2. Sequences and series of real numbers

Exercise 2.1

1. Question

Write the first three terms of the following sequences whose n^{th} terms are given by

(i) $a_n = \frac{n(n-2)}{3}$ (ii) $c_n = (-1)^n 3^{n+2}$ (iii) $z_n = \frac{(-1)^n n(n+2)}{4}$

Answer

(i) Here, $a_n = \frac{n(n-2)}{3}$

For n = 1, $a_1 = \frac{1(1-2)}{3} = \frac{1(-1)}{3} = \frac{-1}{3}$

For n = 2, $a_2 = \frac{2(2-2)}{3} = \frac{2(0)}{3} = \frac{0}{3} = 0$

For n = 3, $a_3 = \frac{3(3-2)}{3} = \frac{3(1)}{3} = \frac{3}{3} = 1$

Hence, the first three terms of the sequence are $\frac{-1}{3}$, 0 and 1.

(ii) Here, $c_n = (-1)^n 3^{n+2}$

For n = 1, $c_1 = (-1)^1 3^{1+2} = (-1) 3^3 = (-1)(27) = -27$

For n = 2, $c_2 = (-1)^2 3^{2+2} = (1) 3^4 = (1)(81) = 81$

For n = 3, $c_3 = (-1)^3 3^{3+2} = (-1) 3^5 = (-1)(243) = -243$

Hence, the first three terms of the sequence are -27, 81 and -243.

(iii) Here, $z_n = \frac{(-1)^n n(n+2)}{4}$

For n = 1, $z_1 = \frac{(-1)^1 1(1+2)}{4} = \frac{(-1)1(3)}{4} = \frac{-3}{4}$

For n = 2, $z_2 = \frac{(-1)^2 2(2+2)}{4} = \frac{(1)2(4)}{4} = \frac{8}{4} = 2$

For n = 3, $z_3 = \frac{(-1)^3 3(3+2)}{4} = \frac{(-1)3(5)}{4} = \frac{-15}{4}$

Hence, the first three terms of the sequence are $\frac{-3}{4}$, 2 and $\frac{-15}{4}$.

2. Question

Find the indicated terms in each of the sequences whose n^{th} terms are given by

(i) $a_n = \frac{n+2}{2n+3}; a_7, a_9$ (ii) $a_n = (-1)^n 2^{n+3} (n+1); a_5, a_8$

(iii) $a_n = 2n^2 - 3n + 1; a_5, a_7$ (iv) $a_n = (-1)^n (1 - n + n^2); a_5, a_8$

Answer

(i) Here, $a_n = \frac{n+2}{2n+3}$

For n = 7, $a_7 = \frac{7+2}{2(7)+3} = \frac{9}{14+3} = \frac{9}{17}$

For n = 9, $a_9 = \frac{9+2}{2(9)+3} = \frac{11}{18+3} = \frac{11}{21}$

(ii) Here, $a_n = (-1)^n\, 2^{n+3}\, (n + 1)$

For n = 5, $a_5 = (-1)^5\, 2^{5+3}\, (5 + 1) = (-1)\, 2^8\, (6) = -6\, (256) = -1536$

For n = 8, $a_8 = (-1)^8\, 2^{8+3}\, (8 + 1) = (1)\, 2^{11}\, (9) = 9\, (2048)$

= 18432

(iii) Here, $a_n = 2n^2 - 3n + 1$

For n = 5, $a_5 = 2(5)^2 - 3(5) + 1 = 2(25) - 15 + 1 = 50 - 15 + 1$

= 36

For n = 7, $a_7 = 2(7)^2 - 3(7) + 1 = 2(49) - 15 + 1 = 98 - 21 + 1$

= 78

(iv) Here, $a_n = (-1)^n\, (1 - n + n^2)$

For n = 5, $a_5 = (-1)^5\, (1 - 5 + 5^2) = (-1)\, (1 - 5 + 25) = (-1)\, (21)$

= -21

For n = 8, $a_5 = (-1)^8\, (1 - 8 + 8^2) = (1)\, (1 - 8 + 64) = (1)\, (57)$

= 57

3. Question

Find the 18th and 25th terms of the sequence defined by

$$a_n = \begin{cases} n(n+3), & \text{if } n \in \mathbb{N} \text{ and n is even} \\ \dfrac{2n}{n^2+1}, & \text{if } n \in \mathbb{N} \text{ and n is odd.} \end{cases}$$

Answer

For n = 18, n is even.

So, $a_{18} = 18(18 + 3) = 18(21) = 378$

For n = 25, n is odd.

So, $a_{25} = \frac{2(25)}{25^2+1} = \frac{50}{625+1} = \frac{50}{626} = \frac{25}{313}$

4. Question

Find the 13th and 16th terms of the sequence defined by

$$b_n = \begin{cases} n^2, & \text{if } n \in \mathbb{N} \text{ and n is even} \\ n(n+2), & \text{if } n \in \mathbb{N} \text{ and n is odd.} \end{cases}$$

Answer

For n = 13, n is odd.

So, $b_n = 13(13 + 2) = 13(15) = 195$

For n = 16, n is even.

So, $b_n = 16^2 = 256$

5. Question

Find the first five terms of the sequence given by $a_1 = 2$, $a_2 = 3 + a_1$ and $a_n = 2a_{n-1} + 5$ for n>2.

Answer

Given that $a_1 = 2$, $a_2 = 3 + a_1$ and $a_n = 2a_{n-1} + 5$ for n > 2.

Now, $a_1 = 2$

⇒ $a_2 = 3 + a_1 = 3 + 2 = 5$

⇒ $a_3 = 2a_2 + 5 = 2(5) + 5 = 10 + 5 = 15$

⇒ $a_4 = 2a_3 + 5 = 2(15) + 5 = 30 + 5 = 35$

⇒ $a_5 = 2a_4 + 5 = 2(35) + 5 = 70 + 5 = 75$

∴ The required terms of sequence are 2, 5, 15, 35 and 75.

6. Question

Find the first six terms of the sequence given by

$a_1 = a_2 = a_3 = 1$ and $a_n = a_{n-1} + a_{n-2}$ for n >3.

Answer

Given that $a_1 = a_2 = a_3 = 1$ and $a_n = a_{n-1} + a_{n-2}$ for n > 3.

Now, $a_1 = 1$

⇒ $a_2 = 1$

⇒ $a_3 = 1$

⇒ $a_4 = a_3 + a_2 = 1 + 1 = 2$

⇒ $a_5 = a_4 + a_3 = 2 + 1 = 3$

⇒ $a_6 = a_5 + a_4 = 3 + 2 = 5$

∴ The required terms of the sequence are 1, 1, 1, 2, 3 and 5.

Exercise 2.2

1. Question

The first term of an A.P. is 6 and the common difference is 5. Find the A.P. and its general term.

Answer

First term, a = 6; Common difference, d = 5

We know that the AP is in the form of a, a + d, a + 2d, a + 3d … a + (n – 1) d, a + nd ….

⇒ AP = 6, 6 + 5, 6 + 2(5), 6 + 3(5) …

= 6, 11, 6 + 10, 6 + 15 …

= 6, 11, 16, 21 …

∴ The required A.P. is 6, 11, 16, 21 …

We know that the general form, $t_n = a + (n – 1) d$.

