13. KINETIC THEORY OF GASES AND THERMODYNAMICS

KINETIC THEORY OF GASES (KTG)

1. INTRODUCTION

The kinetic theory of gases describes a gas as a large number of small particles (atoms or molecules), all of which are in constant, random motion. The rapidly moving particles constantly collide with each other and with the walls of the container. Kinetic theory explains macroscopic properties of gases, such as pressure, temperature, viscosity, thermal conductivity and volume, by considering their molecular composition and motion. The theory postulates that gas pressure is due to the impact of molecules or atoms moving at different velocities on the walls of a container.

But here we will only find the dependency of pressure on velocity and of velocity on temperature.

2. ASSUMPTIONS OF KTG

(a) The collisions b/w gas molecules or between a molecule and the container are completely elastic.

(b) Mass of gas molecules is negligible. So, the effect of gravity and gravitational force on each other can be neglected.

(c) Volume of molecules is considered to be negligible.

(d) Collision b/w molecules is governed by Newton's Law of Motion (i.e. Net force = Change in momentum per unit time)

(e) Interaction force b/w particles is negligible. (That is, they exert no forces on one another.)

(f) Molecules are considered to be in constant, random, and rapid motion. So, average velocity of particles in all the 3-directions will be zero i.e. $\langle v_x \rangle = \langle v_y \rangle = \langle v_z \rangle = 0$.

CONCEPTS

These assumptions must always be kept in mind throughout the chapter.

Vaibhav Gupta (JEE 2009 AIR 54)

3. PRESSURE OF AN IDEAL GAS

Applying Newton's Laws to an ideal gas under the assumptions of kinetic theory allows the determination of the average force on container walls. This treatment assumes that the collisions with the walls are perfectly elastic.

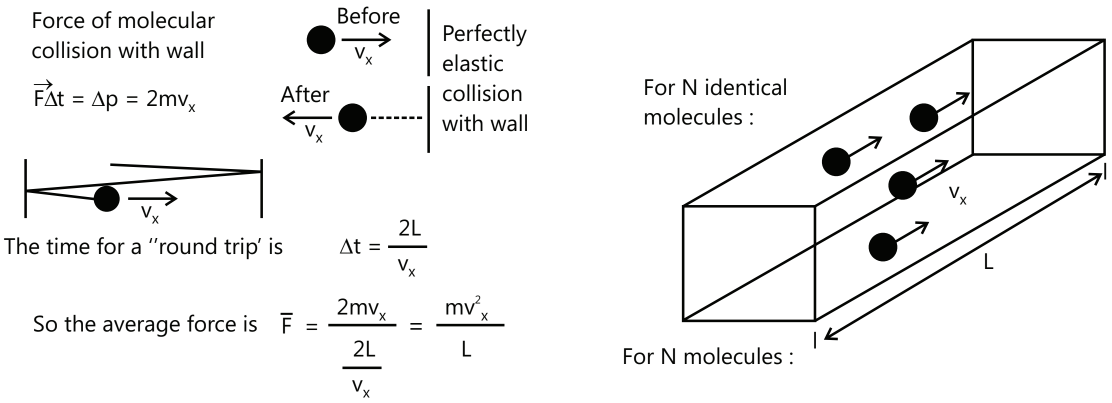


Figure 13.1

An overbar indicates an average quantity. In the expression for the average force per molecule

for N molecules. $\overline{F} = \dfrac{m\left[v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \ldots\ v_{Nx}^2\right]}{L}$

but this can be related to the average: $\overline{v_x^2} = \dfrac{\left[v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \ldots\ v_{Nx}^2\right]}{N}$

It is important to note that it is the average of the square of the velocity which is used, and that this is distinctly different from the square of the average velocity. The average of the square of the velocity is referred as square of root mean square speed.

i.e. $V_{rms} = \sqrt{\overline{v^2}}$; and for N molecules: $\overline{F} = \dfrac{mN\overline{v_x^2}}{L}$

And, since the molecular motion is random according to KTG, so $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$

CONCEPTS

Why are the RMS velocities the same in all the directions? Practically, if we use some pressure measuring device and use it from all directions, we would get the same value of pressure which indicates that RMS velocities must be same in all directions because it is directly proportional to RMS velocity.

Chinmay S Purandare (JEE 2012 AIR 698)

Thus, $\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2} = 3\overline{v_x^2}$;

Now, average force from N molecules can be re-written as $F_{average} = \dfrac{mN\overline{v^2}}{3L}$

Then the pressure in a container can be expressed as $P = \dfrac{F_{avg}}{A} = \dfrac{mN\overline{v^2}}{3LA} = \dfrac{mN\overline{v^2}}{3V} = \dfrac{N}{3V}m\overline{v^2}$

$P=\frac{1}{3}\rho\overline{v^2}$ ρ: density of gas molecules in the container; Therefore, $V_{rms}=\sqrt{3\frac{P}{\rho}}$

Expressed in terms of average molecular kinetic energy: $P=\frac{2N}{3V}\left[\overline{\frac{1}{2}mv^2}\right]$

4. KINETIC INTERPRETATION OF TEMPERATURE

The expression for gas pressure developed from kinetic theory relates pressure and volume to the average molecular kinetic energy. Comparison with the ideal gas law leads to an expression for temperature sometimes referred to as the kinetic temperature.

PV = nRT

And we know from above that: $PV=\frac{2}{3}N\left[\overline{\frac{1}{2}mv^2}\right]$;   Thus, $T=\frac{2}{3}\frac{N}{nR}\left[\overline{\frac{1}{2}mv^2}\right]$

∴ Kinetic energy per mole of gas = $\frac{3}{2}RT$;

As R = Boltzmann's constant x Avogadro number; $R = kN_A$

Thus, Average molecular kinetic energy is: $KE_{avg}=\left[\overline{\frac{1}{2}mv^2}\right]=\frac{3}{2}kT$

Therefore, $V_{rms}=\sqrt{\frac{3RT}{M_m}}$; Where M_m is the molecular weight of the gas.

CONCEPTS

In $V_{rms}=\sqrt{\frac{3RT}{M}}$, M is the molecular weight in Kg.

The temperature at which molecular speed becomes zero is called absolute zero i.e. 0 kelvin

Vaibhav Krishnan (JEE 2009 AIR 22)

Illustration 1: A tank used for filling helium balloons has a volume of $0.3m^3$ and contains 2.0 mol of helium gas at 20.0°C. Assuming that the helium behaves like an ideal gas.

(a) What is the total translational kinetic energy of the molecules of the gas?

(b) What is the average kinetic energy per molecule? **(JEE MAIN)**

Sol: Formula for total kinetic energy and kinetic energy per molecule is used.

(a) Using $(KE)_{Trans}=\frac{3}{2}nRT$

With n = 2.0 mol and T = 293 K, we find that $(KE)_{Trans}=\frac{3}{2}(2.0)(8.31)(293) = 7.3\times10^3$ J

(b) The average kinetic energy per molecule is $\frac{3}{2}kT$.

or $\frac{1}{2}m\overline{v^2} = \frac{1}{2}m\overline{v^2}_{rms} = \frac{3}{2}kT = \frac{3}{2}\left(1.38x10^{-23}\right)(293) = 6.07x10^{-21}J$

Illustration 2: In a circle model of a rotation diatomic molecule of chlorine (Cl_2), the two Cl atoms are 2.0 x 10^{-10} m apart and rotate about their centre of mass with angular speed ω = 2.0 x 10^{12} rad/s. What is the rotational kinetic energy of one molecule of Cl_2, which has a molar mass of 70.0 g/mol? **(JEE ADVANCED)**

Sol: Rotational kinetic energy of each molecule $= \frac{1}{2}I\omega^2$

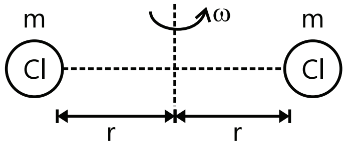


Figure 13.2

Movement of inertia, $I = 2\,(mr^2) = 2mr^2$

Here $m = \frac{70x10^{-3}}{2x6.02x10^{23}} = 5.81x10^{-26}kg$ and $r = \frac{2.0x10^{-10}}{2} = 1.0x10^{-10}m$

$\therefore \quad I = 2(5.81)x10^{-26}(1.0x10^{-10})^2 = 1.16 \text{ x } 10^{-45}$ kg-m²

$\therefore \quad K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}x(1.16x10^{-45})x(2.0x10^{12})^2 = 2.32x10^{-21}J$

Note: At T = 300 K, rotational K.E, should be equal to $\frac{1}{2}kT = \frac{1}{2}x(1.38x10^{-23})x(300) = 2.07x10^{-21}J$

5. TYPES OF SPEED

There are generally three types of speed defined which are physically important:

5.1 Root Mean Square (RMS) Speed v_{rms}

It is the square root of the average of the square of velocities $\overline{v^2}$ i.e. $v_{rms} = \sqrt{\overline{v^2}}$

As has been derived in the previous section, the value of v_{rms} can be derived from the following formula:

$$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}}$$

5.2 Mean/Average Speed v_{av}

It is the mean of the speeds of all the particles i.e. $v_{av} = \frac{v_1 + v_2 + \ldots + v_N}{N}$

From the Maxwellian speed distribution law, we can show that $v_{av} = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi m}} = \sqrt{\frac{8P}{\pi\rho}}$

5.3 Most Probable Speed v_{mps}

It is the mode of the speeds. That is, the speed which is possessed by most of the molecules of a gas at a given temperature T.

$$v_{mp} = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2P}{\rho}}$$

CONCEPTS

The above quantities are easy to derive by some basic knowledge of calculus. However, only knowing them is also fine and observe that

1. $V_{mps}: V_{rms}: V_{av}$:: 1: 1.224: 1.28
2. $V_{mps} =< V_{rms} =< V_{avg}$

Nivvedan (JEE 2009 AIR 113)

Illustration 3: Consider an 1100 particles gas system with speeds distribution as follows:

1000 particles each with speed 100 m/s; 2000 particles each with speed 200 m/s

4000 particles each with speed 300 m/s; 3000 particles each with speed 400 m/s and 1000 particles each with speed 500 m/s Find the average speed, and rms speed. **(JEE MAIN)**

Sol: $\left(V_{avg} = \frac{n_1v_1 + n_2v_2 + n_3v_3 ......}{n_1 + n_2 + n_3 + ...}\right)$ and $V_{rms} = \sqrt{\frac{nv_1^2 + n_2v_2^2 + n_3v_3^2 ....}{n_1 + n_2 + n_3 + ...}}$

The average speed is: $v_{av} = \frac{(1000)(100) + (2000)(200) + (4000)(300) + (3000)(400) + (1000)(500)}{1100}$ = 309 m/s

The rms speed is: $v_{rms} = \sqrt{\frac{(1000)(100)^2 + (2000)(200)^2 + (4000)(300)^2 + (3000)(400)^2 + (1000)(500)^2}{1100}}$ = 328 m/s

Note: Here $\frac{v_{rms}}{v_{av}} \neq \sqrt{\frac{3}{8/\pi}}$ as values and gas molecules are arbitrarily taken.

Illustration 4: Find the rms speed of hydrogen molecules at room temperature (=300 K). **(JEE MAIN)**

Sol: Formula based

Mass of 1 mole of hydrogen gas = 2g = 2 x 10^{-3}kg ⇒ $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3x8.31x300}{2x10^{-3}}} = 1.93x10^3 m/s$

Illustration 5: Find the temperature at which oxygen molecules would have the same rms speed as of hydrogen molecules at 300 K. **(JEE ADVANCED)**

Sol: Compare the V_{rms} of oxygen and hydrogen.

If T be the corresponding temperature $\sqrt{\frac{3RT}{M_o}} = \sqrt{\frac{3R(300)}{M_H}} \Rightarrow T = (300)\left(\frac{M_o}{M_H}\right) = 4800K$

THERMODYNAMICS

1. INTRODUCTION

You would have certainly noticed the whistle of the process cooker in the kitchen. Have you ever thought of reason for that? How small amount of water form large volume of vapours. Think of James Watt and his observation that red to invention of steam engine !

2. GAS LAWS

2.1 Boyle's Law

The Pressure-Volume Law (by Robert Boyle (1627-1691).

Boyle's law or the pressure-volume law states that the volume of a given amount of gas varies inversely with the applied pressure when the temperature and mass are kept constant.

$V \propto \frac{1}{P}$; i.e. $PV = C$; Therefore, $P_1V_1 = P_2V_2$

When pressure goes up, volume goes down and vice-versa.

2.2 Charles' Law

The Temperature-Volume Law (by Jacques Charles (1746 - 1823).

This law states that the volume of a given amount of gas held at constant pressure is directly proportional to the Kelvin temperature.

$v \propto T$; i.e. $V / T = C$; Therefore, $V_1 / T_1 = V_2 / T_2$

As the volume goes up, the temperature also goes up, and vice-versa.

2.3 Gay-Lussac's Law

The Pressure Temperature Law (by Joseph Gay-Lussac (1778-1850).

This law states that the pressure of a given amount of gas held at constant volume is directly proportional to the Kelvin temperature.

$P \propto T$; i.e. $P/T = C$; Therefore, $P_1 / T_1 = P_2 / T_2$

2.4 Avogadro's Law

The Volume Amount Law (Amedeo Avogadro (1776-1856).

Gives the relationship between volume and amount when pressure and temperature are held constant. Remember amount is measured in moles. Also, since volume is one of the variables, that means the container holding the gas is flexible in some way and can expand or contract. If the amount of gas in a container is increased, the volume increases. If the amount of gas in a container is decreased, the volume decreases.

$v \propto n$; i.e. $V/n = C$; Therefore, $V_1 / n_1 = V_2 / n_2$

The combined gas law: The volume of a given amount of gas is proportional to the ratio of its Kelvin temperature and its pressure.

$V \propto \frac{T}{P}$; i.e. $PV/T = C$ Therefore, $P_1 / V_1 = T_1 = P_2V_2 / T_2$

As the pressure goes up, the temperature also goes up, and vice-versa.

The ideal gas law: On combining all the previous laws we get: PV = nRT

Where n: the number of moles and R: constant called the universal gas constant≈ 0.0821 L-atm / mol-K.

CONCEPTS

For solving problems, it is not advisable to individually think over each and every case. Just remember this overall equation. PV = nRT

Nitin Chandrol (JEE 2012 AIR 134)

Illustration 6: An air bubble starts rising from the bottom of a lake. Its diameter is 3.6 mm at the bottom and 4 mm at the surface. The depth of the lake is 250 cm and the temperature at the surface is 40°C. What is the temperature at the bottom of the lake? Given atmospheric pressure = 76 cm of Hg and g = 980 cm/s^2. **(JEE MAIN)**

Sol: Amount of air is constant, hence $\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$ applicable.

At the bottom of the lake, volume of the bubble $V_1=\frac{4}{3}\pi r_1^3=\frac{4}{3}\pi(0.18)^3\,\text{cm}^3$

Pressure on the bubble; P_1 = atmospheric pressure + pressure due to a column on 250 cm of water = 76 x 13.6 x 980 + 250 x 1 x 980 = (76 x 13.6 +250) 980 dyne/cm^2

At the surface of the lake, volume of the bubble $V_2=\frac{4}{3}\pi r_2^3=\frac{4}{3}\pi(0.2)^3\,\text{cm}^3$

Pressure on the bubble; P_2 = atmospheric pressure = (76 x 13.6 x 980) dyne/cm^2

T_2 = 273 + 40°C = 313° K

Now $\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$ or $\frac{(76\text{x}13.6+250)980\text{x}\left(\frac{4}{3}\right)\pi(0.18)^3}{T_1}=\frac{(76\text{x}13.6)\text{x}980\left(\frac{4}{3}\right)\pi(0.2)^3}{313}$

or T_1 = 283.37 K; $\therefore$ T_1 = 283.37 – 273 = 10.37°C

Illustration 7: P-V diagram of n moles of an ideal gas is as shown in figure. Find the maximum temperature between A and B. **(JEE ADVANCED)**

Sol: Find $\frac{dT}{dV}=0,\ \frac{d^2T}{dV^2}<0$ for T_{max} in T/V relation.

Procedure: For given number of moles of a gas, $T \propto PV$

Although $(PV)_A = (PV)_B$ or $T_A = T_B$, still it is not an isothermal process. Because in isothermal process P-V graph is a rectangular hyperbola while it is a straight line. So, to see the behaviour of temperature first we will find either T-V equation or T-P equation and from that equation we can judge how the temperature varies. From the graph first we will write P-V equation, then we will convert it either in T-V equation or in T-P equation.

From the graph the P-V equation can be written as,

$P=-\left(\frac{P_0}{V_0}\right)V+3P_0$ (y = -mx + c) or $PV=-\left(\frac{P_0}{V_0}\right)V^2+3P_0V$

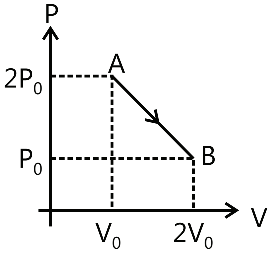


Figure 13.3

or $nRT = 3P_0V - \left(\frac{P_0}{V_0}\right)V^2$ (as PV = nRT) or $T = \frac{1}{nR}\left[3P_0V - \left(\frac{P_0}{V_0}\right)V^2\right]$

This is the required T-V equation. This is quadratic in V. Hence T-V graph is a parabola.

Now, to find maximum or minimum value of T we can substitute.

$$\frac{dT}{dV} = 0 \text{ or } \quad 3P_0 - \left(\frac{2P_0}{V_0}\right)V = 0; \quad \text{or} \quad V = \frac{3}{2}V_0$$

Further $\frac{d^2T}{dV^2}$ is negative at V = $\frac{3}{2}V_0$

Hence, T is maximum at V = $\frac{3}{2}V_0$ and this maximum value is,

$$T_{max} = \frac{1}{nR}\left[(3P_0)\left(\frac{3V_0}{2}\right) - \left(\frac{P_0}{V_0}\right)\left(\frac{3V_0}{2}\right)^2\right]; \quad \text{or} \quad T_{max} = \frac{9P_0V_0}{4nR}$$

Thus, T-V graph is as shown in figure

$$T_A = T_B = \frac{2P_0V_0}{nR} \quad \text{and} \quad T_{max} = \frac{9P_0V_0}{4nR} = 2.25\frac{P_0V_0}{nR}$$

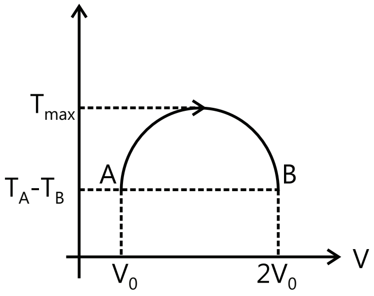


Figure 13.4

3. DEGREE OF FREEDOM (f)

Each independent mode by which a system can absorb energy (Kinetic + Potential) is called a degree of freedom. It is the number of independent motions possible or number of independent coordinates to specify the dynamic position of a body. A molecule can have translational K.E, rotational K.E and vibrational energy (potential, kinetic).

(a) Monatomic Gas: It has 3 degrees of freedom, since it can have translational motion in any direction in space. It can rotate but due to its small moment of inertia, rotational energy is neglected. f = 3

(b) Diatomic gas and linear polyatomic gas: It has 5 degrees of freedom (3 translational + 2 Rotational). Since, the moment of inertia about the axis joining the atoms is negligible, thus, it only has two rotational degrees of freedom. In all, f = 5.

At high temperature, its degree of freedom increases by 2 due to the vibrational energy (1 kinetic + 1 potential vibrational energy). Thus f = 7 at high temperature.

(c) Non-linear Polyatomic gas: It has 6 degrees of freedom (3 translational + 3 rotational). The one rotational degree of freedom which was neglected above, can't be neglected over here. Since, the moment of inertia about any of the three axes is not tending to zero i.e. f = 6.

At high temperature, its degree of freedom increases by 2 due to Vibrational energy

(1 kinetic + 1 potential) Thus, f = 8.

(d) Solid: Solids don't have any translational or rotational degrees of freedom. But at high temperature, it can vibrate along 3 axes. Thus it has 2 degrees of freedom per axis (i.e. potential + kinetic vibrational energy). Thus, at high temperature, solids have 6 degrees of freedom. i.e. f = 6

CONCEPTS

$f_{mix} = (n_1f_1 + n_2f_2 + \ldots\ldots) / (n_1 + n_2 + \ldots\ldots)$ is the equation for effective 'f' for a mixture of gases.

- Degrees of freedom of a diatomic and polyatomic gas depend on temperature and there is no clear cut demarcation line above which vibrational energy becomes significant. Moreover, this temperature varies from gas to gas. On the other hand for a monatomic gas, there is no such confusion. Degree of freedom here is 3 at all temperatures. Unless and until stated in the question you can take f = 3 for a monatomic gas, f = 5 for a diatomic gas and f = 6 for a non-linear polyatomic gas.
- When a diatomic or polyatomic gas dissociates into atoms it behaves as a monatomic gas, whose degrees of freedom are changed accordingly.

B Rajiv Reddy (JEE 2012 AIR 11)

4. INTERNAL ENERGY (U)

(a) In thermodynamics, the internal energy is the total energy contained by a thermodynamic system. It is the energy needed to create the system but excludes any energy due to external force fields (e.g. Internal energy does not include the energy due to the motion of the system as a whole. It further excludes any kinetic or potential energy the body may have because of its location in external gravitational, electrostatic, or electromagnetic fields.).

(b) Internal energy has two major components, kinetic energy (U_{kin}) and potential energy(U_{pot}). The kinetic energy is due to the motion of the system's particles (translations, rotations, vibrations), and the potential energy is associated with the static rest mass energy of the constituents of matter. $U = U_{pot} + U_{kin}$

(c) The internal energy of a system can be changed by heating the system or by doing work on it.

(d) It is an extensive quantity. The internal energy is a state function of a system, because its value depends only on the current state of the system and not on the path taken or process undergone to arrive at this state

(e) The SI unit of energy is the joule (J).

5. LAW OF EQUIPARTITION OF ENERGY

According to the law of equipartition of energy, the total energy of a molecule is equally distributed on the average among all the degrees of freedom. For an ideal gas at absolute temperature T, the energy is: $\frac{1}{2}kT$ per molecule per degree of freedom. k = Boltzmann's constant ; $\frac{1}{2}RT$ per mole, R = gas constant.

Thus, if f be the number of degrees of freedom, the internal energy of n moles of the gas will be $\frac{f}{2}nRT$. Thus, $U = \frac{f}{2}nRT$

6. MOLAR HEAT CAPACITIES (C)

(a) The amount of heat needed to raise the temperature of one mole of a substance by one degree Celsius. i.e.

$$C = \frac{\Delta Q}{n\Delta T} \Rightarrow \Delta Q = nC\Delta T$$

(i) S.I. unit is J/mol K

(ii) The value of C depends on the process through which its temperature is raised.

(b) **Molar heat Capacity at constant Pressure** C_P : The amount of heat needed to raise the temperature of one mole of a substance by one degree Celsius at constant pressure.

(c) **Molar heat Capacity at constant Volume** C_V : The amount of heat needed to raise the temperature of one mole of a substance by one degree Celsius at constant volume.

(i) $\gamma = \text{Ratio of } C_P \text{ to } C_v = \frac{C_P}{C_v}$; γ is also referred to as the adiabatic constant.

CONCEPTS

You Can Write:

$$C_{Vmix} = (n_1Cv_1 + n_2Cv_2 + \ldots\ldots\ldots) / (n_1 + n_2 + \ldots\ldots\ldots)$$

$$C_{Pmix} = (n_1C_{P1} + n_2C_{P2} + \ldots\ldots\ldots) / (n_1 + n_2 + \ldots\ldots\ldots)$$

$$f_{mix} = (n_1f_1 + n_2f_2 + \ldots\ldots\ldots) / (n_1 + n_2 + \ldots\ldots\ldots)$$

But $\gamma_{mix} \neq (n_1\gamma_1 + n_2\gamma_2 + \ldots\ldots\ldots) / (n_1 + n_2 + \ldots\ldots\ldots)$

$\gamma_{mix} = 1 + 2/f_{mix}$ (you will see it later that $\gamma = 1 + 2/f$) = C_{Pmix} / C_{vmix}

Anand K (JEE 2011 AIR 47)

7. ZEROTH LAW OF THERMODYNAMICS

If two bodies A and B are in thermal equilibrium and A and C are also in thermal equilibrium, then B and C are also in thermal equilibrium.

8. FIRST LAW OF THERMODYNAMICS

It is based on the law of conservation of energy. It states that

Heat supplied to the gas = Increment in internal energy + work done by the gas.

$$\Delta Q = \Delta U + \Delta W$$

In differential form, $dQ = dU + dW$

We will be using the following sign convention:

(a) For heat transfer: ΔQ is +ve for heat supplied;

ΔQ is -ve for heat rejected

(b) For Work Done: ΔW=+ve for work done by gas (in expansion of gas)

ΔW=-ve for work done on the gas (in contraction of gas)

We know:

Internal energy: Earlier we have seen that $U = \frac{n}{2}fRT$

Thus, change in internal energy $\Delta U = \frac{n}{2}fR\Delta T$

or in differential from, $dU = \frac{n}{2}fRdT$

Note: Change in Internal Energy is path independent whereas work done or heat energy released are path dependent. Internal Energy depends only on initial and final state of the system.

Illustration 8: Calculate the change in internal energy of 3.0 mol of helium gas when its temperature is increased by 2.0 K. **(JEE MAIN)**

Sol: Formulas based : f degree of freedom for monoatomic gas is 3.

Helium is a monatomic gas. Internal energy of n moles of the gas is,

$$U = \frac{3}{2}nRT\,; \qquad \therefore \quad \Delta U = \frac{3}{2}nR(\Delta T)$$

Substituting the values, $\Delta U = \left(\frac{3}{2}\right)(3)(8.31)(2.0) = 74.8\,J$

Heat transfer: From molar heat capacity, we saw that $\Delta Q = nC\Delta T$ or in differential form, dQ = nCdT

Work done: dW = PdV

For a finite change in volume from V_i to V_f, this equation is then integrated between V_i to V_f to find the net work.

$$W = \int dw = \int_{v_i}^{v_f} PdV$$

Now, there are two methods of finding work done by a gas.

Method 1: This is used when the P-V equation is known to us. Suppose P as a function of V is known to us. P = f(V) then work done can be found by, $W = \int_{v_i}^{v_f} f(V)dV$

Method 2: Since, work done is the integration of pressure w.r.t volume. So, it is clear that work represents the area under P-V graph. But, always take care of the sign of the work done. If the volume increases or in case of cyclic process if it is clockwise then +ve work is done by the gas or vice versa.

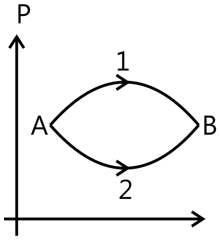


Figure 13.5

Illustration 9: A certain amount of an ideal gas passes from state A to B first by means of process 1, then by means of process 2. In which of the process is the amount of heat absorbed by the gas greater? **(JEE MAIN)**

Sol: ΔV is state function, remains constant. Hence work done directly proportional to heat obserbed.

$$Q_1 = W_1 + \Delta U_1 \text{ and } \qquad Q_2 = W_2 + \Delta U_2$$

U is a state function. Hence, ΔU depends only on the initial and final positions. Therefore, $\Delta U_1 = \Delta U_2$. But $W_1 > W_2$ as the area under 1 is greater than area under 2. Hence, $Q_1 > Q_2$

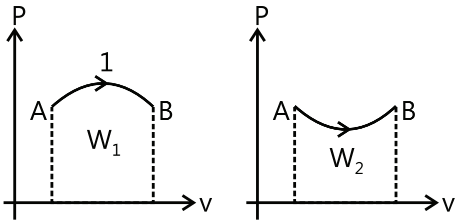


Figure 13.6

9. REVERSIBLE AND IRREVERSIBLE PROCESS

9.1 Reversible Process

A process in which the system and surroundings can be restored to the initial state from the final state without producing any change in the thermodynamic properties of the universe.

E.g. an infinitesimal compression of a gas in a cylinder assuming frictionless surfaces.

9.2 Irreversible Process

In irreversible processes, the system is not in equilibrium throughout the process. The initial state can't be obtained from the final state without producing changes in the universe.

E.g. heat transfer through a finite temperature difference.

10. DIFFERENT THERMODYNAMIC PROCESSES

10.1 Isochoric Process (V = Constant)

C_V is the molar heat capacity for constant volume.

Since $\Delta V = 0$. Therefore, $\Delta W = 0$;

We know that, $nC_v\Delta T = n\frac{f}{2}R\Delta T$ and $= n\frac{f}{2}R\Delta T$

By 1st Law of Thermodynamics: $\Delta Q = \Delta U + \Delta W$; $\Delta Q = \Delta U + 0$;

Thus $nC_v\Delta T = n\frac{f}{2}R\Delta T$; Therefore, $\Delta Q = \Delta U \Rightarrow C_v = \frac{f}{2}R$;

So in isochoric process we have $\Delta Q = \Delta U = nC_v\Delta T$; $\Delta W = 0$

Note: Since, ΔU is an extrinsic property(property that is not inherent). So, $\Delta U = nC_v\Delta T$ can be used for any process.

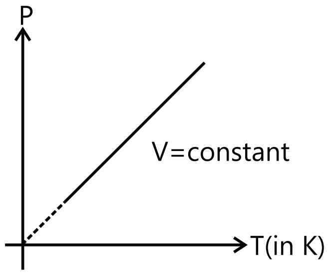


Figure 13.7

10.2 Isobaric Process (P = Constant)

Here $\Delta P = 0$; C_P is the molar heat capacity for this process.

Thus, $dQ = mC_PdT$

Since, dU is an extrinsic property, Thus, $dU = nC_VdT$

We know PV = nRT ; ⇒ PdV + VdP = nRdT ⇒ PdV = nRdT [∵ dP = 0]

And dW = PdV = nRdT

So, by using the 1st law of thermodynamics.

We get , dQ= dU + dW; $\Rightarrow nC_PdT = nC_VdT + nRdT$; $\Rightarrow C_P = C_V + R$

Thus, for the process, we have, $\Delta W = P(V_f - V_i) = nR(T_f - T_i)$; $\Delta Q = nC_P\Delta T$; $\Delta U = nC_v\Delta T$

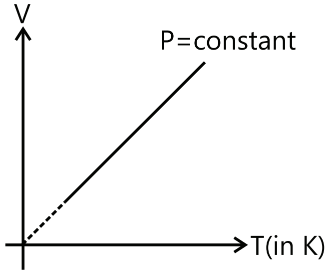


Figure 13.8

Note: We have seen, from above two process that,

$$C_V = \frac{f}{2}R \quad \text{and} \quad C_P = C_V + R \qquad \text{... (i)}$$

$$\text{Thus,} \quad C_P = \left(\frac{f}{2} + 1\right)R \qquad \text{...by (i)}$$

$$\text{We know, } \gamma = \frac{C_P}{C_V} \;;\; \Rightarrow \; \gamma = \frac{\left(\frac{f}{2}+1\right)R}{\left(\frac{f}{2}\right)R} \;;\; \Rightarrow \gamma = 1 + \frac{2}{f} \text{ Thus, } C_V = R/(\gamma - 1) \text{ And } C_P = \gamma R/(\gamma - 1)$$

Thus, for different degrees of freedom (i.e. for different gases) we have:

Types of Gas	**Degrees of freedom**	**Total Energy**	**C_P**	**C_V**	γ
Monatomic	3	$\frac{3}{2}RT$	$\frac{5}{2}R$	$\frac{3}{2}R$	1.67
Diatomic	5	$\frac{5}{2}RT$	$\frac{7}{2}R$	$\frac{5}{2}R$	1.4
Polyatomic(≥ 3)	6	3RT	4R	3R	1.33

Illustration 10: A cylindrical vessel of 28cm diameter contains 20g of nitrogen compressed by a piston supporting a weight of 75kg. The temperature of the gas is 17°C. What work will the gas do, if it is heated to a temperature of 250°C? What amount of heat should be supplied? To what distance will the weight be raised? The process should be assumed to be isobaric; the heating of the vessel and the external pressure are negligible. **(JEE ADVANCED)**

Sol: Use isobaric process equations .

$$\Delta W \text{ (work done)} = \int P dV = P\Delta V = P\times\frac{m}{M}\frac{R\Delta T}{P} = \frac{m}{M}R\Delta T \therefore \Delta W = \frac{20}{28}\times 8.3(250-17) = 1381J$$

$$\Delta Q = n\,C_p\,\Delta T = \frac{7}{2}nR\,\Delta T = 3.5\times\frac{20}{28}\times 8.3\times(250-17) = 4834.7J$$

$$h = \frac{\Delta V}{A} = \frac{nR\Delta T}{P}\times\frac{1}{A}$$

$$P = \left(P_0 + \frac{mg}{A}\right)$$

$$\Rightarrow h = \frac{20\times 8.3\times(250-17)}{28\times 75\times 9.8 + (10^5\times\frac{22}{7}\times 14^2\times 10^{-4})} \Rightarrow h = 0.2\text{ m} = 20\text{cm}$$

10.3 Isothermal Process (T = Constant)

$\Delta T = 0$; $\Delta U = nC_V\Delta T = 0$; $\Rightarrow$ $\Delta W = \Delta Q$;

We know, PV = nRT; $\Rightarrow P = \frac{nRT}{V}$;

$$\therefore \Delta W = \int_{V_i}^{V_f}\frac{nRT}{V}dv = \left[nRT\ln V\right]_{V_i}^{V_f} = nRT\ln\frac{V_f}{V_i}$$

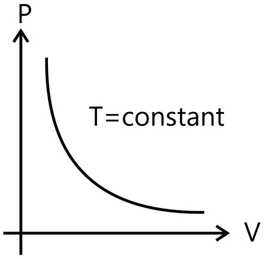


Figure 13.9

Slope of P-V graph: PV=nRT; $\Rightarrow$ (dP)V+P(dV)= nRdT

Since, dT =0;

Thus, (dP)V+P(dV)=0 ; $\Rightarrow$ dP/dV=- P/V

So, in an isothermal process we have: $\Delta U = 0$

$$\Delta Q = \Delta W = nRT\ln\frac{V_f}{V_i} = nRT\ln\frac{P_i}{P_f};\qquad \text{Slope (isothermal)} = -\frac{P}{V}$$

And C (molar heat capacity) tends to infinity since even when temperature change is zero, heat transfer is non-zero.

