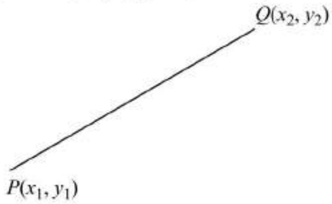
Co-ordinate Geometry

INTRODUCTION

Geometry begins with a point and straight line. Uptil now, we have studied geometry without any use of algebra. In 1637, Descartes used algebra in the study of geometrical relationships. Thus, a new type of geometry was introduced which was given the name analytical geometry or co-ordinate geometry. Thus, co-ordinate geometry is that branch of mathematics in which geometry is studied algebraically, i.e., geometrical figures are studied with the help of equations.

SOME BASIC FORMULAE

1. Distance Formula Distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by



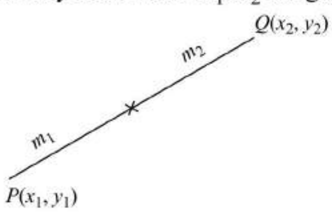
$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Illustration 1 Find the distance between the pair of points $A(2, 5)$ and $B(-3, 7)$.

Solution: $AB = \sqrt{(-3-2)^2 + (7-5)^2} = \sqrt{25+4} = \sqrt{29}$.

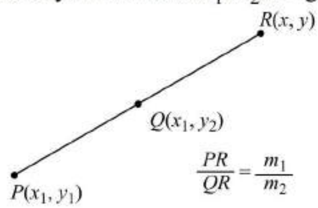
2. Section Formulae

(*a*) *Formula for internal division* The coordinates of the point $R(x, y)$ which divides the join of two given points $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio $m_1:m_2$ are given by



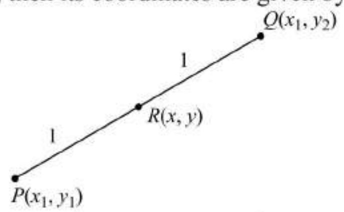
$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right).$$

(*b*) *Formula for external division* The coordinates of the point $R(x, y)$ which divides the join of two given points $P(x_1, y_1)$ and $Q(x_2, y_2)$ externally in the ratio $m_1:m_2$ are given by



$$\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right).$$

(*c*) *Mid-point formula* If R is the mid point of P Q, then its coordinates are given by



$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Illustration 2 Find the coordinates of the point which divides:

(i) the join of (2, 3) and (5, –3) internally in the ratio 1:2

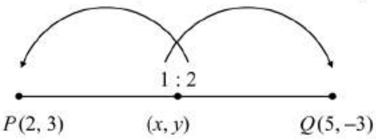
(ii) the join of (2, 1) and (3, 5) externally in the ratio 2:3

Solution: (i) Let (x, y) be the coordinates of the point of division. Then,

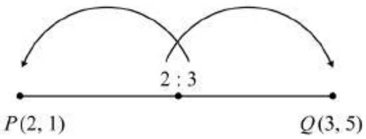
$$x = \frac{1(5)+2(2)}{1+2} = \frac{5+4}{3} = 3$$

$$y = \frac{1(-3)+2(3)}{1+2} = \frac{-3+6}{3} = 1$$

∴ Coordinates of the point of division are (3, 1).



(ii) Let (x, y) be the coordinates of the point of division.



Then, $$x = \frac{2(3)-3(2)}{2-3} = \frac{6-6}{-1} = 0$$

$$y = \frac{2(5)-3(1)}{2-3} = \frac{10-3}{-1} = -7$$

∴ Coordinates of the point of division are (10, –7).

Illustration 3 Find the coordinates of the mid point of the join of points $P(2, -1)$ and $Q(-3, 4)$.

Solution: The coordinates of the mid-point are

$$x = \frac{2-3}{2} = -\frac{1}{2}$$

$$y = \frac{-1+4}{2} = \frac{3}{2}$$

∴ Coordinates of the mid point are $\left(-\frac{1}{2}, \frac{3}{2}\right)$

Note:

If the point R is given and we are required to find the ratio in which R divides the line segment PQ, it is convenient to take the ratio k:1.

Then, the coordinates of R are

$$\left(\frac{kx_2+x_1}{k+1}, \frac{ky_2+y_1}{x+1}\right).$$

Illustration 4 In what ratio does the point (6, – 6) divide the join of (1, 4) and (9, –12)?

Solution: Let the point R (6, – 6) divides the join of P(1, 4) and Q (9, –12) in the ration k:1

By section formula, the coordinates of R are

$$\left(\frac{k(9)+1(1)}{k+1}, \frac{k(-12)+1(4)}{k+1}\right), \text{ i.e., } \left(\frac{9k+1}{k+1}, \frac{-12k+4}{k+1}\right)$$

But the coordinates of R are given to be (6, – 6)

$$\therefore \quad \frac{9k+1}{k+1} = 6 \text{ and } \frac{-12k+4}{k+1} = -6$$

$$\Rightarrow \quad 9k + 1 = 6k + 6 \text{ and } -12k + 4 = -6k - 6$$

$$\Rightarrow \quad 3k = 5 \text{ and } -6k = -10$$

In either case, $k = \frac{5}{3}$ (+ve)

∴ R divides PQ internally in the ratio $\frac{5}{3}$:1

i.e., 5:3

3. Centroid of a Triangle The point of concurrence of the medians of a triangle is called the centroid of triangle. It divides the median in the ratio 2 : 1.

The coordinates of the centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) are given by

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right).$$

Illustration 5 Find the centroid of the triangle whose angular points are (3, – 5), (–7, 4) and (10, –2), respectively.