⇒ $t_n = 6 + (n – 1) 5$

= 6 + 5n – 5

= 5n + 1

∴ The required general term, $t_n = 5n + 1$

2. Question

Find the common difference and 15th term of the A.P. 125, 120, 115, 110 …

Answer

The given A.P. is 125, 120, 115, 110 …

We know that common difference, $d = a_1 - a$.

⇒ d = 120 – 125

= -5

∴ Common difference = -5

We know that nth term of A.P. , $t_n = a + (n - 1) d$.

⇒ $t_{15} = 125 + (15 - 1) (-5)$

= 125 + 14 (-5)

= 125 – 70

= 55

∴ 15^{th} term of A.P. = 55

3. Question

Which term of the arithmetic sequence $24, 23\frac{1}{4}, 22\frac{1}{2}, 21\frac{3}{4}, \ldots$ is 3?

Answer

Here, first term, a = 24

We know that common difference, $d = a_1 - a$.

$$\therefore d = 23\frac{1}{4} - 24 = -\frac{3}{4}$$

We know that nth term of A.P. , $t_n = a + (n - 1) d$.

To find the nth term, here $t_n = 3$

⇒ 3 = 24 + (n – 1) (-3/4)

⇒ 3 – 24 = (n – 1) (-3/4)

⇒ -21 = (n – 1) (-3/4)

⇒ $-21 \times \frac{-4}{3} = n - 1$

⇒ 28 = n – 1

⇒ n = 28 + 1 = 29

∴ t_{29} of the given A.P. is 3.

4. Question

Find the 12th term of the A.P. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \ldots..$

Answer

Given A.P. √2, 3√2, 5√2 …

Here, first term, a = √2

We know that common difference, $d = a_1 - a$.

∴ d = 3√2 - √2 = 2√2

We know that nth term of A.P. , $t_n = a + (n - 1) d$.

⇒ $t_{12} = \sqrt{2} + (12 - 1)(2\sqrt{2})$

= √2 + 11 (2√2)

= √2 + 22 √2

= 23√2

∴ 12th term of A.P. i.e. $t_{12} = 23\sqrt{2}$

5. Question

Find the 17th term of the A.P. 4, 9, 14 …

Answer

Given A.P. 4, 9, 14 …

Here, first term, a = 4

We know that common difference, $d = a_1 - a$.

∴ d = 9 - 4 = 5

We know that nth term of A.P. , $t_n = a + (n - 1) d$.

⇒ $t_{17} = 4 + (17 - 1)(5)$

= 4 + 16 (5)

= 4 + 80

= 84

∴ 17th term of A.P. i.e. $t_{17} = 84$

6. Question

How many terms are there in the following Arithmetic Progressions?

(i) $-1, -\frac{5}{6}, -\frac{2}{3}, \ldots, \frac{10}{3}$. (ii) 7, 13, 19, …, 205.

Answer

(i) Here, first term, a = -1

Last term, $l = \frac{10}{3}$

We know that common difference, $d = a_1 - a$.

∴ $d = \frac{-5}{6} - (-1) = \frac{-5}{6} + 1 = \frac{-5+6}{6} = \frac{1}{6}$

We know that Number of terms, $n = \left(\frac{l-a}{d}\right) + 1$.

⇒ $n = \frac{\frac{10}{3} - (-1)}{\frac{1}{6}} + 1$

$= \frac{\frac{10+3}{3}}{\frac{1}{6}} + 1$

$= \frac{13}{\frac{1}{2}} + 1$

$= (13 \times 2) + 1$

$= 26 + 1$

$= 27$

∴ There are 27 terms in the given A.P.

(ii) Here, first term, $a = 7$

Last term, $l = 205$

We know that common difference, $d = a_1 – a$.

∴ $d = 13 - 7 = 6$

We know that Number of terms, $n = (\frac{l-a}{d}) + 1$.

$\Rightarrow n = \frac{205-7}{6} + 1$

$= \frac{198}{6} + 1$

$= 33 + 1$

$= 34$

∴ There are 34 terms in the given A.P.

7. Question

If 9th term of an A.P. is zero, prove that its 29th term is double (twice) the 19th term.

Answer

Here, $t_9 = 0$

We have to prove that $t_{29} = 2t_{19}$

We know that nth term of A.P. , $t_n = a + (n – 1) d$.

$\Rightarrow t_9 = a + (9 – 1) d = a + 8d$

But $t_9 = 0$

$\Rightarrow a + 8d = 0$

∴ $a = -8d$ … (i)

Now, $t_{29} = a + (29 – 1) d$

$= a + 28d$

$= -8d + 28d$ [From (i)]

$= 20d$

∴ $t_{29} = 20d$ … (1)

Now, $t_{19} = a + (19 – 1) d$

$= a + 18d$

$= -8d + 18d$ [From (i)]

$= 10d$

∴ $t_{19} = 10d$ … (2)

Equating (1) and (2),

⇒ 20d = 10d

⇒ 20d = 10d (2)

∴ $t_{29} = 2(t_{19})$

Hence proved.

8. Question

The 10th and 18th terms of an A.P are 41 and 73 respectively. Find the 27th term.

Answer

Here, $t_{10} = 41$ and $t_{18} = 73$

We know that nth term of A.P. , $t_n = a + (n – 1) d$.

First, $t_{10} = a + (10 – 1) d = 41$

⇒ a + 9d = 41 … (1)

Then, $t_{18} = a + (18 – 1) d = 73$

⇒ a + 17d = 73 … (2)

From (1) and (2),

a + 9d = 41

a + 17d = 73

(-) (-) (-)

-8d = -32

⇒ d = 32/8 = 4

Substituting d = 4 in (1),

⇒ a + 9(4) = 41

⇒ a = 41 – 36 = 5

Now, $t_{27} = 5 + (27 – 1) (4)$

= 5 + 26(4)

= 5 + 104

= 109

∴ The 27^{th} term i.e. $t_{27} = 109$

9. Question

Find n so that the nth terms of the following two A.P.'s are the same.

1, 7, 13, 19,… and 100, 95, 90,… .

Answer

For first A.P. , first term, a = 1; common difference, d = 7 – 1 = 6

For second A.P, first term, a = 100; common difference , d = 95 – 100 = -5

We know that nth term of A.P. , $t_n = a + (n – 1) d$.

First A.P:

$\Rightarrow t_{n1} = 1 + (n – 1) (6)$

$= 1 + 6n – 6$

$\therefore t_{n1} = 6n – 5$

Second A.P:

$\Rightarrow t_{n2} = 100 + (n – 1) (-5)$

$= 100 – 5n + 5$

$\therefore t_{n2} = 105 – 5n$

Given that nth terms of the two A.P.’s are the same.

$\therefore t_{n1} = t_{n2}$

$\Rightarrow 6n – 5 = 105 – 5n$

$\Rightarrow 6n + 5n = 105 + 5$

$\Rightarrow 11n = 110$

$\Rightarrow n = 110/11 = 10$

∴ For the nth terms of the two given A.P.’s to be same, the value of n = 10.

10. Question

How many two digit numbers are divisible by 13?

Answer

The two digits form the A.P.: 10, 11, 12 … 99.

The two digit numbers that are divisible by 13 form the A.P.: 13, 26, 39 … 91.

Here, first term, a = 13

We know that common difference, $d = a_1 – a$.

$\therefore d = 26 – 13 = 13$

And last term, l = 91

We know that $l = a + (n – 1) d$.

