Chapter 2

Beams

CHAPTER HIGHLIGHTS

- *Introduction*
- *Singly reinforced sections*
- *Doubly reinforced section*
- *Flanged beams*

Introduction

Beams and slabs take loads by bending action. Due to bending action, tensile stresses develop in beam which concrete cannot resist as it is weak in tension. Due to this reason, steel which is strong in tension is provided at a place where maximum tensile stresses are developed in beams. Hence, the present chapter outlines the load or moment carrying capacity of singly reinforced sections, doubly reinforced sections, flanged sections and various modes of failure or types of sections are also discussed. The general design requirements for beams are also discussed.

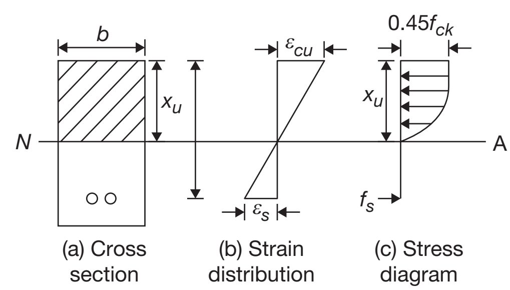
Singly Reinforced Sections

Steel on one side of beam, i.e., on tension side.

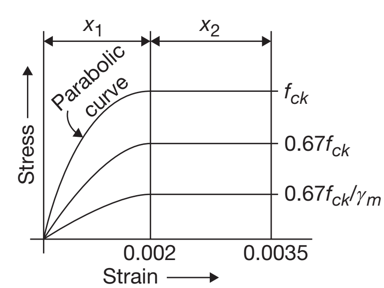
Assumptions Made in Limit State Method of Design in Flexure

IS:456–2000 permits the following assumptions:

1. Plane sections normal to the axis remain plane even after bending. It means, the strain distribution across the depth of cross-section is linear as shown in the following figure.

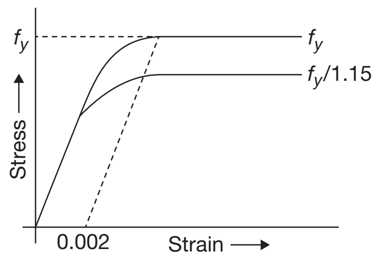


2. At limiting state, the maximum strain in concrete, which occurs at outermost compression fiber is 0.0035.
3. The stress strain curve for concrete is having parabolic shape up to 0.002 strain and, then constant up to limit state of 0.0035. However, IS code do not prevent using other shapes, like rectangle, trapezoidal.

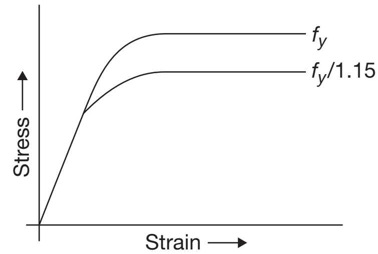


Idealised stress strain curve for concrete

4. The tensile strength of concrete is ignored.
5. The stress in reinforcement is derived from the representative stress-strain curve for the type of steel used as shown below. For design purpose, a partial safety factor of 1.15 is used. Hence, the maximum stress in steel is limited to $\frac{fy}{1.15} = 0.87\,fy$.



(a) Cold worked deformed bar



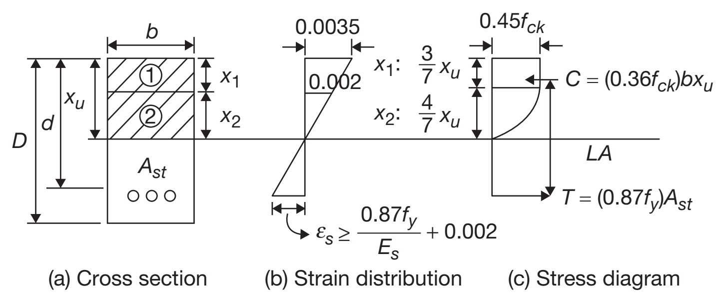
(b) Steel bar with definite yield point (mild steel)

6. The maximum strain in steel (tension reinforcement) at failure shall not be less than $\frac{f_y}{1.15E_s} + 0.002$.

[f_y: characteristic strength of steel, E_s: modulus of elasticity of steel]

Stress Block Parameters

The stress block and strain diagram for a singly reinforced section is shown below.



From similar triangles of strain distribution, $x_1 = \frac{3}{7}x_u$

and $x_2 = \frac{4}{7}x_u$

Tension force in steel $T = (0.87f_y)\,A_{st}$.

Compression force in concrete = $C_1 + C_2$

C_1: Average stress in part (1) × (area)$_1$

$$= (0.45 f_{ck})(bx_1) = 0.45 f_{ck} b\left(\frac{3}{7}x_u\right)$$

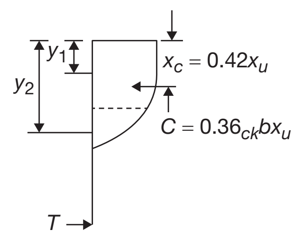
C_2: Average stress in part (2) × (area)$_2$

$$= \left[\frac{2}{3}(0.45 f_{ck})\right](bx_2) = \frac{6}{35} f_{ck} bx_u$$

Total compressive force in concrete is:

$$C = 0.36\, f_{ck}\, bx_u$$

Location of 'C' from top compressed fiber,



y_1: Centroidal distance of rectangular portion from top compressed fiber
y_2: Centroidal distance of parabolic portion from top compressed fiber

$$y_1 = \frac{x_1}{2}, y_2 = x_1 + \frac{3}{8}x_2$$

$$\therefore x_c = \frac{C_1 y_1 + C_2 y_2}{C_1 + C_2}$$

$$x_c = 0.42\, x_u$$

Hence, the total compressive force in concrete is $0.36 f_{ck}\, bx_u$ and, it acts at a distance of $0.42\, x_u$ from the top fiber.

Depth of Neutral Axis (x_u)

It can be obtained by considering equilibrium of internal forces of compression and tension, i.e., ($C = T$).

$$X_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

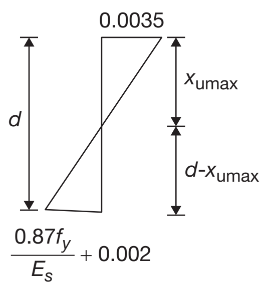
Lever Arm (z)

The distance between line of action of compression and tension forces is called 'lever arm'.

Lever arm, $z = d - 0.42 x_u$

Maximum Depth of Neutral Axis (x_{umax})

The maximum dept of neutral axis can be obtained from similar triangles of strain distribution.