Solution: The coordinates of centroid are

$$\left(\frac{3-7+10}{3}, \frac{-5+4-2}{3}\right) = (2, -1)$$

4. Incentre of a Triangle Incentre of a triangle is the point of concurrence of the internal bisectors of the angles of a triangle.

The coordinates of the incentre of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) are given by

$$\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right).$$

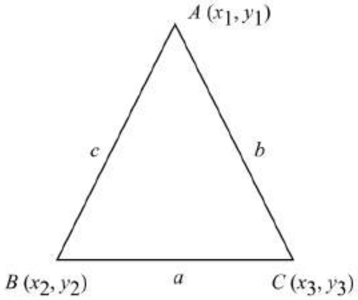


Illustration 6 Find the coordinates of incentre of a triangle having vertices $A(0, 0)$, $B(20, 15)$ and $C(-36, 15)$

Solution: We have,

$$a = BC = \sqrt{(20+36)^2 + (15-15)^2} = 56$$

$$b = AC = \sqrt{(36)^2 + (15)^2} = 39$$

$$c = AB = \sqrt{(20)^2 + (15)^2} = 25$$

$\therefore$ Coordinates of incentre are

$$x = \frac{ax_1 + bx_2 + cx_3}{a+b+c} = \frac{56.0 + 39.20 + 25.(-36)}{56+39+25} = -1.$$

$$y = \frac{ay_1 + by_2 + cy_3}{a+b+c} = \frac{56.0 + 39.15 + 25.15}{56+39+25} = 8$$

Thus, $I \equiv (-1, 8)$

5. Area of a Triangle The area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

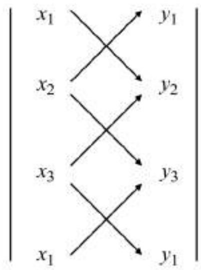
$$A = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Condition of Collinearity of Three Points

The points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ will be collinear (i.e., will lie on a straight line) if the area of the triangle, assumed to be formed by joining them is zero.

Short-cut Method for Finding the Area

1. Write the coordinates of the vertices taken in order in two columns. At the end, repeat the coordinates of the first vertex.



2. Mark the arrow-heads as indicated. Each arrow-head shows the product.
3. The sign of the product remains the same for downward arrows while it changes for an upward arrow.
4. Divide the result by 2.
5. Thus, $\Delta = \frac{1}{2}[(x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2) + (x_3 y_1 - x_1 y_3)]$.

Illustration 7 Find the area of a triangle whose vertices are (4, 4) (3, – 2) and (–3, 16)

Solution: Required area

$$= \frac{1}{2}|-8 - 12 + 48 - 6 - 12 - 64|$$

$$= \frac{1}{2}|-54| = \frac{1}{2}(54) = 27 \text{ sq units.}$$

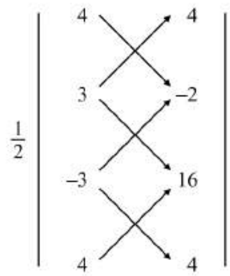
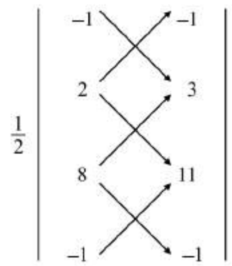


Illustration 8 Show that the three points (–1, –1), (2, 3) and (8, 11) lie on a line

Solution: The area of the triangle whose vertices are (–1, – 1), (2, 3) and (8, 11) is

$$\Delta = \frac{1}{2}|-3 + 2 + 22 - 24 - 8 + 11| = \frac{1}{2}|0| = 0$$

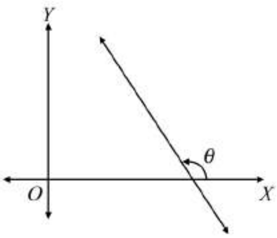
Since the area of the triangle is zero, the given points are collinear.



Slope or Gradient of a Line

The tangent of the angle which a line makes with the positive direction of x-axis is called the slope or the gradient of the line. It is generally denoted by m. If a line makes an angle θ with x-axis, then its slope

$$= \tan\theta, \text{ i.e., } m = \tan\theta.$$



Note:

1. If a line is parallel to x-axis, $m = \tan 0 = 0$.
2. If a line is parallel to y-axis, $m = \tan 90° = \infty$.

Illustration 9 Find the slope of a line whose inclination with x-axis is 30°.

Solution: Slope, $m = \tan 30° = \dfrac{1}{\sqrt{3}}$

Slope of a Line Joining Two Given Points

The slope of the line joining two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 \quad y_1}{x_2 \quad x_1} = \frac{\text{Difference of ordinates}}{\text{Difference of abscissae}}$$

Illustration 10 Find the slope of the line passing through the points (2, 3) and (4, 9).

Solution: Slope of the line $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{9-3}{4-2} = 3$$

Parallel and Perpendicular Lines

(*a*) Two lines are parallel if and only if their slopes m_1, m_2 are equal, i.e., if $m_1 = m_2$.

(*b*) Two lines are perpendicular if and only if their slopes m_1, m_2 satisfy the condition $m_1\, m_2 = -1$.

Illustration 11 Show that the line joining (2, –3) and (–5, 1) is

(*a*) parallel to the line joining (7, –1) and (0, 3)

(*b*) perpendicular to the line joining (4, 5) and (0, –2)

Solution: Let l_1 be the line joining the points (2, –3) and (–5, 1).

$\therefore$ Slope of $l_1 = \dfrac{1-(-3)}{-5-2} = -\dfrac{4}{7}$

(*a*) Let l_2 be the line joining the points (7, –1) and (0, 3).