Illustration 11: Three moles of an ideal gas being initially at a temperature T_0 = 273K were isothermally expanded $\eta = 5.0$ time its initial volume and then isochorically heated so that the pressure in the final state became equal to that in the initial state. The total amount of heat transferred to the gas during the process equals Q = 80 kJ. Find the ratio $\gamma = C_P / C_V$ for this gas. **(JEE ADVANCED)**

Sol: Try draw the process on P/V graph, during isothermal process, the change in internal energy is zero while in isochoric process work dne is zero. Take $C_v = \frac{R}{y-1}$ and proceed to find y.

Procedure:

(1) Find heat expression (Q_1) given in isothermal process

(2) Find heat expression (Q_2) given in isochoric process

(3) Find total heat expression ($Q=Q_1+Q_2$)

(4) Put the value of Q given in question in the expression obtained above

In Isothermal process, the heat transferred to the gas is given by

$$Q_1 = nRT_0 \ln(V_2/V_1) = nRT_0 \ln\eta \qquad \text{... (i)}$$

$$\left[\therefore \eta = (V_2/V_1) = (P_1/P_2)\right]$$

In isochoric process, $Q_2 = \Delta U$ (W=0)

$$\therefore Q_2 = nC_v\Delta T = n\{R/(\gamma-1)\}\Delta T \qquad \text{... (ii)}$$

Now $\frac{P_2}{P_1} = \frac{T_0}{T}$ or $T = T_0\left(\frac{P_1}{P_2}\right) = \eta T_0$ ... (iii)

$$\therefore \quad \Delta T = \eta T_0 - T_0 = (\eta - 1)T_0$$

Substituting the value of ΔT from equation (iii) in equation (ii), we get

$$Q_2 = n\left(\frac{R}{\gamma-1}\right)(\eta-1)T_0; \qquad \therefore \quad Q = nRT_0\ln\eta + n\left(\frac{R}{\gamma-1}\right)(n-1)T_0$$

or $\frac{Q}{nRT_0} - \ln\eta = \left(\frac{\eta-1}{\gamma-1}\right)$; or $\gamma - 1 = \frac{\eta-1}{\frac{Q}{nRT_0} - \ln\eta}$ $\therefore \gamma = 1 + \frac{\eta-1}{\frac{Q}{nRT_0} - \ln\eta}$

Substituting given values, we get $\gamma = 1 + \frac{(5-1)}{\frac{80\text{x}10^2}{3\text{x}8.3\text{x}273} - \ln 5}$; Solving, we get $\gamma = 1.4$

10.4 Adiabatic Process (ΔQ = 0)

For adiabatic process: $\Delta Q = 0$; $\Delta U = -\Delta W$

$$nC_V dT = -Pdv \qquad \text{... (i)}$$

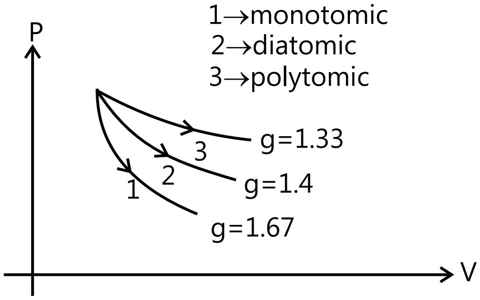
By equation of gas: $PV = nRT$; $\Rightarrow Pdv + Vdp = nRdT$;

$$\therefore ndT = \frac{Pdv + Vdp}{R}$$

$\therefore$ Substituting in (i) we get: $C_v\left(\frac{Pdv + Vdp}{R}\right) = -Pdv$

$$\Rightarrow (C_v + R)Pdv = -C_v\,Vdp; \Rightarrow \frac{C_P d_V}{C_V V} = \frac{-dp}{P} \qquad \left[C_V + R = C_P\right];$$

$$\Rightarrow \gamma\frac{dV}{V} = -\frac{dP}{P}$$



Adiabatic expansion of mono, dia and polyatomic gases

Figure 13.10

On integrating both sides we get,

$$\Rightarrow \gamma\int\int\frac{dv}{V} = -\int_{P_i}^{P_f}\frac{dp}{P}; \Rightarrow \gamma\ln\frac{V_f}{V_i} = \ln\frac{P_i}{P_f}$$

$$\Rightarrow \frac{P_i}{P_f} = \left(\frac{V_f}{V_i}\right)^{\gamma}; \qquad \text{(say)}$$

$$\therefore PV^{\gamma} = \text{Const}$$

The relation found above can be written in many different ways using ideal gas equation (PV=nRT)

$TV^{\gamma-1}$ = constant ; $T^{\gamma}P^{1-\gamma}$ = constant

$$PV^{\gamma} = \text{Const} = K \Rightarrow P = \frac{k}{V^{\gamma}};\ \therefore \Delta W = \int_{V_i}^{V_f} Pdv = \int_{V_i}^{V_f} \frac{k}{V^{\gamma}}dv$$

$$= \frac{KV_f^{1-\gamma} - KV_i^{1-\gamma}}{1-\gamma} = \frac{(P_fV_f^{\gamma})V_f^{1-\gamma} - (P_iV_i^{\gamma})V_i^{1-\gamma}}{1-\gamma}$$

$$\Rightarrow \Delta W = \frac{P_fV_f - P_iV_i}{1-\gamma} = \frac{nR}{1-\gamma}(T_f - T_i)$$

Slope of P-V graph:

We know, PV^{γ} = constant

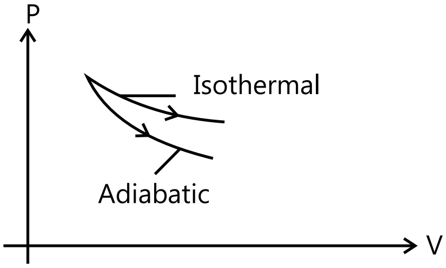
$$\frac{dp}{dV} = \frac{d}{dV}\left(\frac{\text{constant}}{V^{\gamma}}\right) = -\gamma\left(\frac{P}{V}\right);\ \text{Thus, } (\text{Slope})_{\text{adiabatic}} = -\gamma\left(\frac{P}{V}\right)$$

Because $\gamma > 1$, the isothermal curve is not as steep as that for the adiabatic expansion.

So, in adiabatic process we have: $\Delta Q = 0$

$$W = \frac{P_fV_f - P_iV_i}{1-\gamma} = \frac{nR}{1-\gamma}(T_f - T_i)\ ;\ \Delta U = -\Delta W$$

$$(\text{Slope})_{\text{adiabatic}} = -\gamma\left(\frac{P}{V}\right)$$



Adiabatic and isothermal expansion of an idea gas

Figure 13.11

And C (molar heat capacity) is zero, since even when temperature change is not zero, heat transfer is zero.

Illustration 12: An ideal monatomic gas at 300K expands adiabatically to twice its volume. What is the final temperature? **(JEE MAIN)**

Sol: Adiabatic process PV^{γ} = constant or $TV^{\gamma-1}$ = constant.

For an ideal monatomic gas, $\gamma = \frac{5}{3}$

In an adiabatic process, $TV^{\gamma-1} = \text{constant};\ \therefore\ T_fV_f^{\gamma-1} = T_iV_i^{\gamma-1}$

or $$T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = (300)\left(\frac{1}{2}\right)^{\frac{5}{3}-1} = 189K$$

Illustration 13: An ideal gas having initial pressure P, volume V and temperature T is allowed to expand adiabatically until its volume becomes 5.66 V while its temperature falls to T/2.

(a) How many degrees of freedom do the gas molecules have?

(b) Obtain the work done by the gas during the expansion as a function of the initial pressure P and volume V. **(JEE ADVANCED)**

Sol: Apply $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ and hence $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$

$$\omega = \frac{nR}{1-\gamma}\left(\frac{T}{2} - T\right) = \frac{nRT}{2(\gamma-1)} = \frac{nRT}{2\times 0.4} = \frac{PV}{0.8}$$

(a) For adiabatic change,

$$TV^{\gamma-1} = T'(V')^{\gamma-1}\ ;\ T' = T/2, V' = 5.66V$$

$$TV^{\gamma-1} = (T/2)x(5.66\,V)^{\gamma-1};\ (5.66)^{\gamma-1} = 2$$

It shows that the gas is diatomic for which the gas molecules have five degrees of freedom.

(b) Work done by the gas during adiabatic expansion, W, is given by,

$$W = \frac{PV - P'V'}{\gamma - 1}; \text{As } \frac{P'V'}{T} = \frac{PV}{T},\ \ P' = \frac{PVT'}{V'T}$$

$$P' = \frac{1}{2}x\frac{1}{5.66}xP = \frac{P}{11.32};\therefore\ W = \frac{1}{0.4}\left(PV - \frac{P}{11.32}x5.66V\right) = \frac{PV}{0.8} = \ 1.25PV.$$

10.5 Cyclic Process

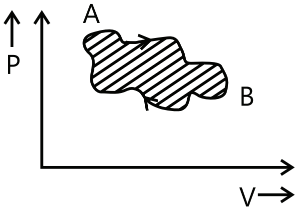


Figure 13.12

In a cyclic process, the thermodynamic process ends at the initial state.

Since, initial and final states are same, there,

$\therefore \Delta Q = \Delta W\ ;\ \therefore \left|\Delta Q\right| = \left|\Delta W\right| =$ Area enclosed by the curve.

Sign of ΔW is '+' if process is clockwise and '-' if process is anti-clockwise.

10.6 Polytropic Process

For a process: **PV^x = constant** where $x \neq 1$, also called a polytropic process.

$$\Delta W = (P_f V_f - P_i V_i)/1 - x = nR\,\Delta T/1 - x\,; \qquad C = C_v + \frac{R}{1-x} = \frac{R}{\gamma-1} + \frac{R}{1-x}$$

Slope of P-V diagram (also known as Indicator diagram) at any point is $\frac{dP}{dV} = -x\frac{P}{V}$

11. FREE EXPANSION

In a free expansion, the gas is allowed to expand into a vacuum. This happens quickly, so there is no heat transferred. No work is done, because the gas does not displace anything. According to the First Law, this means that: $\Delta U = 0$

There is no change in internal energy, so the temperature stays the same.

On a PV diagram all you can do is plot the end-points. The process follows a path on the diagram that is not well-defined because the temperature is constant. The connection between the initial and final states is: $P_iV_i = P_fV_f$

Thus, ΔW=0; ΔQ=0; ΔU=0; T=const. ; thus, PV = constant

Illustration 14: An insulated container is divided into two equal portions. One portion contains an ideal gas at pressure P and temperature T, while the other portion is a perfect vacuum. If a hole is opened between the two portions, find the change in internal energy and temperature of the gas? **(JEE ADVANCED)**

Sol: No opposing force, hence work done is zero. As the system is thermally insulated, $\Delta Q = 0$

Further as here the gas is expanding against vacuum (surroundings) the process is called free expansion and for it, $\Delta W = \int Pdv = 0$ [as for vacuum P = 0]

So in accordance with first law of thermodynamics, i.e. $\Delta Q = \Delta U + \Delta W$, we have $0 = \Delta U + 0$,

i.e. $\Delta U = 0$ or U = constant

So in this problem internal energy of the gas remains constant, i.e. $\Delta U = 0$. Now as for an ideal gas $U = 3/2\mu RT$, i.e. $U \propto T$; so temperature of the gas will also remain constant, i.e. $\Delta T = 0$

CONCEPTS

(a) A non-conducting container with no external source of heat energy will always undergo any change adiabatically.

(b) A conducting (finitely conducting) container with no external source of heat energy will undergo change:

- **(i)** Isothermally if the process is slow
- **(ii)** Adiabatically if the process is fast

(c) An infinitely conducting container with no external source of heat energy will always undergo any change isothermally.

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Illustration 15: A gas undergoes a process such that $P \propto \frac{1}{T}$ If the molar heat capacity for this process is C= 33.24 J/mol-K, find the degree of freedom of the molecules of the gas. **(JEE ADVANCED)**

Sol: Heat capacity = amount of heat required for per unit change of temperature. $C = \frac{Q}{\Delta T}$

As $P \propto \frac{1}{T}$ or PT = constant .... (i)

We have for one mole an ideal gas PV=RT .... (ii)

From Eqs. (i) and (iii)

P^2V = constant or $PV^{1/2} = K$ (say) ... (iii)

From first law of thermodynamics, $\Delta Q = \Delta U + W$ or $C\Delta T = C_V \Delta T + W$

or $C = C_V + \frac{W}{\Delta T}$ ... (iv)

Here, $\Delta W = \int PdV = K\int_{V_f}^{V_f} V^{-1/2} dv$; $= \frac{P_f V_f - P_i V_i}{1-(1/2)} = \frac{R(T_f - T_i)}{1/2} = \frac{RT}{1/2}$; $\therefore$ $\frac{W}{\Delta T} = 2R$

Substituting in Eq. (iv), we have $C = C_v + 2R = \frac{R}{\gamma - 1} + 2R$

Substituting the values, $33.24 = R\left(\frac{1}{\gamma-1} + 2\right) = 8.31\left(\frac{1}{\gamma-1} + 2\right)$

Solving this we get $\gamma = 1.5$; Now, $\gamma = 1 + \frac{2}{F}$ or degree of freedom $F = \frac{2}{\gamma - 1} = \frac{2}{1.5 - 1} = 4$

Alternate Solution: In process PV^x =constant, molar heat capacity is given by $C = \frac{R}{\gamma - 1} + \frac{R}{1 - x}$

The given process is $PV^{1/2}$ = constant or $x = \frac{1}{2}$; $\therefore$ $C = \frac{R}{\gamma - 1} + \frac{R}{1 - \frac{1}{2}} = \frac{R}{\gamma - 1} + 2R$

Now, we may proceed in the similar manner.

Illustration 16: An ideal gas expands isothermally along AB and does 700 J of work.

(a) How much heat does the gas exchange along AB?

(b) The gas then expands adiabatically along BC and does 400 J of work. When the gas returns to A along CA, it exhausts 100J of heat to its surroundings. How much work is done on the gas along this path? **(JEE MAIN)**

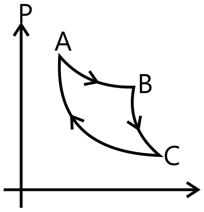


Figure 13.13

Sol: Apply $Q = \Delta U + W$ for the forces AB, BC & then CA separately. Remember ΔU is a state function.

(a) AB is an isothermal process. Hence, $\Delta U_{AB} = 0$ and $Q_{AB} = W_{AB} = 700J$

(b) BC is an adiabatic process. Hence, $Q_{BC} = 0$; $W_{BC} = 400J$ (given)

$\therefore \quad \Delta U_{BC=} - W_{BC} = -400J$

ABC is a cyclic process and internal energy is a state function. Therefore, $(\Delta U)_{\text{whole cycle}} = 0 = \Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA}$

and from first law of thermodynamics, $Q_{AB} + Q_{BC} + Q_{CA} = W_{AB} + W_{BC} + W_{CA}$

Substituting the values, $700 + 0 - 100 = 700 + 400 + \Delta W_{CA}$

$\therefore \quad \Delta W_{CA} = -500J$

Negative sign implies that work is done on the gas.

12. HEAT ENGINE & EFFICIENCY

Heat Engine is a device which converts the input heat energy into mechanical energy by using a cyclic process.

It absorbs a quantity of heat Q_1 from a source and performs an amount of work W and returns to the initial state after rejecting some heat Q_2 to a sink. The working substance which is gas or liquid undergoes a cyclic thermodynamic process. The source is at a higher temperature than the sink.

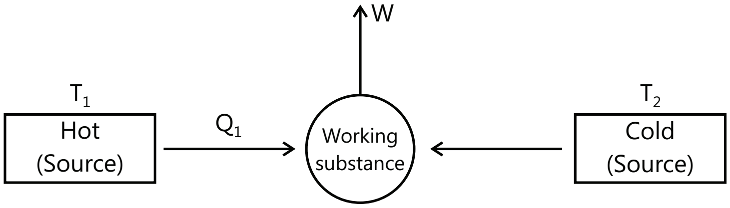


Figure 13.14

The efficiency η, of a heat engine is given by

$$\eta = \left(\frac{\begin{array}{c}\text{Work done by the working substance} \\ \text{(an ideal gas in our case) during a cycle}\end{array}}{\text{Heat supplied to the gas during the cycle}} \right) = \frac{W}{Q_1}$$

From conservation of energy. $Q_1 = W + Q_2$

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}; \quad \text{Thus,} \quad \therefore \quad \text{Efficiency}(\eta) = 1 - \frac{\text{Heat released}}{\text{Heat absorbed}}$$

Illustration 17: The P-V diagram of 0.2 mol of a diatomic ideal gas is shown in figure. Process BC is adiabatic. The value of γ for this gas is 1.4. **(JEE ADVANCED)**

(a) Find the pressure and volume at points A, B and C.

(b) Calculate ΔQ, ΔW and ΔU for each of the three processes.

(c) Find the thermal efficiency of the cycle.

Take 1 atm = 1.0×10^5 N/m^2.

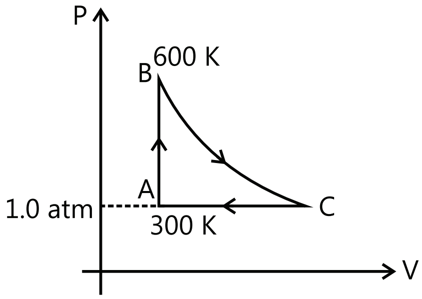


Figure 13.15

Sol: Try to start from A where P_A and T_A are known.

(a) $P_A = P_C = 1$ atm $= 1.01 \times 10^5$ N/m^2

Process AB is an isochoric process.

$$\therefore \quad P \propto T \quad \text{or} \quad \frac{P_B}{P_A} = \frac{T_B}{T_A};$$

$$\therefore \quad P_B = \left(\frac{T_B}{T_A}\right)P_A\left(\frac{600}{300}\right)(1\text{atm}) = 2\,\text{atm} = 2.02 \times 10^5 \text{ N/m}^2$$

From ideal gas equation $V = \frac{nRT}{P}$

$$\therefore \quad V_A = V_B = \frac{nRT_A}{P_A} = \frac{(0.2)(8.31)(300)}{(1.01 \times 10^5)} \approx 5.0 \times 10^{-3}\text{m}^3; = 5 \text{ litre}$$

$$V_C = \frac{nRT_C}{P_C} = \frac{(0.2)(8.31)(455)}{(1.01 \times 10^5)} = 7.6 \times 10^{-3}\text{m}^3 \approx 7.6 \text{ litre}$$

A	1 atm	5 lt
B	2 atm	5 lt
C	1 atm	7.6 lt

(b) **Process AB** is an isochoric process. Hence, $\Delta W_{AB} = 0$

$$\Delta Q_{AB} = \Delta U_{AB} = nC_v\Delta T = n\left(\frac{5}{2}R\right)(T_B - T_A) = (0.2)\left(\frac{5}{2}\right)(8.31)(600 - 300) \approx 1246\,\text{J}$$

Process BC is an adiabatic process. Hence, $\Delta Q_{BC} = 0$

$$\therefore \quad \Delta W_{BC} = -\Delta U_{BC}$$

$$\Delta U_{BC} = nC_v\Delta T = nC_V(T_C - T_B) = (0.2)\left(\frac{5}{2}R\right)(455 - 600) = (0.2)\left(\frac{5}{2}\right)(8.31)(-145)\,\text{J} \approx -602\text{J}$$

$$\therefore \quad \Delta W_{BC} = -U_{BC} = 602\text{J}$$

Process CA is an isobaric process. Hence,

$$\Delta Q_{CA} = nC_P\Delta T = n\left(\frac{7}{2}R\right)(T_A - T_C) = (0.2)\left(\frac{7}{2}\right)(8.31)(300 - 455)$$

12.1 Types of Heat Engine

(a) **External combustion engine:** In which heat is produced by burning the fuel in a chamber outside the main body (working substance) of the engine. Steam engine is an external combustion engine. The thermal efficiency of a steam engine varies from 10 to 20%.

(b) **Internal combustion engine:** In which heat is produced by burning the fuel inside the main body of the engine. Petrol engine and diesel engines are internal combustion engine. Here, we will just discuss about only one heat engine i.e. Carnot engine.

12.2 Carnot Engine

Carnot Cycle is the most efficient heat engine which undergoes a cycle of two isothermal and two adiabatic processes. It absorbs heat Q_1 by expanding isothermally at T_1 along AB and then expands adiabatically along BC to temperature T_2. It is compressed isothermally at T_2 rejecting heat Q_2 along CD and is compressed adiabatically along DA. Its efficiency η is given by:

$$\eta = 1 - \frac{T_2}{T_1}$$

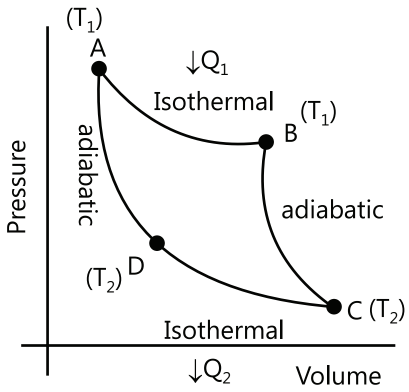


Figure 13.16

Derivation of efficiency in Carnot engine:

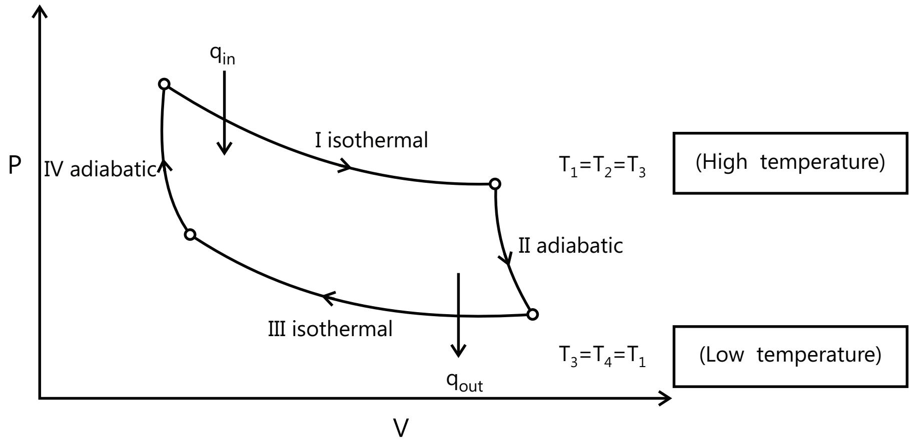


Figure 13.17: P-V diagram of the carnot cycle

CONCEPTS

Try to prove that carnot engine is the most efficient one among all engines working in the same temperature range. You will come across a nice use of the second law of thermodynamics!

GV Abhinav (JEE 2012 AIR 329)

Work, heat, Δ U. and, Δ H($nc_p\Delta T$) in the P-V diagram of the Carnot Cycle.

Process	**w**	**q**	**ΔU**	**ΔH**
I	$-nRT_h \ln\left(\frac{V_2}{V_1}\right)$	$nRT_h \ln\left(\frac{V_2}{V_1}\right)$	0	0
II	$-n\bar{C}_v(T_l - T_h)$	0	$n\bar{C}_v(T_l - T_h)$	$n\bar{C}_p(T_l - T_h)$
III	$-nRT_l \ln\left(\frac{V_4}{V_3}\right)$	$nRT_l \ln\left(\frac{V_4}{V_3}\right)$	0	0

Process	**w**	**q**	**ΔU**	**ΔH**
IV	$-n\bar{C}_v(T_h - T_l)$	0	$n\bar{C}_v(T_h - T_l)$	$n\bar{C}_p(T_h - T_l)$
Cycle	$-nRT_h \ln\left(\frac{V_2}{V_1}\right) - nRT_l \ln\left(\frac{V_4}{V_3}\right)$	$nRT_h \ln\left(\frac{V_2}{V_1}\right) + nRT_l \ln\left(\frac{V_4}{V_3}\right)$	0	0

The efficiency of the Carnot engine is defined as the ratio of the energy output to the energy input.

$$\text{Efficiency} = \frac{\text{Net work done by heat engine}}{\text{Heat absorbed by heat engine}} = \frac{-\text{Wsys}}{q_h} = \frac{nRT_h \ln\left(\frac{V_2}{V_1}\right) + nRT_l\left(\frac{V_4}{V_3}\right)}{nRT_h \ln\left(\frac{V_2}{V_1}\right)}$$

Since processes II (2-3) and IV (4-1) are adiabatic. $\left(\frac{T_2}{T_3}\right)^{C_V/R} = \frac{V_3}{V_2}$, and $\left(\frac{T_1}{T_4}\right)^{C_V/R} = \frac{V_4}{V_1}$

and since $T_1 = T_2$ and $T_3 = T_4$. $\frac{V_3}{V_4} = \frac{V_2}{V_1}$

$$\text{Therefore Efficiency} = \frac{nRT_h \ln\left(\frac{V_2}{V_1}\right) - nRT_l \ln\left(\frac{V_2}{V_1}\right)}{nRT_h \ln\left(\frac{V_2}{V_1}\right)}; \quad \boxed{\text{Efficiency} = \frac{T_h - T_l}{T_h} = 1 - T_l / T_h}$$

CONCEPTS

- By the second law of thermodynamics we can say that, it is impossible to construct an engine, operating in a cycle, which will extract heat from a source and convert the whole of it into work. It implies that it is impossible to make heat flow from a colder body to a hotter body without the help of an external agency or an engine from outside.
- That efficiency of Carnot engine is maximum (not 100%) for given temperatures T_1 and T_2. But still Carnot is not a practical engine because many ideal situations have been assumed while designing this engine which can practically not be obtained.

Anurag Saraf (JEE 2011 AIR 226)

Illustration 18: Carnot's engine takes in a thousand kilo calories of heat from a reservoir at 827°C and exhausts it to a sink at 27°C. How much work does it perform? What is the efficiency of the engine? **(JEE MAIN)**

Sol: $\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$ and for cannot cycle $\eta = 1 - \frac{T_2}{T_1}$, hence $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

Given, $Q_1 = 10^6\,\text{Cal}$ $T_1 = (827 + 273) = 1100\text{K}$ and $T_2 = (27 + 273) = 300\text{K}$

as, $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$ $\therefore$ $Q_2 = \frac{T_2}{T_1} Q_1 = \left(\frac{300}{1100}\right)(10^6)$ = 2.72×10^5 cal

Efficiency of the cycle, $\eta = \left(1 - \frac{T_2}{T_1}\right) x100$ or $\eta = \left(1 - \frac{300}{1100}\right) x100 = 72.72\%$

PROBLEM-SOLVING TACTICS

(a) Most of the problems of T_{max}, P_{max} and V_{max} are solved by differentiating. Sometimes the graph will be given and sometimes, the equation will be given. For problems of finding P_{max} , you will require either the P-V or P-T equation.

(b) From the sections of thermodynamic, graphical questions are usually asked so one must have a thorough understanding of PV diagrams at least. It is generally advised to convert other graphs namely TV and PT graph to PV graph before solving to visualize it easily. Also in many cases we can directly get asked quantities like work done because it involves integration of area under the PV graph.

(c) Silly mistakes must be avoided because there is not much variety in problems and not so much to think. Remember the following points:-

  (i) Be very careful in signs of Q and W while solving questions of thermodynamics.

  (ii) Be careful in noting the type of gas used in question whether it is monatomic or diatomic.

  (iii) Be careful with units of given quantities and asked quantities.

  (iv) You can list more from your personal experience.

(d) It would be helpful to note keywords in a question (like noting down slow and fast changes because slow changes imply reversible change whereas fast change imply irreversible change).

(e) It is always helpful to make a separate list of known quantities and asked quantities and then think of a way, a link between them.

(f) Questions are mostly based on various reversible or irreversible processes which have their own set of formulae. And no more processes can be found other than these. Hence, these concepts must be strong and you should know the algorithm of each one thoroughly.

FORMULAE SHEET

Kinetic Theory of Gases

$$PV = \frac{1}{3} nM_m V_{rms}{}^2 = nRT$$

Kinetic energy per mole of gas = $E = \frac{1}{2} M_m V_{rms}{}^2 = \frac{3}{2} RT$

$$V_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}}$$

$$V_{av} = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8P}{\pi \rho}}$$

M is the molar mass in kilogram per mole.

$$V_{mp} = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2P}{\rho}}$$

$$V_{rms} : V_{av} : V_{mps} :: \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2};$$

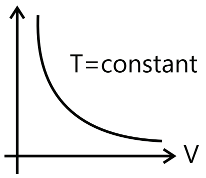
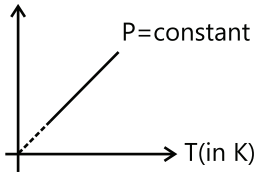
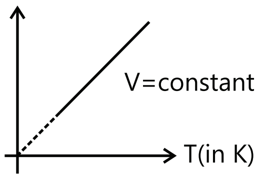
$$V_{mps} : V_{rms} : V_{av} :: 1 : 1.224 : 1.28$$

The mean free path of a gas molecule is the average distance between two successive collisions. It is represented by λ.

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 \rho}$$

Here, σ = Diameter of the molecule
k = Boltzmann's constant

Thermodynamics

(a)	**Boyle's law:** According to this law, for a given mas of a gas the volume of a gas at constant temperature (called isothermal process) is inversely proportional to its pressure, i.e., $V \propto \frac{1}{P}$ (T= constant) or PV = constant	 **Figure 13.18**
(b)	**Charle's law:** According to this law, for a given mass of a gas, the volume of a gas at constant pressure (called isobaric process) is directly proportional to its absolute temperature, i.e., $V \propto T$ (P= constant) or $\frac{V}{T} = \text{constant}$	 **Figure 13.19**
(c)	**Gay Lussac's law or Pressure law:** According to this law, for a given mass of a gas the pressure of a gas at constant volume (called isochoric process) is directly proportional to its absolute temperature, i.e., $P \propto T$ (V= constant) or $\frac{P}{T} = \text{constant}$	 **Figure 13.20**
(d)	**Avogadro's law:** According to this law, at same temperature and pressure equal volumes of all gases contain equal number of molecules.	

Ideal gas equation	$PV = nRT$ OR $\rho = \frac{PM}{RT}$
Degree of freedom of monoatomic gas	f = 3 (all translational)

Degree of freedom of a diatomic and linear polyatomic gas	f = 5 (3 translational + 2 rotational) at room temperatures and f = 7 (3 translational + 2 rotational + 2 vibrational) at high temperatures.
Degree of freedom of nonlinear polyatomic gas	f = 6 (3 translational + 3 rotational) at room temperatures and f = 8 (3 translational + 3 rotational + 2 vibrational) at high temperatures.
Degree of freedom of solid	An atom in a solid has no degrees of freedom for translational and rotational motion. At high temperatures due to vibration along 3 axes it has 3 x 2 = 6 degrees of freedom. f = 6 (all vibrational) at high temperatures.

$$C = \frac{\Delta Q}{n\Delta T} \quad \text{or} \quad \Delta Q = nC\Delta T$$

For a gas the value of C depends on the process through which its temperature is raised.

Types of gas	**Degrees of freedom**	**Total energy**	C_P	C_V	γ
Monoatomic	3	$\frac{3}{2}RT$	$\frac{5}{2}R$	$\frac{3}{2}R$	1.67
Diatomic	5	$\frac{5}{2}RT$	$\frac{7}{2}R$	$\frac{5}{2}R$	1.4
Polyatomic(≥ 3)	6	3RT	4R	3R	1.33

$$\text{Ratio of specific heat :- } \gamma = \frac{C_P}{C_V} = \frac{f+2}{f} \begin{cases} \xrightarrow{\text{monoatomic}} 5/3 = 1.67 \\ \xrightarrow{\text{diatomic}} 7/5 = 1.4 \\ \xrightarrow{\text{polyatomic}} 4/3 = 1.33 \end{cases}$$

Note: C of a gas depends on the process of that gas, which can be infinite in types.

and $f = \frac{2}{\gamma - 1}$; $C_v = \frac{R}{\gamma - 1}$; $C_P = \frac{\gamma R}{\gamma - 1}$

Often the first law must be used in its differential form, which is dU = dQ – dW

This can also be written as dQ = dU + dW

Δ Q is +ve for heat supplied

Δ Q is -ve for heat rejected

1. Δ W = +ve for work done by the gas (in expansion of gas)

   Δ W = -ve for work done on the gas (in contraction of gas)

2. For change in internal energy of the gas, apply $\Delta U = nC_v\Delta T$ or in differential form, $dU = nC_v dt = \frac{f}{2}nRdT$

3. For heat transfer, apply $Q = nC\Delta T$

   or in differential form $dQ = nCdT$

   $dW = PdV$

$$W = \int dW = \int_{V_i}^{V_f} PdV \quad = \text{Area under P-V curve}$$

Isothermal	**Q = W**	**0**	$nRT\ln\left(\frac{V_f}{V_i}\right) = nRT\ln\left(\frac{P_i}{P_f}\right)$
Adiabatic	0	$nC_V\Delta T$	$\frac{P_iV_i - P_fV_f}{\gamma-1} = -\Delta = \frac{nR\Delta T}{\gamma-1}$
Isobaric	$nC_P\Delta T$	$nC_V\Delta T$	$P(V_f - V_i)$
Isochoric	$Q = \Delta U$ $= nC_V\Delta T$	$nC_V\Delta T$	0

$(\text{Slope})_{\text{isothermal}} = -\frac{P}{V}$ $\qquad$ $(\text{Slope})_{\text{adiabatic}} = -\gamma\left(\frac{P}{V}\right)$

$$\eta = \left(\frac{\begin{array}{c}\text{Work done by the working substance}\\ \text{(an ideal gas in our case) during a cycle}\end{array}}{\text{Heat supplied to the gas during the cycle}}\right) \times 100 = \frac{W_{\text{Total}}}{Q_{+ve}} \times 100 = \left\{1 - \left|\frac{Q_{-ve}}{Q_{+ve}}\right|\right\} \times 100$$

For Carnot Engine: $\eta = 1 - \frac{T_2}{T_1}$

Solved Examples

JEE Main/Boards

Example 1: An electric bulb of volume 250cm^3 was sealed off during manufacture at the pressure of 10^{-3}mm of Hg at 27°C. Find the number of air molecules in the bulb.

