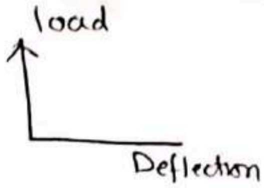
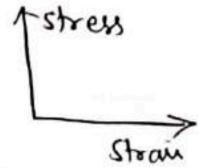
STRENGTH OF MATERIAL

Toughness	area of Load-Deflection diagram till fracture
Resilience	———— till elastic limit
modulus of toughness	area of stress-strain diagram till fracture
modulus of Resilience	———— till elastic limit



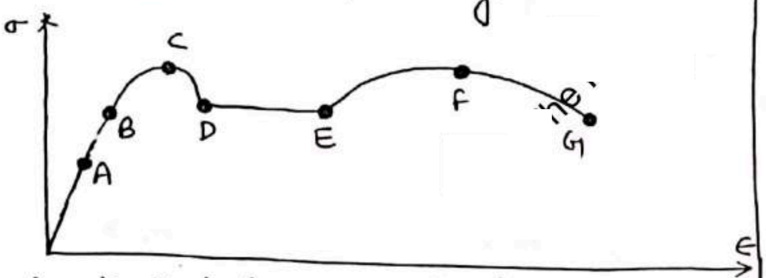


Homologus temp ⇒ at which creep uncontrollable

Brinell hardness no = $\dfrac{P}{\frac{\pi D}{2}\left[D-\sqrt{D^2-d^2}\right]}$

D → Steel ball dia. } mm
d → Indent dia }

mildsteel → tension loading



A → limit of Proportionality (Hook's law valid)

B → elastic limit

C → $\boxed{\text{upper yield point}}$ → not depends on length

↓ depends on

$\boxed{\text{cross-sectional area}}$ $\boxed{\text{shape}}$ $\boxed{\text{equipment}}$

D → lower yield point

E → strain hardening starts

F → ultimate point

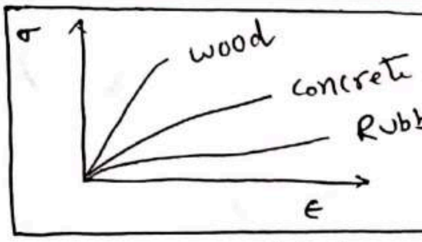
G → fracture point

Proof stress :- Stress required to cause a permanent extension equal to defined % of gauge length like 0.2 % of strain at → whatever stress is known as proofstress.

Presence of free graphite → material is Cast Iron

note :- steel has no graphite

Percentage carbon ↑ ⇒ yield stress ↑ But fracture strain ↓



material

- $\boxed{\text{ductile}}$ — Post elastic strain > 5%
  - Ex. lead, mildsteel, Copper
- $\boxed{\text{Brittle}}$ — < 5%
  - Ex. Rubber, glass, castIron

note :- In Brittle material → $\boxed{\text{No Plastic Zone}}$

v.v.imp

material

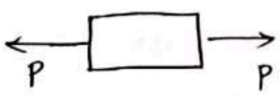
- ductile
  - weak in shear
- Brittle
  - weak in brittle

order :- shear < tensile ≤ compressive
↓
(ductile material weakest in shear)

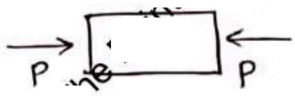
order :-
Tensile < shear < compressive
↓
(Brittle material like concrete weak in tension)

Ductile material

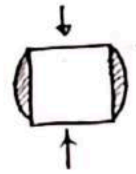
- tensile loading
- compressive loading



∵ ductile weak in shear ∴ fails in shear [shear max. at 45°]
⇓
{"CUP-CONE FRACTURE" failure plane @ 45°}



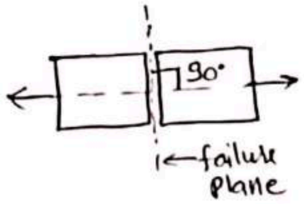
↓
BULZING failure
↓



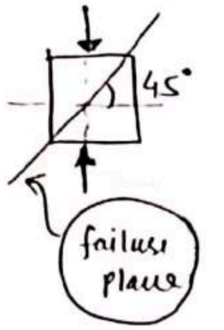
Brittle material

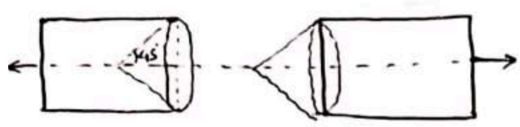
- Tensileloading
- Compressive Loading

Tensile loading ↓
∵ Brittle weak in tension hence 90° from longitudinal axis failure plane.



Compressive loading ↓
∵ strong in compression hence fail in shear @45°



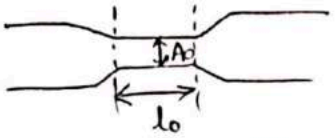


Torsion applied {basically Torque T}

- Ductile material shaft
  ↓
  Plane fracture
- Brittle material shaft
  ↓
  helicoidal surface

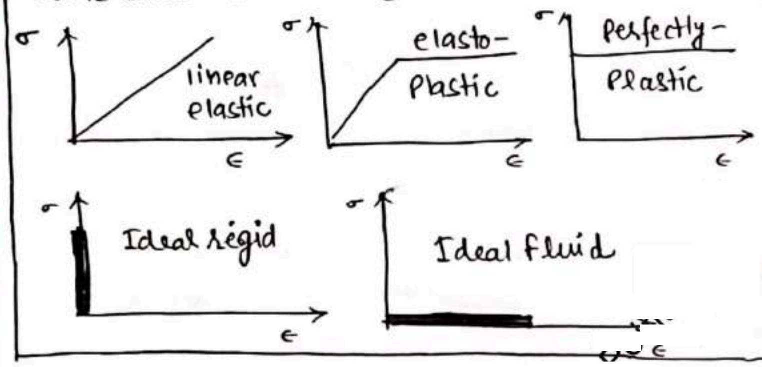
Ductility measurement

gauge length $= 5.65\sqrt{A_0}$
l_0



Tenacity → Resist fracture under tensile load

Some stress-strain diagram :-



viscoelastic material → time dependent stress-strain curve

Isotropic material → Identical properties in all direction.

Homogeneous material → similar properties throughout volume.

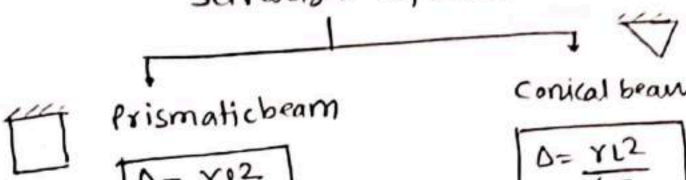
material	no. of Independent elastic constant
homogeneous Isotropic	2
orthotropic (wood)	9
Anisotropic	21

note:- Anisotropic material → their Properties changes when measured from different directions.

orthotropic → subset of Anisotropic material

∵ their Properties differ along 3 Mutually direction.

Selfweight Deflation



Prismatic beam

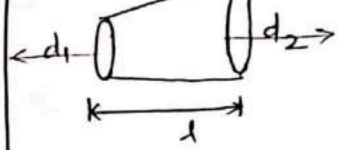
$$\Delta = \frac{\gamma l^2}{2E}$$

Conical beam

$$\Delta = \frac{\gamma l^2}{6E}$$

v.v.imp

not depend on cross sectional area.

