)$

is the fundamental period

Let $f(x) = \cos(\sin x) + \cos(\cos x)$

$f(\pi + x) = \cos(\sin x) + \cos(\cos x)$

$$f\left(\frac{\pi}{2}+x\right) = \cos(\cos x) + \cos(\sin x)$$

$\Rightarrow$ fundamental period is $\frac{\pi}{2}$

12. (a, b, c, d) $f(x) = \frac{|x-1|}{x^2} = \begin{cases} \frac{1-x}{x^2}, & x<1, x \neq 0 \\ \frac{x-1}{x^2}, & x \geq 1 \end{cases}$

$$f'(x) = \begin{cases} \frac{x-2}{x^3}, & x<1, x\neq 0 \\ \frac{2-x}{x^3}, & x>1 \end{cases}$$

$$f'(x) < 0 \Rightarrow \left.\begin{matrix} \frac{x-2}{x^3} < 0, & x<1, x\neq 0 \\ \frac{2-x}{x^3} < 0, & x>1 \end{matrix}\right\}$$

$\Rightarrow 0 < x < 1$ or $x > 2$

$\therefore f$ is decreasing in $(0, 1) \cup (2, \infty)$ and so one-one.

$$f'(x) > 0 \Rightarrow \left.\begin{matrix} \frac{x-2}{x^3} > 0, & x<1, x\neq 0 \\ \frac{2-x}{x^3} > 0 & x>1 \end{matrix}\right\}$$

$\Rightarrow x < 0$ or $1 < x < 2$

$\therefore f$ is increaisng in $(-\infty, 0) \cup (1, 2)$ and so one-one.

13. (b,c,d) Let $f(x) + \frac{a}{30} = 0$ ...(1)

where $f(x) = \frac{x^3 - 6x^2 + 11x - 6}{x^3 + x^2 - 10x + 8}$

$$= \frac{(x-1)(x-2)(x-3)}{(x-1)(x-2)(x+4)} = \frac{x-3}{x+4}, \quad x \neq 1, 2, -4$$

Range of $f(x) = R - \left\{1, -\frac{2}{5}, -\frac{1}{6}\right\}$

So equation (1) does not have solution if

$\frac{a}{30} = -1, \frac{2}{5}, \frac{1}{6}$

$a = -30, 12, 5$

E MATRIX-MATCH TYPE

1. A-r; B-q; C-r,s; D-p

(A) $\sin^{-1}(5x)$ is defined if

$$-1 \leq 5x \leq 1 \Rightarrow -\frac{1}{5} \leq x \leq \frac{1}{5}$$

(B) Clearly $0 \leq \sqrt{1-25x^2} \leq 1$

$\Rightarrow$ Range of $\sqrt{1-25x^2}$ is $[0, 1]$

(C) $f(x) = \frac{x+1}{x-1}$

$$\Rightarrow fof(x) = \frac{\left(\frac{x+1}{x-1}\right)+1}{\left(\frac{x+1}{x-1}\right)-1} = \frac{x+1+x-1}{x+1-x+1} = \frac{2x}{2} = x$$

$\Rightarrow f \, o \, f(x) = x, x \neq 1$

Period of $x - [x]$ is 1.

2. A-r; B-s; C-t; D-p

(A) Range of $a\cos x + b\sin x + c$ is

$$[c - \sqrt{a^2+b^2}, c + \sqrt{a^2+b^2}] = [-\sqrt{2}, \sqrt{2}]$$

(B) $-1 \leq -\cos 3x \leq 1 \Rightarrow 1 \leq 2 - \cos 3x \leq 3$

$$\Rightarrow 1 \geq \frac{1}{2-\cos 3x} \geq \frac{1}{3}$$

(C) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$

$$\Rightarrow \tan^{-1} x - \cot^{-1} x = \tan^{-1} x - \left(\frac{\pi}{2} - \tan^{-1} x\right)$$

$$= 2\tan^{-1} x - \frac{\pi}{2}$$

$$\therefore -\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2} \Rightarrow -\pi < 2\tan^{-1} x < \pi$$

$$\Rightarrow -\frac{3\pi}{2} < 2\tan^{-1} x - \frac{\pi}{2} < \frac{\pi}{2}$$

(D) Let $y = \frac{\tan x}{\tan 3x} = \frac{1 - 3\tan^2 x}{3 - \tan^2 x}$ $(\tan x \neq 0)$

$$\Rightarrow 3y - 1 = (y-3)\tan^2 x$$

$$\Rightarrow \tan^2 x = \frac{3y-1}{y-3} = \frac{(3y-1)(y-3)}{(y-3)^2} > 0$$

$$\Rightarrow y \in \left(-\infty, \frac{1}{3}\right) \cup (3, \infty)$$

3. **A-q,t; B-s; C-p,q,r; D-p**

(A) The equation implies that $(x^2-1)\cos x \ge 0$

$$\Rightarrow x^2-1\ge 0 \text{ and } \cos x \ge 0 \; or \; x^2-1 \le 0 \text{ and } \cos x \le 0$$

$$\Rightarrow -\frac{\pi}{2} \le x \le -1 \text{ or } 1 \le x \le \frac{\pi}{2}$$

