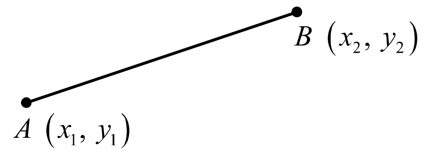
Straight line

Chapter 24

DISTANCE FORMULA

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

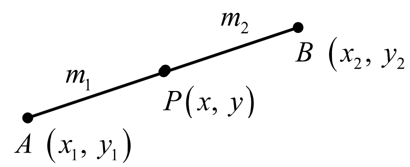
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



SECTION FORMULA

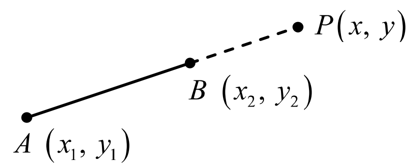
1. The coordinates of the point $P(x, y)$, dividing the line segment joining the two points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m_1 : m_2$, are given by

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2},\ y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}, \qquad \frac{AP}{BP} = \frac{m_1}{m_2}$$



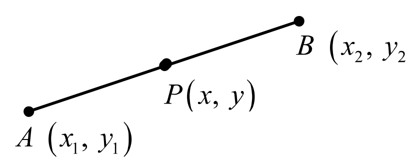
2. The coordinates of the point $P(x, y)$, dividing the line segment joining the two points $A(x_1, y_1)$ and $B(x_2, y_2)$ externally in the ratio $m_1 : m_2$, are given by

$$x = \frac{m_1 x_2 - m_2 x_1}{m_1 - m_2},\ y = \frac{m_1 y_2 - m_2 y_1}{m_1 - m_2}, \qquad \frac{AP}{BP} = \frac{m_1}{m_2}$$



3. The coordinates of the mid point of the line segment joining the two points $A(x_1, y_1)$ and $B(x_2, y_2)$ are given by

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

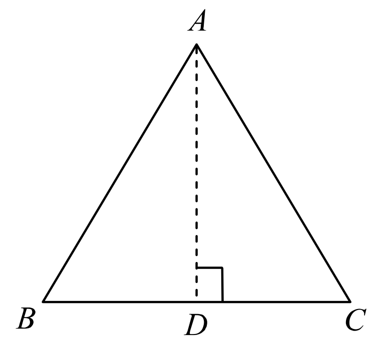


AREA OF A TRIANGLE

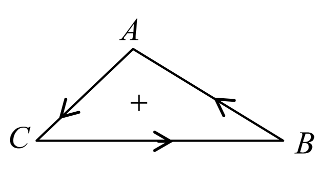
Let $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) respectively be the coordinates of the vertices A, B, C of a triangle ABC. Then the are of triangle ABC, is

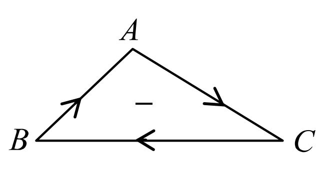
$$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right] \qquad \ldots (1)$$

$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \qquad \ldots (2)$$



While using formula (1) or (2), order of the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) has not been taken into account. If we plot the points $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$, then the area of the triangle as obtained by using formula (1) or (2) will be positive or negative as the points A, B, C are in anti-clockwise or clockwise directions,





So, while finding the area of triangle $ABC,$ we use the formula :

$$\text{Area of } \Delta ABC = \frac{1}{2}\left| x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)\right| = \frac{1}{2}\left|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right|$$

Note :

(a) If the three points A, B, C are collinear then area of $\Delta\ ABC$ is zero.