∴ From the strain diagram:

$$\frac{x_{umax}}{d} = \frac{0.0035}{\frac{0.87 f_y}{E_s} + 0.0055}$$

Values of X_{umax}/d for Different Grades of Steel

f_y(N/mm^2)	$\frac{x_{umax}}{d}$
250	0.53
415	0.48
500	0.46

Modes of Failure/Types of Section

Balanced Section

- The section is called 'balanced section', when the strains in both concrete and steel increases simultaneously and reaches to their maximum values at the same time.
- The moment of resistance for a balanced section is called 'limiting moment of resistance'.
- In these sections, $x_u = x_{umax}$

Moment of resistance for a balanced section:

M_{ulim} = Compression force × LA

$M_{ulim} = 0.36 f_{ck}\, b x_{umax} (d - 0.42\, x_{umax})$

M_{ulim} = Tension force × LA

$M_{ulim} = 0.87 f_y b A_{st} (d - 0.42\, x_{umax})$

Where LA is lever arm

Limiting Moment of Resistance for Singly Reinforced Section

f_y(N/mm^2)	M_{ulim}
250	$0.148 f_{ck} bd^2$
415	$0.138 f_{ck} bd^2$
500	$0.133 f_{ck} bd^2$

Under-reinforced Section

- Amount of steel in a section is less than that of the required amount for a balanced section.
- Neutral axis will shift upwards to maintain the equilibrium between forces.
- Failure of section is initiated by steel reaching its yield value (primary tension failure).
- In these section, $x_u < x_{umax.}$
- Moment of resistance of under-reinforced section is given by:

$$M_u = \text{Compression force} \times LA$$
$$M_u = 0.36 f_{ck}\; b x_u (d - 0.42 x_u)$$

or

$$M_u = \text{Tension force} \times LA$$
$$M_u = 0.87\; f_y A_{st} (d - 0.42\; x_u)$$

Over-reinforced Section

- Amount of steel in a section is more than that of the amount required for a balanced section.
- Neutral axis will shift downwards to maintain the equilibrium.
- In these sections, strain in concrete reaches to its ultimate value prior to steel reaching its yield value (i.e., primary compression failure).
- These sections are avoided because of sudden failure of concrete (brittle) without giving warnings.
- In these sections, take or consider

$$x_u = x_{umax.}$$

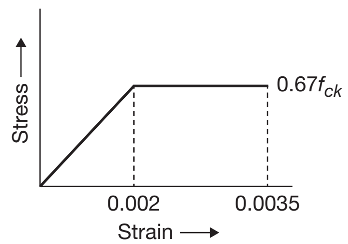
- Moment of resistance:

$$M_u = \text{Compression force} \times LA$$
$$= 0.36\; f_{ck} b x_{umax} (d - 0.42\; x_{umax})$$

SOLVED EXAMPLE

Direction for solved examples 1 and 2:

Assume straight line instead of parabola for stress-strain curve of concrete as given below, and partial safety factor as 1.0.



A rectangular under-reinforced section of 300 mm width and 500 mm effective depth is reinforced with 3 bars of grade Fe415 each of 16 mm diameter. Concrete mix is M20. **[GATE, 2000]**

Example 1

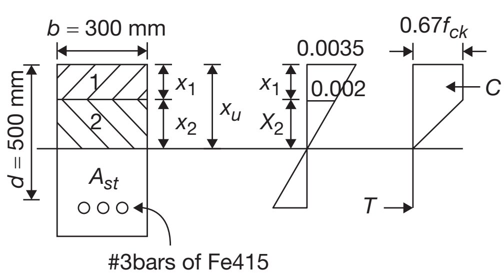
The depth of neutral axis from the compression fiber is:

(A) 76 mm (B) 81 mm
(C) 87 mm (D) 100 mm

Solution

No change in strain distribution diagram

$$\therefore x_1 = \frac{3}{7}x_u \; x_2 = \frac{4}{7}x_u$$



C: Compressive force in concrete

$$= c_1 + c_2$$

C_1: Average stress in part (1) × A_1

$$= (0.67 f_{ck})bx_1$$

$$C_1 = 0.67 f_{ck} b \frac{3}{7} x_u$$

C_2: Average stress in part (2) × A_2

$$= \frac{1}{2}(0.67 f_{ck})bx_2$$

$$C_2 = \frac{1}{2} 0.67 f_{ck} b \frac{4}{7} x_u$$

$$C : C_1 + C_2$$

$$= 0.67 f_{ck}\, b \frac{3}{7} x_u + \frac{1}{2} 0.67 f_{ck}\, b \frac{4}{7} x_u$$

$$C = (0.478) f_{ck} b x_u$$

To calculate x_u:

$$C = T \Rightarrow 0.478 f_{ck} b x_u = 0.87 f_y A_{st}$$

$$X_u = \frac{0.87 f_y A_{st}}{0.478 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 3 \times \frac{\pi}{4} \times 16^2}{0.478 \times 20 \times 300}$$

$$X_u = 75.93 \text{ mm} \approx 76 \text{ mm}$$

Hence, the correct answer is option (A).

Example 2

The depth of neutral axis obtained as per IS:456–2000 differs from the depth of neutral axis as obtained in the above question by:

(A) 15 mm (B) 20 mm
(C) 25 mm (D) 32 mm

Solution

Depth of neutral axis as per IS:456–2000 is:

$$X_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 3 \times \frac{\pi}{4} \times 16^2}{0.36 \times 20 \times 300}$$

$$X_u = 100.82 \text{ mm} \approx 101 \text{ mm}$$

∴ Difference in neutral axis depth

$$= 101 - 76 = 25 \text{ mm}$$

Hence, the correct answer is option (C).

General Design Requirements for Beams

1. **Effective span**
   - For simply supported beam: Lesser of two values
     (a) Clear span + effective depth of beam
     (b) Centre-to-centre distance between supports, whichever is less.
   - For cantilever beam: Clear overhang $+\left(\frac{d}{2}\right)$

2. **Limiting stiffness:** Basic values of span to effective depth (for spans not exceeding 10 m) are given below [as per clause 23.2 of IS:456–2000]

Cantilevers – 7

Simply supported – 20

Continuous – 26

- The above values should be modified by multiplying with modification factors depending on the amount and type of steel.
- For spans above 10 m, the above values may be multiplied by 10/span in m.

3. **Minimum reinforcement:** Minimum area of tension reinforcement should not be less than the following (as per clause 26.5.1 of IS:456–2000]:

$$\boxed{\frac{(A_{st})_{\min}}{b \cdot d} = \frac{0.85}{f_y}}$$

4. **Limiting percentage of steel:** The percentage of tensile reinforcement corresponding to limiting moment of resistance is known as limiting percentage of steel.

$$P_{t,} \lim = \frac{(A_s t_1)_{\lim}}{b\,d} \times 100$$

$$= \left[\frac{0.36\ f_{ck}}{0.87\ f_y} \frac{x_{u\max}}{d}\right] \times 100$$

5. **Maximum tension reinforcement:** The maximum area of tension reinforcement should not exceed 4% of the gross cross-sectional area.

$$\boxed{(A_{st})\max = (0.04)(bD)}$$

6. **Cover to reinforcement/nominal cover/clear cover**
   - The distance from extreme fiber of shear stirrup to bottom tension fiber is known as cover/clear cover.
   - It is based on serviceability or environmental conditions.