(B) $[x^2] = x + 2\{x\} \Rightarrow [x]^2 = [x] + 3\{x\}$

$$\Rightarrow \{x\} = \frac{[x]^2-[x]}{3}$$

Thus

$$0 \le \frac{[x]^2-[x]}{3} < 1 \Rightarrow [x] \in \left(\frac{1-\sqrt{13}}{2}, 0\right] \cup \left[1, \frac{1+\sqrt{13}}{2}\right)$$

$\therefore [x] = -1, 0, 1, 2$

$\{x\} = \frac{2}{3}, 0, 0, \frac{2}{3} \Rightarrow x = -\frac{1}{3}, 0, 1, \frac{8}{3}$

(C) We have $-1 \le x \le 1 \Rightarrow -\frac{\pi}{4} \le \tan^{-1} x \le \frac{\pi}{4}$

and $f(x) = \frac{\pi}{2} + \tan^{-1} x \Rightarrow \frac{\pi}{4} \le f(x) \le \frac{3\pi}{4}$

$\therefore [f(x)] = 0, 1 \text{ or } 2$

(D) $\ln(\cos(\sin x)) \ge 0 \Rightarrow \cos(\sin x) \ge 1$

$\Rightarrow \cos(\sin x) = 1$. Hence

$\ln(\cos(\sin x)) = 0 \Rightarrow f(x) = 0 \;\forall\; x \in$ Domain .

4. **A-p, r, s; B-r,s; C-t; D-q,t**

(A) Put $x = y = 0$, then $2f(0) = 2\{f(0)\}^2 \Rightarrow f(0) = 1$. Now put $x = 0$, then $f(y) + f(-y) = 2f(0)f(y)$

$\therefore f(y) = f(-y) \Rightarrow f(x)$ is even $\Rightarrow f(x)$ is many one

Again $f(0) = 1 \Rightarrow f(x)$ cannot be onto

(B) If $x \le 0$, then $f(x) = 0 \Rightarrow f(x)$ is many one and neither even nor odd. Clearly $f(x)$ is not onto, for example $f(x) \ne -1$ for any x.

(C) $f(x)$ is either $-x^n+1$ or x^n+1. But $f(3) = 28$.

$\therefore f(x) = x^3 + 1$, which is neither even nor odd but one-one and onto both.

(D) Obviously $f(x)$ is odd and $f'(x) = 2 + \cos x > 0$, so $f(x)$ is one-one.

Also,

$f(x) \to \infty$ if $x \to \infty$ and $f(x) \to -\infty$ as $x \to -\infty$,

so f(x) is onto.

5. **A-r, B- s; C-q; D-p**

(A) $f(x) = \log(ax^3 + (b+a)x^2 + (b+c)x + c)$

$= \log(x(ax^2+bx+c) + ax^2+bx+c)$

$= \log\{(x+1)(ax^2+bx+c)\}$

as $b^2 - 4ac < 0, a > 0 \Rightarrow ax^2+bx+c > 0 \quad \forall\, x \in R$.

So, $x + 1 > 0 \Rightarrow x > -1$

Domain of $f(x)$ $(-1, \infty)$

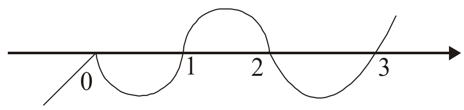
Range of $f(x)$ is clearly R.

(B) $f(x) = \ell n \tan^{-1}(x^3 - 6x^2 + 11x - 6)\, x\,(e^x - 1)$

$\Rightarrow x(e^x-1)(x^3-6x^2+11x-6) > 0$

$\Rightarrow x(e^x-1)(x-1)(x-2)(x-3) > 0$

Domain of $f(x)$ is $(1, 2) \cup (3, \infty)$. Range of $f(x)$ is clearly R.