(b) **The area of a quadrilateral**, whose vertices are $A\left(x_1,\ y_1\right),\ B\left(x_2,\ y_2\right),\ C\left(x_3,\ y_3\right)$ and $D\left(x_4,\ y_4\right),$ is

$$= \frac{1}{2}\left|\left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & y_3 \\ x_4 & y_4 \end{vmatrix} + \begin{vmatrix} x_4 & y_4 \\ x_1 & y_1 \end{vmatrix}\right]\right|$$

(c) **The area of a polygon** of n sides with vertices $A_1\left(x_1,\ y_1\right),\ A_2\left(x_2,\ y_2\right),....,\ A_n\left(x_n,\ y_n\right)$ is

$$= \frac{1}{2}\left|\left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \ldots + \begin{vmatrix} x_{n-1} & y_{n-1} \\ x_n & y_n \end{vmatrix} + \begin{vmatrix} x_n & y_n \\ x_1 & y_1 \end{vmatrix}\right]\right|$$

(d) If $a_1x+b_1y+c_1=0,\ a_2x+b_2y+c_2=0$ and $a_3x+b_3y+c_3=0$ are the equations of the sides of a triangle, then the area of the triangle is $= \dfrac{1}{2\left|C_1C_2C_3\right|}\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2$

where $C_1,\ C_2,\ C_3$ are the cofactors of $c_1,\ c_2,\ c_3$ in the determinant

i.e. $C_1 = a_2b_3 - a_3b_2,\ C_2 = a_3b_1 - a_1b_3$ and $C_3 = a_1b_2 - a_2b_1.$

LOCUS

When a point moves in a plane under certain geometrical conditions, the point traces out a path. This path of a moving point is called its locus.

EQUATION OF LOCUS

The equation to a locus is the relation which exists between the coordinates of any point on the path and which holds for no other point except those lying on the path.

PROCEDURE FOR FINDING THE EQUATION OF THE LOCUS OF A POINT

(i) If we are finding the equation of the locus of the locus of a point $P,$ assign coordinates $\left(h,\ k\right)$ to $P.$

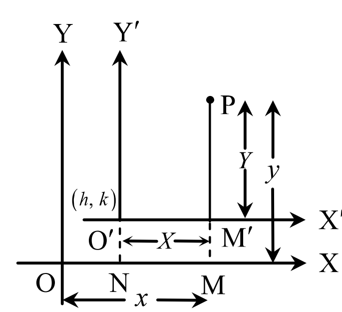
(ii) Express the given conditions as equations in terms of the known quantities to facilitate calculations. We sometimes include some unknown quantities known as parameters.

(iii) Eliminate the parameters, so that the eliminate contains only $h,\ k$ and known quantities.

(iv) Replace h by $x,$ and k by $y,$ in the eliminate. The resulting equation would be the equation of the locus of $P.$

(v) If x and y coordinates of the moving point are obtained in terms of a third variable t (called the parameter), eliminate t to obtain the relation in x and y and simplify this relation. This will give the required equation of locus.

TRANSLATION OF AXES



The translation of axes involves the shifting of the origin to a new point, the new axes remaining parallel to the original axes.

Let $OX,\ OY$ be the original axes and O' be the new origin. Let coordinates of O' referred to original axes i.e. $OX,\ OY$ be $\left(h,\ k\right).$

Let $O'X'$ and $O'Y'$ be drawn parallel to and in the same direction as OX and OY respectively. Let P be any point in the plane having coordinates (x, y) referred to old axes and (X, Y) referred to new axes.

Then, $x = OM = ON + NM = ON + O'M'$ and $y = MP = MM' + M'P = NO' + M'P$

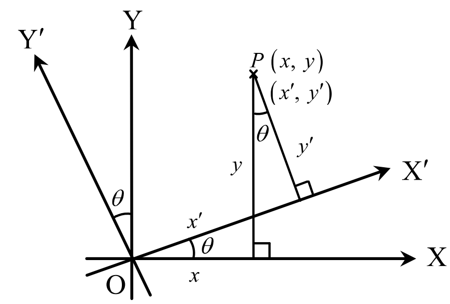
$= h + X = X + h$ $= k + Y = Y + k.$

Thus, we get $x = X + h,\ y = Y + k \Rightarrow X = x - h,\ Y = y - k.$

Thus, the point whose coordinates were (x, y) has new coordinates $(x - h, y - k)$.

