05

Work, Energy and Power

TOPIC 1
Work and Energy

01 A block moving horizontally on a smooth surface with a speed of 40 m/s splits into two parts with masses in the ratio of 1:2. If the smaller part moves at 60 m/s in the same direction, then the fractional change in kinetic energy is **[2021, 31 Aug Shift-II]**

(a) 1/3 (b) 2/3 (c) 1/8 (d) 1/4

Ans. *(c)*

Let a block of mass M splits into two masses m_1 and m_2 in ratio 1:2.

Then, mass of smaller part,

$$m_1 = \frac{M}{1+2} = \frac{M}{3}$$

Mass of bigger part, $m_2 = 2M/3$

Given, speed of mass M, $v = 40$ m/s

Speed of mass m_1, $v_1 = 60$ m/s

Let speed of mass m_2, $v_2 = v$

Using conservation of linear momentum,

$$\mathbf{p}_i = \mathbf{p}_f$$

$$M\mathbf{v} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$$

$$M \times 40 = \frac{M}{3} \times 60 + \frac{2M}{3} \times v \Rightarrow v = 30 \text{ m/s}$$

$\therefore$ The fractional change in kinetic energy,

$$\frac{\Delta K}{K} = \frac{K_f - K_i}{K_i} = \frac{K_f}{K_i} - 1$$

$$= \frac{\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2}{\frac{1}{2}Mv^2} - 1$$

$$= \frac{\frac{1}{2}\left[\frac{M}{3} \times (60)^2 + \frac{2M}{3} \times (30)^2\right]}{\frac{1}{2}M \times (40)^2} - 1$$

$$= \frac{9}{8} - 1 = \frac{1}{8}$$

02 A block moving horizontally on a smooth surface with a speed of 40 ms^{-1} splits into two equal parts. If one of the parts moves at 60 ms^{-1} in the same direction, then the fractional change in the kinetic energy will be $x : 4$, where x is **[2021, 31 Aug Shift-I]**

Ans. *(5)*

Given, initial speed of block, $u = 40\,\text{ms}^{-1}$

Let total mass of block $= m$

$\therefore$ Broken masses, $m_1 = m_2 = \frac{m}{2}$

Final speed of m_1, $v_1 = 60\,\text{ms}^{-1}$

As we know that, kinetic energy,

$$\text{KE} = 1/2\, mv^2$$

$\therefore$ Initial kinetic energy $\text{KE}_i = \frac{1}{2}mu^2$

$$= \frac{1}{2}m(40)^2 = 800\,m \quad \dots\text{(i)}$$

By using law of conservation of momentum,

$$mu = m_1v_1 + m_2v_2$$

$$\therefore \quad v_2 = \frac{m \times 40 - m/2 \times 60}{m/2}$$

$$= \frac{10\,m}{m/2} = 20\,\text{ms}^{-1}$$

So, final kinetic energy

$$\text{KE}_f = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$= \frac{1}{2}\frac{m}{2}(v_1^2 + v_2^2) = \frac{m}{4}(60^2 + 20^2)$$

$$= m/4\,(3600 + 400)$$

$$= 1000\,m \quad \dots\text{(ii)}$$

$\therefore$ Divide Eq (ii) by Eq. (i), we get

$$\frac{\text{KE}_f}{\text{KE}_i} = \frac{1000\,m}{800} = \frac{5}{4}m$$

So, $x = 5$

03 Two persons A and B perform same amount of work in moving a body through a certain distance d with application of forces acting at angle 45° and 60° with the direction of displacement respectively. The ratio of force applied by person A to the force applied by person B is $\frac{1}{\sqrt{x}}$. The value of x is .... **[2021, 27 Aug Shift-I]**

Ans. *(2)*

Given, work done by both person is same,

$$W_A = W_B$$

Direction of first force with displacement, $\theta_1 = 45°$

Direction of second force with displacement, $\theta_2 = 60°$

Ratio of force applied by person A to force applied by person B is $F_A : F_B = 1 : \sqrt{x}$.

Work done by both person is same,

$W_A = W_B \Rightarrow F_A d\cos\theta_1 = F_B d\cos\theta_2$

($\because$ distance is same for both $= d$)

$$\frac{F_A}{F_B} = \frac{\cos\theta_1}{\cos\theta_2} = \frac{\cos 60°}{\cos 45°}$$

$$\Rightarrow \quad \frac{F_A}{F_B} = \frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{F_A}{F_B} = \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2}} \Rightarrow x = 2$$

Thus, the value of x is 2.

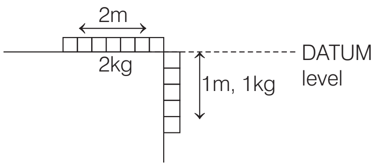
04 A uniform chain of length 3 m and mass 3 kg overhangs a smooth table with 2 m laying on the table. If k is the kinetic energy of the chain in joule as it completely slips off the table, then the value of k is ....

(Take, $g = 10\,\text{m/s}^2$)

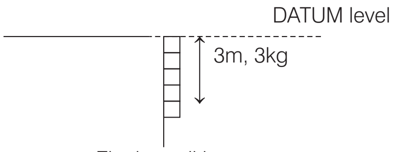
[2021, 26 Aug Shift-I]

Ans. *(40)*

The given situation is shown below as



Initial condition



Final condition

Applying law of conservation of energy, we get

$$K_{\text{initial}} + U_{\text{initial}} = K_{\text{final}} + U_{\text{final}}$$

$$K_{\text{initial}} = 0$$

$\Rightarrow U_{\text{initial}} = -1 \times 10 \times \dfrac{1}{2}$ (1 m lie below DATUM level, so negative)

$$K_{\text{final}} = ?$$

$$U_{\text{final}} = -3 \times 10 \times \frac{3}{2}$$

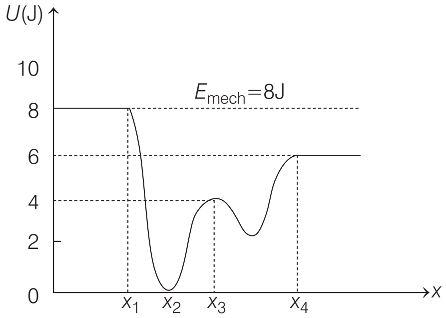
(3 m lie below DATUM level, so negative)

$$0 - 1 \times 10 \times \frac{1}{2} = K_{\text{final}} - 3 \times 10 \times \frac{3}{2}$$

$$\Rightarrow \quad -5 = K_{\text{final}} - 45$$

$$\Rightarrow \quad K_{\text{final}} = 45 - 5 = 40\,\text{J}$$

05 Given below is the plot of a potential energy function $U(x)$ for a system, in which a particle is in one-dimensional motion, while a conservative force $F(x)$ acts on it. Suppose that $E_{\text{mech}} = 8\,\text{J}$, the incorrect statement for this system is **[2021, 27 July Shift-II]**



[where, KE = kinetic energy]

(a) at $x > x_4$, KE is constant throughout the region.

(b) at $x < x_1$, KE is smallest and the particle is moving at the slowest speed.

(c) at $x = x_2$, KE is greatest and the particle is moving at the fastest speed.

(d) at $x = x_3$, KE = 4 J.

Ans. *(b)*

From the diagram given in question,

Case I When $x > x_4$;

$$U = \text{constant} = 6\,\text{J}$$

$$K = E_{\text{mech}} - U = 2\,\text{J} = \text{constant}$$

Case II When $x < x_1$;

$$U = \text{constant} = 8\,\text{J}$$

$$K = E_{\text{mech}} - U = 8 - 8 = 0$$

It means that particle is at rest.

Case III When $x = x_2$; $U = 0$

$$\Rightarrow \quad K = E_{\text{mech}} = 8\,\text{J}$$

It means that KE is greatest and particle is moving at the fastest spee(d)

Case IV When $x = x_3$; $U = 4\,\text{J}$

$$\Rightarrow \quad U + K = 8\,\text{J}$$

$$\Rightarrow \quad K = 4\,\text{J}$$

06 A force of $F = (5y + 20)\hat{\mathbf{j}}$ N acts on a particle. The work done by this force when the particle is moved from $y = 0$ m to $y = 10$ m is ...... J. **[2021, 25 July Shift-II]**

Ans. *(450)*

Given, force, $\mathbf{F} = (5y + 20)\hat{\mathbf{j}}$ N

Since, work done $(W) = \int_0^{10} F \cdot dy$

$$\therefore \quad W = \int_0^{10} (5y + 20)\,dy$$

$$= \frac{5y^2}{2}\Big|_0^{10} + 20y\Big|_0^{10}$$

$$= \frac{5}{2}(10^2 - 0^2) + 20(10 - 0)$$

$$= \frac{5}{2}(100) + 200$$

$$= 250 + 200 = 450\,\text{J}$$

07 A porter lifts a heavy suitcase of mass 80 kg and at the destination lowers it down by a distance of 80 cm with a constant velocity. Calculate the work done by the porter in lowering the suitcase. [Take, $g = 9.8\,\text{ms}^{-2}$] **[2021, 22 July Shift-II]**

(a) -62720.0 J (b) -627.2 J

(c) $+627.2$ J (d) 784.0 J

Ans. *(b)*

Given, mass of suitcase, $m = 80$ kg

Let height lowered by porter, $h = 80$ cm

$$= 80 \times 10^{-2}\,\text{m}$$

As we know that,

Work $(W) = -mgh$

$$\therefore \quad W = -80 \times 9.8 \times 80 \times 10^{-2}$$

$$= -627.2\,\text{J}$$

08 If the kinetic energy of a moving body becomes four times of its initial kinetic energy, then the percentage change in its momentum will be **[2021, 20 July Shift-II]**

(a) 100% (b) 200%

(c) 300% (d) 400%

Ans. *(a)*

We know that,

Kinetic energy, $K = \dfrac{p^2}{2m} \Rightarrow$

$$p \propto \sqrt{K} \quad \text{...(i)}$$

where, p = linear momentum of the body

and m = mass of the body.

Considering Eq. (i), we can write

$$\frac{p_2}{p_1} = \frac{\sqrt{K_2}}{\sqrt{K_1}} \quad \text{...(ii)}$$

According to question,

$$\Rightarrow \quad K_2 = 4K_1 \quad \text{...(iii)}$$

Putting the value of K_2 in Eq. (ii), we get

$$\frac{p_2}{p_1} = \frac{\sqrt{4K_1}}{\sqrt{K_1}} \Rightarrow \frac{p_2}{p_1} = 2$$

$$\Rightarrow \quad p_2 = 2p_1$$

$\therefore$ Percentage change in momentum

$$= \frac{p_2 - p_1}{p_1} \times 100$$

$$= \frac{2p_1 - p_1}{p_1} \times 100$$

$$= \frac{p_1}{p_1} \times 100 = 100\%$$

09 In a spring gun having spring constant 100 N/m a small ball B of mass 100 g is put in its barrel (as shown in figure) by compressing the spring through 0.05 m. There should be a box placed at a distance d on the ground, so that the ball falls in it. If the ball leaves the gun horizontally at a height of 2 m above the ground. The value of d is .... m.

(Take, $g = 10\,\text{m/s}^2$)

[2021, 20 July Shift-I]

Ans. *(0.003)*

Given, $k = 100\,\text{N/m}$

$m = 100\,\text{g} = 0.1\,\text{kg}$

$x = 0.05\,\text{m}$ and $H = 2\,\text{m}$

By energy conservation,

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = x\sqrt{\frac{k}{m}}$$

$$= 0.05 \times \sqrt{\frac{100}{0.1}} = 0.5\sqrt{10}\ \text{ms}^{-1} \quad \text{...(i)}$$

Time of flight of ball, $t = \sqrt{\frac{2H}{g}}$

$$\Rightarrow \quad t = \sqrt{\frac{2 \times 2}{10}} = \frac{2}{\sqrt{10}}\,\text{s} \quad \text{...(ii)}$$

$\therefore$ Range of ball, $d = vt$

$$= 0.5\sqrt{10} \times \left(\frac{2}{\sqrt{10}}\right)$$

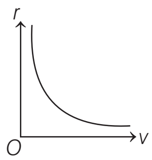
[From Eqs. (i) and (ii)]

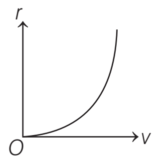
$$= 0.5 \times 2 = 30 \times 10^{-3}\,\text{m} = 0.003\,\text{m}$$

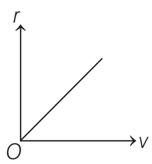
10 A particle of mass m moves in a circular orbit under the central potential field, $U(r) = \frac{-C}{r}$, where C is a positive constant.

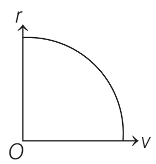
The correct radius-velocity graph of the particle's motion is

[2021, 18 March Shift-II]

(a) 

(b) 

(c) 

(d) 

Ans. *(a)*

The central potential field when particle moves in circular orbit,

$$U(r) = -\frac{C}{r}$$

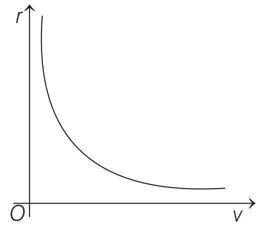
We know that,

$$F = -\frac{dU}{dr}$$

$$\Rightarrow \quad F = -\frac{d}{dr}\left(-\frac{C}{r}\right)$$

$$\Rightarrow \quad |F| = -\frac{C}{r^2}$$

$$\Rightarrow \quad \frac{mv^2}{r} = -\frac{C}{r^2} \Rightarrow v^2 \propto \frac{1}{r}$$

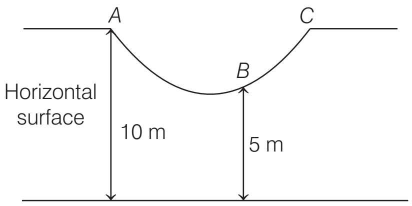


The graph between velocity and radius is hyperbolic.

11 As shown in the figure, a particle of mass 10 kg is placed at a point a. When the particle is slightly displaced to its right, it starts moving and reaches the point b. The speed of the particle at B is x m/s. (Take, $g = 10\,\text{m/s}^2$)

The value of x to the nearest integer is ....

[2021, 18 March Shift-I]



Ans. *(10)*

Given,

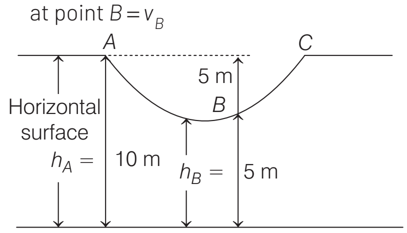
The mass of the particle, $m = 10\,\text{kg}$

The speed of the particle at point A,

$v_A = 0\,\text{m/s}$

The elevation of the point A from the point B, $h_A = 5 + h_B$

Let's consider the speed of the particle at point $B = v_B$



Using the law of conservation of energy.

Energy at point A = Energy at point B

$$\frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B$$

Substituting the values in the above equations, we get

$$\frac{1}{2}(10)(0)^2 + 10 \times 10 \times (5 + h_B)$$

$$= \frac{1}{2}(10)v_B^2 + 10 \times 10 \times h_B$$

$$v_B = 10\,\text{m/s} = x\,\text{m/s}$$

$\therefore \quad x = 10$

12 Two particles having masses 4 g and 16 g respectively are moving with equal kinetic energies. The ratio of the magnitudes of their linear momentum is $n : 2$. The value of n will be ....

[2021, 25 Feb Shift-II]

Ans. *(1)*

Given, mass of particle A, $m_A = 4\,\text{g}$

Mass of particle B, $m_B = 16\,\text{g}$

Kinetic energy of A and B is same

i.e. $\text{KE}_A = \text{KE}_B$

As, kinetic energy (KE) $= p^2/2m$

where, p is momentum and m is mass.

$$\therefore \quad \frac{(p_A)^2}{m_A} = \frac{(p_B^2)}{m_B} \Rightarrow \frac{p_A^2}{4} = \frac{p_B^2}{16} \Rightarrow \frac{p_A}{p_B} = \frac{1}{2}$$

$\because$ linear momentum is $n : 2$.

$\therefore \quad n = 1$

13 Two solids A and B of mass 1 kg and 2 kg respectively are moving with equal linear momentum. The ratio of their kinetic energies $(\text{KE})_A : (\text{KE})_B$ will be $\frac{A}{1}$, so the value of A will be ....

[2021, 24 Feb Shift-II]

Ans. *(2)*

Given, $m_A = 1\,\text{kg}$, $m_B = 2\,\text{kg}$,

$(\text{KE})_A : (\text{KE})_B = A : 1$

Linear momentum of A and B are equal.

$\Rightarrow \quad p_A = p_B$

$\because$ Kinetic energy (KE) $= p^2/2m$

$$\therefore \quad \frac{\text{KE}_A}{\text{KE}_B} = \frac{m_B}{m_A} = \frac{2}{1} = \frac{A}{1} \Rightarrow A = 2$$

14 A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine, so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizontally a distance of 0.2 m, while launching the ball, the value of F (in N) is (Take, $g = 10\,\text{ms}^{-2}$) ....

[2020, 3 Sep Shift-I]

Ans. *(150)*

Work done by bowling machine = Initial kinetic energy of ball = Final potential energy of ball.

$\Rightarrow$ Force × displacement $= mgh$

$$\Rightarrow \quad F(0.2) = (0.15)(10)(20)$$

$$F = 150\,\text{N}$$

15 A particle moving in the xy-plane experiences a velocity dependent force $\mathbf{F} = k(v_y \hat{\mathbf{i}} + v_x \hat{\mathbf{j}})$, where v_x and v_y are the x and y components of its velocity $\mathbf{v}$.

If $\mathbf{a}$ is the acceleration of the particle, then which of the following statements is true for the particle?

[2020, 6 Sep Shift-II]

(a) Quantity $\mathbf{v} \times \mathbf{a}$ is constant in time.
(b) $\mathbf{F}$ arises due to a magnetic field.
(c) Kinetic energy of particle is constant in time.
(d) Quantity $\mathbf{v} \cdot \mathbf{a}$ is constant in time.

Ans. *(a)*

Given that, force, $\mathbf{F} = k(v_y \hat{\mathbf{i}} + v_x \hat{\mathbf{j}})$

Acceleration, $\mathbf{a} = \frac{k}{m}(v_y \hat{\mathbf{i}} + v_x \hat{\mathbf{j}})$

or $a_x \hat{\mathbf{i}} + a_y \hat{\mathbf{j}} = \frac{k}{m}(v_y \hat{\mathbf{i}} + v_x \hat{\mathbf{j}})$

$$\frac{dv_x}{dv_y} = \frac{k}{m} v_y \text{ and } \frac{dv_y}{dt} = \frac{k}{m} v_x$$

On dividing, $\frac{dv_x}{dv_y} = \frac{v_y}{v_x}$ or $v_x dv_x = v_y dv_y$

Integrating both sides, we get

$$v_x^2 = v_y^2 + c \quad \text{...(i)}$$

Option (a)

$$\mathbf{v} \times \mathbf{a} = (v_x \hat{\mathbf{i}} + v_y \hat{\mathbf{j}}) \times \frac{k}{m}(v_y \hat{\mathbf{i}} + v_x \hat{\mathbf{j}})$$

$$= \frac{k}{m}(v_x^2 - v_y^2)\hat{\mathbf{k}} = \left(\frac{k}{m} \times c\right)\hat{\mathbf{k}}$$

[From Eq. (i)]

which is constant.

