3 CHAPTER

Arches

Q.1 The bending moment in a three-hinged arch when compared with a simply supported beam is usually
(a) larger
(b) smaller
(c) can be larger or smaller
(d) none of these

Q.2 The bending moment in an arch is proportional to
(a) vertical ordinate of the arch.
(b) intercept between the arch axis and the funicular polygon.
(c) vertical ordinate of the funicular polygon.
(d) none of the above.

Q.3 A suspended cable hinged at two ends is a
(a) statically unstable structure.
(b) statically indeterminate structure.
(c) geometrically unstable structure.
(d) geometrically inadmissible structure.

Q.4 A shallow parabolic cable when subjected to a uniformly distributed load infinitely long develops
(a) zero bending moment.
(b) infinite bending moment.
(c) zero axial force.
(d) none of these.

Q.5 An arch in the form of a parabola with axis vertical has hinges at the abutments and the crown. The abutments are at different levels. The height of the crown above abutments are h_1 and h_2. The arch has a total horizontal span L. The horizontal thrust due to a uniformly distributed load w per unit length is

(a) $\dfrac{wL^2}{2\left[h_1^{1/2}+h_2^{1/2}\right]^2}$ (b) $\dfrac{wL^2}{\left[h_1^{1/2}+h_2^{1/2}\right]^2}$

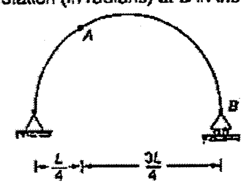
(c) $\dfrac{wL^2}{2\left[h_1^{1/3}+h_2^{1/3}\right]^3}$ (d) $\dfrac{wL^2}{\left[h_1^{1/3}+h_2^{1/3}\right]^3}$

Q.6 A parabolic two hinged arch is loaded with a concentrated load W at the crown. The horizontal thrust is equal to

(a) $\dfrac{54}{128}\dfrac{WL}{h}$ (b) $\dfrac{50}{128}\dfrac{WL}{h}$

(c) $\dfrac{25}{128}\dfrac{WL}{h}$ (d) $\dfrac{36}{128}\dfrac{WL}{h}$

Q.7 For the semicircular two-hinged arch shown in the figure below, a moment of 50 t-cm applied at B produces a displacement of 0.5 cm at A. If a concentrated load of 10 t is applied at A, the rotation (in radians) at B in the arch will be

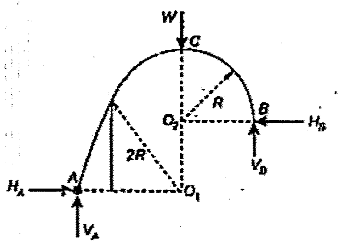


(a) 0.1 (b) 0.001
(c) 0.0001 (d) 0.01

Q.8 In a two hinged arch, an increase in temperature induces
(a) no bending moment in the arch rib.
(b) uniform bending moment in the arch rib.
(c) maximum bending at the crown.
(d) maximum bending moment at the crown.

Q.9 Which one of the following is associated with the rib shortening in arches either due to change in temperature or lack of fit to cause stresses in the arch members?
(a) Only two-hinged arches and not three-hinged arches.
(b) Two and three-hinged arches.
(c) Two-hinged arches made of reinforced concrete only.
(d) Only three-hinged arches but not two-hinged arches.

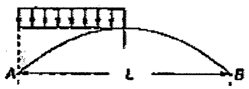
Q.10 A three-hinged circular arch *ACB* is formed by two quadrants of circles *AC* and *BC* of radii 2*R* and *R* respectively with *C* as crown, as shown in the figure. Consider the following in respect of the reactive forces developed at supports *A* and *B* due to concentrated load '*W*' at the crown:

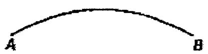


1. Line of action of reaction R_A at *A* is inclined at 45° to the horizontal.
2. $V_A = V_B = W/2$
3. $H_B = 2H_A$
4. $H_A = H_B = W/2$

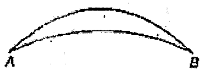
Which of the above is/are correct?
(a) 1 and 3 (b) 2 and 3
(c) 1, 2 and 4 (d) 4 only

Q.11 For the two-hinged parabolic arch as shown in the figure, which one of the following diagram represents the shape of the bending moment variation.