ROTATION OF AXES

ROTATION OF AXES WITHOUT CHANGING THE ORIGIN

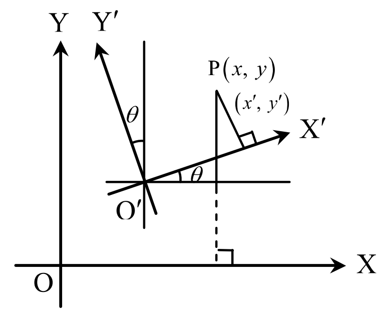


Let OX, OY be the original axes and OX', OY' be the new axes obtained by rotating OX and OY through an angle θ in the anticlockwise sense. Let P be any point in the plane having coordinates (x, y) w.r.t. axes OX and OY and (x', y') w.r.t. axes OX' and OY'.

Then,

$$\begin{cases} x = x'\cos\theta - y'\sin\theta \\ y = x'\sin\theta + y'\cos\theta \end{cases} \quad \text{and} \quad \begin{cases} x' = x\cos\theta + y\sin\theta \\ y' = -x\sin\theta + y\cos\theta \end{cases}.$$

CHANGE OF ORIGIN AND ROTATION OF AXES



If origin is changed to $O'(h, k)$ and axes are rotated about the new origin O' by angle θ in the anticlockwise sense such that the new coordinates of $P(x, y)$ become (x', y') then the equations of transformation will be

$$x = h + x'\cos\theta - y'\sin\theta \quad \text{and} \quad y = k + x'\sin\theta + y'\cos\theta.$$

GENERAL EQUATION OF A STRAIGHT LINE

An equation of the form $ax + by + c = 0$, where a, b, c are constants and a, b are not simultaneously zero, always represents a straight line.

SLOPE OF A LINE

If a line makes an angle $\theta\left(\theta \neq \frac{\pi}{2}\right)$ with the positive direction of x-axis, then $\tan\theta$ is the slope or gradient of that line. It is usually denoted by m. *i.e.* $m = \tan\theta.$

Slope of the line joining two points (x_1, y_1) and (x_2, y_2) $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{y_2 - y_1}{x_2 - x_1}.$

INTERCEPT OF A LINE ON THE AXES

(i) **Intercept of a line on *x*-axis**

If a line cuts x-axis at $(a, 0)$, then a is called the intercept of the line on x-axis. $|a|$ is called the length of the intercept of the line on x-axis. Intercept of a line on x-axis may be positive or negative.

(ii) **Intercept of a line on *y*-axis**

If a line cuts y-axis at $(0, b)$, then b is called the intercept of the line on y-axis and $|b|$ is called the length of the intercept of the line on y-axis. Intercept of a line on y-axis may be positive or negative.

EQUATIONS OF LINES PARALLEL TO AXES

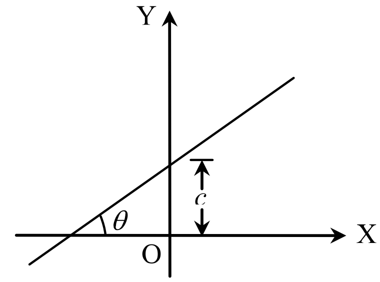
1. **Equation of *x*-axis :** The equation of x-axis is $y = 0$.
2. **Equation of *y*-axis :** The equation of y-axis is $x = 0$.
3. **Equation of a line parallel to *y*-axis :** The equation of the straight line parallel to y-axis at a distance a from it on the positive side of x-axis is $x = a$.

4. **Equation of a line parallel to *x*-axis :** The equation of the straight line parallel to x-axis at a distance b from it on the positive side of y-axis is $y = b$.

EQUATION OF A STRAIGHT LINE IN VARIOUS FORMS

1. **Slope-intercept form**

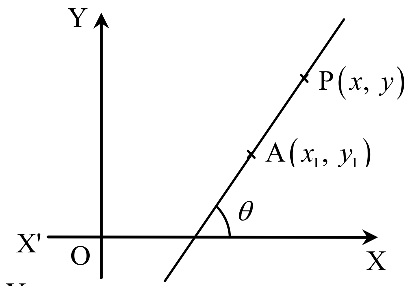
   The equation of a straight line whose slope is m and which cuts an intercept c on the y-axis is given by $y = mx + c$.