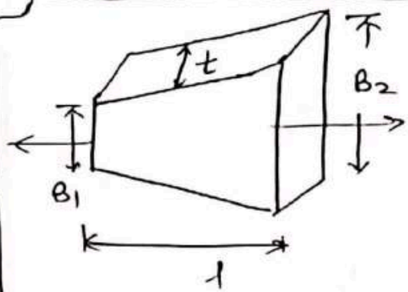


$$\Delta = \frac{Pl}{\left(\frac{\pi d_1 d_2}{4}\right)E}$$

note:- $\Delta_{actual} > \Delta_{avg.dia}$

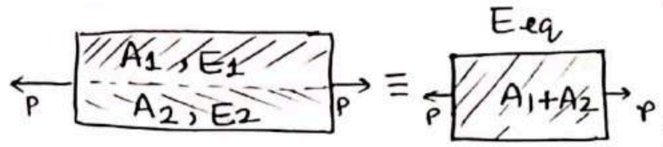
d_1, d_2 → +11.11 % error

$d = \frac{d_1 + d_2}{2}$



$$\Delta = \frac{Pl}{(B_2 - B_1)tE} \ln\left(\frac{B_2}{B_1}\right)$$

Composite Bar :-



$$\boxed{P = P_1 + P_2} \quad \text{—— (I)}$$

$$\boxed{\Delta_1 = \Delta_2} \quad \text{—— (II)}$$

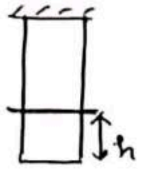
$$\Delta_1 = \frac{P_1 L}{A_1 E_1}, \quad \Delta_2 = \frac{P_2 L}{A_2 E_2}$$

∴

$$E_{eq} = \frac{A_1 E_1 + A_2 E_2}{A_1 + A_2}$$

$$P_1 = P \frac{A_1 E_1}{A_1 E_1 + A_2 E_2}$$

Impact loading :-



$$\boxed{\sigma_{dynamic} = \sigma_{static}\left[1 + \sqrt{1 + \frac{2h}{\Delta_{static}}}\right]}$$

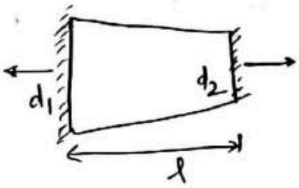
$$\boxed{\underset{(IF)}{\text{Impact factor}} = \frac{\sigma_{dynamic}}{\sigma_{static}} \geq 2}$$

$\boxed{\text{Sudden Impact } h=0 \;\therefore \text{ Impact factor} = 2}$

Dilation = sum of strain in all direction

$$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$$

tapered bar :-

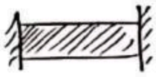


ΔT↑ Temperature Increase

$$\alpha l \Delta T - \frac{P l}{\left(\frac{\pi d_1 d_2}{4}\right)E} = 0$$

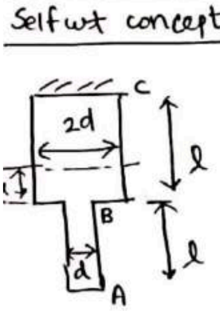
$$\boxed{\begin{aligned} \therefore \sigma_{max} &= \alpha T E\left(\frac{d_1}{d_2}\right) \\ \sigma_{min} &= \alpha T E\left(\frac{d_2}{d_1}\right) \end{aligned}}$$

Special case $d_1 = d_2 = d$



$$\sigma = \alpha T E$$

Selfwt conceptual :-



$$\boxed{\Delta = \Delta_{AB} + \Delta_{BC}}$$

$$\Delta_{AB} = \frac{\gamma_{AB} l_{AB}^2}{2E_{AB}}$$

Δ_{BC} = Selfwt of BC + due to wt of AB

$$\frac{\gamma_{BC} l_{BC}^2}{2E_{BC}} + \frac{\underbrace{\left[\gamma_{AB}\frac{\pi}{4}d_{AB}^2 l_{AB}\right]}_{\text{wt. of AB}} \times l_{BC}}{A_{BC}\cdot E_{BC}}$$

$$A_{BC} \rightarrow \frac{\pi}{4}(2d)^2$$

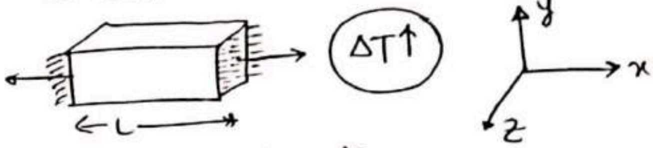
Method-1

Δ_{BC} — M_2 method-2

$$\frac{\int_0^l \left[\gamma_{AB} l_{AB} A_{AB} + \gamma A_x \cdot x\right] dx}{\underbrace{\frac{\pi}{4}(2d)^2}_{A_{BC}} \cdot E_{BC}}$$

Restraint Problems

(I) Restrain from 1 side



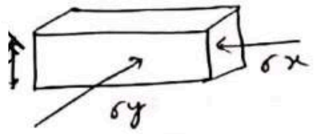
$\sigma_x \longrightarrow$ generate
$\sigma_y \longrightarrow$ zero
$\sigma_z \longrightarrow$ zero } ∵ free

∴ Δ in x direction zero { ∵ fixed }

$$L\alpha\Delta T - \frac{\sigma_x}{E} l = 0$$

$$\boxed{\sigma_x = \alpha T E}$$

(II) Restrain from 2 side



∴ $\Delta x = 0$
$\Delta y = 0$

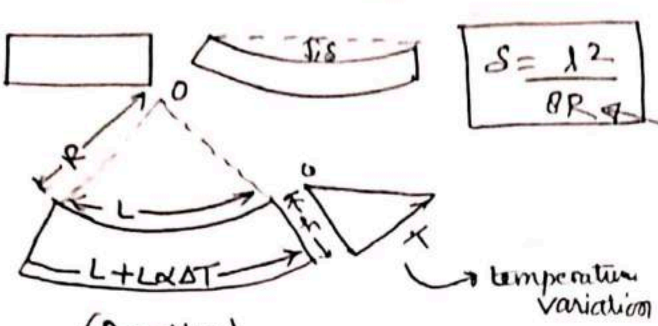
$$\Delta x = l\alpha\Delta T - \frac{\sigma x}{E} l + \frac{\mu \sigma y}{E} \times l = 0$$

$$\Delta y = b\alpha\Delta T - \frac{\sigma y}{E} \times b + \frac{\mu \sigma x}{E} b = 0$$

Solve $\boxed{\sigma x = \sigma y = \frac{\alpha T E}{1-\mu}}$

(III) Restrain from all 3 sides

$$\boxed{\sigma x = \sigma y = \sigma z = \frac{\alpha T E}{1 - 2\mu}}$$



(Bending)

$$\boxed{\delta = \frac{l^2}{8R}}$$

$$\frac{L}{R} = \frac{L+L\alpha\Delta T}{R+h} \quad \therefore R = h/\alpha T$$

$$\therefore \boxed{\delta = \frac{\alpha T l^2}{8h}} \quad **$$

$\boxed{\mu = 0.25 \text{ to } 0.42 \text{ elastic material}}$

$\boxed{\mu = 0.15 \quad \text{concrete}}$

$$\boxed{E = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}} \quad \text{modulus of elasticity}$$

$$\boxed{G = \frac{\text{Shear stress}}{\text{Shear strain}}} \quad \text{modulus of rigidity}$$

$$\boxed{K = \frac{\text{direct normal stress}}{\text{volumetric strain}}} \quad \text{Bulk modulus}$$

$$\boxed{\mu = -\frac{\text{lateral strain}}{\text{longitudinal strain}}} \quad \text{Poison ratio}$$

$$\boxed{\mu = \frac{3K-2G}{6K+2G}} \qquad \boxed{\frac{E}{3} < G < \frac{E}{2}}$$

Stress matrix
$$\underbrace{\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}}_{\text{Symmetric matrix}}$$