Structural Elements	Minimum Cover
Slab (with mild exposure)	20 mm
Beams	25 mm
Columns	40 mm
Footings	50 mm

7. **Spacing of bars:** The horizontal distance between main bars shall not be less than the greatest among the following:

(a) Diameter of the bar if diameter are equal.

(b) Diameter of the largest bar if the bars are unequal.

(c) 5 mm more than the nominal maximum size of coarse aggregate.

If needle vibrator is used to compact concrete, $\frac{2}{3}$ of nominal maximum size of coarse aggregate.

The vertical distance between bars shall be the greatest among the following:

(a) 15 mm

(b) $\frac{2}{3}$ of nominal maximum size of coarse aggregate

(c) Maximum size of bars.

8. **Side face reinforcement:**
   - If d >750 mm, side face reinforcement (A_{sf}) is provided.

$$\boxed{A_{sf} = 0.1\%\,(A_g)}$$

If beam is of T section,

A_g = Web area

d = Depth of web

- A_{sf} shall be distributed equally on both sides and maximum spacing between them shall not exceed 300 mm c/c.

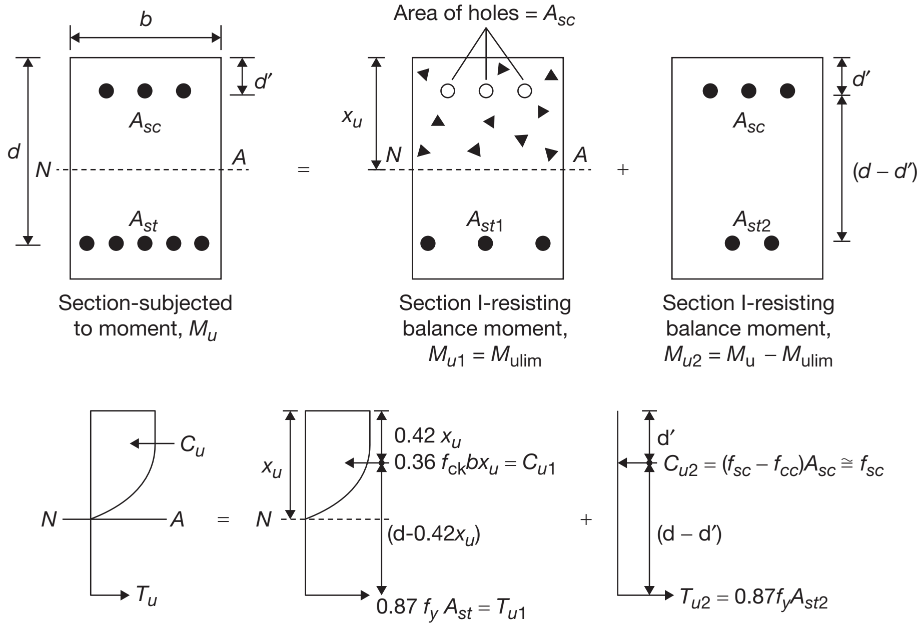
Doubly Reinforced Section

Beams with reinforcement in both compression and tension sides are called 'doubly reinforced beams'.

Situations Where Doubly Reinforced Beams are Used

- Restriction in the depth of beam due to architectural or any construction-related problem.
- At the sections where bending moment changes its sign.
- Members subjected to change in direction of wind loads and in earthquake regions.
- To resist moment higher than the limiting moment of resistance of a singly-reinforced section.

Analysis of Doubly-reinforced Beams



Doubly-reinforced beam

Moment of resistance of doubly-reinforced beam (M_u) = (A singly-reinforced section with M_{ulim}) + (A section with compression steel and additional tension steel to resist additional moment M_{u2})

$$\boxed{M_u = M_{ulim} + M_{u2}}$$

Additional moment of resistance M_{u2} is given by:

$$\boxed{M_{u2} = (f_{sc}\ A_{sc})(d - d') = 0.87\ f_y\ A_{st2}(d - d')}$$

Limiting moment of resistance of singly-reinforced section is given by:

$$M_{ulim} = 0.148\, f_{ck} bd^2 \quad \text{for Fe250}$$
$$= 0.138\, f_{ck} bd^2 \quad \text{for Fe415}$$
$$= 0.133\, f_{ck} bd^2 \quad \text{for Fe500}$$

Where

f_{sc} = Stress in compression steel.

d' = Distance between the top compression fiber to centroid of compression reinforcement.

A_{sc} = Area of compression reinforcement required to resist M_{u2}.

A_{st2} = Area of additional tensile reinforcement to balance compression steel.

A_{st1} = Area of tensile reinforcement for a balanced singly-reinforced section.

Neutral axis: By equating total force of compression to total force of tension.

$$C_{u1} + C_{u2} = T_u\ 0.36\, f_{ck}\, b\, x_u + f_{sc}\, A_{sc} = 0.87\, f_y A_{st}$$

$$\boxed{x_u = \frac{0.87 f_y (A_{st1} + A_{st2}) - (f_{sc} A_{sc})}{0.36\, f_{ck}\, b}}$$

Area of compression steel (A_{sc}):

$$\boxed{A_{sc} = \frac{M_{u2}}{(f_{sc})(d - d')}}$$

Area of tension steel (A_{st}): A_{st1} is based on limiting moment of resistance of a singly-reinforced section.

$$M_{ulim} = 0.87\, f_y\, A_{st1}\ (d - 0.42 x_{umax})$$

$$\boxed{A_{st1} = \frac{M_{ulim}}{0.87\, f_y\, (d - 0.42 x_{umax})}}$$

A_{st2} is based on additional moment of resistance.

$$M_{u2} = M_u - M_{ulim} = 0.87\, f_y\, A_{st2}\ (d - d')$$

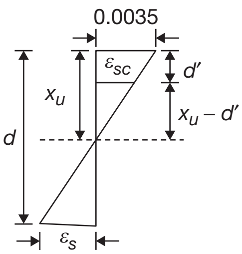
$$\boxed{A_{st2} = \frac{M_{u2}}{0.87\, f_y\, (d - d')}}$$

Total area of tension steel:

$$\boxed{A_{st} = A_{st1} + A_{st2}}$$

Stress in compression steel (f_{sc}): By knowing the strain at its level, the stress in compression steel can be obtained.