2. **Point-slope form**

   The equation of a straight line passing through the point (x_1, y_1) and having slope m is given by

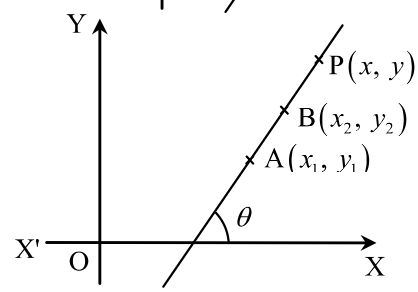
   $$y - y_1 = m(x - x_1).$$



3. **Two-point form**

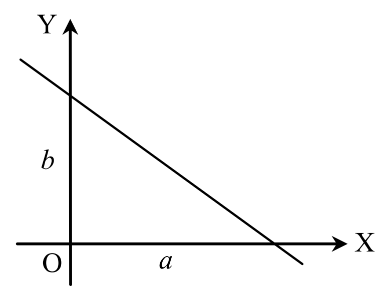
   The equation of a straight line passing through two points (x_1, y_1) and (x_2, y_2) is given by

   $$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$



4. **Intercept form**

   The equation of a straight line which cuts off intercepts a and b on x-axis and y-axis respectively is given by $\frac{x}{a} + \frac{y}{b} = 1.$

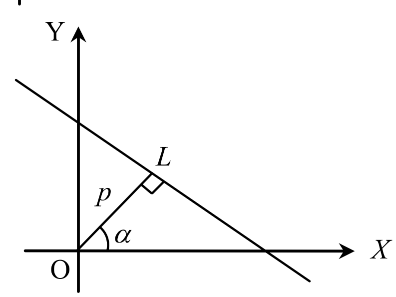


5. **Normal form (or perpendicular form)**

   The equation of a straight line upon which the length of the perpendicular from the origin is p and the perpendicular makes an angle α with the positive direction of x-axis is given by

   $$x\cos\alpha + y\sin\alpha = p.$$

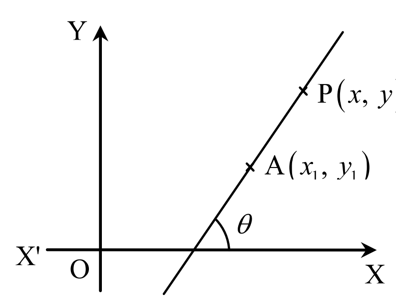
   Note : In normal form of equation of a straight line p is always taken as positive and α is measured from positive direction of x-axis in anticlockwise direction between 0 and 2π .



6. **Parametric form or symmetric form**

   The equation of a straight line passing through the point (x_1, y_1) and making an angle θ with the positive direction of x-axis is

   $$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r \qquad \text{where } \; 0 \le \theta < \pi$$



where r is the distance of the point (x, y) from the point (x_1, y_1).

Note : The coordinates of any point on the line at a distance r from the point $A(x_1, y_1)$ can be taken as $(x_1 + r\cos\theta, y_1 + r\sin\theta)$ or $(x_1 - r\cos\theta, y_1 - r\sin\theta)$ and $0 \le \theta \le \pi$

REDUCTION OF THE GENERAL EQUATION TO DIFFERENT STANDARD FORMS

1. **Slope-intercept form :** To reduce the equation $Ax + By + C = 0$ to the form $y = mx + c$.

   Given equation is $Ax + By + C = 0$ or $y = -\frac{A}{B}x - \frac{C}{B}$

   which is of the form $y = mx + c$,

   where $m = -\frac{A}{B}$ and $c = -\frac{C}{B}$.

   Note : Slope of the line $Ax + By + C = 0$ is $m = -\frac{\text{coefficient of } x}{\text{coefficient of } y} = -\frac{A}{B}$.

2. **Intercept form :** To reduce the equation $Ax + By + C = 0$ to the form $\frac{x}{a} + \frac{y}{b} = 1$.

   This reduction is possible only when $C \ne 0$.
   Given equation is $Ax + By + C = 0$

   $\Rightarrow -\frac{A}{C}x - \frac{B}{C}y = 1$, where $C \ne 0$ or $\frac{x}{-C/A} + \frac{y}{-C/B} = 1$, which is of the form $\frac{x}{a} + \frac{y}{b} = 1$,

   where $a = -\frac{C}{A}$ and $b = -\frac{C}{B}$.

