

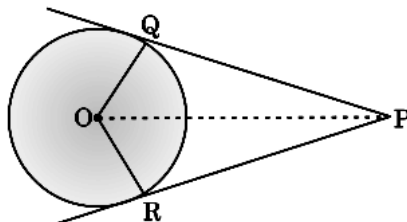
CHAPTER – 10 CIRCLES

THEOREMS

- 1) The tangent to a circle is perpendicular to the radius through the point of contact.
- 2) The lengths of tangents drawn from an external point to a circle are equal.

Given : A circle $C(O, r)$ and two tangents say PQ and PR from an external point P .

To prove : $PQ = PR$.



Construction : Join OQ , OR and OP .

Proof : In $\triangle OQP$ and $\triangle ORP$

$$OQ = OR$$

(radii of the same circle)

$$OP = OP$$

(Common)

$$\angle Q = \angle R = \text{each } 90^\circ \text{ (The tangent at any point of a circle is perpendicular to the radius through the point of contact)}$$

$$\text{Hence } \triangle OQP \cong \triangle ORP$$

(By RHS Criterion)

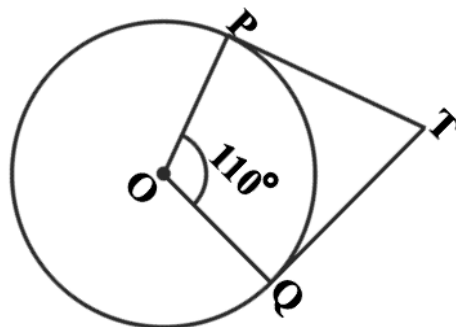
$$\therefore PQ = PR$$

(By CPCT)

Hence Proved.

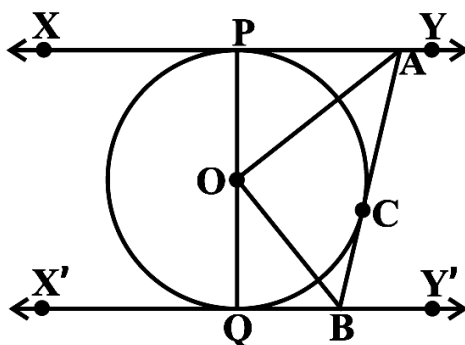
IMPORTANT QUESTIONS

1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle
2. In the below figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110^\circ$, then find $\angle PTQ$.

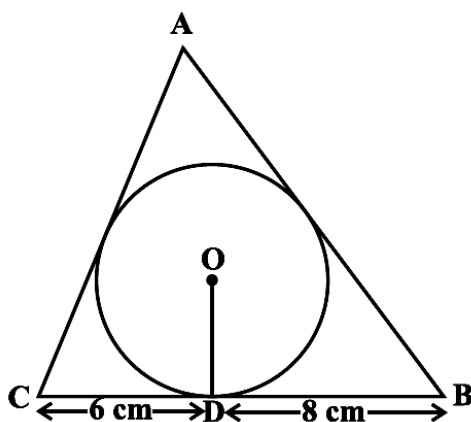


3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80° , then find $\angle POA$
4. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.
5. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.
6. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that $AB + CD = AD + BC$
7. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.
8. Prove that the parallelogram circumscribing a circle is a rhombus.
9. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

10. Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.
11. XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and $X'Y'$ at B . Prove that $\angle AOB = 90^\circ$.

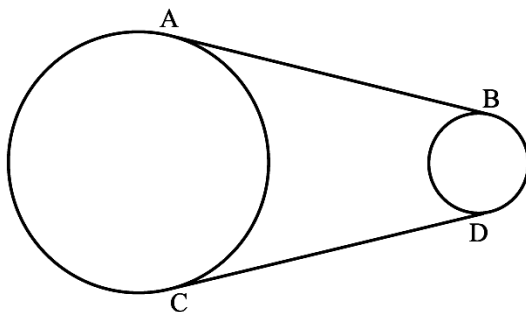


12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC .

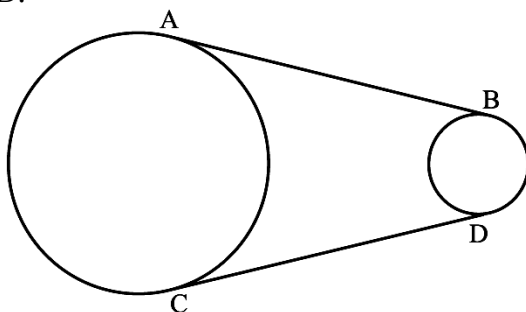


13. Two tangents TP and TQ are drawn to a circle with centre O from an external point T . Prove that $\angle PTQ = 2 \angle OPQ$.
14. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T . Find the length TP .
15. Two tangents PQ and PR are drawn from an external point to a circle with centre O . Prove that $QORP$ is a cyclic quadrilateral.
16. If from an external point B of a circle with centre O , two tangents BC and BD are drawn such that $\angle DBC = 120^\circ$, prove that $BC + BD = BO$, i.e., $BO = 2BC$.
17. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.
18. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A .
19. From an external point P , two tangents, PA and PB are drawn to a circle with centre O . At one point E on the circle tangent is drawn which intersects PA and PB at C and D , respectively. If $PA = 10$ cm, find the perimeter of the triangle PCD .
20. In a right triangle ABC in which $\angle B = 90^\circ$, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P . Prove that the tangent to the circle at P bisects BC .
21. If d_1, d_2 ($d_2 > d_1$) be the diameters of two concentric circles and c be the length of a chord of a circle which is tangent to the other circle, prove that $d_2^2 = c^2 + d_1^2$.
22. If a, b, c are the sides of a right triangle where c is the hypotenuse, prove that the radius r of the circle which touches the sides of the triangle is given by $r = \frac{a+b+c}{2}$.
23. Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.

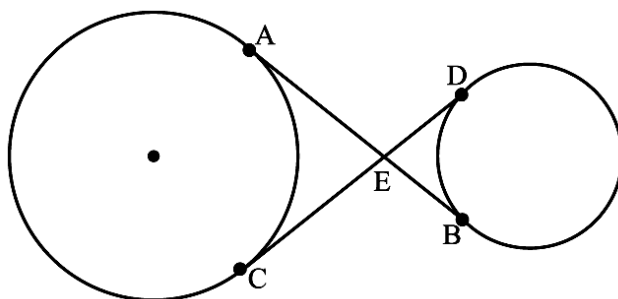
24. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.
25. If from an external point B of a circle with centre O, two tangents BC and BD are drawn such that $\angle DBC = 120^\circ$, prove that $BC + BD = BO$, i.e., $BO = 2BC$.
26. Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.
27. In below figure, AB and CD are common tangents to two circles of unequal radii. Prove that $AB = CD$.



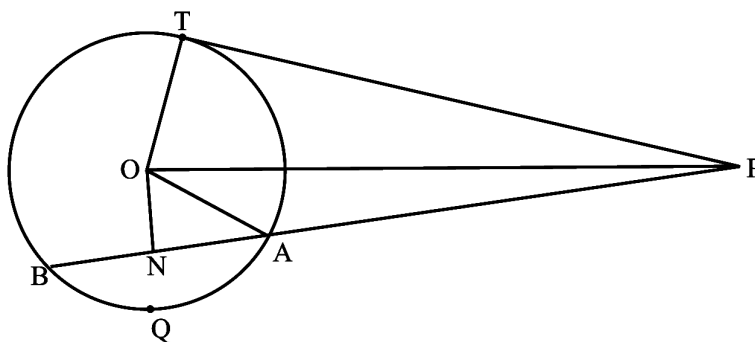
28. In below figure, AB and CD are common tangents to two circles, if radii of the two circles are equal, prove that $AB = CD$.



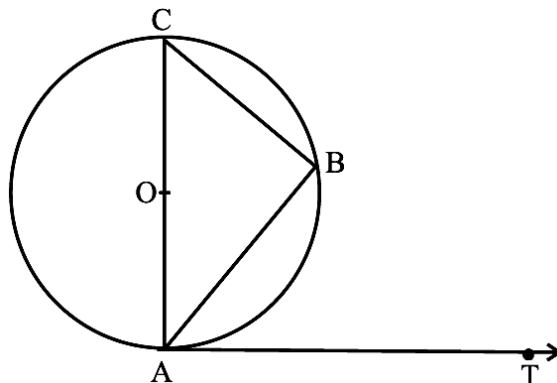
29. In below figure, common tangents AB and CD to two circles intersect at E. Prove that $AB = CD$.