ε_{sc}—strain at level of compression steel



From similar triangles of strain distribution,

$$\frac{\varepsilon_{sc}}{(x_u - d')} = \frac{0.0035}{x_u}$$

$$\boxed{\varepsilon_{sc} = (0.0035)\left[1 - \frac{d'}{x_u}\right]}$$

For mild steel, $f_{sc} = [\varepsilon_{sc}]E_s$

$$= (0.0035)\left[1 - \frac{d'}{x_u}\right](2 \times 10^5)$$

$$f_{sc} = \left[1 - \frac{d'}{x_u}\right]$$

Subject to a maximum of $0.87\, f_y$

For HYSD bars, f_{sc} is based on d'/d values:

Grade of Steel	d'/d			
	0.05	0.10	0.15	0.20
Fe 415	355	353	342	329
Fe 500	424	412	395	370

Direction for questions 3 and 4:

A doubly-reinforced rectangular concrete beam has a width of 300 mm and an effective depth of 500 mm. The beam is reinforced with 2200 mm^2 of steel in tension and 628 mm^2 of steel in compression. The effective cover for compression steel is 50 mm. Assume that both tension and compression steel yield. The grades of concrete and steel used are M20 and Fe250, respectively. The stress block parameters (rounded off to first two decimal places) for concrete shall be bland as per IS:456–2000. **[GATE, 2010]**

Example 3

The depth of neutral axis is:

(A) 205.30 mm (B) 184.56 mm
(C) 160.91 mm (D) 145.30 mm

Solution

Given, $b = 300$ mm $d = 500$ mm; $d' = 50$ mm

$A_{st} = 2200$ mm^2 $A_{sc} = 628$ mm^2

Both tension and compression yield $f_{sc} = 0.87 f_y$

$f_{ck} = 20$ N/mm^2, $f_y = 250$ N/mm^2

To calculate depth of neutral axis:

Total compression force = Total tension force

$$C_{u1} + C_{u2} = T_u$$

$$0.36\, f_{ck}\, bx_u + f_{sc}A_{sc} = 0.87 f_y A_{st}$$

$$\therefore x_u = \frac{0.87\, f_y A_{st} - f_{sc}A_{sc}}{0.36\, f_{ck}\, b}$$

$$= \frac{0.87 \times 250 \times 2200 - 0.87 \times 250 \times 628}{0.36 \times 20 \times 300}$$

$$x_u = 158\ 29 \text{ mm.}$$

Hence, the correct answer is option (C).

Example 4

The moment of resistance of section is

(A) 206.00 kN/m (B) 209.20 kN/m
(C) 237.80 kN/m (D) 251.90 kN/m

Solution

$X_{umax} = 0.53 \times d = 0.43 \times 500 = 265$ mm

$X_u < x_{umax} \Rightarrow$ under Reinforced section

$\therefore M_u = 0.36 f_{ck} bx_u(d - 0.42x_u) + f_{sc}A_{sc}(d - d')$

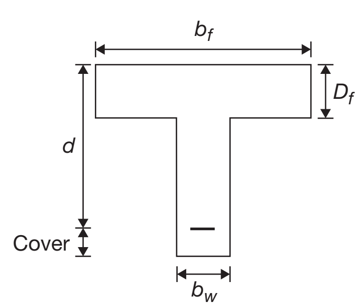
$= 0.36 \times 20 \times 300 \times 160.91(500 - 0.42 \times 160.91) + 0.87 \times 250 \times 628\ (500 - 50)$

$M_u = 209.20$ kN/m.

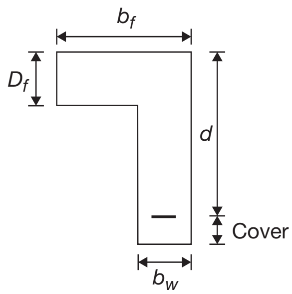
Hence, the correct answer is option (B).

Flanged Beams

T-beams and L-beams are the examples of flanged beams.



T-beam



L-beam

These sections are treated as flanged sections only when the following requirements are met:

1. Beams and slabs are always cast monolithically. [i.e., slab shall be cast integrally with the web]
2. Transverse reinforcement shall not be less than 60% of main reinforcement at mid-span of slab if main reinforcement is parallel to beam.

Effective Width of Flange

For T-beams, $b_f = \frac{l_0}{6} + b_w + 6D_f \leq c$

For L-beams, $b_f = \frac{l_0}{12} + b_w + 3D_f \leq \frac{c}{2}$

For isolated beams

For T-beam, $b_f = \frac{l_0}{\left(\frac{l_0}{b} + 4\right)} + b_w \leq b$

For L-beam, $b_f = \frac{0.5 l_0}{\left(\frac{l_0}{b} + 4\right)} + b_w \leq b$

Where

l_0 = Distance between points of zero moments in beam (for simply supported beam: $l_0 = l$, for continuous or fixed beam: $l_0 = 0.70l$)

l = Effective span

b_w = Breadth of web or width of web

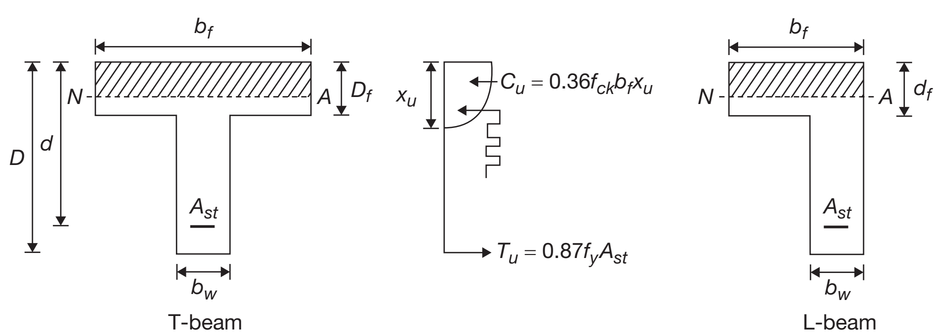
D_f = Thickness of flange or thickness of slab

b = Actual width of flange

Moment of Resistance of Singly-reinforced Flanged Sections

Based on the position of neutral axis, the following three cases arise.

Case 1: Neutral axis is within flange ($x_u \leq D_f$).



When neutral axis is within flange, the flanged beam can be treated like a rectangular beam of width $b = b_f$.

Moment of resistance in case 1 ($x_u \leq D_f$)

$$M_u = 0.36\, f_{ck}\, b_f\, x_u (d - 0.42\, x_u)$$
$$M_u = 0.87\, f_y A_{st}\, (d - 0.42\, x_u)$$

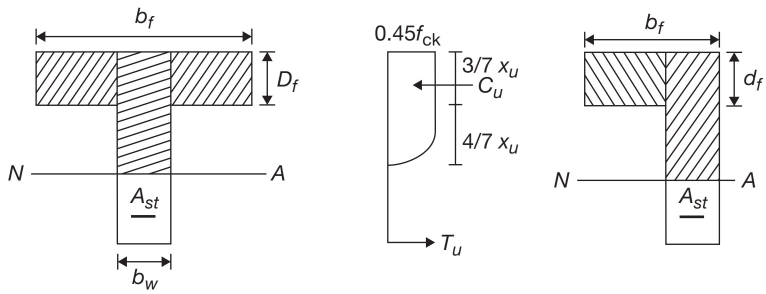
Case 2: Neutral axis below the flange, i.e., in web ($x_u > D_f$) and $D_f > \frac{3}{7} x_u$ or $\frac{D_f}{d} > 0.2$.