Sol: $PV = \frac{N}{A}RT = N\frac{R}{A}T = NkT$

Let N be the number of air molecules in the bulb.

$V_1 = 250\text{cm}^3$, $P_1 = 10^{-3}$mm of Hg,

$T_1 = 273 + 27 = 300°\text{K}$

As $P_1V_1 = NkT$ where k is constant, then

$10^{-3} \text{x } 250 = \text{N.k.}300$ ... (i)

At N.T.P., one mole of air occupies a volume of 22.4 litre,

$V_0 = 22400\,\text{cm}^3, P_0 = 760$ mm of Hg,

T = 273° K and $N_0 = 6 \times 10^{23}$ molecules

$\therefore 760 \times 22400 = 6\text{x}10^{23} \times k \times 273$ ... (ii)

Dividing equation (i) by equation (ii), we get

$$\frac{10^{-3} \times 250}{760 \times 22400} = \frac{N \times 300}{6 \times 10^{23} \times 273}$$

$$N = \frac{10^{-3} \times 250 \times 6 \times 10^{23} \times 273}{760 \times 22400 \times 300} = 8.02\text{x}10^{15} \text{ mole}$$

Example 2: One gram-mole of oxygen at 27°C and one atmospheric pressure is enclosed in a vessel.

(a) Assuming the molecules to be moving with V_{rms}, find the number of collisions per second which molecules make with one square meter area of the vessel wall.

(b) The vessel is next thermally insulated and moved with a constant speed v_0. It is then suddenly stopped.

The process results in a rise of the temperature of the gas by 1°C. Calculate the speed v_0.

Sol: Formula based: $n = \frac{P}{kT}$ & $V_{rms} = \sqrt{\frac{3N_AkT}{M_n}}$. Recall the assumption of KTG. Kinetic energy changed to internal energy.

(a) n = P/kT

where k = Boltzmann constant =

1.38×10^{-23} J/mol/K

P = 1 atmosphere = 1.01×10^{5} n/m^2,

T = 27°C = 300°K

$$n=\frac{1.01x10^{5}}{1.38x10^{-23}x300}=2.44x10^{25}m^{-3}$$

The root mean square velocity V_{rms} is given by

$$V_{rms}=\sqrt{\left(\frac{3RT}{M_m}\right)}=\sqrt{\left(\frac{3N_AkT}{M_m}\right)} \quad (\because R=N_Ak)$$

N_A= Avogadro number = 6.02×10^{23} molecule/mole

M_m = 32gm = 32×10^{-3} kg.

$$V_{rms}=\sqrt{\left(\frac{36.02x10^{23}x1.38x10^{-23}x300}{32x10^{-3}}\right)}=483.4\,m/s.$$

Since each molecule may be moving at a given instant, along any of the six directions (i.e. $\pm$X, $\pm$Y and $\pm$Z) only (1/6)th of the total molecules contained in the volume along positive and negative x, y, z directions, move towards the wall. The number of collisions per second with one square meter area of the vessel wall will be

$$=\frac{1}{6}xn\times V_{rms}=\frac{1}{6}\times 2.44\times 10^{25}\times 483.4 = 1.97\times 10^{27}.$$

(b) K.E. $=\frac{1}{2}mv_0^{\,2}$

Heat energy gained = $C_v\Delta T=C_v\,x1=C_v$

$C_p-C_v=R$ or $C_v=\frac{R}{\gamma-1}$

$\therefore \gamma=1.41\ ;\ \frac{1}{2}mv_0^{\,2}=C_v=\frac{R}{\gamma-1}$

$$v_0=\sqrt{\left(\frac{2R}{m(\gamma-1)}\right)}=\sqrt{\left(\frac{2x8.31x1}{32x10^{-3}x141-1}\right)}=35.6\,m/sec.$$

Example 3: An ideal gas having initial pressure P, volume V and temperature T is allowed to expand adiabatically until its volume be-comes 5.66 V while its temperature fall to T/2.

(a) How many degrees of freedom do the gas molecules have?

(b) Obtain the work done by the gas during the expansion as a function of the initial pressure P and volume V.

Sol: (a) $y=\frac{\frac{f}{2}+1}{f/2}$ or $y=1+\frac{2}{f}$

(b) $W=\frac{nR}{1-\gamma}(T_f-T_i)$

(a) For adiabatic process

$TV^{\gamma-1}=T_1V_1^{\gamma-1};\ V_1=5.66V$ and $T_1=T/2$

$\therefore\quad TV^{\gamma-1}=\frac{T}{2}x(5.66V)^{\gamma-1};\ (5.66)^{\gamma-1}=2$

$(\gamma-1)\log 5.66=\log 2$

$\gamma-1=\frac{\log 2}{\log 5.66};\ \gamma-1=\frac{0.3010}{0.7528}=0.4$

$\therefore\quad \gamma=1.4\,(\text{diatomic gas})$

The degrees of freedom of gas molecules = 5

(b) Work done during adiabatic change, W, is given by

$W=\frac{1}{1-\gamma}[P_1V_1-PV]$

For an ideal gas equation,

$\frac{PV}{T}=\frac{P_1V_1}{T_1}$ or $P_1=\frac{T_1V}{TV_1}P\ ;\ P_1=\frac{1}{2}x\frac{1}{5.66}P=\frac{P}{11.32}$

$$W=\frac{1}{1-1.4}\left[\frac{P}{11.32}x5.66V-PV\right]$$

$$=\frac{PV}{0.4}\left[1-\frac{5.66}{1132}\right]=\frac{PV}{0.8}\ =1.25\ PV.$$

Example 4: Two moles of helium gas with γ equal to $\frac{5}{3}$ are initially at temperature 27°C and occupy a volume of 20 litres. The gas is first expanded at constant pressure until the volume is doubled. Then it undergoes an adiabatic change until the temperature returns to its initial value.

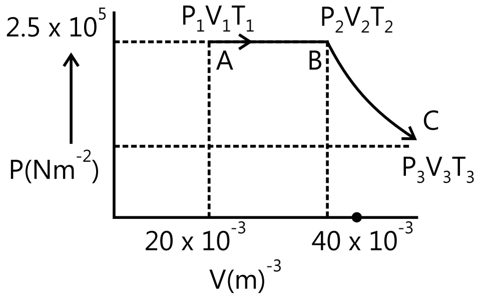
(a) Sketch the process on P-V diagram.

(b) Find the final value of volume and pressure of the gas.

(c) Find the work done by the gas under isobaric and adiabatic processes and total work done.

Sol: Relate P_1 V_1 and T_1 with P_2, V_2, T_2 and P_3, V_3, T_3. Calculate the work alone of process AB, BC separately.

(a) The sketch of P-V diagram is shown in the figure.



(b) At A, $V_1 = 20l = 20\times10^{-3}\,m^3$ $T_1 = 27°C = 300K$; $P_1 = ?$

For an ideal gas, PV = nRT.

$$P_1 = \frac{nRT_1}{V_1} = \frac{2\times8.31\times300}{20\times10^{-3}} = 2.5\times10^5\,Nm^{-2}$$

At B, $P_2 = P_1 = 2.5\times10^5\,Nm^{-2}$, $V_2 = 40\times10^{-3}\,m^3$

For isobaric process,

$$\frac{T_2}{T_1} = \frac{V_2}{V_1} = \frac{40\times10^{-3}}{20\times10^{-3}} = 2 \quad T_2 = 2T_1 = 2\times300 = 600K$$

The gas now undergoes adiabatic expansion from B to C.

$$T_3V_3^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\frac{V_3}{V_2} = \left(\frac{T_2}{T_3}\right)^{\frac{1}{\gamma-1}} = \left(\frac{600}{300}\right)^{1/(5/3-1)} = 2^{3/2} = 2\sqrt{2} = 2.83$$

Final Volume =

$V_3 = V_2 \times 2.82 = 40\times10^{-3} \times 2.83 = 113\times10^{-3}\,m^3$ V

Final pressure P_3 is given by $P_3V_3^{\gamma} = P_2V_2^{\gamma}$

$$P_3 = P_2 \times \left(\frac{V_2}{V_3}\right)^{\gamma} = 2.5\times10^5 \times \left(\frac{40\times10^{-3}}{113\times10^{-3}}\right)^{5/3}$$

$$= 2.5\times10^5 \times (0.353)^{5/3} = 2.5\times10^5 \times 0.176$$

$$= 4.410^5\,N.m^{-2}$$

(c) Work done during isobaric process along AB = P $(V_2 - V_1) = 2.5\times10^5 \times 20\times10^{-3} = 5000J$

Work done during adiabatic process along

$$BC = \frac{nR(T_2 - T_3)}{\gamma - 1} = \frac{2\times831\times(600-300)}{\frac{5}{3}-1}$$

Total work done = 5000 + 7479 = 11479 J.

Example 5: Two moles of helium gas undergo a cyclic process as shown in the figure.

Assuming the gas to be ideal, calculate the following quantities in this process.

(a) The net change in the heat energy

(b) The net work done

(c) The net change in internal energy

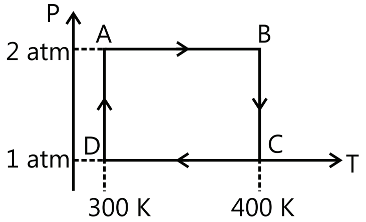
Sol: Calculate for isobaric and isothermal process only.

(a) AB is isobaric process. The work done during this process from A to B:

$$W_{AB} = P(V_2 - V_1) = nR(T_2 - T_1)$$

or $W_{AB} = 2\times8.3\times(400-300) = 1160$ joule

Work done during isothermal process from B to C:



$$W_{AB} = nRT_C\log_e(V_2/V_1) = nRT_C\log_e(P_2/P_1)$$

$$= 2\times8.3\times400\times2.303\log_{10}2$$

$$= 2\times8.3\times400\times2.303\times0.3010 = 4602.9 \text{ joule}$$

Work done during isobaric process from C to D:

$$W_{CD} = nR(T_D - T_C) = 2\times8.3\times(300-400) = -1660 \text{ joule}$$

Work done during isothermal process from D to A:

$$W_{DA} = mRT_D\log_e(P_D/P_A) = nRT_D\log_e 2$$

$$= 2\times8.03\times300\times2.0303\times0.3010 = -3452.2 \text{ Joule}$$

Net workdone = $W_{AB} + W_{BC} + W_{CD} + W_{DA}$

= 1660 + 4602.9 – 1660 – 3452.2

= 1150.7 joule

(b) First law of thermodynamics gives

$\Delta Q = \Delta U + \Delta W$

As $\Delta U = 0$, in cyclic process

$\therefore\ \Delta Q = \Delta W = 1150.7$ joule

The heat given to the system = 1150.7 joule

(c) As the gas returns to its original state, there is no change in internal energy.

Example 6: An ideal gas is taken a cyclic thermodynamic process through four steps. The amount of heat involved in these steps are $Q_1 = 5960\,J, Q_2 = -5585\,J, Q_3 = -2980\,J$ and $Q_4 = 3645\,J$ respectively. The corresponding works involved are $W_1 = 2200\,J$, $W_2 = -825\,J$, respectively.

(a) Find the value of W_4

(b) What is the efficiency of the cycle?

Sol: $Q_{Total} = W_{Total}$ as $\Delta U = 0$ in cyclic process,

$$\eta = \frac{W_T}{(Q \text{ absorbed})} \text{ only}$$

As the process is cyclic, $\Delta U=0$

Net heat absorbed by the system

$$Q = Q_1 + Q_2 + Q_3 + Q_4$$

$= 5960 - 5585 - 2980 + 3645 = 1040$ J

Net work performed

$$W = W_1 + W_2 + W_3 + W_4$$

$= 2200 - 825 - 1100 + W_4 = 275 + W_4$

According to the first law of thermodynamics

$Q = \Delta U + W$; $1040 = 0 + 275 + W_4$

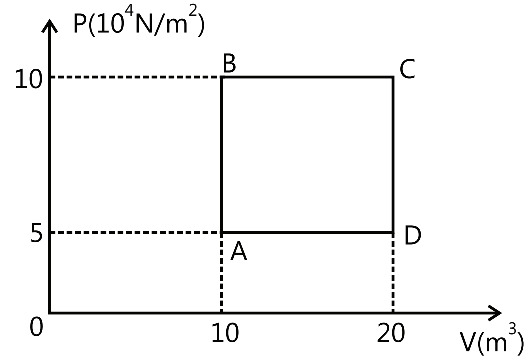
$\therefore\ W_4 = 1040 - 275 = 765$ joule.

$$\text{Efficiency } \eta = \frac{\text{Work done}(W)}{\text{Heat absorbed}(Q_1 + Q_4)}$$

$$= \frac{275 + 765}{5960 + 3645} = \frac{1040}{9605} = 0.1082$$

Percentage efficiency = 10.82%

Example 7: A sample of 2 kg of monoatomic Helium (assumed ideal) is taken through the process ABC and another sample of 2 kg of the same gas is taken through the process ADC as shown in the figure.



Given molecular mass of Helium = 4

(a) What is the temperature of Helium in each of the states A, B, C and D?

(b) Is there any way of telling afterwards which sample of Helium went through the process ABC and which went through the process ADC? Write Yes or No.

(c) How much is the heat involved in each of the processes ABC and ADC?

Sol: Work = Area under P–V curve hence, work done in ABC is more than in ADC so is the heat (Q). At C, system's thermodynamic states are same, it can't be determined how they are achieved.

(a) $n = \dfrac{2 x 10^3}{4} = 500$

At A, $P_A V_A = nRT_A$ or $T_A = (P_A V_A / nR)$

$$\therefore \quad T_A = \frac{(5x10^4)x10}{500x831} = 120.33K$$

$$\text{Similarly, } T_B = \frac{(10x10^4)x10}{500x831} = 240.66K$$

$$T_C = \frac{(10x10^4)x20}{500x831} = 481.32K$$

$$T_D = \frac{(5x10^4)x20}{500x831} = 240.66K$$

(b) No

(c) For process ABC:

Change in internal energy $\Delta U = nC_v \Delta T$

$$(\Delta U)_{ABC} \qquad = n\left(\frac{3}{2}R\right)\Delta T$$

$$= 500x\left(\frac{3}{2}x8.3\right)x[481.32 - 120.33] = 2.25x10^6 \text{ J}$$

Work done $(\Delta W)_{ABC} = 10$ x (10 x 10^4)

$= 10^6$ J $(\Delta Q)_{ABC} = (dU)_{ABC} + (\Delta W)_{ABC}$

$= 2.25x10^6 + 10^6 = 3.25x10^6$ J

For process ADC:

$(\Delta W)_{ADC}$ = 5 x 10^4 [20-10] = 0.5 x 10^6 J

$(\Delta U)_{ADC}$ = 2.25 x 10^6

$\therefore (\Delta Q)_{ADC}$ = (2.25 x 10^6) + (0.5 x 10^6)

= 2.75 x 10^6 J

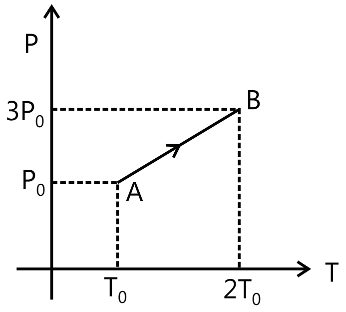
Example 8: Pressure versus temperature graph of an ideal gas is as shown in figure. Density of the gas at point A is ρ_0. Density at B would be

Sol: Use relation between density and temperature, given below.

$$\rho = \frac{PM}{RT} \propto \frac{P}{T}$$

Pressure

$$\left(\frac{P}{T}\right)_A = 2\left(\frac{P_0}{T_0}\right)$$



and $\left(\frac{P}{T}\right)_B = \frac{3}{2}\left(\frac{P_0}{T_0}\right) = \frac{3}{4}\left(\frac{P}{T}\right)_A$

$$\therefore \ \rho = \frac{3}{4}\rho_A = \frac{3}{4}\rho_0$$

Example 9: The root mean square (rms) speed of hydrogen molecules at a certain temperature is 300 m/s. If the temperature is doubled and hydrogen gas dissociates into atomic hydrogen the rms speed will become

Sol: Formula based $V_{rms} \propto \sqrt{T}$.

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

T is doubled and M is halved. Therefore, rms speed will become two times or 600 m/s.

Example 10: The changes in temperature of an ideal gas, when its volume changes from V to 2V in the process P = aV, is (Here a is a positive constant)

Sol: Use relation between P and V.

$P \propto V \qquad (P=aV)$

Therefore, pressure and volume both are doubled or temperature becomes four times ($T \propto PV$)

JEE Advanced/Boards

Example 1: A gaseous mixture enclosed in a vessel of volume V consists of one gram mole of a gas A with $\gamma(C_P/C_V) = 5/3$ and another has B with $\gamma = 7/5$ at a certain temperature T. The gram molecular weights of the gases A and B are 4 and 32 respectively. The gases A and B do not react with each other and are assumed to be ideal. The gaseous mixture follows the equation $PV^{19/13}$ = constant, in adiabatic processes.

(a) Find the number of gram moles of the gas B in the gaseous mixture.

(b) Compute the speed of sound in the gaseous mixture at T=300K.

(c) If T is raised by 1 K from 300 K, find the percentage change in the speed of sound in the gaseous mixture.

(d) The mixture is combined adiabatically to 1/5th of its initial volume V. Find the change in its adiabatic compressibility in terms of the given quantities.

Sol: γ of the mixture is known from equation of process. Therefore C_V can be known. Compare C_V (mixture)

$$= \frac{n_A C_{V_A} + n_B (C_V)_B}{(n_A + n_B)}$$

(a) As the gaseous mixture follows the equation $PV^{19/13}$ = constant, then for the mixture of the gas $\gamma = 19/13$.

$$\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_v} = \frac{19}{13}$$

$$\frac{R}{C_V} = \frac{6}{13}, \ \therefore \ C_V = \frac{13R}{6}$$

$$C_P = C_V + R = \frac{19}{6}R$$

For gas A, $\gamma_A = 5/3$; For gas B, $\gamma_B = 7/5$.

$$(C_V)_A = \frac{3}{2}R \text{ and} (C_V)_B = \frac{5}{2}R$$

$$(C_P)_A = \frac{5}{2}R \text{ and} (C_P)_B = \frac{7}{2}R$$

Let n_A and n_B be the number of kg moles in gas A and gas B respectively.

n_A =1gm mole = 10^{-3} kg.mole

As the gases have fixed volume, no work is done by the gas and vessel system. In the adiabatic process, no heat is exchanged with the surroundings, the internal energy of the system will remain constant.

$$\therefore \ (n_A + n_B) C_V \, dT = n_A (C_V)_A dT + n_B (C_V)_B dT$$

$$(n_A + n_B) C_V = n_A (C_V)_A + n_B (C_V) B$$

$$(1x10^{-3} + n_B)\left(\frac{13}{6}R\right) = (1x10^{-3})\frac{3}{2}R + n_B\left(\frac{5}{6}R\right)$$

$$13x10^{-3} + 13n_B = 9x10^{-3} + n_B(15)$$

$$2n_B = 4x10^{-3}$$

$$\therefore \quad n_B = \frac{4x10^{-3}}{2} = 2x10^{-3}\text{kgmole} = 2\text{gm mole}$$

(b) The speed of sound in gaseous mixture is given by

$$\nu = \sqrt{\left(\frac{\gamma RT}{M}\right)}$$

where R is gas constant and M is equivalent gram molecular weight of gaseous mixture. Let M_A and M_B be the gram molecular weights of gases A and B respectively, then

$$n_A M_A + n_B M_B = (n_A + n_B)M$$

$$\text{or} \quad M = \frac{n_A M_A + n_B M_B}{n_A n_B} = \frac{(1x4)+(2x32)}{1+2}$$

$$= \frac{68}{3}\text{gm} = \frac{68}{3}x10^{-3}\text{ kg}$$

$$\therefore \nu = \sqrt{\left[\left(\frac{19}{13}\right)x\frac{8.3x300x3}{68x10^{-3}}\right]} = 400.7\text{m / s}$$

(c) At temperature T, $\nu = \sqrt{\left(\frac{\gamma RT}{M}\right)}$

$$T' = (300+1)K, \nu' = \sqrt{\left(\frac{\gamma RT'}{M}\right)}$$

$$\therefore \quad \frac{\nu'}{\nu} = \sqrt{\left(\frac{T'}{T}\right)} = \sqrt{\left(\frac{301}{300}\right)} \text{or} \frac{\nu'}{\nu} - 1 = \left[\sqrt{\left(\frac{301}{300}\right)} - 1\right]$$

The percentage change in speed of sound = $\frac{\nu' - \nu}{\nu}x100$

$$= \left[\sqrt{\frac{301}{300}} - 1\right]x100 = 0.17\%$$

(d) Adiabatic compressibility =

$$\beta_{mt} = -\frac{1}{V}\left(\frac{dv}{dp}\right)_{mt} = -\frac{1}{\gamma P}$$

$$\Delta\beta = \frac{1}{\gamma P}\left[1 - \frac{1}{(5)^{\gamma}}\right] = \frac{13}{3x19}\frac{V}{RT}\left(1 - \frac{1}{(5)^{\gamma}}\right) = 2.487x10^{-3}.$$

Example 2: At 27°C, two moles of an ideal monoatomic gas occupy a volume V. The gas expands adiabatically to a volume 2V. Calculate

(a) The final temperature of the gas

(b) Change in its internal erergy

(c) The workdone by the gas during this process

Sol: Formula TV^{γ} = constant, $\Delta U = n\,C_v\,\Delta T \quad W = -\Delta U$ used.

(a) For adiabatic expansion

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}; \; 300x(V)^{\gamma-1} = T_2(2V)^{\gamma-1}$$

$$T_2 = 300x\frac{1}{(2)^{\gamma-1}} = \frac{300}{(2)^{2/3}} \quad (\because \gamma = 5/3)$$

$$\therefore \; T_2 = 188.99K$$

(b) Change in thermal energy is given by

$$\Delta U = nC_V(T_2 - T_1) = 2x(3R/2)(188.99 - 300)$$

$$= 2x\left(\frac{3x8.3}{2}\right)x(-111.09) = 2767.5\,J$$

The negative sign indicates that there is a decrease in the internal energy.

(c) For adiabatic process,

$$\Delta W + \Delta U = 0 \text{ or } \Delta W = -\Delta U$$

The workdone by the gas is given by

$$\Delta W = -\frac{nR(T_2 - T_1)}{\gamma - 1} = -\frac{2x8.3x(188.99 - 300)}{(5/3 - 1)}$$

$$= 2764.2\,J$$

Example 3: Two moles of an ideal monoatomic gas, initially at pressure p_1 and volume V_1 undergo an adiabatic compression until its volume is V_2. Then the gas is given heat Q at constant volume V_2.

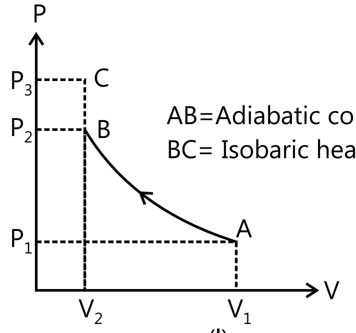
(a) Sketch the complete process on a P-V diagram.

(b) Find the total workdone by the gas, the total change in its internal energy and the final temperature of the gas.

[Give your answers in terms of p_1, V_1, V_2, Q and R].

Sol: Use formula $TV^{\gamma-1}$ = constant and $W = \frac{nR\Delta T}{1-\gamma}$

(a) P-V diagram is shown in the figure where AB is adiabatic compression and BC is isobaric heating



...(I)

(b) Let T_1 and T_2 be the initial temperature and the temperature after adiabatic compression respectively.

For adiabatic compression

$$V_1^{\gamma-1}T_1 = V_2^{\gamma-1}T_2$$

$$T_1 = \frac{p_1V_1}{2R}, \gamma = \frac{5}{3} \text{ for monoatomic gas}$$

$$\therefore\ V_1^{2/3}\left(\frac{p_1V_1}{2R}\right) = V_2^{2/3}T_2\ \therefore\ T_2\left(\frac{p_1V_1^{5/3}}{2RV_2^{2/3}}\right) \qquad \text{....(i)}$$

For isochoric process at temperature T_3, heat supplied is Q.

$$\therefore\ Q = nC_V dT\ ;\ Q = 2.3/2R(T_3 - T_2)$$

$$\frac{Q}{3R} = T_3 - T_2\,;\therefore\ T_3 = \frac{Q}{3R} + T_2 = \frac{Q}{3R} + \frac{p1V_1^{5/3}}{2RV_2^{2/3}} \qquad \text{.... (ii)}$$

The total work done by the gas is equal to the work done in adiabatic process plus the work done in isochoric process when $W_{BC} = 0$

$$\therefore\ W_{Total} = W_{AB} + W_{BC} = W_{AB}$$

$$W_{Total} = \frac{nR(T_1 - T_2)}{\gamma - 1} =$$

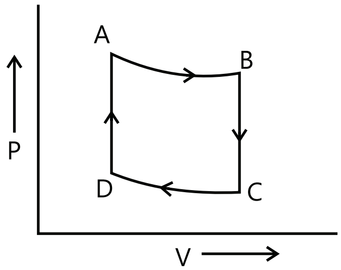
$$= \frac{2R}{2/3}\left[\frac{p_1V_1}{2R} - \frac{p_1v_1^{5/3}}{2RV_2^{2/3}}\right] = \frac{3}{2}p_1V_1\left[1 - \left(\frac{V_1}{V_2}\right)^{2/3}\right]$$

Change in internal energy $\Delta U = nC_V(T_3 - T_1)$

$$\therefore\ \Delta U = 2x\left(\frac{3}{2}R\right)\left[\frac{Q}{3R} + \frac{p_1V_1^{5/3}}{2RV_2^{2/3}} - \frac{p_1V_1}{2R}\right]$$

$$= Q + \frac{3}{2}\frac{p_1V_1^{5/3}}{V_2^{2/3}} - \frac{3}{2}p_1V_1 = Q + \frac{3}{2}p_1V_1\left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right]$$

Example 4: One mole of monoatomic ideal gas is taken through the cycle shown in figure.



$A \to B$	Adiabatic expansion
$B \to C$	Cooling at constant volume
$C \to D$	Adiabatic compression
$D \to A$	Heating at constant volume

The pressure and temperature at A, B etc., are denoted by $P_A, T_A; P_B, T_B$ etc. respectively.

Given $T_A = 1000K, P_B = (2/3)P_A \,\&\, P_C = (1/3)P_A$. Calculate

(a) The work done by the gas in the process $A \to B$

(b) The heat lost by the gas in the process $B \to C$ and

(c) Temperature T_D given (2/3) = 0.85 and R-8.31 J/mol K.

Sol: Use the relation for the respective processes. Such as T/P relation in adiabatic process.

(a) As for adiabatic change

$PV^\gamma = \text{constant}$

$$\text{i.e.}\quad P\left(\frac{nRT}{P}\right)^\gamma = \text{constant} \qquad [\text{as } PV = nRT]$$

$$\text{i.e.}\quad \frac{T^\gamma}{P^{\gamma-1}} = \text{constant}$$

$$\text{where } \gamma = \frac{5}{3}$$

$$\text{i.e.}\quad T_B = T_A\left(\frac{2}{3}\right)^{1-\frac{1}{\gamma}} = 1000\left(\frac{2}{3}\right)^{2/5} = 850K$$

$$\text{so}\quad W_{AB} = \frac{nR[T_i - T_f]}{\gamma - 1} = \frac{1x8.3[1000 - 850]}{[(5/3) - 1]}$$

i.e. $W_{AB} = (3/2)x8.31x150 = 1869.75J$

(b) For B → C, V = constant so $\Delta W = 0$

so from first law of thermodynamics

$\Delta Q = \Delta U + \Delta W = \mu C_V \Delta T + 0$

$$\text{or}\quad \Delta Q = 1x\left(\frac{3}{2}R\right)(T_C - 850) \qquad \text{as } C_V = \frac{3}{2}R$$

Now along path BC, V = constant; $P \propto T$

$$\text{i.e.} \frac{P_C}{P_B} = \frac{T_C}{T_B},\ T_C = \frac{(1/3)P_A}{(2/3)P_A} x T_B = \frac{T_B}{2} = \frac{850}{2} = 425K \qquad \text{....(ii)}$$

$$\text{So, } \Delta Q = 1x\frac{3}{2}x8.31(425 - 850) = -5297.625J$$

[Negative heat means heat is lost by the sys.]

(c) $D \to A$ process is isochoric

$$\frac{P_D}{P_A} = \frac{T_D}{T_A},\ \text{i.e.}\quad P_D = P_A\frac{T_D}{T_A}$$

But C and D are on the same adiabatic

$$\left(\frac{T_D}{T_C}\right)^{\gamma} = \left(\frac{P_D}{P_C}\right)^{\gamma-1} = \left(\frac{P_A T_D}{P_C T_A}\right)^{\gamma-1}$$

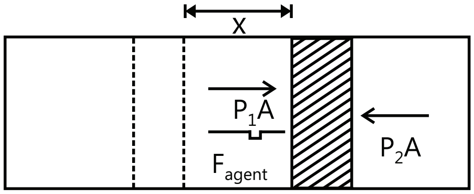
or $(T_D)^{1/\gamma} = T_C\left[\frac{P_A}{P_C T_A}\right]^{1-\frac{1}{\gamma}}$,

i.e. $T_C^{3/5} = \left(\frac{T_B}{2}\right)\left[\frac{P_A}{(1/3)P_A 1000}\right]^{2/5}$

i.e. $T_D^{3/5} = \left[\frac{1}{2}\left(\frac{2}{3}\right)^{2/3} x1000\right]\left[\frac{3}{1000}\right]^{2/5}$

i.e. $T_D = 500K$

Example 5: A piston can freely move inside a horizontal cylinder closed from both ends. Initially, the piston separates the inside space of the cylinder into two equal part of volume V_0, in which an ideal gas is contained under the same pressure P_0 and at the same temperature.



What work has to be performed in order to increase isothermally the volume of one of gas η times compared to that of the other by slowly moving piston?

Sol: Apply isothermal condition on both compartments. Then, proceed to find V_f (left)/V_f (right).

Let the agent move as shown.

In equilibrium position, $P_1A + F_{agent} = P_2A$

$F_{agent} = (P_2 - P_1)A$

Elementary work done by the agent

$$F_{agent}\,dx = (P_2 - P_1)Axdx = (P_2 - P_1)dV \quad \text{... (i)}$$

Applying PV = constant for two parts, we have

$P_1(V_0 + Ax) = P_0 V_0$ and $P_2(V_0 - Ax) = P_0 V_0$

$P_1 = \frac{P_0V_0}{(V_0 + Ax)}$ and $P_2 = \frac{P_0V_0}{(V_0 - Ax)}$

$$\therefore \quad P_2 - P_1 = \frac{P_0V_0(2Ax)}{V_0^2 - A^2x^2} = \frac{2P_0V_0V}{V_0^2 - V^2}$$

When the volume of the left end is η times the volume of right end, we have $(V_0 + V) = \eta(V_0 - V)$

$$V = \left(\frac{\eta-1}{\eta+1}\right)V_0 \quad \text{... (ii)}$$

The work done by the agent is given by

$$W = \int_0^V (P_2 - P_1)dV = \int_0^V \frac{2P_0V_0V}{V_0^2 - V^2}dV$$

$$= -P_0V_0[\mathrm{In}(V_0^2 - V^2)^V]_0^v = -P_0V_0[\mathrm{In}(V_0^2 - V^2) - \mathrm{In}V_0^2]$$

$$= -P_0V_0\left[\mathrm{In}\left\{V_0^2 - \left(\frac{\eta-1}{\eta+1}\right)^2 V_0^2\right\} - \mathrm{In}V_0^2\right]$$

$$= -P_0V_0[\mathrm{In}\{4\eta/(\eta+1)^2\}] = P_0V_0\,\mathrm{In}\left[\frac{(\eta+1)^2}{4\eta}\right].$$

Example 6: An ideal gas has a density of 1.78 kg / m^3 is contained in a volume of 44.8 x 10^{-3} m^3. The temperature of the gas is 273 K. The pressure of the gas is 0.01 x 10^5 Pa. The gas constant R = 8.31 J-K^{-1} mole $^{-1}$.

(a) What is the root mean square velocity of the air molecules?

(b) How many moles of gas are present?

(c) What is the gas?

(d) What is the internal energy of the gas?

Sol: Use relation V_{rms} and P given. m = ρV = nM also.

(a) $P = \frac{1}{3}\rho V^2 \quad \Rightarrow$

$$V_{rms} = \left(\frac{3P}{\rho}\right)^{1/2} = \left(\frac{(3)(1.01x10^5\,N/m^2)}{1.78\,kg/m^3}\right)^{1/2}$$

= 4.13 x 10^2 m/s

(b) $PV = n_mRT \quad \Rightarrow$

$$n_m = \frac{PV}{RT} = \frac{(1.01x10^5Nm^{-2})(44.8x10^{-3}\,m^3)}{(8.31JK^{-1}mole^{-1})(273K)}$$

= 2.0 moles

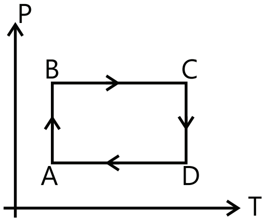
$$M_{molar} = \frac{\rho V}{n_m} = \frac{(1.78kg/m^3)(44.8x10^{-3}m^3}{(2.0\,moles)}$$

$= 340.0x10^{-3}$ kg / mole

(c) This ideal gas is Argon.

(d) Internal energy of monoatomic gas=3/2 nRT.

Example 7: Plot P-V, V-T and ρ -T graph corresponding to the P-T graph for an ideal gas shown in the figure.



Sol: Look for parameter which is constant in the each process.

Process AB is an isothermal process with T = constant and $P_B > P_A$.