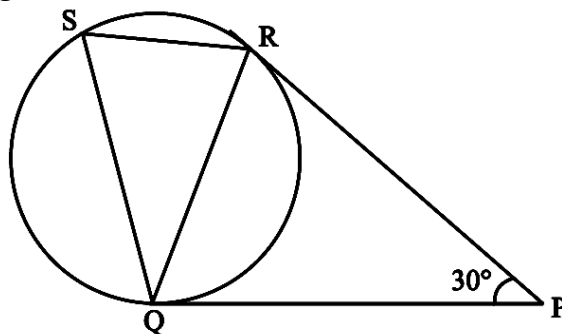
30. In below figure, from an external point P, a tangent PT and a line segment PAB is drawn to a circle with centre O. ON is perpendicular on the chord AB. Prove that :
- $PA \cdot PB = PN^2 - AN^2$
 - $PN^2 - AN^2 = OP^2 - OT^2$
 - $PA \cdot PB = PT^2$



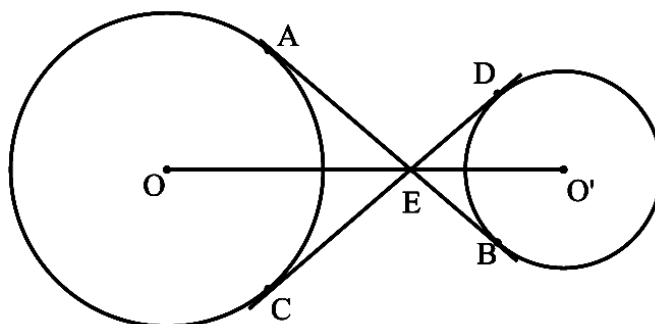
31. If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that $AQ = \frac{1}{2}(BC + CA + AB)$
32. If a hexagon ABCDEF circumscribe a circle, prove that $AB + CD + EF = BC + DE + FA$.
33. Let s denote the semi-perimeter of a triangle ABC in which $BC = a$, $CA = b$, $AB = c$. If a circle touches the sides BC, CA, AB at D, E, F, respectively, prove that $BD = s - b$.
34. From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At one point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. If $PA = 10$ cm, find the perimeter of the triangle PCD.
35. If AB is a chord of a circle with centre O, AOC is a diameter and AT is the tangent at A as shown in below figure. Prove that $\angle BAT = \angle ACB$



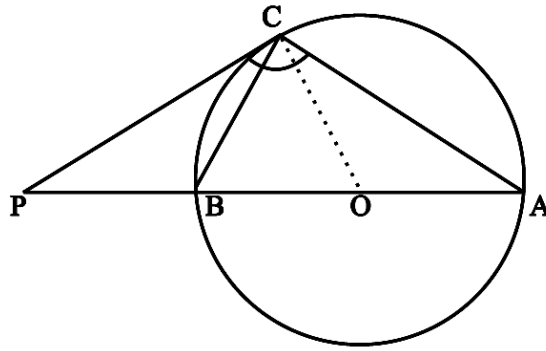
36. In below figure, tangents PQ and PR are drawn to a circle such that $\angle RPQ = 30^\circ$. A chord RS is drawn parallel to the tangent PQ. Find the $\angle RQS$.



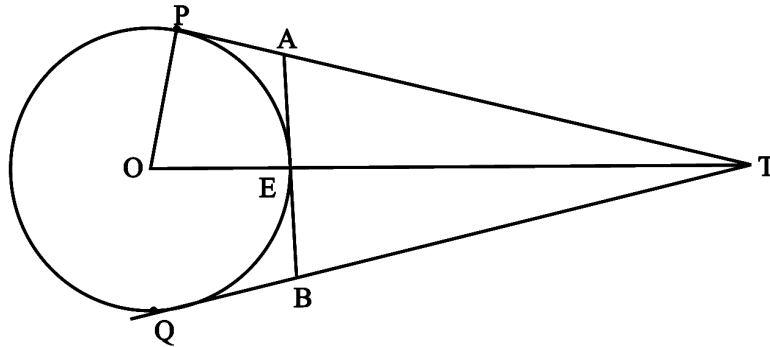
37. AB is a diameter and AC is a chord of a circle with centre O such that $\angle BAC = 30^\circ$. The tangent at C intersects extended AB at a point D. Prove that $BC = BD$.
38. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.
39. A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ.
40. In below figure, the common tangent, AB and CD to two circles with centres O and O' intersect at E. Prove that the points O, E, O' are collinear.