In this case, entire flange is subjected to a uniform stress of $0.45\, f_{ck}$.

Moment of resistance in case 2:

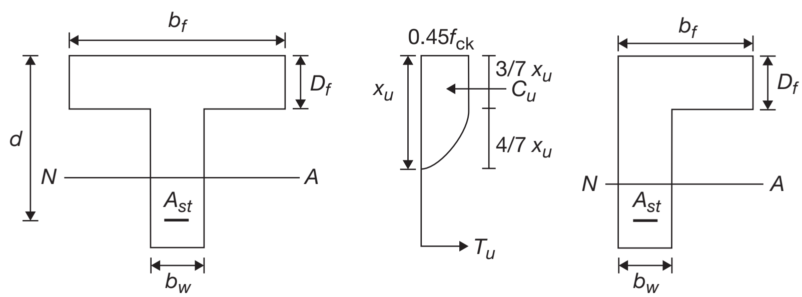
$$\left(x_u > D_f \;\&\; \frac{D_f}{d} \leq 0.2 \right)$$

$$M_u = 0.36\, f_{ck}\, b_w\, x_u (d - 0.42\, x_u) + 0.45\, f_{ck}\, (b_f - b_w) D_f \left(d - \frac{D_f}{2} \right)$$



Case 3: NA below the flange ($x_u > D_f$), i.e., in web and $D_f > \frac{3}{7}x_u$ or $\frac{D_f}{d} > 0.2$.

In this case, the compressive stress is not uniform across the depth of flange. IS:456–2000 has recommended the modified thickness of flange equal to y_f. The compressive stress over this depth y_f is assumed to be uniform equal to $0.45\,f_{ck}$.



Moment of resistance in case 3 $(x_u > D_f)$ and $\frac{D_f}{d} > 0.2$

$$M_u = 0.36\,f_{ck}\,b_w\,x_u(d - 0.42x_u) + 0.45\,f_{ck}\,(b_f - b_w)y_f\left(d - \frac{y_f}{2}\right)$$

Where

$$Y_f = 0.15\,x_u + 0.65\,D_f$$

but not more than D_f.

Minimum and Maximum Reinforcement

Minimum percentage of reinforcement to be provided in flanged beams is:

$$\frac{A_{st}}{b_w d} = \frac{0.85}{f_y}$$

Maximum area of tension reinforcement should not exceed 4% of the gross cross-sectional area ($b_w d$).

Moment of Resistance of Doubly-reinforced Flanged Section

Case 1: If $x_u \le D_f$:

$$M_u = 0.36\,f_{ck}\,b_f\,x_u(d - 0.42\,x_u) + f_{sc}A_{sc}(d - d')$$

Case 2: If $x_u > D_f$ and $\frac{D_f}{d} \le 0.2$ or $D_f \le \frac{3}{7}x_u$

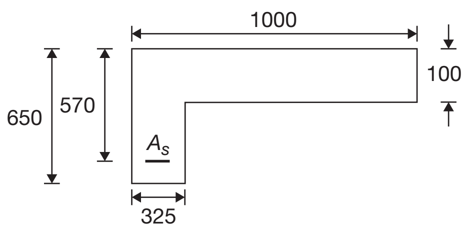
$$M_u = 0.36\,f_{ck}\,b_w\,x_u(d - 0.42\,x_u) + 0.45\,f_{ck}(b_f - b_w)\left(d - \frac{D_f}{2}\right) + f_{sc}\,A_{sc}(d - d')$$

Case 3: If $x_u > D_f$ and $\frac{D_f}{d} > 0.2$ or $D_f > \frac{3}{7}x_u$

M_u is based on the above equation only except replacing D_f by y_f.

Direction for questions 5 and 6:

The cross-section at mid-span of a beam at the edge of a slab is shown in the sketch. A portion of the slab is considered as the effective flange width for the beam. The grades of concrete and reinforcing steel are M25 and Fe415, respectively. The total area of reinforcing bars (A) is 4000 mm^2. At the ultimate limit state, x_u denotes the depth of neutral axis from the top fiber. Treat the section as under-reinforced and flanged ($x_u > 100$ mm).



All dimensions are in 'mm'. **[GATE, 2012]**

Example 5

The value of x_u (in mm) computed as per the limit state method of IS:456–2000 is:

(A) 200
(B) 223.3
(C) 236.3
(D) 273.6

Solution

$$D_f = 100 \text{ mm},$$
$$d = 570 \text{ mm}$$

$$\frac{D_f}{d} = \frac{100}{570} = 0.175 \leq 0.2$$

So, it comes under case (2).

To compute x_u; Equate compressive force to tensile force $C = T$

$$0.36f_{ck}b_w x_u + 0.45f_{ck}(b_f - b_w)D_f = 0.87f_y A_{st}$$

$$\therefore x_u = \frac{0.87 f_y A_{st} - 0.45 f_{ck}(b_f - b_w)D_f}{0.36 f_{ck} b_w}$$

$$= \frac{0.87 \times 415 \times 4000 - 0.45 \times 25 \times (1000 - 325)100}{0.36 \times 25 \times 325}$$

$$X_u = 234.12 \text{ mm} \sim 236.3 \text{ mm}$$

Hence, the correct answer is option (C).

Example 6

The ultimate moment capacity (in kNm) of the section as per the limit state method IS:456–2000 is:

(A) 475.2
(B) 717.0
(C) 756.4
(D) 762.5

Solution

$$M_u = 0.36f_{ck}b_w x_u(d - 0.42x_u) + 0.45f_{ck}(b_f - b_w)D_f\left(d - \frac{D_f}{2}\right)$$

$$= 0.36 \times 25 \times 325 \times 236.3(570 - 0.42(236.3)) + 0.45 \times 25 \, (1000 - 325)\ 100\left(570 - \frac{100}{2}\right)$$

$M_u = 717.0$ kN/m

Hence, the correct answer is option (B).