$\Rightarrow 91 = 13 + (n – 1) (13)$

$\Rightarrow 91 – 13 = (n – 1) (13)$

$\Rightarrow 78 = (n – 1) (13)$

$\Rightarrow 78/13 = (n – 1)$

$\Rightarrow 6 = n – 1$

$\Rightarrow n = 6 + 1 = 7$

∴ There are 7 two digit numbers that are divisible by 13.

11. Question

A TV manufacturer has produced 1000 TVs in the seventh year and 1450 TVs in the tenth year. Assuming that the production increases uniformly by a fixed number every year, find the number of TVs produced in the first year and in the 15th year.

Answer

Here, $t_7 = 1000$ and $t_{10} = 1450$

We know that nth term of A.P. , $t_n = a + (n – 1) d$.

First, $t_7 = a + (7 – 1) d = 1000$

⇒ a + 6d = 1000 … (1)

Then, $t_{10} = a + (10 – 1) d = 1450$

⇒ a + 9d = 1450 … (2)

From (1) and (2),

a + 6d = 1000

a + 9d = 1450

(-) (-) (-)

-3d = -450

⇒ d = 450/3 = 150

Substituting d = 150 in (1),

⇒ a + 6(150) = 1000

⇒ a = 1000 – 900 = 100

Now, $t_1 = 100 + (1 – 1) (150)$

= 100 + 0

= 100

And $t_{15} = 100 + (15 – 1) (150)$

= 100 + 14 (150)

= 100 + 2100

= 2200

∴ Number of TVs produced in the first year are 100 and in the 15^{th} year are 2200.

12. Question

A man has saved ₹640 during the first month, ₹720 in the second month and ₹800 in the third month. If he continues his savings in this sequence, what will be his savings in the 25th month?

Answer

The savings form the A.P.: 640, 720, 800 …

Here, first term, a = 640

We know that common difference, $d = a_1 – a$.

∴ d = 720 – 640 = 80

We know that nth term of A.P. , $t_n = a + (n – 1) d$.

⇒ $t_{25} = 640 + (25 – 1) (80)$

= 640 + (24) (80)

= 640 + 1920

= 2560

∴ The man’s savings in the 25^{th} month are Rs. 2560.

13. Question

The sum of three consecutive terms in an A.P. is 6 and their product is –120. Find the three numbers.

Answer

We know that the three consecutive terms of an A.P. may be taken as a – d, a, a + d.

Given, a – d + a + a + d = 6

⇒ 3a = 6

⇒ a = 6/3 = 2

Also given (a – d) (a) (a + d) = -120

We know that (a – b) (a + b) = $a^2 - b^2$

⇒ $(a^2 - d^2)$ (2) = -120

⇒ $(2^2 - d^2)$ = -120/2

⇒ 4 – d^2 = -60

⇒ d^2 = 60 + 4 = 64

⇒ d = 8

So, the numbers are

⇒ a – d = 2 – 8 = -6

⇒ a = 2

⇒ a + d = 2 + 8 = 10

∴ The three consecutive numbers are -6, 2, 10.

14. Question

Find the three consecutive terms in an A. P. whose sum is 18 and the sum of their squares is 140.

Answer

We know that the three consecutive terms of an A.P. may be taken as a – d, a, a + d.

Given, a – d + a + a + d = 18

⇒ 3a = 18

⇒ a = 18/3 = 6

Also given $(a - d)^2 + a^2 + (a + d)^2 = 140$

We know that $(a - b)^2 = a^2 - 2ab + b^2$

And $(a + b)^2 = a^2 + 2ab + b^2$

⇒ $a^2 - 2ad + d^2 + a^2 + a^2 + 2ad + d^2 = 140$

⇒ $3a^2 + 2d^2 = 140$

⇒ $3(6)^2 + 2d^2 = 140$

⇒ $108 + 2d^2 = 140$

⇒ $2d^2 = 140 - 108 = 32$

⇒ $d^2 = 32/2 = 16$

⇒ d = 4

So, the numbers are

⇒ a – d = 6 – 4 = 2

⇒ a = 6

⇒ a + d = 6 + 4 = 10

∴ The three consecutive numbers are 2, 6, 10.

15. Question

If m times the mth term of an A.P. is equal to n times its nth term, then show that the (m + n)th term of the A.P. is zero.

Answer

We know that nth term of A.P. , $t_n = a + (n - 1)\, d$.

⇒ First, $t_n = a + (n - 1)\, d$

⇒ Then, $t_m = a + (m - 1)\, d$

Given, $mt_m = nt_n$

⇒ $m\,[a + (m - 1)\, d] = n\,[a + (n - 1)\, d]$

⇒ $ma + m^2d - md = na + n^2d - nd$

⇒ $ma - na + m^2d - n^2d - md + nd = 0$

⇒ $a\,(m - n) + d\,(m^2 - n^2) - d\,(m - n) = 0$

We know that $a^2 - b^2 = (a - b)\,(a + b)$

⇒ $(m - n)\,[a + d\,(m + n) - d] = 0$

⇒ $[a + d\,(m + n) - d] = 0$

⇒ $a + (m + n - 1)\, d = 0$

∴ (m + n)th term, $t_{m+n} = 0$

Hence proved.

16. Question

A person has deposited ₹25,000 in an investment which yields 14% simple interest annually. Do these amounts (principal + interest) form an A.P.? If so, determine the amount of investment after 20 years.

Answer

Yes, the amounts form an A.P.

Given principal, p = Rs. 25,000

Simple Interest, r = 14%

Time, t = 20 years

We know that Total amount = $p\,(1 + \frac{r \times t}{100})$

⇒ Total Amount = $25000\,(1 + \frac{14 \times 20}{100})$

= $25000\,(1 + \frac{14}{5})$

= $25000\,(\frac{19}{5})$

= 95000

∴ Amount of investment after 20 years = Rs. 95, 000

17. Question

If a, b, c are in A.P. then prove that $(a - c)^2 = 4(b^2 - ac)$.

Answer

Given, a, b and c are in A.P.

We know that when t_1, t_2, t_3 … are in A.P., $t_3 - t_2 = t_2 - t_1$

$\Rightarrow c - b = b - a$

$\Rightarrow 2b = a + c$

Squaring on both sides,

$\Rightarrow (2b)^2 = (a + c)^2$

We know that $(a + b)^2 = a^2 + 2ab + b^2$.

$\Rightarrow 4b^2 = a^2 + 2ac + c^2$

Subtracting 4ac on both sides,

$\Rightarrow 4b^2 - 4ac = a^2 + 2ac + c^2 - 4ac$

$\Rightarrow 4(b^2 - ac) = a^2 - 2ac + c^2$

$\therefore 4(b^2 - ac) = (a - c)^2$

Hence proved.

18. Question

If a, b, c are in A.P. then prove that $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are also in A.P.

Answer

Given, a, b, c are in A.P.