41. The tangent at a point C of a circle and a diameter AB when extended intersect at P. If $\angle PCA = 110^\circ$, find $\angle CBA$



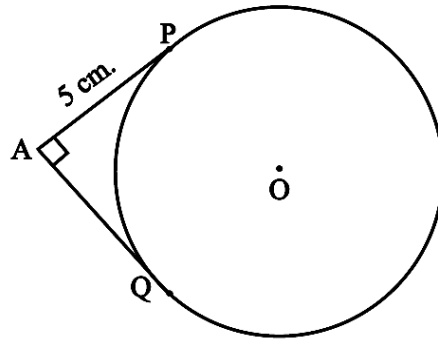
42. In below figure. O is the centre of a circle of radius 5 cm, T is a point such that $OT = 13$ cm and OT intersects the circle at E. If AB is the tangent to the circle at E, find the length of AB.



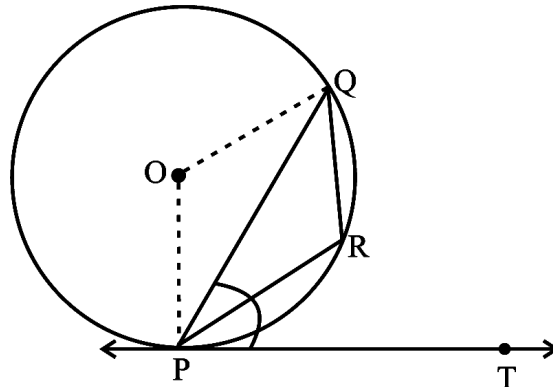
43. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.
 44. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.
 45. If an isosceles triangle ABC, in which $AB = AC = 6$ cm, is inscribed in a circle of radius 9 cm, find the area of the triangle.
 46. Two circles with centres O and O' of radii 3 cm and 4 cm, respectively intersect at two points P and Q such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ.
 47. In a right triangle ABC in which $\angle B = 90^\circ$, a circle is drawn with AB as diameter intersecting the hypotenuse AC and P. Prove that the tangent to the circle at P bisects BC.
 48. A is a point at a distance 13 cm from the centre O of a circle of radius 5 cm. AP and AQ are the tangents to the circle at P and Q. If a tangent BC is drawn at a point R lying on the minor arc PQ to intersect AP at B and AQ at C, find the perimeter of the $\triangle ABC$.

MCQ QUESTIONS (1 mark)

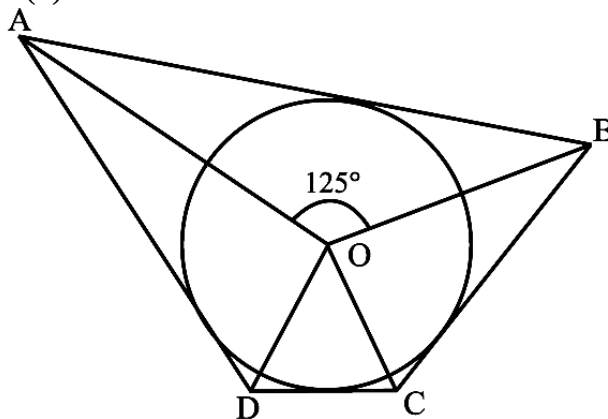
- If angle between two radii of a circle is 130° , the angle between the tangents at the ends of the radii is :
 (a) 90° (b) 50° (c) 70° (d) 40°
- If radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is tangent to the other circle is
 (a) 3 cm (b) 6 cm (c) 9 cm (d) 1 cm
- In the below figure, the pair of tangents AP and AQ drawn from an external point A to a circle with centre O are perpendicular to each other and length of each tangent is 5 cm. Then the radius of the circle is
 (a) 10 cm (b) 7.5 cm (c) 5 cm (d) 2.5 cm



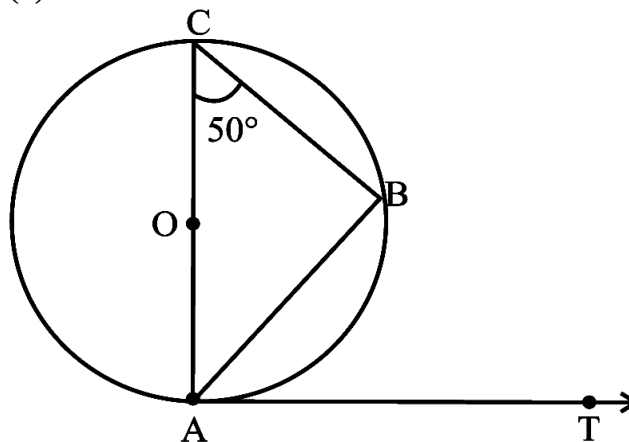
4. In below figure, PQ is a chord of a circle and PT is the tangent at P such that $\angle QPT = 60^\circ$. Then $\angle PRQ$ is equal to
 (a) 135° (b) 150° (c) 120° (d) 110°