$\therefore$ Slope of $l_2 = \dfrac{3-(-1)}{0-7} = -\dfrac{4}{7}$

$\because$ Slope of l_1 = slope of l_2 (each = – 4/7)

$\therefore$ Lines l_1 and l_2 are parallel.

(*b*) Let l_3 be the line joining the points (4, 5) and (0, –2).

$\therefore$ Slope of $l_3 = \dfrac{-2-5}{0-4} = \dfrac{-7}{-4} = \dfrac{7}{4}$

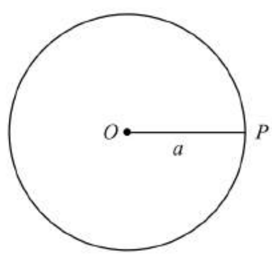
$\because$ Slope of $l_1 \times$ slope of $l_3 = -\dfrac{4}{7} \times \dfrac{7}{4} = -1$

$\therefore$ the lines l_1 and l_3 are perpendicular

Locus

When a point moves so that it always satisfies a given condition or conditions, the path traced out by it is called its locus under these conditions.

Illustration 12 Let O be a given point in the plane of the paper and let a point P move on the paper so that its distance from O is constant and is equal to a. All the positions of the moving point must lie on a circle whose centre is O and radius is a. This circle, therefore, the *locus* of P when it moves under the condition that its distance from O is equal to a constant a.



Short-cut Method to Find the Locus

1. Take a point on the locus and suppose that its coordinates are (x, y).
2. Apply the given conditions(s) to (x, y) and simplify the algebraic equation so formed.
3. The simplified equation is the required equation of the locus.

Illustration 13 A point moves so that its distance from (3, 0) is twice its distance from (–3, 0). Find the equation of its locus

Solution: Let $P(x, y)$ be any point on the locus. And, A (3, 0) and B (–3, 0) be the given points.

By the given condition, $PA = 2\,PB$

$$\Rightarrow \sqrt{(x-3)^2+(y-0)^2} = 2\sqrt{(x+3)^2+(y-0)^2}$$

Squaring both sides,

$$x^2 + y^2 - 6x + 9 = 4\,(x^2 + y^2 + 6x + 9)$$

or, $3x^2 + 3y^2 + 30x + 27 = 0$

or, $x^2 + y^2 + 10x + 9 = 0$,

which is the required equation of the locus

Practice Exercises

Difficulty Level-1 (Based on Memory)

1. The equation of the line through the point of intersection of $3x - y - 1 = 0$ and $x - 3y + 5 = 0$, passing through the point (1, 5) is:

   (*a*) $2x - y + 5 = 0$ (*b*) $2x + y + 5 = 0$

   (*c*) $x + y = 0$ (*d*) $x = 1$

   [Based on MAT, 2001]

2. The points (0, 8/3), (1, 3) and (82, 30) are the vertices of:

   (*a*) An obtuse-angled triangle

   (*b*) A right-angled triangle

   (*c*) An isosceles triangle

   (*d*) None of these

   [Based on MAT, 2002]

3. The line segment joining (–3, –4) and (1, –2) is divided by *y*-axis in the ratio:

   (*a*) 1:3 (*b*) 2:3

   (*c*) 3:1 (*d*) 3:2

4. The ratio in which (4, 5) divides the join of (2, 3), (7, 8) is:

   (*a*) –2:3 (*b*) –3:2

   (*c*) 3:2 (*d*) 2:3

5. The locus of the point, the sum of whose distances from the coordinate axes is 9 is:

   (*a*) $x^2 - y^2 = 9$ (*b*) $x^2 - y^2 = -9$

   (*c*) $y^2 - x^2 = 9$ (*d*) None of these

6. The equation of the line passing through the point (1, 1) and perpendicular to the line $3x + 4y - 5 = 0$ is

   (*a*) $3x + 4y - 7 = 0$ (*b*) $3x + 4y + k = 0$

   (*c*) $4x - 3y + 1 = 0$ (*d*) $4x - 3y - 1 = 0$

   [Based on FMS, 2006]

7. The point (22, 23) divider the join of P(7, 5) and Q externally in the ratio 3:5, then Q =

   (*a*) (3, 7) (*b*) (–3, 7)

   (*c*) (3, –7) (*d*) (–3, –7)

8. The centroid of a triangle formed by (7, p), (q, –6), (9, 10) is (6, 3), then (p, q) =

   (*a*) (4, 5) (*b*) (5, 4)

   (*c*) (–5, –2) (*d*) (5, 2)

9. The base vertices of a right angled isosceles triangle are (2, 4) and (4, 2) then its third vertex is:

   (*a*) (1, 1) or (2, 2) (*b*) (2, 2) or (4, 4)

   (*c*) (1, 10 or (3, 3) (*d*) (2, 2) or (3, 3)

10. Mid-points of the sides AB and AC of ΔABC are (3, 5) and (–3, –3) respectively, then the length of BC =

    (*a*) 10 (*b*) 15

    (*c*) 20 (*d*) 30

11. ABC is an isosceles triangle with $B \equiv (1, 3)$ and $C \equiv (-2, 7)$ then A =

    (*a*) (5/6, 6) (*b*) (6, 5/6)

    (*c*) (7, 1/8) (*d*) None of these.