Here, first term = a

Common difference, $d_1 = b - a$ … (1)

and $d_2 = c - b$ … (2)

Consider $\frac{1}{bc}, \frac{1}{ca}$ and $\frac{1}{ab}$,

Common difference, $d_3 = \frac{1}{ca} - \frac{1}{bc}$

$= \frac{bc-ca}{abc}$

$= \frac{b-a}{abc}$ … (3)

$\Rightarrow d_4 = \frac{1}{ab} - \frac{1}{ca}$

$= \frac{ca-ab}{abc}$

$= \frac{c-b}{abc}$ … (4)

From (1) and (2),

$\Rightarrow d_1 = d_2$

$\Rightarrow b - a = c - b$

Dividing both sides by abc,

$\Rightarrow \frac{b-a}{abc} = \frac{c-b}{abc}$

$\Rightarrow d_3 = d_4$ [From (3) and (4)]

Hence, $\frac{1}{bc}$, $\frac{1}{ca}$ and $\frac{1}{ab}$ are in A.P.

19. Question

If a^2, b^2, c^2 are in A.P. then show that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are also in A.P.

Answer

Given, a^2, b^2 and c^2 are in A.P.

We know that when t_1, t_2, t_3 … are in A.P., $t_3 - t_2 = t_2 - t_1$

$\Rightarrow b^2 - a^2 = c^2 - b^2$

We know that $a^2 - b^2 = (a - b)(a + b)$

$\Rightarrow (b - a)(b + a) = (c - b)(c + b)$

$\Rightarrow \frac{(b-a)}{(c+b)} = \frac{(c-b)}{(b+a)}$

Dividing by (c + a) on both sides,

$\Rightarrow \frac{(b-a)}{(c+b)(c+a)} = \frac{(c-b)}{(b+a)(c+a)}$

$\Rightarrow \frac{(b+c-c-a)}{(c+b)(c+a)} = \frac{(c+a-a-b)}{(b+a)(c+a)}$

$\Rightarrow \frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$

Hence, $\frac{1}{b+c}$, $\frac{1}{c+a}$ and $\frac{1}{a+b}$ are in A.P.

20. Question

If $a^x = b^y = c^z$, $x \neq 0$, $y \neq 0$, $z \neq 0$ and $b^2 = ac$, then show that $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

Answer

Let $a^x = b^y = c^z = k$.

We know that if $a^m = k$, then $a = k^{\frac{1}{m}}$.

$\Rightarrow a = k^{\frac{1}{x}}$, $b = k^{\frac{1}{y}}$, $c = k^{\frac{1}{z}}$

Given, $b^2 = ac$

$\Rightarrow {k^{\frac{1}{y}}}^2 = k^{\frac{1}{x}} \times k^{\frac{1}{z}}$

We know that $(a^m)^n = a^{mn}$ and $a^m \times a^n = a^{m+n}$.

$\Rightarrow k^{\frac{2}{y}} = k^{\frac{1}{x}+\frac{1}{z}}$

Bases are same, so we equate the powers.

$\Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$

$\Rightarrow \frac{1}{y} + \frac{1}{y} = \frac{1}{x} + \frac{1}{z}$

$\Rightarrow \frac{1}{y} - \frac{1}{x} = \frac{1}{z} - \frac{1}{x}$

We know that when t_1, t_2, t_3 ... are in A.P., $t_3 - t_2 = t_2 - t_1$

$\therefore \frac{1}{x}, \frac{1}{y}$ and $\frac{1}{z}$ are in A.P.

Exercise 2.3

1 A. Question

Find out which of the following sequences are geometric sequences. For those geometric sequences, find the common ratio.

0.12, 0.24, 0.48,….

Answer

(Note: Sequences are in G.P. if they have common ratios

i.e., if

$\frac{a_{n+1}}{an} = \frac{a_n}{a_{n-1}}$)

Given: $a_1 = 0.12$, $a_2 = 0.24$, $a_3 = 0.48$

Now,

$\Rightarrow \frac{a_2}{a_1} = \frac{0.24}{0.12}$

$\Rightarrow \frac{a_2}{a_1} = 2$

And,

$\Rightarrow \frac{a_3}{a_2} = \frac{0.48}{0.24}$

$\Rightarrow \frac{a_3}{a_2} = 2$

Therefore,

$\Rightarrow \frac{a_2}{a_1} = \frac{a_3}{a_2} = 2$

Now, ∵ the ration is same,

⇒ 0.12, 0.24, 0.48, … is a G.P with common ratio = 2

1 B. Question

Find out which of the following sequences are geometric sequences. For those geometric sequences, find the common ratio.

0.004, 0.02, 0.1,….

Answer

(Note: Sequences are in G.P. if they have common ratios

i.e., if

$\frac{a_{n+1}}{an} = \frac{a_n}{a_{n-1}}$)

Given: $a_1 = 0.004$, $a_2 = 0.02$, $a_3 = 0.1$

Now,

$$\Rightarrow \frac{a_2}{a_1} = \frac{0.02}{0.004}$$

$$\Rightarrow \frac{a_2}{a_1} = 5$$

And,

$$\Rightarrow \frac{a_3}{a_2} = \frac{0.1}{0.02}$$

$$\Rightarrow \frac{a_3}{a_2} = 5$$

Therefore,

$$\Rightarrow \frac{a_2}{a_1} = \frac{a_3}{a_2} = 5$$

Now, ∵ the ration is same,

⇒ 0.004, 0.02, 0.1, ... is a G.P with common ratio = 5

1 C. Question

Find out which of the following sequences are geometric sequences. For those geometric sequences, find the common ratio.

$$\frac{1}{2}, \frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \ldots\ldots .$$

Answer

(Note: Sequences are in G.P. if they have common ratios

i.e., if

$\frac{a_{n+1}}{an} = \frac{a_n}{a_{n-1}}$)

Given:

$$a_1 = \frac{1}{2}, a_2 = \frac{1}{3}, a_3 = \frac{2}{9}, a_4 = \frac{4}{27}$$

Now,

$$\Rightarrow \frac{a_2}{a_1} = \frac{\frac{1}{3}}{\frac{1}{2}}$$

$$\Rightarrow \frac{a_2}{a_1} = \frac{1}{3} \times 2$$

$$\Rightarrow \frac{a_2}{a_1} = \frac{2}{3}$$

And,

$$\Rightarrow \frac{a_3}{a_2} = \frac{\frac{2}{9}}{\frac{1}{3}}$$

$$\Rightarrow \frac{a_3}{a_2} = \frac{2}{9} \times 3$$

$$\Rightarrow \frac{a_3}{a_2} = \frac{2}{3}$$

And,

$$\Rightarrow \frac{a_4}{a_3} = \frac{\frac{4}{27}}{\frac{2}{9}}$$

$$\Rightarrow \frac{a_4}{a_3} = \frac{4}{27} \times \frac{9}{2}$$

$$\Rightarrow \frac{a_4}{a_3} = \frac{2}{3}$$

Therefore,

$$\Rightarrow \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \frac{2}{3}$$

Now, ∵ the ration is same,

$$\Rightarrow \frac{1}{2}, \frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \ldots \ldots \text{ is a G.P with common ratio } = \frac{2}{3}$$

1 D. Question

Find out which of the following sequences are geometric sequences. For those geometric sequences, find the common ratio.

$$12, 1, \frac{1}{12}, \ldots$$

Answer

(Note: Sequences are in G.P. if they have common ratios

i.e., if

$\frac{a_{n+1}}{an} = \frac{a_n}{a_{n-1}}$)