3. **Normal form :** To reduce the equation $Ax + By + C = 0$ to the form $x\cos\alpha + y\sin\alpha = p$.
   Given equation is $Ax + By + C = 0$ or $Ax + By = -C$ … (1)

CASE 1. When $C < 0$, i.e. $-C > 0$, dividing both sides of equation (1) by $\sqrt{A^2 + B^2}$, we get

$$\frac{A}{\sqrt{A^2 + B^2}}x + \frac{B}{\sqrt{A^2 + B^2}}y = -\frac{C}{\sqrt{A^2 + B^2}}$$

which is of the form $x\cos\alpha + y\sin\alpha = p$,

where $\cos\alpha = \frac{A}{\sqrt{A^2 + B^2}}$, $\sin\alpha = \frac{B}{\sqrt{A^2 + B^2}}$ and $p = -\frac{C}{\sqrt{A^2 + B^2}}$

CASE 2. When $C > 0$ i.e. $-C < 0$; from (1) $-Ax - By = C$

or $$\frac{-A}{\sqrt{A^2 + B^2}}x - \frac{B}{\sqrt{A^2 + B^2}}y = \frac{C}{\sqrt{A^2 + B^2}}$$

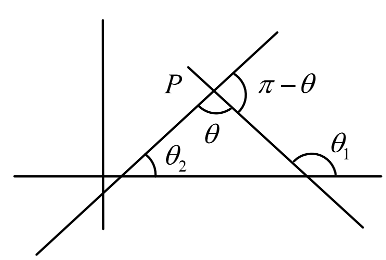
which is of the form $x\cos\alpha + y\sin\alpha = p$,

where $\cos\alpha = -\frac{A}{\sqrt{A^2 + B^2}}$, $\sin\alpha = -\frac{B}{\sqrt{A^2 + B^2}}$ and $p = \frac{C}{\sqrt{A^2 + B^2}}$.

ANGLE BETWEEN TWO INTERSECTING LINES

The angle θ between the lines $y = m_1x + c_1$ and $y = m_2x + c_2$ is given by

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2},$$



provided no line is $\perp$ to x-axis and the acute angle θ is given by $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$.

Note :

(a) If both the lines are $\perp$ to x-axis then the angle between them is $0°$.

(b) If any of the two lines is perpendicular to x-axis, then the slope of that line is infinite.

Let $m_1 = \infty$. Then $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{1 - \frac{m_2}{m_1}}{\frac{1}{m_1} + m_2}\right| = \left|\frac{1}{m_2}\right|$

or $\theta = \left|90° - \alpha\right|$ where $\tan\alpha = m_2$

(c) The two lines are parallel if and only if $m_1 = m_2$.

(d) The two lines are $\perp$ if and only if $m_1 \times m_2 = -1$.

CONDITION FOR TWO LINES TO BE COINCIDENT, PARALLEL, PERPENDICULAR OR INTERSECTING

Two lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are

(i) **Coincident,** if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$; (ii) **Parallel,** if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$;

(iii) **Perpendicular,** if $a_1 a_2 + b_1 b_2 = 0$;

(iv) **Intersecting,** if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e. if they are neither co-incident nor parallel.

EQUATION OF A LINE PARALLEL TO A GIVEN LINE

The equation of a line parallel to a given line $ax + by + c = 0$ is $ax + by + k = 0$, where k is a constant.

Thus to write the equation of any line parallel to a given line, do not change the coefficient of x and y and change the constant term only.

EQUATION OF A LINE PERPENDICULAR TO A GIVEN LINE

The equation of a line perpendicular to a given line $ax + by + c = 0$ is $bx - ay + k = 0$, where k is a constant. Thus to write the equation of any line perpendicular to a given line interchange the coefficients of x and y then change the sign of any one of them and finally change the constant term.

POINT OF INTERSECTION OF TWO GIVEN LINES

Let the two given lines be $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$.