Option (b)

Magnetic force can never change the speed of a charged particle. So, force (F) does not arise due to magnetic field.

Option (c)

The given force is not central force. So, work has to be done by this force and it will bring the change in kinetic energy.

Option (d)

$$\mathbf{v} \cdot \mathbf{a} = (v_x \hat{\mathbf{i}} + v_y \hat{\mathbf{j}}) \times \frac{k}{m}(v_y \hat{\mathbf{i}} + v_x \hat{\mathbf{j}})$$

$$= \frac{2k}{m} v_x v_y$$

which is not constant as force is not zero.

Hence, correct option is (a).

16 A person pushes a box on a rough horizontal platform surface. He applies a force of 200 N over a distance of 15 m. Thereafter, he gets progressively tired and his applied force reduces linearly with distance to 100 N. The total distance through which the box has been moved is 30 m. What is the work done by the person during the total movement of the box?

[2020, 4 Sep Shift-II]

(a) 5250 J (b) 2780 J
(c) 3280 J (d) 5690 J

Ans. *(a)*

For $0 \le x \le 15$ m, $F = 200$ N ...(i)

For $15 \text{ m} < x \le 30$ m,

Force F is linearly decreasing from 200 N to 100 N.

So, using two-point form of straight line, we have

$$(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$(F - F_1) = \left(\frac{F_2 - F_1}{x_2 - x_1}\right)(x - x_1)$$

Here, $x_1 = 15$ m, $F_1 = 200$ N
$x_2 = 30$ m, $F_2 = 100$ N

So, $(F - 200) = \left(\frac{100 - 200}{30 - 15}\right)(x - 15)$

$$F - 200 = \frac{-100}{15}(x - 15)$$

$$F = 200 - \frac{20}{3}(x - 15)$$

$$F = 200 - \frac{20}{3}x + 100$$

$$F = \left(300 - \frac{20}{3}x\right)\text{N} \quad \text{...(ii)}$$

Therefore,

$$F = \begin{cases} 200\,\text{N}; & 0 \le x \le 15\,\text{m} \\ \left(300 - \frac{20}{3}x\right)\text{N}; & 15\,\text{m} < x \le 30\,\text{m} \end{cases}$$

Now, work done during the complete movement of the box,

$$W = \int_0^{30} F\,dx$$

$$= \int_0^{15} 200\,dx + \int_{15}^{30}\left(300 - \frac{20x}{3}\right)dx$$

$$= 200[x]_0^{15} + \left[300x - \frac{20}{3}\frac{x^2}{2}\right]_{15}^{30}$$

$$= 200[15 - 0] + \left[300x - \frac{10}{3}x^2\right]_{15}^{30}$$

$$= 200 \times 15 + \left[\left\{300(30) - \frac{10}{3}(30)^2\right\} - \left\{300(15) - \frac{10}{3}(15)^2\right\}\right]$$

$$= 3000 + [\{9000 - 3000\} - \{4500 - 750\}]$$

$$= 3000 + [6000 - 3750]$$

$$= 3000 + 2250 = 5250 \text{ J}$$

Hence, option (a) is correct.

17 If the potential energy between two molecules is given by $U = -\frac{A}{r^6} + \frac{B}{r^{12}}$, then at equilibrium, separation between molecules and the potential energy are

[2020, 6 Sep Shift-I]

(a) $\left(\frac{B}{2A}\right)^{1/6}, -\frac{A^2}{2B}$ (b) $\left(\frac{B}{A}\right)^{1/6}, 0$

(c) $\left(\frac{2B}{A}\right)^{1/6}, -\frac{A^2}{4B}$ (d) $\left(\frac{2B}{A}\right)^{1/6}, -\frac{A^2}{2B}$

Ans. *(c)*

Potential energy between two molecules is given by

$$U = -\frac{A}{r^6} + \frac{B}{r^{12}}$$

From the relation between force and potential energy,

Force acting between them, $F = -\frac{dU}{dr}$

$$\therefore \quad F = -\frac{d}{dr}\left(-\frac{A}{r^6} + \frac{B}{r^{12}}\right)$$

$$= -\left(\frac{6A}{r^7} - \frac{12B}{r^{13}}\right)$$

$$= \frac{6}{r^7}\left(-A + \frac{2B}{r^6}\right)$$

At equilibrium, $F = 0$

$$\Rightarrow \quad \frac{6}{r^7}\left(-A + \frac{2B}{r^6}\right) = 0$$

$$\Rightarrow \quad -A + \frac{2B}{r^6} = 0 \quad (\because r \ne 0)$$

$$\Rightarrow \quad r^6 = \frac{2B}{A}$$

or $$r = \left(\frac{2B}{A}\right)^{\frac{1}{6}}$$

The above calculated value of r is the separation between molecules at equilibrium.

Now, putting this value in the expression of potential energy, we get

$$U = -\frac{A}{\left\{\left(\frac{2B}{A}\right)^{\frac{1}{6}}\right\}^6} + \frac{B}{\left\{\left(\frac{2B}{A}\right)^{\frac{1}{6}}\right\}^{12}}$$

$$= -\frac{A^2}{2B} + \frac{A^2}{4B}$$

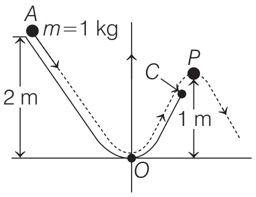
$$\therefore \quad U = -\frac{A^2}{4B}$$

18 A particle ($m = 1$ kg) slides down a frictionless track (AOC) starting from rest at a point A (height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaching its highest point P (height 1 m), the kinetic energy of the particle (in J) is : (Figure drawn is schematic and not to scale; take $g = 10\,\text{ms}^{-2}$) .... **[2020, 7 Jan Shift-I]**

Ans. *(10)*

Following is the situation given :

Since, the given path AOC is frictionless. So, it starting from point A, the particle during the path AOC will attain maximum height at P.

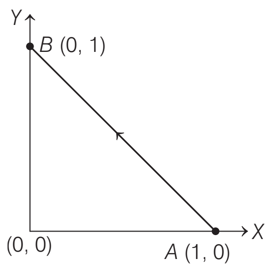


Energy conservation at A and P gives

PE at A = (PE + KE) at P

$$\Rightarrow \quad U_A = U_P + K_P$$
$$mgh_A = mgh_P + K_P$$
$$1 \times 10 \times 2 = 1 \times 10 \times 1 + K_P$$
$$\therefore \quad K_P = 10\,\text{J}$$

19 Consider a force $\mathbf{F} = -x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$. The work done by this force in moving a particle from point $A(1,0)$ to $B(0,1)$ along the line segment is (all quantities are in SI units) **[2020, 9 Jan Shift-I]**



(a) $\frac{3}{2}$ (b) 2 (c) 1 (d) $\frac{1}{2}$

Ans. *(c)*

Work done by a variable force on the particle,

$$W = \int \mathbf{F} \cdot d\mathbf{r} = \int \mathbf{F} \cdot (dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}})$$

$\therefore$ In two dimension, $d\mathbf{r} = dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}$

and it is given $\mathbf{F} = -x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$.

$$\therefore \quad W = \int (-x\hat{\mathbf{i}} + y\hat{\mathbf{j}}) \cdot (dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}})$$
$$= \int -x\,dx + y\,dy = \int -x\,dx + \int y\,dy$$

As particle is displaced from $A(1, 0)$ to $B(0, 1)$, so x varies from 1 to 0 and y varies from 0 to 1.

So, with limits, work will be

$$W = \int_1^0 -x\,dx + \int_0^1 y\,dy$$
$$= \left[\frac{-x^2}{2}\right]_1^0 + \left[\frac{y^2}{2}\right]_0^1$$
$$= \frac{1}{2}(0 - (-1)^2 + (1)^2 - 0) = 1\,\text{J}$$

20 A force acts on a 2 kg object, so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds? **[2019, 9 Jan Shift-II]**

(a) 850 J (b) 900 J (c) 950 J (d) 875 J

Ans. *(b)*

Here, the displacement of an object is given by

$$x = (3t^2 + 5)\,\text{m}$$

Therefore, velocity $(v) = \frac{dx}{dt} = \frac{d(3t^2 + 5)}{dt}$

or $\quad v = 6t$ m/s $\quad$ ...(i)

The work done in moving the object from $t = 0$ to $t = 5$ s

$$W = \int_{x_0}^{x_5} F \cdot dx \qquad \text{...(ii)}$$

The force acting on this object is given by

$$F = ma = m \times \frac{dv}{dt}$$
$$= m \times \frac{d(6t)}{dt} \quad [\text{using Eq. (i)}]$$
$$F = m \times 6 = 6m = 12\,\text{N}$$

Also, $\quad x_0 = 3t^2 + 5 = 3 \times (0)^2 + 5 = 5$ m

and at $t = 5$ s,

$$x_5 = 3 \times (5)^2 + 5 = 80\,\text{m}$$

Put the values in Eq. (ii), we get

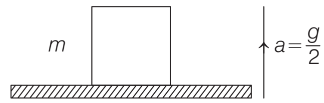
$$W = 12 \times \int_{x_0}^{x_5} dx = 12\,[80 - 5]$$
$$W = 12 \times 75 = 900\,\text{J}$$

Alternate Solution

To using work - kinetic energy theorem is,

$$W = \Delta \text{K} \cdot \text{E} = \frac{1}{2}m(v_f^2 - v_i^2)$$
$$= \frac{1}{2}m \times (30^2 - 0^2)$$
$$= \frac{1}{2} \times 2 \times 900 = 900\,\text{J}$$

21 A block of mass m is kept on a platform which starts from rest with constant acceleration $\frac{g}{2}$ upwards as shown in figure. Work done by normal reaction on block in time t is **[2019, 10 Jan Shift-I]**

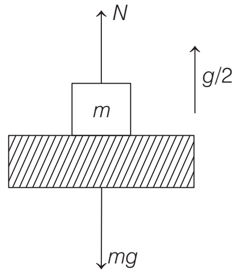


(a) $\frac{mg^2t^2}{8}$ (b) $\frac{3mg^2t^2}{8}$

(c) 0 (d) $-\frac{mg^2t^2}{8}$

Ans. *(b)*

Normal reaction force on the block is



$$N = ma_{\text{net}}$$

where, a_{net} = net acceleration of block.

$$= g + a$$
$$= g + \frac{g}{2} = \frac{3g}{2}$$
$$\Rightarrow \quad N = m\left(g + \frac{g}{2}\right) = \frac{3mg}{2}$$

Now, in time 't' block moves by a displacement s given by

$$s = 0 + \frac{1}{2}at^2$$
$$= \frac{1}{2}\left(\frac{g}{2}\right)t^2 \qquad (\because u = 0)$$

Here, $\quad a = \frac{g}{2}$ (given)

$\therefore$ Work done = Force $\times$ Displacement

$$\Rightarrow \quad W = \frac{3mg}{2} \times \frac{gt^2}{4}$$
$$= \frac{3mg^2t^2}{8}$$

22 A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is **[JEE Main 2018]**

(a) $-\frac{k}{4a^2}$ (b) $\frac{k}{2a^2}$

(c) zero (d) $-\frac{3}{2}\frac{k}{a^2}$

Ans. *(c)*

$\therefore$ Force $= -\frac{dU}{dr}$

$\Rightarrow \quad F = -\frac{d}{dr}\left(\frac{-k}{2r^2}\right) = -\frac{k}{r^3}$

As particle is on circular path, this force must be centripetal force.

$\Rightarrow \quad |F| = \frac{mv^2}{r}$

So, $\frac{k}{r^3} = \frac{mv^2}{r} \Rightarrow \frac{1}{2}mv^2 = \frac{k}{2r^2}$

$\therefore$ Total energy of particle $=$ KE $+$ PE

$= \frac{k}{2r^2} - \frac{k}{2r^2} = 0$

Total energy $= 0$

23 A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10$ ms^{-1}. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be **[JEE Main 2017]**

(a) 10^{-3} kgs^{-1} (b) 10^{-4} kgm^{-1}

(c) 10^{-1} kgm^{-1}s^{-1} (d) 10^{-3} kgm^{-1}

Ans. *(b)*

Given, force, $F = -kv^2$

$\therefore$ Acceleration, $a = \frac{-k}{m}v^2$

or $\frac{dv}{dt} = \frac{-k}{m}v^2 \Rightarrow \frac{dv}{v^2} = -\frac{k}{m}.dt$

Now, with limits, we have

$$\int_{10}^{v}\frac{dv}{v^2} = -\frac{k}{m}\int_0^t dt$$

$\Rightarrow \quad \left(-\frac{1}{v}\right)_{10}^{v} = -\frac{k}{m}t \Rightarrow \frac{1}{v} = 0.1 + \frac{kt}{m}$

$\Rightarrow \quad v = \frac{1}{0.1 + \frac{kt}{m}} = \frac{1}{0.1 + 1000k}$

$\Rightarrow \quad \frac{1}{2} \times m \times v^2 = \frac{1}{8}mv_0^2 \Rightarrow v = \frac{v_0}{2} = 5$

$\Rightarrow \quad \frac{1}{0.1 + 1000\,k} = 5$

$\Rightarrow \quad 1 = 0.5 + 5000\,k$

$\Rightarrow \quad k = \frac{0.5}{5000}$

$\Rightarrow \quad k = 10^{-4}$ kg/m

24 A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m, 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipate(d) How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted into mechanical energy with a 20% of efficiency rate. (Take, $g = 9.8\ \text{ms}^{-2}$) **[JEE Main 2016 (Offline)]**

(a) 2.45×10^{-3} kg

(b) 6.45×10^{-3} kg

(c) 9.89×10^{-3} kg

(d) 12.89×10^{-3} kg

Ans. *(d)*

Given, potential energy burnt by lifting weight

$= mgh = 10 \times 9.8 \times 1 \times 1000 = 9.8 \times 10^4$

If mass lost by a person be m, then energy dissipated

$= m \times \frac{2}{10} \times 3.8 \times 10^7$ J

$\Rightarrow \quad 9.8 \times 10^4 = m \times \frac{1}{5} \times 3.8 \times 10^7$

$\Rightarrow \quad m = \frac{5}{3.8} \times 10^{-3} \times 9.8$

$= 12.89 \times 10^{-3}$ kg

25 When a rubber band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$, where a and b are constants. The work done in stretching the unstretched rubber band by L is **[JEE Main 2014]**

(a) $aL^2 + bL^3$ (b) $\frac{1}{2}(aL^2 + bL^3)$

(c) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (d) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

Ans. *(c)*

Key Idea As, we know that change in potential energy of a system corresponding to a conservative internal force as

$$U_f - U_i = -W = -\int_i^t F.d\,r$$

Given, $F = ax + bx^2$

According to work-energy theorem, we know that work done in stretching the rubber band by t is

$|dW| = Fdx$

$|W| = \int_0^L (ax + bx^2)\,dx$

$= \left[\frac{ax^2}{2}\right]_0^L + \left[\frac{bx^3}{3}\right]_0^L$

$= \left[\frac{aL^2}{2} - \frac{a \times (0)^2}{2}\right] + \left[\frac{b \times L^3}{3} - \frac{b \times (0)^3}{3}\right]$

$\therefore \quad |W| = \frac{aL^2}{2} + \frac{bL^3}{3}$

26 This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

If two springs S_1 and S_2 of force constants k_1 and k_2 respectively are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement I If stretched by the same amount, work done on S_1 will be more than that on S_2.

Statement II $k_1 < k_2$ **[AIEEE 2012]**

(a) Statement I is false, Statement II is true

(b) Statement I is true, Statement II is false

(c) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I

(d) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

Ans. *(a)*

As no relation between k_1 and k_2 is given in the question, that is why, nothing can be predicted about Statement I. But as in Statement II, $k_1 < k_2$.

Then, for same force,

$$W = F \cdot x = F \cdot \frac{F}{k} = \frac{F^2}{k} \Rightarrow W \propto \frac{1}{k}$$

i.e., $W_1 > W_2$

But for same displacement,

$$W = F \cdot x = \frac{1}{2}kx \cdot x = \frac{1}{2}kx^2 \Rightarrow W \propto k$$

i.e., $W_1 < W_2$

Thus, Statement II is true and Statement I is false.

27 An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range **[AIEEE 2008]**

(a) 200 J – 500 J
(b) 2×10^5 J – 3×10^5 J
(c) 20000 J – 50000 J
(d) 2000 J – 5000 J

Ans. *(d)*

The given question is somewhat based on approximations. Let mass of athlete be 65k g.

Approx velocity from the given data is 10m /s.

So, $\quad \text{KE} = \dfrac{65\times100}{2} = 3250\text{ J}$

So, option (d) is the most probable answer.

28 A block of mass 0.50 kg is moving with a speed of 2 .00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is **[AIEEE 2008]**

(a) 0.16 J (b) 1.00 J
(c) 0.67 J (d) 0.34 J

Ans. *(c)*

From law of conservation of momentum, we have

$m_1v_1 + m_2v_2 = (m_1 + m_2)v$

Given, $\quad m_1 = 0.50\text{ kg}, v_1 = 2\text{ ms}^{-1},$

$m_2 = 1\text{ kg}, v_2 = 0 \quad$ [at rest]

$0.5\times2 + 1\times0 = 1.5\times v$

[assumed that 2nd body is at rest]

$\Rightarrow \quad v = \dfrac{2}{3}$

$\therefore \quad \Delta K = K_f - K_i$

$$= \frac{1.5\times\left(\frac{2}{3}\right)^2}{2} - (0.5)\times\frac{2^2}{2} = -\frac{2}{3}\text{J}$$

$= -0.67$ J

So, energy lost is 0.67 J.