Given: $a_1 = 12$, $a_2 = 1$, $a_3 = \frac{1}{12}$

Now,

$$\Rightarrow \frac{a_2}{a_1} = \frac{1}{12}$$

And

$$\Rightarrow \frac{a_3}{a_2} = \frac{\frac{1}{12}}{1}$$

$$\Rightarrow \frac{a_3}{a_2} = \frac{1}{12}$$

Therefore,

$$\Rightarrow \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{1}{12}$$

Now, ∵ the ration is same,

$\Rightarrow\ 12, 1, \frac{1}{12}, \ldots$ is a G.P with common ratio $= \frac{1}{12}$

1 E. Question

Find out which of the following sequences are geometric sequences. For those geometric sequences, find the common ratio.

$$\sqrt{2}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}, \ldots$$

Answer

(Note: Sequences are in G.P. if they have common ratios

i.e., if

$\frac{a_{n+1}}{an} = \frac{a_n}{a_{n-1}}$)

Given:

$a_1 = \sqrt{2}, a_2 = \frac{1}{\sqrt{2}}, a_3 = \frac{1}{2\sqrt{2}}$

Now,

$$\Rightarrow \frac{a_2}{a_1} = \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}}$$

$$\Rightarrow \frac{a_2}{a_1} = \frac{1}{\sqrt{2} \times \sqrt{2}}$$

$$\Rightarrow \frac{a_2}{a_1} = \frac{1}{2}$$

And

$$\Rightarrow \frac{a_3}{a_2} = \frac{\frac{1}{2\sqrt{2}}}{\frac{1}{\sqrt{2}}}$$

$$\Rightarrow \frac{a_3}{a_2} = \frac{1}{2\sqrt{2}} \times \sqrt{2}$$

$$\Rightarrow \frac{a_3}{a_2} = \frac{1}{2}$$

Thherefore,

$$\Rightarrow \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{1}{2}$$

Now, ∵ the ration is same,

$\Rightarrow\ \sqrt{2}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}, \ldots.$ is a G.P with common ratio $= \frac{1}{2}$

1 F. Question

Find out which of the following sequences are geometric sequences. For those geometric sequences, find the common ratio.

$4, -2, -1, -\frac{1}{2}, \ldots$

Answer

(Note: Sequences are in G.P. if they have common ratios

i.e., if

$\frac{a_{n+1}}{an} = \frac{a_n}{a_{n-1}}$)

Given: a_1 = 4, a_2 = -2, a_3 = -1, $a_4 = -\frac{1}{2}$

Now,

$\Rightarrow \frac{a_2}{a_1} = \frac{2}{4}$

$\Rightarrow \frac{a_2}{a_1} = \frac{1}{2}$

And,

$\Rightarrow \frac{a_3}{a_2} = \frac{-1}{2}$

And,

$\Rightarrow \frac{a_4}{a_3} = \frac{-\frac{1}{2}}{-1}$

$\Rightarrow \frac{a_4}{a_3} = \frac{1}{2}$

Therefore,

$\Rightarrow \frac{a_2}{a_1} = \frac{a_4}{a_3} \neq \frac{a_3}{a_2}$

Now, ∵ the ration is not same,

$\Rightarrow -4, -2, -1, -\frac{1}{2}, \ldots .\text{is not a G.P.}$

2. Question

Find the 10th term and common ratio of the geometric sequence $\frac{1}{4}, -\frac{1}{2}, 1, -2, \ldots$.

Answer

Given:

$a_1 = \frac{1}{4}, a_2 = -\frac{1}{2}, a_3 = 1, a_4 = -2 \text{ forms G.P}$

$\Rightarrow \text{common ratio(r)} = \frac{a_{n+1}}{a_n}$; (where, n = 1,2,......n-1)

Now, taking n = 3.

$\Rightarrow r = \frac{a_4}{a_3}$

$\Rightarrow r = -\frac{2}{1}$

$\Rightarrow r = -2$

Also, $a_n = a_1 r^{n-1}$ (n = no. of term and a_1 = first term of G.P)

$\therefore 10^{th}$ term $(a_{10} = a_1 r^9)$

$$\Rightarrow a_{10} = (\frac{1}{4})(-2)^9$$

$$\Rightarrow a_{10} = \left(\frac{1}{2^2}\right) \times 2^9 \times (-1)^9$$

$\Rightarrow a_{10} = -2^7$

$\Rightarrow$ common ratio = -2 and 10^{th} term = -2^7

3. Question

If the 4th and 7th terms of a G.P. are 54 and 1458 respectively, find the G.P.

Answer

$a_4 = 54$ and $a_7 = 1458$

$\because a_n = a_1 r^{n-1}$ (n = no. of term, a_1 = first term of G.P, r = common ratio)

$\Rightarrow a_4 = a_1 r^3$

$\Rightarrow 54 = a_1 r^3$ ....(1)

Also, $\Rightarrow a_7 = a_1 r^6$

$\Rightarrow 1458 = a_1 r^6$ ....(2)

Dividing equation (1) & (2), we get-

$$\Rightarrow \frac{54}{1458} = \frac{r^3}{r^6}$$

$\Rightarrow r^3 = 27$

$\Rightarrow r^3 = 3^3$

$\Rightarrow r = 3$

Now, putting value of r in equation (1), we get-

$\Rightarrow 54 = a_1 3^3$

$$\Rightarrow a_1 = \frac{54}{27}$$

$\Rightarrow a_1 = 2$

Now, G.P will be-

$\Rightarrow a_1$, $a_1 r$,$a_1 r^2$, $a_1 r^3$,.........

$\Rightarrow 2, 2 \times 3, 2 \times 3^2, 2 \times 3$...

$\Rightarrow$ 2, 6, 18, 54,... is the G.P.

4. Question

In a geometric sequence, the first term is $\frac{1}{3}$ and the sixth term is $\frac{1}{729}$, find the G.P.

Answer

$a_1 = \frac{1}{3}$ and $a_6 = \frac{1}{729}$

∵ $a_n = a_1r^{n-1}$ (n = no. of term, a_1 = first term of G.P, r = common ratio)

⇒ $a_6 = a_1r^5$

⇒ $\frac{1}{729} = \frac{1}{3}r^5$ ....(1)

⇒ $r^5 = \frac{3}{729}$

⇒ $r^5 = \frac{1}{3^5}$

⇒ $r = \frac{1}{3}$

Now, G.P will be-

⇒ $a_1, a_1r, a_1r^2, a_1r^3$,.........

⇒ $\frac{1}{3}, \frac{1}{3} \times \frac{1}{3}, \frac{1}{3} \times \frac{1}{3}^2$, ...

⇒ $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$, ... is the G. P.

5. Question

Which term of the geometric sequence,

(i) 5, 2, $\frac{4}{5}, \frac{8}{25}$, ... , is $\frac{128}{15625}$? (ii) 1, 2, 4, 8,..., is 1024 ?

Answer

(i) $a_1 = 5$, $a_2 = 2$

⇒ $\text{common ratio}(r) = \frac{a_{n+1}}{a_n}$

⇒ $r = \frac{a_2}{a_1}$

⇒ $r = \frac{2}{5}$

∵ $a_n = a_1r^{n-1}$ (n = no. of term, a_1 = first term of G.P, r = common ratio)

⇒ $\frac{128}{15625} = 5 \times \left(\frac{2}{5}\right)^{n-1}$

⇒ $\frac{128}{78125} = \left(\frac{2}{5}\right)^{n-1}$

⇒ $\left(\frac{2}{5}\right)^7 = \left(\frac{2}{5}\right)^{n-1}$

⇒ n-1 = 7

⇒ n = 8

$\therefore \frac{128}{15625}$ is the 8th term of G.P.