P-V graph: $P \propto \frac{1}{V}$ i.e., P-V graph is a hyperbola with $P_B > P_A$ and $V_B > V_A$.

V-T graph: T= constant. Therefore, V-T graph is a straight line parallel to V-axis with $V_B > V_A$.

$\rho - T$ graph : $\rho = \frac{PM}{RT}$ or $\rho \propto P$.

As T is constant. Therefore, $\rho - T$ graph is a straight line parallel to ρ -axis with $\rho_B > \rho_A$ as $P_B > P_A$.

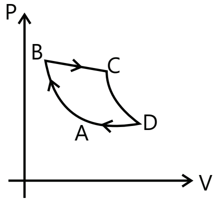
Process BC is isobaric process with P = constant and $T_C > T_B$.

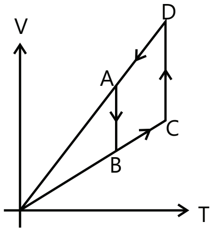
P-V graph: As P is constant. Therefore, P-V graph is a straight line parallel to V-axis with $V_C > V_B$ (because V $\propto$ T in an isobaric process)

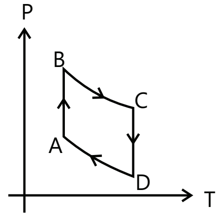
V-T graph: In an isobaric process $V \propto T$, i.e., V-T graph is a straight line passing through the origin, with $T_C > T_B$ and $V_C > V_B$.

ρ-T graph: $\rho \propto \frac{1}{T}$ (when P = constant), i.e., ρ -T graph is a hyperbola with $T_C > T_B$ and $\rho_C > \rho_B$.

There is no need to discuss C-D and D-A processes as they are opposite to AB and BC respectively. The corresponding three graphs are shown above.







JEE Main/Boards

Exercise 1

Q.1 Although the r.m.s. speed of gas molecules is of the order of the speed of sound in that gas yet on opening a bottle of ammonia in one corner of a room, its smell takes time in reaching the other corner. Explain Why?

Q.2 The pressure of a gas at – 173°C temperature is 1 atmosphere, keeping the volume constant, to what temperature should the gas be heated so that its pressure becomes 2 atmosphere.

Q.3 Explain (i) Boyle's law (ii) Charle's law. Why they are not applicable to real gases at all states?

Q.4 State and explain (i) Guy Isac's law and (ii) Gas equation. Distinguish clearly between R and r for a gas.

Q.5 State the postulates of Kinetic Theory of gases. Explain the pressure exerted by an ideal gas.

Q.6 Find an expression for the pressure exerted by a gas and establish its relation with kinetic energy of the gas.

Q.7 From Kinetic Theory of gases, explain kinetic interpretation of temperature and absolute zero.

Q.8 Explain the concept of mean free path.

Q.9 Explain what is meant by Brownian Motion?

Q.10 The density of water is 1000kg/m^3. The density of water vapour at 100°C and 2 atmospheric pressure is 0.6kg m^3. The volume of a molecule multiplied by the total number gives what is called, molecular volume. Estimate the ratio (or fraction) of the molecular volume to the total volume occupied by the water vapour under the above conditions of temperature and pressure.

Q.11 A 3000cm^3 tank contains oxygen at 20°C and a gauge pressure of 2.5 x 10^{6}Pa. Find the mass of the oxygen in the tank. Take 1 atm = 10^5 Pa.

Q.12 Calculate the r.m.s. velocity of air molecules at N.T.P. Given that 22400 c.c. of gas at N.T.P. weight 64 gm.

Q.13 How many collisions per second does each molecule of a gas make, when average speed of the molecule is 500ms^{-1} and mean free path is 2.66 x 10^{-7} m?

Q.14 Calculate the mean free path of gas molecules, if number of molecules per cm^3 is 3 x 10^{19} and diameter of each molecule is 2Å.

Q.15 The diameter of a gas molecules is 2.4 x 10-^{10}m. Calculate the mean free path at N.T.P. Given Boltzmann constant k = 1.38 x 10^{-23} J molecule^{-1} K^{-1}.

Q.16 Which molecules, ice at 0°C or water 0°C have greater potential energy and why?

Q.17 An ideal gas is compressed at a constant temperature, will its internal energy increases of decrease?

Q.18 Which type of motion of the molecules is responsible for internal energy of a monoatomic gas?

Q.19 The volume of an ideal gas is V at a pressure P. On increasing the pressure by Δ P, the change in volume of the gas is (ΔV_1) under isothermal conditions and (ΔV_2) under adiabatic conditions, Is $\Delta V_1 > \Delta V_2$ or vice-versa and why?

Q.20 200 joule of work is done on a gas to reduce its volume by coming it. If this change is done under adiabatic conditions, find out the change in internal energy of the gas and also the amount of heat absorbed by the gas?

Q.21 Give briefly the concept of internal energy.

Q.22 Define the four thermodynamic processes. What is meant by indicator diagram?

Q.23 State the sign conventions used in all thermodynamic processes.

Q.24 What do you learn by applying first law of thermodynamics to isothermal and adiabatic processes?

Q.25 Explain what is meant by isothermal operations. Give some examples.

Q.26 What are adiabatic operations? Enumerate some examples. State equations representing these operations.

Q.27 Obtain an expression for work done by a gas in isothermal expansion.

Q.28 Derive an expression for work done in an adiabatic process.

Q.29 What are cyclic and non cyclic processes? Calculate work done in such processes.

Q.30 What are reversible and irreversible processes? Give some examples of each.

Q.31 What is a heat engine? Obtain an expression for its efficiency.

Q.32 A tyre pumped to a pressure of 3 atmosphere suddenly bursts. Calculate the fall in temperature due to adiabatic expansion. The temperature of air before expansion is 27°C and value of γ = 1.4.

Q.33 A quantity of air at 27°C and atmospheric pressure is suddenly compressed to half its original volume. Find the final (i) pressure and (ii) temperature. Given γ for air = 1.42.

Q.34 A Cylinder containing one gram mole of gas was put on boiling water bath and compressed adiabatically till its temperature rose by 70°C. Calculate the work done and increase in energy of the gas, γ = 1.5, R = 2 cal. mole^{-1} K^{-1}.

Q.35 One gram mole of an ideal gas at S.T.P. is subjected to reversible adiabatic expansion to double its volume. Find the change in internal energy in the process. Take γ =1.4.

Q.36 If 1 gram oxygen at 760mm pressure and 0°C has it volume doubled in an adiabatic change, calculate the change in internal energy. Take R=2 cal. mole^{-1} K^{-1}, J=4.2 J cal^{-1} and γ =1.4.

Exercise 2

Single Correct Choice Type

Q.1 Find the approx. number of molecules contained in a vessel of volume 7 litres at 0°C at 1.3 x 10^{5}Pascal

(A) 2.4 x 10^{23} (B) 3 x 10^{23}

(C) 6 x 10^{23} (D) 4.8 x 10^{23}

Q.2 An ideal gas mixture filled inside a balloon expands according to the relation $PV^{2/3}$ = constant. The temperature inside the balloon is

(A) Increasing (B) Decreasing

(C) Constant (D) Can't be said

Q.3 A rigid tank contains 35 kg of nitrogen at 6 atm. Sufficient quality of oxygen is supplied to increase the pressure to 9 atm, while the temperature remains constant. Amount of oxygen supplied to the tank is:

(A) 5 kg (B) 10 kg (C) 20 kg (D) 40 kg

Q.4 At temperature T K, the pressure of 4.0g argon in bulb is p. The bulb is put in a bath having temperature higher by 50K than the first one. 0.8g of argon gas had to be removed to maintained original pressure. The temperature T is equal to

(A) 510 K (B) 200 K (C) 100 K (D) 73 K

Q.5 When 2 gms of a gas are introduced into an evacuated flask kept at 25°C the pressure is found to be one atmosphere. If 3 gms of another gas added to the same flask the pressure becomes 1.5 atmosphere. The ratio of the molecular weights of these gases will be

(A) 1: 3 (B) 3: 1 (C) 2: 3 (D) 3: 2

Q.6 During an experiment, an ideal gas obeys an addition equation of state P^2V = constant. The initial temperature and pressure of gas are T and V respectively. When it expands to volume 2 V, then its temperature will be?

(A) T (B) $\sqrt{2}$ T (C) 2 T (D) 2$\sqrt{2}$ T

Q.7 A barometer tube, containing mercury, is lowered in a vessel containing mercury until only 50 cm of the tube is above the level of mercury in the vessel. If the atmospheric pressure is 75 cm of mercury, what is the pressure at the top of the tube?

(A) 33.3 kPa (B) 66.7 kPa

(C) 3.33 MPa (D) 6.67 MPa

Q.8 A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing one mole of He gas (molar mass 4) at a temperature 3T has a pressure of

(A) P/8 (B) P (C) 2P (D) 8P

Q.9 The ratio of average translational kinetic energy to rotational kinetic energy of a diatomic molecule temperature T is

(A) 3 (B) 7/5 (C) 5/3 (D) 3/2

Q.10 One mole of an ideal gas at STP is heated in an insulated closed container until the average speed of its molecules is doubled. Its pressure would therefore increase by factor.

(A) 1.5 (B) $\sqrt{2}$ (C) 2 (D) 4

Q.11 One mole of an ideal gas is contained within a cylinder by a frictionless piston and is initially at temperature T. The pressure of the gas is kept constant while it is heated and its volume doubles. If R is molar gas constant, the work done by the gas in increasing its volume is?

(A) RT In2 (B) 1/2RT (C) RT (D) 3/2 RT

Q.12 A polyatomic gas with six degrees of freedom does 25J work when it is expanded at constant pressure. The heat given to the gas is?

(A) 100J (B) 150J (C) 200J (D) 250J

Q.13 In thermodynamic process pressure of a fixed mass of gas is changed in such a manner that the gas release 30 joule of heat and 18 joule of work was done on the gas. It the initial internal energy of the gas was 60 joule, then, the final internal energy will be?

(A) 32 joule (B) 48 joule

(C) 72 joule (D) 96 joule

Q.14 An ideal gas undergoes an adiabatic process obeying the relation $PV^{4/3}$ = constant. If its initial temperature is 300 K and then its pressure is increased upto four times its initial value, then the final temperature is (in Kelvin)?

(A) $300\sqrt{2}$ (B) $300\sqrt[3]{2}$

(C) 600 (D) 1200

Q.15 1 kg of a gas does 20kJ of work and receives 16kJ of heat when it its expanded between two states. A second kind of expansion can be found between the initial and final state which requires a heat input of 9 kj. The work done by the gas in the second expansion is:

(A) 32 kJ (B) 5 kJ

(C) -4 kJ (D) 13 kJ

Q.16 A mixture of ideal gases 7 kg of nitrogen and 11 kg of CO_2. Then (Take γ for nitrogen and CO_2 as 1.4 and 1.3 respectively)

(A) Equivalent molecular weight of the mixture is 36.

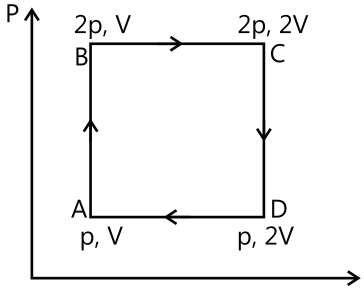
(B) Equivalent molecular weight of the mixture is 18.

(C) γ for the mixture is 5/2

(D) γ for the mixture is 4/3

Previous Years' Questions

Q.1 An ideal mono-atomic gas is taken round the cycle ABCD as sown in the P-V diagram (see figure). The work done during the cycle is ***(1983)***



(A) PV (B) 2 PV

(C) $\frac{1}{2}$pV (D) Zero

Q.2 At room temperature, the rms speed of the molecules of a certain diatomic gas is found to be 1930 m/s. The gas is ***(1984)***

(A) H_2 (B) F_2 (C) O_2 (D) Cl_2

Q.3 70 cal of heat are required to raise the temperature of 2 moles of an ideal diatomic gas at constant pressure from 30°C to 35°C. The amount of heat required (in calorie) to raise the temperature of the same gas through the same range (30°C to 35°C) at constant volume is ***(1985)***

(A) 30 (B) 50 (C) 70 (D) 90

Q.4 If one mole of a monatomic gas ($\gamma = 5/3$) is mixed with one mole of a diatomic gas ($\gamma = 7/5$), the value of γ for the mixture is ***(1988)***

(A) 1.40 (B) 150 (C) 1.53 (D) 3.07

Q.5 The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K the root mean square velocity of the gas molecules is v, at 480 K it becomes ***(1996)***

(A) 4 v (B) 2 v (C) v/2 (D) v/4

Q.6 The average translational energy and the rms speeds of molecules in a sample of oxygen gas at 300 K are 6.21×10^{-21}J and 484 m/s respectively. The corresponding values at 600 K are nearly (assuming ideal gas behaviour) ***(1997)***

(A) $12.42\text{x}10^{-21}$ J, 968 m/s

(B) $8.78\text{x}10^{-21}$ J, 684 m/s

(C) $6.21\text{x}10^{-21}$ J, 968 m/s

(D) $12.42\text{x}10^{-21}$ J, 684 m/s

Q.7 A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is p. An identical vessel containing one mole of the gas (molar mass 4) at a temperature 2T has a pressure of ***(1997)***

(A) p/8 (B) p (C) 2 p (D) 8 p

Q.8 Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume 2V. The changes in the pressure in A and B are found to be ΔP and 1.5 Δp respectively. Then ***(1998)***

(A) $4m_A = 9m_B$ (B) $2m_A = 3m_B$

(C) $3m_A = 2m_B$ (D) $9m_A = 4m_B$

Q.9 A mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is **(1999)**

(A) 4 RT (B) 15 RT (C) 9 RT (D) 11RT

Q.10 A monoatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 are the lengths of the gas column before and after expansion respectively, then T_1/T_2 is given by ***(2000)***

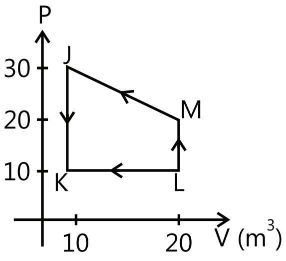
(A) $(L_1/L_2)^{2/3}$ (B) (L_1/L_2)

(C) L_2/L_1 (D) $(L_2/L_1)^{2/3}$

Q.11 An ideal gas is expanding such that pT^2 = constant. The coefficient of volume expansion of the gas is ***(2008)***

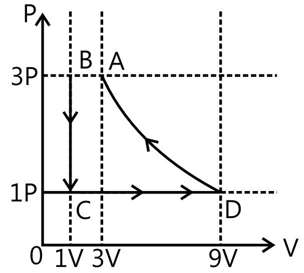
(A) $\frac{1}{T}$ (B) $\frac{2}{T}$ (C) $\frac{3}{T}$ (D) $\frac{4}{T}$

Q.12 Match the following for the given process ***(2006)***



Column I	Column II
(A) Process $J \rightarrow K$	(p) Q>0
(B) Process $K \rightarrow L$	(q) W<0
(C) Process $L \rightarrow K$	(r) W>0
(D) Process $M \rightarrow J$	(s) Q<0

Q.13 One mole of a monatomic ideal gas is taken through a cycle ABCDA as shown in the P-V diagram. column II gives the characteristics involved in the cycle. Match them with each of the processes given in column I. ***(2011)***



Column I	Column II
(A) Process $A \rightarrow B$	(p) Internal energy decreases
(B) Process $B \rightarrow C$	(q) Internal energy increases
(C) Process $C \rightarrow D$	(r) Heat is lost
(D) Process $D \rightarrow A$	(s) Heat is gained
	(t) Work is done on the gas

Q.14 For an ideal gas

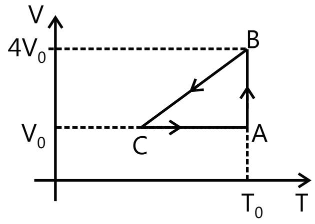
(A) The change in internal energy in a constant pressure process from temperature T_1 to T_2 is equal to $nC_V(T_2-T_1)$, where C_V is the molar heat capacity at constant volume and n the number of moles of the gas

(B) The change in internal energy of the gas and the work done by the gas are equal in magnitude in an adiabatic process

(C) The internal energy does not change in an isothermal process

(D) No heat is added or removed in an adiabatic process. ***(1989)***

Q.15 One mole an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in figure. Its pressure at A is P_0. Choose the correct option(s) from the following ***(2010)***



(A) Internal energies at A and B are the same

(B) Work done by the gas in process AB is P_0V_0 In 4

(C) Pressure at C is $\frac{P_0}{4}$

(D) Temperature at C is $\frac{T_0}{4}$

Q.16 The speed of sound in oxygen (O_2) at a certain temperature is 460 ms^{-1}. The speed of sound in helium (He) at the same temperature will be (assumed both gases to be ideal) ***(2008)***

(A) 460 ms^{-1} (B) 500 ms^{-1}

(C) 650 ms^{-1} (D) 1420 ms^{-1}

Q.17 An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure P_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure P_2 and temperature T_2. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be ***(2008)***

(A) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_2+P_2V_2T_1}$ (B) $\frac{P_1V_1T_1+P_2V_2T_2}{P_1V_1+P_2V_2}$

(C) $\frac{P_1V_1T_2+P_2V_2T_1}{P_1V_1+P_2V_2}$ (D) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_1+P_2V_2T_2}$

Q.18 One kg of a diatomic gas is at a pressure of 8×10^4 N/m^2. The density of the gas is 4 kg m^{-3}. What is the energy of the gas due to its thermal motion? ***(2009)***

(A) 3×10^4 J (B) 5×10^4 J

(C) 6×10^4 J (D) 7×10^4 J

Q.19 Assuming the gas to be ideal the work done on the gas in taking it from A to B is ***(2009)***

(A) 200 R (B) 300 R (C) 400 R (D) 500 R

Q.20 The work done on the gas in taking it from D to A is ***(2009)***

(A) – 414 R (B) + 414 R

(C) – 690 R (D) + 690 R

Q.21 The net work done on the gas in the cycle ABCDA is ***(2009)***

(A) Zero (B) 276 R

(C) 1076 R (D) 1904 R

Q.22 A diatomic ideal gas is used in a Car engine as the working substance. If during the adiabatic expansion part of the cycle, volume of the gas increases from V to 32V the efficiency of the engine is ***(2010)***

(A) 0.5 (B) 0.75 (C) 0.99 (D) 0.25

Q.23 The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is D = [U(x = ∞) – $U_{\text{at equilibrium}}$], D is ***(2010)***

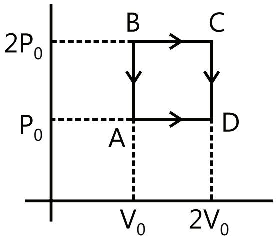
(A) $\frac{b^2}{2a}$ (B) $\frac{b^2}{12a}$ (C) $\frac{b^2}{4a}$ (D) $\frac{b^2}{6a}$

Q.24 Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively: ***(2011)***

(A) 372 K and 330 K (B) 330 K and 268 K

(C) 310 K and 248 K (D) 372 K and 310 K

Q.25 Helium gas goes through a cycle ABCDA (consisting of two isochoric and two isobaric lines) as shown in figure. Efficiency of this cycle is nearly: (Assume the gas to be close to ideal gas) ***(2012)***



(A) 15.4% (B) 9.1% (C) 10.5% (D) 12.5%

Q.26 A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be ***(2012)***

(A) Efficiency of Carnot engine cannot be made larger than 50%

(B) 1200 K

(C) 750 K

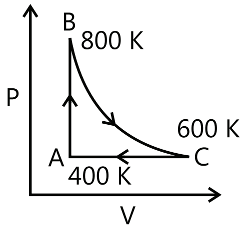
(D) 600 K

Q.27 An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency: ***(2013)***

(A) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ (B) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$

(C) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$

Q.28 One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800 K and 600 K respectively. Choose the correct statement ***(2014)***



(A) The change in internal energy in whole cyclic process is 250R

(B) The change in internal energy in the process CA is 700R

(C) The change in internal energy in the process AB is –350R

(D) The change in internal energy in the process BC is – 500R

Q.29 Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $P = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is ***(2015)***

(A) $T \propto e^{-R}$ (B) $T \propto e^{-3R}$

(C) $T \propto \frac{1}{R}$ (D) $T \propto \frac{1}{R^3}$

Q.30 A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways:

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy changes of the body in the two cases respectively is ***(2015)***

(A) ln 2, 4 ln2 (B) ln 2, ln 2

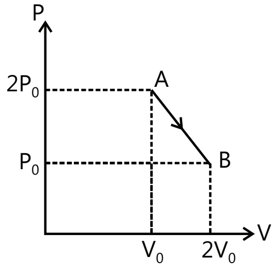
(C) ln 2, 2 ln 2 (D) 2 ln 2, 8 ln 2

Q.31 Consider an ideal gas confined in an isolated closed chamber. As the gas undegoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is : $\left(\gamma = \frac{C_P}{C_V}\right)$ ***(2015)***

(A) $\frac{3\gamma+5}{6}$ (B) $\frac{3\gamma-5}{6}$

(C) $\frac{\gamma+1}{2}$ (D) $\frac{\gamma-1}{2}$

Q.32 'n' moles of an ideal gas undergoes a process A →B as shown in the figure. The maximum temperature of the gas during the process will be: ***(2016)***



(A) $\frac{3P_0V_0}{2nR}$ (B) $\frac{9P_0V_0}{2nR}$ (C) $\frac{9P_0V_0}{nR}$ (D) $\frac{9P_0V_0}{4nR}$

Q.33 An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_p and C_V are molar specific heat at constant pressure and constant volume, respectively): ***(2016)***

(A) $n = \frac{C - C_p}{C - C_v}$ (B) $n = \frac{C_p - C}{C - C_v}$

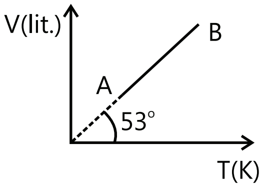
(C) $n = \frac{C - C_v}{C - C_p}$ (D) $n = \frac{C_p}{C_v}$

JEE Advanced/Boards

Exercise 1

Q.1 A closed vessel of volume V_0 contains oxygen at a pressureP_0 and temperatureT_0. Another closed vessel of the same volume V_0 contains helium at a pressure P_0 and temperature$T_0/2$. Find ratio of the masses of oxygen to the helium.

Q.2 V-T curve for 2 moles of a gas is straight line as shown in the graph here. Find the pressure of gas at A.



Q.3 A gas is undergoing an adiabatic process. At a certain stage A, the values of volume and temperature = (V_0, T_0) and the magnitude of the slope of V-T curve is m. Find the value of C_P and C_V.

Q.4 Find the molecular mass of a gas if the specific heats of the gas are $C_P = 0.2$ cal/gm°C and $C_V = 0.15$ cal/gm°C. [Take R = 2 cal/mole°C]

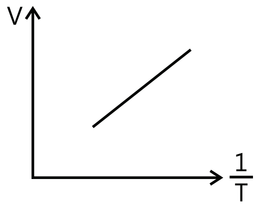
Q.5 The average degrees of freedom per molecules for a gas is 6. The gas performs 25 J of work when it expands at constant pressure. Find the heat absorbed by the gas.

Q.6 A mixture of 4gm helium and 28 gm of nitrogen in enclosed in a vessel of constant volume 300 K. Find the quantity of heat absorbed by the mixture to double the room mean velocity of its molecules. (R = Universal gas constant)

Q.7 One mole of an ideal gas is compressed from 0.5 lit to 0.25 lit. During the compression, 23.04×10^2 J of work is done on the gas and heat is removed to keep the temperature of the gas constant at all times. Find the temperature of the gas. (Take universal gas constant R = 8.31 J $mol^{-1}K^{-1}$)

Q.8 Ideal diatomic gas is taken through a process $\Delta Q = 2\Delta U$. Find the molar heat capacity for the process (where ΔQ is the heat supplied and ΔU is change in internal energy)

Q.9 An ideal gas has a molar heat capacity C_V at constant volume. Find the molar heat capacity of this gas as a function of volume, if the gas undergoes the process: $T = T_0 e^{\alpha V}$.



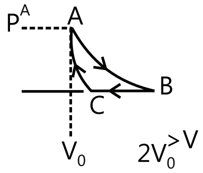
Q.10 One mole of an ideal monoatomic gas undergoes a process as shown in the figure. Find the molar specific heat of the gas in the process.

Q.11 The pressure of an ideal gas changes with volumes as P = aV where 'a' is a constant. One mole of this gas is expanded to 3 times its original volume V_0. Find

(i) The heat transferred in the process.

(ii) The heat capacity of the gas

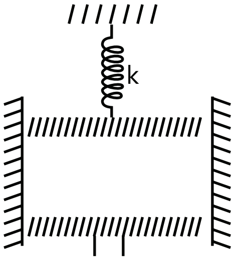
Q.12 In a cycle ABCA consisting of isothermal expansion AB, isobaric compression BC and adiabatic compression CA, find the efficiency of cycle.



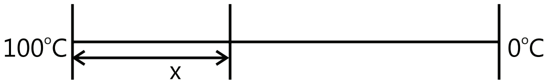
(Given: $T_A = T_B = 400K, \gamma = 1.5$)

Q.13 A highly conduction solid cylinder of radius a and length ℓ is surrounded by co-axial layer of a material having thermal conductivity K and negligible heat capacity. Temperature of surrounding space (out side the layer) is T_0, which is higher than temperature of the cylinder. If heat capacity per unit volume of cylinder material is s and outer radius of the layer is b, calculate time required to increase temperature of the cylinder from T_1 to T_2. Assume and faces to be thermally insulated.

Q.14 A vertical brick duct (tube) is filled with cast iron. The lower end of the duct is maintained at a temperature T_1 which is greater than the melting point T_m of cast iron and the upper end at a temperature T_2 which is less than the temperature of the melting point of cast iron. It is given that the conductivity of liquid cast iron is equal to k times the conductivity solid cast iron. Determine the fraction of the duct filled with molten metal.



Q.15 A lagged stick of cross section area $1cm^2$ and length 1 m is initially at a temperature of 10°C. It is then kept between 2 reservoirs of temperature 100°C and 0°C. Specific heat capacity is 10 J/kg°C and linear mass density is 2 kg/m. Find



(a) Temperature gradient along the rod in steady state.

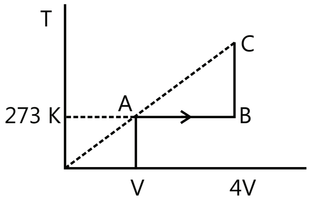
(b) Total heat absorbed by the rod to reach steady state.

Q.16 A cylindrical block of length 0.4 m and area of cross-section $0.04m^2$ is placed coaxially on a thin metal disc of mass 0.4 kg and of the same cross-section. The upper face of the cylinder is maintained at a constant temperature of 400K and the initial temperature of the disc is 300K. If the thermal conductivity of the material of the cylinder is 10 watt/m-K and the specific heat of the material of the disc in 600 J/kg-K, how long will it take for the temperature of the disc to increase to 350K? Assume, for purposes of calculation, the thermal conductivity of the disc to be very high and the system to be thermally insulated expect for the upper face of the cylinder.

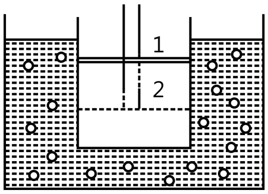
Q.17 A liquid takes 5 minutes to cool from 80°C to 50°C. How much time will it take to cool from 60°C to 30°C? The temperature of surrounding is 20°C. Use exact method.

Q.18 An ideal gas at NTP is enclosed in an adiabatic vertical cylinder having area of cross section A = 27 cm², between two light movable pistons as shown in the figure. Spring with force constant k = 3700 N/m is in a relaxed state initially. Now the lower piston is moved upwards a height h/2, h being the initial length of gas column. It is observed that the upper piston moves up by a distance h/16. Find h taking γ for the gas to be 1.5. Also find the final temperature of the gas.

Q.19 At a temperature of T_0 = 273°K, two moles of an ideal gas undergoes a process as shown. The total amount of heat imparted to the gas equals Q = 27.7 kJ. Determine the ratio of molar specific heat capacities.



Q.20 A cylinder containing a gas is closed by a movable piston. The cylinder is submerged in an ice-water mixture. The piston is quickly pushed down from position 1 to position 2. The piston is held at position 2 until the gas is again at 0°C and then slowly raised back to position 1. Represent the whole process on P-V diagram. If m = 100 gm of ice are melted during the cycle, how much work is done on the gas, Latent heat of ice = 80 cal/gm.

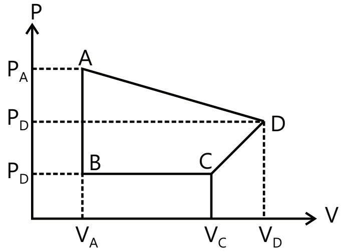


Q.21 A parallel beam of particles of mass m moving with velocities v impinges on a wall at an angle θ to its normal. The number of particles per unit volume in the beam is n. If the collision of particles with the wall is elastic, then find the pressure exerted by this beam on the wall.

Q.22 For the thermodynamic process shown in the figure.

$P_A = 1x10^5 Pa; P_B = 0.3x10^5 Pa$

$P_D = 0.6x10^5 Pa; V_A = 0.20 litre\ V_D = 1.30 litre$



(a) Find the work performed by the system along path AD.

(b) If the total work done by the system along the path ADC is 85J find the volume at point C.

(c) How much work is performed by the system along the path CDA?

Exercise 2

Single Correct Choice Type

Q.1 A perfect gas of a given mass is heated first in small vessel and then in a large vessel, such that their volumes remain unchanged. The P-T curves are

(A) Parabolic with same curvature

(B) Parabolic with different curvature

(C) Linear with same slopes

(D) Linear with different slopes

Q.2 An open and wide glass tube is immersed vertically in mercury in such a way that length 0.05 m extends above mercury level. The open end of the tube is closed and the tube is raised further by 0.43 m. The length of air column above mercury level in the tube will be? Take $P_{atm} = 76cm$ of mercury.

(A) 0.215 m (B) 0.2 m (C) 0.1 m (D) 0.4 m

Q.3 A container X has volume double that of container Y and both are connected by a thin tube. Both contains same ideal gas. The temperature of X is 200K and that of Y is 400 K. If mass of gas in X is m then Y it will be:

(A) m/8 (B) m/6 (C) m/4 (D) m/2

Q.4 An ideal gas of Molar mass M is contained in a vertical tube of height H, closed at both ends. The tube is accelerating vertically upwards with acceleration g. Then, the ratio of pressure at the bottom and the mid point of the tube will be

(A) exp[2MgH/RT] (B) exp[-2MgH/RT]

(C) exp[MgH/RT] (D) MgH/RT

Q.5 Two monoatomic ideal gas at temperature T_1 and T_2 are mixed. There is no loss of energy. If the masses of molecules of the two gases are m_1 and m_2 and number of their molecules are n_1 and n_2 respectively. The temperature of the mixture will be?

(A) $\frac{T_1+T_2}{n_1+n_2}$ (B) s $\frac{T_1}{n_1}+\frac{T_2}{n_2}$

(C) $\frac{n_2T_1+n_1T_2}{n_1+n_2}$ (D) $\frac{n_1T_1+n_2T_2}{n_1+n_2}$

Q.6 At temperature T, N molecules of gas A each having mass m and at the same temperature 2N molecules of gas B each having mass 2m are filled in a container. The mean square velocity of molecules of gas B is v^2 and mean square of x component of velocity of molecules of gas A is w^2. The ratio of w^2 / v^2 is?

(A) 1 (B) 2 (C) 1/3 (D) 2/3

Q.7 A reversible adiabatic path on a P-V diagram for an ideal gas passes through state A where $P=0.7x10^5$ N/m^{-2} and v = $0.0049m^3$. The ratio of specific heat of the gas is 1.4. The slope of path at A is?

(A) $2.0x10^7 Nm^{-5}$ (B) $1.0x10^7 Nm^{-5}$

(C) $-2.0x10^7 Nm^{-5}$ (D) $-1.0x10^7 Nm^{-5}$

Q.8 A cylinder made of perfectly non conducting material closed at both ends is divided into two equal parts by a heat proof piston. Both parts of the cylinder contain the same masses of a gas at a temperature $t_0 = 27°C$ and pressure $P_0 = 1$ atm. Now if the gas in one of the parts is slowly heated to t = 57°C while the temperature of first part is maintained at t_0 the distance moved by the piston from the middle of the cylinder will be (length of the cylinder = 84cm)

(A) 3cm (B) 5cm (C) 2cm (D) 1cm

Q.9 A vessel contains an ideal monoatomic gas which expands at constant pressure, when heat Q is given to it. Then the work done in expansion is:

(A) Q (B) $\frac{3}{5}Q$ (C) $\frac{2}{5}Q$ (D) $\frac{2}{3}Q$

Multiple Correct Choice Type

Q.10 Two bodies A and B have thermal emissivities of 0.81 respectively. The outer surface areas of the two bodies are the same. The two bodies radiate energy at the same rate. The wavelength λ_B, corresponding to the maximum spectral radiancy in the radiation from B, is shifted from the wavelength corresponding to the maximum spectral radiancy in the radiation from A by 1.00 μm. If the temperature of A is 5802 K,

(A) The temperature of B is 1934K

(B) $\lambda_B = 1.5\ \mu m$

(C) The temperature of B is 11604 K

(D) The temperature of B is 2901 K

Q.11 During an experiment, an ideal gas is found to obey a condition VP^2 = constant. The gas is initially at a temperature T, pressure P and volume V. The gas expands to volume 4V.

(A) The pressure of gas changes to $\frac{P}{2}$

(B) The temperature of gas changes to 4T

(C) The graph of above process on the P-T diagram is parabola

(D) The graph of above process on the P-T diagram is hyperbola.

Q.12 The total kinetic energy of translatory motion of all the molecules of 5 litres of nitrogen exerting a pressure P is 3000 J.