Solving these two equations, the point of intersection of the given two lines is given by

$$\left(\frac{b_1 c_2 - b_2 c_1}{a_1 b_2 - a_2 b_1}, \frac{c_1 a_2 - c_2 a_1}{a_1 b_2 - a_2 b_1}\right).$$

INTERIOR ANGLES OF A TRIANGLE : To find the interior angles of a triangle arrange the slopes of the sides in decreasing order i.e., $m_1 > m_2 > m_3$. Then apply

$$\tan\alpha = \frac{m_1 - m_2}{1 + m_1 m_2}, \quad \tan\beta = \frac{m_2 - m_3}{1 + m_2 m_3}, \quad \tan\gamma = \frac{m_3 - m_1}{1 + m_3 m_1}$$

LINES THROUGH THE INTERSECTION OF TWO GIVEN LINES

The equation of any line passing through the point of intersection of the lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ is $(a_1 x + b_1 y + c_1) + k(a_2 x + b_2 y + c_2) = 0$, where k is a parameter. The value of k can be obtained by using one more conditions which the required line satisfies.

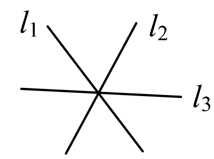
CONDITIONS OF CONCURRENCE

The family of given lines are said to be concurrent if they meet in a point.

WORKING RULE TO PROVE THAT THREE GIVEN LINES ARE CONCURRENT

1. The three lines $a_1x+b_1y+c_1=0, \quad a_2x+b_2y+c_2=0, \quad a_3x+b_3y+c_3=0$

   are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$



2. The three lines $P=0, Q=0$ and $R=0$ are concurrent if there exist constants l, m and n, not all zero at the same time, such that $lP+mQ+nR=0$.

   This method is particularly useful in theoretical results.

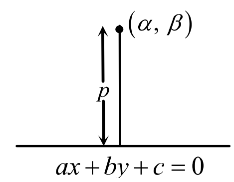
POSITION OF TWO POINTS RELATIVE TO A LINE

Two points (x_1, y_1) and (x_2, y_2) are on the same side or on opposite sides of the line $ax+by+c=0$ according as the expressions ax_1+by_1+c and ax_2+by_2+c have same sign or opposite signs.

LENGTH OF PERPENDICULAR FROM A POINT ON A LINE

The length of the perpendicular from the point (α, β) to the line $ax+by+c=0$ is given by

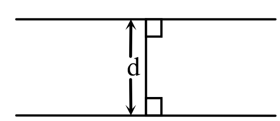
$$p=\frac{|a\alpha+b\beta+c|}{\sqrt{a^2+b^2}}.$$



DISTANCE BETWEEN TWO PARALLEL LINES

The distance between two parallel lines $ax+by+c_1=0$ and $ax+by+c_2=0$ is given by

$$d=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}.$$



Note : (a) The distance between two parallel lines can also be obtained by taking a suitable point (take $y=0$ and find x or take $x=0$ and find y) on one straight line and then finding the length of the perpendicular from this point to the second line.

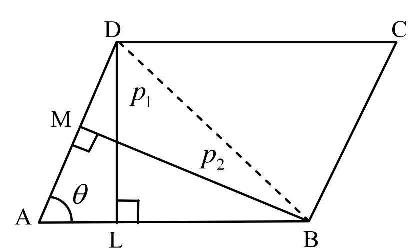
(b) Area of a parallelogram or a rhombus, equations of whose sides are given, can be obtained by using the following formula

$$Area=\frac{p_1p_2}{\sin\theta}=p_1p_2\operatorname{cosec}\theta$$

where $p_1 = DL =$ distance between lines AB and CD,

$p_2 = BM =$ distance between lines AD and BC,

θ = angle between adjacent sides AB and AD.



In the case of a rhombus, $p_1=p_2$

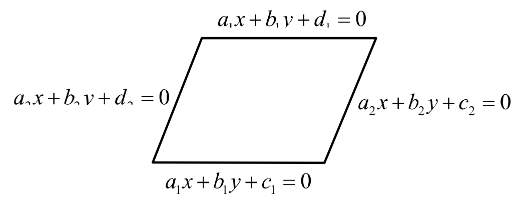
$\therefore$ Area of rhombus $=\dfrac{p_1^2}{\sin\theta}$.