(ii) $a_1 = 1, a_2 = 2$

$\Rightarrow$ common ratio(r) $= \frac{a_{n+1}}{a_n}$

$\Rightarrow r = \frac{a_2}{a_1}$

$\Rightarrow r = \frac{2}{1}$

$\because a_n = a_1r^{n-1}$ (n = no. of term, a_1 = first term of G.P, r = common ratio)

$\Rightarrow 1024 = 1 \times 2^{n-1}$

$\Rightarrow 2^{10} = 2^{n-1}$

$\Rightarrow$ n-1 = 10

$\Rightarrow$ n = 11

$\therefore$ 1024 is the 11th term of G.P.

6. Question

If the geometric sequences 162, 54, 18,.... and $\frac{2}{81}, \frac{2}{27}, \frac{2}{9}$,...have their nth term equal, find the value of n.

Answer

Given:For 1st G.P-

$a_1 = 162$ and $a_2 = 54$,

And for 2nd G.P-

$A_1 = \frac{2}{81}, A_2 = \frac{2}{27}$.

$\Rightarrow$ common ratio (for 1st G.P)(r) $= \frac{54}{162}$

$r = \frac{1}{3}$

$\Rightarrow$ common ratio (for 2nd G.P)(R) $= \frac{\frac{2}{27}}{\frac{2}{81}}$

R = 3

$\because a_n = a_1r^{n-1}$ (n = no. of term, a_1 = first term of G.P, r = common ratio)

And $\because$ nth term of both G.P are equal,

$\Rightarrow a_n = A_n$.

$\Rightarrow a_1r^{n-1} = A_1R^{n-1}$

$\Rightarrow 162\left(\frac{1}{3}\right)^{n-1} = \frac{2}{81}(3)^{n-1}$

$\Rightarrow \left(\frac{1}{9}\right)^{n-1} = \frac{2}{162 \times 81}$

$\Rightarrow \left(\frac{1}{9}\right)^{n-1} = \left(\frac{1}{9}\right)^{4}$

⇒ n-1 = 4

⇒ n = 5.

7. Question

The fifth term of a G.P. is 1875. If the first term is 3, find the common ratio.

Answer

a_1 = 3 and a_5 = 1875

∵ $a_n = a_1r^{n-1}$ (n = no. of term, a_1 = first term of G.P, r = common ratio)

⇒ $a_5 = a_1r^4$

⇒1875 = $3r^4$ ....(1)

$\Rightarrow r^4 = \frac{1875}{3}$

⇒ r^5 = 625

⇒ $r^5 = 5^5$

⇒ r = 5 is the common ratio.

8. Question

The sum of three terms of a geometric sequence is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

Answer

Let the first term of G. P $= \frac{a}{r}$.

⇒ second term = a and third term = ar.

(where, r is common ratio)

$\because$ sum of three terms $= \frac{39}{10}$

$\Rightarrow a(\frac{1}{r} + 1 + r) = \frac{39}{10}$

$\Rightarrow a = \frac{39r}{10(1+r+r^2)}$ ....(1)

Also, their product is 1.

$\Rightarrow \frac{a}{r} \times a \times ar = 1$

⇒ a^3 = 1

⇒ a = 1

Substituting a = 1 in equation (1)

$\Rightarrow 1 = \frac{39r}{10(1 + r + r^2)}$

⇒ 10 + 10r + $10r^2$ = 39r

$\Rightarrow 10r^2-29r + 10 = 0$

$\Rightarrow 10r^2-25r-4r + 10 = 0$

$\Rightarrow 5r(2r-5)-2(2r-5)$

$\Rightarrow (5r-2)(2r-5) = 0$

$\Rightarrow r = \frac{5}{2} \text{or} \frac{2}{5}$

Now, G.P will be-

$\frac{a}{r}, a, ar, \ldots \ldots \ldots$

$\Rightarrow \frac{1}{\frac{5}{2}}, 1, \frac{1}{\frac{2}{5}}, \ldots \ldots \text{or } \frac{1}{\frac{2}{5}}, 1, \frac{1}{\frac{5}{2}}, \ldots \ldots$

$\Rightarrow \frac{2}{5}, 1, \frac{5}{2}, \ldots \ldots . \text{or } \frac{5}{2}, 1, \frac{2}{5}, \ldots \ldots$ is the G.P with common ratio $\frac{5}{2}$ and $\frac{2}{5}$ respectively.

$\Rightarrow \frac{2}{5}, 1, \frac{5}{2}$ or $\frac{5}{2}, 1, \frac{2}{5}$, are the terms.

9. Question

If the product of three consecutive terms in G.P. is 216 and sum of their products in pairs is 156, find them.

Answer

Let the first term of G.P be $\frac{a}{r}$.

$\Rightarrow$ second term = a and third term = ar.

(where, r is the common ratio)

∵ sum of three terms is 156

$\Rightarrow \frac{a}{r} \times a \times ar = 216$

$\Rightarrow a^3 = 216$

$\Rightarrow a = 6.$ ....(1)

Also,sum of their product in pairs is 216.

$\Rightarrow \left(\frac{a}{r} \times a\right) + (a \times ar) + (ar \times \frac{a}{r}) = 156$

$\Rightarrow a^2(\frac{1}{r} + r + 1) = 156$ ....(2)

Substituting (1) in (2), we get-

$\Rightarrow 6^2\left(\frac{1}{r} + r + 1\right) = 156$

$\Rightarrow 36(1 + r + r^2) = 156r$

$\Rightarrow 36 + 36r + 36r^2 = 156r$

$\Rightarrow 36 -120r + 36r^2 = 0$

$\Rightarrow 12(3r^2-10r + 3) = 0$

$\Rightarrow 3r^2-10r + 3 = 0$

$\Rightarrow 3r^2-9r-1r + 3 = 0$

$\Rightarrow 3r(r-3) -1(r-3) = 0$

$\Rightarrow (3r-1)(r-3) = 0$

$\Rightarrow\ r\ =\ \frac{1}{3}\text{or } 3$

Now, G.P will be-

$\frac{a}{r}, a, ar, \dots\dots\dots$

$\Rightarrow \frac{6}{\frac{1}{3}}, 6, 6 \times \frac{1}{3}, \dots\dots \text{or} \frac{6}{3}, 6, 6 \times 3, \dots\dots$

$=\ 18, 6, 2, \dots \dots.$ or $2, 6, 18 \dots \dots.$ is the G.P with common ratio $\frac{1}{3}$ and 3 respectively.

⇒18, 6, 2 or 2, 6, 18 are the three consecutive terms.

10. Question

Find the first three consecutive terms in G.P. whose sum is 7 and the sum of their reciprocals is $\frac{7}{4}$

Answer

Let the first term of G. P be $\frac{a}{r}$.

⇒ second term = a and third term = ar.

(where, r is the common ratio)

∵ sum of three terms is 7

$\Rightarrow\ a(\frac{1}{r}\ +\ 1\ +\ r)\ =\ 7$

$\Rightarrow\ a\ =\ \frac{7r}{(1+r+r^2)}$ ....(1)

Also, sum of their reciprocal $=\ \frac{7}{4}$.