$\because \tau_{xy} = \tau_{yx}$

$\tau_{xz} = \tau_{zx}$

$\tau_{yz} = \tau_{zy}$

Strain matrix
$$\begin{bmatrix} \epsilon_{xx} & \frac{\phi_{xy}}{2} & \frac{\phi_{xz}}{2} \\ \frac{\phi_{yx}}{2} & \epsilon_{yy} & \frac{\phi_{yz}}{2} \\ \frac{\phi_{zx}}{2} & \frac{\phi_{zy}}{2} & \epsilon_{zz} \end{bmatrix}$$

** $\epsilon_{xx} = \frac{\partial u}{\partial x}$ = normal strain in x direction

ϕ_{xy} = shear strain $= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$

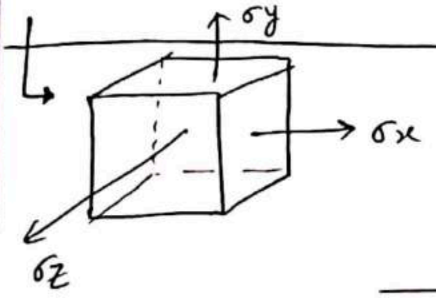
v.v.Imp ***

$$E = 2G(1+\mu) = 3K(1-2\mu) = \frac{9KG}{3K+G}$$

v.v.Imp

Volumetric strain $\frac{\Delta V}{V} = \epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z = \frac{\sigma_x+\sigma_y+\sigma_z}{E}(1-2\mu)$ ▶ for perfectly plastic material $\Delta V = 0$

$\therefore \epsilon_v = 0 \therefore \mu = 0.5$

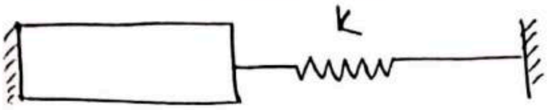


$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E}$$

Same

ϵ_y, ϵ_z

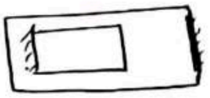
$\boxed{\text{Wrough Iron} \longrightarrow \text{Purest form}}$

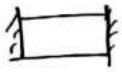


$$\boxed{\sigma = \frac{\alpha T E}{1 + \left(\frac{AE/L}{k}\right)}}$$

$\frac{AE}{L} \rightarrow$ Axial stiffness

Rigid $k = \infty$ | Space $k = 0$





$\sigma = \alpha T E$

$\therefore \sigma = 0$] no stress generate

$$\boxed{\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{1}{R} = \text{curvature}}$$

$R \rightarrow$ Radius of curvature

↓ differentiate

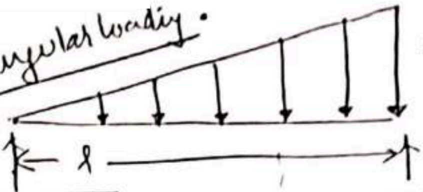
$$\boxed{EI\frac{d^3y}{dx^3} = \frac{dM}{dx} = S \longrightarrow \text{shear force}}$$

$\Longrightarrow$ Slope of Bending Moment diagram gives you $\rightarrow$ shear force.

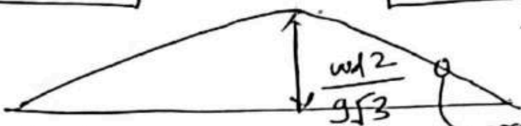
↓ differentiate

$$\boxed{EI\frac{d^4y}{dx^4} = \frac{dS}{dx} = -w \rightarrow \text{KN/m Load Intensity}}$$

Triangular loading.

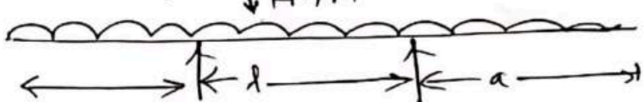


$\boxed{R_A = \frac{wl}{6}}$ $\boxed{R_B = \frac{wl}{3}}$



$\boxed{\text{BMD}}$

3rd degree curve



$$\text{max (+)ve BM} = \frac{w}{8}\left(l^2 - (2a)^2\right)$$

$$\text{max (-)ve BM} = \frac{wa^2}{2}$$

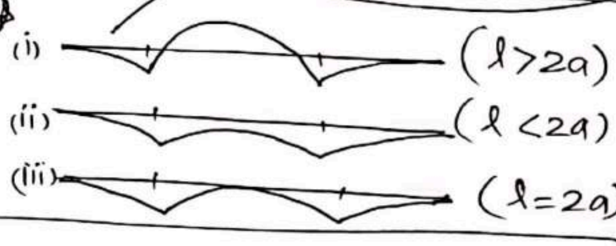
if both equal then $\boxed{a = \frac{l}{2\sqrt{2}}}$

$$\text{overhanging part} = \frac{a}{l+2a}$$

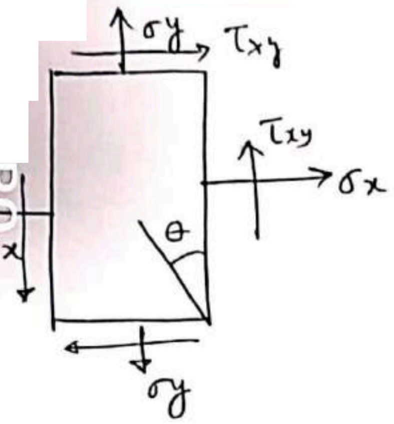
(20.71%)

2 contraflexure point

BMD



Principal stress/ strain :-



$$\boxed{\sigma_x' = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta} \quad \text{— (1)}$$

$$\boxed{\sigma_x' + \sigma_y' = \sigma_x+\sigma_y = \sigma_1+\sigma_2}$$

$\boxed{\sigma_y' \Rightarrow \text{ put } \theta \longrightarrow 90+\theta \text{ in eq}^n \text{ (1)}}$

$$\boxed{\sigma_1/\sigma_2 = \left(\frac{\sigma_x+\sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + (\tau_{xy})^2}}$$

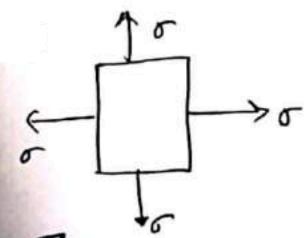
$\boxed{\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}}$ $\boxed{\theta_p \longrightarrow \text{angle of principal plane}}$

Std. mohr circle :-

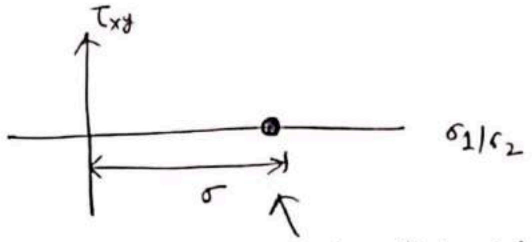
centre $= \left(\frac{\sigma_x+\sigma_y}{2}, 0\right)$

radius $= \left|\frac{\sigma_1-\sigma_2}{2}\right| = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + (\tau_{xy})^2}$

(I)



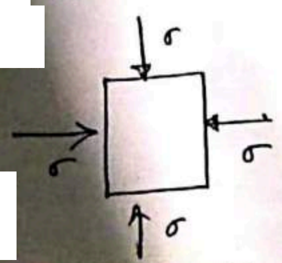
$\longrightarrow$



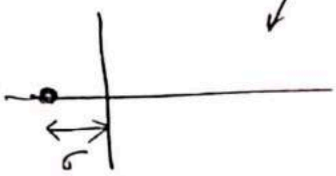
[both tensile & equal magnitude]

mohr circle will be a point

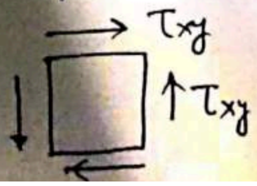
(II)



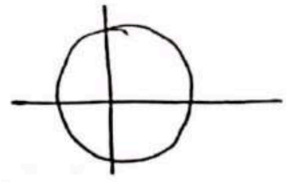
$\longrightarrow$



(III)