12. The third vertex of an equilateral triangle whose two vertices are (2, 4), (2, 6) is:

    (*a*) ($\sqrt{3}$, 5) (*b*) (2$\sqrt{3}$, 5)

    (*c*) (2 + $\sqrt{3}$, 5) (*d*) (2, 5)

13. P(3, 4), Q(7, 7) are collinear with the point R where PR = 10. Then, R =

    (*a*) (5, 2) (*b*) (–5, 2)

    (*c*) (–5, –2) (*d*) (5, –2)

14. The nearest point from origin is:

(*a*) (2, –3) (*b*) (5, 0)

(*c*) (2, –1) (*d*) (1, 3)

15. A line is of length 10 and one end is (2, –3). If the abscissa of the other end is 10 then its ordinate is:

(*a*) 3 or 9 (*b*) –3 or –9

(*c*) 3 or –9 (*d*) –3 or 9

16. The distance between the two points is 5. One of them is (3, 2) and the ordinate of the second is –1, then its *x*-coordinate is:

(*a*) 7, –1 (*b*) –7, 1

(*c*) –7, –1 (*d*) 7, 1

17. The equation of second degree

$$x^2 + 2\sqrt{2}xy + 2y^2 + 4x + 4\sqrt{2}y + 1 = 0$$

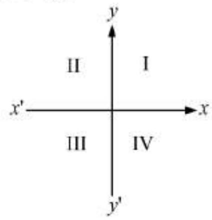
represents a pair of straight lines. The distance between them is:

(*a*) 4 (*b*) $4\sqrt{3}$

(*c*) 2 (*d*) $2\sqrt{3}$

[Based on FMS (Delhi), 2002]

18. In a rectangular coordinate system shown above, which quadrant, if any, contains, no point (*x*, *y*) that satisfies the inequality $2x - 3y \le -6$?



(*a*) I (*b*) II

(*c*) III (*d*) IV

[Based on ATMA, 2008]

19. A point divides internally the line segment joining the points (8, 9) and (– 7, 4) in the ratio 2:3. What are the coordinates of the point?

(*a*) (7, 2) (*b*) (2, 3)

(*c*) (3, 2) (*d*) (2, 7)

[Based on ATMA, 2008]

20. The coordinates of *A*, *B*, *C* are (– 1, 7), (3, 1) and (5, 7) respectively and *D*, *E*, *F* are the mid points of *BC*, *CA* and *AB* respectively. The area of the Δ *DEF* is (square units):

(*a*) 8 (*b*) 7.5

(*c*) 9 (*d*) 4

[Based on ATMA, 2008]

Difficulty Level-2
(Based on Memory)

1. The course of an enemy submarine as plotted on a set of rectangular axes gives the equation $2x + 3y = 5$. On the same axes, the course of a destroyer is indicated by the equation $x - y = 10$. The point (*x*, *y*) at which the submarine can be destroyed is:

(*a*) (7, –3) (*b*) (–3, 7)

(*c*) (–7, 3) (*d*) (3, –7)

[Based on FMS (Delhi), 2003]

2. The three lines $ax + by + c = 0$, $bx + cy + a = 0$ and $cx + ay + b = 0$ are concurrent, only when:

(*a*) $a^2 + b^2 + c^2 = 2abc$

(*b*) $a^3 + b^3 + c^3 = 3abc$

(*c*) $a^2 + b^2 + c^2 = ab + bc + ca$

(*d*) $a^3 + b^3 + c^3 = 3(ab + bc + ca)$

[Based on FMS (Delhi), 2002]

3. If $x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0$ represents a pair of straight lines, then the value of λ is:

(*a*) 4 (*b*) 3

(*c*) 2 (*d*) $2\sqrt{3}$

[Based on FMS (Delhi), 2002]

4. The curve described parametrically by

$x = t^2 + t + 1$ and $y = t^2 - t + 1$ represents:

(*a*) A pair of straight lines

(*b*) An ellipse

(*c*) A parabola

(*d*) A hyperbola

[Based on REC Tiruchirapalli, 2002]

5. A point moves so that its distance from *y*-axis is half of its distance from the origin. The locus of point is:

(*a*) $2x^2 - y^2 = 0$ (*b*) $x^2 - 3y^2 = 0$

(*c*) $3x^2 - y^2 = 0$ (*d*) $x^2 - 2y^2 = 0$

6. What is the locus of the point of intersection of two tangents to the parabola $y^2 = 4ax$, which are at right angles to each other?

(*a*) $x - a = 0$ (*b*) $x + a = 0$

(*c*) $x - a = 4$ (*d*) None of these

[Based on FMS, 2009]

7. Find the minimum value of $\sqrt{x^2+y^2}$ if $5x + 2y = 60$.

(*a*) $\frac{60}{13}$ (*b*) $\frac{13}{5}$

(*c*) $\frac{13}{12}$ (*d*) 1

[Based on FMS, 2010]

8. Which one of the following points is not on the graph of $y = \frac{x}{x+1}$?

(*a*) (0, 0) (*b*) $\left(-\frac{1}{2}, -1\right)$

(*c*) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (*d*) (–1, 1)

[Based on FMS, 2010]

9. Given the four equations:

1. $3y - 2x = 12$ **2.** $-2x - 3y = 10$

3. $3y + 2x = 12$ **4.** $2y + 3x = 10$

The pair representing perpendicular lines is:

(*a*) (1) and (4) (*b*) (1) and (3)

(*c*) (1) and (2) (*d*) (2) and (4)

[Based on FMS, 2010]

10. The points (6, 12) and (0, –6) are connected by a straight line. Another point on this line is:

(*a*) (3, 3) (*b*) (2, 1)

(*c*) (7, 16) (*d*) (–1, –4)

[Based on FMS, 2011]

11. In a rhombus *ABCD* the diagonals *AC* and *BD* intersect at the point (3, 4). If the point '*A*' is (1, 2), the diagonal *BD* has the equation:

(*a*) $x - y - 1 = 0$ (*b*) $x - y + 1 = 0$

(*c*) $x + y - 1 = 0$ (*d*) $x + y - 7 = 0$

[Based on IIFT, 2005]

12. In rectangular coordinates system, the curve $x^2y^2 = 20$ has real points:

(*a*) on all the four quadrants

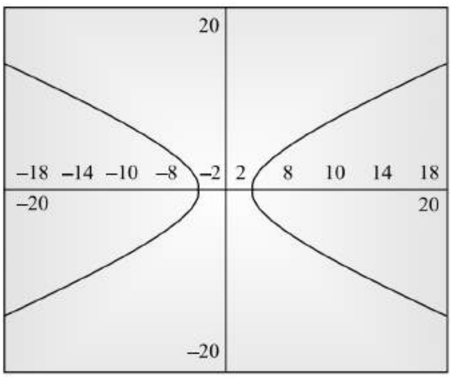
(*b*) on only the first and third quadrants

(*c*) on only the second and third quadrants

(*d*) none of above

[Based on IIFT, 2005]

13. Which equation can be graphically represented as follows?



(*a*) $8x^2 - 15y^2 = 169$ (*b*) $9x^2 - 16y^2 = 144$

(*c*) $|(x-8)(y-15)| = 12$ (*d*) $|(x-9)(y-16)| = 13$

[Based on XAT, 2007]

14. Triangle *ABC* has vertices *A*(0, 0), *B*(0, 6) and *C*(9, 0). The points *P* and *Q* lie on side. *AC* such that *AP* = *PQ* = *QC*. Similarly, the points *R* and *S* lie on side *AB* such that *AR* = *RS* = *SB*. If the line segments *PB* and *RC* intersect at *X*, then the slope on the line *AX* is:

(*a*) 2/3 (*b*) – 2/3

(*c*) 3/2 (*d*) – 3/2

[Based on XAT, 2007]

15. In a plane rectangular coordinate system, points *L*, *M*, *N* and *O* are represented by the coordinates (–5, 0), (1, –1), (0, 5), and (–1, 5) respectively. Consider a variable point *P* in the same plane. The minimum value of *PL* + *PM* + *PN* + *PO* is:

(*a*) $1+\sqrt{37}$ (*b*) $5\sqrt{2}+2\sqrt{10}$

(*c*) $\sqrt{41}+\sqrt{37}$ (*d*) $\sqrt{41}+1$

[Based on XAT, 2011]

16. In a rectangular coordinate system, triangle *ABC* is drawn so that one side of the triangle connects two points on the *y*-axis, *A*(0, 2) and *B*(0, – 4). If point *C* has coordinates (*c*, 0) and the area of Δ*ABC* is 21, then *c* is equal to:

(*a*) $\sqrt{53}$ (*b*) 7

(*c*) $\frac{7}{3}$ (*d*) 21

[Based on NMAT, 2005]

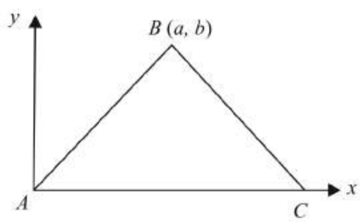
17. In the *xy*-coordinate system, if (*a*, *b*) and (*a* + 3), (*b* + *k*) are two points on the line defined by the equation *x* = 3*y* – 7, then *k* is equal to:

(*a*) 3 (*b*) 2

(*c*) 1/3 (*d*) 9

[Based on ATMA, 2006]

18. If the area of the triangle given be low is 20, then what are the coordinates of the point *C*?



(*a*) (0, 40/*a*) (*b*) $(a^2 + b^2, 0)$

(*c*) (20/*b*, 0) (*d*) (40/*b*, 0)

[Based on SCMHRD Ent. Exam., 2003]

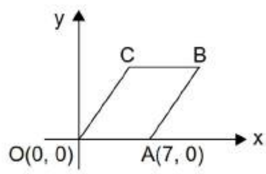
19. Coordinates of the points X, Y and Z are (6, 4), (−3,5) and (2, −4), respectively. Find the coordinates of a point that divides the medians from all the three vertices in the ratio 2:1.

(a) $\left(\frac{5}{3}, \frac{5}{3}\right)$ (b) $\left(\frac{5}{3}, \frac{5}{2}\right)$

(c) $\left(\frac{7}{4}, \frac{3}{2}\right)$ (d) Data insufficient

[Based on CAT, 2006]

20. In the figure, $OABC$ is a parallelogram. The area of the parallelogram is 21 sq units and the point C lies on the line $x = 3$. Find the coordinates of B.



(*a*) (3, 10)
(*b*) (10, 3)
(*c*) (10, 10)
(*d*) (8, 3)

[Based on CAT, 2012]

Answer Keys

Difficulty Level-1

1. (*d*) **2.** (*d*) **3.** (*c*) **4.** (*d*) **5.** (*d*) **6.** (*d*) **7.** (*d*) **8.** (*d*) **9.** (*b*) **10.** (*c*) **11.** (*a*) **12.** (*c*) **13.** (*c*)
14. (*c*) **15.** (*c*) **16.** (*a*) **17.** (*c*) **18.** (*d*) **19.** (*d*) **20.** (*d*)

Difficulty Level-2

1. (*a*) **2.** (*b*) **3.** (*c*) **4.** (*a*) **5.** (*c*) **6.** (*b*) **7.** (*a*) **8.** (*d*) **9.** (*a*) **10.** (*a*) **11.** (*d*) **12.** (*a*) **13.** (*b*)
14. (*a*) **15.** (*a*) **16.** (*b*) **17.** (*c*) **18.** (*d*) **19.** (*a*) **20.** (*b*)

Explanatory Answers

Difficulty Level-1

1. (*d*) The equation of the line through the intesection of $3x - y - 1 = 0$ and $x - 3y + 5 = 0$ is given by

$$(3x - y - 1) + k(x - 3y + 5) = 0 \qquad (1)$$

where k is a constant.