29 A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes a uncompressed spring and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10000 N/m. The spring compresses by **[AIEEE 2007]**

(a) 5.5 cm (b) 2.5 cm
(c) 11.0 cm (d) 8.5 cm

Ans. *(a)*

According to work-energy theorem

Loss in kinetic energy = Work done against friction + Potential energy of spring

$$\frac{1}{2}mv^2 = f\,x + \frac{1}{2}kx^2$$

$\Rightarrow \quad \dfrac{1}{2}\times2(4)^4 = 15\,x + \dfrac{1}{2}\times10000\,x^2$

$\Rightarrow \quad 5000\,x^2 + 15x - 16 = 0$

$\therefore \quad x = 0.055\text{ m} = 5.5\text{ cm}$

30 A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is **[AIEEE 2007]**

(a) K (b) zero
(c) $\dfrac{K}{4}$ (d) $\dfrac{K}{2}$

Ans. *(c)*

Kinetic energy at highest point,

$(\text{KE})_H = \dfrac{1}{2}mv^2\cos^2\theta$

$= K\cos^2\theta = K(\cos60°)^2 = K/4$

31 The potential energy of a 1 kg particle free to move along the x-axis is given by

$$V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)\text{J}$$

The total mechanical energy of the particle is 2 J. Then, the maximum speed (in ms^{-1}) is **[AIEEE 2006]**

(a) $\dfrac{3}{\sqrt2}$ (b) $\sqrt2$
(c) $\dfrac{1}{\sqrt2}$ (d) 2

Ans. *(a)*

Given, $\quad V(x) = \left(\dfrac{x^4}{4} - \dfrac{x^2}{2}\right)$

For minimum value of V, $\dfrac{dV}{dx} = 0$

$\Rightarrow \quad \dfrac{4x^3}{4} - \dfrac{2x}{2} = 0$

$\Rightarrow \quad x = 0, \quad x = \pm1$

So, $\quad V_{\min}(x=\pm1) = \dfrac{1}{4} - \dfrac{1}{2} = \dfrac{-1}{4}$ J

Now, $K_{\max} + V_{\min}$ = Total mechanical energy

$\Rightarrow \quad K_{\max} = \left(\dfrac{1}{4}\right) + 2 \;$ or $\; K_{\max} = \dfrac{9}{4}$

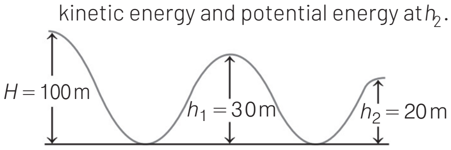
or $\quad \dfrac{mv^2}{2} = \dfrac{9}{4} \;$ or $\; v = \dfrac{3}{\sqrt2}\text{ ms}^{-1}$

32 A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is **[AIEEE 2005]**

(a) 40 m/s (b) 20 m/s
(c) 10 m/s (d) $10\sqrt{30}$ m/s

Ans. *(a)*

According to conservation of energy, potential energy at height H is sum of kinetic energy and potential energy at h_2.



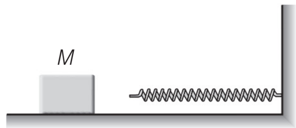
$\therefore \quad mgH = \dfrac{1}{2}mv^2 + mgh_2$

$\Rightarrow \quad mg(H - h_2) = \dfrac{1}{2}mv^2$

or $\quad v = \sqrt{2g(100-20)}$

or $\quad v = \sqrt{2\times10\times80} = 40$ m/s

33 The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant k and compresses it by length L. The maximum momentum of the block after collision is **[AIEEE 2005]**



(a) $\sqrt{Mk}\,L$ (b) $\dfrac{kL^2}{2M}$
(c) zero (d) $\dfrac{ML^2}{k}$

Ans. *(a)*

Momentum would be maximum when KE would be maximum and this is the case when total elastic PE is converted into KE.

According to conservation of energy,

$$\frac{1}{2}kL^2 = \frac{1}{2}Mv^2$$

$\Rightarrow \quad kL^2 = \dfrac{(Mv)^2}{M}$

or $\quad MkL^2 = p^2 \quad [\because p = Mv]$

$\Rightarrow \quad p = L\sqrt{Mk}$

34 A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to **[AIEEE 2004]**

(a) x^2 (b) e^x (c) x (d) $\log_e x$

Ans. *(a)*

From given information $a = -kx$, where a is acceleration, x is displacement and k is a proportionality constant,

$$\frac{vdv}{dx} = -kx \Rightarrow v\,dv = -kx\,dx$$

Let for any displacement from 0 to x, the velocity changes from v_0 to v.

$$\Rightarrow \int_{v_0}^{v} v\,dv = -\int_0^x kx\,dx$$

$$\Rightarrow \left[\frac{v^2}{2}\right]_{v_0}^{v} = -k\left[\frac{x^2}{2}\right]_0^x$$

$$\Rightarrow \frac{v^2 - v_0^2}{2} = -\frac{kx^2}{2}$$

$$\Rightarrow m\left(\frac{v^2 - v_0^2}{2}\right) = -\frac{mkx^2}{2}$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = -\frac{mkx^2}{2}$$

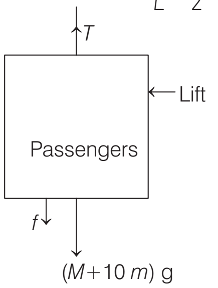
$$\Rightarrow \Delta K \propto x^2 \quad [\Delta K \text{ is loss in KE}]$$

35 A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table? **[AIEEE 2004]**

(a) 7.2 J (b) 3.6 J
(c) 120 J (d) 1200 J

Ans. *(b)*

Mass per unit length $= \frac{M}{L} = \frac{4}{2} = 2$ kg/m



The mass of 0.6 m of chain $= 0.6 \times 2 = 1.2$ kg

The height of centre of mass of hanging part

$$h = \frac{0.6 + 0}{2} = 0.3 \text{ m}$$

Hence, work done in pulling the chain on the table = Work done against gravity force

i.e, $W = mgh$

$$= 1.2 \times 10 \times 0.3 = 3.6 \text{ J}$$

36 A force $\mathbf{F} = (5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$ N is applied over a particle which displaces it from its origin to the point $\mathbf{r} = (2\mathbf{i} - \mathbf{j})$ m. The work done on the particle in joule is **[AIEEE 2004]**

(a) −7 (b) +7 (c) +10 (d) +13

Ans. *(b)*

Work done in displacing the particle

$$W = \mathbf{F} \cdot \mathbf{r}$$

$$= (5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) \cdot (2\mathbf{i} - \mathbf{j})$$

$$= 5 \times 2 + 3 \times (-1) + 2 \times 0 = 10 - 3 = 7 \text{ J}$$

37 A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then, the work required to stretch it further by another 5 cm is **[AIEEE 2003]**

(a) 12.50 N-m (b) 18.75 N-m
(c) 25.00 N-m (d) 6.25 N-m

Ans. *(b)*

Work done to stretch the spring by 5 cm from mean position $= W_1 = \frac{1}{2}kx_1^2$

$$= \frac{1}{2} \times 5 \times 10^3 \times (5 \times 10^{-2})^2$$

$$= 6.25 \text{ J}$$

Work done to stretch the spring by 10 cm from mean position, $W_2 = \frac{1}{2}k(x_1 + x_2)^2$

$$= \frac{1}{2} \times 5 \times 10^3 (5 \times 10^{-2} + 5 \times 10^{-2})^2 = 25 \text{ J}$$

Net work done to stretch the spring from 5 cm to 10 cm $= W_2 - W_1$

$$= 25 - 6{\cdot}25$$

$$= 18.75 \text{ J} = 18.75 \text{ N-m}$$

38 Consider the following two statements.

I. Linear momentum of a system of particles is zero.
II. Kinetic energy of a system of particles is zero. Then, **[AIEEE 2003]**

(a) I does not imply II and II does not imply I
(b) I implies II but II does not imply I
(c) I does not imply II but II implies I
(d) I implies II and II implies I

Ans. *(c)*

Here, II is implying I but I is not implying II as kinetic energy of a system of particles is zero means speed of each and every particle is zero which says that momentum of every particle is zero.

But statement I means linear momentum of a system of particles is zero, which may be true even, if particles have equal and opposite momentums and hence having non-zero kinetic energy.

39 A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is **[AIEEE 2002]**

(a) 16 J (b) 8 J (c) 32 J (d) 24 J

Ans. *(b)*

The work done on the spring is stored as the PE of the body and is given by

$$U = \int_{x_1}^{x_2} F_{\text{ext}}\,dx$$

or $$U = \int_{x_1}^{x_2} kx\,dx = \frac{1}{2}k(x_2^2 - x_1^2)$$

$$= \frac{800}{2}[(0.15)^2 - (0.05)^2]$$

$$= 400(0.2 \times 0.1) = 8 \text{ J}$$

40 A ball whose kinetic energy is E, is projected at an angle of 45° to the horizontal. The kinetic energy of the ball at the highest point of its flight will be **[AIEEE 2002]**

(a) E (b) $\frac{E}{\sqrt{2}}$ (c) $\frac{E}{2}$ (d) zero

Ans. *(c)*

At the highest point of its flight, vertical component of velocity is zero and only horizontal component is left which is

$$u_x = u\cos\theta$$

Given, $\theta = 45°$

$$\therefore \quad u_x = u\cos 45° = \frac{u}{\sqrt{2}}$$

Hence, at the highest point, kinetic energy is

$$E' = \frac{1}{2}mu_x^2 = \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2}m\left(\frac{u^2}{2}\right)$$

$$= \frac{E}{2} \qquad \left[\because \frac{1}{2}mu^2 = E\right]$$

TOPIC 2

Work energy theorem, Power and Vertical Circle

41 A body of mass m dropped from a height h reaches the ground with a speed of $0.8\sqrt{gh}$. The value of workdone by the air-friction is **[2021, 1 Sep Shift-II]**

(a) −0.68 mgh (b) mgh
(c) 1.64 mgh (d) 0.64 mgh

Ans. *(a)*

Given, the mass of the body $= m$

The height from which the body dropped $= h$

The speed of the body when reached the ground, $v_f = 0.8\sqrt{gh}$

Initial velocity of the body, $v = 0$ m/s

Using the work-energy theorem,

Work done by gravity + Work done by air-friction = Final kinetic energy − Initial kinetic energy

$$W_{mg} + W_{\text{air-friction}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Here, work done by gravity $= mgh$

$\Rightarrow$

$$mgh + W_{\text{air-friction}} = \frac{1}{2}m(0.8\sqrt{gh})^2 - \frac{1}{2}m(0)^2$$

$$\Rightarrow \quad W_{\text{air-friction}} = \frac{0.64\,mgh}{2} - mgh$$

$$= 0.32mgh - mgh$$

$$= -0.68\,mgh$$

The value of the work done by the air friction is $-0.68\,mgh$.

42 An engine is attached to a wagon through a shock absorber of length 1.5 m. The system with a total mass of 40000 kg is moving with a speed of 72 kmh^{-1}, when the brakes are applied to bring it to rest. In the process of the system being brought to rest, the spring of the shock absorber gets compressed by 1.0 m. If 90% of energy of the wagon is lost due to friction, the spring constant is .... $\times 10^5$ N/m. **[2021, 1 Sep Shift-II]**

Ans. *(16)*

Given, the length of the shock absorber, $l = 1.5$ m

The total mass of the system, $M = 40000$ kg

The speed of the wagon, $v = 72$ km/h

When brakes are applied, the final velocity, $v_f = 0$

The compressed spring of the shock absorber, $x = 1$ m

Applying the work-energy theorem,

Work done by the system = Change in kinetic energy

$$W = \Delta \text{KE}$$

$$W_{\text{friction}} + W_{\text{spring}} = \frac{1}{2}mv_f^2 + \frac{1}{2}mv_i^2$$

$$-\frac{90}{100}\left(\frac{1}{2}mv^2\right) + W_{\text{spring}} = 0 - \frac{1}{2}mv_i^2$$

($\because$ 90% energy lost due to friction)

$$W_{\text{spring}} = -\frac{10}{100} \times \frac{1}{2}mv^2$$

$$-\frac{1}{2}kx^2 = \frac{1}{20}mv^2$$

$$k = \frac{mv^2}{10 \times x^2}$$

Substituting the values in the above equation, we get

$$k = \frac{40000 \times \left(72 \times \frac{5}{18}\right)^2}{10(1)^2}$$

$$= 16 \times 10^5 \text{ N/m}$$

Comparing the spring constant,

$$k = x \times 10^5$$

The value of the $x = 16$.

43 An automobile of mass m accelerates starting from origin and initially at rest, while the engine supplies constant power P. The position is given as a function of time by **[2021, 27 July Shift-II]**

(a) $\left(\frac{9P}{8m}\right)^{\frac{1}{2}} \cdot t^{\frac{3}{2}}$ (b) $\left(\frac{8P}{9m}\right)^{\frac{1}{2}} \cdot t^{\frac{2}{3}}$

(c) $\left(\frac{9m}{8P}\right)^{\frac{1}{2}} \cdot t^{\frac{3}{2}}$ (d) $\left(\frac{8P}{9m}\right)^{\frac{1}{2}} \cdot t^{\frac{3}{2}}$

Ans. *(d)*

According to given situation energy supplied in delivering the constant power P is equal to the kinetic energy of the automobile.

$\therefore$ Energy supplied = Kinetic energy

$$\Rightarrow \quad Pt = \frac{1}{2}mv^2 \quad \Rightarrow v = \left(\frac{2Pt}{m}\right)^{1/2}$$

or $$\frac{ds}{dt} = \left(\frac{2Pt}{m}\right)^{1/2} \quad \left[\because v = \frac{ds}{dt}\right]$$

Integrating both sides of above equation, we get

$$\int_s^0 ds = \left(\frac{2P}{m}\right)^{1/2} \int_0^t t^{1/2} \cdot dt$$

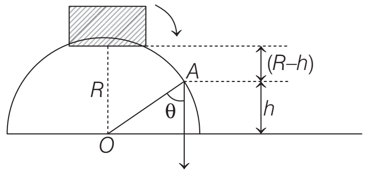
$$\Rightarrow \quad s = \left(\frac{2P}{m}\right)^{1/2} \cdot \frac{2}{3}t^{3/2}$$

$$\Rightarrow \quad s = \frac{2}{3}\left(\frac{2P}{m}\right)^{1/2} \cdot t^{3/2}$$

$$\Rightarrow \quad s = \left(\frac{8P}{9m}\right)^{1/2} \cdot t^{3/2}$$

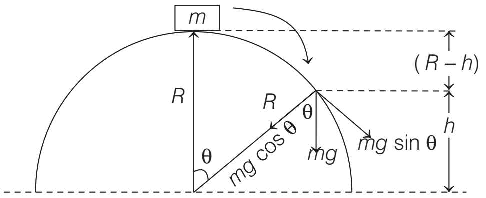
44 A small block slides down from the top of hemisphere of radius $R = 3$ m as shown in the figure. The height h at which the block will lose contact with the surface of the sphere is .... m.

(Assume there is no friction between the block and the hemisphere) **[2021, 27 July Shift-II]**



Ans. *(2)*

The given figure can also be represented as



$\because$ From work - energy theorem, we have

Work done = Change in kinetic energy

$\Rightarrow \quad W = \Delta\text{KE}$

$\Rightarrow \quad Mg(R - R\cos\theta) = \frac{1}{2}mv^2$

$\Rightarrow \quad v = \sqrt{2gR(1-\cos\theta)}$ ...(i)

To loose contact,

$$\frac{mv^2}{R} = mg\cos\theta \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{m[2gR(1-\cos\theta)]}{R} = mg\cos\theta$$

$\Rightarrow \quad 2(1-\cos\theta) = \cos\theta$

$\Rightarrow \quad 2 - 2\cos\theta = \cos\theta$

$\Rightarrow \quad 2 = 3\cos\theta$

$\Rightarrow \quad \cos\theta = \frac{2}{3}$

$\because \quad \cos\theta = \frac{h}{R}$ (using the figure)

$\Rightarrow \quad \cos\theta = \frac{2}{3} = \frac{h}{R} = \frac{h}{3}$

$\Rightarrow \quad h = 2$ m

45 A pendulum bob has a speed of 3 m/s at its lowest position. The pendulum is 50 cm long. The speed of bob when the length makes an angle of 60° to the vertical will be .... m/s. (Take, $g = 10$ m/s^2) **[2021, 25 July Shift-I]**

Ans. *(2)*

Given, speed of bob at lowest point,

$$v_1 = 3\text{ms}^{-1}$$

Length of string, $l = 50 \text{ cm} = 50 \times 10^{-2}$ m

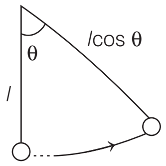
Angle, $\theta = 60^\circ$

Let v_2 be the speed of bob at 60° from mean position.

By using law of conservation of energy

$$\text{KE}_i + \text{PE}_i = \text{KE}_f + \text{PE}_f$$

$$\Rightarrow \quad \frac{1}{2}mv_1^2 + 0 = \frac{1}{2}mv_2^2 + mgl(1-\cos\theta)$$



$$\Rightarrow \quad \frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 - mgl(1-\cos\theta)$$

$$\Rightarrow \quad v_2^2 = v_1^2 - 2gl(1-\cos\theta)$$

$$\Rightarrow$$

$$v_2 = \sqrt{3^2 - 2\times 10\times 50\times 10^{-2}\,(1-\cos 60^\circ)}$$

$$= \sqrt{9 - 10\left(1-\frac{1}{2}\right)} = \sqrt{9-5}$$

$$= \sqrt{4} = 2 \text{ ms}^{-1}$$

46 A body at rest is moved along a horizontal straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to

[2021, 20 July Shift-II]

(a) $t^{\frac{3}{2}}$ (b) $t^{\frac{1}{2}}$ (c) $t^{\frac{1}{4}}$ (d) $t^{\frac{3}{4}}$

Ans. *(a)*

According to the quesiton, we can say that, the energy of the machine is delivering a constant power which is further used up in moving a body (from rest) along a horizontal straight line.

$\therefore$ Energy supply $= Pt$

where, $P =$ power

and $t =$ time taken.

$$\Rightarrow \quad \frac{1}{2}mv^2 = Pt$$

$$\Rightarrow \quad v^2 \propto t \Rightarrow v \propto \sqrt{t}$$

or $\quad \dfrac{ds}{dt} = c\sqrt{t}$ ...(i)

where, c is any constant.

Now, integrating Eq. (i), we get

$$\int_0^S ds = \int_0^t c\sqrt{t}\,dt \Rightarrow s = c\int_0^t t^{\frac{1}{2}} \cdot dt$$

$$\Rightarrow \quad s = \frac{2c}{3}t^{\frac{3}{2}} \Rightarrow s \propto t^{3/2}$$

So, the distance moved by the body in time t is proportional to $t^{\frac{3}{2}}$.