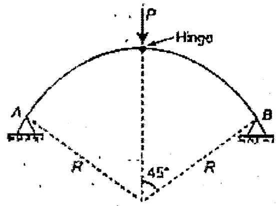
(a)


(b)


(c)


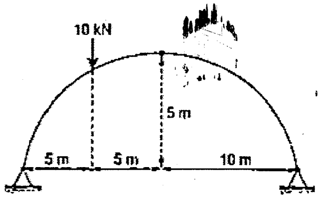
(d)


Q.12 A three hinged arch is subjected to point load at crown. The magnitude of horizontal reaction at A is



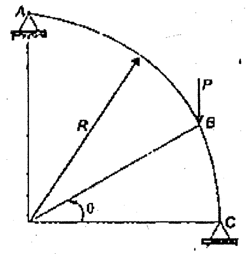
(a) $\frac{P}{2(\sqrt{2}-1)}$ (b) $\frac{P}{2}$
(c) $(\sqrt{2}-1)$ (d) $\frac{P}{2\sqrt{2}}$

Q.13 A three hinged parabolic arch having a span of 20 m and a rise of 5 m carries a point load of 10 kN at quarter span from the left end as shown in the figure. The resultant reaction at the left support and its inclination with the horizontal are respectively



(a) 9.01 kN and 56.31°
(b) 9.01 kN and 33.69°
(c) 7.50 kN and 56.31°
(d) 2.50 kN and 33.69°

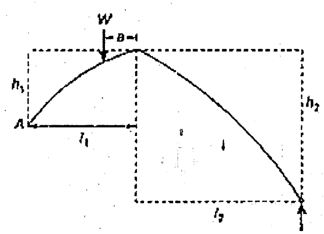
Q.14 A three hinged arch, shown in the figure is quarter of a circle. If the vertical and horizontal components of reaction at A are equal, the value of θ is



(a) 60° (b) 45°
(c) 30° (d) None of these

Q.15 A three hinged arch of span 30 m and rise 6 m is subjected to a rise of temperature of 40°C. The change in the rise of the arch is
(Take $\alpha = 12 \times 10^{-6}$ per °C)
(a) 20.9 mm (b) 30.9 mm
(c) 10.9 mm (d) 40.9 mm

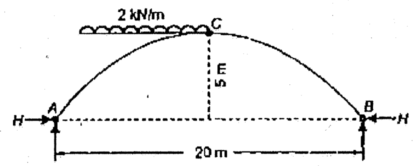
Q.16 Find the horizontal thrust at each support for the three hinged arch as shown in the figure.



(a) $\frac{Wl_1(l_2-a)}{h_1l_2+h_2l_1}$ (b) $\frac{Wl_2(l_1-a)}{h_1l_2+h_2l_1}$
(c) $\frac{Wl_1(l_2-a)}{h_1l_1+h_2l_2}$ (d) $\frac{Wl_2(l_1-a)}{h_1l_1+h_2l_2}$

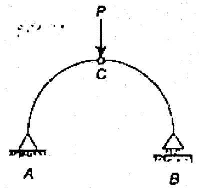
Q.17 A two hinged semicircular arch of radius *R* carries a concentrated load W at the crown. The horizontal thrust is
(a) $\frac{W}{2\pi}$ (b) $\frac{W}{\pi}$
(c) $\frac{2W}{3\pi}$ (d) $\frac{4W}{3\pi}$

Q.18 The horizontal thrust at support *A* in a three hinged arch shown in the given figure is