Since it passes through the point (1, 5), therefore

$$-3 + k(-9) = 0$$

$$\Rightarrow \qquad k = -1/3$$

$$\therefore (1) \Rightarrow (3x - y - 1) - \frac{1}{3}(x - 3y + 5) = 0 \Rightarrow x = 1.$$

2. (*d*) Let $A\left(0, \frac{8}{3}\right)$, B (1, 3) and C (82, 30) be the three points

$$AB = \sqrt{(1-0)^2 + \left(3 - \frac{8}{3}\right)^2}$$

$$= \sqrt{1 + \frac{1}{9}} = \frac{\sqrt{10}}{3}$$

$$BC = \sqrt{(82-1)^2 + (30-3)^2}$$

$$= \sqrt{6561 + 729}$$

$$= \sqrt{7290} = 27\sqrt{10}$$

$$CA = \sqrt{(82-0)^2 + (30 - 8/3)^2}$$

$$= \sqrt{\frac{10 \times (82)^2}{9}} = \frac{82}{3}\sqrt{10}$$

Here $AB + BC = CA \Rightarrow ABC$ is a straight line.

3. (*c*)

4. (*d*)

5. (*d*) Sum of the distances from the axis

$$= |x| + |y| = 9.$$

6. (*d*) Let perpendicular line be $4x - 3y + k = 0$

By putting (1, 1) in the given equation,

$\therefore\ 4 - 3 + k = 0 \Rightarrow k = -1$

$\therefore$ Equation is $4x - 3y - 1 = 0$

7. (*d*) $\left(\frac{35-3x}{5-3}, \frac{25-3y}{5-3}\right) = (22, 23) \Rightarrow Q = (-3, -7)$.

8. (*d*)

9. (*b*) $A = (2, 4), B = (4, 2)$, then C

$$= \left\{\frac{(2+4)\pm(4-2)}{2}, \frac{(4+2)\mp(2-4)}{2}\right\}$$

$$= \left(\frac{6\pm 2}{2}, \frac{6\pm 2}{2}\right) = (2, 2) \text{ or } (4, 4).$$

10. (*c*) Let $D = (3, 5), E = (-3, -3)$, then

$$DE = \sqrt{(3+3)^2 + (5+3)^2} = \sqrt{36+64}$$

$$= \sqrt{100} = 10$$

$$\Rightarrow \quad BC = 2\,DE = 2 \times 10 = 20.$$

11. (*a*) $$AB^2 = \left(1 - \frac{5}{6}\right)^2 + (3-6)^2$$

$$= \frac{1}{36} + 9 = \frac{325}{36}.$$

12. (*c*) Third vertex

$$= \left\{\frac{(2+2)\pm\sqrt{3}(6-4)}{2}, \frac{6+4\pm\sqrt{3}(2-2)}{2}\right\}$$

$$= (2 \pm \sqrt{3}, 5).$$

13. (*c*)

14. (*c*) $\sqrt{(2-0)^2 + (-1-0)^2} = \sqrt{5}$

$\Rightarrow (2, -1)$ is the nearest point.

15. (*c*) $$\sqrt{(10-2)^2 + (y+3)^2} = 10$$

$$\Rightarrow \quad 64 + y^2 + 9 + 6y - 100 = 0$$

$$\Rightarrow \quad y^2 + 6y - 27 = 0$$

$$\Rightarrow \quad y = -9, 3.$$

16. (*a*) $$\sqrt{(3-x)^2 + (2+1)^2} = 5$$

$$\Rightarrow \quad (3-x)^2 + 9 = 25$$

$$\Rightarrow \quad 9 + x^2 - 6x + 9 - 25 = 0$$

$$\Rightarrow \quad x^2 - 6x - 7 = 0$$

$$\Rightarrow \quad x = 7, -1.$$

17. (*c*) The given equation can be written as

$$x(x + \sqrt{2}y + 4) + \sqrt{2}y(x + \sqrt{2}y + 4) + 1 = 0$$

or, $(x + \sqrt{2}y)(x + \sqrt{2}y + 4) + 1 = 0$

Let, $(x + \sqrt{2}y) = z$

$$\Rightarrow \quad z(z+4) + 1 = 0$$

$$\Rightarrow \quad z^2 + 4z + 1 = 0$$

$$\Rightarrow \quad z = \frac{-4 \pm \sqrt{16-4}}{2}$$

$$= \frac{-4 \pm 2\sqrt{3}}{2}$$

Thus, the two lines are $(x + \sqrt{2} + 2 - \sqrt{3}) = 0$ and $(x + \sqrt{2} + 2 + \sqrt{3}) = 0$, which are parallel, because their equations differ by a constant only.

$\therefore$ The distance between the parallel lines is

$$\frac{(2+\sqrt{3}) - (2-\sqrt{3})}{\sqrt{1^2 + (\sqrt{2})^2}} = \frac{2\sqrt{3}}{\sqrt{1+2}} = 2.$$

18. (*d*) $2x - 3y \le -6$

In quadrant I,we have $x = 1$ and $y = 3$ i.e., (1, 3) to satisfy the equation.

In quadrant II, we have $x = -1$ and $y = 3$ i.e., (–1, 2) to satisfy the equation.

In quadrant III, we have $x = -8$ and $y = -2$ i.e., (–8, – 2) to satisfy the equation.