47 A constant power delivering machine has towed a box, which was initially at rest, along a horizontal straight line. The distance moved by the box in time t is proportional to

[2021, 18 March Shift-I]

(a) $t^{2/3}$ (b) $t^{3/2}$ (c) t (d) $t^{1/2}$

Ans. *(b)*

We know that,

Power = force × velocity [$\because$ Given, power = constant]

$\therefore$ Force × velocity = constant

$\Rightarrow \quad ma \times v =$ constant

$\Rightarrow \quad a \times v =$ constant

$$\Rightarrow \quad \left(v\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right) = C \Rightarrow \int v\,dv = \int C\,dt$$

$$\Rightarrow \quad \frac{v^2}{2} = Ct \Rightarrow v = \sqrt{2Ct}$$

$$\Rightarrow \quad \frac{dx}{dt} = \sqrt{2Ct} \qquad \left[\because v = \frac{dx}{dt}\right]$$

$$\Rightarrow \quad \int dx = \int \sqrt{2Ct}\,dt$$

$$\Rightarrow \quad x = \sqrt{2C}\left(\frac{t^{3/2}}{3/2}\right) \Rightarrow x \propto t^{3/2}$$

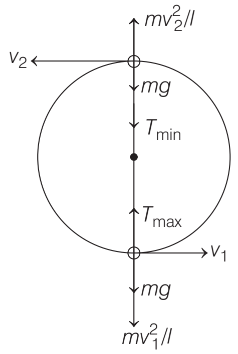
48 A small bob tied at one end of a thin string of length 1m is describing a vertical circle, so that the maximum and minimum tension in the string are in the ratio 5 : 1. The velocity of the bob at the highest position is .... m/s. (Take, $g = 10 \text{ m/s}^2$)

[2021, 25 Feb Shift-I]

Ans. *(5)*

Given, length of string, $l = 1$m

$T_{\max}$ and $T_{\min}$ be the tension in string and v_1 and v_2 be the velocities of bob at bottom and top in vertical circle.



$$T_{\max} = mg + mv_1^2/l$$

and $\quad T_{\min} = mv_2^2/l - mg$

$$\because \quad \frac{T_{\max}}{T_{\min}} = \frac{mg + mv_1^2/l}{mv_2^2/l - mg} = \frac{5}{1} \quad \text{(given)}$$

$$\Rightarrow \quad mg + mv_1^2/l = 5mv_2^2/l - 5mg$$

Here, $\quad v_1 = \sqrt{v_2^2 + 4gl}$

$$\Rightarrow \quad mg + \frac{m}{l}(v_2^2 + 4gl) = \frac{5mv_2^2}{l} - 5mg$$

$$\Rightarrow \quad g + \frac{v_2^2 + 4gl}{l} = \frac{5v_2^2}{l} - 5g$$

$$\Rightarrow \quad 6gl = 5v_2^2 - v_2^2 - 4gl$$

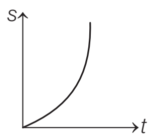
$$\Rightarrow \quad 10gl = 4v_2^2$$

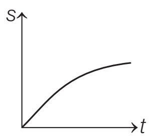
$$\Rightarrow \quad v_2 = \sqrt{\frac{10\,gl}{4}} = \sqrt{\frac{10\times 10\times 1}{4}}$$

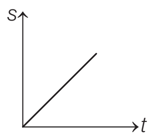
$$= \sqrt{25} = 5 \text{ m/s}$$

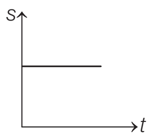
49 A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s)-time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale)

[2020, 3 Sep Shift-II]

(a) 

(b) 

(c) 

(d) 

Ans. *(a)*

As, power $= \dfrac{d}{dt}(\text{KE}) =$ constant

$$\Rightarrow \quad \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = P \Rightarrow \frac{1}{2}m\cdot 2v\frac{dv}{dt} = P$$

$$\Rightarrow \quad v\,dv = \frac{P}{m}\cdot dt$$

Integrating both sides, we get

$$\int v\,dv = \frac{P}{m}\int dt \Rightarrow \frac{v^2}{2} = \frac{P}{m}\cdot t$$

$$\Rightarrow \quad v = \sqrt{\frac{2P}{m}}\cdot t^{1/2} \Rightarrow \frac{ds}{dt} = \sqrt{\frac{2P}{m}}\cdot t^{1/2}$$

$$\Rightarrow \quad ds = \sqrt{\frac{2P}{m}}\cdot t^{1/2}dt$$

Again, integrating both sides, we get

$$s = \int ds = \int\sqrt{\frac{2P}{m}}\cdot t^{1/2}dt \Rightarrow s = C\cdot t^{3/2}$$

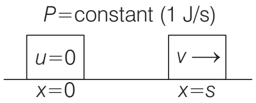
where, C is a constant $= \dfrac{2}{3}\sqrt{\dfrac{2P}{m}}$.

Hence, displacement (s)-time (t) graph is correctly represented in option (a).

50 A body of mass 2 kg is driven by an engine delivering a constant power of 1 J/s. The body starts from rest and moves in a straight line. After 9 s, the body has moved a distance (in m) .... **[2020, 5 Sep Shift-II]**

Ans. *(18)*

Let s be the required distance.



From Work-Energy theorem,

Work = Change in kinetic energy

$\Rightarrow \quad$ Power × Time = ΔK

i.e., $\quad Pt = \Delta K \Rightarrow Pt = \frac{1}{2}mv^2 \quad$ ...(i)

Given, $\quad P = 1\,\text{Js}^{-1}$, $t = 9\,\text{s}$, $m = 2\,\text{kg}$

Substituting all the given values in eq. (i), we get

$$1 \times 9 = \frac{1}{2}(2)v^2$$

$$v^2 = 9 \Rightarrow v = 3\,\text{m/s} \quad (\text{at } t = 9\,\text{s})$$

As, $\quad Fv = P \Rightarrow (ma)v = P \quad [\because F = ma]$

$$\Rightarrow \quad m\left[\frac{dv}{dt}\right]v = P \Rightarrow m\left[\frac{ds}{dt}\frac{dv}{ds}\right]v = P$$

$$\Rightarrow \quad m\left[v\frac{dv}{ds}\right]v = P$$

$$\Rightarrow \quad 2v^2 dv = ds \quad \{\because P = 1\,\text{J/s and } m = 2\,\text{kg}\}$$

Integrating both sides,

$$\int_0^3 2v^2 dv = \int_0^s ds \Rightarrow \frac{2}{3}[v^3]_0^3 = s$$

$$\frac{2}{3}[27 - 0] = s \Rightarrow s = 18\,\text{m}$$

Hence, after 9 s, the body has moved a distance of 18 m.

51 A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to (Take, 1 HP = 746 W, $g = 10\,\text{ms}^{-2}$) **[2020, 7 Jan Shift-I]**

(a) $2.0\,\text{ms}^{-1}$ (b) $1.5\,\text{ms}^{-1}$
(c) $1.9\,\text{ms}^{-1}$ (d) $1.7\,\text{ms}^{-1}$

Ans. *(c)*

At maximum load, force provided by motor to pull the lift,

F = weight carried + friction = $mg + f$

$= (2000 \times 10) + 4000 = 24000\,\text{N}$

Power delivered by motor at speed v of load, $P = F \times v$

$$\Rightarrow \quad v = \frac{P}{F} = \frac{60 \times 746}{24000} = 1.865 = 1.9\,\text{ms}^{-1}$$

52 An elevator in a building can carry a maximum of 10 persons with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed of 3 m/s. The frictional force opposing the motion is 6000 N.

If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator ($g = 10\,\text{m/s}^2$) must be at least **[2020, 7 Jan Shift-II]**

(a) 62360 W (b) 48000 W
(c) 56300 W (d) 66000 W

Ans. *(d)*

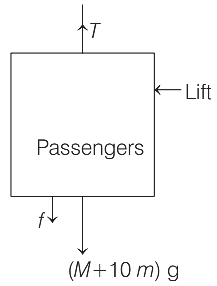
Mass of elevator, $M = 920$ kg

Mass of all '10' passengers carried by elevator $= 10 \times m = 10 \times 68 = 680$ kg

Total weight of elevator and passengers

$= (M + 10\,m)g = (920 + 680) \times 10$

$= 16000$ N



Force of friction = 6000 N

Total force (T) applied by the motor of elevator

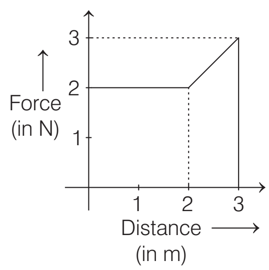
$= 16000 + 6000 = 22000$ N

Power delivered by elevator's motor,

$P = F \cdot v = 22000 \times 3 \quad [\because v = 3\,\text{ms}^{-1}]$

$= 66000$ W

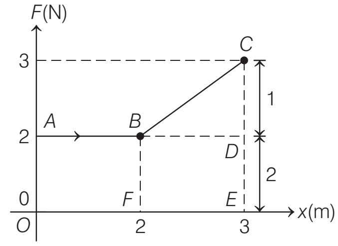
53 A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is **[2019, 8 April Shift-I]**



(a) 4 J (b) 2.5 J
(c) 6.5 J (d) 5 J

Ans. *(c)*

Key Idea Area under force-displacement graph gives the value of work done.



$\therefore$ Work done on the particle

= Area under the curve ABC

W = Area of square $ABFO$ + Area of ΔBCD + Area of rectangle $BDEF$

$= 2 \times 2 + \frac{1}{2} \times 1 \times 1 + 2 \times 1$

$= 6.5$ J

Now, from work-energy theorem,

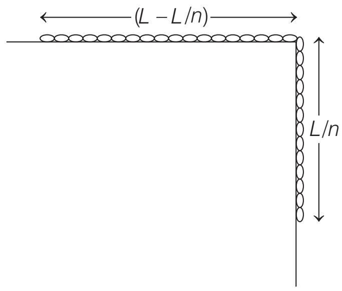
$$\Delta W = K_f - K_i$$

$$\Rightarrow \quad K_f = \Delta W = 6.5\,\text{J} \quad [\because K_i = 0]$$

54 A uniform cable of mass M and length L is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)$th part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be **[2019, 9 April Shift-I]**

(a) $\frac{2MgL}{n^2}$ (b) $nMgL$
(c) $\frac{MgL}{n^2}$ (d) $\frac{MgL}{2n^2}$

Ans. *(d)*

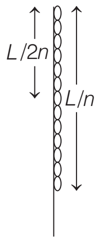


Given, mass of the cable is M.

So, mass of $\frac{1}{n}$th part of the cable, i.e. hanged part of the cable is $= M/n \quad$ ...(i)

Now, centre of mass of the hanged part will be its middle point.

So, its distance from the top of the table will be $L/2n$.



$\therefore$ Initial potential energy of the hanged part of cable,

$$U_i = \left(\frac{M}{n}\right)(-g)\left(\frac{L}{2n}\right)$$

$$\Rightarrow \quad U_i = -\frac{MgL}{2n^2} \quad \text{...(ii)}$$

When whole cable is on the table, its potential energy will be zero.

$$\therefore \quad U_f = 0 \quad \text{...(iii)}$$

Now, using work-energy theorem,

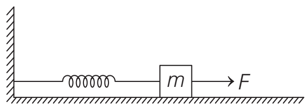
$$W_{\text{net}} = \Delta U = U_f - U_i$$

$$\Rightarrow \quad W_{\text{net}} = 0 - \left(-\frac{MgL}{2n^2}\right)$$

[using Eqs. (ii) and (iii)]

$$\Rightarrow \quad W_{\text{net}} = \frac{MgL}{2n^2}$$

55 A block of mass m lying on a smooth horizontal surface is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed as shown in the figure. The block is initially at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is **[2019, 9 Jan Shift-I]**



(a) $\frac{\pi F}{\sqrt{mk}}$ (b) $\frac{F}{\sqrt{mk}}$

(c) $\frac{2F}{\sqrt{mk}}$ (d) $\frac{F}{\pi\sqrt{mk}}$

Ans. ***(b)***

In a spring-block system, when a block is pulled with a constant force F, then its speed is maximum at the mean position. Also, it's acceleration will be zero.

In that case, force on the system is given as,

$$F = kx \quad \text{...(i)}$$

where, x is the extension produced in the spring.

or $$x = \frac{F}{k}$$

Now we know that, for a system vibrating at its mean position, its maximum velocity is given as,

$$v_{\max} = A\omega$$

where, A is the amplitude and ω is the angular velocity.

Since, the block is at its mean position.

So, $$A = x = \frac{F}{k}$$

$$v_{\max} = \frac{F}{k}\sqrt{\frac{k}{m}} \quad \left[\because \omega = \sqrt{\frac{k}{m}}\right]$$

$$= \frac{F}{\sqrt{km}}$$

Alternate Solution

According to the work-energy theorem, net work done = change in the kinetic energy

Here, net work done = work done due to external force (W_{ext}) + work done due to the spring (W_{spr}).

As, $$W_{\text{ext}} = F \cdot x$$

and $$W_{\text{spr}} = \frac{-1}{2}kx^2$$

$$\Rightarrow \quad \Delta\text{KE} = F \cdot x + \left(-\frac{1}{2}kx^2\right)$$

$$(\Delta\text{KE})_f - (\Delta\text{KE})_i = F \cdot x - \frac{1}{2}kx^2$$

$$\Rightarrow \frac{1}{2}mv_{\max}^2 - \frac{1}{2}m(0)^2 = F \cdot \left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2$$

[using Eq. (i)]

$$\Rightarrow \quad \frac{1}{2}mv_{\max}^2 = \frac{F^2}{k} - \frac{F^2}{2k} = \frac{F^2}{2k}$$

or $$v_{\max}^2 = \frac{F^2}{km}$$

$$\Rightarrow \quad v_{\max} = F/\sqrt{km}$$

56 A particle which is experiencing a force, is given by $\mathbf{F} = 3\hat{\mathbf{i}} - 12\hat{\mathbf{j}}$, undergoes a displacement of $\mathbf{d} = 4\hat{\mathbf{i}}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement ? **[2019, 10 Jan Shift-II]**

(a) 9 J (b) 15 J

(c) 12 J (d) 10 J

Ans. ***(b)***

We know that, work done in displacing a particle at displacement $\mathbf{d}$ under force $\mathbf{F}$ is given by $\Delta W = \mathbf{F} \cdot \mathbf{d}$

By substituting given values, we get

$$\Rightarrow \quad \Delta W = (3\hat{i} - 12\hat{j}) \cdot (4\hat{i})$$

$$\Rightarrow \quad \Delta W = 12 \text{ J} \quad \text{... (i)}$$

Now, using work-energy theorem, we get

work done (ΔW) = change in kinetic energy (ΔK)

or $$\Delta W = K_2 - K_1 \quad \text{... (ii)}$$

Comparing Eqs. (i) and (ii), we get

$$K_2 - K_1 = 12 \text{ J}$$

or $$K_2 = K_1 + 12 \text{ J}$$

Given, initial kinetic energy, $K_1 = 3$ J

$\therefore$ Final kinetic energy,

$$K_2 = 3 \text{ J} + 12 \text{ J}$$

$$= 15 \text{ J}$$

57 A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 s will be **[JEE Main 2017]**

(a) 22 J (b) 9 J

(c) 18 J (d) 4.5 J

Ans. ***(d)***

From Newton's second law,

$$\frac{\Delta p}{\Delta t} = F \Rightarrow \Delta p = F\Delta t$$

$$\therefore \quad p = \int dp = \int_0^1 F\,dt$$

$$\Rightarrow \quad p = \int_0^1 6t\,dt = 3\,\text{kg}\left(\frac{\text{m}}{\text{s}}\right)$$

Also, change in kinetic energy

$$\Delta k = \frac{\Delta p^2}{2m} = \frac{3^2}{2 \times 1} = 4.5$$

From work-energy theorem,

work done = change in kinetic energy.

So, work done = $\Delta k = 4.5$ J

58 At time $t = 0$, particle starts moving along the x-axis. If its kinetic energy increases uniformly with time t, the net force acting on it must be proportional to **[AIEEE 2011]**

(a) $\sqrt{t}$ (b) constant

(c) t (d) $1/\sqrt{t}$

Ans. ***(d)***

Given, $k \propto t \Rightarrow \frac{dk}{dt} = \text{constant}$

$$\Rightarrow \quad K \propto t$$

$$\frac{1}{2}mv^2 \propto t \Rightarrow v \propto \sqrt{t}$$

Also, $$P = Fv = \frac{dK}{dt} = \text{constant}$$

$$\Rightarrow \quad F \propto \frac{1}{v}$$

$$\Rightarrow \quad F \propto \frac{1}{\sqrt{t}}$$

Alternate Solution

$$K \propto t\; v \propto \sqrt{t},$$

$$F = ma = m\frac{dv}{dt}$$

As, $a = \frac{dv}{dt} = k\frac{1}{2}t^{-1/2}$

$a \propto \frac{1}{\sqrt{t}}$, so $F \propto \frac{1}{\sqrt{t}}$

59 A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it until the string makes an angle of 45° with the initial vertical direction is **[AIEEE 2006]**

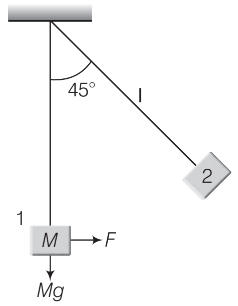
(a) $Mg(\sqrt{2}+1)$ (b) $Mg\sqrt{2}$

(c) $\frac{Mg}{\sqrt{2}}$ (d) $Mg(\sqrt{2}-1)$

Ans. *(d)*

Here, the constant horizontal force required to take the body from position 1 to position 2 can be calculated by using work-energy theorem.

Let us assume that body be taken slowly, so that its speed does not change, then



$$\Delta K = 0 = W_F + W_{Mg} + W_{\text{tension}}$$

[symbols have their usual meanings]

$$W_F = F \times l \sin 45° = \frac{Fl}{\sqrt{2}}$$

$$W_{Mg} = Mg(l - l\cos 45°),\ W_{\text{tension}} = 0$$

$\therefore$ $F = Mg(\sqrt{2}-1)$

60 A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest, assuming that it faces constant resistance to motion? **[AIEEE 2005]**

(a) 3.0 cm (b) 2.0 cm

(c) 1.5 cm (d) 1.0 cm

Ans. *(d)*

According to work-energy theorem,

Total work done = Change in kinetic energy

$$W = \Delta K$$

Case I $-F \times 3 = \frac{1}{2}m\left(\frac{v_0}{2}\right)^2 - \frac{1}{2}mv_0^2$

where, F is resistive force and v_0 is initial spee(d)

Case II Let the further distance travelled by the bullet before coming to rest be s.

$\therefore$ $-F(3+s) = K_f - K_i = -\frac{1}{2}mv_0^2$

$\Rightarrow$ $-\frac{1}{8}mv_0^2(3+s) = -\frac{1}{2}mv_0^2$

or $\frac{1}{4}(3+s) = 1$ or $\frac{3}{4} + \frac{s}{4} = 1$

or $s = 1$ cm

61 A body of mass m is accelerated uniformly from rest to a speed v in a time T. The instantaneous power delivered to the body as a function of time, is given by **[AIEEE 2005, 04]**

(a) $\frac{mv^2}{T^2}t$ (b) $\frac{mv^2}{T^2}t^2$

(c) $\frac{1}{2}\frac{mv^2}{T^2}t$ (d) $\frac{1}{2}\frac{mv^2}{T^2}t^2$

Ans. *(a)*

A body of mass m with uniform acceleration, then force

$$F = ma = \frac{mv}{T} \quad \left[\therefore a = \frac{v-0}{T}\right]$$

Instantaneous power $= Fv = mav$

$$= \frac{mv}{T}\cdot at$$

$$= \frac{mv}{T}\cdot\frac{v}{T}\cdot t = \frac{mv^2}{T^2}t$$

62 A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane, it follows that **[AIEEE 2004]**

(a) its velocity is constant

(b) its acceleration is constant

(c) its kinetic energy is constant

(d) it moves in a straight line

Ans. *(c)*

When a force of constant magnitude acts on velocity of particle perpendicularly, then there is no change in the kinetic energy of particle. Hence, kinetic energy remains constant.