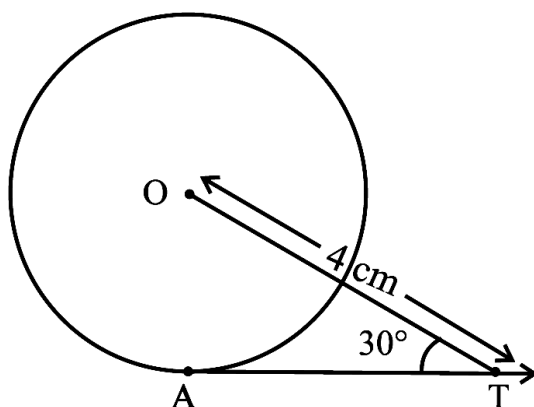
5. In the below figure, if $\angle AOB = 125^\circ$, then $\angle COD$ is equal to
 (a) 62.5° (b) 45° (c) 35° (d) 55°



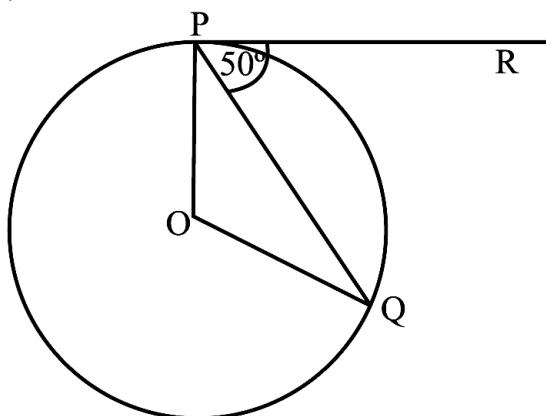
6. In the below figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50^\circ$. If AT is the tangent to the circle at the point A, then $\angle BAT$ is equal to
 (a) 65° (b) 60° (c) 50° (d) 40°



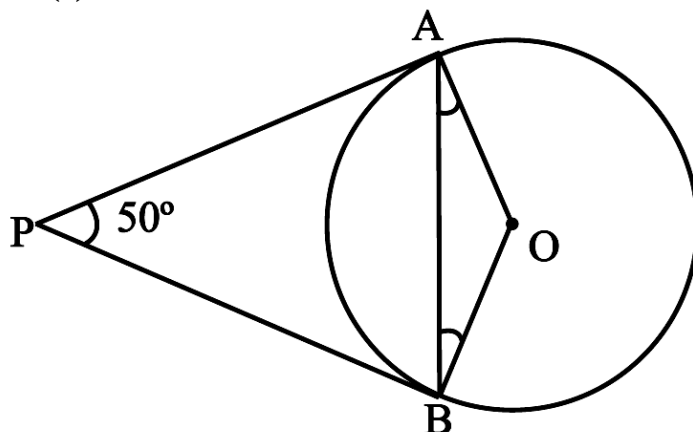
7. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is
(a) 60 cm^2 (b) 65 cm^2 (c) 30 cm^2 (d) 32.5 cm^2
8. At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is
(a) 4 cm (b) 5 cm (c) 6 cm (d) 8 cm
9. In below figure, AT is a tangent to the circle with centre O such that $OT = 4 \text{ cm}$ and $\angle OTA = 30^\circ$. Then AT is equal to
(a) 4 cm (b) 2 cm (c) $2\sqrt{3} \text{ cm}$ (d) $4\sqrt{3} \text{ cm}$



10. In below figure, if O is the centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then $\angle POQ$ is equal to
(a) 100° (b) 80° (c) 90° (d) 75°



11. In below figure, if PA and PB are tangents to the circle with centre O such that $\angle APB = 50^\circ$, then $\angle OAB$ is equal to
(a) 25° (b) 30° (c) 40° (d) 50°



12. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then length of each tangent is equal to

- (a) $\frac{3}{2}\sqrt{3}$ cm (b) 6 cm (c) 3 cm (d) $3\sqrt{3}$ cm

13. In below figure, if PQR is the tangent to a circle at Q whose centre is O, AB is a chord parallel to PR and $\angle BQR = 70^\circ$, then $\angle AQB$ is equal to

- (a) 20° (b) 40° (c) 35° (d) 45°