But in quadrant IV, there is not any set of value of (x and y) which can satisfy the equation i.e., in general, we can say that the quadrant in which x is Positive and Negative, we can have no value to satisfy the inequality.

19. (*d*) Here, $x_1 = 8, x_2 = -7$

$y_1 = 9, y_2 = 4$

and, $m : n = 2 : 3$

i.e., $m = 2, n = 3$

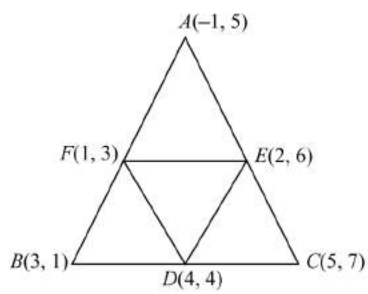
$\therefore$ Required coordinates of the point

$$= \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$$= \left(\frac{2\times(-7) + 3\times 8}{2+3}, \frac{2\times 4 + 3\times 9}{2+3}\right)$$

$$= (2, 7).$$

20. (*d*) Since, (D) is the mid point of BC



$\therefore$ Coordinate of $D = \left(\frac{3+5}{2}, \frac{1+7}{2}\right)$

$= (-4, 4)$

Similarly, coordinate of E and F are (2, 6) and (1, 3) respectively.

$\therefore$ Area of

$\Delta DEF\ \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[1(4-6) + 4(6-3) + 2(4-3)]$

$= \frac{1}{2}(-2+12+2-2) = \frac{1}{2} \times 8 = 4$ sq unit.

Difficulty Level-2

1. (*a*) $2x + 3y = 5, x - y = 10$

$\Rightarrow x = 7, y = -3.$

2. (*b*) The condition that the three straight lines

$$a_1x + b_1y + c_1 = 0,$$
$$a_2x + b_2y + c_2 = 0,$$
$$a_3x + b_2y + c_3 = 0$$

may meet in a point is given by

$$a_1(b_2c_3 - b_3c_2) + b_1(c_2a_3 - c_3a_2) + c_1(a_2b_3 - a_3b_2) = 0 \quad (1)$$

Put $a_1 = a, b_1 = b, c_1 = c$

$a_2 = b, b_2 = c, c_2 = a$

$a_3 = c, b_3 = a, c_3 = b$

$\therefore$ (1) gives

$a(cb - a^2) + b(ac - b^2) + c(ab - c^2) = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc.$

3. (*c*) General equation of the second degree

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of straight lines

$\Rightarrow abc + 2fgh - af^2 - bg^2 - ch^2 = 0 \quad (1)$

In our case,

$a = 1, 2h = -3, b = \lambda, 2g = 3, 2f = -5, c = 2$

$\therefore$ From (1)

$\Rightarrow 2\lambda + 2 \times \left(-\frac{5}{2}\right) \times \left(\frac{3}{2}\right) \times \left(-\frac{3}{2}\right) - \frac{25}{4} - \lambda \times \frac{9}{4} - 2 \times \frac{9}{4} = 0$

$\Rightarrow -\frac{\lambda}{4} + \frac{45}{4} - \frac{25}{4} - \frac{18}{4} = 0$

$\Rightarrow \frac{\lambda}{4} = \frac{1}{2}$

$\Rightarrow \lambda = 2.$

4. (*a*) Eliminating t from the given equations, we get $x^2 + y^2 - 2xy - 2x - 2y + 4 = 0$, which is an equation of the pair of straight lines.

5. (*c*) $x = \frac{1}{2}\sqrt{x^2 + y^2}$

$\Rightarrow 2x = \sqrt{x^2 + y^2}$

$\Rightarrow 4x^2 = x^2 + y^2$

$\Rightarrow 3x^2 - y^2 = 0.$

6. (*b*) Equation of tangent to the given parabola is

$$y = mx + \frac{a}{m}$$

$\Rightarrow m^2x - ym + a = 0$

Now, the equation will have two roots say m_1 and m_2 which are the slopes of the tangents.

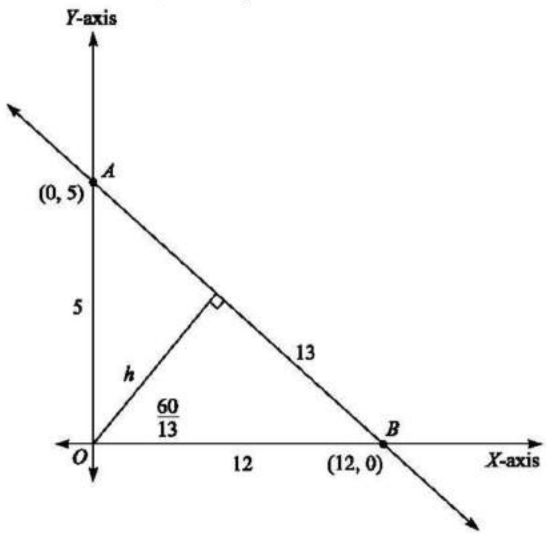
Now, the product of the slopes should be –1

$\therefore m_1 m_2 = -1 \Rightarrow \frac{a}{x} = -1$

$\Rightarrow a + x = 0.$

7. (*a*) $\sqrt{x^2 + y^2}$ is the radius of a circle such that $5x + 12y = 60$

The line $5x + 12y = 60$ is plotted as follows



All the points on the line AB satisfy $5x + 12y = 60$

$\sqrt{x^2 + y^2}$ will be minimum when the distance between O and line AB is minimum.