63 A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to **[AIEEE 2003]**

(a) $t^{3/4}$ (b) $t^{3/2}$

(c) $t^{1/4}$ (d) $t^{1/2}$

Ans. *(b)*

Delivering power of a machine

$$P = \text{constant}$$

$$P = F\cdot u$$

$$\left[\because \text{Power} = \frac{w}{t} = F\cdot\frac{s}{t} = F\cdot v\right]$$

$$= mav = m\frac{dv}{dt}v$$

$$P = mv\frac{dv}{dt}$$

$$vdv = \frac{P}{m}dt$$

Integrating on both sides, we get

$$\int_0^v v\,dv = \int_0^t \frac{P}{m}dt$$

$$\frac{v^2}{2} = \frac{Pt}{m},\ v = \left(\frac{2Pt}{m}\right)^{1/2}$$

$$v = \frac{ds}{dt} = \left(\frac{2Pt}{m}\right)^{1/2}$$

$$ds = \left(\frac{2Pt}{m}\right)^{1/2}dt$$

$$\int_0^s ds = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}\,dt$$

$$s = \sqrt{\frac{2P}{m}}\frac{t^{3/2}}{3/2}$$

$\therefore$ $s \propto t^{3/2}$

TOPIC 3
Collision

64 A body of mass M moving at speed v_0 collides elastically with a mass m at rest. After the collision, the two masses move at angles θ_1 and θ_2 with respect to the initial direction of motion of the body of mass M. The largest possible value of the ratio M/m, for which the angles θ_1 and θ_2 will be equal, is **[2021, 31 Aug Shift-I]**

(a) 4 (b) 1 (c) 3 (d) 2

Ans. *(c)*

Given, mass of body 1 = M

Mass of body 2 = m

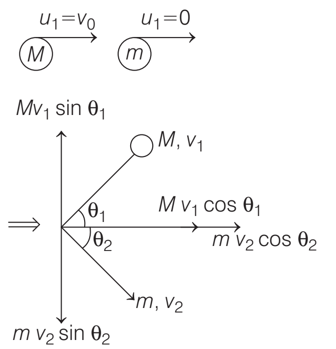
Initial speed of body 1, $u_1 = v_0$

Initial speed of body 2, $u_1 = 0$

Final speed of body 1 and 2 = v_1 and v_2

Angle made by body 1 and 2 after collision with respect to initial direction

$$= \theta_1, \theta_2$$



By using law of conservation of momentum,

Along X-axis

$$Mv_0 + m\cdot 0 = Mv_1 \cos\theta_1 + mv_2 \cos\theta_2$$

If $\theta_1 = \theta_2 = \theta$

$$\therefore \quad Mv_0 = Mv_1 \cos\theta + mv_2 \cos\theta \quad \text{... (i)}$$

Along Y-axis,

$$Mv_1 \sin\theta_1 = mv_2 \sin\theta_2$$

[since, $\theta_1 = \theta_2 = \theta$]

$$v_2 = \frac{Mv_1}{m} \quad \text{...(ii)}$$

Substituting the value of v_2 in Eq. (i), we get

$$Mv_0 = Mv_1 \cos\theta + m\left(\frac{Mv_1}{m}\right)\cos\theta$$

$$= 2Mv_1 \cos\theta$$

$$v_1 = \frac{v_0}{2\cos\theta} \quad \text{...(iii)}$$

By using law of conservation of energy along X-axis

$$\frac{1}{2}Mv_0^2 = \frac{1}{2}Mv_1^2 + \frac{1}{2}mv_2^2$$

$$\Rightarrow \quad Mv_0^2 = Mv_1^2 + m\left(\frac{M}{m}v_1\right)^2$$

[from Eq. (ii)]

$$\Rightarrow \quad Mv_0^2 = Mv_1^2 + \frac{M^2v_1^2}{m} = \frac{Mv_1^2}{m}(m + M)$$

$$\Rightarrow \quad v_0^2 = \left(\frac{v_0}{2\cos\theta}\right)^2\left(\frac{m+M}{m}\right)$$

[From Eq. (iii)]

$$\Rightarrow \quad v_0^2 = \frac{v_0^2}{4\cos^2\theta}\left(1 + \frac{M}{m}\right)$$

$$\Rightarrow \quad 4\cos^2\theta = 1 + \frac{M}{m}$$

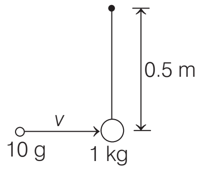
$$\Rightarrow \quad \frac{M}{m} = 4\cos^2\theta - 1$$

$$\left[\text{For largest possible value of } \frac{M}{m}, \theta = 0\right]$$

$$= 4\cos^2 0° - 1 = 4 - 1 = 3$$

65 A bullet of 10 g, moving with velocity v, collides head-on with the stationary bob of a pendulum and recoils with velocity 100 m/s. The length of the pendulum is 0.5 m and mass of the bob is 1 kg. The minimum value of v .... m/s, so that the pendulum describes a circle. (Assume, the string to be inextensible and $g = 10$ m/s^2)

[2021, 27 Aug Shift-II]



Ans. ***(400)***

Given, mass of bullet $(m_b) = 10$ g

$$= 10 \times 10^{-3} \text{ kg}$$

Initial speed of bullet is v.

Length of pendulum, $l = 0.5$ m

Mass of bob, $m = 1$ kg

Initial speed of bob, $u = 0$

Final speed of bullet, $v_b = 100 \text{ ms}^{-1}$

Final speed of bob for making complete circle at bottom, $v' = \sqrt{5gl}$

Acceleration due to gravity, $g = 10 \text{ ms}^{-2}$

By using law of conservation of momentum

$$m_b u_b + mu = -m_b v_b + mv'$$

$$\Rightarrow \quad \frac{10}{1000}v + 0 = \frac{-10}{1000} \times 100 + 1\sqrt{5gl}$$

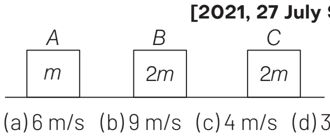
$$\Rightarrow \quad \frac{v}{100} = -1 + \sqrt{5 \times 10 \times \frac{5}{10}}$$

$$\Rightarrow \quad \frac{v}{100} = -1 + 5$$

$$\Rightarrow \quad v = 4 \times 100 = 400 \text{ ms}^{-1}$$

66 Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. The masses of A, B and C are m, $2m$ and $2m$, respectively. A moves towards B with a speed of 9 m/s and makes an elastic collision with it. There after B makes a completely inelastic collision with C. All motions occur along same straight line. The final speed of C is

[2021, 27 July Shift-I]



(a) 6 m/s (b) 9 m/s (c) 4 m/s (d) 3 m/s

Ans. ***(d)***

If v_A' and v_B' be the velocity of body A and B after first collision, then by conservation of linear momentum

$$mv_A = mv_A' + 2mv_B'$$

$$v_A = v_A' + 2v_B' \quad \text{...(i)}$$

Again, $e = \dfrac{v_A' - v_B'}{0 - v_A}$

$$1 = \frac{v_A' - v_B'}{-v_A}$$

[For elastic collision, $e = 1$]

$$\Rightarrow \quad -v_A = v_A' - v_B' \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii), we have

$$0 = 2v_A' + v_B'$$

$$\Rightarrow \quad v_B' = -2v_A' \quad \text{...(iii)}$$

From Eqs. (ii) and (iii), we get

$$-v_A = v_A' - (-2v_A')$$

$$\Rightarrow \quad -v_A = v_A' + 2v_A' = 3v_A'$$

$$\Rightarrow \quad v_A' = -\frac{v_A}{3} = -\frac{9}{3} = -3 \text{ m/s}$$

$\therefore$ From Eq. (iii), $v_B' = -2(-3) = 6$ m/s

Again, after 2nd collision,

$$2mv_B' = (2m + 2m)v_c = 4mv_c$$

$$\Rightarrow \quad v_c = \frac{v_B'}{2} = \frac{6}{2} = 3 \text{ m/s}$$

67 A ball of mass 4 kg, moving with a velocity of 10 ms^{-1}, collides with a spring of length 8 m and force constant 100 Nm^{-1}. The length of the compressed spring is x m. The value of x to the nearest integer, is ....

[2021, 18 March Shift-II]

Ans. ***(6)***

Given, mass of the ball, $m = 4$ kg

Velocity of the ball, $v = 10$ m/s

Force constant, $k = 100$ N/m

The length of the spring, $x = 8$ m

Let x is the compressed length of the spring.

Using the work-energy theorem,

"It states that kinetic energy of the ball is converted into the stored energy of spring".

$$\frac{mv^2}{2} = \frac{kx^2}{2} \quad \Rightarrow \quad \frac{4(10)^2}{2} = \frac{(100)\,x^2}{2}$$

$$\Rightarrow \quad x = 2 \text{m}$$

The final length (compressed) of the spring

$$= 8 - 2 = 6 \text{ m}$$

Hence, the value of x to the nearest integer is 6.

68 An object of mass m_1 collides with another object of mass m_2, which is at rest. After the collision, the objects move with equal speeds in opposite direction. The ratio of the masses $m_2 : m_1$ is

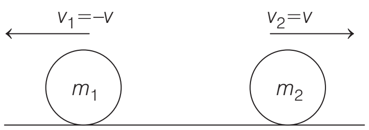
[2021, 18 March Shift-II]

(a) 3 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 1

Ans. *(a)*

The mass m_1 is moving with speed u_1 initially and mass m_2 is at rest. After the collision, the mass m_1 and m_2 move with speed v in opposite directions.

Before collision



After collision

Using the law of conservation of linear momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow \quad m_1u_1 + m_2(0) = m_1(-v) + m_2v$$

$$\Rightarrow \quad m_1u_1 = (-m_1 + m_2)v \quad \text{...(i)}$$

Since, the collision is elastic because they move with same speed after the collision. Hence, coefficient of restitution, $e = 1$

$$\therefore \quad e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$\Rightarrow \quad 1 = \frac{v-(-v)}{u_1 - 0}$$

$$u_1 = 2v \quad \text{...(ii)}$$

Putting the above value in Eq. (i), we get

$$m_1(2v) = (-m_1 + m_2)v$$

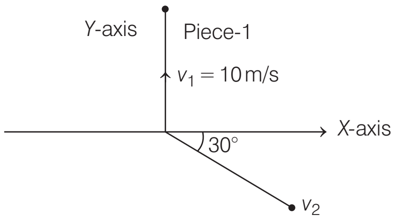
$$\Rightarrow \quad 3m_1 = m_2 \Rightarrow \frac{m_2}{m_1} = \frac{3}{1}$$

$$\Rightarrow \quad m_2 : m_1 = 3:1$$

69 A ball of mass 10 kg moving with a velocity $10\sqrt{3}$ m/s along the X-axis, hits another ball of mass 20 kg which is at rest. After the collision, first ball comes to rest while the second ball disintegrates into two equal pieces. One piece starts moving along Y-axis with a speed of 10 m/s.

The second piece starts moving at an angle of 30° with respect to the X-axis. The velocity of the ball moving at 30° with X-axis is x m/s. The configuration of pieces after collision is shown in the figure below. The value of x to the nearest integer is ....

[2021, 18 March Shift-I]



Ans. *(20)*

Given,

The mass of the first ball, $m_1 = 10$ kg

The mass of the second ball, $m_2 = 20$ kg

The initial velocity of the first ball,

$$u_1 = 10\sqrt{3} \text{ m/s}$$

The initial velocity of the second ball,

$$u_2 = 0 \text{ m/s}$$

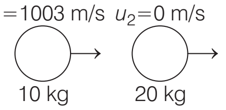
The final velocity of the first ball,

$$v_1 = 0 \text{ m/s}$$

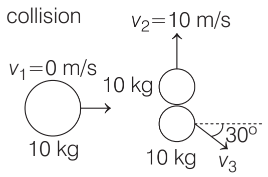
The final velocity of the first piece of the second ball, $v_2 = 10$ m/s

Let's consider the final velocity of the second piece of the second ball $= v_3$

As shown in the figure,



Before collision



After collision

The net external force on the system is to be zero. Hence, we can use the law of conservation of linear momentum in both directions.

In x-direction the linear momentum conserved,

$$m_1u_{1x} + m_2u_{2x} = m_1v_{1x} + m_2v_{2x} + m_3v_{3x}$$

$$\Rightarrow$$

$$10(10\sqrt{3}) + 0 = 10(0) + 0 + 10v_3\cos 30^\circ$$

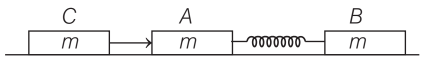
$$\Rightarrow \quad v_3 = 20 \text{ m/s}$$

Hence, the velocity of the ball moving at 30° with respect to the X-axis is 20 m/s.

So, the value of x to the nearest integer is 20.

70 Two identical blocks A and B each of mass m resting on the smooth horizontal floor are connected by a light spring of natural length L and spring constant k. A third block C of mass m moving with a speed v along the line joining A and B collides with A. The maximum compression in the spring is

[2021, 17 March Shift-II]



(a) $v\sqrt{\frac{m}{2k}}$ (b) $\sqrt{\frac{mv}{2k}}$ (c) $\sqrt{\frac{mv}{k}}$ (d) $\sqrt{\frac{m}{2k}}$

Ans. *(a)*

Let v is the speed of the third block C.

The velocity of centre of mass of A and B is

$$v_{CM} = \frac{v}{2}$$

The spring is compressed maximum by x distance.

Using the law of conservation of energy.

$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{1}{4}mv^2 = \frac{1}{2}kx^2 \Rightarrow x = \sqrt{\frac{mv^2}{2k}}$$

$$\Rightarrow \quad x = v\sqrt{\frac{m}{2k}}$$

71 A rubber ball is released from a height of 5 m above the floor. It bounces back repeatedly, always rising to $\frac{81}{100}$ of the height through which it falls. Find the average speed of the ball.
(Take, $g = 10 \text{ ms}^{-2}$)

[2021, 17 March Shift-II]

(a) 3.0 ms^{-1} (b) 3.5 ms^{-1}
(c) 2.0 ms^{-1} (d) 2.5 ms^{-1}

Ans. *(d)*

Velocity of rubber ball when it strikes to the ground, $v_0 = \sqrt{2gh_0}$

Using the formula of coefficient of restitution,

$$e = \frac{\text{velocity after collision}}{\text{velocity before collision}}$$

$$\Rightarrow \quad e = \frac{v_1 - 0}{v_0 - 0} \Rightarrow e = \frac{v_1}{\sqrt{2gh_0}}$$

$$\Rightarrow \quad v_1 = e\sqrt{2gh_0} \quad \text{...(i)}$$

As, initial height of the ball $= h_0$.

$\therefore$ The first height of the rebound, $h_1 = \frac{v_1^2}{2g}$

$$\Rightarrow \quad h_1 = e^2h_0 \quad \text{[Using Eq. (i)]}$$

The nth height of the ball to the rebound,

$$h_n = \frac{v_n^2}{2g} \Rightarrow h_n = e^{2n}h_0$$

The velocity of the ball after nth rebound, $v_n = e^n v_0$

Now, the total distance travelled by the ball after nth rebound is

$$H = h_0 + 2h_1 + 2h_2 + 2h_3 \ldots$$

$$H = h_0 + 2e^2 h_0 + 2e^4 h_0 + 2e^6 h_0 \ldots$$

$$H = h_0[1 + 2e^2(1 + e^2 + e^4 + e^6 \ldots)]$$

Using the formula,

$$1 + e^2 + e^4 + \ldots = \frac{1}{1 - e^2}$$

$$\Rightarrow \quad H = h_0\left[1 + 2e^2\left(\frac{1}{1 - e^2}\right)\right]$$

$$\Rightarrow \quad H = h_0\left(\frac{1 + e^2}{1 - e^2}\right)$$

$$\Rightarrow \quad H = 5\left(\frac{1 + (0.81)}{1 - (0.81)}\right)\left(\because e^2 = \frac{h_1}{h_0} = \frac{81}{100}\right)$$

$$H = 47.6 \text{ m}$$

Now, the total time taken by the ball to come to rest

$$T = t_0 + 2t_1 + 2t_2 + \ldots$$

$$T = \sqrt{\frac{2h_0}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + \ldots$$

$$T = \sqrt{\frac{2h_0}{g}}[1 + 2e + 2e^2 + \ldots]$$

$$T = \sqrt{\frac{2h_0}{g}}[1 + 2e(1 + e + e^2 + \ldots)]$$

$$T = \sqrt{\frac{2h_0}{g}}\left(\frac{1 + e}{1 - e}\right)$$

$$\Rightarrow \quad T = \sqrt{\frac{2(5)}{10}}\left(\frac{1 + \sqrt{0.81}}{1 - \sqrt{0.81}}\right)$$

$$T = 19 \text{ s}$$

The average velocity,

$$v_{\text{avg}} = \frac{H}{T} = \frac{47.6}{19}$$

$$= 2.5 \text{ m/s}$$

Hence, the average velocity is 2.5 m/s.

72 A boy is rolling a 0.5 kg ball on the frictionless floor with the speed of 20 ms^{-1}. The ball gets deflected by an obstacle on the way. After deflection it moves with 5% of its initial kinetic energy. What is the speed of the ball now ?