$\Rightarrow \frac{r}{a} + \frac{1}{a} + \frac{1}{ar} = \frac{7}{4}$

$\Rightarrow \frac{1}{a}(1\ +\ r\ +\ \frac{1}{r})\ =\ \frac{7}{4}$

$\Rightarrow \frac{4}{a}\ =\ \frac{7r}{(1+r+r^2)}$ ....(2)

Now, ∵ R.H.S of equation (1) & (2) is equal-

⇒ L.H.S of equation (1) = L.H.S of equation (2)

$\Rightarrow\ a\ =\ \frac{4}{a}$

$\Rightarrow a^2 = 4$

$\Rightarrow a = \sqrt{4}$

⇒ a = 2

Now substituting a = 1 in (2), we get-

$$\Rightarrow\ 2\ =\ \frac{7r}{(1\ +\ r\ +\ r^2)}$$

$\Rightarrow 2 + 2r + 2r^2 = 7r$

$\Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow 2r^2 - 4r - r + 2 = 0$

$\Rightarrow (2r-1)(r-2) = 0$

$$\Rightarrow r\ =\ \frac{1}{2}\ \text{or}\ 2$$

Now, G.P will be-

$$\frac{a}{r}, a, ar, \dots \dots \dots$$

$$\Rightarrow \frac{2}{\frac{1}{2}}, 2, 2 \times \frac{1}{2}, \dots \dots \text{or} \frac{2}{2}, 2, 2 \times 2, \dots \dots$$

$$=\ 4, 2, 1, \dots \dots.\ \text{or}\ 1, 2, 4 \dots \dots.\ \text{is the G.P with common ratio}\ \frac{1}{2}\ \text{or}\ 2.$$

⇒ 4, 2, 1 or 1, 2, 4 are the three consecutive terms.

11. Question

The sum of the first three terms of a G.P. is 13 and sum of their squares is 91. Determine the G.P.

Answer

Let the first term of G.P be a

⇒ second term = ar and third term = ar^2.

(where, r is the common ratio)

∵ sum of three terms is 13

$\Rightarrow a(1 + r + r^2) = 13r$ ....(1)

Also, sum of their squares is 91.

$\Rightarrow a^2\ (1 + r^2 + r^4) = 91r^2$ ....(2)

Now, Squaring (1) dividing by (2)

$$\Rightarrow \frac{(a^2(1\ +\ r\ +\ r^2)^2)}{a^2\ (\ 1\ +\ r^2\ \ +\ r^4\ )}\ =\ \frac{169r^2}{91r^2}$$

$$\Rightarrow \frac{(1\ +\ r\ +\ r^2)^2}{(\ 1\ +\ r^2\ )^2 - r^2}\ =\ \frac{13}{7}.$$

$$\Rightarrow \frac{(1\ +\ r\ +\ r^2)^2}{(\ 1\ +\ r^2\ \ +\ r)(\ 1\ +\ r^2\ -\ r)}\ =\ \frac{13}{7}$$

$$\Rightarrow \frac{(\ 1\ +\ r^2\ \ +\ r)}{1\ +\ r^2\ -\ r}\ =\ \frac{13}{7}$$

$\Rightarrow 7(\ 1 + r^2 + r) = 13(\ 1 + r^2 - r)$

$\Rightarrow (7 + 7r^2 + 7r) = (\ 13 + 13r^2 - 13\ r)$

$\Rightarrow 6r^2 - 20r + 6 = 0$

$\Rightarrow 2(3r^2 - 10r + 3) = 0$

$\Rightarrow 3r^2 - 10r + 3 = 0$

$\Rightarrow 3r^2 - 9r - r + 3 = 0$

$\Rightarrow (3r - 1)(r - 3) = 0$

$\Rightarrow r = 3 \text{ or } \frac{1}{3}$

Substituting r in equation (1), we get-

$\Rightarrow a(1 + 3 + 9) = 13 \times 3$

And,

$a\left(3 + 1 + \frac{1}{9}\right) = 13$

$\Rightarrow 13a = 13$

And

$\frac{13a}{9} = 13$

$\Rightarrow$ a = 1 and a = 9.

Now, G.P is-

a, ar, ar^2,....

$\Rightarrow$ If r = 3 and a = 1 then,

$\Rightarrow 1, 1\times3, 1\times3^2$, ...

= 1, 3 , 9 are the first three terms.

And, If $r = \frac{1}{3}$ and a = 9 then,

$\Rightarrow 9, 9\times\frac{1}{3}, 9\times\frac{1}{3}^2, \ldots \ldots$

= 9, 3, 1 are the first three terms.

$\Rightarrow$ 1,3,9,... ... or 9,3,1,... ... is the G.P.

12. Question

If ₹1000 is deposited in a bank which pays annual interest at the rate of 5% compounded annually, find the maturity amount at the end of 12 years.

Answer

∵ there is annual compounding interest

$\text{i.e.,} 1000\left(1 + \frac{5}{100}\right)$ in the first year, $1000\left(1 + \frac{5}{100}\right)^2$ in the second year,

$1000\left(1 + \frac{5}{100}\right)^3$ in the third year, and so on ....

$\Rightarrow$ consecutive amounts are forming G.P.

$a_1 = 1000\left(1 + \frac{5}{100}\right)$ and $r = 1 + \frac{5}{100}$

Now, a_{12} = ?

∵ $a_n = a_1 r^{n-1}$ (n = no. of term, a_1 = first term of G.P, r = common ratio)

$$\Rightarrow a_{12} = 1000 \times (1 + \frac{5}{100}) \times \left(1 + \frac{5}{100}\right)^{11}$$

$$\Rightarrow a_{12} = 1000 \times \left(\frac{105}{100}\right)^{12}$$

$$\therefore \text{, the maturity amount at the end of 12 years} = ₹\,1000\left(\frac{105}{100}\right)^{12}$$

13. Question

A company purchases an office copier machine for ₹50,000. It is estimated that the copier depreciates in its value at a rate of 15% per year. What will be the value of the copier after 15 years?

Answer

∵ there is annual depreciation at a constant rate per year

$$\text{i. e., } 50{,}000\left(1 - \frac{15}{100}\right) \text{ in the first year, } 50{,}000\left(1 - \frac{15}{100}\right)^2 \text{ in the second year,}$$

$$50{,}000\left(1 - \frac{15}{100}\right)^3 \text{ in the third year and so on..}$$

⇒ consecutive value are forming G.P.

$$\Rightarrow a_1 = 50{,}000\left(1 - \frac{15}{100}\right) \text{ and } r = 1 - \frac{15}{100}$$

Now, a_{15} = ?

∵ $a_n = a_1 r^{n-1}$ (n = no. of term, a_1 = first term of G.P, r = common ratio)

$$\Rightarrow a_{15} = 50{,}000 \times (1 - \frac{15}{100}) \times \left(1 - \frac{15}{100}\right)^{14}$$

$$\Rightarrow a_{15} = 50{,}000 \times \left(\frac{85}{100}\right)^{15}$$

$$\therefore \text{, the value of the copier after 15 years} = ₹\,50{,}000\left(\frac{85}{100}\right)^{15}$$

14. Question

If a, b, c, d are in a geometric sequence, then show that (a - b + c) (b + c + d) = ab + bc + cd.

Answer

Proof:∵ a, b, c, d are in G.P

⇒ a = a, b = ar, c = ar^2, d = ar^3.