(A) The total K.E. of 10 litres of N_2 at a pressure of 2P is 3000 J

(B) The total K.E. of 10 litres of He at a pressure of 2P is 3000 J

(C) The total K.E. of 10 litres of Q_2 at a pressure of 2P is 20000 J

(D) The total K.E. of 10 litres of Ne at a pressure of 2P is 12000 J

Q.13 A container holds 10^{26} molecules/m^3, each of mass 3 x 10^{-27} kg. Assume that 1/6 of the molecules move with velocity 2000 m/s directly towards one wall of the container while the remaining 5/6 of the molecules move either away from the wall or in perpendicular direction, and all collisions of the molecules with the wall are elastic

(A) Number of molecules hitting 1 m^2 of the wall every second is 3.33 x 10^{28}.

(B) Number of molecules hitting 1 m^2 of the wall every second is 2 x 10^{29}.

(C) Pressure exerted on the wall by molecules is 24 x 10^5 Pa.

(D) Pressure exerted on the wall by molecules is 4 x 10^5 Pa.

Q.14 Two gases have the same initial pressure, volume and temperature. They expand to the same final volume, one adiabatically and the other isothermally

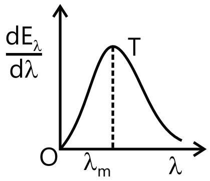
(A) The final temperature is greater for the isothermal process

(B) The final pressure is greater for the isothermal process

(C) The work done by the gas is greater for the isothermal process.

(D) All the above options are incorrect

Q.15 The figure shows a radiant energy spectrum graph for a black body at a temperature T. Choose the correct statement(s)



(A) The radiant energy is not equally distributed among all the possible wavelengths

(B) For a particular wavelength the spectral intensity is maximum

(C) The area under the curve is equal to the total rate at which heat is radiated by the body at that temperature

(D) None of these

Q.16 Two metallic sphere A and B are mode of same material and have got identical surface finish. The mass of sphere A is four times that of B. Both the spheres are heated to the same temperature and placed in a room having lower temperature but thermally insulated from each other.

(A) The ratio of heat loss of A to that of B is $2^{4/3}$.

(B) The ratio of heat loss of A to that of B $2^{2/3}$.

(C) The ratio of the initial rate of cooling of A to that of B is $2^{-2/3.}$

(D) The ratio of the initial rate of cooling of A to that of B is $2^{-4/3.}$

Q.17 50 gm ice at = 10°C is mixed with 20 gm steam at 100°C. When the mixture finally reaches its steady state inside a calorimeter of water equivalent 1.5 gm then: [Assume calorimeter was initially at 0°C, take latent heat of vaporization of water = 1 cal/gm-°C, specific heat capacity of ice = 0.5 cal/gm°C]

(A) Mass of water remaining is: 67.4 gm

(B) Mass of water remaining is: 67.87 gm

(C) Mass of steam remaining is: 2.6 gm

(D) Mass of steam remaining is: 2.13 gm

Q.18 A gas expands such that it is initial and final temperature are equal. Also, the process followed by the gas traces a straight line on the P-V diagram:

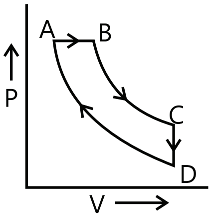
(A) The temperature of the gas remains constant throughout.

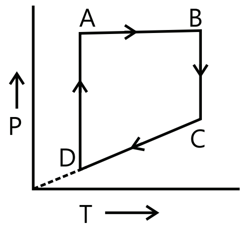
(B) The temperature of the gas first increases and then decreases.

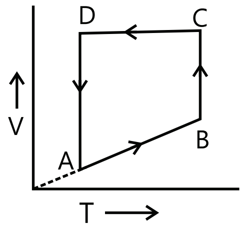
(C) The temperature of the gas first decreases and then increases.

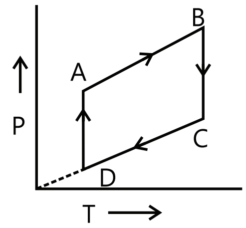
(D) The straight line has a negative slopes

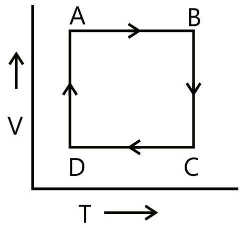
Q.19 A cyclic process ABCD is shown in the P-V diagram. Which of the following curves represents the same process if BC & DA are isothermal processes.



(A)


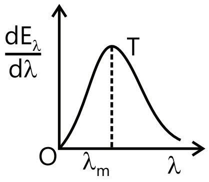
(B)


(C)


(D)


Comprehension Type

Paragraph 1:



Q.20 The figure shows a radiant energy spectrum graph for a black body at a temperature T.

Choose the correct statement (s)

(A) The radiant energy is not equally distributed among all the possible wave lengths

(B) For a particular wavelength the spectral intensity is maximum

(C) The area under the curve is equal to the total rate at which heat is radiated by the body at that temperature.

(D) None of these

Q.21 If the temperature of the body is raised to higher temperature T′, then choose the correct statement(s)

(A) The intensity of radiation for every wavelength increases

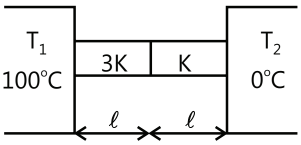
(B) The maximum intensity occurs at a shorter wavelength

(C) The area under the graph increases

(D) The area under the graph is proportional to the fourth power of temperature

Paragraph 2:

Two rods A and B of same cross-sectional area A and length / connected in series between a source (T_1 =100°C) and a sink (T_2 = 0°C) as shown in figure. The rod is laterally insulated



Q.22 The ratio of the thermal resistance of the rod is

(A) $\frac{R_A}{R_B} = \frac{1}{3}$ (B) $\frac{R_A}{R_B} = 3$

(C) $\frac{R_A}{R_B} = \frac{3}{4}$ (D) $\frac{4}{3}$

Q.23 If T_A and T_B are the temperature drops across the rod A and B, then

(A) $\frac{T_A}{T_B} = \frac{3}{1}$ (B) $\frac{T_A}{T_B} = \frac{1}{3}$

(C) $\frac{T_A}{T_B} = \frac{3}{4}$ (D) $\frac{T_A}{T_B} = \frac{4}{3}$

Q.24 If G_A and G_B are the temperature gradients across the rod A and B, then

(A) $\frac{G_A}{G_B} = \frac{3}{1}$ (B) $\frac{G_A}{G_B} = \frac{1}{3}$

(C) $\frac{G_A}{G_B} = \frac{3}{4}$ (D) $\frac{G_A}{G_B} = \frac{4}{3}$

Paragraph 3:

In fluids heat transfer tales place and molecules of the medium takes very active part. The molecules take energy from high temperature zone and move towards low temperature zone. This method is known as convection, when we require heat transfer with fast phase, we use some mechanism to make the flow of fluid on the body fast. The rate of loss of heat is proportional to velocity of fluid (v), and temperature difference (ΔT) between the body any fluid, of course more the surface area of body more the rate of loss of heat. We can write the rate of loss of heat as $\frac{dQ}{dt} = KAv\Delta T$

Where K is positive constant.

Now answer the following questions

Q.25 A body is being cooled with fluid. When we increase the velocity of fluid 4 times and decrease the temperature difference 1/2 time, the rate of loss of heat increases.

(A) Four times (B) Two times

(C) Six times (D) No change

Q.26 In the above question if mass of the body increased two times, without change in any of the other parameters, the rate of cooling

(A) Decreases

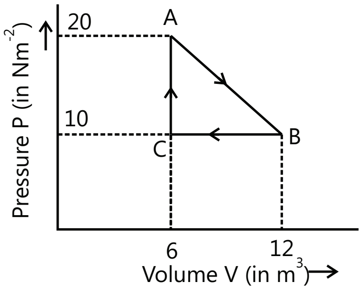
(B) Increases

(C) No effect of change of mass

(D) None of these

Paragraph 4:

When a thermo-dynamic process is shown on P-V diagram, area under curve represents work done during process. During cyclic process work done is area enclosed. The P-V graph for a thermodynamically system is shown in figure.



Q.27 The work done by the system in the process A to B is

(A) 90 J (B) 60 J (C) 0 J (D) 30J

Q.28 The work done in the process B to C is

(A) -90 J (B) -60 J (C) 0 J (D) -30J

Q.29 The work done in the complete cycle ABCA is

(A) 90 J (B) 60 J (C) 0 J (D) 30 J

Paragraph 5:

Five moles of helium are mixed with two moles of hydrogen to form a mixture. Take molar mass of helium $M_1 = 4g$ and that of hydrogen $M_2 = 2g$

Q.30 The equivalent molar mass of the mixture is

(A) 6g (B) $\frac{13g}{7}$

(C) $\frac{18g}{7}$ (D) None

Q.31 The equivalent degree of freedom f of the mixture is

(A) 3.57 (B) 1.14 (C) 4.4 (D) None

Q.32 The equivalent value of γ is

(A) 1.59 (B) 1.53 (C) 1.56 (D) None

Q.33 If the internal energy of He sample of 100J and that of the hydrogen sample is 200J, then the internal energy of the mixture is

(A) 900J (B) 128.5J

(C) 171.4J (D) 300J

Match the Column

Q.34 An ideal gas at pressure P and volume V is expanded to volume 2V. Column I represents the thermodynamic processes used during expansion. column II represents the work during these processes in the random order.

Column I	Column II
(p) isobaric	(x) $\frac{PV(1-2^{1-\gamma})}{\gamma-1}$
(q) isothermal	(y) PV
(r) adiabatic	(z) PV/n 2

The correct matching of column I and column II is given by:

(A) p-y, q-z, r-x (B) p-y, q-x, r-z

(C) p-x, q-y, r-z (D) p-z, q-y, r-x

Previous Years' Questions

Q.1 When the ideal diatomic gas is heated at constant pressure the fraction of the heat energy supplied which increases the internal energy of the gas is ***(1990)***

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{3}{7}$ (D) $\frac{5}{7}$

Q.2 A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300 K. The ratio of the average rotational kinetic energy per O_2 molecule to per N_2 molecule is ***(1998)***

(A) 1: 1

(B) 1: 2

(C) 2: 1

(D) Depends on the moment of inertia of the two molecules

Q.3 Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then the rise in temperature of the gas in B is ***(1998)***

(A) 30 K (B) 18 K (C) 50 K (D) 42 K

Q.4 The ratio of the speed of sound in nitrogen gas to that in helium gas, at 300 K is ***(1999)***

(A) $\sqrt{(2/7)}$ (B) $\sqrt{(1/7)}$

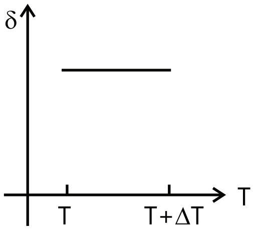
(C) $(\sqrt{3})/5$ (D) $(\sqrt{6})/5$

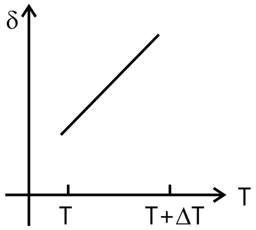
Q.5 Starting with the same initial conditions, and ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal, W_2 if purely isobaric and W_3 if purely adiabatic, then ***(2000)***

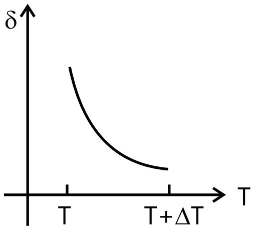
(A) $W_2 > W_1 > W_3$; (B) $W_2 > W_3 > W_1$

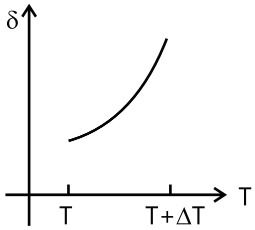
(C) $W_1 > W_2 > W_3$; (D) $W_1 > W_3 > W_2$

Q. 6 An ideal gas is initially at temperature T and volume V. Its volume is increased by ΔV due to an increase in temperature ΔT, pressure remaining constant. The quantity $\delta = V / V\Delta T$ varies with temperature as ***(2000)***

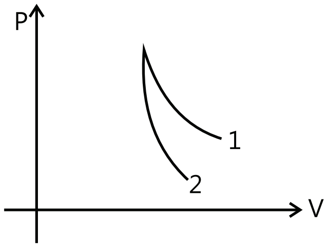








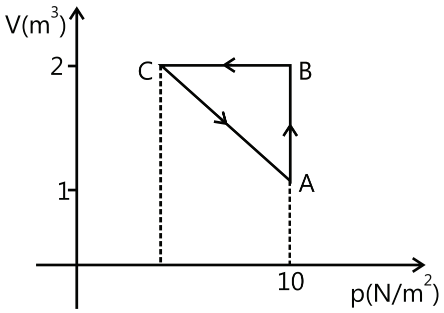
Q.7 P-V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to ***(2001)***



(A) He and O_2 (B) O_2 and He

(C) He and Ar (D) O_2 and N_2

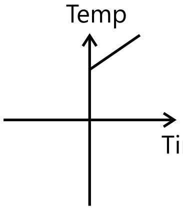
Q.8 An ideal gas is taken through the cycle $A \to B \to C \to A$, as shown in the figure. If the net heat supplied to the gas in the cycle is 5J, the work done by the gas in the process $C \to A$ is ***(2002)***



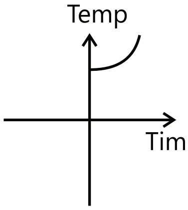
(A) -5J (B) -10 J (C) -15 J (D) -20 J

Q.9 Liquid oxygen at 50 K is heated to 300 K at constant pressure of 1 atm. The rate of heating is constant. Which of the following graphs represent the variation of temperature with time? ***(2004)***

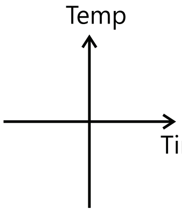
(A)



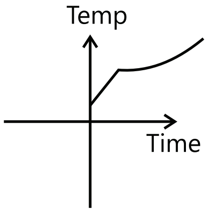
(B)



(C)



(D)



Q.10 An ideal gas expands isothermally from a volume V_1 to V_2 and then compressed to original volume V_1 adiabatically. Initial pressure is P_1 and final pressure is P_3. The total work done is W. Then, ***(2004)***

(A) $P_3 > P_1, W > 0$ (B) $P_3 = P_1, W > 0$

(C) $P_3 > P_1, W < 0$ (D) $P_3 = P_1, W = 0$

Q.11 Statement-I The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume

Statement-II The molecules of a gas collide with each other and the velocities of the molecules change due to the collision. ***(2007)***

(A) If statement-I is true, statement-II is true; statement-II is the correct explanation for statement-I

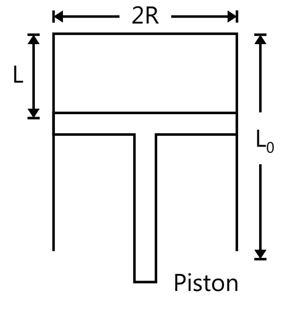
(B) If statement-I is true, statement-II is true; statement-II is not a correct explanation for statement-I

(C) If Statement-I is true; statement true is false

(D) If Statement-I is false; statement-II is true

Paragraph 1:

A fixed thermally conducting cylinder has a radius R and height L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface as shown in the figure. The atmospheric pressure is P_0.



Q.12 The piston is now pulled out slowly and held at a distance 2L from the top. The pressure in the cylinder between its top and the piston will then be ***(2007)***

(A) P_0 (B) $\frac{P_0}{2}$

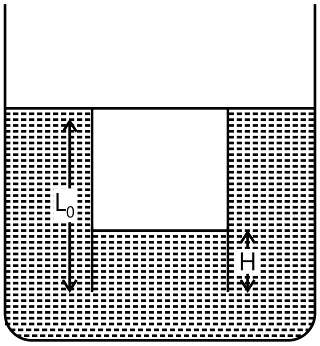
(C) $\frac{P_0}{2} + \frac{Mg}{\pi R^2}$ (D) $\frac{P_0}{2} - \frac{Mg}{\pi R^2}$

Q.13 Whilethe piston is at a distance 2L from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is ***(2007)***

(A) $\left(\frac{2P_0\pi R^2}{\pi R^2 P_0 + Mg}\right)(2L)$ (B) $\left(\frac{P_0\pi R^2 - Mg}{\pi R^2 P_0}\right)(2L)$

(C) $\left(\frac{P_0\pi R^2 + Mg}{\pi R^2 P_0}\right)(2L)$ (D) $\left(\frac{P_0\pi R^2}{\pi R^2 P_0 - Mg}\right)(2L)$

Q.14 The piston is take completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of the water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies ***(2007)***



(A) $\rho g(L_0 - H)^2 + p_0(L_0 - H) + L_0 p_0 = 0$

(B) $\rho g(L_0 - H)^2 - p_0(L_0 - H) - L_0 p_0 = 0$

(C) $\rho g(L_0 - H)^2 + p_0(L_0 - H) - L_0 p_0 = 0$

(D) $\rho g(L_0 - H)^2 - p_0(L_0 - H) + L_0 p_0 = 0$

Q.15 C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then, ***(2009)***

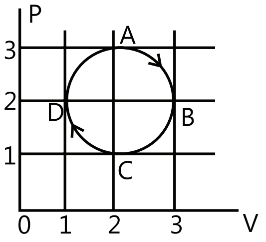
(A) $C_P - C_V$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(B) $C_P + C_V$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(C) $\frac{C_P}{C_V}$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(D) $C_P * C_V$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

Q.16 The figure shows the P-V plot an ideal gas taken through a cycle ABCDA. The part ABC is a semi-circle and CDA is half of an ellipse. Then, ***(2009)***



(A) The process during the path $A \to B$ is isothermal

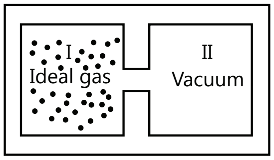
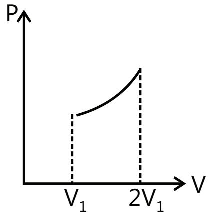
(B) Heat flows out of the gas during the path $B \to C \to D$

(C) Work done during the path $A \to B \to C$ is zero

(D) Positive work is done by the gas in the cycle ABCDA

Match the Columns

Q.17 Column I contains a list of processes involving expansion of an ideal gas. Match this with column II describing the thermodynamic change during this process. Indicate your answer by darkening the appropriate bubbles of the 4 x 4 matrix given in the ORS. ***(2008)***

Column I	**Column II**
(A) An insulated container has two chambers separated by a valve. Chamber I contains an ideal gas and the Chamber II has vacuum. The valve is opened.  	(p) The temperature of the gas decreases
(B) An ideal monoatomic gas expands to twice its original volume such that its pressure $p \propto \frac{1}{V^2}$, where V is the volume of the gas.	(p) The temperature of the gas increases or remains constant
(C) An ideal monoatomic gas expands to twice its original volume such that its pressure $p \propto \frac{1}{V^{4/3}}$, where V is its volume.	(q) The gas loses heat
(D) An ideal monoatomic gas expands such that its pressure p and volume V follows the behaviour shown in the graph.  	(r) The gas gains heat

Q.18 A real gas behaves like an ideal gas if its ***(2010)***

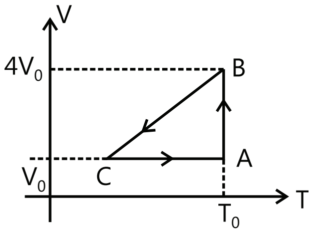
(A) Pressure and temperature are both high

(B) Pressure and temperature are both low

(C) Pressure is high and temperature is low

(D) Pressure is low and temperature is high

Q.19 One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is P_0. Choose the correct option(s) from the following: ***(2010)***



(A) Internal energies at Aand B are the same

(B) Work done by the gas in process AB is $P_0V_0 \ln 4$

(C) Pressure at C is $\frac{P_0}{4}$

(D) Temperature at C is $\frac{T_0}{4}$

Q.20 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is ***(2011)***

(A) $\frac{9}{8}RT_1$ (B) $\frac{3}{2}RT_1$ (C) $\frac{15}{8}RT_1$ (D) $\frac{9}{2}RT_1$

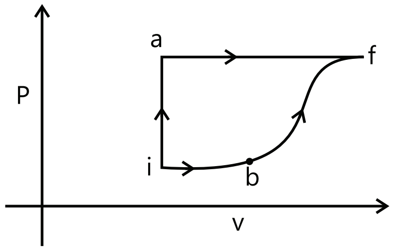
Q.21 A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{V_{rms}(\text{helium})}{V_{rms}(\text{argon})}\right)$ is ***(2012)***

(A) 0.32 (B) 0.45 (C) 2.24 (D) 3.16

Q.22 Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is ***(2013)***

(A) 1 : 4 (B) 1 : 2 (C) 6 : 9 (D) 8 : 9

Q.23 A thermodynamic system is taken from an initial state i with internal energy U_i = 100 J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are W_{af} = 200 J, W_{ib} = 50 J and W_{bf} = 100 J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is U_b = 200 J and Q_{iaf} = 500 J, the ratio Q_{bf} / Q_{ib} is ***(2014)***



Q.24 A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is(are) ***(2015)***

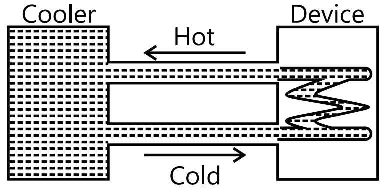
(A) The average energy per mole of the gas mixture is 2RT.

(B) The ratio of speed of sound in the gas mixture to that in helium gas is.

(C) The ratio of the rms speed of helium atoms to that of hydrogen molecules is 1/2.

(D) The ratio of the rms speed of helium atoms to that of hydrogen molecules is.

Q.25 A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is



(Specific heat of water is 4.2 kJ kg^{-1} K^{-1} and the density of water is 1000 kg m^{-3}) ***(2016)***

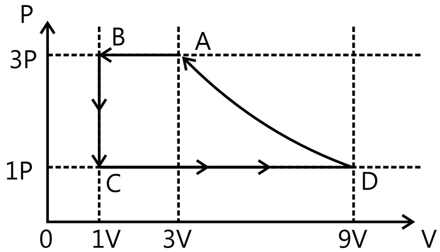
(A) 1600 (B) 2067 (C) 2533 (D) 3933

Q.26 A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it upto height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice.

[Take atmospheric pressure = 1.0 × 10^5 N/m^2, density of water = 1000 kg/m^3 and g = 10 m/s^2. Neglect any effect of surface tension.] ***(2009)***

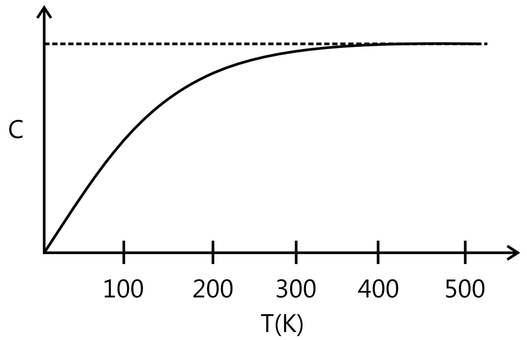
Q.27 A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. If the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, the value of a is ***(2010)***

Q.28 One mole of a monatomic gas is taken through a cycle ABCDA as shown in the P-V diagram. column II give the characteristics involved in the cycle. Match them with each of the processes given in column I. ***(2011)***



Column I	Column II
(A) Process A → B	(p) Internal energy decreases
(B) Process B → C	(q) Internal energy increases.
(C) Process C → D	(r) Heat is lost
(D) Process D → A	(s) Heat is gained
	(t) Work is done on the gas

Q.29 The figure shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation. ***(2013)***



(A) The rate at which heat is absorbed in the range 0-100 K varies linearly with temperature T.

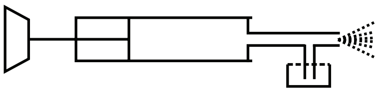
(B) Heat absorbed in increasing the temperature from 0-100 K is less than the heat required for increasing the temperature from 400 – 500 K.

(C) There is no change in the rate of heat absorption in range 400 – 500 K.

(D) The rate of heat absorption increases in the range 200 – 300 K.

Paragraph 1:

A spray gun is shown in the figure where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere. ***(2014)***



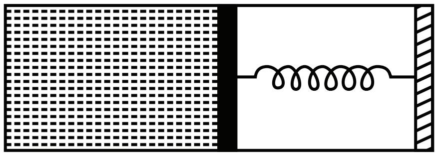
Q.30 If the piston is pushed at a speed of 5 mms^{-1}, the air comes out of the nozzle with a speed of

(A) 0.1 ms^{-1} (B) 1 ms^{-1} (C) 2 ms^{-1} (D) 8 ms^{-1}

Q.31 If the density of air is ρ_a and that of the liquid ρ_ℓ, then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to

(A) $\sqrt{\frac{\rho_a}{\rho_\ell}}$ (B) $\sqrt{\rho_a\rho_\ell}$ (C) $\sqrt{\frac{\rho_\ell}{\rho_a}}$ (D) ρ_ℓ

Q.32 An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state.



The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statement(s) is(are) ***(2015)***

(A) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\frac{1}{4}P_1V_1$

(B) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1V_1$

(C) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3}P_1V_1$

(D) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\frac{17}{6}P_1V_1$

Q.33 A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $P_i = 10^5$ Pa and volume $V_i = 10^{-3}$ m^3 changes to a final state at $P_f = (1/32) \times 10^5$ Pa and $V_f = 8 \times 10^{-3}$ m^3 in an adiabatic quasi-static process, such that P^3V^5 = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at P_i followed by an isochoric (isovolumetric) process at volume V_f. The amount of heat supplied to the system in the two-step process is approximately ***(2016)***

(A) 112 J (B) 294 J (C) 588 J (D) 813 J

Questions

JEE Main/Boards

Exercise 1

Q. 11 Q.20 Q.32
Q.33 Q.34 Q.35
Q.36

Exercise 2

Q.4 Q.5 Q.15
Q.16 Q.17 Q.20
Q.28 Q.29 Q.30

JEE Advanced/Boards

Exercise 1

Q.12 Q.15 Q.18
Q.19 Q.22

Exercise 2

Q.8 Q.10 Q.11

Answer Key

JEE Main/Boards

Exercise 1

Q.2 $P_1 / T_1 = P_2 / T_2$ or $T_2 = P_2 T_1 / P_1 = \frac{2x(273-173)}{1} = 200\text{K} = -73°\text{C}$

Q.10 6×10^{-4}

Q.11 0.103 kg

Q.13 $1.88 \times 10^{9}\ \text{s}^{-1}$

Q.14 1.87×10^{-7} m

Q.15 1.47×10^{-7} m

Q.17 No, because internal energy of an ideal gas depends only on temperature of the gas

Q.18 Translational motion of molecules.

Q.19 $\frac{\Delta V_1}{\Delta V_2} = \gamma$. As $\gamma > 1$, $\therefore (\Delta V_1) > (\Delta V_2)$

Q.20 Internal energy increasing by 200 J. Heat absorbed is zero.

Q.32 80.8°C

Q.33 (i) 2.675 atm (ii) 128.3°C

Q.34 -1176 joule, -280 cal

Q.35 – 1374 J

Q.36 – 43.38 J

Exercise 2

Single Correct Choice Type

Q.1 A   **Q.2** A   **Q.3** C   **Q.4** B   **Q.5** A   **Q.6** B

Q.7 A   **Q.8** C   **Q.9** D   **Q.10** D   **Q.11** C   **Q.12** A

Q.13 B   **Q.14** A   **Q.15** D   **Q.16** A

Previous Years' Questions

Q.1 A   **Q.2** A   **Q.3** B   **Q4** B   **Q.5** B   **Q.6** D

Q.7 C   **Q.8** C   **Q.9** D   **Q.10** D   **Q.11** C

Q.12 A → s; B → r, p; C → p; D → s   **Q.13** A → p; r, t; B → p, r; C → q, s; D → r, t   **Q.14** A, B, C, D

Q.15 A, B   **Q.16** D   **Q.17** A   **Q.18** B   **Q.19** C   **Q.20** A

Q.21 B   **Q.22** B   **Q. 23** C   **Q.24** D   **Q.25** A   **Q.26** C

Q.27 C   **Q.28** D   **Q.29** C   **Q.30** B   **Q.31** C   **Q.32** D

Q.33 A

JEE Advanced/Boards

Exercise 1

Q.1 4:1

Q.2 1.25×10^4 Pa

Q.3 $\frac{mRT_0}{V_0}\left(1+\frac{T_0 m}{V_0}\right)R$

Q.4 The molar mass of the gas is 40 gm,

Q.5 100J

Q.6 3600 R

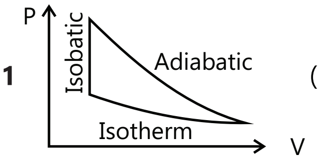
Q.7 400 K

Q.8 5R

Q.9 $C_V + \frac{R}{\alpha V}$

Q.10 $\frac{R}{2}$

Q.11



(i) $\left(\frac{\gamma+1}{\gamma-1}\right)4aV_0^2$, (ii) $\left(\frac{\gamma+1}{\gamma-1}\right)\frac{R}{2}$

Q.12 $1-\frac{3\left(1-\frac{1}{2^{1/3}}\right)}{\ln 2}$

Q.13 $\frac{a^2 s}{2K}\log_e\left(\frac{b}{a}\right)\log_e\left(\frac{T_0-T_1}{T_0-T_2}\right)$

Q.14 $\frac{I_1}{I}=\frac{k(T_1-T_m)}{k(T_1-T_m)+(T_m-T_2)}$

Q.15 (a) -100°C/m, (b) 1000 J

Q.16 166.3 sec

Q.17 10 minutes

Q.18 1.6 m, 364 K

Q.19 1.63

Q.20 8000 cal.

Q.21 $2mv^2 \cos^2\theta$

Q.22 (a) $W_{AD} = 88$ J, (b) $V_C = 1.23$ litre,
(c) $W_{CDA} = -85$ J

Exercise 2

Single Correct Choice Type

Q.1 D **Q.2** C **Q.3** C **Q.4** C **Q.5** D **Q.6** D

Q.7 C **Q.8** C **Q.9** C

Multiple Correct Choice Type

Q.10 A, B **Q.11** A, D **Q.12** C, D **Q.13** A, D **Q.14** A, B, C **Q.15** A, B

Q.16 A, C **Q.17** A, C **Q.18** B, D **Q.19** A, B

Comprehension Type

Paragraph 1: **Q.20** A, B **Q.21** A, B, C, D

Paragraph 2: **Q.22** A **Q.23** B **Q.24** B

Paragraph 3: **Q.25** B **Q.26** A

Paragraph 4: **Q.27** A **Q.28** B **Q.29** D

Paragraph 5: **Q.30** D **Q.31** A **Q.32** C **Q.33** D

Match the Column

Q.34 A

Previous Years' Questions

Q.1 D **Q.2** A **Q.3** D **Q.4** C **Q.5** A **Q.6** C

Q.7 B **Q.8** A **Q.9** C **Q.10** C **Q.11** B **Q.12** A

Q.13 D **Q.14** C **Q.15** B, D **Q.16** B, D

Q.17 A → q; B → p, r; C → p, s; D → q, s **Q.18** D **Q.19** A, B **Q.20** A

Q.21 D **Q.22** D **Q.23** 2 **Q.24** A, B, D **Q.25** B **Q.26** 6

Q.27 4 **Q.28** A → p, r, t; B → p, r; C → q, s; D → r, t **Q.29** A, B, C, D **Q.30** C

Q.31 A **Q.32** B or A, B, C **Q.33** C

Solutions

JEE Main/Boards

Exercise 1

Sol 1: The speed of molecules follows directly from measuring the pressure and density-you don't need to know the size of molecules. In standard kinetic theory, collisions with other molecules have always been ignored, because the molecules were tiny. Though, they aren't. So, the molecules are not going in straight line. Thus, it takes time for the smell to spread in the room.

Sol 2: Initial

P_i = 1 atm

T_0 = -173°C = 100 k

V_i = V

Finally

P_f = 2 atm

T_f = ?

$V_f = V$ = const.

By ideal gas law

PV = nRT

$\Rightarrow V = \frac{T}{P}$ = constant

$\therefore \frac{T_i}{P_i} = \frac{T_f}{P_f}$

$\Rightarrow T_f = \frac{T_i \times P_f}{P_i} = \frac{100 \times 2}{1}$

= 200 K = -73°C

Sol 3: Boyle's law states that absolute pressure exerted by a given mass of an ideal gas is inversely proportional to the volume it occupies if the temperature and amount of gas remains unchanged within a closed system.

i.e. $P \propto \frac{1}{V}$

[Keeping temp. and amt. of gas fixed]

Charles's law

Law of volumes: - $V \propto T$

[Given pressure and amount of gas remains constant]

These laws are not applicable to real gases since in real gases size of particles is not negligible, collisions of particles are not elastic and there are attractive forces between particles.

Sol 4: (i) Guy Isaac's law: - The pressure of a gas of fixed mass and fixed volume is directly proportional to the gas absolute temperature.

i.e. $P \propto T$.

[Keeping volume and amount of gas fixed]

$\Rightarrow \frac{P}{T}$ = constant

(ii) Ideal gas equation: - PV = nRT

R → Gas constant

r → Radius of gas molecule

Sol 5: Postulates of Kinetic Theory Gases (KTG):

1. Gases consist of particles in random motion.
2. Volume of molecule is negligible.
3. Attraction or repulsion forces between any two particles are negligible
4. The collisions between gas molecules or molecule and wall of container are completely elastic meaning no energy is gained or lost from collisions.
5. The time it takes to collide is negligible
6. All gases at a given temperature have same Kinetic Energy.
7. Motion of particles is random
8. Effect of gravity on gas molecule is neglected
9. Average momentum of gas molecule is zero.