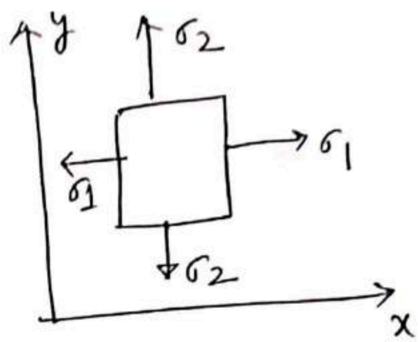


$\longrightarrow$



circle origin at (0,0)
radius = τ_{xy}

Analysis of strain :- {when σ_1, σ_2 given}



major principal strain $\boxed{\epsilon_1 = \frac{\sigma_1}{E} - \frac{\mu \sigma_2}{E}}$

minor " " $\boxed{\epsilon_2 = \frac{\sigma_2}{E} - \frac{\mu \sigma_1}{E}}$

solve them

$$\boxed{\sigma_1 = \frac{E}{1-\mu^2}\left[\epsilon_1 + \mu \epsilon_2\right]}$$

$$\boxed{\sigma_2 = \frac{E}{1-\mu^2}\left[\epsilon_2 + \mu \epsilon_1\right]}$$

- mohr circle ⟶ strain Analysis ⟶ put $\tau_{xy} \rightarrow \frac{\phi_{xy}}{2}$ rest will be same

($\theta \rightarrow$ from horizontal)

total strain energy per unit volume $= \frac{1}{2}\sigma_1 \epsilon_1 + \frac{1}{2}\sigma_2 \epsilon_2 + \frac{1}{2}\sigma_3 \epsilon_3$

put ϵ_1 & ϵ_2 in terms of σ_1 & σ_2

then

3D case

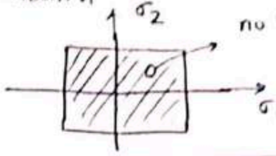
$$\boxed{\frac{SE}{\text{volume}} \leftarrow U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right]}$$

Theory of failure

For no Failure condition

(I) maximum Principal stress theory
(Rankine Theory)

$$\boxed{\sigma_{max} \text{ or } \sigma_1 \le (\sigma_y / Fos)}$$



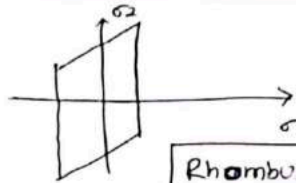
Rectangular

note:- this Theory ⟶ Best for Brittle
∵ Brittle material fails under tension
(Brittle failure)

(II) maximum principal strain Theory
(St. Venant's Theory)

$$\boxed{\epsilon_1 \le \frac{(\sigma_y/Fos)}{E}}$$

$$\left\{ \text{where } \epsilon_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E} - \frac{\mu\sigma_3}{E} \right\}$$



Rhombus

note:- This theory overestimate the elastic strength of ductile material

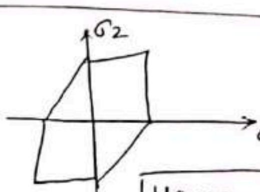
Brittle + ductile

(III) maximum shear stress theory
(Guest & Tresca's theory)

$$\boxed{\tau_{max} \le \frac{(\sigma_y/Fos)}{2}}$$

$$\left\{ \tau_{max} = \left\{ \left|\frac{\sigma_1 - \sigma_2}{2}\right| \text{ or } \left|\frac{\sigma_1}{2}\right| \text{ or } \left|\frac{\sigma_2}{2}\right| \right\} \right\}$$

↓
basically absolute max. shear stress



Hexagon

note:-
- most conservative theory
- not for hydrostatic loading because $\sigma_1 = \sigma_2 = \sigma_3$ ∴ $\tau_{max} = 0$
- ductile + shaft

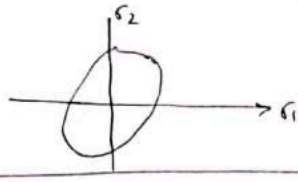
(IV) maximum strain energy Theory
(Haigh's theory)

$$\boxed{SE/\text{volume} \le \frac{(\sigma_y/Fos)^2}{2E}}$$

$$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_3\sigma_2 + \sigma_3\sigma_1) \right] \le \frac{(\sigma_y/Fos)^2}{2E}$$

note:- ductile + Pressure vessel Best

ellipse

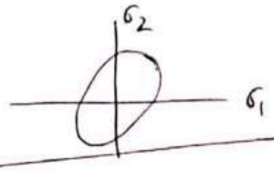


(V) max. shear strain energy theory
(von-mises theory) / distortion theory

$$\frac{1}{12G}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] = \frac{(\sigma_y/Fos)^2}{6G}$$

note:- ductile + pure shear

ellipse



* Shear strain energy $= \frac{1}{2}\, \sigma_{avg}\, \epsilon_v$

σ_{avg} → volumetric stress, ϵ_v → Volumetric strain

$$\sigma_{avg} = \left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{3}\right)$$

$$\epsilon_v = \frac{\sigma_1 + \sigma_2 + \sigma_3}{E}(1 - 2\mu)$$

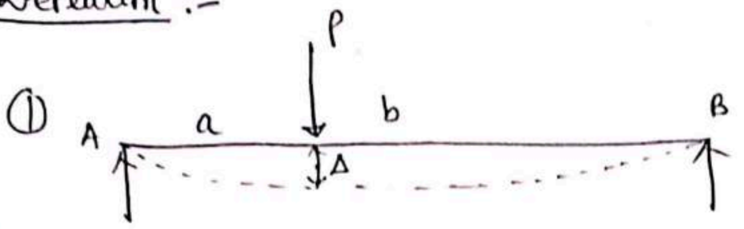
note: when $\sigma_1 >>>>>>>>> \sigma_2, \sigma_3$
theory gives same result

Mayur 19/2/2020

REDMI NOTE

Defletion :-

(1)



$$\boxed{l = a+b}$$

$$\boxed{\theta_A = \frac{Pab(l+b)}{6EIl}}$$

$$\boxed{\theta_B = \frac{Pab(l+a)}{6EIl}}$$

$$\boxed{\Delta_{under\ load} = \frac{Pa^2b^2}{3EIl}}$$

$$\boxed{\Delta_{max} = \frac{Pb(l^2-b^2)^{3/2}}{9\sqrt{3}EIl}}$$ @ $\sqrt{\frac{l^2-b^2}{3}}$ { $\Delta_{max} \rightarrow$ between of (mid section of beam and load) }

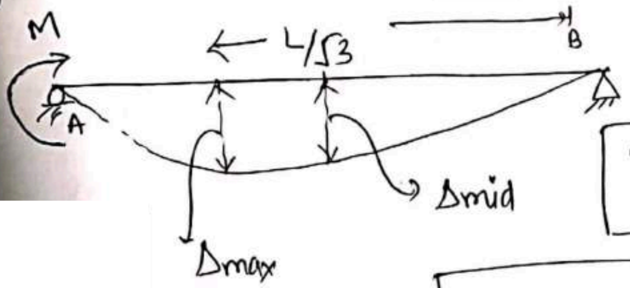
$\Rightarrow$ put $a=b=l/2$,

$$\boxed{\theta_A = \frac{Pl^2}{16EI} = \theta_B}$$

$$\boxed{\Delta = \frac{Pl^3}{48EI}}$$

---

(2)



$$\boxed{\theta_B = \frac{ML}{6EI}}$$

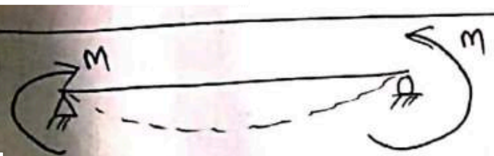
$$\boxed{\Delta_{mid} = \frac{ML^2}{16EI}}$$

$$\boxed{\theta_A = \frac{ML}{3EI}}$$

$$\boxed{\Delta_{max} = \frac{ML^2}{9\sqrt{3}EI}}$$ @ $L/\sqrt{3}$ from oposite end.