(a) 5 kN (b) 10 kN
(c) 15 kN (d) 20 kN

Q.19 The vertical reaction at support '*A*' for a three hinged parabolic arch as shown in the figure is



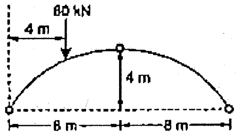
(a) 0 (b) $\frac{P}{2}$
(c) *P* (d) None of these

Q.20 A symmetrical two-hinged parabolic arch when subjected to uniformly distributed load on the entire horizontal span, is subjected to
(a) radial shear alone
(b) normal thrust alone
(c) normal thrust and bending moment
(d) normal thrust, radial shear and bending

Q.21 Three semicircular symmetrically three hinged arch have radii 5 m, 7.5 m and 10 m respectively, and each arch supports a point load W at its own crown. What is the ratio of horizontal thrusts in these arches?

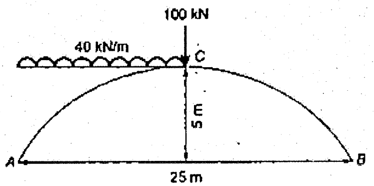
(a) 1 : 2 : 3 (b) 1 : 1.5 : 3
(c) 1 : 1 : 1 (d) 1 : 1.5: 2

Q.22 The three hinged arch shown as in the given figure will have the value of horizontal reaction as



(a) 20 kN (b) 30 kN
(c) 40 kN (d) 50 kN

Q.23 The horizontal thrust at A for the two hinged arch shown in given figure, will be



(a) 312.5 kN (b) 410.16 kN
(c) 97.66 kN (d) 315 kN

Q.24 The shape of influence line diagram for the horizontal thrust of three hinged arch is

(a) Rectangular (b) Triangular
(c) Parabolic (d) Cubic

Q.25 **Assertion A:** The bending moment at any section in an arch is less than that at a similar point in a corresponding simply supported beam with the same loading.

Reason (R): The horizontal reaction at the support induces a moment opposite in nature to that caused by beam action.

(a) Both Assertion (A) and Reason (R) are individually true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are individually true but Reason (R) is NOT the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

Q.26 For a two-hinged parabolic arch, let V_x is sum of the vertical forces on the left hand side of the section and H is the horizontal thrust. If α is the angle of tangent at the point on arch with horizontal, the axial thrust at section from left hand side is given by

(a) $V_x \cos\alpha - H \sin\alpha$
(b) $V_x \sin\alpha - H \cos\alpha$
(c) $V_x \cos\alpha + H \sin\alpha$
(d) $V_x \sin\alpha + H \cos\alpha$

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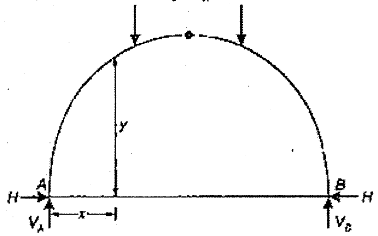
Answers Arches

1. (b)	2. (b)	3. (c)	4. (a)	5. (a)	6. (c)	7. (a)	8. (c)	9. (a)	10. (c)
11. (b)	12. (a)	13. (a)	14. (b)	15. (a)	16. (b)	17. (b)	18. (b)	19. (a)	20. (b)
21. (d)	22. (c)	23. (b)	24. (b)	25. (a)	26. (d)				

Explanations Arches

1. (b)

Take simply supported beam and 3 hinged arch of same span and same load.



$$M_x = V_A x - H.y$$

∴ BM in 3 hinged arch is less

2. (b)

Eddy's theorem states that bending moment in an arch is proportional to intercept between arch axis and funicular polygon.

4. (a)

A parabolic arch having UDL on entire span carries zero moment at any section that is why a parabolic arch subjected to UDL is designed for normal thrust only.