[2021, 17 March Shift-I]

(a) 19.0 ms^{-1} (b) 4.47 ms^{-1}

(c) 14.41 ms^{-1} (d) 1.00 ms^{-1}

Ans. *(b)*

Given, mass of rolling ball, $m = 0.5$ kg

Speed of ball, $u = 20\text{ms}^{-1}$

Before deflection,

Initial kinetic energy, $\text{KE}_i = \frac{1}{2}mu^2$

$$= \frac{1}{2} \times 0.5 \times (20)^2$$

$$= \frac{1}{2} \times 0.5 \times 400 = \frac{1}{2} \times \frac{1}{2} \times 400 = 100 \text{ J}$$

It is given in the question that after deflection the ball moves with 5% of its initial kinetic energy

$$\text{KE}_f = 5\% \text{ of KE}_i \Rightarrow \text{KE}_f = \frac{5}{100} \times 100 = 5 \text{ J}$$

If the final speed of the ball is v ms^{-1}, then

$$\text{KE}_f = \frac{1}{2}mv^2$$

$$\Rightarrow \quad 5 = \frac{1}{2} \times 0.5 \times v^2 \Rightarrow 10 = 0.5 \times v^2$$

$$\Rightarrow \quad v^2 = \frac{10}{0.5} = \frac{100}{5} \Rightarrow v^2 = 20$$

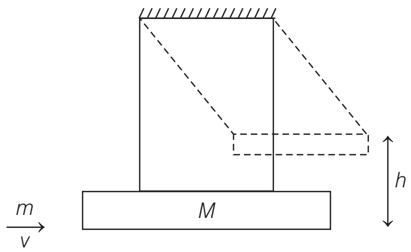
$$\Rightarrow \quad v = \sqrt{20} = 4.47 \text{ ms}^{-1}$$

$$\Rightarrow \quad v = 4.47 \text{ ms}^{-1}$$

73 A large block of wood of mass $M = 5.99$ kg is hanging from two long massless cords. A bullet of mass $m = 10$ g is fired into the block and gets embedded in it. The system (block + bullet) then swing upwards, their centre of mass rising a vertical distance $h = 9.8$ cm before the (block + bullet) pendulum comes momentarily to rest at the end of its arc. The speed of the bullet just before collision is

(Take $g = 9.8 \text{ ms}^{-2}$)

[2021, 16 March Shift-II]



(a) 841.4 m/s (b) 811.4 m/s

(c) 831.4 m/s (d) 821.4 m/s

Ans. *(c)*

Given,

Mass of large block of wood, $M = 5.99$ kg

Mass of bullet, $m = 10$ g

Height at which their centre of mass rise, $h = 9.8$ cm

From the law of conservation of energy,

Energy of the system when bullet gets embedded = Energy of the system till it momentarily comes to rest.

$$\Rightarrow \quad \frac{1}{2}(M + m)v_1^2 = (M + m)gh$$

where, v_1 = velocity of bullet + block system

$$\Rightarrow \quad v_1 = \sqrt{2gh} \quad \ldots \text{(i)}$$

According to law of conservation of momentum,

Momentum before collision = Momentum after collision.

$$\Rightarrow \quad mv = (M + m)v_1$$

[where, v = velocity of bullet before collision]

$$\Rightarrow mv = (M + m)\sqrt{2gh} \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad v = \left(\frac{M + m}{m}\right)\sqrt{2gh}$$

$$\Rightarrow$$

$$v = \frac{(5.99 + 0.01)}{10 \times 10^{-3}} \times \sqrt{2 \times 9.8 \times 9.8 \times 10^{-2}}$$

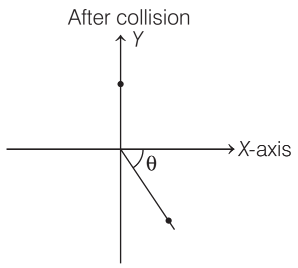
$$\Rightarrow \quad v = 831.55 \text{ ms}^{-1}$$

74 A ball of mass 10 kg moving with a velocity $10\sqrt{3}$ ms^{-1} along X-axis, hits another ball of mass 20 kg, which is at rest. After collision, the first ball comes to rest and the second one disintegrates into two equal pieces. One of the pieces starts moving along Y-axis at a speed of 10 m/s. The second piece starts moving at a speed of 20 m/s at an angle θ (degree) with respect to the X-axis.

The configuration of pieces after collision is shown in the figure.

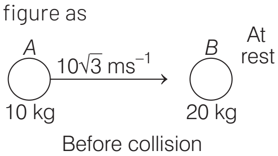
The value of θ to the nearest integer is ....

[2021, 16 March Shift-I]

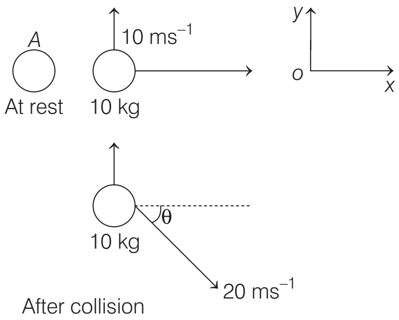


Ans. *(30)*

We can represent the given situation in figure as



Before collision



It means linear momentum is conserved along X-axis.

According to the law of conservation of linear momentum,

$$\mathbf{p}_i = \mathbf{p}_f$$

$$m_A u_A + m_B u_B = m_A v_A + (m_B v_B \cos\theta)$$

$$= m_A v_A + m_{B_1} v_{B_1} \cos 90° + m_{B_2} v_{B_2} \cos\theta$$

$$\Rightarrow 10 \times 10\sqrt{3} + 20 \times 0 = 10 \times 0 + 10 \times 10 \times 0 + 10 \times 20 \cos\theta$$

$$\Rightarrow \quad 10 \times 10\sqrt{3} = 200 \cos\theta$$

where, $\cos\theta$ being the horizontal component i.e., along X-axis

$$\Rightarrow \quad \cos\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \theta = 30° \qquad [\because \cos 30° = \sqrt{3}/2]$$

75 Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion (A) Body P having mass M moving with speed u has head-on collision elastically with another body Q having mass m initially at rest. If $m << M$, body Q will have a maximum speed equal to $2u$ after collision.

Reason (R) During elastic collision, the momentum and kinetic energy are both conserved. In the light of the above statements, choose the most appropriate answer from the options given below.

[2021, 26 Feb Shift-I]

(a) A is not correct but R is correct.
(b) Both A and R are correct but R is not the correct explanation of A.
(c) Both A and R are correct and R is the correct explanation of A.
(d) A is correct but R is not correct.

Ans. *(c)*

Let v_1 and v_2 are the speed of P and Q after collision.

By using law of conservation of mementum,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow \quad Mu + m \cdot 0 = Mv_1 + mv_2$$

$$\Rightarrow \quad \frac{M(u - v_1)}{m} = v_2 \qquad \text{...(i)}$$

and by using law of conservation of energy,

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\Rightarrow \quad Mu^2 + 0 = Mv_1^2 + mv_2^2$$

$$\Rightarrow \quad M(u^2 - v_1^2) = mv_2^2$$

$$\Rightarrow \quad M(u - v_1)(u + v_1)/m = v_2^2 \qquad \text{...(ii)}$$

Substituting the value of $M\frac{(u - v_1)}{m}$ from Eq. (i) in Eq. (ii), we get

$$v_2(u + v_1) = v_2^2$$

$$\Rightarrow \quad u + v_1 = v_2$$

$$\because \quad M >> m$$

$$\therefore \quad v_1 = u$$

and $\quad v_2 = 2u$

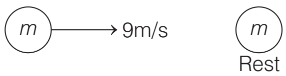
Hence, option (c) is the correct.

76 A ball with a speed of 9 m/s collides with another identical ball at rest. After the collision, the direction of each ball makes an angle of 30° with the original direction. The ratio of velocities of the balls after collision is $x : y$, where x is ....

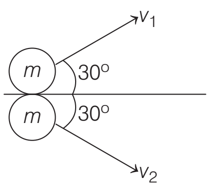
[2021, 24 Feb Shift-I]

Ans. *(1)*

The situation is shown below



Before collision



After collision

Using conservation of linear momentum in y-direction,

$$p_i = p_f$$

As, $\quad p_i = 0$

and $\quad p_f = mv_1 \sin 30° - mv_2 \sin 30°$

$$\Rightarrow \quad 0 = m \times \frac{1}{2} v_1 - m \times \frac{1}{2} v_2$$

$$\Rightarrow \quad v_1 = v_2 \text{ or } v_1 : v_2 = 1 : 1$$

Since, $v_1 : v_2 = x : y$ (gi ven)

$$\therefore \quad x = 1$$

77 A particle of mass m with an initial velocity $u\hat{\mathbf{i}}$ collides perfectly elastically with a mass $3m$ at rest. It moves with a velocity $v\hat{\mathbf{j}}$ after collision, then v is given by

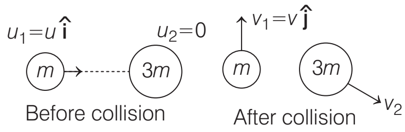
[2020, 2 Sep Shift-I]

(a) $v = \sqrt{\frac{2}{3}} u$ (b) $v = \frac{1}{\sqrt{6}} u$

(c) $v = \frac{u}{\sqrt{2}}$ (d) $v = \frac{u}{\sqrt{3}}$

Ans. *(c)*

As collision is elastic as shown below, both momentum and KE are conserve(d)



Momentum conservation gives,

$$mu\hat{\mathbf{i}} = mv\hat{\mathbf{j}} + 3m\mathbf{v}_2$$

$$\Rightarrow \quad \mathbf{v}_2 = \frac{1}{3}(u\hat{\mathbf{i}} - v\hat{\mathbf{j}})$$

$$\Rightarrow \quad |\mathbf{v}_2| = \sqrt{\frac{u^2 + v^2}{9}}$$

or $\quad v_2^2 = (u^2 + v^2)/9 \qquad \text{...(i)}$

Kinetic energy conservation gives,

$$\Rightarrow \quad \frac{1}{2} mu^2 = \frac{1}{2} mv^2 + \frac{1}{2} 3mv_2^2$$

$$\Rightarrow \quad u^2 = v^2 + 3v_2^2 \qquad \text{...(ii)}$$

Substituting value of v_2 from Eq (i) into Eq (ii), we get

$$u^2 = v^2 + 3\left(\frac{u^2 + v^2}{9}\right)$$

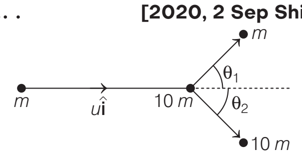
$$\Rightarrow \quad \frac{2}{3} u^2 = \frac{4}{3} v^2$$

$$\Rightarrow \quad v = \frac{u}{\sqrt{2}}$$

Hence, correct option is (c).

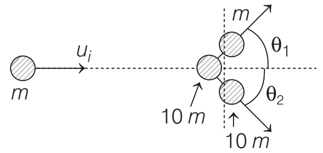
78 A particle of mass m is moving along the X-axis with initial velocity $u\hat{\mathbf{i}}$. It collides elastically with a particle of mass 10 m at rest and then moves with half its initial kinetic energy (see figure). If $\sin\theta_1 = \sqrt{n} \sin\theta_2$, then value of n is ....

[2020, 2 Sep Shift-II]



Ans. *(10)*

Given, impact is shown below,



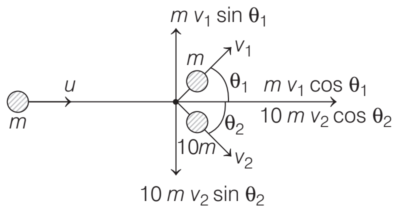
For particle 1, final KE is equals to half of its initial value,

$\Rightarrow \quad K_f = \frac{1}{2} K_i$

$\Rightarrow \quad \frac{1}{2} m_1 v_1^2 = \left(\frac{1}{2} m_1 u^2 \right) \times \frac{1}{2}$

$\Rightarrow$ Final velocity of m_1 will be, $v_1 = \frac{u}{\sqrt{2}}$

Momentum in y-direction is conserved,



i.e. $\quad 10 m v_2 \sin\theta_2 = m v_1 \sin\theta_1$

Here, $v_1 = \frac{u}{\sqrt{2}}$ and $\sin\theta_1 = \sqrt{n} \sin\theta_2$

So, we have

$$10 m v_2 \cdot \sin\theta_2 = \frac{mu}{\sqrt{2}} \cdot \sqrt{n} \sin\theta_2$$

$$\Rightarrow \quad v_2 = \frac{u\sqrt{n}}{10\sqrt{2}} \quad \text{...(i)}$$

Also collision is elastic, so KE is conserve(d)

$$\Rightarrow \quad \frac{1}{2} m u^2 = \frac{1}{2} m v_1^2 + \frac{1}{2} (10m) v_2^2$$

Substituting values of v_1 and v_2, we have

$$u^2 = \left(\frac{u}{\sqrt{2}} \right)^2 + 10 \left(\frac{u^2 n}{100 \times 2} \right)$$

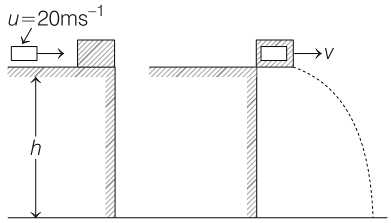
$$\Rightarrow \quad \frac{u^2}{2} = \frac{u^2 n}{10 \times 2} \Rightarrow n = 10$$

79 A block of mass 1.9 kg is at rest at the edge of a table of height 1 m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the collision, then the kinetic energy just before the combined system strikes the floor, is (Take, $g = 10\ \text{m/s}^2$ and assume there is no rotational motion and loss of energy after the collision is negligible) **[2020, 3 Sep Shift-II]**

(a) 20 J
(b) 19 J
(c) 21 J
(d) 23 J

Ans. *(c)*

When the bullet undergoes an inelastic collision with block, a part of KE of bullet is lost.



When bullet + block system falls from height h, then its total energy (kinetic + potential) becomes kinetic energy, so kinetic energy of bullet + block system at bottom just before collision is equal to total energy just after collision.

Now, by law of conservation of momentum, we have

$$mu = (m + M)v$$

$$\Rightarrow \quad v = \frac{mu}{m + M} = \frac{0.1 \times 20}{(0.1 + 1.9)} = 1\text{ms}^{-1}$$

Total energy of bullet and block just after collision

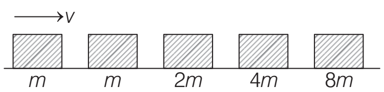
$$= \text{KE} + \text{PE} = \frac{1}{2}(m + M)v^2 + (m + M)gh$$

$$= \frac{1}{2} \times 2 \times 1^2 + 2 \times 10 \times 1$$

$$= 1 + 20 = 21\text{J}$$

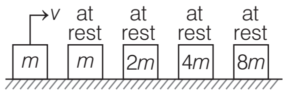
Hence, correct option is (c).

80 Blocks of masses m, $2m$, $4m$ and $8m$ are arranged in a line on a frictionless floor. Another block of mass m, moving with speed v along the same line (see figure) collides with mass m in perfectly inelastic manner. All the subsequent collisions are also perfectly inelasti(c) By the time, the last block of mass $8m$ starts moving, the total energy loss is p% of the original energy. Value of p is close to **[2020, 4 Sep Shift-I]**



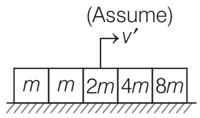
(a) 77 (b) 87
(c) 94 (d) 37

Ans. *(c)*



Before first collision

$\Rightarrow$



After last collision

Since, all the collisions are perfectly inelastic, so after the final collision, all blocks will be moving together. Let their final velocity be v'.

By law of conservation of linear momentum,

$$(\mathbf{p}_{\text{sys}})_i = (\mathbf{p}_{\text{sys}})_f$$

$$\Rightarrow \quad mv + m(0) + 2m(0) + 4m(0) + 8m(0) = (m + m + 2m + 4m + 8m)v'$$

$$\Rightarrow \quad mv = 16mv'$$

$$\Rightarrow \quad v' = \frac{v}{16} \quad \text{...(i)}$$

Now, initial kinetic energy of system,

$$(K_{\text{sys}})_i = \frac{1}{2} mv^2 + 0 + 0 + 0 + 0 = \frac{1}{2} mv^2$$

And final kinetic energy of system,

$$(K_{\text{sys}})_f = \frac{1}{2}(m + m + 2m + 4m + 8m)(v')^2$$

$$= \frac{1}{2} \times 16m \times \left(\frac{v}{16} \right)^2$$

$$= \frac{1}{2} \times 16m \times \frac{v^2}{256}$$

$$= \frac{1}{32} mv^2$$

Loss in kinetic energy,

$$(\Delta K_{\text{sys}})_{\text{loss}} = (K_{\text{sys}})_i - (K_{\text{sys}})_f = \frac{1}{2} mv^2 - \frac{1}{32} mv^2 = \frac{15}{32} mv^2$$

% loss in kinetic energy,

$$\%(\Delta K_{\text{sys}})_{\text{loss}} = \frac{(\Delta K_{\text{sys}})_{\text{loss}}}{(K_{\text{sys}})_i} \times 100\%$$

$$= \frac{\frac{15}{32} mv^2}{\frac{1}{2} mv^2} \times 100\% = \frac{15}{16} \times 100\%$$

$$= 93.75\%$$

Given that,

$$\% (\Delta K_{\text{sys}})_{\text{loss}} = p\%$$

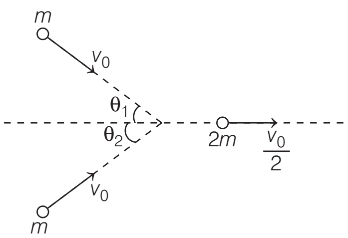
so, $\quad p = 93.75 \simeq 94$

Hence, correct option is (c).

81 Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial speed. The angle between the initial velocities of the two bodies (in degree) is ………… **[2020, 6 Sep Shift-I]**

Ans. *(120)*

Let initial velocities of two bodies are making angle θ_1 and θ_2 with horizontal direction as shown in figure.



Initial momentum, $\mathbf{p}_i = \mathbf{p}_1 + \mathbf{p}_2$

$$= \{mv_0 \cos\theta_1 \hat{\mathbf{i}} + mv_0 \sin\theta_1 \hat{\mathbf{j}}\} + \{mv_0 \cos\theta_2 \hat{\mathbf{i}} + mv_0 \sin\theta_2 - \hat{\mathbf{j}}\}$$

$$= mv_0(\cos\theta_1 + \cos\theta_2)\hat{\mathbf{i}} + mv_0(\sin\theta_1 - \sin\theta_2)\hat{\mathbf{j}}$$

Final momentum, $\mathbf{p}_f = (2m)\left(\frac{v_0}{2}\right)\hat{\mathbf{i}}$

$\Rightarrow \qquad \mathbf{p}_f = mv_0\hat{\mathbf{i}}$

In collision momentum remains conserved, so, applying momentum conservation,

$$\mathbf{p}_f = \mathbf{p}_i$$

or $\quad mv_0\hat{\mathbf{i}} = mv_0(\cos\theta_1 + \cos\theta_2)\hat{\mathbf{i}} + mv_0(\sin\theta_1 - \sin\theta_2)\hat{\mathbf{j}}$

$\Rightarrow \quad \sin\theta_1 - \sin\theta_2 = 0$

$\Rightarrow \quad \theta_1 = \theta_2$

and $\quad mv_0 = 2mv_0 \cos\theta$

or $\quad \cos\theta = \frac{1}{2}$ or $\theta = 60°$

But angle between initial velocities is $\theta_1 + \theta_2$ which is equal to $60° + 60° = 120°$.