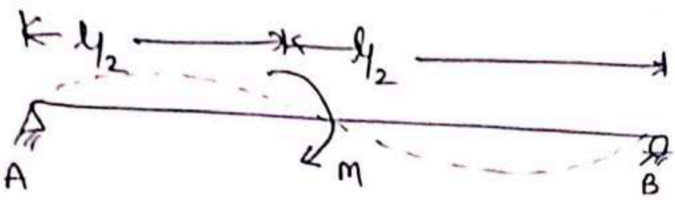
---

(3)

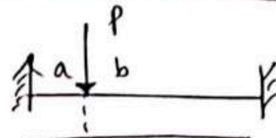


$$\boxed{\theta_A = \theta_B = \frac{ML}{2EI}}$$

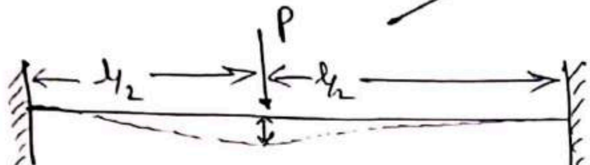
$$\boxed{\Delta_{mid} = \Delta_{max} = \frac{ML^2}{8EI}}$$

(4) 

$$\theta_A = \theta_B = \frac{ML}{12EI}$$



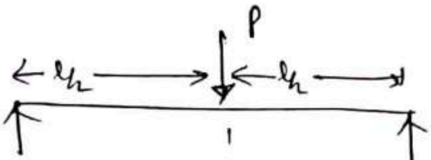
$$\Delta = \frac{Pa^3b^3}{3EIl}$$

(5) 

$\theta_A = 0$ $\theta_B = 0$

atmid

$$\Delta_{fix} = \frac{Pl^3}{192EI} = \frac{\Delta_{SSB}}{4}$$



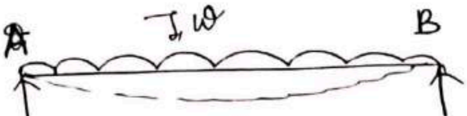
$$\theta_A = \frac{Pl^2}{16EI} \qquad \theta_B = \frac{Pl^2}{16EI}$$

$$\Delta = \frac{Pl^3}{48EI}$$

(6) w kN/m = W (kN)

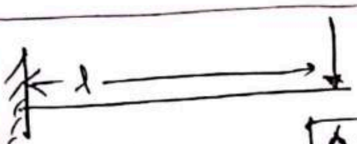
$\theta_A = 0$ $\theta_B = 0$

$$\Delta_{mid} = \frac{wl^4}{384EI} = \frac{\Delta_{SSB}}{5}$$



$$\theta_A = \frac{wl^3}{24EI} = \theta_B$$

$$\Delta_{mid} = \frac{5wl^4}{384EI}$$

(7) 

$\theta = 0$
$\Delta = 0$

$$\theta = \frac{Pl^2}{2EI}$$

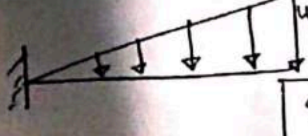
$$\Delta = \frac{Pl^3}{3EI}$$

(8) 

$\theta = 0$
$\Delta = 0$

$$\theta = \frac{ML}{EI}$$

$$\Delta = \frac{ML^2}{2EI}$$

(9) 

$$\theta = \frac{wl^3}{8EI}$$

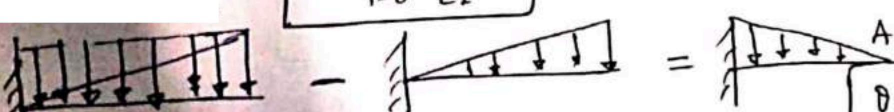
$$\Delta = \frac{11}{120}\frac{wl^4}{EI}$$

** most

(11) 

$$\Delta_{AV} = \left(\frac{Pr^3}{EI}\right)\frac{\pi}{4}$$

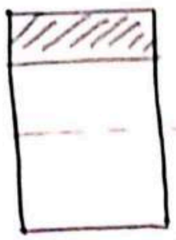
$$\Delta_{AH} = \left(\frac{Pr^3}{EI}\right) \times \frac{1}{2}$$

(10) 

$$\theta_A = \frac{wl^3}{24EI}$$

$$\Delta_A = \frac{wl^4}{30EI}$$

Shear stress :-



SF (shear force) taken by this section $= \boxed{\dfrac{\sigma_{max}}{y_{max}}(A\bar{y})}$

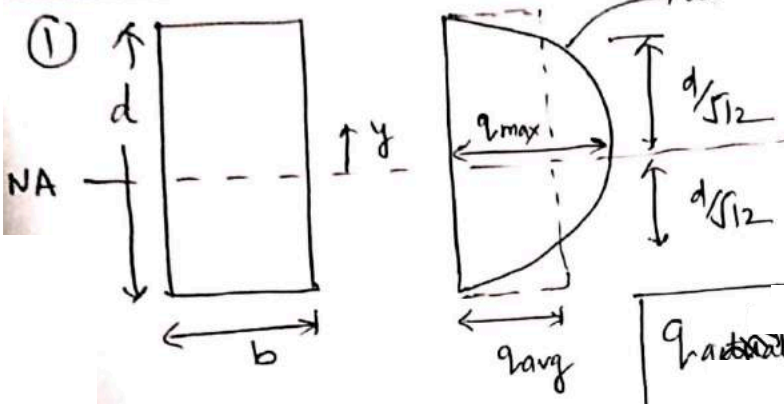
BM (Bending moment) $= \boxed{\dfrac{\sigma_{max}}{y_{max}}\left(I_{\text{area about N.A}}\right)}$

Std. result **

$$\boxed{q = \frac{SA\bar{y}}{Ib}}$$

---

(1)



$$\boxed{q = \frac{6S}{bd^3}\left[\frac{d^2}{4} - y^2\right]}$$

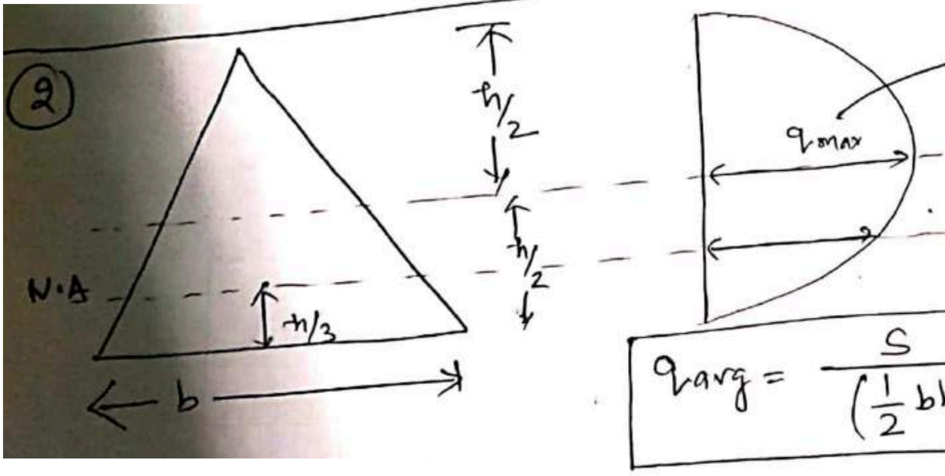
$y \longrightarrow$ from N.A

$$\boxed{q_{\cancel{actual}} = q_{avg} \text{ @ } y = \frac{d}{\sqrt{12}}}$$

at NA $\longrightarrow$ $\boxed{q_{max} = \frac{3}{2} q_{avg}}$ $\boxed{q_{avg} = \frac{S}{bd}}$