Expression for the pressure and v_{rms} of a gas-

$v^2 = v_x^2 + v_y^2 + v_z^2$

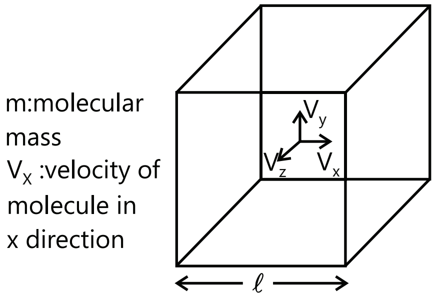
By postulates of KTG we know

$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle$

{<> means average}

$\therefore \langle v^2 \rangle = 3 \langle v_x^2 \rangle$

Since collision is elastic



Change in momentum in one collision

$(\Delta P) = 2m\, v_x$

Time interval between collision $(\Delta t) = \frac{2\ell}{v_x}$

$\therefore$ Force by one molecule $= \frac{\Delta P}{\Delta t} = \frac{m{v_x}^2}{\ell}$

$\therefore$ Total force $= \frac{m}{\ell}\left[v_{x1}^2 + v_{x2}^2 + ...v_{xn}^2\right]$

[Assuming n molecules in the vessel]

$$= \frac{mn}{\ell}\frac{\left[v_{x1}^2 + v_{x2}^2 + ...v_{xn}^2\right]}{n} = \frac{M}{\ell} < v_1^2 >$$

M = mn : - Total mass

$\Rightarrow$ Total force $= \frac{M}{3\ell} \langle v^2 \rangle$

Pressure (P) = $\frac{\text{Force}}{\text{Area}}$

Assume vessel to be a cube.

$$\therefore\ P = \frac{\frac{1}{3}\frac{M}{\ell}<v^2>}{\ell\times\ell} = \frac{1}{3}\ \frac{M}{\ell^3}\ <v^2>$$

$$\Rightarrow P = \frac{1}{3}\rho v^2_{\ rms}$$

[Since volume = λ^3 $\therefore$ $(M/\lambda^3) = \rho$ and $v_{rms} = \sqrt{<v^2>}$

Sol 6: Expression for pressure has been found in previous question.

i.e. $P = \frac{1}{3}\ \rho v^2_{rms}$

$$P = \frac{1}{3}\left(\frac{M}{V}\right)v^2_{rms}$$

$$\Rightarrow PV = \frac{1}{3}\ n\ M_m\ v^2_{rms}$$

[M_m: molecular weight]

Using gas equation we get

$$PV = \frac{1}{3}\ n\ M_m\ V_{rms}^{\ 2}$$

$$\Rightarrow \frac{1}{2}\ M_m\ v^2_{rms} = \frac{3}{2}\ PV_m\ ;\ V_m \text{ is molar volume}$$

L.H.S. is the expression for kinetic energy per mole of gas.

Sol 7: We know that molar kinetic energy

$$= \frac{3}{2}PV_m = \frac{3}{2}\ RT \text{ [From previous question]}$$

$$\therefore \text{ Kinetic energy per molecule} = \frac{1}{N_A}\ [\frac{3}{2}\ RT]$$

$$= \frac{1}{N_A}\times\frac{3}{2}\ KN_A\ T = \frac{3}{2}\ KT$$

{N_A: Avogadro's number

K: Boltzmann constant}

$$\Rightarrow \frac{1}{2}\ m\ v^2_{rms} = \frac{3}{2}\ kT \text{ [m: molecular mass]}$$

$$\therefore\ T = \frac{mv_{rms}^{\ 2}}{3k} = \frac{2}{3k}\times \text{kinetic energy per molecule.}$$

∴ The average energy of the molecule is proportional to absolute temperature. Absolute zero is the temperature at which the kinetic energy of the molecules becomes zero. i.e. they stop.

Sol 8: Mean free path is the average distance travelled by the molecules between successive collisions.

Sol 9: Brownian motion is the random motion of particles in a fluid. Resulting from their collision with the other atoms or molecules in the fluid.

Sol 10: Volume of m kg of molecules = $\frac{m}{1000}$

Volume occupied by water vapour = $\frac{m}{0.6}$

$$\therefore \text{ Required ratio} = \frac{\frac{m}{1000}}{\frac{m}{0.6}} = 6\times 10^{-4}$$

Note: Here water is assumed to be completely compact. Therefore, by using density of water we calculate molecular volume.

Sol 11: $V = 3000\ cm^3 = 3000\times 10^{-6}\ m^3$

$T = 20°\ C = 293\ k$

$P = 2.5\times 10^6\ Pa + 10^5\ Pa = 2.6\times 10^6\ Pa$

$$n = \frac{PV}{RT} = \frac{2.6\times 10^6\times 3000\times 10^{-6}}{8.31\times 293}$$

$\simeq$ 3.20 c/moles

∴ Mass of O_2 in tank

$= 3.204\times 32 \simeq 102.5$ gm $= \simeq 0.103$ kg

Sol 12: $V_{rms} = \sqrt{\frac{3RT}{M_m}} = \sqrt{\frac{3\times 8.314\times 273}{64\times 10^{-3}}}$

$\simeq$ 326.18 m/s

Sol 13: Average time per collision

$$= \frac{2.66\times 10^{-7}m}{500m\ sec^{-1}} = 5.32\times 10^{-10}\ sec$$

∴ No. of collision per second

$$= \frac{1}{5.32\times 10^{-10}} \simeq 1.88\times 10^9\ sec$$

Sol 14: Mean free path estimate = $\frac{1}{\sqrt{2}\pi d^2 n_v}$

d: diameter of molecule, n_v molecules per unit volume

$n_V = 3\times 10^{19}\ cm^{-3} = 3\times 10^{25}\ m^{-3}$

$d = 2\times 10^{-10}$ m

$\therefore$ Mean free path

$$= \frac{1}{1.414 \times 3.14 \times (2 \times 10^{-10})^2 \times 3 \times 10^{25}}$$

$= 0.0187 \times 10^{-5}$ m

$\simeq 1.87 \times 10^{-7}$ m

Sol 15: Mean free path $(\overline{x}) = \dfrac{1}{\sqrt{2}\pi d^2 n_v}$

$n_v = \dfrac{nN_A}{V}$

n: number of moles in volume V

N_A = Avogadro's number = 6.02×10^{23}

By gas equation: - $\dfrac{n}{V} = \dfrac{P}{RT}$

$\therefore n_v = \dfrac{PN_A}{RT}$

$\therefore \overline{x} = \dfrac{RT}{\sqrt{2}\pi d^2 PNA}$

$$= \frac{8.314 \times 273}{1.414 \times 3.14 \times (2.4 \times 10^{-10})^2 \times 10^5 \times 6.02 \times 10^{23}}$$

$\simeq 1.47 \times 10^{-7}$ m

Sol 16: Water has higher potential energy. As we already know that kinetic energy depends on temperature. So, as we cool water to ice its K.E. remains constant at 0°C. Therefore, the heat energy removed from water, is accounted as decrease in potential energy of ice.

Sol 17: We know that internal energy is a state function and depends on temperature.

$\therefore \Delta U = 0 \rightarrow$ No change

Sol 18: Translational motion of molecules.

Sol 19: For isothermal

PV = constant

$\Rightarrow P\Delta V_1 + V\Delta P = 0$

$$\Rightarrow \frac{\Delta V_1}{\Delta P} = -\frac{-V}{P} \qquad \text{......(i)}$$

For adiabatic process

PV^{γ} = constant

$\Rightarrow V^{\gamma}\Delta P + P\gamma V^{\gamma-1}\Delta V_2 = 0$

$$\Rightarrow \frac{\Delta V_2}{\Delta P} = +\frac{1}{\gamma}\left(\frac{-V}{P}\right) \qquad \text{.......(ii)}$$

By (i) and (ii) we get

$$\frac{\Delta V_2}{\Delta P} = \frac{1}{\gamma}\left(\frac{\Delta V_1}{\Delta P}\right)$$

$$\Rightarrow \frac{\Delta V_1}{\Delta V_2} = \gamma$$

As r > 1

$\therefore \Delta V_1 > \Delta V_2$

Sol 20: ΔW = -200 J

ΔQ = 0 [adiabatic conditions]

By 1st law of thermodynamics:-

$\Delta Q = \Delta U + \Delta W$

$\Rightarrow \Delta U$ = + 200 J

Sol 21: The internal energy is the total energy contained by a thermodynamics system. It is the energy needed to create the system but excludes the energy due to external force fields. E.g. Kinetic, energy if the sample is moving or potential energy if the sample is at a height from the ground. Internal energy has two major components i.e. kinetic and internal potential energy.

Sol 22: Four thermodynamics process are:-

(i) Isothermal process: Temperature remains constant. i.e. PV = constant [$\because \Delta T = 0$]

$\therefore \Delta U = nC_V \Delta T = 0$

(ii) Adiabatic process:- No flow of heat either into or out from the system.

i.e. $\Delta Q = 0 = \Delta U + W$

$\therefore \Delta U = -W$

(iii) Isochoric process:- Volume of the system remains constant.

i.e. V = constant

$\Rightarrow \Delta V = 0$

$\therefore W = \int PdV = 0$

Thus, $\Delta\theta = \Delta U$

(iv) Isobaric process:- Pressure remains constant

i.e. P = constant

$\Delta P = 0$

$W = P(V_f - V_i)$

Indicator diagram: are graphical representation of cyclic variations of pressure and volume within a heat engine.

Sol 23: Sign Convention

$\Delta Q \rightarrow$ +ve for heat supplied to gas

$\rightarrow$ -ve for heat rejected by gas

$\Delta W \rightarrow$+ve for work done by gas

$\rightarrow$ -ve for work done on gas

Sol 24: Isothermal process

$\Delta T = 0$

$\therefore \Delta U = nC_v \Delta T = 0$

By 1st law of thermodynamics:-

$\therefore \Delta Q = \Delta U + \Delta W \Rightarrow \Delta Q = \Delta W$

No change in internal energy

Work done by system = heat supplied to the system.

Adiabatic Process:-

$\Delta Q = 0$

So, $\Delta Q = \Delta U + \Delta W$

$\Rightarrow \Delta U = -\Delta W$

No heat is supplied to the system

Change in internal energy = Work done on gas

Sol 25: Isothermal Operation: Operations/ process in which temperature stays constant.

Example:- System immersed in a large constant temperature ice-water bath.

Sol 26: Adiabatic operations are those operations in which neither heat enters or leaves a system.

Example:- A system having a perfectly non-conducting boundary.

$Q = \Delta U + W$

$\because Q = 0$

$\therefore \Delta U + W = 0 \Rightarrow \Delta U = -W$

Sol 27: $\Delta T = 0$

$\Delta U = nC_V\Delta T = 0$

$\therefore \Delta W = \Delta Q - \Delta U \Rightarrow \Delta W = \Delta Q$

$$\Delta W = \int_{V_i}^{V_f} PdV$$

We know, $PV = nRT \Rightarrow P = \dfrac{nRT}{V}$

$$\therefore \Delta W = \int_{V_i}^{V_f} \frac{nRTdV}{V} = nRT \ln V\Big|_{V_i}^{V_f} = nRT \ln \frac{V_f}{V_i}$$

$$\Rightarrow \Delta W = nRT \ln \frac{V_f}{V_i} = nRT \ln \frac{P_i}{P_f}$$

Sol 28: For adiabatic process

$\Delta Q = 0$

$\therefore \Delta U = -\Delta W$

$\therefore nC_VdT = -PdV$ ...(i)

By equation of gas:- $PV = nRT$

$\Rightarrow PdV + VdP = nRdT$

$$\therefore ndT = \frac{PdV + VdP}{R}$$

$\therefore$ Substituting in (i) we get :-

$$C_V\left(\frac{PdV + VdP}{R}\right) = -PdV$$

$\Rightarrow (C_V+R)\, PdV = -C_V\, VdP$

$$\Rightarrow \frac{C_PdV}{C_VV} = \frac{-dP}{P} \quad [\because C_V + R = C_P]$$

$$\Rightarrow \gamma\frac{dV}{V} = \frac{-dP}{P}$$

On integrating both sides, we get

$$\gamma\int_{V_i}^{V_f}\frac{dV}{V} = -\int_{P_i}^{P_f}\frac{dP}{P} \Rightarrow \gamma \ln\frac{V_f}{V_i} = \ln\frac{P_i}{P_f}$$

$$\Rightarrow \frac{P_i}{P_f} = \left(\frac{V_f}{V_i}\right)^{\gamma} \Rightarrow P_iV_i^{\gamma} = P_f\, V_f^{\gamma} = k \text{ (say)}$$

$$\therefore PV^{\gamma} = \text{const.} = k \Rightarrow P = \frac{k}{V^{\gamma}}$$

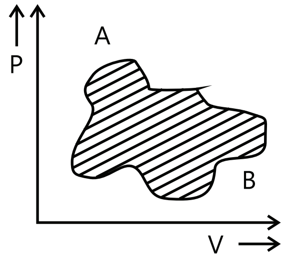
$$\therefore W = \int_{V_i}^{V_f} PdV = \int_{V_i}^{V_f}\frac{k}{V^{\gamma}}\, dV$$

$$\Rightarrow W = k\left.\frac{V^{-\gamma+1}}{-\gamma+1}\right|_{V_i}^{V_f} = \frac{kV_f^{1-\gamma} - KV_i^{1-\gamma}}{1-\gamma}$$

$$= \frac{(P_fV_f^{\gamma})V_f^{1-\gamma} - (P_iV_i^{\gamma})V_i^{1-\gamma}}{1-\gamma}$$

$$\Rightarrow W = \frac{P_fV_f - P_iV_i}{1-\gamma} = \frac{nR}{1-\gamma}(T_f - T_i)$$

Sol 29: Cyclic processes:- In cyclic process, thermodynamic process start from the same state and end at the initial state.



Since, initial and final states are same,

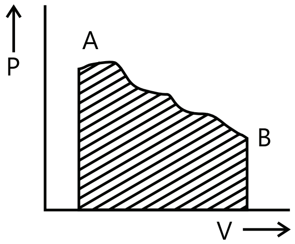
Thus $\Delta U = 0$,

$\therefore \Delta Q = \Delta W$

$\therefore |\Delta Q| = |\Delta W|$ = Area enclosed by the curve.

Sign of ΔW is '+' if process is clockwise and '–' if process in anti-clockwise.

Non-cyclic process: If initial and final states are not same.



$\Delta W = \int_A^B PdV$ = Area enclosed by P-V curve

Sol 30: Reversible process: - The process in which the system and surroundings can be restored to the initial state from the final state without producing any change in the thermodynamic properties of the universe.

Ex: - An infinitesimal compression of a gas in a cylinder.

Irreversible process:- In irreversible process system is not in equilibrium throughout the process. Initial state can't be obtained from final state without producing changes in the universe.

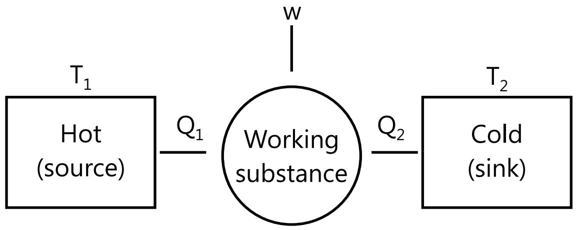
Example:- (i) Processes having friction.

(ii) Heat transfer through a finite temperature difference.

Sol 31: Heat engine is a device which convert heat energy into mechanical energy by using a cyclic process.

$W = Q_1 - Q_2$

Efficiency: $\eta = \dfrac{W}{Q_{given}}$



Here, Q_1 is the heat given to the substance whereas Q_2 is the heat released.

$\therefore Q_{given} = Q_1$

$$\therefore \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_\perp} = 1 - \frac{Q_2}{Q_1}$$

$$\therefore \text{Efficiency } (\eta) = 1 - \frac{\text{Heat released}}{\text{Heat absorbed}}$$

Two types of heat engines:-

(i) Internal combustion engines

(ii) External combustion engines

Sol 32: For adiabatic process

PV^γ = constant

We know $V = \dfrac{nRT}{P}$

$$\therefore P\left(\frac{nRT}{P}\right)^\gamma = \text{const.}$$

$\Rightarrow P^{1-\gamma}T^\gamma$ = const.

P_i = 3 atm; P_f = 1 atm

T_i = 300 K; T_f = ?

$$\therefore T_f^\gamma = \frac{P_i^{1-\gamma}T_i^\gamma}{P_f^{1-\gamma}}$$

$$\Rightarrow T_f^\gamma = \left(\frac{3}{1}\right)^{1-1.4}(300)^{1.4}$$

$$\Rightarrow T_f = 3^{\frac{-0.4}{1.4}} \times 300 \simeq 219.2\text{ K}$$

$\therefore$ Fall in temperature = $T_i - T_f$ = 80.8° C

Sol 33: T_i = 27°C = 300 K; T_f = ?

P_i = 1 atm; P_f = ?

$V_i = V$; $V_f = V/2$

(i) PV^γ = constant

$$\therefore P_f = \left(\frac{V_i}{V_f}\right)^\gamma P_i = (2)^{1.42} \times 1 \simeq 2.675\text{ atm}$$

(ii) PV^{γ} = constant

$\Rightarrow \frac{nRT}{V} V^{\gamma}$ = constant

$\Rightarrow TV^{\gamma-1}$ = constant

$\therefore T_f = \left(\frac{V_i}{V_f}\right)^{\gamma-1} T_i$

$= (2)^{0.42} \times 300$

$\Rightarrow T \approx 128.38\ °C$

Sol 34: $n = 1$

$T_i = T; T_f = (T + 70)$ K

$W = \frac{nR}{1-\gamma}[T_f - T_i] = \frac{1\times 2}{1-1.5}[70] = -280$ Cal

$\therefore$ Work done = -1176 joules

$\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = 280$ Cal

Work done on gas can be seen by increase in temperature of the gas, which accounts for 280 calories of energy.

Sol 35: $n = 1$

$V_i = V; \Delta Q = 0$

$V_f = 2V_i; \Delta U = -\Delta W = \frac{-nR}{1-\gamma}[T_f - T_i]$

$T_i = 273K$

$TV^{\gamma-1}$ = constant

$\therefore T_f = \left(\frac{V_i}{V_f}\right)^{\gamma-1} \times T_i$

$\Rightarrow T_f = (0.5)^{0.4} \times 273 = 206.89$ K

$\therefore \Delta U = -\frac{1\times 8.314}{1-1.4}[206.89-273]$

$\Rightarrow \Delta U \simeq -1374$ Joule

Sol 36: $V_i = V; V_f = 2V$

P_i = 760 mm of Hg= 1 atm $\simeq 10^5$ Pascal

$T_i = 273$ K, $n = \frac{1}{32}$

$\because \Delta Q = 0$

For adiabatic process:-

$TV^{\gamma-1}$ = constant

$\therefore T_f = \left(\frac{V_i}{V_f}\right)^{\gamma-1} T_i$

$= \left(\frac{1}{2}\right)^{1.4-1} \times 273 = 206.89$ K

$\therefore \Delta U = \frac{nR}{1-\gamma}(T_f - T_i)$

$= \frac{-\left(\frac{1}{32}\right)\times(2\times 4.2)}{1-1.4}(206.89 - 273)$

$\simeq -43.38$ Joule

Exercise 2

Sol 1: (A) $V = 7L = 7 \times 10^{-3}\ m^3$

$T = 273$ K

$P = 1.3 \times 10^5$ Pa

$\therefore m = \frac{PV}{RT} = \frac{1.3\times 10^5 \times 7\times 10^{-3}}{8.314\times 273} \simeq 0.4$ moles

$\therefore$ No. of molecules = $0.4 \times 6.02 \times 10^{23}$

$= 2.4 \times 10^{23}$ molecules

Sol 2: (A) $PV^{2/3}$ =const.

$\Rightarrow \frac{nRT}{V} V^{2/3}$ = const.

$\Rightarrow TV^{-1/3}$ = const.

$\Rightarrow T$ = conts. $\times V^{1/3}$

$\therefore$ On increasing volume, temperature increase

Sol 3: (C) T = constant, V = constant,

$n_i = \frac{35}{28} = 1.25$ Kmoles

$\therefore \frac{P}{n} = \frac{RT}{V}$ = const.

$\therefore \frac{P_f}{n_f} = \frac{P_i}{n_i}$

$\Rightarrow n_f = \frac{P_f}{P_i} \times n_i = \frac{9}{6} \times \frac{35}{28} \approx 1.875$ kmoles

$\therefore$ Moles of O_2 supplied

= 1.875 – 1.25 = 0.625 kmoles

$\therefore$ Amount of oxygen = 20 kg

Sol 4: (B) $PV = \frac{4}{M}RT$ .....(i)

$PV = \frac{(4-0.8)}{M}R(T+50) = \frac{3.2R}{M}(T+50)$ .....(ii)

By (i) and (ii) we get,

$$\frac{4}{M}RT = \frac{3.2R}{M}(T+50) \Rightarrow 4T = 3.2T + 3.2 \times 50$$

$\Rightarrow T = 200$ K

Sol 5: (A) For P = 1 atm, m_1 = 2gms, T=298 K

$\therefore\ 1\times V = \frac{2R}{M_1}\times 298$ ....(i)

For P = 1.5 atm, m_1=2 gm, m_2=3 gm, T=298 K.

$\therefore\ 1.5\times V = \left[\frac{2}{M_1}+\frac{3}{M_2}\right]R\times 298$ ....(ii)

Using (ii)/(i) we get,

$$1.5 = \frac{\left[\frac{2}{M_1}+\frac{2}{M_2}\right]}{\frac{2}{M_1}} \Rightarrow 3 = \left[2+\frac{3M_1}{M_2}\right] \Rightarrow \frac{M_1}{M_2} = \frac{1}{3}$$

Sol 6: (B) $P = \frac{nRT}{V}$

$$\therefore\ \left(\frac{nRT}{V}\right)^2 V = \text{constant} \Rightarrow \frac{T^2}{V} = \text{constant}$$

$$\therefore\ T_f = \left(\frac{V_f}{V_i}\right)^{\frac{1}{2}} \times T_i = \sqrt{2}T$$

Sol 7: (A) $P_{\text{at top of tube}} + P_{\text{due to 50 cm of Hg}} = P_{\text{atmospheric}}$

$\Rightarrow P_{\text{at top of tube}} = (75 - 50)$ cm of Hg

= 25 cm of Hg

$= 25 \times \frac{10^5}{75}$ Pa = 33.3 k Pa

Sol 8: (C)

For O_2	For H
n = 1	n = 1
T	2T
P	P′
V	V
$\therefore\ PV = 1\times RT$	$\therefore P'V = 1\times R\times(2T)$
$\Rightarrow PV = RT$	$\Rightarrow P' = 2P$

Sol 9: (D) For diatomic molecule

Translational degree of freedoms = 3

Rotational degree of freedoms = 2

Ratio $= \frac{3KT}{2KT} = \frac{3}{2}$

Sol 10: (D) We know

$v_{rms} = 1.085\ v_{avg}$

$\Rightarrow \sqrt{\frac{3P}{\rho}} = 1.085\ v_{avg}$; ρ is density

In this case ρ is constant

$\therefore\ P \propto (v_{avg})^2$

$$\therefore\ \frac{P_f}{P_i} = \left[\frac{(v_{avg})_f}{(v_{avg})_i}\right]^2 = 4$$

Sol 11: (C) Frictionless piston means, no generation of heat due to the motion of piston.

n = 1

$T_i = T$

P = const.

$V_i = V$ $V_f = 2V$

Initially:- PV = RT

Work done = $P\Delta V = P(2V - V) = PV = RT$

Sol 12: (A) f = 6

$\Delta W = 25 \Rightarrow P\Delta V = 25 \Rightarrow nR\Delta T = 25$ J

[By using gas equation]

$\therefore$ P = constant

$\Delta Q = \Delta U + \Delta W$ [By 1st law of thermodynamics]

and $\Delta U = \frac{nf}{2}R\Delta T = \frac{6}{2}\times 25 = 75$ J

$\therefore\ \Delta Q = 100$ J

Alternative Method

At constant pressure, $\Delta Q = nC_P\,\Delta T = C_P\frac{P\Delta V}{R}$

[$\Rightarrow PV = nRT$

$P\Delta V = nR\Delta T$ at const. pressure]

$\Delta W = P\Delta V = 25$ J

$f = 6; \qquad C_P = \left(1+\frac{f}{2}\right) R = 4R$

$\therefore \Delta Q = (4R)\left[\frac{25}{R}\right] = 100$ J

Sol 13: (B) $\Delta Q = -30$ J

$\Delta W = -18$ J

By 1st law of thermodynamics

$\Delta Q = \Delta U + \Delta W$

$\Rightarrow \Delta U = -30 - (-18) = -12$ J

$\Rightarrow U_B - U_i = -12$

$\therefore U_B = -12 + 60 = 48$ Joules

Sol 14: (A) $T_i = 300$ K $\qquad T_f = ?$

$P_i = P \qquad P_f = 4P$

By gas equation we know: - $V = \frac{nRT}{P}$

$\therefore PV^{4/3}$ = constant

$\Rightarrow P\left(\frac{nRT}{P}\right)^{\frac{4}{3}} = \text{constant} \Rightarrow \frac{T^{\frac{4}{3}}}{P^{\frac{1}{3}}} = \text{constant}$

$\therefore T_f = \left(\frac{P_f}{P_i}\right)^{\frac{1}{3}\div\frac{4}{3}} \times T_i = \left(\frac{4P}{P}\right)^{\frac{1}{4}} \times 300$ K

$\Rightarrow T_f = 300\sqrt{2}$ K

Sol 15: (D) For first kind of expansion:-

$\Delta W = 20$ KJ

$\Delta Q = 16$ KJ

$\therefore \Delta U = \Delta Q - \Delta W = -4$ KJ

Since, U is a state function. Therefore, value of ΔU in both expansions remain same.

Thus, for second expansion:-

$\Delta U = -4$KJ, $\Delta Q = 9$ KJ

$\therefore$ By first law of the thermodynamics:-

$\Delta W = \Delta Q - \Delta U = 13$ KJ

Sol 16: (A) $n_{N_2} = \frac{7\text{kg}}{28} = 0.25$ k moles

$f_{N_2} = \frac{2}{\gamma-1} = \frac{2}{1.4-1} = 5$

$n_{CO_2} = \frac{11\text{kg}}{44} = 0.25$ k moles

$f_{CO_2} = \frac{2}{1.3-1} = \frac{2}{0.3} = \frac{20}{3}$

$\therefore$ Equivalent molecular weight

$= \frac{n_1M_1 + n_2M_2}{n_1+n_2} = \frac{m_1+m_2}{n_1+n_2}$

$= \frac{7+11}{0.25+0.25} = 36$ gm

$\gamma_{mix} \neq \frac{n_1\gamma_1 + n_2\gamma_2 + ...}{n_1+n_2+.....}$

$f_{mix} = \frac{n_1f_1 + n_2f_2 + ...}{n_1+n_2}$

$= \frac{\left(0.25\times 5 + 0.25\times\frac{20}{3}\right)}{0.25+0.25} \simeq \frac{35}{6}$

$\therefore \gamma_{mix} = 1 + \frac{2}{f_{mix}} = 1 + \frac{2}{35/6} = \frac{47}{35} = 1.34$

Previous Years' Questions

Sol 1: (A) Work done in a cyclic process = area between the cycle

= AB × BC = (2P – P) × (2V – V) = PV

Note if cycle is clockwise (p on y-axis and V on x-axis) work done is positive and if it is anticlockwise work done is negative.

Sol 2: (A) $v_{rms} = \sqrt{\frac{3RT}{M}}$

Room temperature T = 300 K

$\therefore 1930 = \sqrt{\frac{3\times 8.31\times 10^3 \times 300}{M}}$

$\therefore$ M = 2.0 g/mol or the gas is H_2.

Sol 3: (B) $Q_1 = nC_P\,\Delta T$, $Q_2 = nC_V\,\Delta T$, $\frac{Q_2}{Q_1} = \frac{C_V}{C_P} = \frac{1}{\gamma}$

or $Q_2 = \frac{Q_1}{\gamma} = \frac{70}{1.4} = 50$ cal

Sol 4: (B) $\gamma_1 = \dfrac{5}{3}$ means gas is monatomic or $C_{V_1} = \dfrac{3}{2}R$

$\gamma_2 = \dfrac{7}{5}$ means gas is diatomic or $C_{V_2} = \dfrac{5}{2}R$

C_V (of the mixture)

$$= \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2} = \frac{(1)\left(\frac{3}{2}R\right) + (1)\left(\frac{5}{2}\right)R}{1+1} = 2R$$

C_P (of the mixture) = $C_V + R = 3R$

$$\therefore \gamma_{\text{mixture}} = \frac{C_P}{C_V} = \frac{3R}{2R} = 1.5$$

Sol 5: (B) $v_{rms} = \sqrt{\dfrac{3RT}{M}}$

i.e., $V_{rms} \propto \sqrt{T}$

When temperature is increased from 120 K to 480 K (i.e., four times), the root mean square speed will become $\sqrt{4}$ or 2 times i.e., 2v.

Sol 6: (D) The average translational KE = $\dfrac{3}{2}$ kT which is directly proportional to T, while rms speed of molecules is given by

$v_{rms} = \sqrt{\dfrac{3RT}{M}}$ i.e., $v_{rms} \propto \sqrt{T}$

When temperature of gas is increased from 300 K to 600 K (i.e., 2 times), the average translational KE will increase to 2 times and rms speed to $\sqrt{2}$ or 1.414 times.

$\therefore$ Average translational KE = $2 \times 6.21 \times 10^{-21}$ J

$= 12.42 \times 10^{-21}$ J

And $v_{rms} = (1.414)(484)\text{m/s} \approx 684$ m/s

Sol 7: (C) PV = nRT or $P = \dfrac{nRT}{V}$ or $P \propto T$

If V and n are same. Therefore, if T is doubled pressure also becomes two times i.e., 2p.

Sol 8: (C) Process is isothermal. Therefore

T = constant. $\left(p \propto \dfrac{1}{V}\right)$

Volume is increasing, therefore, pressure will decrease.

In chamber A →

$$-\Delta P = (p_A)_i - (p_A)_f = \frac{n_ART}{V} - \frac{n_ART}{2V}$$

$$= \frac{n_ART}{2V} \qquad \text{..... (i)}$$

In chamber B →

$$-1.5\Delta P = (p_B)_i - (p_B)_f = \frac{n_BRT}{V} - \frac{n_BRT}{2V}$$

$$= \frac{n_BRT}{2V} \qquad \text{....... (ii)}$$

From Equation (i) and (ii)

$\dfrac{n_A}{n_B} = \dfrac{1}{1.5} = \dfrac{2}{3}$ or $\dfrac{m_A/M}{m_B/M} = \dfrac{2}{3}$ or $\dfrac{m_A}{m_B} = \dfrac{2}{3}$ or $3m_A = 2m_B$

Sol 9: (D) Internal energy of n moles of an ideal gas at temperature T is given by

$$U = n\left(\frac{f}{2}RT\right)$$

where, f = degrees of freedom

= 5 for O_2 and 3 for Ar

Hence, $U = U_{O_2} + U_{Ar}$

$$= 2\left(\frac{5}{2}RT\right) + 4\left(\frac{3}{2}RT\right) = 11\ RT$$

Sol 10: (D) During adiabatic expansion, we know

$TV^{\gamma-1}$ = constant

or $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

For a monoatomic gas, $\gamma = \dfrac{5}{3}$

$$\therefore \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{AL_2}{AL_1}\right)^{\frac{5}{3}-1}$$

(A = Area of cross-section of piston)

$$= \left(\frac{L_2}{L_1}\right)^{2/3}$$

Sol 11: (C) pT^2 = constant

$\therefore \left(\dfrac{nRT}{V}\right)T^2$ = constant or T^3V^{-1}= constant

Differentiating the equation, we get

$$\frac{3T^2}{V}.dT - \frac{T^3}{V^2}\ dV = 0$$

or $3dT = \frac{T}{V}.dV$ .... (i)

From the equation, $dV = V\gamma\, dT$

γ = coefficient of volume expansion of gas $= \frac{dV}{V.dT}$

From Equation (i) $\gamma = \frac{dV}{V.dT} = \frac{3}{T}$

Sol 12: A → s; B → r, p; C → p; D → s

In process J → K V is constant whereas p is decreasing. Therefore, T should also decrease.

∴ W = 0, ΔU = – ve and Q < 0

In process K → L p is constant while V is increasing. Therefore, temperature should also increase.

W > 0, ΔU > 0 and Q > 0

In process L → M This is inverse of process J → K.

∴ W = 0, ΔU > 0 and Q > 0

In process M → J

V is decreasing. Therefore, W < 0

$(PV)_J < (PV)_M$

∴ $T_f < T_M$ or $\Delta U < 0$

Therefore, Q < 0

Sol 13: A → p, r, t; B → p, r; C → q, s; D → r, t

Internal energy $\propto T \propto PV$

This is because $U = \frac{nf}{2}RT = \frac{f}{2}(PV)$

Here, n = number of moles, f = degree of freedom.

∴ If the product PV increases then internal energy will increase and if product decreases the internal energy will decrease.

Further, work is done on the gas, if volume of gas decreases. For heat exchange,

$Q = W + \Delta U$

Work done is area under p–V graph. If volume increases work done by gas is positive and if volume decrease work done by gas is negative. Further ΔU is positive if product of PV is increasing and ΔU is negative if product of PV is decreasing.

If heat is taken by the gas, Q is positive and if heat is lost by the gas, Q is negative.

Keeping the above points in mind the answer to this question is as under.

(A) → (p, r, t); (B) → (p, r);

(C) → (q, s); (D) → (r, t)

Sol 14: (A) $\Delta U = nC_V\, \Delta T = nC_V\,(T_2 - T_1)$ in all processes.

(B) In adiabatic process $\Delta Q = 0$

∴ $\Delta U = -\Delta W$ or $|\Delta U| = |\Delta W|$

(C) In isothermal process $\Delta T = 0$

∴ $\Delta U = 0$ (as $\Delta U = nC_V\Delta T$)

(D) In adiabatic process $\Delta Q = 0$

Sol 15: (A, B) $T_A = T_B$ ∴ $U_A = U_B$

$$W_{AB} = (1)(R)T_0 \ln\left(\frac{V_f}{V_i}\right) = RT_0 \ln\left(\frac{4V_0}{V_0}\right) = p_0V_0 \ln(4)$$

Information regarding p and T at C cannot be obtained from the given graph. Unless it is mentioned that line BC passes through origin or not.