Area of $\Delta ABC = \frac{1}{2} \times OB \times AO = \frac{1}{2} \times AB \times h$

$\therefore \quad \frac{1}{2} \times 12 \times 5 = \frac{1}{2} \times 13 \times h$

$\therefore \quad h = \frac{60}{13}.$

8. (*d*) $y = \frac{x}{x+1}$

Substituting options, we find that $(-1, 1)$ gives us 0 in the denominator.

$\therefore$ $(-1, 1)$ cannot lie on the graph of

$$y = \frac{x}{x+1}$$

9. (*a*) Slope of the line $ax + by + c = 0$ is $-a/b$

(1) $3y - 2x = 12$

$\therefore$ Slope $= 2/3$

(2) $-2x - 3y = 10$

$\therefore$ Slope $= -2/3$

(3) $3y + 2x = 12$

$\therefore$ Slope $= -2/3$

(4) $2y + 3x = 10$

$\therefore$ Slope $= -3/2$

For two lines to be perpendicular, product of their slopes is -1.

$\therefore$ (1) and (4) represent perpendicular lines.

10. (*a*) Slope of the line joining the points (6, 12) and (0, – 6)

$$= \frac{12 - (-6)}{6 - 0} = 3$$

The equation of line $= y + 6 = 3x$

$\Rightarrow \quad y = 3x - 6$

Now, check with the options, only point (3, 3) satisfies this equation.

11. (*d*) Here point of intersection = (3, 4)

Since, the diagonal BD must pass through this point.

Now, by option, we find that the point (3, 4) satisfy the equation $x + y - 7 = 0$ and $x - y + 1 = 0$.

But the point A (1, 2) satisfy the equation $x - y + 1 = 0$. Hence, the required diagonal is $x + y - 7 = 0$.

12. (*a*) $x^2 y^2 = 20$

$\Rightarrow \quad y = \pm\frac{\sqrt{20}}{x}$

If x is negative, then

$$y = \mp\frac{\sqrt{20}}{x}$$

If x is positive, then

$$y = \pm\frac{\sqrt{20}}{x}$$

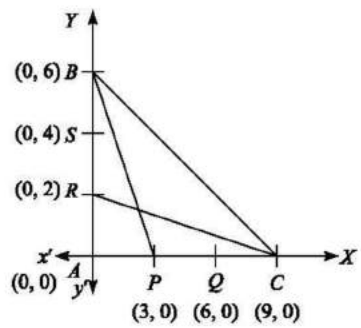
$\therefore$ Point (x, y) can be in all the four quadrants.

13. (*b*) Since, the graph passes through (– 4, 0) and (4, 0)

Perfect answer is (*b*).

$\because \quad 9 \times 16 - 16 \times 0 = 144.$

14. (*a*)



Equation of line RC,

$$(y - 0) = \left(\frac{2-0}{0-9}\right)(x - 9)$$

$\Rightarrow \quad 2x + 9y - 18 = 0 \quad (1)$

Equation of line PB,

$$(y - 0) = \left(\frac{6-0}{0-3}\right)(x - 3)$$

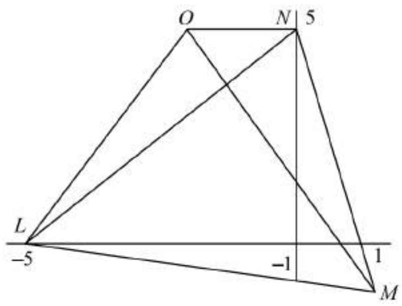
$\Rightarrow \quad 2x + y - 6 = 0 \quad (2)$

The point of intersection of lines is $\left(\frac{9}{4}, \frac{3}{2}\right)$.

The slope of line $AX = \frac{\left(\frac{3}{2} - 0\right)}{\left(\frac{9}{4} - 0\right)} = \frac{2}{3}.$

15. (*a*) $PL + PN$ will be minimum if P lies on LN, also

$PO + PM$ will be minimum if P lies on OM. P must be the intersection point of the diagonals of the quadrilateral.



min $(PL + PM + PN + PO)$ = min $(PL + PN)$ + min $(PO + PM) = LN + OM = 5\sqrt{2} + 2\sqrt{10}$.

16. (*b*) Distance between A and $B = 2 - (-4) = 6$

Area of the triangle $= \frac{1}{2} \times$ base $\times$ height

$\Rightarrow \quad 21 = \frac{1}{2} \times 6 \times c$

$\therefore \quad c = 7.$

17. (*c*) $\quad x - 3y + 7 = 0$

By substituting the points, we get

$$a - 3b + 7 = 0 \quad (1)$$

$$a + 3 - 3b - 3k + 7 = 0$$

$$\Rightarrow \quad a - 3b - 3k + 10 = 0$$

$$\Rightarrow \quad a - 3(b + k) + 10 = 0 \quad (2)$$

from solving Eqs. (1) and (2), we get

$k = 1/3.$

18. (*d*) $\quad$ Area $= \frac{1}{2} \times AC \times b = 20$

$\therefore \quad AC = \frac{40}{b}$

$\Rightarrow$ Coordinates of the point C are $\left(\frac{40}{b}, 0\right)$.

19. (*a*) The point that divides all the medians in the ratio 2:1 is the centroid of the triangle.

Centroid of $\Delta XYZ = 0 \equiv \left(\frac{6-3+2}{3}, \frac{4+5-4}{3}\right)$

$\equiv \left(\frac{5}{3}, \frac{5}{3}\right)$

20. (b) The coordinates of the point C are $(3, x)$.

Therefore, the height of the parallelogram is 3 units.

As the area of the parallelogram is 21 sq. units, we get,

$x \times 7 = 21$

or, $x = 3$

Thus, the coordinates of C are (3, 3).

The coordinates of D are (10, 3) as $CO = 7$.