Also, area of rhombus $= \frac{1}{2} d_1 d_2$

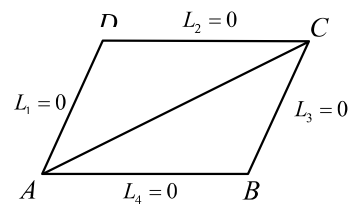
where d_1 and d_2 are the lengths of two $\perp$ diagonals of a rhombus.

(c) If the equation of sides of a parallelogram are as shown then its area is given by

$$\left| \frac{(c_1 - d_1)(c_2 - d_2)}{a_1 b_2 - a_2 b_1} \right|$$



(d) If equation of sides of a parallelogram are $L_1 = 0,\ L_2 = 0,\ L_3 = 0,\ L_4 = 0$ then $L_2L_3 - L_1L_4 = 0$ represents the diagonal *BD*. Again $L_1L_2 - L_3L_4 = 0$ represents the diagonal AC.



EQUATIONS OF STAIGHT LINES PASSING THROUGH A GIVEN POINT AND MAKING A GIVEN ANGLE WITH A GIVEN LINE

The equations of the straight lines which pass through a given point (x_1, y_1) and make a given angle α with the given straight line $y = mx + c$ are $y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$

EQUATIONS OF THE BISECTORS OF THE ANGLES BETWEEN THE LINES

The equations of the bisectors of the angles between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are given by

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}.$$

TO FIND THE EQUATION OF THE BISECTOR OF THE ACUTE AND OBTUSE ANGLE BETWEEN TWO LINES

Let the equations of the two lines be $a_1x + b_1y + c_1 = 0$ … (1)

and $a_2x + b_2y + c_2 = 0$ … (2)

where $c_1 > 0$ and $c_2 > 0$.

Then the equation $\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = + \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$

is the equation of bisector containing origin.

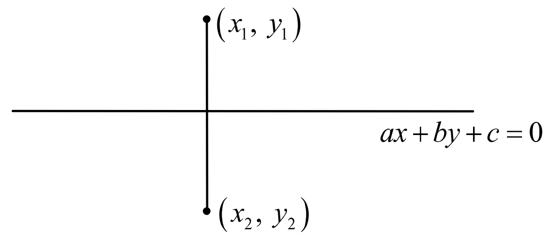
Similarly, the equation $\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = - \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$

is the equation of bisector not containing origin.

Note : If $a_1a_2 + b_1b_2 > 0$, then the origin lies in obtuse angle i.e., the bisector containing origin is obtuse angle bisector and if $a_1a_2 + b_1b_2 < 0$, then the origin lies in acute angle i.e., the bisector containing origin is acute angle bisector.

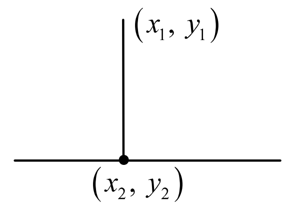
1. **IMAGE OF A POINT :** The co-ordinate of image (x_2, y_2) of a given point (x_1, y_1) in the line $ax+by+c=0$ are given by

$$\frac{x_2-x_1}{a}=\frac{y_2-y_1}{b}=\frac{-2(ax_1+by_1+c)}{a^2+b^2}$$



2. **FOOT OF THE PERPENDICULAR :** The co-ordinate of foot of perpendicular (x_2, y_2) of a given point (x_1, y_1) on the line $ax+by+c=0$ are given by

$$\frac{x_2-x_1}{a}=\frac{y_2-y_1}{b}=\frac{-(ax_1+by_1+c)}{a^2+b^2}$$



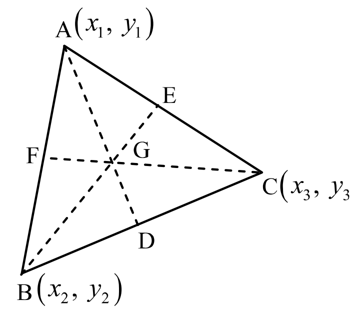
STANDARD POINTS OF A TRIANGLE

Centroid of a Triangle

The point of intersection of the medians of the triangle is called the centroid of the triangle. The centroid divides the medians in the ratio $2:1$.