Sol 16: (D) No option is correct

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{\gamma_1 M_2}{\gamma_2 M_1}} = \sqrt{\frac{\frac{7}{5}\times 4}{\frac{5}{3}\times 32}}$$

$$\frac{460}{v_2} = \sqrt{\frac{21}{25\times 8}} \Rightarrow v_2 = \frac{460\times 5\times 2\sqrt{2}}{\sqrt{21}} = 1420 \text{ m/s}$$

Sol 17: (A) $U = U_1 + U_2$

$$T = \frac{(P_1V_1 + P_2V_2)T_1T_2}{(P_1V_1T_2 + P_2V_2T_1)}$$

Sol 18: (B) Thermal energy corresponds to internal energy

Mass = 1 kg

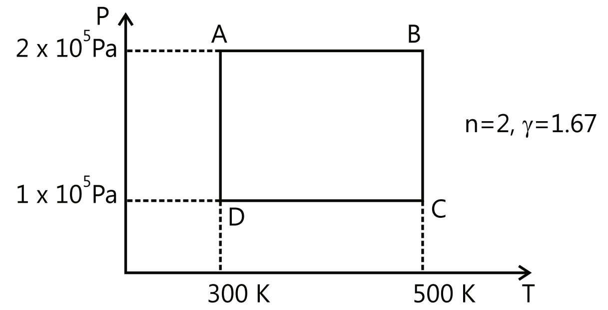
Density = 8 kg/m^3

$\Rightarrow$ Volume $= \frac{\text{mass}}{\text{density}} = \frac{1}{8}\text{m}^3$

Pressure = 8×10^4 N/m^2

∴ Internal Energy $= \frac{5}{2}P \times V = 5 \times 10^4$ J

Sol 19: (C) $W_{AB} = \Delta Q - \Delta U = {}_nC_p dT - {}_nC_v dT$ (at constant pressure)



$= {}_n(C_p - C_v)dt$

$= nRdT = 2 \times R \times (500 - 300) = 400\ R$

Sol 20: (A) At constant temperature (isothermal process)

$$W_{DA} = nRT\ln\left(\frac{P_1}{P_2}\right) = 2.303 \times 2R \times 300\log\left(\frac{10^5}{2\times10^5}\right)$$

$$= 2.303 \times 600\log\left(\frac{1}{2}\right)$$

$$= 0.693 \times 600lR = -414R$$

Sol 21: (B) Net work done in a cycle = $W_{AB} + W_{BC} + W_{CB} + W_{BA}$

$= 400\ R + 2 \times 2.303 \times 500\ R \ln 2 - 400R - 414\ R$

$= 1000R \times \ln 2 - 600R \times \ln 2 = 400R \times \ln 2 = 276R$

Sol 22: (B) The efficiency of cycle is

$$\eta = 1 - \frac{T_2}{T_1}$$

for adiabatic process

$TV^{\gamma-1} = \text{constant}$

For diatomic gas $\gamma = \frac{7}{5}$

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$T_1 = T_2(32)^{\frac{7}{5}-1} = T_2(2^5)^{2/5} = T_2 \times 4$$

$T_1 = 4T_2$

$$\eta = \left(1 - \frac{1}{4}\right) = \frac{3}{4} = 0.75$$

Sol 23: (C) $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$

$U(x = \infty) = 0$

as, $F = -\frac{dU}{dx} = -\left[\frac{12a}{x^{13}} + \frac{6b}{x^7}\right]$

At equilibrium, F = 0

$\therefore x^6 = \frac{2a}{b}$

$$\therefore U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = \frac{-b^2}{4a}$$

$$\therefore D = [U(x = \infty) - U_{\text{at equilibrium}}] = \frac{b^2}{4a}$$

Sol 24: (D) $\eta_1 = \frac{T_1 - T_2}{T_1} = \frac{1}{6}$

and $\eta_2 = \frac{T_1 - (T_2 - 62)}{T_1} = \frac{1}{3} \Rightarrow \frac{T_1 - T_2}{T_1} + \frac{62}{T_1} = \frac{1}{3}$

$\frac{1}{6} + \frac{62}{T_1} = \frac{1}{3} \Rightarrow \frac{62}{T_1} = \frac{1}{6}$

$\therefore T_1 = 62 \times 6 = 372K$

$\frac{T_1 - T_2}{T_1} = \frac{1}{6} \quad \Rightarrow 1 - \frac{T_2}{T_1} = \frac{1}{6}$

$\Rightarrow \frac{T_2}{372} = \frac{5}{6} \quad \Rightarrow T_2 = 310K$

Sol 25: (A) Work done in complete cycle = Area under P–V graph = P_0V_0

From A to B, heat given to the gas

$= {}_nC_v\Delta T = n\frac{3}{2}R\Delta T = \frac{3}{2}V_0\Delta P = \frac{3}{2}P_0V_0$

From B to C, heat given to the system

$$= {}_nC_v\Delta T = n\left(\frac{5}{2}R\right)\Delta T = \frac{5}{2}(2P_0)\Delta V = 5P_0V_0$$

From C to D and D to A, heat is rejected.

$$\text{Efficiency, } \eta = \frac{\text{Work done by gas}}{\text{Heat given to the gas}} \times 100$$

$$\eta = \frac{P_0V_0}{\frac{3}{2}P_0V_0 + 5P_0V_0} = 15.4\%$$

Sol 26: (C) $\frac{40}{100}=\frac{500-T_S}{500}, T_S=300K$

$$\frac{600}{100}=\frac{T-300}{T}\Rightarrow T=750K$$

Sol 27: (C)

$\frac{Mg}{A}=P_0$ $\qquad$ $P_0V_0^{\gamma}=PV^{\gamma}$

$Mg=P_0A$.............(i) $\qquad$ $P_0AX_0^{\gamma}=PA(x_0-x)^{\gamma}$

Let piston is displaced by x $\qquad$ $P=\frac{P_0Ax_0^{\gamma}}{(x_0-x)^{\gamma}}$

$$Mg=\left(\frac{P_0x_0^{\gamma}}{(x_0-x)^{\gamma}}\right)A=F_{restoring}$$

$$P_0A\left(1-\frac{x_0^{\gamma}}{(x_0-x)^{\gamma}}\right)=F_{restoring} \qquad [x_0-x\approx x_0]$$

$$F=\frac{\gamma P_0Ax}{x_0}$$

$$\therefore f=\frac{1}{2\pi}\sqrt{\frac{\gamma P_0A}{x_0M}} \qquad =\frac{1}{2\pi}\sqrt{\frac{\gamma P_0A^2}{MV_0}}$$

Sol 28: (D) $\Delta U={}_nC_V\Delta T=1\times\frac{5R}{2}\Delta T$

For BC, ΔT = –200 K

$\Rightarrow \Delta U$ = –500R

Sol 29: (C) $p=\frac{1}{3}\frac{U}{V}$

$$\frac{nRT}{V}\propto\frac{1}{3}T^4$$

$$VT^3=\text{const}$$

$$\frac{4}{3}\pi R^3T^3=\text{const}$$

$$TR=\text{const}\Rightarrow\ T\propto\frac{1}{R}$$

Sol 30: (B) Since entropy is a state function, therefore change in entropy in both the processes should be same.

Sol 31: (C) Since $\tau=\frac{1}{n\pi\sqrt{2}v_{rms}d^2}$

$n\propto\frac{1}{V}$ and $v_{rms}\propto\sqrt{T}$

$$\Rightarrow\tau\propto\frac{V}{\sqrt{T}}$$

$$n=C_1V^{-1}\ \ <v>=C_2T^{1/2}$$

Since $TV^{\gamma-1}=$ constant $\Rightarrow\tau\propto V^{\frac{\gamma+1}{2}}$

Sol 32: (D) $P-P_0=-\frac{P_0}{V_0}(V-2V_0)$

$$P=3P_0-\frac{P_0}{V_0}V \qquad \text{...(i)}$$

$$\frac{nRT}{V}=3P_0-\frac{P_0}{V_0}V$$

$$nRT=3P_0V-\frac{P_0}{V_0}V^2$$

Differentiate w.r.t. Volume

$$3P_0-\frac{2P_0}{V_0}V=0\ \Rightarrow\ V=\frac{3V_0}{2}$$

Putting in equ (i)

$$P=3P_0-\frac{P_0}{V_0}\left(\frac{3V_0}{2}\right)=\frac{3P_0}{2}$$

Now, PV = xRT

$$\frac{9P_0V_0}{4}=nRT\Rightarrow T=\frac{9}{4}\frac{P_0V_0}{xR}$$

Sol 33: (A) $C=C_V+\frac{R}{1-n}$

$$C-C_V=\frac{C_P-C_v}{1-n};\ 1-n=\frac{C_P-C_v}{C-C_v}$$

$$n=1-\frac{C_P-C_v}{C-C_v}=\frac{C-C_P}{C-C_v}$$

JEE Advanced/Boards

Exercise 1

Sol 1:

For O_2	For He
$P = P_0$	$P = P_0$
$V = V_0$	$V = V_0$
$T = T_0$	$T = T_0/2$
M = 32 gm	M = 4gm
	PV = n RT

$$\Rightarrow \frac{m}{M}\,T = \frac{PV}{R} = \text{constant}$$

$$\therefore \frac{m_{O_2}}{M_{O_2}}\,T_{O_2} = \frac{m_{He}}{M_{He}}\,T_{He}$$

$$\Rightarrow \frac{m_{O_2}}{32} \times T_0 = \frac{m_{He}}{4} \times T_0/2 \Rightarrow \frac{m_{O_2}}{m_{He}} = 4$$

Sol 2: We have, $V = \left(\frac{nR}{P}\right)$ T and we know that V – T curve is a straight line.

$\therefore$ Pressure is constant and its slope = $\frac{nR}{P} = \tan 53^{o}$

$$\Rightarrow \frac{2 \times 8.314 \times 10^3 \text{Pa L / moleK}}{P} = \frac{4}{3}$$

$\Rightarrow P \approx 1.25 \times 10^4$ Pa

Sol 3: PV^{γ} = constant

$\Rightarrow TV^{\gamma-1}$ = constant

$\Rightarrow V^{\gamma-1} dT + (\gamma\text{-}1)\ V^{\gamma-2} T\, dV = 0$

$$\Rightarrow \frac{dV}{dt} = \frac{-V}{(\gamma-1)T} \Rightarrow \left|\frac{dV}{dT}\right| = \frac{V}{(\gamma-1)T}$$

$$\Rightarrow m = \frac{+V_0}{(\gamma-1)T_0} \Rightarrow \frac{1}{\gamma-1} = +\frac{mT_0}{V_0}$$

$$\therefore C_V = \frac{R}{\gamma-1} = +\frac{mT_0R}{V_0}$$

And $C_p = C_V + R$

$$= \frac{(V_0 + mT_0)R}{V_0} = \frac{mRT_0}{V_0}\left(1 + \frac{T_0 m}{V_0}\right)R$$

Sol 4: C_P' and C_v' be molar specific heats.

$\therefore C_P' = C_P \times M = 0.2$ M cal/mole °C

$C_V' = C_V \times M = 0.15$ M cal/mole °C

and $C_P' - C_V' = R$

$\Rightarrow$ 0.2 M - 0.15 M = 2

$$\Rightarrow M = \frac{2}{0.05} = 40 \text{ gms}$$

Sol 5: f = 6

At constant pressure

$\Delta Q = nC_P\, \Delta T$

$$= C_P \times \frac{P\Delta V}{R}$$

[$PV = nRT \Rightarrow P\,\Delta V = nR\,\Delta T$ at constant pressure]

$$= \frac{C_P}{R} \times \Delta W$$

$$C_P = \left(1 + \frac{f}{2}\right) R = 4R$$

And ΔW = 25 J

$$\Delta Q = \frac{4R}{R} \times 25 = 100 \text{ J}$$

Sol 6:

He	N_2
m = 4 gm	m_{N_2} = 28 gm
$\therefore n_{He} = \frac{4}{4} = 1$ mole	$n_{N2} = \frac{28}{28} = 1$ mole
f = 3	f = 5
$\therefore C_V = \frac{3}{2}$	$\therefore C_V = 5/2\ R$

$$\therefore C_{V\,mix} = \frac{n_{He}C_{V_{He}} + n_{N_2}C_{V_{N_2}}}{n_{He} + n_{N_2}}$$

$$\therefore C_{V\,mix} = \frac{\frac{3}{2}R + \frac{5}{2}R}{1+1} = 2R$$

We know $v_{rms} = \sqrt{\frac{3RT}{M}}$

$\therefore$ To double the v_{rms} temperature must be made 4 times the original temperature.

$\therefore T_f = 4\ T_i = 1200$ K

$\therefore \Delta T = 900$ K

$\therefore$ $\Delta Q = n\,C_V\,\Delta T$ [At constant volume]

$$= (n_{He} + n_{N_2})\,C_{V_{mix}}\,\Delta T$$

$$= 2 \times 2R \times 900 = 3600\ R$$

Sol 7: T = constant

$\therefore \Delta U = 0$

Thus $\Delta W = -23.04 \times 10^2$ J

$$\Rightarrow nRT \ln \frac{V_f}{V_i} = -23.04 \times 10^2$$

$$\Rightarrow 8.31 \times T \ln \frac{0.25}{0.50} = -23.04 \times 10^2$$

$$\therefore T = \frac{+23.04 \times 10^2}{8.31 \times \ln 2} \approx 400\ k$$

Sol 8: Let C be the molar heat capacity of this gas.

$\because \Delta Q = nC\Delta T$

And we know $\Delta U = nC_V\,\Delta T$

$\therefore \Delta Q = 2\Delta U$

$\therefore nC\,\Delta T = 2nC_V\,\Delta T$

$$\Rightarrow C = 2C_V = 2\left(\frac{5}{2}R\right) = 5R$$

Sol 9: C → Molar heat capacity of the process

By 1st law of thermodynamics:-

$dQ = dU + dW$

$\Rightarrow nC\,dT = nC_v\,dT + dW$ .....(i)

$dW = PdV$

$$= \frac{nRT}{V}dV = nR\frac{T_0e^{\alpha V}}{V}dV$$

$T = T_0\,e^{\alpha V}$

$\therefore dT = \alpha T_0\,e^{\alpha V}\,dV$

$$\Rightarrow dV = \frac{dT}{\alpha T_0 e^{\alpha V}}$$

$$\Rightarrow dW = \frac{nR}{\alpha V}\,dT$$

Substituting value of dW in (i) we get

$$nCdT = nC_V\,dT + \frac{nR}{\alpha V}dT$$

$$\therefore C = C_V + \frac{R}{\alpha V}$$

Sol 10: From graph, it is clear that V and 1/T have a linear relationship

$$\therefore V = \frac{m}{T}$$

m is the slope of straight line

$\Rightarrow VT$ = constant

$\Rightarrow PV^2$ = constant = k (say)

$$\Delta W = \int_{V_i}^{V_f} PdV = \int_{V_i}^{V_f} kV^{-2}dV$$

$$= -kV^{-1}\Big|_{V_i}^{V_f} = -[k\,V_f^{-1} - kV_i^{-1}]$$

$$= -[(P_fV_f^2)\,v_f^{-1} - (P_iV_i^2)\,V_i^{-1}] = -[P_fV_f - P_iV_i]$$

$= -nR\Delta T.$

Thus $\Delta Q = \Delta U + \Delta W$

$\Rightarrow nC\,\Delta T = nC_V\,\Delta T - nR\,\Delta T$

$\Rightarrow C = C_V - R$

$$\Rightarrow C = \frac{3}{2}R - R = \frac{R}{2}$$

Sol 11: (i) $\Delta W = \int_{V_i}^{V_f} PdV = \int_{V_i}^{V_f} aVdV$

$$= \frac{aV^2}{2}\Bigg|_{V_i}^{V_f} = \frac{aV_f^2 - aV_i^2}{2} = \frac{\left(\frac{P_f}{V_f}\right)V_f^2 - \left(\frac{P_i}{V_i}\right)V_i^2}{2}$$

$$\left(a = \frac{P_f}{V_f} = \frac{P_i}{V_i}\right)$$

$$\Rightarrow \Delta W = \frac{P_fV_f - P_iV_i}{2} = \frac{nR\Delta T}{2}$$

$$\Delta U = nC_V\,\Delta T = \frac{nR}{\gamma - 1}\,\Delta T$$

$\Delta Q = \Delta W + \Delta U$

$$= \frac{nR\Delta T}{2} + \frac{nR}{\gamma-1}\,\Delta T = nR\Delta T\left[\frac{1}{2} + \frac{1}{\gamma-1}\right]$$

$$= \frac{nR\Delta T}{2}\left[\frac{\gamma+1}{\gamma-1}\right] \quad ....(i)$$

We know P = aV

$$\Rightarrow \frac{nRT}{V} = aV \Rightarrow T = \frac{aV^2}{nR}$$

$$\therefore T_i = \frac{aV_0^2}{R} \text{ ad } T_f = \frac{9aV_0^2}{R}$$

$$\therefore\ \Delta T = \frac{8aV_0^2}{R} \qquad .....(ii)$$

Substituting (ii) in (i)

Thus $\Delta Q = \frac{R}{2} \times \frac{8aV_0^2}{R}\left(\frac{\gamma+1}{\gamma-1}\right) = \left(\frac{\gamma+1}{\gamma-1}\right) 4aV_0^2$

(ii) C → Heat capacity of gas

So $\Delta Q = nC\Delta T$

Using (i) we get

$$nC\Delta T = \frac{nR\Delta T}{2}\left(\frac{\gamma+1}{\gamma-1}\right)$$

$$\Rightarrow C = \left(\frac{\gamma+1}{\gamma-1}\right)\frac{R}{2}$$

Sol 12: At A

$V_A = V_0$, $T_A = 400$ K

$$\therefore\ P_A = \frac{400nR}{V_0}$$

At B

$V_B = 2V_0$, $T_B = 400$ K, $P_B = \frac{400nR}{2V_0}$

At C

$V_C = ?$, $T_C = ?$, $P_C = \frac{400nR}{2V_0}$

$\because$ AC is adiabatic process

$\therefore\ PV^\gamma = \text{constant}$

$$\Rightarrow V_C = \left(\frac{P_A}{P_C}\right)^{\frac{1}{\gamma}} V_A = (2)^{\frac{1}{1.5}} V_0 \Rightarrow V_C = 2^{2/3}V_0$$

$$\Rightarrow T_C = \frac{P_CV_C}{nR} = 400 \times (2)^{-1/3}$$

For process AB

$\Delta U = 0$

$$\Delta Q = \Delta W = nR \times (400) \times \ln\left(\frac{2V_0}{V_0}\right)$$

$= 400\ nR \ln 2$

For process BC

$$\Delta U = nC_V\,\Delta T = \frac{nR}{\gamma-1} \times [400\,(2)^{-1/3} - 400]$$

$= -2 \times 400\ n\ R\ [1-2^{-1/3}]$

$$\Delta Q = nC_P\Delta T = -\frac{n\gamma R}{\gamma-1} \times 400\ [1-2^{-1/3}]$$

$= -3 \times 400\ n\ R\ [1-2^{-1/3}]$

$\Delta W = -400\ n\ R\ [1-2^{-1/3}]$

For process AC

$\Delta Q = 0$

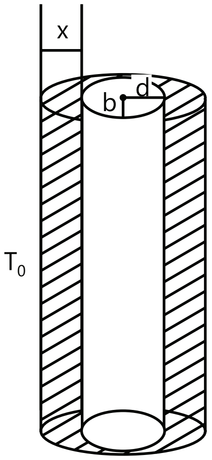
$\Delta U = nC_V\Delta T = 2 \times 400\ nR\ [1 - 2^{-1/3}]$

$\Delta W = -\Delta U = -2 \times 400\ nR\ [1-2^{-1/3}]$

$$\text{Efficiency (e)} = 1 - \frac{Q_{\text{released}}}{Q_{\text{absorbed}}}$$

$$= 1 - \frac{3 \times 400nR[1-2^{-1/3}]}{400nR\ \ln 2} = 1 - \frac{3(1-2^{-1/3})}{\ln 2}$$

Sol 13: $\frac{dQ}{dt} = \frac{-kAdT}{dx} = \frac{-k[2\pi(b-x)\times \ell]dT}{dx}$



On integrating with respect to dx we get

$$\Rightarrow \int_0^{b-a} \frac{dQ}{dt} \times \frac{1}{2\pi(b-x)}\,dx = \int_a^{b-a} -k\ell\frac{dT}{dx} \times dx$$

$$\Rightarrow \frac{dQ}{dt} \times \frac{-\ln}{2\pi}(b-x)\Big|_0^{b-a} = -\,k\ell\Delta T$$

$$\frac{dQ}{dt}.\ln\frac{b}{a} = -2\pi kl(T-T_0)$$

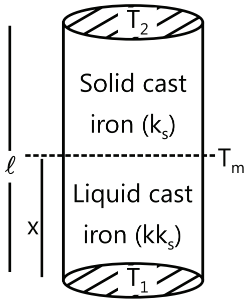
$$\left[\left(\pi a^2 l\right)3\frac{dT}{dx}\right]\ln\left(\frac{b}{a}\right) = -2\pi kl(T-T_0)$$

⇒ On integrating w.r.t. t we get

$$\Rightarrow \frac{-\pi a^2 ls\ln\left(\frac{b}{a}\right)}{2\pi kl}\int_{T_\perp}^{T_2}\frac{dT}{T-T_0} = \int_0^t dt$$

$$\Rightarrow \frac{a^2s}{2K}\ln\left(\frac{b}{a}\right)\ \ln\left(\frac{T_0-T_1}{T_0-T_2}\right) = t$$

Sol 14: $k_S \to$ Conductivity of solid cast iron. At the point of junction of solid cast iron and liquid cast iron, temperature would be equal to melting point of iron.



$$\left(\frac{dQ}{dt}\right)_{\text{solid cast iron}} = \left(\frac{dQ}{dt}\right)_{\text{liquid cast iron}}$$

$$\Rightarrow \frac{-k_sA[T_2-T_m]}{\ell - x} = \frac{-kk_s[T_m-T_1]}{x}$$

$$\Rightarrow \frac{x}{\ell - x} = \frac{k[T_m-T_1]}{T_2-T_m}$$

$$\Rightarrow \frac{x}{\ell} = \frac{k[T_m-T_1]}{(T_2-T_m)+k(T_m-T_1)} = \frac{k(T_1-T_m)}{k(T_1-T_m)+(T_m-T_2)}$$

Sol 15: (a) Temperature gradient = $\frac{dT}{dx}$

$$= \frac{(0-100)^\circ C}{(1-0)m} = -100^\circ C/m$$

(b) $\frac{dT}{dx} = \frac{T-100}{x-0} = -100^\circ C/m$

$\Rightarrow T = 100\,(1 - x)\ ^\circ C$

Total heat absorbed

$$= \int \text{mass} \times \text{specific heat} \times \text{change of temp}$$

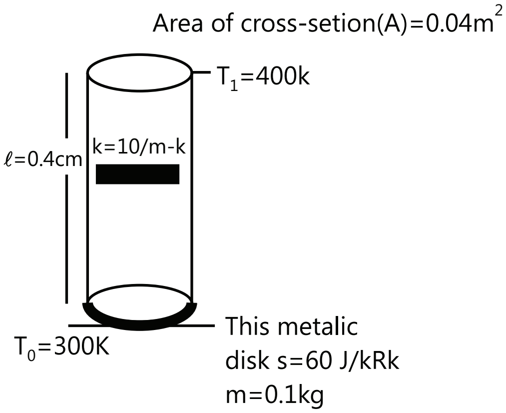
$$= \int_0^1 (2dx)(10)(T-0) = 20\int_0^1 100(1-x)dx$$

$$= 2000\left[x - \frac{x^2}{2}\right]_0^1 = 1000\text{ J}$$

Sol 16: $\frac{dQ}{dt} = \frac{-kA(T-400)}{\ell}$

$$\Rightarrow ms\,\frac{dT}{dt} = \frac{-kA(T-400)}{\ell}$$

$$\Rightarrow \frac{ms\ell}{kA}\,\frac{dT}{T-400} = -dt$$



On integrating we get

$$\Rightarrow -\frac{ms\ell}{kA}\int_{300}^{350}\frac{dT}{T-400} = \int_0^t dt$$

$$\Rightarrow t = \frac{-0.4\times 600\times 0.4}{10\times 0.04}\ \lambda n\left|\frac{350-400}{300-400}\right|$$

≈ 166.3 sec

Sol 17: $(T - T_0) = (T_1 - T_0)\, e^{-kt}$

$$\Rightarrow t = \frac{1}{k}\ \lambda n\left(\frac{T_1-T_0}{T-T_0}\right)$$

T_0 : Surrounding temp

T_1 : Initial temp of object

T : Final temp of object

$$\therefore\ 5 = \frac{1}{k}\ \text{In}\left(\frac{80-20}{50-20}\right) = \frac{1}{R}\ \text{In}\ (2)$$

$$\text{and }\ t = \frac{1}{k}\ \text{In}\left(\frac{60-20}{30-20}\right) = \frac{1}{k}\text{In}(2)^2 = 2\times\frac{1}{k}\ \text{In}2$$

$\therefore$ t = 10 min

Sol 18: A = 27 cm^2 = 27 × 10^{-4} m^2

k = 3700 N/m

P_i = 1 atm = 10^5 Pa

T_i = 273 K

V_i = Ah.

Initial contraction in spring

$$= \frac{P_i\times A}{k} = \frac{10^5\times 27\times 10^{-4}}{3700} = \frac{27}{370}m$$

P_f = ?

T_f = ?

$V_f = \frac{9hA}{16}$

Since, spring contract by h/16 length

$\therefore$ Force exerted by spring = $k\left(\frac{h}{16}+\frac{27}{370}\right)$

Avg. pressure due to spring (P_0)

$= \frac{k}{A}\left(\frac{h}{16}+\frac{27}{370}\right)$

Since process is adiabatic

$\therefore P_f = \left(\frac{V_i}{V_f}\right)^{\gamma} P_i$

$\left(\frac{16}{09}\right)^{3/2} 10^5 = \frac{4\times16}{27} \times 10^5$

P_f must be equal to P_0 for equilibrium

Thus, $\frac{k}{A}\left[\frac{h}{16}+\frac{27}{370}\right] = \frac{4\times16}{27} \times 10^5$

$\Rightarrow \frac{h}{16} + \frac{27}{370} = \frac{4\times16\times10^5\times27\times10^{-4}}{27\times3700}$

$\Rightarrow h = \frac{4\times(16)^2}{370} - \frac{27\times16}{370}$

$\Rightarrow h = 1.6$ m

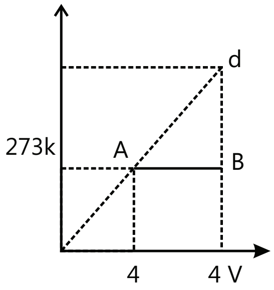
In adiabatic process $TV^{\gamma-1}$ = constant

$\Rightarrow T_f = \left(\frac{V_i}{V_f}\right)^{\gamma-1} T_i$

$= \left(\frac{16}{9}\right)^{1/2} \times 273$

$\Rightarrow T_f = 364$ K

Sol 19: Process AC is isobaric



$\therefore T_C = \frac{V_C}{V_A} T_A = \frac{4V}{V} \times T_A$

$\Rightarrow T_C = 4T_A$

$\Delta Q_{AB} = \Delta U + \Delta W$

$= nRT_A \lambda n\ 4$

$\Delta Q_{BC} = \Delta U + \Delta W$

$= nC_V [T_C - T_A] = 3n\ C_V\ T_A$

$\therefore \Delta Q_{AB} + \Delta Q_{BC} = 27.7\times 10^3$ J

$\Rightarrow nRT_A \lambda n\ 4 + 3\ nC_V\ T_A = 27700$

$\Rightarrow R\ \lambda n\ 4 + 3C_V = \frac{27700}{2\times273} \approx 50.73$

$\therefore C_V \approx 13.068$ J/mol-K

$\frac{C_P}{C_V} = \frac{C_V+R}{C_V} = 1 + \frac{R}{C_V} \approx 1.63$

Sol 20: Initially

$T_i = 0^0C = 273K = T_1$

$P_i = P_1$

$V_i = V_1$

From (1) to (2)

Process is adiabatic

$\Delta Q = 0$

$\Delta U = -\Delta W$

$P = P_2$

$T = T_2$

$V = V_2 < V_1$

At (2)

Process is isochoric

$\therefore$ V = constant

$\therefore V = V_2$

$\therefore P = P_3 < P_1$

$T = 273\ K = T_1 < T_2$

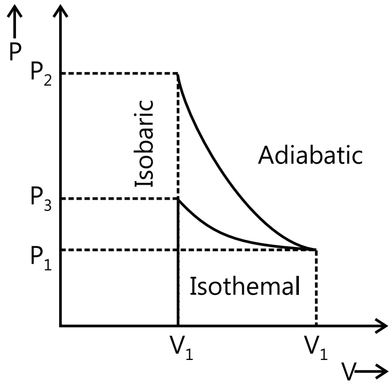
From (2) to (1)

Process is isothermal

$T = 273\ K = T_1$

$V = V_1$

$\therefore P = P_1$



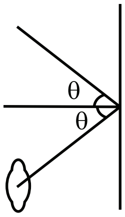
$\Delta Q_{process} = 100 \times 80 = 8000$ cal

$\Delta U_{process} = 0$ cal

$\therefore \Delta W + \Delta U = \Delta Q$

$\therefore \Delta W = 8000$ cal

Sol 21: Number of molecules hitting 1m^2 of wall per second (N)== nv cos θ $\frac{\text{molecules}}{m^2 \text{ sec}}$



Change in momentum of 1 molecule

= m (2v cos θ) =2mv cos θ

∴ Pressure exerted = N ×change of momentum of 1 molecule

= nv cos θ × 2 mv cos θ = 2mv^2 cos^2 θ

Sol 22: (a) Work along path A-D

= Area under curve AD

$= \frac{1}{2}\ [P_A + P_D] \times [V_D - V_A]$

$= \frac{1}{2} \times 1.6 \times 10^5 \times 1.1 \times 10^{-3} = 88$ J

(b) $\Delta W_{ADC} = \Delta W_{AD} + \Delta W_{DC} \Rightarrow \Delta W_{DC} = -3$J

$\therefore -[\frac{1}{2}[P_D + P_B] \times [V_D - V_C]] = -3$

$\Rightarrow 0.9 \times 10^5\ (1.3 - V_c) \times 10^{-3} = 6$

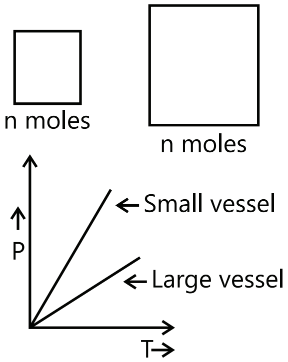
$\Rightarrow V_C \approx 1.23$ L

(c) $W_{CDA} = -W_{ADC} = -85$ J

Exercise 2

Single Correct Choice Type

Sol 1: (D)



V = constant

and PV = nRT

$\Rightarrow P = \left(\frac{nR}{V}\right) T$

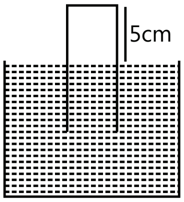
∴ P = mT where m is a constant

∴ P-T curve for both vessels will be linear but with different slopes, since the constant volumes have different values in both cases.

Sol 2: (C) Initially

$P_i = 76$ cm of Hg

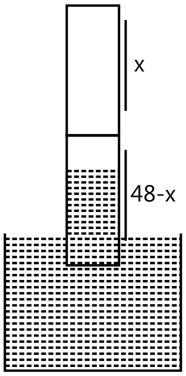
$V_i = 5A$



Finally

$P_f = (76) - (48 - x) = (28 + x)$ cm of Hg

$V_f = x\ A$



Since, temperature remains constant

$\therefore P_iV_i = P_fV_f$

$\Rightarrow 76 \times 5A = (28 + x) \times A$

$\Rightarrow x^2 + 28x - 380 = 0$

$\Rightarrow (x + 38)(x - 10) = 0$

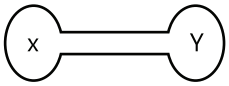
$\Rightarrow x = 10$ cm or $x = -38$ cm

$x = -38$ rejected since x can't be -ve

Sol 3: (C)

Container X Container Y

2V V



Process (P) in both vessels will be same.

Thus $V \propto nT$

$\therefore n \propto \frac{V}{T}$

$\therefore n_X = k\frac{2V}{200}$ [k is some constant]

$n_Y = \frac{kV}{400}$

$\therefore \frac{n_X}{n_Y} = \frac{4}{1}$

Since gas in both container is same

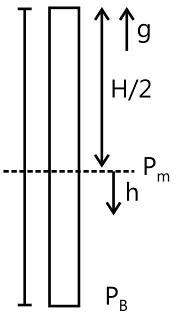
$\therefore \frac{m_X}{m_Y} = \frac{n_X}{n_Y} = 4$

$\therefore m_Y = \frac{m_X}{4} = \frac{m}{4}$

Sol 4: (C) Pressure gradient will develop due to the upward acceleration so

$$\frac{dP}{dh} = 2\rho g$$

$$\frac{dP}{dh} = \frac{PM}{RT}2g$$



$$\int_{P_m}^{P_B} \frac{dP}{P} = \int_0^{H/2} \frac{2Mg}{RT} dh$$

$$\lambda n\left(\frac{P_B}{P_m}\right) = \frac{MgH}{RT}$$

$$\frac{P_B}{P_m} = \exp(MgH/RT)$$

Sol 5: (D) Since there is no loss of energy

$\therefore$ Sum of change of internal energies must be zero.

i.e. $\Delta U_1 + \Delta U_2 = 0$

$\Rightarrow n_1 C_v [T - T_1] + n_2 C_v [T - T_2] = 0$

$\Rightarrow (n_1 + n_2) T = n_1 T_1 + n_2 T_2$

$\Rightarrow T = \frac{n_1T_1 + n_2T_2}{n_1 + n_2}$

Sol 6: (D) $v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow v_{rms}^2 \propto \frac{1}{M}$

$$\therefore \frac{(v_{rms}^2)_B}{(v_{rms}^2)_A} = \frac{M_A}{M_B} = \frac{m \times N_A}{2m \times N_A} = \frac{1}{2} \quad \text{....(i)}$$

Since by the postulate of KTG that, molecules move in random motions.