5. (a)

I. A 3-hinged parabolic arch consist of two different parabolic arch.

A. Carrying UDL

Horizontal thrust

$$H = \frac{wL^2}{2\left[\sqrt{h_1} + \sqrt{h_2}\right]^2}$$

Vertical thrust

$$V_A = \frac{w\sqrt{h_1}}{\left[\sqrt{h_1} + \sqrt{h_2}\right]}$$

$$V_B = \frac{w\sqrt{h_2}}{\left[\sqrt{h_1} + \sqrt{h_2}\right]}$$

B. Carrying concentrated Load W at crown

$$H = \frac{WL}{\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}$$

II. 3 hinged circular arch consists of two different circular arch with concentrated load 'W' at crown

$$H = V_a = V_b = \frac{W}{2} \rightarrow \text{Independent of radius of arch.}$$

6. (c)

Parabolic two hinge arch

$$H = \frac{wl^2}{8h} \rightarrow \text{Subjected to UDL}$$

$$H = \frac{25}{128}\frac{WL}{h} \rightarrow \text{Subjected to concentrated load}$$

Semi circular 2 hinge arch

$$H = \frac{4}{3}\frac{wR}{\pi} \rightarrow \text{Subjected to UDL}$$

$$H = \frac{W}{\pi} \rightarrow \text{Subjected to concentrated load}$$

7. (a)

From Maxwell-Betti theorem

$$P_A \Delta_{AB} = M_{BA} \theta_{BA}$$

$$\therefore \quad \theta_{BA} = \frac{10 \times 0.5}{50} = 0.1 \text{ radians}$$

8. (c)

Increase in temperature in a two hinged arch will cause horizontal thrust only. Moment due to horizontal thrust is $-Py$.

So maximum bending moment will be at crown as crown has highest value of y.

9. (a)

In the case of two hinged and fixed arches rib shortens by an amount $\frac{HL}{AE}$.

There is always horizontal compressive thrust due to temperature increase or lack of fit in the case of two hinged arches. While in three hinged arches no such thrust exists and only the shape will change at crown.

10. (c)

The moment at crown is zero. Consider portion AC.

$H_A.2R = V_A.2R \Rightarrow H_A = V_A$

For BC $H_B.R = V_B.R \Rightarrow H_B = V_B$

As $H_A = H_B$

So $V_A = V_B$

Now $V_A + V_B = W$

So $V_A = V_B = \frac{W}{2}$

11. (b)

The horizontal thrust influence line in two-hinged arch is a fourth order parabola.

However bending moment at any point

= Beam B.M. – Thrust B.M.

Horizontal thrust due to udl throughout the span

$= \frac{wl^2}{8h}$

For half span loading shown in the figure, the symmetry shows that horizontal thrust

$H = \frac{wl^2}{16h}$

B.M. for loaded half portion taking x positive from A.

$$M = \frac{3}{8}wlx - \frac{wx^2}{2} - \frac{wl^2}{16h}\cdot\frac{4h}{l^2}x(l-x)$$

$$= \frac{wlx}{8} - \frac{wx^2}{4} = \frac{wx}{8}(l-2x)x \le \frac{l}{2}$$

For right half $M = -\frac{wx}{8}(l-2x)$

with x positive from B.

These expressions are same as that for three hinged arch in the same case.

12. (a)

Vertical reaction at both support $= \frac{P}{2}$ (by symmetry)

Now, taking moment about hinge at crown from LHS = 0

$$\Rightarrow \quad \frac{P}{2}\times\frac{R}{2} - H\left(R - \frac{R}{\sqrt{2}}\right) = 0$$

$$\Rightarrow \quad H = \frac{P}{2(\sqrt{2}-1)}$$

13. (a)

Let the vertical reactions at left and right support be V_L and V_R upwards respectively.

Taking moments about right support, we get

$V_L \times 20 - 10 \times 15 = 0$

$\Rightarrow \quad V_L = 7.5$ kN

$\therefore \quad V_R = 10 - 7.5 = 2.5$ kN

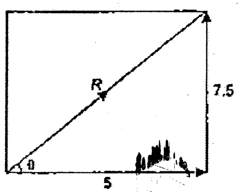
Let the horizontal reaction at left support be H from left to right.