82 Particle A of mass m_1 moving with velocity $(\sqrt{3}\,\hat{\mathbf{i}} + \hat{\mathbf{j}})\text{ms}^{-1}$ collides with another particle B of mass m_2 which is at rest initially. Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be the velocities of particles A and B after collision, respectively. If $m_1 = 2m_2$ and after collision $\mathbf{v}_1 = (\hat{\mathbf{i}} + \sqrt{3}\,\hat{\mathbf{j}})\text{ms}^{-1}$, then the angle between $\mathbf{v}_1$ and $\mathbf{v}_2$ is **[2020, 6 Sep Shift-II]**

(a) 15° (b) 60°
(c) −45° (d) 105°

Ans. *(d)*

Given that, $\mathbf{u}_1 = (\sqrt{3}\hat{\mathbf{i}} + \hat{\mathbf{j}})$ m/s, $\mathbf{u}_2 = 0$

$\mathbf{v}_1 = (\hat{\mathbf{i}} + \sqrt{3}\hat{\mathbf{j}})$ m/s and $m_1 = 2m_2$

Using conservation of linear momentum,

$$\mathbf{p}_i = \mathbf{p}_f$$

$$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$$

$$2m_2(\sqrt{3}\hat{\mathbf{i}} + \hat{\mathbf{j}}) + m_2(0) = 2m_2(\hat{\mathbf{i}} + \sqrt{3}\hat{\mathbf{j}}) + m_2\mathbf{v}_2$$

$$\mathbf{v}_2 = 2(\sqrt{3}\hat{\mathbf{i}} + \hat{\mathbf{j}}) - 2(\hat{\mathbf{i}} + \sqrt{3}\hat{\mathbf{j}})$$

$$= 2(\sqrt{3} - 1)(\hat{\mathbf{i}} - \hat{\mathbf{j}}) \text{ m/s}$$

Let the angle between $\mathbf{v}_1$ and $\mathbf{v}_2$ be θ, then

$$\cos\theta = \frac{\mathbf{v}_1 \cdot \mathbf{v}_2}{v_1 v_2} = \frac{(\hat{\mathbf{i}} + \sqrt{3}\hat{\mathbf{j}}) \cdot 2(\sqrt{3} - 1)(\hat{\mathbf{i}} - \hat{\mathbf{j}})}{2 \cdot 2\sqrt{2}(\sqrt{3} - 1)}$$

$$[\because v_1 = 2 \text{ m/s}, v_2 = 2\sqrt{2}(\sqrt{3} - 1) \text{ m/s}]$$

$$= \frac{2(\sqrt{3} - 1) - 2\sqrt{3}(\sqrt{3} - 1)}{4\sqrt{2}(\sqrt{3} - 1)}$$

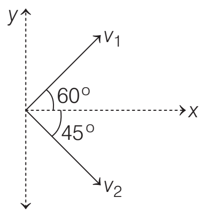
$$= \frac{2(\sqrt{3} - 1)(1 - \sqrt{3})}{4\sqrt{2}(\sqrt{3} - 1)}$$

$$\cos\theta = \frac{1 - \sqrt{3}}{2\sqrt{2}} = -0.259 \Rightarrow \theta = 105°$$

Alternate solution

Directly observing the direction of $\mathbf{v}_1$ and $\mathbf{v}_2$.

$$\theta_1 = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 60°$$



$\Rightarrow \quad \theta_2 = \tan^{-1}(-1) = 45°$

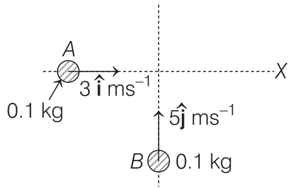
$\therefore \quad \theta = \theta_1 + \theta_2 = 60° + 45° = 105°$

Hence, correct option is (d).

83 A body A, of mass $m = 0.1$ kg has an initial velocity of $3\hat{\mathbf{i}}$ ms^{-1}. It collides elastically with another body, B of the same mass which has an initial velocity of $5\hat{\mathbf{j}}$ ms^{-1}. After collision, A moves with a velocity $\mathbf{v} = 4(\hat{\mathbf{i}} + \hat{\mathbf{j}})$. The energy of B after collision is written as $\frac{x}{10}$ J. The value of x is ……… . **[2020, 8 Jan Shift-I]**

Ans. *(1)*

Given situation in as shown in the figure.



Total initial momentum,

$$\mathbf{p}_i = m_A v_A + m_B v_B$$

$$= 0.1 \times 3\hat{\mathbf{i}} + 0.1 \times 5\hat{\mathbf{j}}$$

$$= 0.3\,\hat{\mathbf{i}} + 0.5\,\hat{\mathbf{j}} \text{ (kg-ms}^{-1}\text{)}$$

Final velocity of A, $\quad v_A = 4(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

Let final velocity of B is v_B.

Then, final momentum after collision,

$$\mathbf{p}_f = m_A v_A + m_B v_B$$

$$= 0.1 \times 4(\hat{\mathbf{i}} + \hat{\mathbf{j}}) + 0.1 \times v_B$$

Now, by conservation of momentum, we have

$$\mathbf{p}_i = \mathbf{p}_f$$

$\Rightarrow \quad 0.3\hat{\mathbf{i}} + 0.5\hat{\mathbf{j}} = 0.4\hat{\mathbf{i}} + 0.4\hat{\mathbf{j}} + 0.1 v_B$

$\Rightarrow \quad v_B = -\hat{\mathbf{i}} + \hat{\mathbf{j}}$

Kinetic energy of B after collision will be

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.1 \times \left|(-\hat{\mathbf{i}} + \hat{\mathbf{j}})\right|^2$$

$$= \frac{1}{2} \times 0.1 \times 2 = 0.1\text{s J}$$

It is given, energy of B after collision is $\frac{x}{10}$.

So, $\frac{x}{10} = 0.1$ or $x = 1$

84 A particle of mass m is dropped from a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, then the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is **[2020, 8 Jan Shift-II]**

(a) $\sqrt{\frac{1}{2}}$ (b) $\frac{1}{2}$
(c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{\frac{3}{4}}$

Ans. *(c)*

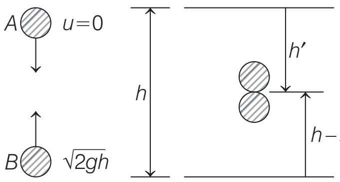
Let particles collide at some distance h' from top at time t_0. Then,

$$h' = \frac{1}{2}gt_0^2 \qquad \text{...(i)}$$

(for particle A)

and $h - h' = \sqrt{2gh} \cdot t_0 - \frac{1}{2} g t_0^2$ ....(ii)

(for particle B)



From these equations, particles meet after time t_0 given by

$$t_0 = \frac{h}{\sqrt{2gh}} = \sqrt{\frac{h}{2g}}$$

Velocities of particles A and B at instant of collision are $v_A = gt_0$

and $v_B = \sqrt{2gh} - gt_0$.

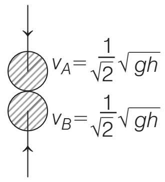
Hence,

$$v_A = g \times \sqrt{\frac{h}{2g}} = \sqrt{\frac{1}{2} gh} = \frac{1}{\sqrt{2}} \sqrt{gh}$$

and $v_B = \sqrt{2gh} - g\sqrt{\frac{h}{2g}} = \left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)\sqrt{gh}$

$$= \frac{1}{\sqrt{2}} \times \sqrt{gh}$$

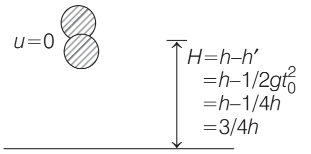
So, particles collide as shown in the figure.



From momentum conservation, we can see that particles stuck, $p_{\text{initial}} = p_{\text{final}}$.

This means the combined system of particles comes to rest ($v_{\text{combined mass}} = 0$) instantaneously.

Now, we have to calculate time of fall of combined mass.



Combined mass starts with $u = 0$ and its height above earth's surface is $H = \frac{3}{4} h$.

So, time taken by combined mass to reach ground is given by

$$H = ut + \frac{1}{2} at^2$$

$$\Rightarrow \quad \frac{3}{4} h = \frac{1}{2} g \times t^2$$

$$\Rightarrow \quad \sqrt{\frac{3}{2}} \sqrt{\frac{h}{g}} = t$$

85 Two particles of equal mass m have respective initial velocities $u\hat{\mathbf{i}}$ and $u\left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{2}\right)$. They collide completely inelastically. The energy lost in the process is

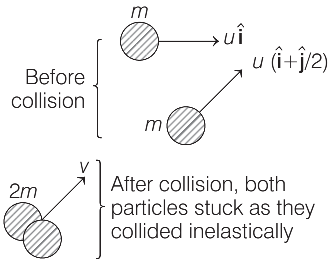
[2020, 9 Jan Shift-I]

(a) $\frac{3}{4} mu^2$ (b) $\sqrt{\frac{2}{3}} mu^2$

(c) $\frac{1}{3} mu^2$ (d) $\frac{1}{8} mu^2$

Ans. ***(d)***

Collision between particles are as shown in the figure.



From momentum conservation, we have

$$m(u\hat{\mathbf{i}}) + mu\left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{2}\right) = 2m\mathbf{v}$$

$$\Rightarrow \quad \mathbf{v} = \frac{u}{2}\hat{\mathbf{i}} + u\left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{4}\right)$$

$$= \frac{3}{4} u\hat{\mathbf{i}} + \frac{1}{4} u\hat{\mathbf{j}}$$

Initial kinetic energy of particles,

$$K_1 = \frac{1}{2} mu^2 + \frac{1}{2} m\left(\frac{u}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2} mu^2 + \frac{1}{4} mu^2 = \frac{3}{4} mu^2$$

Final kinetic energy of combined particles,

$$K_2 = \frac{1}{2}(2m) v^2$$

$$= \frac{1}{2} \times 2m \times \left(\sqrt{\left(\frac{3}{4}u\right)^2 + \left(\frac{1}{4}u\right)^2}\right)^2$$

$$= \frac{5}{8} mu^2$$

Change in kinetic energy or energy lost

$$= K_1 - K_2$$

$$= \frac{3}{4} mu^2 - \frac{5}{8} mu^2 = \frac{1}{8} mu^2$$

86 A particle of mass m is projected with a speed u from the ground at an angle $\theta = \frac{\pi}{3}$ w.r.t. horizontal (X-axis). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\hat{\mathbf{i}}$. The horizontal distance covered by the combined mass before reaching the ground is

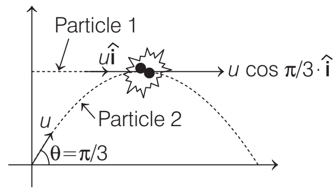
[2020, 9 Jan Shift-II]

(a) $\frac{3\sqrt{3}}{8} \frac{u^2}{g}$ (b) $\frac{3\sqrt{2}}{4} \frac{u^2}{g}$

(c) $\frac{5}{8} \frac{u^2}{g}$ (d) $2\sqrt{2} \frac{u^2}{g}$

Ans. ***(a)***

Collision is as shown in the figure.



Velocity of the particle projected from origin at its topmost point,

$$\mathbf{u}_2 = u \cos\frac{\pi}{3} \cdot \hat{\mathbf{i}} = \frac{u}{2} \hat{\mathbf{i}}$$

By conservation of momentum (velocity of combined mass after collision (v)), we have

$$mu\hat{\mathbf{i}} + m\frac{u}{2}\hat{\mathbf{i}} = 2m\mathbf{v}$$

$$\Rightarrow \quad \mathbf{v} = \frac{3}{4} u\hat{\mathbf{i}}$$

Time of fall of combined mass from $h_{\max}$,

$$t = \frac{u \sin\theta}{g} = \frac{u \sin\frac{\pi}{3}}{g} = \frac{\sqrt{3}}{2} \frac{u}{g}$$

During this time, combined particle keeps on moving with a horizontal speed of $|\mathbf{v}| = \frac{3}{4} u$.

So, horizontal distance covered by combined mass before reaching the ground,

$$R = \text{speed} \times \text{time} = \frac{3}{4} u \times \frac{\sqrt{3}}{2} \frac{u}{g} = \frac{3\sqrt{3}}{8} \cdot \frac{u^2}{g}$$

87 A body of mass m_1 moving with an unknown velocity of $v_1\hat{\mathbf{i}}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{\mathbf{i}}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{\mathbf{i}}$ and $v_4\hat{\mathbf{i}}$, respectively.

If $m_2 = 0.5 m_1$ and $v_3 = 0.5 v_1$, then v_1 is **[2019, 8 April Shift-II]**

(a) $v_4 + v_2$ (b) $v_4 - \frac{v_2}{4}$

(c) $v_4 - \frac{v_2}{2}$ (d) $v_4 - v_2$

Ans. *(d)*

Key Idea Total linear momentum is conserved in all collisions, i.e. the initial momentum of the system is equal to final momentum of the system.

Given,

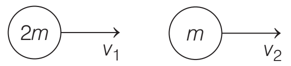
$$m_2 = 0.5 m_1 \Rightarrow m_1 = 2m_2$$

Let $m_2 = m$, then, $m_1 = 2m$

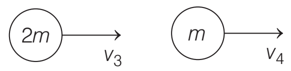
Also, $v_3 = 0.5 v_1$

Given situation of collinear collision is as shown below

Before collision,



After collision,



∴ According to the conservation of linear momentum, Initial momentum = Final momentum

$$m_1 v_1 \hat{\mathbf{i}} + m_2 v_2 \hat{\mathbf{i}} = m_1 v_3 \hat{\mathbf{i}} + m_2 v_4 \hat{\mathbf{i}}$$

$$\Rightarrow \quad 2m v_1 \hat{\mathrm{i}} + m v_2 \hat{\mathrm{i}} = 2m(0.5 v_1)\hat{\mathrm{i}} + m v_4 \hat{\mathbf{i}}$$

$$\Rightarrow v_4 = v_1 + v_2 \Rightarrow v_1 = v_4 - v_2$$

88 A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one-fourth of its original speed. What is the mass of the second body? **[2019, 9 April Shift-I]**

(a) 1.5 kg (b) 1.2 kg

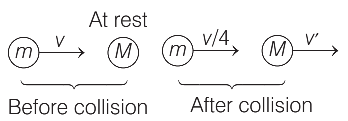
(c) 1.8 kg (d) 1.0 kg

Ans. *(b)*

Key Idea For an elastic collision, coefficient of restitution (*e*), i.e. the ratio of relative velocity of separation after collision to the relative velocity of approach before collision is 1.

Given, mass of small body, $m = 2$ kg

Given situation is as shown



Using momentum conservation law for the given system,

$$(\text{Total momentum})_{\text{before collision}} = (\text{Total momentum})_{\text{after collision}}$$

$$\Rightarrow m(v) + M(0) = m\left(\frac{v}{4}\right) + M(v') \quad \text{...(i)}$$

$\because \quad e = 1$ and we know that,

$$e = -\frac{v_2 - v_1}{u_2 - u_1}$$

$$\Rightarrow \quad 1 = -\frac{v' - v/4}{0 - v} \Rightarrow v = v' - v/4$$

or $\quad v' = 5v/4 \quad$ ...(ii)

Using value from Eq. (ii) into Eq. (i), we get

$$mv = \frac{mv}{4} + M\left(\frac{5v}{4}\right)$$

$$\Rightarrow \quad m\left(v - \frac{v}{4}\right) = M\left(\frac{5v}{4}\right)$$

$$\Rightarrow \quad \frac{3}{4} mv = \frac{5}{4} Mv$$

$$M = \frac{3}{5} m = \frac{3}{5} \times 2 = 1.2 \text{ kg}$$

89 A particle of mass *m* is moving with speed 2*v* and collides with a mass 2*m* moving with speed *v* in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass *m*, which move at angle 45° with respect to the original direction.

The speed of each of the moving particle will be **[2019, 9 April Shift-II]**

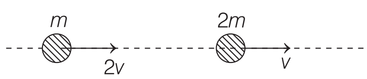
(a) $\sqrt{2}\, v$ (b) $\frac{v}{\sqrt{2}}$

(c) $\frac{v}{(2\sqrt{2})}$ (d) $2\sqrt{2}\, v$

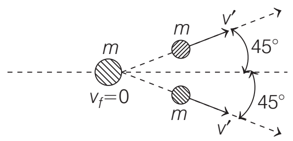
Ans. *(d)*

According to the questions,

Initial condition,



Final condition,



As we know that, in collision, linear momentum is conserved in both *x* and *y* directions separately.

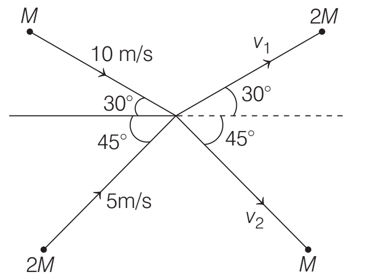
So, $\quad (p_x)_{\text{initial}} = (p_x)_{\text{final}}$

$$m(2v) + 2m(v) = 0 + mv' \cos 45° + mv' \cos 45°$$

$$\Rightarrow \quad 4mv = \frac{2m}{\sqrt{2}} v'$$

$$\Rightarrow \quad v' = 2\sqrt{2} v$$

90 Two particles of masses *M* and 2*M*, moving as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speed v_1 and v_2 are nearly **[2019, 10 April Shift-I]**



(a) 6.5 m/s and 3.2 m/s

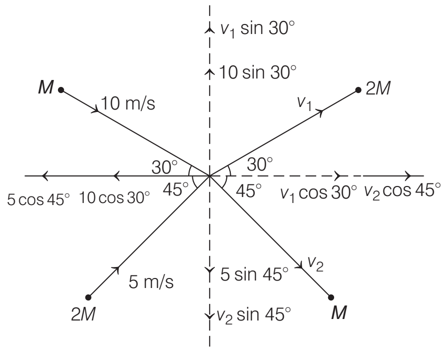
(b) 3.2 m/s and 6.3 m/s

(c) 3.2 m/s and 12.6 m/s

(d) 6.5 m/s and 6.3 m/s

Ans. *(d)*

The given condition can be drawn as shown below



Applying linear momentum conservation law in *x*-direction, we get

Initial momentum = Final momentum

$$(M \times 10 \cos 30°) + (2M \times 5 \cos 45°) = (M \times v_2 \cos 45°) + (2M \times v_1 \cos 30°)$$

$$\Rightarrow \left(M \times 10 \times \frac{\sqrt{3}}{2}\right) + \left(2M \times 5 \times \frac{1}{\sqrt{2}}\right) = \left(M \times v_2 \times \frac{1}{\sqrt{2}}\right) + \left(2M \times v_1 \times \frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad 5\sqrt{3} + 5\sqrt{2} = \frac{v_2}{\sqrt{2}} + v_1\sqrt{3} \quad \text{...(i)}$$

Similarly, applying linear momentum conservation law in *y*-direction, we get

$$(M \times 10 \sin 30°) - (2M \times 5 \sin 45°) = (M \times v_2 \sin 45°) - (2M \times v_1 \sin 30°)$$

$$\Rightarrow \left(M \times 10 \times \frac{1}{2}\right) - \left(2M \times 5 \times \frac{1}{\sqrt{2}}\right)$$

$$= \left(M \times v_2 \times \frac{1}{\sqrt{2}}\right) - \left(2M \times v_1 \times \frac{1}{2}\right)$$

$$\Rightarrow \quad 5 - 5\sqrt{2} = \frac{v_2}{\sqrt{2}} - v_1 \quad \text{...(ii)}$$

Subtracting Eq. (ii) from Eq. (i), we get

$$(5\sqrt{3} + 5\sqrt{2}) - (5 - 5\sqrt{2}) = \left(\frac{v_2}{\sqrt{2}} + v_1\sqrt{3}\right) - \left(\frac{v_2}{\sqrt{2}} - v_1\right)$$

$$\Rightarrow \quad 5\sqrt{3} + 10\sqrt{2} - 5 = v_1\sqrt{3} + v_1$$

$$\Rightarrow \quad v_1 = \left(\frac{5\sqrt{3} + 10\sqrt{2} - 5}{1 + \sqrt{3}}\right) = \frac{8.66 + 14.142 - 5}{1 + 1.732} = \frac{17.802}{2.732}$$

$$\Rightarrow \quad v_1 = 6.516 \text{ m/s} \approx 6.5 \text{ m/s} \quad \text{...(iii)}$$

Substituting the value from Eq. (iii) in Eq. (i), we get

$$5\sqrt{3} + 5\sqrt{2} = \frac{v_2}{\sqrt{2}} + 6.51 \times \sqrt{3}$$

$$\Rightarrow \quad v_2 = (5\sqrt{3} + 5\sqrt{2} - 6.51 \times \sqrt{3})\sqrt{2}$$

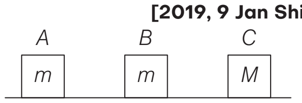
$$v_2 = (8.66 + 7.071 - 11.215)\, 1.414$$

$$\Rightarrow \quad v_2 = 4.456 \times 1.414$$

$$\Rightarrow \quad v_2 \approx 6.3 \text{ m/s}$$

91 Three blocks A, B and C are lying on a smooth horizontal surface as shown in the figure. A and B have equal masses m while C has mass M. Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of $\frac{M}{m}$?