$\therefore \langle v_x^2 \rangle = \langle v_Y^2 \rangle = \langle v_X^2 \rangle$ and $v^2 = v_x^2 + v_y^2 + v_z^2$

$\therefore \langle v^2 \rangle = 3\langle v_x^2 \rangle$

Thus $(v_{rms}^2)_A = 3w^2$ & $(v_{rms}^2)_B = v^2$

Thus, by (i) we get: - $\frac{v^2}{3w^2} = \frac{1}{2} \Rightarrow \frac{w^2}{v^2} = \frac{2}{3}$

Sol 7: (C) $P = 0.7 \times 10^5$ N/m^2

$V = 0.0049$ m^3

$\gamma = \frac{C_P}{C_V} = 1.4$ = ratio of specific heats

$\frac{C_V}{C_P} \neq 1.4$ since, we know $C_P > C_V$

In reversible adiabatic process

PV^γ = constant

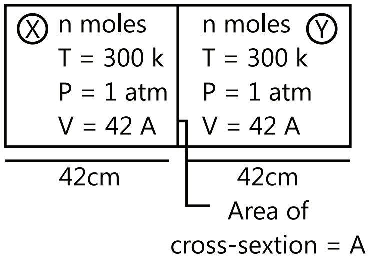
$\therefore \quad V^\gamma \Delta P + P\gamma V^{\gamma-1}\Delta V = 0$

$$\Rightarrow \frac{\Delta P}{\Delta V} = \frac{-\gamma P}{V} = -\frac{1.4 \times 0.7 \times 10^5}{0.0049}$$

$$\Rightarrow \quad \frac{dP}{dV} = -2 \times 10^7 \text{ Nm}^{-5}$$

Sol 8: (C) Equal mass of same gas $\Rightarrow$ Equal moles

Initially



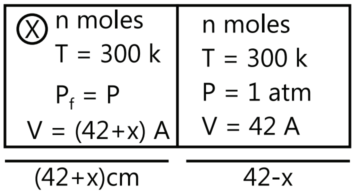
Using gas equation we get

$PV = nRT$

$\Rightarrow 42\ A = nR\ 300$

$$\Rightarrow \frac{nR}{A} = \frac{42}{300} \qquad \text{......(i)}$$

Finally



Using gas equation we get

$$P(42 + x)A = nR\ 330 \qquad \text{.....(ii)}$$

$$P(42 - x)A = nR\ 300 \qquad \text{.....(iii)}$$

(ii)/(iii) gives

$$\frac{42+x}{42-x} = \frac{330}{300}$$

$$\Rightarrow \frac{2x}{42-x} = \frac{30}{300}$$

$\Rightarrow 20x = 42 - x$

$\Rightarrow x = 2$ cm

Sol 9: (C) At constant pressure

$W = P\Delta V$

For monoatomic: - $C_P = \frac{5}{2}R$

and $Q = nC_P\,\Delta T$

$$\Rightarrow Q = n \times \frac{5R}{2} \times \frac{P\Delta V}{nR} = \frac{5}{2}W$$

$$\therefore W = \frac{2}{5}Q$$

Multiple Correct Choice Type

Sol 10: (A, B) $e_A = 0.01$ and $e_B = 0.81$

$A_A = A_B$

$E_A = E_B$

$\Rightarrow e_A\sigma A_A T_A^4 = e_B\,\sigma A_B T_B^4$

$\Rightarrow 0.01\,T_A^4 = 0.81\,T_B^4$

$$\Rightarrow T_B = \frac{1}{3} \times T_A$$

$$\Rightarrow T_B = \frac{1}{3} \times 5802 = 1934\text{ K}$$

By Wien's displacement law

$\lambda_m T = \text{constant} = 2.93 \times 10^{-3}$ mK

$\therefore \quad \lambda_{m_A} = 0.5\ \mu$m

Since, it is given in the question that

$\lambda_{m_B} = 1\ \mu\text{m} + \lambda_{m_A}$

$\therefore \quad \lambda_{m_B} = 1.5\ \mu$m

Sol 11: (A, D) P^2V = const.

$$\therefore P_f = \left(\frac{V_i}{V_f}\right)^{1/2} \times P_i = \left(\frac{V}{4V}\right)^{1/2} P = P/2$$

$$P^2\left(\frac{nRT}{P}\right) = \text{constant}$$

$\Rightarrow PT$ = constant

$\therefore$ P – T curve is hyperbola

$$\text{and } T_f = \left(\frac{P_i}{P_f}\right) T_i = 2T$$

Sol 12: (C, D) P = 3000 J

$V = 5\,\ell$

$$v_{rms} = \sqrt{\frac{3RT}{M_m}}$$

$$\Rightarrow \frac{1}{2}(nM_m)\,v_{rms}^2 = \frac{3}{2}nRT = \frac{3}{2}PV$$

Translational K.E. of all molecules $= \frac{3}{2}PV$

$$\therefore \frac{3}{2}P \times 5\,\ell = 3000\text{ J}$$

$\therefore$ P = 400 J/l

Total K.E. of 10 ℓ of gas at Pressure 2P for a monoatomic gas = $\frac{3}{2}$ × 800 × 10 = 12000 J

Total K.E. of 10 ℓ, 2P pressure for a diatomic gas = $\frac{5}{2}$ PV

= $\frac{5}{2}$ × 800 ×10 = 2000 Joules

Note: In options its asked total K.E. not translational K.E.

Sol 13: (A, D) No. of molecules moving towards wall per unit volume = $\frac{1}{6} \times 10^{26} \frac{\text{molecules}}{\text{m}^3}$

∴ No. of molecules hitting 1m^2 of the wall every sec (n)

$$= \left(\frac{1}{6}\times 10^{26}\right) \times (2000)$$

$$= 3.33 \times 10^{28} \frac{\text{molecules}}{\text{m}^2\,\text{sec}}$$

Pressure exerted = (change in momentum) × no. of molecules putting wall per unit area per sec

= (3 × 10^{-27} kg/molecules) × (2 × 2000 m/s) × 3.33 × 10^{28} molecules/m^2 sec)

≈ 4 × 10^5 Pa

Sol 14: (A, B, C) Isothermal process Adiabatic process

PV = constant $\qquad$ PV$^\gamma$ = constant

$\Rightarrow \frac{T}{V}$ = constant $\qquad \Rightarrow TV^{r-1}$ =constant

$\therefore T_f = \frac{V_f}{V_i} \times T_i \qquad \Rightarrow T_f = \left(\frac{V_i}{V_f}\right)^{\gamma-1} T_i$

(Since there is expansion)

$\therefore \frac{V_f}{V_i} > 1 \qquad \Rightarrow \frac{V_i}{V_f} < 1$

Thus $\left(\frac{V_i}{V_f}\right)^{\gamma-1} < 1$

Since γ-1 is +ve

(∴ Isothermal temp. > adiabatic temp.)

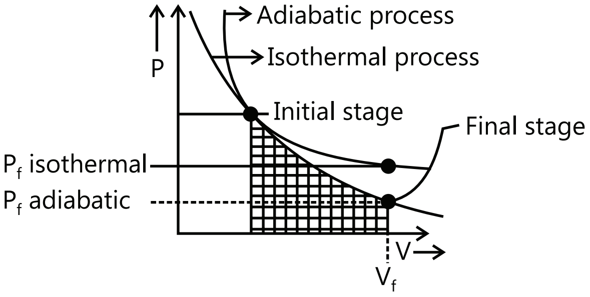
$P_f = \frac{V_i}{V_f} \times P_i \qquad P_f = \left(\frac{V_i}{V_f}\right)^{\gamma} \times P_i$

(Since $\gamma > 1$ and $\frac{V_i}{V_f} < 1$)

$$\therefore \frac{V_i}{V_f} > \left(\frac{V_i}{V_f}\right)^{\gamma}$$

∴ Isothermal pressure > Adiabatic pressure

Alternative method:



From graph, it is very clear, that

$P_{f\text{ isothermal}} > P_{f\text{ adiabatic}}$

and for same volume and moles of gas

$T \propto P\ [\because PV = nRT]$

Thus, $T_{f\text{ isothermal}} > T_{f\text{ adiabatic}}$

Work done = $\int_{V_i}^{V_f} P\,dV$ = area under P-V diag

Thus, (work done)$_{\text{isothermal}}$ > (work done)$_{\text{adiabatic}}$

Sol 15: (A, B) Area under the curve gives the rate at which heat per unit surface is radiated by the body.

i.e. total rate of heat radiation = (Area under the curve) × (Surface area of the body)

Sol 16: (A, C) $\rho = \frac{m}{V}$

$\Rightarrow \rho \times \frac{4}{3}\pi r^3 = m$

$\Rightarrow r \propto (m)^{1/3}$

And area of sphere (A) $\propto r^2$

$\therefore A \propto (m)^{2/3}$

$\therefore \frac{A_A}{A_B} = (4)^{2/3} = (2)^{4/3}$

$\therefore$ Ratio of heat loss = $\frac{eA_A\sigma(T-T_0)^4}{eA_B\sigma(T-T_0)^4}$

$= \frac{A_A}{A_B} = (2)^{4/3}$

By Newton's law of cooling:

$\frac{dQ}{dt} = ms(COOH)_2 = -k\,(T - T_0)$

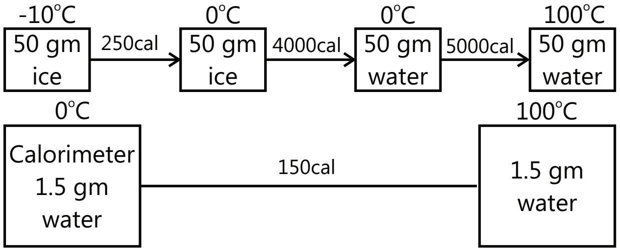
$\Rightarrow \frac{dT}{dt} = \frac{-k}{ms}(T - T_0)$

Where $k = 4eA\sigma T_0^3$

$$\therefore \frac{dT}{dt} \propto \frac{A}{m}$$

$$\therefore \frac{\left(\frac{dT}{dt}\right)_A}{\left(\frac{dT}{dt}\right)_B} = \frac{\frac{A_A}{m_A}}{\frac{A_B}{m_B}} = \frac{(2)^{4/3}}{4} = 2^{-2/3}$$

Sol 17: (A, C)



$\therefore$ Heat absorbed by ice and calorimeter to reach 100° C water

= 250 + 4000 + 5000 + 150 = 9400 cal

$\therefore$ Amount of steam converted into water

$$= \frac{9400}{540} = 17.4 \text{ gm}$$

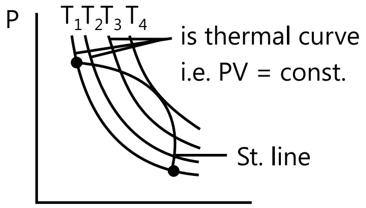
$\therefore$ Amount of water remaining = 50 + 17.40 = 67.4 gm

$\therefore$ Amount of steam remaining = 20 – 17.4 = 2.6 gm

Sol 18: (B, D) $\Delta T = 0$

The slope of straight line can't be +ve. Since, $T \propto PV$ and if slope is +ve, then both P and V are increasing. Therefore, temperature will always increase.

$T_1 < T_2 < T_3 < T_4$



Thus, from graph it can be seen, that temperature first increases and then decrease.

Sol 19: (A, B) Process AB is isochoric: - i.e. P = constant

PV = nRT

$$\Rightarrow V = \left(\frac{nR}{P}\right)T$$

$\therefore$ V is a linear function of T with slope $\frac{nR}{P}$ and passing through origin in V-T curve.

Secondly for process CD: P-T will be a linear curve passing through origin.

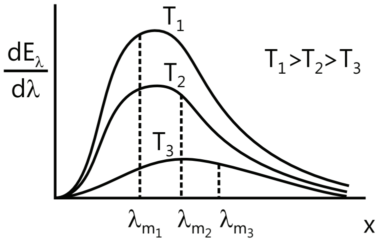
Comprehension Type

Paragraph 1:

Sol 20: (A, B) Area under the curve gives the rate at which heat per unit surface is radiated by the body.

i.e. total rate of heat radiation = (Area under the curve) × (Surface area of the body)

Sol 21: (A, B, C, D) $\lambda_m T$ = constant [By Wien's Displacement Law]



Area under graph = $E_\lambda = e\sigma T^4 \propto T^4$

Paragraph 2:

Sol 22: (A)

$A_A = A$ $\quad$ $A_B = A$

$\ell_A = l$ $\quad$ $\ell_B = l$

$k_A = 3k$ $\quad$ $k_B = k$

$$\therefore R_A = \frac{\ell_A}{k_A A_A} = \frac{\ell}{3kA} \quad R_B = \frac{\ell_B}{k_B A_B} = \frac{\ell}{k_A}$$

$$\therefore \frac{R_A}{R_B} = \frac{1}{3}$$

Sol 23: (B) Rate at which heat flows from A

= Rate at which heat flows from B

$$\Rightarrow \left(\frac{dQ}{dt}\right)_A = \left(\frac{dQ}{dt}\right)_B$$

$$\Rightarrow \frac{T_A}{R_A} = \frac{T_B}{R_B} \Rightarrow \frac{T_A}{T_B} = \frac{R_A}{R_B} = \frac{1}{3}$$

Sol 24: (B) $G_A = \frac{T_A}{\ell_A} = \frac{T_A}{\ell}$ and $G_B = \frac{T_B}{\ell_B} = \frac{T_B}{\ell}$

$\therefore \frac{G}{G_B} = \frac{T_A}{T_B} = \frac{1}{3}$

Paragraph 3:

Sol 25: (B) $\left(\frac{dQ}{dt}\right)_{\text{initially}} = kAV\,\Delta T$

$\left(\frac{dQ}{dt}\right)_{\text{finally}} = kA(4V)\left(\frac{\Delta T}{2}\right) = 2\left(\frac{dQ}{dt}\right)_{\text{initially}}$

Sol 26: (A) If all the parameters are kept constant then

$\frac{dQ}{dt} = ms\frac{dT}{dt} = kA\,v\,\Delta T$

$\therefore \frac{dT}{dt} = \frac{kAv\Delta T}{ms}$

Paragraph 4:

Sol 27: (A) w_{ab}= Area below curve AB

$= \frac{1}{2}[10 + 20] \times (12 - 6)$

$= \frac{1}{2} \times 30 \times 6 = 90$ J

Sol 28: (B) w_{BC} = -Area below curve BC

$= -10[12 - 6] = -60$ J

Note:- w_{BC} is –ve, since volume is decreasing

Sol 29: (D) $w_{CA} = 0$ since volume is constant

$\therefore w_{ABCA} = w_{AB} + w_{BC} + w_{CA} = (90) + (-60) + 0 = 30$ J

Paragraph 5:

$n_{He} = 5$ moles $n_{H_2} = 2$ moles

$M_{He} = 4$g $M_{H_2} = 2$ g

Sol 30: (D) Equivalent molar mass $= \frac{n_{He}M_{He} + n_{H_2}M_{H_2}}{n_{He} + n_{H_2}}$

$= \frac{5\times4+2\times2}{5+2} = \frac{24}{7}$gm

Sol 31: (A) $f_{H_2} = 5$ and $f_{He} = 3$

$\therefore f_{mix} = \frac{n_{H_2}f_{H_2} + n_{He}f_{He}}{n_{H_2} + n_{He}}$

$= \frac{2\times5+5\times3}{2+5} = \frac{25}{7} = 3.57$

Sol 32: (C) $\therefore r_{mix} = 1+\frac{2}{f_{mix}} = 1+\frac{2}{25/7} = 1.56$

Sol 33: (D) Since internal energy is an extensive property

$\therefore U_{mix} = U_{He} + U_{H_2} = 200+100=300$ J

Match the Columns

Sol 34: (A) $V_i = V$ $V_f = 2V$

$P_i = P$

(P) → (y) isobaric process

$W = P\Delta V = PV$

(q) → (z) isothermal

$W = nRT\ln\frac{V_f}{V_i} = PV\ln 2$

(r) → (x) Adiabatic

$P_f = \left(\frac{V_i}{V_f}\right)^{\gamma} P_i = 2^{\gamma}P$

$W = \frac{P_fV_f - P_iV_i}{1-\gamma} = \frac{(2^{-\gamma}P)(2V) - PV}{1-\gamma}$

$\Rightarrow W = \frac{PV(1-2^{1-\gamma})}{\gamma-1}$

Previous Years' Questions

Sol 1: (D) The desired fraction is

$f = \frac{\Delta U}{\Delta Q} = \frac{nC_V\Delta T}{nC_p\Delta T} = \frac{C_V}{C_p} = \frac{1}{\gamma}$

or $f = \frac{5}{7}\left(\text{as } \gamma = \frac{7}{5}\right)$

Sol 2: (A) Average kinetic energy per molecule per degree of freedom = $\frac{1}{2}$kT. Since, both the gases are diatomic and at same temperature (300 K), both will have the same number of rotational degree of freedom i.e., two. Therefore, both the gases will have the same average rotational kinetic energy per molecule

$= 2 \times \frac{1}{2}kT$ or kT

Thus, ratio will be 1: 1.

Sol 3: (D) A is free to move, therefore, heat will be supplied at constant pressure

$\therefore dQ_A = nC_P\, dT_A$ ...... (i)

B is held fixed, therefore, heat will be supplied at constant volume.

$\therefore dQ_B = nC_V dT_B$ ...... (ii)

But $dQ_A = dQ_B$ (given)

$nC_p\Delta T_A = nC_V\Delta T_B \therefore dT_B = \left(\frac{C_p}{C_V}\right)dT_A$

$= \gamma(dT_A)$ [γ = 1.4 (diatomic)]

(dT_A = 30 K)

= (1.4)(30 K)

$\therefore dT_B$ = 42 K

Sol 4: (C) Speed of sound in an ideal gas is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$\therefore v \propto \sqrt{\frac{\gamma}{M}}$ (T is same for both the gases)

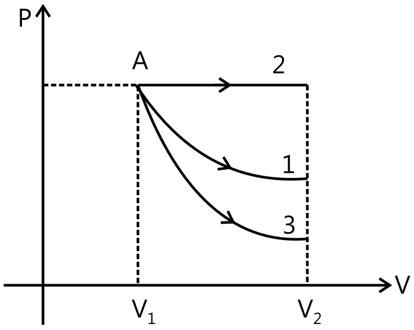
$$\therefore \frac{v_{N_2}}{v_{He}} = \sqrt{\frac{\gamma_{N_2}}{\gamma_{He}}.\frac{M_{He}}{M_{N_2}}} = \sqrt{\frac{7/5}{5/3}\left(\frac{4}{28}\right)} = \sqrt{3}/5$$

$\gamma_{N_2} = \frac{7}{5}$ (Diatomic)

$\gamma_{He} = \frac{5}{3}$ (Monoatomic)

Sol 5: (A) The corresponding p–V graphs (also called indicator diagram) in three different processes will be as shown:

Area under the graph gives the work done by the gas



$(Area)_2 > (Area) > (Area)_3$

$\therefore W_2 > W_1 > W_3$

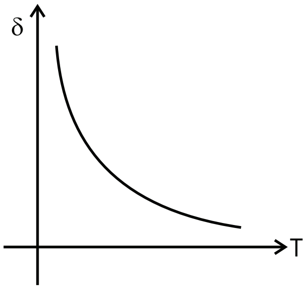
Sol 6: (C) For an ideal gas : PV = nRT

For P = constant

$P\Delta V = nR\Delta T$

$$\therefore \frac{\Delta V}{\Delta T} = \frac{nR}{p} = \frac{nR}{\frac{nRT}{V}} = \frac{V}{T}$$

$$\therefore \frac{\Delta V}{V\Delta T} = \frac{1}{T} \text{ or } \delta = \frac{1}{T}$$



Therefore, δ is inversely proportional to temperature T. i.e., when T increases, δ decreases and vice-versa.

Hence, δ-T graph will be a rectangular hyperbola as shown in the above figure.

Sol 7: (B) In adiabatic process

Slope of P-V graph, $\frac{dP}{dV} = -\gamma\frac{P}{V}$

Slope $\propto \gamma$ (with negative sign)

From the given graph,

$(slope)_2 > (slope)_1$

$\therefore \gamma_2 > \gamma_1$

Therefore, 1 should correspond to O_2 (γ = 1.4) and 2 should correspond to He (γ = 1.67).

Sol 8: (A) $\Delta W_{AB} = P\Delta V = (10)(2 - 1) = 10$ J

$\Delta W_{BC} = 0$ (as V = constant)

From first law of thermodynamics

$\Delta Q = \Delta W + \Delta U$

$\Delta U = 0$ (process ABCA is cyclic)

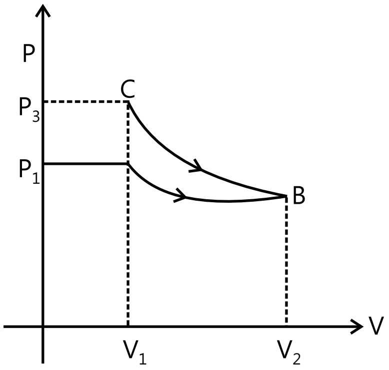
$\therefore\ \Delta Q = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$

$\therefore\ \Delta W_{CA} = \Delta Q - \Delta W_{AB} - \Delta W_{BC}$

$= 5 - 10 - 0 = -5$ J

Sol 9: (C) Temperature of liquid oxygen will first increase in the same phase. Then, phase change (liquid to gas) will take place. During which temperature will remain constant. After that temperature of oxygen in gaseous state will further increase.

Sol 10: (C) Slope of adiabatic process at a given state (P, V, T) is more than the slope of isothermal process. The corresponding P-V graph for the two processes is as shown in figure.



In the graph, AB is isothermal and BC is adiabatic.

W_{AB} = positive (as volume is increasing)

and W_{BC} = negative (as volume is decreasing) plus,

$|W_{BC}| > |W_{AB}|$, as area under p–V graph gives the work done.

Hence, $W_{AB} + W_{BC} = W < 0$

From the graph itself, it is clear that $P_3 > P_1$

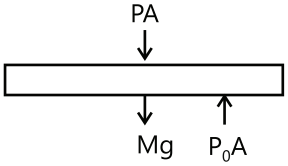
Note: At point B, slope of adiabatic (process BC) is greater than the slope of isothermal (process AB)

Sol 11: (B) Total translational kinetic energy

$$= \frac{3}{2}nRT = \frac{3}{2}PV = 1.5\ PV$$

Sol 12: (A) Since it is open from top. Pressure will be P_0.

Sol 13: (D) Let p be the pressure in equilibrium.



Then, $P_A = P_0A - Mg$

$$P = P_0 - \frac{Mg}{A} = p_0 - \frac{Mg}{\pi R^2}$$

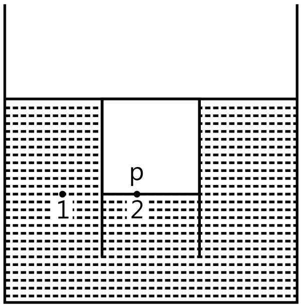
Applying, $P_1V_1 = P_2V_2$

$\therefore\ P_0\,(2AL) = (P)(AL')$

$$\therefore\ L' = \frac{2P_0L}{P} = \left(\frac{P_0}{P_0 - \frac{Mg}{\pi R^2}}\right)(2L)$$

$$= \left(\frac{P_0\pi R^2}{\pi R^2 P_0 - Mg}\right)(2L)$$

Sol 14: (C)



$p_1 = p_2$

$P_0 + \rho g\,(L_0 - H) = P$ ....(i)

Now, applying $P_1V_1 = P_2V_2$ for the air inside the cylinder, we have

$P_0(L_0) = P(L_0 - H)$

$$\therefore\ p = \frac{P_0L_0}{L_0 - H}$$

Substituting in Equation (i), we have

$$P_0 + \rho g(L_0 - H) = \frac{P_0L_0}{L_0 - H}$$

or $\rho g(L_0 - H)^2 + P_0(L_0 - H) - P_0L_0 = 0$

Sol 15: (B, D) In case of free expansion under adiabatic conditions, change in internal energy $\Delta U = 0$

$\therefore$ Internal energy and temperature will remain constant.

(B) $P \propto \frac{1}{V^2}$

$\therefore PV^2 = \text{constant}$ .... (i)

or $\left(\frac{nRT}{V}\right)V^2 = \text{constant}$

$\therefore T \propto \frac{1}{V}$ .... (ii)

If volume is doubled, temperature will decrease as per equation (ii).

Further, molar heat capacity in process PV^x = constant is

$$C = C_v + \frac{R}{1-x}$$

From equation (i) , x = 2

$$\therefore C = \frac{3}{2}R + \frac{R}{1-2} = +\frac{R}{2}$$

Since, molar heat capacity is positive, according to $Q = nC\Delta T$, Q will be negative if ΔT is negative. Or gas loses heat if temperature is decreasing.

(C) $P \propto \frac{1}{V^{4/3}}$

$PV^{4/3} = \text{constant}$

$\therefore \left(\frac{nRT}{V}\right)V^{4/3} = \text{constant}$

$\therefore T \propto \frac{1}{V^{1/3}}$

Further, with increase in volume temperature will decrease.

Here, $x = \frac{4}{3}$

$$\therefore C = \frac{3}{2}R + \frac{R}{1-\frac{4}{3}} = -1.5\,R$$

As molar heat capacity is negative, Q will be positive if ΔT is negative. Or gas gains heat with decrease in temperature.

(D) $T \propto PV$

In expansion from V_1 to $2V_1$, product of PV is increasing. Therefore, temperature will increase. Or ΔU = +ve. Further, in expansion work done is also positive.

Hence, $Q = W + \Delta U$ = +ve or, gas gains heat.

Sol 16: (B, D) For monoatomic gas

$$C_v = \frac{3}{2}R,\ C_p = \frac{5}{2}R$$

For diatomic gas, $C_v = \frac{5}{2}R,\ C_p = \frac{7}{5}R$

Sol 17: A → q; B → p, r; C → p, s; D → q, s

(A) P-V graph is not rectangular hyperbola. Therefore, process A – B is not isothermal.

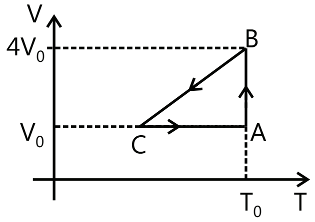
(B) In process BCD, product of PV (therefore temperature and internal energy) is decreasing. Further, volume is decreasing. Hence, work done is also negative. Hence, Q will be negative or heat will flow out of the gas.

(C) W_{ABC} = positive

(D) For clockwise cycle on p- v diagram with P on y-axis, net work done is positive.

Sol 18: (D) At low pressure and high temperature inter molecular forces become ineffective. So a real gas behaves like an ideal gas.

Sol 19: (A, B)



$\because U = \frac{f}{2}nRT$

Where f, n, R are constants. Also temperature T is same at A and B.

$\therefore U_A = U_B$

Also,

$$\Delta W_{AB} = nRT_0 \ln\left(\frac{V_f}{V_i}\right) = nRT_0 \ln\frac{4V_0}{V_0} = nRT_0 \ln 4 = P_0V_0 \ln 4$$

Sol 20: (A) $TV\lambda^{-1} = C$

$T_1(5.6)^{2/3} = T_2\,(0.7)^{2/3} \Rightarrow T_2 = T_1(8)^{2/3} = 4T_1$

$\therefore \Delta w(\text{work done on the system}) = \frac{nR\Delta T}{\gamma - 1} = \frac{9}{8}RT_1$

Sol 21: (D) $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$\text{Required ratio} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3.16$$

Sol 22: (D) $PV = nRT = \frac{m}{M}RT$

$\Rightarrow PM = \rho RT$

$$\frac{\rho_1}{\rho_2} = \frac{P_1M_1}{P_2M_2} = \left(\frac{P_1}{P_2}\right)\times\left(\frac{M_1}{M_2}\right) = \frac{4}{3}\times\frac{2}{3} = \frac{8}{9}$$

Here ρ_1 and ρ_2 are the densities of gases in the vessel containing the mixture.

Sol 23: (2) $U_b = 200$ J, $U_i = 100$ J

Process iaf

Process	W (in Joule)	ΔU (in Joule)	Q (in Joule)
ia		0	
af		200	
Net	300	200	500

$\Rightarrow U_f = 400$ Joule

Process ibf

Process	W (in Joule)	ΔU (in Joule)	Q (in Joule)
ia	100	50	150
af	200	100	300
Net	300	150	450

$$\Rightarrow \frac{Q_{bf}}{Q_{ib}} = \frac{300}{150} = 2$$

Sol 24: (A, B, D) $U = {}_nC_{V_1}T + {}_nC_{V_2}T$

$$= 1\times\frac{5}{2}RT + 1\times\frac{3}{2}RT = 4RT$$

$\Rightarrow 2C_{V_{mix}}T = 4RT$

Average energy per mole $= 2RT \Rightarrow C_{V_{mix}} = 2R$

$$\frac{C_{mix}}{C_{He}} = \sqrt{\left(\frac{\lambda_{mix}}{\lambda_{He}}\right)\left(\frac{M_{He}}{M_{mix}}\right)} = \sqrt{\frac{3}{2}\times\frac{3}{5}\times\frac{4}{3}} = \sqrt{\frac{6}{5}}$$

$$\frac{V_{rms\,He}}{V_{rms\,H_2}} = \sqrt{\frac{M_{H_2}}{M_{he}}} = \frac{1}{\sqrt{2}}$$

Sol 25: (B) Rate of heat generated $\frac{dQ}{dt} = 3kW$

Let at any time 't', temperature of cooler = T

Rate of cooling:

$$ms\frac{dT}{dt} = 3kW - P$$

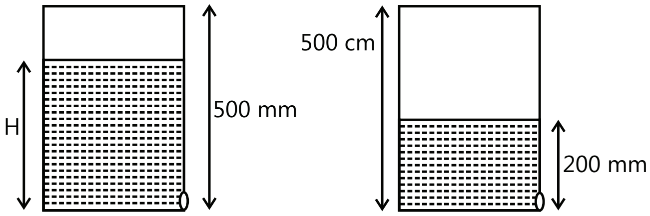
$$\int_{10}^{30} dT = \frac{(3kW - P)}{ms}\int_0^3 dt$$

$$30 - 10 = \frac{(3kW - P)\times 3\times 3600}{120\times 4.2\times 10^3}$$

$$3kW - P = \frac{20\times 120\times 42}{3\times 36} = \frac{2800}{3}$$

P = 3000 - 933 = 2067 W

Sol 26: (6) $P = P_0 - \rho gh = 98\times 10^3$ N/m^2



$P_0V_0 = PV$

$10^5[A(500 - H)] = 98\times 10^3[A(500 - 200)]$

H = 206 mm

Level fall = 206 − 200 = 6 mm

Sol 27: (4) $TV^{\gamma-1} = \text{constant}$

$$TV^{7/5-1} = aT\left(\frac{v}{32}\right)^{7/5-1}$$

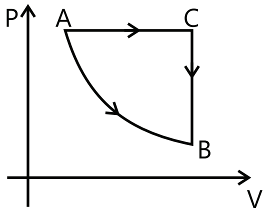
$\therefore a = 4$

Sol 28: A → p, r, t; B → p, r; C → q, s; D → r, t

Process A → B	→ Isobaric compression
Process B → C	→ Isochoric process
Process C → D	→ Isobaric expansion
Process D → A	→ Polytropic with $T_A = T_D$

Sol 29: (A, B, C, D) Option (A) is correct because the graph between (0 – 100 K) appears to be a straight line upto a reasonable approximation.

Option (B) is correct because area under the curve in the temperature range (0 - 100 K) is less than in range (400 - 500 K.)



Option (C) is correct because the graph of C versus T is constant in the temperature range (400 - 500 K)

Option (D) is correct because in the temperature range (200 – 300 K) specific heat capacity increases with temperature.

Sol 30: (C) By $A_1V_1 = A_2V_2$

$$\Rightarrow \pi(20)^2 \times 5 = \pi(1)^2 V_2 \Rightarrow V_2 = 2m/s^2$$

Sol 31: (A) $\frac{1}{2}\rho_a V_a^2 = \frac{1}{2}\rho_\ell V_\ell^2$

For given V_a

$$V_\ell \propto \sqrt{\frac{\rho_a}{\rho_\ell}}$$

Sol 32: (B or A, B, C) P (pressure of gas) $= P_1 + \frac{kx}{A}$

$$W = \int PdV = P_1(V_2 - V_1) + \frac{kx^2}{2} = P_1(V_2 - V_1) + \frac{(P_2 - P_1)(V_2 - V_1)}{2}$$

$$\Delta U = nC_V\Delta T = \frac{3}{2}(P_2V_2 - P_1V_1)$$

$$Q = W + \Delta U$$

Case I: $\Delta U = 3P_1V_1, W = \frac{5P_1V_1}{4}, Q = \frac{17P_1V_1}{4}, U_{spring} = \frac{P_1V_1}{4}$

Case II: $\Delta U = \frac{9P_1V_1}{2}, W = \frac{7P_1V_1}{3}, Q = \frac{41P_1V_1}{6}, U_{spring} = \frac{P_1V_1}{3}$

Note: A and C will be true after assuming pressure to the right of piston has constant value P_1.

Sol 33: (C) For adiabatic process

P^3V^5 = constant

$PV^{\frac{5}{3}}$ = constant

$\gamma = \frac{5}{3}$ gas is monoatomic

For process AC

$$\Delta Q_1 = {}_nC_p\Delta T = n\left(\frac{5}{2}R\right)\Delta T = \frac{5}{2}P.\Delta V$$

$$\Delta Q_1 = \frac{5}{2} \times 10^5 \times (8-1) \times 10^{-3}$$

$$\Delta Q_1 = 17.5 \times 10^2 J = 1750J$$

For process CD

$$\Delta Q_2 = {}_nC_V\Delta T = n\left(\frac{3}{2}R\right)\Delta T = \frac{3}{2}V(\Delta P)$$

$$\Delta Q_2 = \frac{3}{2} \times 8 \times 10^{-3} \times \left(-1 + \frac{1}{32}\right) \times 10^5$$

$$\Delta Q_2 = \frac{-93}{8} \times 10^2 = -11.625 \times 10^2$$

$$\Delta Q_{net} = 1750 - 1162 = 588 J$$