---

(2)



at mid hight

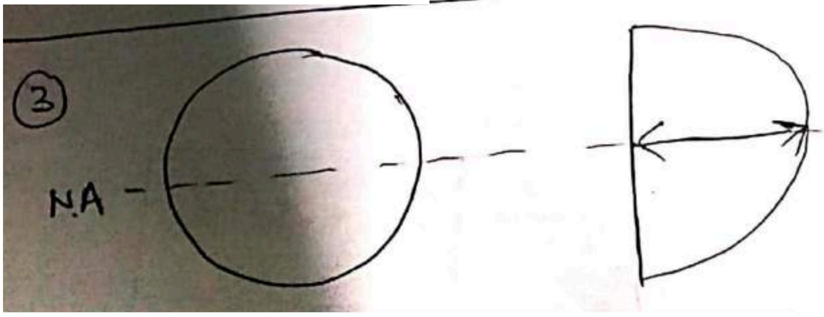
$$\boxed{q_{max} = \frac{3}{2} q_{avg}}$$

$$\boxed{q_{NA} = \frac{8}{3}\frac{S}{bh}}$$

$$\boxed{q_{avg} = \frac{S}{\left(\frac{1}{2}bh\right)}}$$

---

(3)

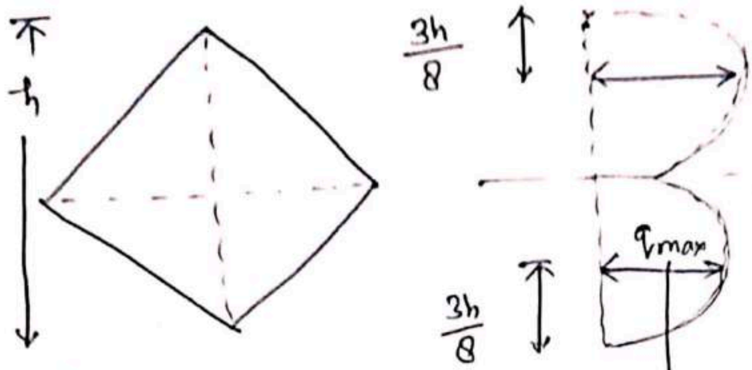


$$\boxed{q_{max} = \frac{4}{3} q_{avg}} \quad \boxed{q_{avg} = \frac{S}{\pi R^2}}$$

$$\boxed{q = \frac{4}{3}\frac{S}{\pi R^4}\left(R^2 - y^2\right)}$$

$y \longrightarrow$ from NA

④



$$\boxed{q_{max} = \frac{9}{8} q_{avg}}$$

$$\boxed{q_{avg} = \frac{S}{Area}}$$

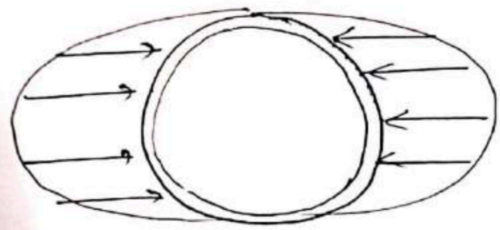
[diagonal section]

[1 diagonal ⟶ vertical]

(from $\frac{3h}{8}$ dist from top and bottom)

---

⑤ Thin ~~Tub~~. circular Tube :-



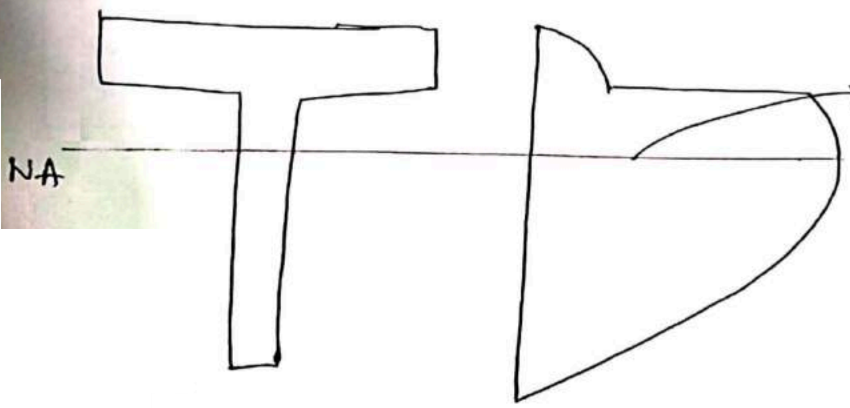
$$\boxed{q_{max} = 2\, q_{avg}}$$

$$\boxed{A = (2\pi R_m)\, t_m}$$

$$\boxed{I = \pi R_m^3\, t_m}$$

---

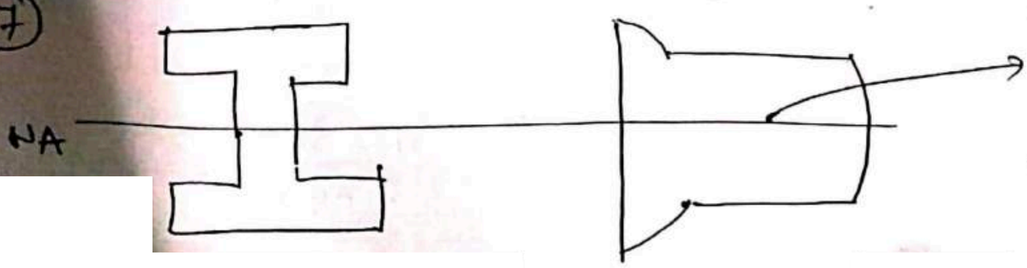
⑥



$\boxed{q_{max}\text{. at N.A}}$

---

⑦



$\boxed{q_{max}\text{ at NA}}$

Torque :-

eqn of Torsion $\boxed{\frac{\tau}{r} = \frac{T}{I_P} = \frac{G\theta}{L}}$ → valid for circular section.

$\boxed{I_P = I_{xx} + I_{yy}}$

$\frac{GI_P}{L}$ → Torsional rigidity $\frac{(GI)}{L}$

$\frac{GI_P}{L}$ → Torsional stiffness

EI → Flexural rigidity

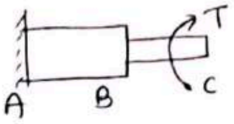
$\frac{EI}{L}$ → flexural Stiffness

AE → axial rigidity

$\frac{AE}{L}$ → axial stiffness.

Compound shaft

Series Connection

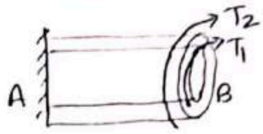


$\boxed{T_{AB} = T_{BC} = T}$

(torque same)

$\boxed{\theta_{AC} = \theta_{AB} + \theta_{BC}}$

parallel connection



$\boxed{\theta_{AB_1} = \theta_{AB_2}}$

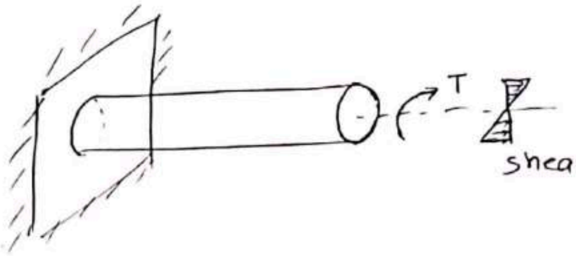
angular Deflection same

$\boxed{T = T_1 + T_2}$

Assumption :- (I) circular section remains circular after twisting { ~~no warping~~ }

(II) stress strain variation linear

(III) Axis of shaft loaded by twisting couple is perpendicular to plane of couple.



(IV) twisting along shaft → uniform

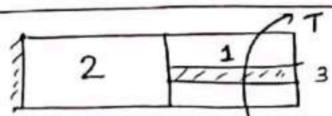
(V) Plane section remain plain, does not warp.