[2019, 9 Jan Shift-I]

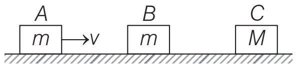


(a) 4 (b) 2
(c) 3 (d) 5

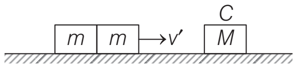
Ans. *(a)*

Key Idea For a perfectly inelastic collision, the momentum of the system remains conserved but there is some of loss of kinetic energy. Also, after collision the objects of the system are stuck to each other and move as a combined system.

Initially, block A is moving with velocity v as shown in the figure below,



Now, A collides with B such that they collide inelastically. Thus, the combined mass (say) move with the velocity 'v'' as shown below,



Then, if this combined system is collided inelastically again with the block C. So, now the velocity of system be v'' as shown below.

Thus, according to the principle of conservation of momentum,

initial momentum of the system
= final momentum of the system

$$\Rightarrow \quad mv = (2m + M)v''$$

$$\text{or} \quad v'' = \left(\frac{mv}{2m + M}\right) \quad \text{...(i)}$$

Initial kinetic energy of the system,

$$(KE)_i = \frac{1}{2}mv^2 \quad \text{...(ii)}$$

Final kinetic energy of the system, $(KE)_f$

$$= \frac{1}{2}(2m + M)(v'')^2 = \frac{1}{2}(2m + M)\left(\frac{mv}{2m + M}\right)^2$$

[$\because$ using Eq. (i)]

$$= \frac{1}{2} \cdot \frac{v^2 m^2}{(2m + M)} \quad \text{...(iii)}$$

Dividing Eq. (iii) and Eq. (ii), we get

$$\frac{KE)_f}{KE)_{ii}} = \frac{\frac{\frac{1}{2}m^2v^2}{(2m + M)}}{\frac{1}{2}mv^2} = \frac{m}{2m + M} \quad \text{...(iv)}$$

It is given that $\frac{5}{6}$th of (KE) is lost in this process.

$$\Rightarrow \quad KE)_f = \frac{1}{6}KE)_i$$

$$\Rightarrow \quad \frac{KE)_f}{KE)_i} = \frac{1}{6} \quad \text{...(v)}$$

Comparing Eq. (iv) and Eq. (v), we get

$$\frac{m}{2m + M} = \frac{1}{6} \Rightarrow 6m = 2m + M$$

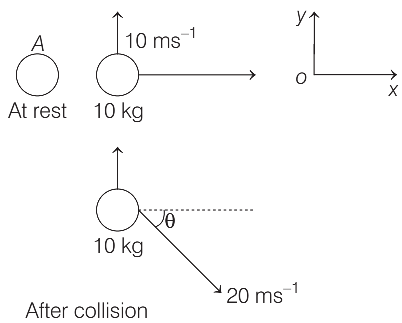
$$4m = M \Rightarrow \frac{M}{m} = 4$$

92 A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward with a velocity 100 ms^{-1} from the ground. The bullet gets embedded in the wood. Then, the maximum height to which the combined system reaches above the top of the building before falling below is (Take, $g = 10\, ms^{-2}$)

[2019, 10 Jan Shift-I]

(a) 20 m (b) 30 m
(c) 10 m (d) 40 m

Ans. *(d)*



After collision

Key Idea As bullet gets embedded in the block of wood so, it represents a collision which is perfectly inelastic and hence only momentum of the systemis conserved.

Velocity of bullet is very high compared to velocity of wooden block so, in order to calculate time for collision, we take relative velocity nearly equal to velocity of bullet.

So, time taken for particles to collide is

$$t = \frac{d}{v_{rel}} = \frac{100}{100} = 1\text{s}$$

Speed of block just before collision is;

$$v_1 = gt = 10 \times 1 = 10\, ms^{-1}$$

Speed of bullet just before collision is

$$v_2 = u - gt = 100 - 10 \times 1 = 90\, ms^{-1}$$

Let v = velocity of bullet + block system, then by conservation of linear momentum, we get

$$-(0.03 \times 10) + (0.02 \times 90) = (0.05)\, v$$

$$\Rightarrow \quad v = 30\, ms^{-1}$$

Now, maximum height reached by bullet and block is

$$h = \frac{v^2}{2g} \Rightarrow h = \frac{30 \times 30}{2 \times 10}$$

$$\Rightarrow \quad h = 45\text{m}$$

$\therefore$ Height covered by the system from point of collision = 45 m

Now, distance covered by bullet before collision in 1 s.

$$= 100 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 95\text{ m}$$

Distance of point of collision from the top of the building

$= 100 - 95 = 5\,m$

$\therefore$ Maximum height to which the combined system reaches above the top of the building before falling below

$= 45 - 5 = 40\,m$

93 A body of mass 1 kg falls freely from a height of 100 m on a platform of mass 3 kg which is mounted on a spring having spring constant $k = 1.25 \times 10^6$ N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that $g = 10\,ms^{-2}$, the value of x will be close to **[2019, 11 Jan Shift-I]**

(a) 8 cm (b) 4 cm
(c) 40 cm (d) 80 cm

Ans. *(*)*

Initial compression of the spring,

$$mg = k\left(\frac{x_0}{100}\right) \quad (x_0 \text{ in cm})$$

$$\Rightarrow \quad x_0 = \frac{3 \times 10 \times 100}{1.25 \times 10^6} = \frac{3}{1250}$$

Which is very small and can be neglected.

Applying conservation of momentum before and after the collision i.e., momentum before collision = momentum after collision.

$m \times \sqrt{2gh} = (m + M)v$ ($\because$ velocity of the block just before the collision is

$$v^2 - 0^2 = 2gh$$

or $\quad v = \sqrt{2gh}$)

After substituting the given values, we get

$$1 \times \sqrt{2 \times 10 \times 100} = 4v$$

or $\quad 4v = 20\sqrt{5}$

So, $\quad v = 5\sqrt{5}\,m/s$

Let this be the maximum velocity, then for the given system, using

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$\therefore \frac{1}{2} \times 4 \times 125 = \frac{1}{2} \times 1.25 \times 10^6 \times \left(\frac{x}{100}\right)^2$$

$$\Rightarrow \quad 4 = 10^4 \times \frac{x^2}{10^4} \text{ or } x = 2\text{ cm}$$

$\therefore$ No option given is correct.

94 A simple pendulum is made of a string of length l and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. Then, M is given by **[2019, 12 Jan Shift-I]**

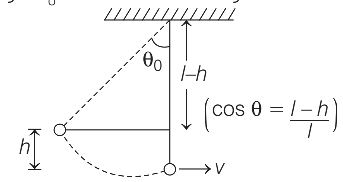
(a) $m\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$ (b) $\frac{m}{2}\left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}\right)$

(c) $m\left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}\right)$ (d) $\frac{m}{2}\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$

Ans. *(a)*

Pendulum's velocity at lowest point just before striking mass m is found by equating it's initial potential energy (PE) with final kinetic energy (KE).

Initially, when pendulum is released from angle θ_0 as shown in the figure below,



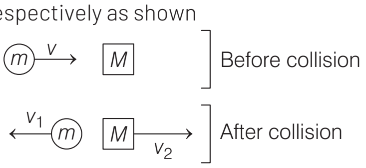
We have,

$$mgh = \frac{1}{2}mv^2$$

Here, $\quad h = l - l\cos\theta_0$

So, $\quad v = \sqrt{2gl\,(1 - \cos\theta_0)}$ ...(i)

With velocity v, bob of pendulum collides with block. After collision, let v_1 and v_2 are final velocities of masses m and M respectively as shown



Then if pendulum is deflected back upto angle θ_1, then

$$v_1 = \sqrt{2gl\,(1 - \cos\theta_1)} \quad \text{...(ii)}$$

Using definition of coefficient of restitution to get

$$e = \frac{|\text{velocity of separation}|}{|\text{velocity of approach}|}$$

$$1 = \frac{v_2 - (-v_1)}{v - 0} \Rightarrow v = v_2 + v_1 \quad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$\Rightarrow \quad \sqrt{2gl(1 - \cos\theta_0)} = v_2 + \sqrt{2gl(1 - \cos\theta_1)}$$

$$\Rightarrow \quad v_2 = \sqrt{2gl}\,(\sqrt{1 - \cos\theta_0} - \sqrt{1 - \cos\theta_1}) \quad \text{...(iv)}$$

According to the momentum conservation, initial momentum of the system = final momentum of the system

$$\Rightarrow \quad mv = Mv_2 - mv_1$$

$$\Rightarrow \quad Mv_2 = m(v + v_1)$$

$$Mv_2 = m\sqrt{2gl}\,(\sqrt{1 - \cos\theta_0} + \sqrt{1 - \cos\theta_1})$$

Dividing Eq. (v) and Eq. (iv), we get

$$\Rightarrow \quad \frac{M}{m} = \frac{\sqrt{1 - \cos\theta_0} + \sqrt{1 - \cos\theta_1}}{\sqrt{1 - \cos\theta_0} - \sqrt{1 - \cos\theta_1}}$$

$$= \frac{\sqrt{\sin^2\left(\frac{\theta_0}{2}\right)} + \sqrt{\sin^2\left(\frac{\theta_1}{2}\right)}}{\sqrt{\sin^2\left(\frac{\theta_0}{2}\right)} - \sqrt{\sin^2\left(\frac{\theta_1}{2}\right)}}$$

$$\frac{M}{m} = \frac{\sin\left(\frac{\theta_0}{2}\right) + \sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_0}{2}\right) - \sin\left(\frac{\theta_1}{2}\right)}$$

For small θ_0, we have

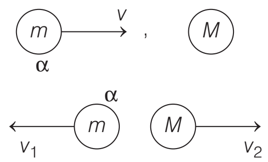
$$\frac{M}{m} = \frac{\frac{\theta_0}{2} + \frac{\theta_1}{2}}{\frac{\theta_0}{2} - \frac{\theta_1}{2}} \text{ or } M = m\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$$

95 An α-particle of mass m suffers one-dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing 64% of its initial kinetic energy. The mass of the nucleus is **[2019, 12 Jan Shift-II]**

(a) 1.5 m (b) 4 m
(c) 3.5 m (d) 2 m

Ans. *(b)*

We have following collision, where mass of α particle $= m$ and mass of nucleus $= M$



Let α particle rebounds with velocity v_1, then

Given; final energy of α = 36% of initial energy

$$\Rightarrow \quad \frac{1}{2}mv_1^2 = 0.36 \times \frac{1}{2}mv^2$$

$$\Rightarrow \quad v_1 = 0.6\,v \quad \text{...(i)}$$

As unknown nucleus gained 64% of energy of α, we have

$$\frac{1}{2}Mv_2^2 = 0.64 \times \frac{1}{2}mv^2$$

$$\Rightarrow \quad v_2 = \sqrt{\frac{m}{M}} \times 0.8\,v \quad \text{...(ii)}$$

From momentum conservation, we have

$$mv = Mv_2 - mv_1$$

Substituting values of v_1 and v_2 from Eqs. (i) and (ii), we have

$$mv = M\sqrt{\frac{m}{M}} \times 0.8v - m \times 0.6v$$

$$\Rightarrow \quad 1.6\,mv = \sqrt{mM} \times 0.8v \quad \Rightarrow 2m = \sqrt{mM}$$

$$\Rightarrow \quad 4m^2 = mM \quad \Rightarrow \quad M = 4m$$

96 In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles after collision, is **[JEE Main 2018]**

(a) $\frac{v_0}{4}$ (b) $\sqrt{2}\, v_0$

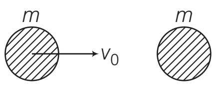
(c) $\frac{v_0}{2}$ (d) $\frac{v_0}{\sqrt{2}}$

Ans. *(c)*

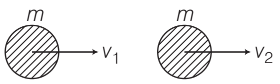
Key Idea Momentum is conserved in all type of collisions,

Final kinetic energy is 50% more than initial kinetic energy

$$\Rightarrow \quad \frac{1}{2}mv_2^2 + \frac{1}{2}mv_1^2 = \frac{150}{100} \times \frac{1}{2}mv_0^2 \quad \text{...(i)}$$



Before collision



After collision

Conservation of momentum gives,

$$mv_0 = mv_1 + mv_2$$

$$v_0 = v_2 + v_1 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$v_1^2 + v_2^2 + 2v_1v_2 = v_0^2 \Rightarrow 2v_1v_2 = \frac{-v_0^2}{2}$$

$$\therefore \quad (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2 = 2v_0^2$$

or $$v_{\text{rel}} = \sqrt{2}v_0$$

97 A particle of mass m moving in the x-direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y-direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to **[JEE Main 2015]**

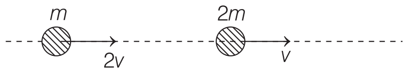
(a) 44% (b) 50%

(c) 56% (d) 62%

Ans. *(c)*

Key Idea Conservation of linear momentum can be applied but energy is not conserved.

Consider the movement of two particles as shown below.



Conserving linear momentum in x-direction

$$(p_i)x = (p_f)x$$

or $$2mv = (2m + m)\,v_x$$

or $$v_x = \frac{2}{3}v$$

Conserving linear momentum in y-direction

$$(p_i)y = (p_f)y \text{ or } 2mv = (2m + m)\,v_y$$

or $$v_y = \frac{2}{3}v$$

Initial kinetic energy of the two particles system is

$$E_i = \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)(v)^2$$

$$= \frac{1}{2} \times 4mv^2 + \frac{1}{2} \times 2mv^2$$

$$= 2mv^2 + mv^2 = 3mv^2$$

Final energy of the combined two particles system is

$$E_f = \frac{1}{2}(3m)(v_x^2 + v_y^2)$$

$$= \frac{1}{2}(3m)\left[\frac{4v^2}{9} + \frac{4v^2}{9}\right]$$

$$= \frac{3m}{2}\left[\frac{8v^2}{9}\right] = \frac{4mv^2}{3}$$

Loss in the energy $\Delta E = E_i - E_f$

$$= mv^2\left[3 - \frac{4}{3}\right] = \frac{5}{3}mv^2$$

Percentage loss in the energy during the collision

$$\frac{\Delta E}{E_i} \times 100 = \frac{(5/3)\,mv^2}{3mv^2} \times 100 = \frac{5}{9} \times 100$$

$$\simeq 56\%$$

98 This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement I A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$, then $f = \left(\frac{m}{M+m}\right)$.

Statement II Maximum energy loss occurs when the particles get stuck together as a result of the collision. **[JEE Main 2013]**

(a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I

(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

(c) Statement I is true, Statement II is false

(d) Statement I is false, Statement II is true

Ans. *(d)*

Energy $E = \frac{p^2}{2m}$, where p is momentum, m is the mass moving of the particle. Maximum energy loss occurs when the particles get stuck together as a result of the collision.

Maximum energy loss (ΔE)

$$= \frac{p^2}{2m} - \frac{p^2}{2(m+M)}$$

where, $(m + M)$ is the resultant mass when the particles get stuck.

$$\Delta E = \frac{p^2}{2m}\left[1 - \frac{m}{m+M}\right] = \frac{p^2}{2m}\left[\frac{M}{m+M}\right]$$

Also, $p = mv$

$$\therefore \quad \Delta E = \frac{m^2v^2}{2m}\left[\frac{M}{m+M}\right] = \frac{mv^2}{2}\left[\frac{M}{m+M}\right]$$

Comparing the expression with

$$\Delta E = f\left(\frac{1}{2}mv^2\right), f = \frac{M}{m+M}$$

99 **Statement I** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement II Principle of conservation of momentum holds true for all kinds of collisions. **[AIEEE 2010]**

(a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I

(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

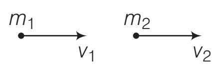
(c) Statement I is true, Statement II is true

(d) Statement I is true, Statement II is false

Ans. *(a)*

If it is a completely inelastic collision, then

$$m_1v_1 + m_2v_2 = m_1v + m_2v$$



$$v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2},$$

$$\text{KE} = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$$

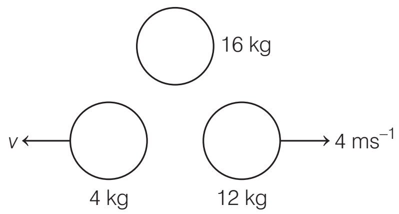
As p_1 and p_2 both simultaneously cannot be zero. Therefore, total KE cannot be lost.

100 A bomb of mass 16 kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is 4ms^{-1}. The kinetic energy of the other mass is **[AIEEE 2006]**

(a) 144 J (b) 288 J
(c) 192 J (d) 96 J

Ans. ***(b)***

Here, momentum of the system is remaining conserved as no external force is acting on the bomb (system).



Initial momentum (before explosion)
= Final momentum (after explosion)

Let velocity of 4 kg mass be v ms^{-1}. From momentum conservation, we can say that its direction is opposite to velocity of 12 kg mass.

From $\quad p_i = p_f$

$\Rightarrow \quad 0 = 12 \times 4 - 4 \times v$

or $\quad v = 12$ m/s

$$\therefore \text{KE of 4 kg mass} = \frac{4 \times (12)^2}{2} = 288 \text{ J}$$

101 A mass m moves with a velocity v and collides inelastically with another identical mass. After collision, the 1st mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the second mass after collision. **[AIEEE 2005]**

(a) v (b) $\sqrt{3}v$
(c) $\frac{2}{\sqrt{3}}v$ (d) $\frac{v}{\sqrt{3}}$

Ans. ***(c)***

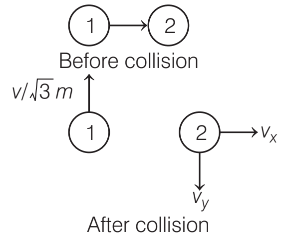
In x-direction,

Apply conservation of momentum, we get

$$mu_1 + 0 = mv_x$$

$\Rightarrow \quad mv = mv_x$

$\Rightarrow \quad v_x = v$



In y-direction, apply conservation of momentum, we get

$$0 + 0 = m\left(\frac{v}{\sqrt{3}}\right) - mv_y \Rightarrow v_y = \frac{v}{\sqrt{3}}$$

Velocity of second mass after collision

$$v' = \sqrt{\left(\frac{v}{\sqrt{3}}\right)^2 + v^2} = \sqrt{\frac{4}{3}v^2}$$

or $\quad v' = \frac{2}{\sqrt{3}}v$