Taking moments about the crown from left, we get

$7.5 \times 10 - 10 \times 5 - H \times 5 = 0$

$\Rightarrow \quad H = 5$ kN

Let the resultant reaction at the left support makes an angle θ with the horizontal.



$\tan\theta = \frac{7.5}{5}$

$\Rightarrow \quad \theta = \tan^{-1}(1.5)$

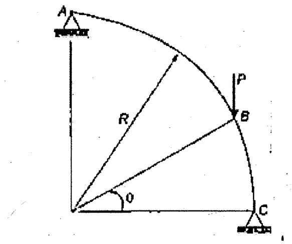
$\Rightarrow \quad \theta = 56.31°$

Resultant reaction,

$R = \sqrt{H^2 + V_L^2}$

$= \sqrt{5^2 + (7.5)^2} = 9.01$ kN

14. (b)



Values of horizontal and vertical components of reaction at A are equal at θ = 45°. Since the vertical and horizontal components are given by sinθ and cosθ respectively, and sin θ = cos θ at 45° thus the components are equal.

15. (a)

Change in rise of the arch

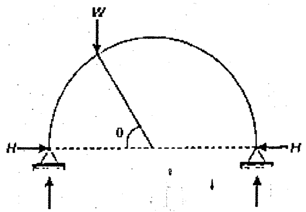
$$\delta = \frac{l^2 + 4h^2}{4h}\alpha T$$

$$= \frac{900 + 4\times 6^2}{4\times 6}\times 12\times 10^{-6}\times 40$$

$= 0.0209$ m = 20.9 mm

17. (b)

For two hinged semicircular arch with load W applied at any section, the radius vector makes an angle θ with the horizontal.



$\therefore \quad H = \frac{W}{\pi}\sin^2\theta$

When load is at crown.

$\theta = \frac{\pi}{2}$

$\therefore \quad H = \frac{W}{\pi}$

Hence option (b) is correct.

18. (b)

$\Sigma F_y = 0$

$\therefore \; R_A + R_B = 2 \times 10 = 20$ ...(i)

$\Sigma M_B = 0; \; \therefore \; R_A \times 20 - 2 \times 10 \times 15 = 0$

$\therefore \quad R_A = 15$ kN (↑)

and $\quad R_B = 5$ kN (↑)

$M_C = 0$ from left

$R_A \times 10 - H \times 5 - 10 \times 2 \times 5 = 0$

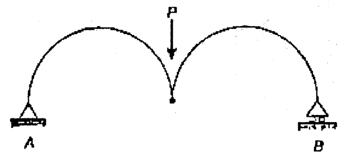
$15 \times 10 - 10 \times 2 \times 5 = 5H$

$\therefore \quad H = 10$ kN (→)

Hence option (b) is correct.

19. (a)

Structure is unstable. It will get deflect as shown in the figure below



Hence, vertical reaction at 'A' will be zero.

22. (c)

Let vertical and horizontal reactions at right support be V and H respectively.

Taking moment about the left hinge

$V_R \times 16 = 80 \times 4$

$\therefore \quad V_R = 20$ kN

Taking moment about the top hinge

$H \times 4 = 20 \times 8$

$\therefore \quad H = 40$ kN

23. (b)

Due to udl, $H_1 = \frac{wl^2}{16h} = \frac{40\times 25^2}{16\times 5} = 312.5$ kN

Due to point load,

$H_2 = \frac{25}{128}\frac{Wl}{h} = 97.66$ kN

Total thrust $= H_1 + H_2 = 410.16$ kN

26. (d)

Shear force at a section on left side

$V = V_r \cos\alpha - H\sin\alpha$

Axial (Radial) thrust, $N = V_r \sin\alpha + H\cos\alpha$.

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