The coordinates of the centroid of a triangle with vertices $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right).$$



(i) If P is any point in the plane of the triangle ABC and G is the centroid then

$$PA^2+PB^2+PC^2=GA^2+GB^2+GC^2+3PG^2$$

(ii) If G is the centroid of the triangle ABC, then

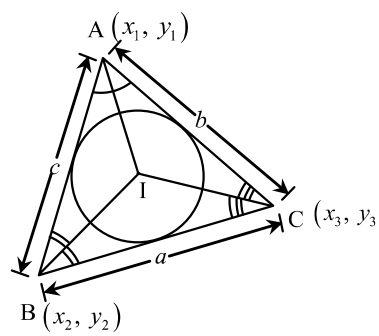
$$AB^2+BC^2+CA^2=3\left[GA^2+GB^2+GC^2\right]$$

Incentre of a Triangle

The point of intersection of the internal bisectors of the angles of a triangle is called the *incentre of the triangle*.

The coordinates of the incetre of a triangle with vertices $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are

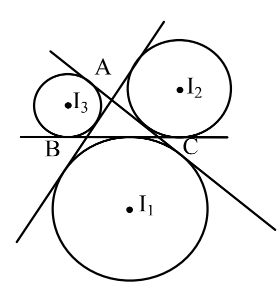
$$\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right).$$



Ex-centres of a Triangle

A circle touches one side outside the triangle and the other two extended sides then circle is known as excircle.

Let ABC be a triangle then there are three excircles, with three excentres I_1, I_2, I_3 opposite to vertices A, B and C respectively. If the vertices of triangle are $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ then

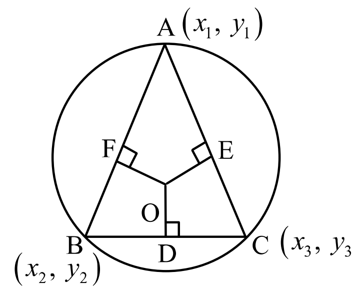


$$I_1 = \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c} \right)$$

$$I_2 = \left(\frac{ax_1 - bx_2 + cx_3}{a-b+c}, \frac{ay_1 - by_2 + cy_3}{a-b+c} \right)$$

$$I_3 = \left(\frac{ax_1 + bx_2 - cx_3}{a+b-c}, \frac{ay_1 + by_2 - cy_3}{a+b-c} \right).$$

CIRCUMCENTRE

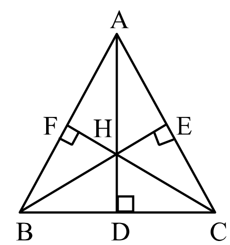


The circumcentre of a triangle is the point of intersection of the perpendicular bisectors of the sides of a triangle. It is the centre of the circle which passes through the vertices of the triangle and so its distance from the vertices of the triangle is same and this distance is known as the circum-radius of the triangle.

If angles of triangle ΔABC i.e. A, B, C and vertices of triangle $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are given, then circumcentre of the triangle ABC is

$$\left(\frac{x_1 \sin 2A + x_2 \sin 2B + x_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}, \frac{y_1 \sin 2A + y_2 \sin 2B + y_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C} \right)$$

ORTHOCENTRE



The orthocentre of a triangle is the point of intersection of altitudes.

If angles of a ΔABC, i.e. A, B and C and vertices of triangle $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are given, then orthocentre of ΔABC is

$$\left(\frac{x_1 \tan A + x_2 \tan B + x_3 \tan C}{\tan A + \tan B + \tan C}, \frac{y_1 \tan A + y_2 \tan B + y_3 \tan C}{\tan A + \tan B + \tan C} \right)$$

Note :

(i) If H is the orthocentre of ΔABC then

orthocentre of ΔAHB is C

orthocentre of ΔBHC is A

orthocentre of ΔAHC is B

(ii) If any two lines out of three lines i.e. AB, BC and CA are perpendicular, then orthocentre is the point of intersection of two perpendicular lines i.e., in right angled Δ the right angled vertex is the orthocentre of Δ and mid-point of hypotneuse is circumcentre