Exercises

1. In a singly reinforced beam, the tensile steel reaches its maximum allowable stress earlier than concrete. What is such a section known as?
   (A) Under-reinforced section
   (B) Over-reinforced section
   (C) Balanced section
   (D) Economic section

Direction for questions 2 and 3:

In the design of beams for the limit state of collapse in flexure as per IS:456–2000, let the maximum strain in concrete be limited to 0.0025 (in place of 0.0035). For this situation, consider a rectangular beam section with breadth as 250 mm, effective depth as 350 mm, area of tension steel as 1500 mm^2 and characteristic strength of concrete and steel as 30 Mpa and 250 Mpa respectively

2. The depth of neutral axis for the balanced failure is
   (A) 140 mm (B) 156 mm
   (C) 168 mm (D) 185 mm
3. At the limiting state of collapse in flexure, the force acting on the compression zone of the section is
   (A) 326 kN (B) 389 kN
   (C) 424 kN (D) 542 kN

Direction for questions 4 and 5:

A reinforced concrete beam, size 200 mm wide and 300 mm deep overall is simply supported over a span of 3 m. It is subjected to two point loads '*P*' of equal magnitude placed at middle third points. The two loads are gradually increased simultaneously. Beam is reinforced with 2 HYSD bars of 16 mm diameter placed at an effective cover of 40 mm on bottom face and nominal shear reinforcement. The characteristic compressive strength and the bending tensile strength of the concrete are 20.0 N/mm^2 and 2.2 N/mm^2 respectively.

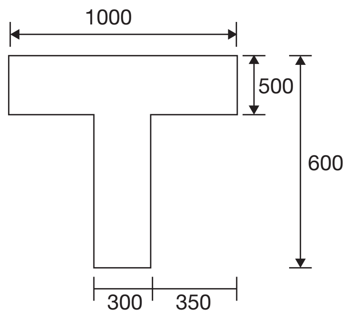
4. Ignoring the presence of tension reinforcement, find the value of *P*, in kN when the first flexure crack will occur in the beam
   (A) 4.5 (B) 5.0
   (C) 6.6 (D) 7.5
5. In the theoretical failure load of the beam for attainment of limit state of collapse in flexure is
   (A) 23.7 kN (B) 25.6 kN
   (C) 27.8 kN (D) 31.6 kN
6. Maximum strains in an extreme fiber in concrete and in tension reinforcement (Fe 415 grade and E_s = 200 kN/mm^2) in a balanced section at limit state of flexure are respectively?
   (A) 0.0035 and 0.0038 (B) 0.002 and 0.0018
   (C) 0.0035 and 0.0041 (D) 0.002 and 0.0031
7. As per the provision of IS:456–2000, in limit state method for design of beams, the limiting value of depth of neutral axis in a reinforced concrete beam of effective depth '*d*' is given as
   (A) 0.53 *d*
   (B) 0.48 *d*
   (C) 0.46 *d*
   (D) any of the above depending on different grades of steel
8. IS:456–2000 recommends providing certain minimum steel in a RCC beam
   (A) to ensure compression failure.
   (B) to avoid rupture of steel in case a flexure failure occurs.
   (C) to hold the stirrup steel in position.
   (D) to provide enough ductility to the beam.
9. The following two statements are made with reference to a simply supported under reinforced RCC beam:
   I. Failure takes place by crushing of concrete before steel has yielded
   II. The neutral axis moves up as the load is increased

   With reference to the above statements, which of the following applies
   (A) Both the statements are false
   (B) I is true but II is false
   (C) Both the statements are true
   (D) I is false but II is true
10. Maximum strain at level of compression steel for a rectangular section having effective cover to compression steel as *d*' and neutral axis depth from compression face x_u is
    (A) $0.0035\left(1-\frac{d'}{x_u}\right)$ (B) $0.002\left(1-\frac{d'}{x_u}\right)$
    (C) $0.0035\left(1-\frac{x_u}{d'}\right)$ (D) $0.002\left(1-\frac{x_u}{d'}\right)$
11. A floor slab of thickness, *t*, is cast monolithically to a rectangular continuous beam of span, L_1 and width, *B*. If the distance between two consecutive points of contraflexure is L_0, the effective width of compression flange at a continuous support is
    (A) *B* (B) *L*/3
    (C) $B + 12t$ (D) $B + 6t + L_0/6$
12. The effective width of a reinforced concrete T-beam flange under compression, according to IS:456–1978, given l_0 is the distance between the adjacent zero moment points, *b* is the breadth of rib and *D* is the thickness of flange, is

(A) $\frac{l_0}{6} + B + 6D$ (B) $l_0 + 6D$

(C) $\frac{l_0}{6} + 6D$ (D) $\frac{l_0}{6} + b$

13. An isolated T-beam is used as a walkway. The beam is simply supported with an effective span of 6 m. The effective width of flange, for the cross-section shown in figure, is



(A) 900 mm (B) 1000 mm
(C) 1259 mm (D) 1500 mm

14. Flexural collapse in over reinforced beams is due to
(A) primary compression failure.
(B) secondary compression failure.
(C) primary tension failure.
(D) bond failure.

15. In an RCC beam, side face reinforcement is provided if its depth exceeds
(A) 300 mm (B) 500 mm
(C) 700 mm (D) 750 mm

16. The effective width 'b_f' of flange of a continuous T-beam in a floor system is given by:

$$b_f = \frac{L_0}{6} + b_w + 6D_f$$

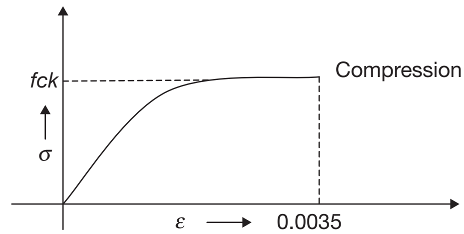
Where L_0 represents the
(A) distance between points of contraflexure in a span.
(B) effective span of beams.
(C) clear span of beams.
(D) spacing between beams.

17. Minimum tension steel in RC beam needs to be provided to
(A) prevent sudden failure.
(B) arrest crack width.
(C) control excessive deflection.
(D) prevent surface hair cracks.

18. Balanced neutral axis depth for a singly reinforced cross section depends on
(A) grade of concrete.
(B) grade of steel.
(C) amount of steel reinforcement.
(D) All the these

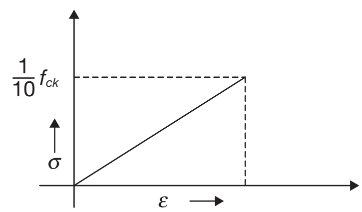
19. A rectangular concrete beam of width 230 mm and effective depth 300 mm is reinforced with 4–12 mm bars in tension zone. M20 Grade concrete and Fe415 steel are used. Find the moment of resistance of the beam.
(A) 42.23 kN-m
(B) 35.26 kN-m
(C) 64.13 kN-m
(D) 72.54 kN-m

20. If the following figure represents the idealized σ–ε curve of concrete in compression.

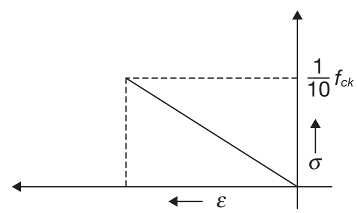


Then the σ–ε curve of concrete in tension is

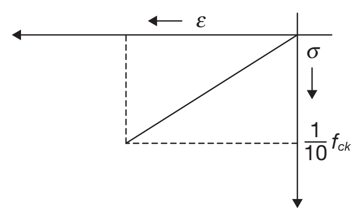
(A)



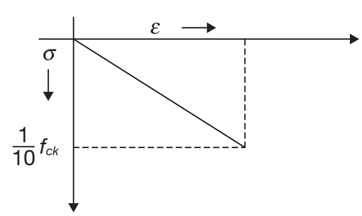
(B)



(C)



(D)



Direction for questions 21 and 22:

A reinforced concrete beam, size 250 mm wide and 400 mm deep effective is simply supported over a span of 6 m. It is subjected to a point load of P at centre of the beam. The point load is increased gradually. Beam is reinforced with 5 HYSD bars of Fe415 grade of 12 mm diameter placed at an effective cover of 40 mm on bottom face and nominal shear reinforcement. The characteristic compressive strength and bending tensile strength of concrete are 20 MPa and 2.2 MPa respectively.