(VI) radial lines remains radial during torsion

(VII) stress induced within elastic limit.

note:- This topic → conventional / mains**

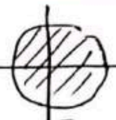
Ex.-



c/s

$\boxed{T = T_1 + T_3}$ $\boxed{\theta_1 = \theta_3}$

$I_P\ solid = \frac{\pi}{32} D^4$



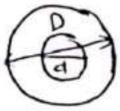
$I_{xx} = \frac{\pi}{64} D^4$

$I_{yy} = \frac{\pi}{64} D^4$

note:- **

tube $I_P = 2\pi R^3 t$

$I_P\ hollow = \frac{\pi}{32}(D^4 - d^4)$



$P = \frac{2\pi N T}{60}$

P → (Watt), N → rpm, T → Nm

Strain energy due to Torsion

$\Rightarrow \frac{1}{2} T\theta$

$\Rightarrow \frac{1}{2} T\left[\frac{TL}{GI_P}\right]$

$\Rightarrow \frac{T^2L}{2GI_P}$

{ same style like SE due to Bending moment $= \frac{M^2L}{2EI}$

$$\boxed{\text{Shear Resilience} = \frac{SE}{\text{Volume}} = \frac{1}{2}\tau\phi}$$

Solid shaft $\boxed{\frac{SE}{\text{volume}} = \frac{\tau_{max}^2}{4G}}$

hollow shaft $\boxed{\frac{SE}{\text{volume}} = \frac{\tau_{max}^2}{4G}\left[\frac{R_o^2 + R_i^2}{R_o^2}\right]}$

$\tau_{max} = \frac{16T}{\pi D_s^3}$ ⟶ Solid shaft D_s $\left\{ \frac{\tau}{r} = \frac{T}{I_P} \right\}$

$\tau_{max} = \frac{16T}{\pi D^3\left[1 - \frac{1}{n^4}\right]}$ ⟶ hollow shaft

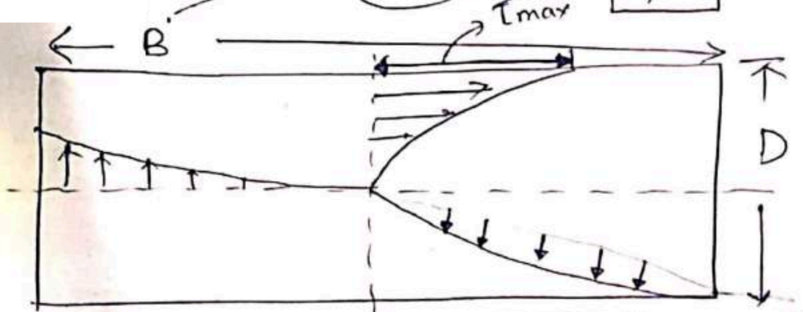
$\boxed{n = \frac{D}{d} = \frac{\text{outer dia}}{\text{inner ,,}}}$

$\boxed{\frac{T_H}{T_S} = \frac{n + 1/n}{\sqrt{n^2 - 1}}}$ torque comparison b/w solid & hollow shaft

$\boxed{\frac{W_H}{W_S} = \frac{(n^2-1)n^{2/3}}{(n^4-1)^{2/3}}}$ weight comparison b/w hollow & solid shaft

Shear stress distribution for Rectangular Bar :-

(B>D) $\boxed{B>D}$



conclusion :- ① Corners $\boxed{\tau = 0}$

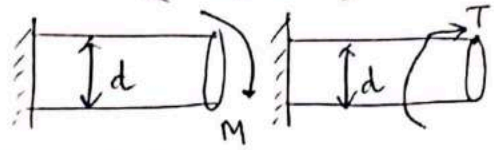
(II) τ_{max}. at boundary surface but overall max. ⟶ middle of longer side (B)

$\boxed{\text{Shear flow } q = \frac{T}{2\pi A_m}}$

$\Downarrow$

Shear stress × thickness $= q$
τ (t)

(Solid shafts)



$\boxed{\sigma = \frac{32M}{\pi d^3}}$ $\boxed{\tau_{max} = \frac{16T}{\pi d^3}}$

$\boxed{\frac{\sigma}{\tau_{max}} = \frac{2M}{T}}$

combined effect of bending & Torsion

$\boxed{\frac{\sigma_1}{\sigma_2} = \sigma_1/\sigma_2 = \frac{16}{\pi D^3}\left[M \pm \sqrt{M^2+T^2}\right]}$

⟶ Principal stresses $\{\sigma_1, \sigma_2\}$ + −

$\boxed{\tau_{max} = \frac{16}{\pi D^3}\left[\sqrt{M^2+T^2}\right]}$

⟶ max. shear stress.

$\boxed{\text{equivalent BM} = \frac{1}{2}\left[M + \sqrt{M^2+T^2}\right]}$
M'

$\frac{32M'}{\pi d^3} = \frac{16}{\pi d^3}\left(M + \sqrt{M^2+T^2}\right)$

Same

$\boxed{\text{equivalent torque} = \sqrt{M^2+T^2}}$

⟶ if M, T given then Shaft strength will be checked by this formula.

19/2/20

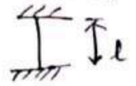
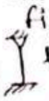
Column :-

Euler Theory → valid only $\boxed{\text{for long column}}$

↳ in this theory direct axial compressive stress neglected hence not used for short column.

$$\boxed{P_{cr} = \frac{\pi^2 E I_{min}}{(l_{eff})^2}} = P_e$$

$\boxed{l_{eff}}$

Both end fixed	Both end hinge	one end hinge + one end fix	one end fixed, other free
			
$l_{eff} = \frac{l}{2}$	$l_{eff} = l$	$l_{eff} = l/\sqrt{2}$	$l_{eff} = 2l$

$$\boxed{\underset{\lambda}{\text{Slenderness Ratio}} = \frac{l_{eff}}{r_{min}}}$$

r_{min} → min radius of Gyration

$$\boxed{r_{min} = \sqrt{\frac{I_{min}}{A}}}$$

$$\therefore P_{cr} = \frac{\pi^2 E I_{min}}{\lambda^2 r_{min}^2} = \frac{\pi^2 E I_{min}}{\lambda^2 \times (I_{min}/A)}$$

$$\frac{P_{cr}}{A} = \sigma_{cr} = \text{euler critical stress or Buckling strength} = \frac{\pi^2 E}{\lambda^2}$$

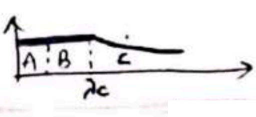
Check :- validity of Euler's Theory

$$\sigma_{cr} \leq \sigma_y$$

$$\frac{\pi^2 E}{\lambda^2} \leq \sigma_y$$

$$\therefore \lambda \geq \sqrt{\frac{\pi^2 E}{\sigma_y}}$$

$\lambda_c = 90$ for mild steel



A → safe short column
C → safe long column
B → Intermediate safe column.