(C) $f(x) = \frac{x^2-3x+2}{x^2+x-6} = \frac{(x-1)(x-2)}{(x+3)(x-2)} = \frac{x-1}{x+3}, x \ne 2, -3$

Let $y = \frac{x-1}{x+3} \Rightarrow xy + 3y = x - 1$

$x(y-1) = -(3y+1) \Rightarrow x = \frac{3y+1}{1-y} \Rightarrow y \ne 1$

Also, $x \to 2, \Rightarrow y \to \frac{1}{5}$

So, Range = $R - \left\{\frac{1}{5}, 1\right\}$

Domain = $R - \{-3, 2\}$

(D) $f(x) = \sin^2\frac{x}{4} + \cos\frac{x}{4}$

Clearly domain is R.

$f(x) = \sin^2\frac{x}{4} + \cos\frac{x}{4}$

$f(x) = 1 - \cos^2\frac{x}{4} + \cos\frac{x}{4}$

$$= 1 - \left(\cos^2\frac{x}{4} - \cos\frac{x}{4} + \frac{1}{4}\right) + \frac{1}{4} = \frac{5}{4} - \left(\cos\frac{x}{4} - \frac{1}{2}\right)^2$$

Range of $f(x)$ is $\left[-1, \frac{5}{4}\right]$.

6. **A-r; B-p; C-t; D-q**

(A) $f(x) = \sin^{-1}(x^2+1) + \cos^{-1}(x^2+1) + [1+x^2]^{1/x}$

$f(x)$ is defined if $x^2 + 1 \le 1 \Rightarrow x = 0 \Rightarrow f(x) = \frac{\pi}{2} + 1$

(B) $\cot\frac{\pi}{2}[x+T] = \cos\frac{\pi}{2}[x] = \begin{cases} 0 & \text{if } [x] \text{ is odd} \\ \text{not defined, if } [x] \text{ is even} \end{cases}$

$\therefore$ Perod of $f(x) = 2$

(C) $f\left(\frac{2\tan x}{1+\tan^2 x}\right) = \frac{(\cos 2x+1)(\sec^2 x + 2\tan x)}{2}$

$= \frac{(2\cos^2 x)(1+\tan x)^2}{2} = \frac{(1+\tan x)^2}{\sec^2 x}$

$= \frac{1+\tan^2 x + 2\tan x}{1+\tan^2 x} = 1 + \frac{2\tan x}{1+\tan^2 x}$

$\therefore f(x) = 1 + x$, whenever defined

(D) $f(x) = \tan^{-1}(2a - 2x - x^3)$ is onto $\Rightarrow$ Range $= \left(-\frac{\pi}{2}, 0\right]$

$\Rightarrow 2a - 2x - x^2 \le 0 \Rightarrow x^2 + 2x - 2a \ge 0 \;\forall\; x \in R$

$\Rightarrow (x+1)^2 - (1+2a) \ge 0 \;\forall\; x \in R \Rightarrow a = -\frac{1}{2}$

F NUMERIC/INTEGER ANSWER TYPE

1. **Ans. : 0**

Let $y = f(x) = \frac{x-1}{c-x^2+1}$

Take $y = -t$, where $t \in \left[\frac{1}{3}, 1\right]$,

$\therefore \quad -t = \frac{x-1}{c-x^2+1}$

$\Rightarrow \quad x^2 - c - 1 = \frac{x-1}{t} \Rightarrow x^2 - \frac{1}{t}x + \frac{1}{t} - c - 1 = 0$

As $-t \in \left[-1, -\frac{1}{3}\right]$, hence the above must not possess real solution

$\therefore \quad \left(\frac{1}{t}\right)^2 - 4\left(\frac{1}{t} - c - 1\right) < 0 \Rightarrow \frac{1}{t^2} - \frac{4}{t} + 4 < -4c$

$\Rightarrow c < -\frac{1}{4}\left(\frac{1}{t} - 2\right)^2$

Now, $\frac{1}{3} \le t \le l \Rightarrow 1 \le \frac{1}{t} - 2 \le 1 \Rightarrow -\frac{1}{4} \le \frac{1}{4}\left(\frac{1}{t} - 2\right)^2 \le 0$

Hence, $c \in \left(-\infty, -\frac{1}{4}\right]$

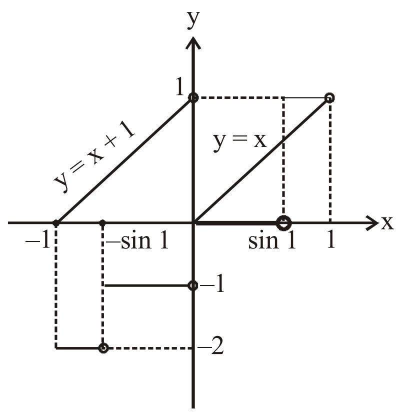
2. **Ans. : 1**

$\sin^{-1}x$ is defined if $-1 \le x \le 1$ we have

If $-1 \le x \le -\sin 1$ then $[\sin^{-1} x] = -2$

If $-\sin 1 \le x < 0$ then $[\sin^{-1} x] = -1$

If $0 \le x < \sin 1$, $[\sin^{-1} x] = 0$ and if $\sin 1 \le x < 1$, then $[\sin^{-1} x] = 1$