21. Ignoring the pressure on tension reinforcement, find the value of load P in kN when the first flexural crack will develop on the beam
(A) 26.4 kN (B) 11.83 kN
(C) 16.4 kN (D) 35.5 kN

22. The theoretical failure load of the beam for attainment of limit sate of collapse in flexure is
(A) 33 kN (B) 48 kN
(C) 52 kN (D) 64 kN

23. A reinforced beam of size 230 mm width and 350 mm overall depth is subjected to a working moment of 65 kN-m. If M20 grade concrete and Fe415 grade steel are used, it is to be designed as (effective cover = 50 mm)
(A) balanced section
(B) singly reinforced section
(C) doubly reinforced section
(D) over reinforced section

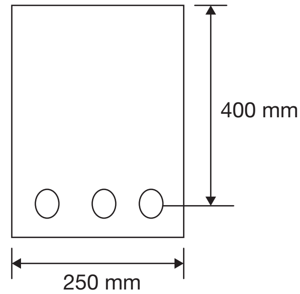
24. A reinforced concrete beam of 12 m effective span and 1 m effective depth is simply supported. If the total UDL of the beam is 10 MN/m the design shear force for the beam is
(A) 20 MN (B) 30 MN
(C) 40 MN (D) 50 MN

25. A T-beam roof section has the following particulars. Width of flange = 600 mm, Thickness of slab = 120 mm. Width of web = 250 mm depth of web = 300 mm Effective cover = 50 mm. If 3–20ϕ bars reinforcement is provided and M20 Grade concrete and Fe415 steel are used. Find the moment of Resistance of the beam.
(A) 92 kN-m
(B) 106 kN-m
(C) 114 kN-m
(D) 138 kN-m

26. The ratio of the vertical deflection limit for a continuous and a simply supported beams (of length ≤ 10 m) is ______.
(A) 1.1 (B) 1.3
(C) 1.6 (D)1.8

27. For a ring beam of a water tank, side face reinforcement should be provided as per IS:456–2000 if the depth exceeds ______.
(A) 750 mm (B) 500 mm
(C) 450 mm (D) 300 mm

28. A simply supported RC beam having clear span 5 m and support width 300 mm has the cross section as shown.



What is the effective span of the beam as per IS:456?
(A) 5400 (B) 5300
(C) 5250 (D) 5200

29. For a continuous RC beam, match List I (Condition) with List II (Placement of live load)

	List I		List II
a.	For maximum sagging moment in a span	**1.**	The span adjoining the span as well as alternate span
b.	For maximum hogging moment at a support	**2.**	The same span as well as alternate spans
c.	For maximum hogging moment in a span	**3.**	The adjacent spans on both sides of this support as well as spans alternate to these
		4.	Spans next to the adjacent spans of the support plus alternate spans

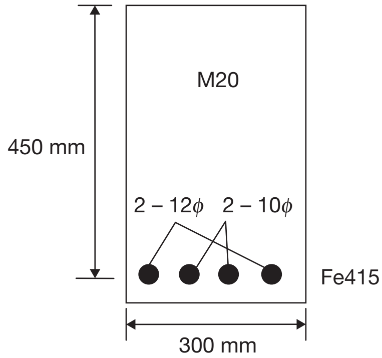
Codes:

	a	b	c		a	b	c
(A)	1	2	2	(B)	2	3	1
(C)	2	4	1	(D)	1	3	2

30. If a simply supported concrete beam, pre stressed with a force of 2500 kN is designed by load balancing concept for an effective span of 10 m and to carry a total load of 50 kN-m, the central dip of the cable profile (in mm) should be ______.
(A) 200 (B) 220
(C) 250 (D) 290

31. In a reinforced concrete section, the stress at the extreme fibre in compression is 6.2 Mpa. The depth of neutral axis is 65 mm and the grade of concrete is M25. Assuming linear elastic behavior of concrete, the effective curvature of the section (per mm) is
(A) 2.82×10^{-6}
(B) 3.82×10^{-6}
(C) 4.82×10^{-6}
(D) 5.82×10^{-6}

32. For the given singly reinforced beam section, find the moment of resistance by Limit state method as per IS 456 in kN-m?



(A) 39 kN-m
(B) 46 kN-m
(C) 59 kN-m
(D) 72 kN-m

33. At the limit state of collapse an RC beam is subjected to flexural moment of 200 kN-m and torque of 10 kN-m. If the beam is of 300 mm wide and gross depth of 472 mm with an effective cover of 35 mm, the equivalent flexural moment in kN-m will be _____.
(A) 215 kN-m (B) 235 kN-m
(C) 265 kN-m (D) 285 kN-m

34. A T-beam roof section has following particulars:
Thickness of slab 100 mm; width of rib 300 mm; depth of beam 500 mm; center to centre distance of beams 3 m; effective span of beams 6 m; distance between points of contra flexure 3.6 m; The effective flange width in meters is ______.
(A) 1.1 m (B) 1.5 m
(C) 2.2 m (D) 2.6 m

35. A reinforced concrete beam of 10 m effective span and 1m effective depth is simply supported. If the total UDL on the beam is 10 kN-m, the design shear force for the beam is ____.
(A) 50 kN (B) 47.5 kN
(C) 32.5 kN (D) 40 kN

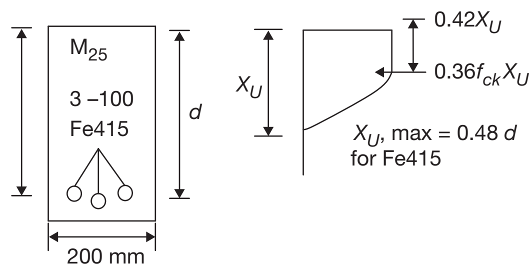
36. The maximum percentage of moment redistribution allowed in RCC beams is
(A) 40% (B) 30%
(C) 20% (D) 10%

37. In a cantilever beam carrying gravity load, main Reinforcement is provided.
(A) Above the neutral axis
(B) As vertical stirrups
(C) As a helical reinforcement
(D) Below the neutral axis

38. A T-beam roof section has the flowing particulars
Thickness of slab : 120 mm
Width of rib : 300 mm
Depth of beam : 550 mm
Centre to centre distance of beams : 3.2 m
Effective span of beams : 6.5 m
Distance between points of contra flexure : 3.60 m
The effective width of flange of the beam is
(A) 3000 mm
(B) 1620 mm
(C) 2520 mm
(D) 6500 mm

39. A singly reinforced rectangular concrete beam has a width of 150 mm and an effective depth of 350 mm. What is the likely approximation for the limiting value of the moment of resistance of the beam?
(Take f_{ck} : 25 MPa and f_y : 415 MPa)
(A) 60 kN-m
(B) 40 kN-m
(C) 30 kN-m
(D) 50 kN-m

40. Consider the singly reinforced beam section given below (left figure). The stress block parameters for the cross-section from IS:456–2000 are also given below (right figure). The moment of resistance for the given section by the limit state method is __________ kN-m.