Rankine formula :- valid → $\boxed{\text{for short \& long column}}$

{buckling & crushing considered}

$$\boxed{\frac{1}{P_R} = \frac{1}{P_c} + \frac{1}{P_e}}$$

P_c → crushing load; P_e → euler load

$$\boxed{P_c = f_c A}$$

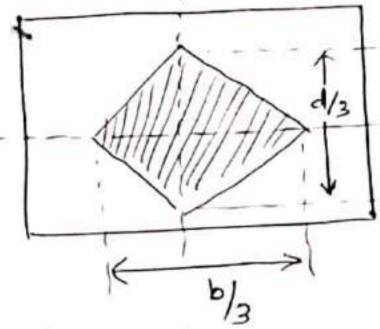
$$\therefore \boxed{P_R = \frac{f_c A}{1 + \alpha \lambda^2}}$$

$$\left\{\alpha = \text{Rankine constant} = \frac{f_c}{\pi^2 E}\right\}$$

Shape of kern in eccentric loading

(I) Rectangular & I section	Rhombus
(II) Square	Square
(III) Circular	Circular
(IV) Hollow circular section	"

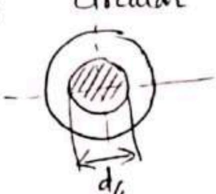
(I) Rectangular & I section



for no tension
$e < b/6$
$e < d/6$
} ∴ called $\frac{1}{3}$rd Rule

(II) Square

(III) Circular



$e < d/8$ } ∴ $\frac{1}{4}$th rule

(IV) Hollow circular section

$$\boxed{e < \frac{D^2 + d^2}{8D}}$$

eccentricity; for diameter → multiply by 2

Spring

- Closed coil
- Open coil

	Closed coil	Open coil
direct shear	✔	✔
Bending [redacted] Stress	Neglected	✔
shear stress	✔	neglected

Spring ⟶ Trick → like capacitor

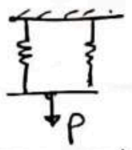
Series

$\Delta = \Delta_1 + \Delta_2$

$\frac{P}{k_{eq}} = \frac{P}{k_1} + \frac{P}{k_2}$

$$\boxed{\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}}$$

Parallel



$\Delta_1 = \Delta_2$

$P = P_1 + P_2$

$$\boxed{k_{eq} = k_1 + k_2}$$

flexibility (mm/N) $= 1/k$ $\quad \Delta = P/k \quad k \to \frac{N}{mm}$

Closed coil :- (Bending stress → neglected)

① $\tau_{max} = \text{innerside} = \underbrace{\frac{P}{\frac{\pi}{4}d^2}}_{\text{direct stress}} + \underbrace{\frac{16(PR)}{\pi d^3}}_{\text{shear stress due to torque}}$

$$\boxed{\begin{aligned} \tau_{max} &= \text{Inner} = \frac{16PR}{\pi d^3}\left[1 + \frac{d}{4R}\right] \\ \tau_{min} &= \text{outer} = \frac{16PR}{\pi d^3}\left[1 - \frac{d}{4R}\right] \end{aligned}}$$

$\because \frac{d}{4R} <<<< 1$ hence direct shear we can also neglect.

② strain energy stored in spring

$U = \frac{1}{2}T\theta = \frac{1}{2}T\left(\frac{TL}{GI_p}\right)$

यहाँ $T = PR \quad L = 2\pi R n \quad I_p = \frac{\pi d^4}{32}$

$$\therefore \boxed{U = \frac{32P^2R^3n}{Gd^4}}$$

③ axial deflection $\boxed{\Delta = \frac{\partial U}{\partial P} = \frac{64PR^3n}{Gd^4}}$

④ $\boxed{\text{stiffness } k = P/\Delta = \frac{Gd^4}{64R^3n}}$

flexibility $= 1/k$

⑤ Proof load :- max. load carrying capacity of spring without getting permanently distorted.

Let f_s ⟶ allowable shear stress of material of spring

$\therefore f_s = \frac{16(PR)}{\pi d^3}$

$\therefore P_{max} = \text{Proof Load} = \frac{\pi d^3 f_s}{16R}$

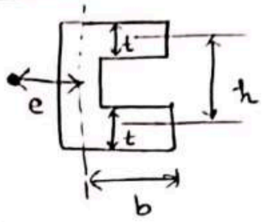
leaf spring :-

$$\boxed{\text{max stress } f_{max} = \frac{3Wl}{2bnt^2}}$$ *

Shear centre ⊙S
'or'
(Centre of flexure)

$\Big\}\rightarrow$ no torsion

formula



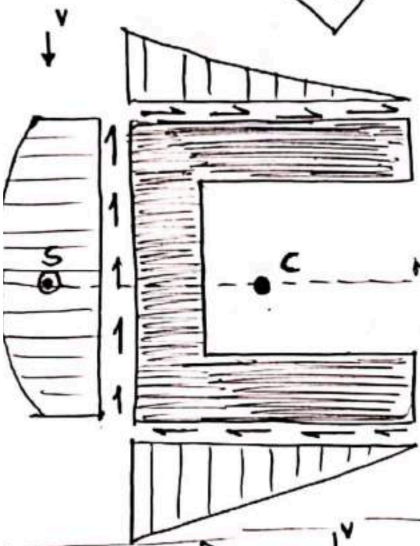
$$\boxed{e = \frac{b^2 h^2 t}{4 I_{NA}}}$$

e → shear centre distance

Shear flow in common section for vertical load ↓ through section

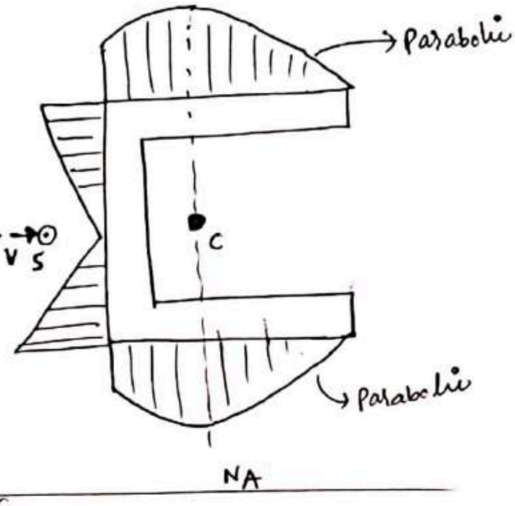
in figure

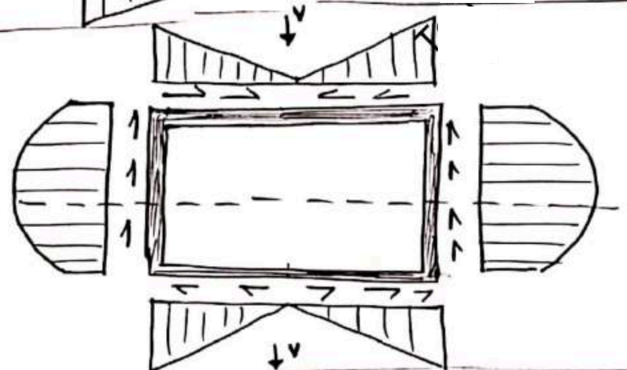
Shear centre ⊙S
Centroid •C

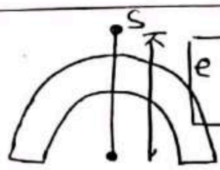


Shear flow tends to max. at NA and tends to zero at a free edge.

when load direction horizontal

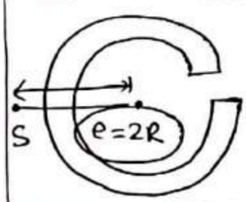




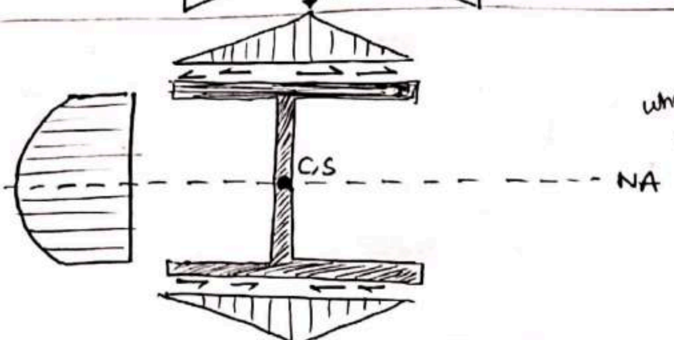


$$\boxed{e = \frac{4R}{\pi}}$$

Semicircular Section ✓



✓



when load direction horizontal