So, the graph of $y = [\sin^{-1} x]$ is shown, Also

$y = x - [x] = \begin{cases} x+1, & \text{if } -1 \le x < 0 \\ x, & \text{if } 0 \le x < 1 \\ 0, & \text{if } x = 1 \end{cases}$

Clearly from graph, the two curves meet at $x = 0$ only.

3. **Ans : 15**

$f(3\pi + x) = f(x) \Rightarrow \cos(3n\pi + nx)$

$\sin\left(\frac{15\pi + 5x}{n}\right) = \cos(nx)\cos\left(\frac{5x}{n}\right)$

Put $x = 0$, then $\cos 3n\pi \sin\left(\frac{15\pi}{n}\right) = 0$

Now $\cos 3n\pi \ne 0$ for any $n \in$ I, so,

$\sin\frac{15\pi}{n} = 0 \Rightarrow \frac{15\pi}{n} = k\pi, k \in I$

$\therefore \; n = \frac{15}{k} \Rightarrow$ The largest value of $n = 15$.

(c) Put $y = 0$ in the given relation, then

$$f\left(\frac{x}{2}\right)=\frac{f(x)+f(0)}{2}$$

$$\therefore \quad 2f\left(\frac{x}{2}\right)=f(x)+1 \Rightarrow f(2x)=2f(x)-1 \;.....(1)$$

Now,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\lim_{h\to 0}\frac{1}{h}\left[f\left\{\frac{2(x+h)}{2}\right\}-f(x)\right]$$

$$\lim_{h\to 0}\frac{1}{h}\left[\frac{f(2x)+f(2h)-2f(x)}{2}\right]=\lim_{h\to 0}\left[\frac{f(2h)-1}{2h}\right]$$

[from (1)]

$$\lim_{h\to 0}\frac{f(2h)-f(0)}{2h}=f'(0)=-1$$

$\therefore \quad f(x)=-x+c$. Put $x=0 \Rightarrow 1=0+c \Rightarrow c=1$

$\therefore \quad f(x)=-x+1 \quad \Rightarrow f(-1)=2$

5. Ans.: 1

$fog\,(x)=f(g(x))=f(4x\,(1-x))$

$\Rightarrow \dfrac{1-4x(1-x)}{1+4x(1-x)}$ when $0 \le 4x(1-x) \le 1$ and $0 \le x \le 1$

But $4x - 4x^2 \ge 0 \Rightarrow 0 \le x \le 1$

$4x - 4x^2 \le 1 \Rightarrow (2x-1)^2 \ge 0 \Rightarrow x \in R$

Hence $fog\,(x) = \dfrac{1-4x+4x^2}{1+4x-4x^2}$, $\quad 0 \le x \le 1$

Let $y = \dfrac{4x^2-4x+1}{-(4x^2-4x)+1}$, $\quad 0 \le x \le 1$

Put $4x^2 - 4x = t \quad t \in [-1, 0]$

$$y=\frac{1+t}{1-t}, \frac{dy}{dt}=\frac{1-t+1+t}{(1-t)^2}>0$$

Range of $fog\,(x) = [0, 1]$

$\Rightarrow \alpha + \beta = 1$

6. Ans.: 5

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{f\left(x\left(1+\frac{h}{x}\right)\right)-f(1.x)}{h}$$

$$=\lim_{h\to 0}\frac{f(x).f\left(1+\frac{h}{x}\right)+2-f(x)-f\left(1+\frac{h}{x}\right)-f(x).f(1)-2+f(x)+f(1)}{h}$$

$$=\lim_{h\to 0}\frac{(f(x)-1).f\left[\left(1+\frac{h}{x}\right)-f(1)\right]}{\frac{h}{x}.x}=\frac{f(x)-1}{x}.f'(1)$$

$$f'(x)=2\left\{\frac{f(x)-1}{x}\right\}$$

$$\Rightarrow \frac{f'(x)}{f(x)-1}=\frac{2}{x}$$

$\ell n\,(f(x)-1) = 2\,\ell n\, x + \ell n\, c$

$f(x) - 1 = cx^2$

$f(x) = cx^2 + 1$

as $f(1) = 2 \Rightarrow 2 = c + 1 \Rightarrow c = 1$

$\Rightarrow f(x) = x^2 + 1$