(A) 19.5 kN-m
(B) 21.5 kN-m
(C) 42.5 kN-m
(D) 30 kN-m

41. For a beam of cross section, width = 250 mm and effective depth = 420 mm, the number of rebars of 8 mm diameter required to satisfy minimum tension reinforcement required specified by IS:456–2000 (assuming grade of steel reinforcement as Fe415) is _____. (round off to the nearest higher value)
(A) 3 (B) 4
(C) 5 (D) 6

42. A concrete beam of rectangular cross section of 250 mm 350 mm is prestressed with a force of 250 kN at eccentricity 150 mm. The maximum compressive stress in the concrete is
(A) 7.5 N/mm^2
(B) 12.5 N/mm^2
(C) 15 N/mm^2
(D) 10 N/mm^2

Previous Years' Questions

Direction for questions 1 and 2:
A singly reinforced rectangular concrete beam has a width of 150 mm and an effective depth of 330 mm. The characteristic compressive strength of concrete is 20 Mpa and the characteristic tensile strength of steel is 415 Mpa. Adopt the stress block for concrete as given in IS:456–2000 and take limiting values of depth of neutral axis as 0.48 times the effective depth of beam **[GATE, 2007]**

1. The limiting value of moment of resistance of beam in kN-m is
(A) 0.14 (B) 0.45
(C) 45.08 (D) 156.82

2. From the above data, the limiting area of tension steel in mm^2 is
(A) 473.9 (B) 412.3
(C) 373.9 (D) 312.3

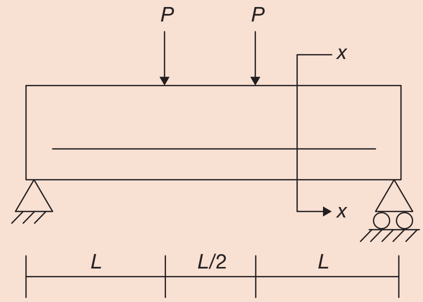
3. As per IS:456–2000, in the limit state design of a flexural member, the strain in reinforcing bars under tension at ultimate state should not be less than **[GATE, 2012]**
(A) f_y/E_s (B) $\dfrac{f_y}{E_s}+0.002$
(C) $\dfrac{f_y}{1.15E_s}$ (D) $\dfrac{f_y}{1.15E_s}+0.002$

4. The plane section remains plane assumption in bending theory implies **[GATE, 2013]**
(A) strain profile is linear.
(B) stress profile is linear.
(C) both strain and stress profiles are linear.
(D) shear deformations are neglected.

5. For a beam of cross section, width = 230 mm and effective depth = 500 mm, the number of rebars of 12 mm diameter required to satisfy minimum tension reinforcement requirement specified by IS:456–2000 (assuming grade of reinforcement as Fe500) **[GATE, 2014]**

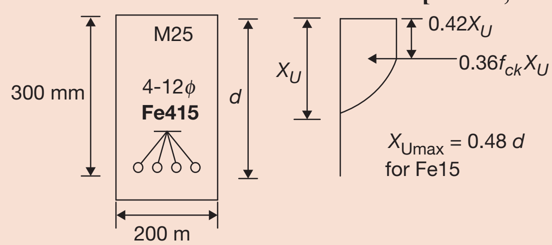
6. In a reinforced concrete section, the stress at the extreme fiber in compression is 5.80 Mpa. The depth of neutral axis in the section is 58 mm and the grade of concrete is M25. Assuming linear elastic behavior of the concrete, the effective curvature of the section (in per mm) is **[GATE, 2014]**
(A) 2.0×10^{-6} (B) 3.0×10^{-6}
(C) 4.0×10^{-6} (D) 5.0×10^{-6}

7. Consider the singly Reinforced beam shown in the figure below



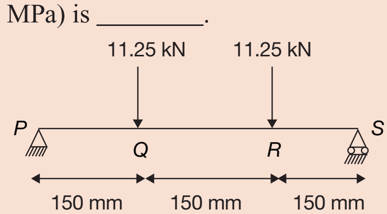
At cross-section *x-x* which of the following statements is TRUE at the limit state? **[GATE, 2015]**
(A) The variation of stress is linear and that of strain is non linear.
(B) The variation of strain is linear and that of stress is non linear.
(C) The variation of both stress and strain is linear.
(D) The variation of both stress and strain is non linear.

8. Consider the singly reinforced beam section given below (left figure). The stress block parameters for the cross section from IS:456–2000 are also given below (right figure). The moment of resistance for the given section by the limit state method in kN-m **[GATE, 2015]**



9. A reinforced concrete (RC) beam with width of 250 mm and effective depth of 400 mm is reinforced with Fe415 steel. As per the provisions of IS:456–2000, the minimum and maximum amount of tensile reinforcement (expressed in mm^2) for the section are, respectively **[GATE, 2016]**
(A) 250 and 3500
(B) 205 and 4000
(C) 270 and 2000
(D) 300 and 2500

10. A 450 mm long plain concrete prism is subjected to the concentrated vertical loads as shown in the figure. Cross-section of the prism is given as 150 mm × 150 mm. Considering linear stress distribution across the cross-section, the modulus of rupture (expressed in MPa) is ________. **[GATE, 2016]**



Answer Keys

Exercises

1. A	**2.** B	**3.** B	**4.** C	**5.** D	**6.** A	**7.** D	**8.** D	**9.** D	**10.** A
11. D	**12.** A	**13.** A	**14.** B	**15.** D	**16.** A	**17.** A	**18.** B	**19.** A	**20.** C
21. B	**22.** B	**23.** C	**24.** D	**25.** C	**26.** B	**27.** C	**28.** B	**29.** B	**30.** C
31. B	**32.** C	**33.** A	**34.** B	**35.** D	**36.** B	**37.** A	**38.** B	**39.** A	**40.** A
41. C	**42.** D								

Previous Years' Questions

1. C	**2.** A	**3.** D	**4.** A	**5.** 2	**6.** C	**7.** B	**8.** 42.77	**9.** B	**10.** 3