⇒ L.H.S = $(a - ar + ar^2)(ar + ar^2 + ar^3)$

= $a^2r(1 + r^2 + r^4)$

And, R.H.S = $a^2r + a^2r^3 + a^2r^5$

= $a^2r(1 + r^2 + r^4)$

⇒ L.H.S = R.H.S

Hence, proved that-

(a - b + c) (b + c + d) = ab + bc + cd.

15. Question

If a, b, c, d are in a G.P., then prove that a + b, b + c, c + d, are also in G.P.

Answer

Proof: ∵ a, b, c, d are in G.P

⇒ a = a, b = ar ,c = ar^2,d = ar^3.

To prove: a + b, b + c, c + d, are also in G.P, if-

⇒ $\frac{b + c}{a + b} = \frac{c + d}{b + c}$

⇒ (a + b) (c + d) = $(b + c)^2$

Now, we need to prove : (a + b) (c + d) = $(b + c)^2$

L.H.S. = (a + ar)(ar^2 + ar^3)

= a(1 + r) ar^2 (1 + r)

= a^2r^2 $(1 + r)^2$

R.H.S. = $(ar + ar^2)^2$

= $(ar(1 + r))^2$

= a^2r^2 $(1 + r)^2$

⇒ L.H.S = R.H.S

Hence, proved that-

(a + b) (c + d) = $(b + c)^2$

⇒ a + b, b + c, c + d, are also in G.P

Exercise 2.4

1. Question

Find the sum of the first (i) 75 positive integers (ii) 125 natural numbers.

Answer

(i) In the A.P.

First term = 1

No. of terms = 75

Common difference = 1

Sum of terms = $\frac{n}{2}(2a + (n-1)d)$

⇒ Sum of terms = $\frac{75}{2}(2 \times 1 + (75-1) \times 1)$

⇒ Sum of terms = $\frac{75}{2} \times (76)$

⇒ Sum of terms = 2850

(ii) In the A.P.

First term = 1

No. of terms = 125

Common difference = 1

Sum of terms = $\frac{n}{2}(2a + (n-1)d)$

⇒ Sum of terms = $\frac{125}{2}(2 \times 1 + (125-1) \times 1)$

⇒ Sum of terms = $\frac{125}{2} \times (126)$

⇒ Sum of terms = 7875

2. Question

Find the sum of the first 30 terms of an A.P. whose nth term is 3 + 2n.

Answer

In the A.P.

First term = 3 + 2×1 = 5

No. of terms = 30

Last term = 3 + 2×30 = 63

Sum of terms = $\frac{n}{2}(a + l)$

⇒ Sum of terms = $\frac{30}{2}(5 + 63)$

⇒ Sum of terms = $\frac{30}{2} \times (68)$

⇒ Sum of terms = 1020

3. Question

Find the sum of each arithmetic series

(i) 38 + 35 + 32 + ... + 2 (ii) $6 + 5\frac{1}{4} + 4\frac{1}{2} + ...25$ terms.

Answer

(i) In the A.P.

First term = 38

Last term = 2

Common difference = 35–38 = –3

Nth term = a + (n–1) d

⇒ 2 = 38 + (n–1)(–3)

⇒ –36 = (n–1)(–3)

⇒ 12 = (n–1)

⇒ n = 13

Sum of terms = $\frac{n}{2}(a + l)$

⇒ Sum of terms = $\frac{13}{2}(38 + 2)$

⇒ Sum of terms = $\frac{13}{2} \times (40)$

⇒ Sum of terms = 260

(ii) In the A.P.

First term = 6

No. of terms = 25

Common difference = 21/4 – 6 = –3/4

Sum of terms = $\frac{n}{2}(2a + (n-1)d)$

⇒ Sum of terms = $\frac{25}{2}\left(2\times 6 + (25-1)\times(\frac{-3}{4})\right)$

⇒ Sum of terms = $\frac{25}{2}\times(12 + (-18))$

⇒ Sum of terms = –75

4. Question

Find the S_n for the following arithmetic series described.

(i) a = 5, n = 30, l = 121 (ii) a = 50, n = 25, d = – 4

Answer

(i) Sum of terms = $\frac{n}{2}(a + l)$

⇒ Sum of terms = $\frac{30}{2}(5 + 121)$

⇒ Sum of terms = $\frac{30}{2}\times(126)$

⇒ Sum of terms = 1890

(ii) Sum of terms = $\frac{n}{2}(2a + (n-1)d)$

⇒ Sum of terms = $\frac{25}{2}(2\times 50 + (25-1)\times(-4))$

⇒ Sum of terms = $\frac{25}{2}\times(4)$

⇒ Sum of terms = 50

5. Question

Find the sum of the first 40 terms of the series $1^2 - 2^2 + 3^2 - 4^2 + \ldots$.

Answer

Let the sum of n terms be,

$S_n = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 \ldots$

$= (1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) + (7^2 - 8^2) \ldots$..

= (1–4) + (9–16) + (25–36) + (49–64)........

= –3 –7 –11 –15.................. [No. of terms $m = \frac{n}{2}$]

This represents an A.P.

(NOTE: Here the number of terms has been halved. If n= no. of terms in original series and m= no. of terms in new A.P. then $m = \frac{n}{2}$)

Now, here a= –3, d= (–7 – (–3)) = –4

Sum of m terms = $\frac{m}{2}[2a + (m-1)d]$

$= \frac{n}{4}\left[2a + \left(\frac{n}{2} - 1\right)d\right]$ $[\because m = \frac{n}{2}]$

$= \frac{n}{4}[2(-3) + \left(\frac{n}{2} - 1\right)(-4)]$

$= \frac{n}{4}[-6 - 2n + 4]$

$= \frac{n}{4}[-2 - 2n]$

$= -\frac{n(n+1)}{2}$

As n= 40,

Required sum = $-\frac{40(40+1)}{2}$

= –820

6. Question

In an arithmetic series, the sum of first 11 terms is 44 and that of the next 11 terms is 55. Find the arithmetic series.

Answer

Sum of terms = $\frac{n}{2}(2a + (n-1)d)$

⇒ 44 = $\frac{11}{2}(2a + (11-1)d)$

⇒ 8 = 2a + 10d

⇒ 2a + 10d = 8 ....(1)

Now sum of next 11 terms is 55

Sum of first 22 terms = Sum of first 11 terms + Sum of next 11 terms

⇒ Sum of first 22 terms = 44 + 55 = 99

Sum of terms = $\frac{n}{2}(2a + (n-1)d)$

⇒ 99 = $\frac{22}{2}(2a + (22-1)d)$

⇒ 9 = 2a + 21d

⇒ 2a + 21d = 9 ....(1)

Subtracting (1) from (2), we get

⇒ 11d = 1

⇒ d = 1/11

Putting value of d in (1), we get

⇒ 2a + 10(1/11) = 8

⇒ 2a = 8–(10/11)

⇒ 2a = 78/11

⇒ a = 39/11

Therefore, the series is $\frac{39}{11}, \frac{40}{11}, \frac{41}{11}, \ldots \ldots \ldots$

7. Question

In the arithmetic sequence 60, 56, 52, 48,..., starting from the first term, how many terms are needed so that their sum is 368?

Answer

In the A.P.

First term = 60

Common difference = 56 – 60 = –4

Sum of terms = $\frac{n}{2}(2a + (n-1)d)$

⇒ 368 = $\frac{n}{2}(2 \times 60 + (n-1) \times (-4))$

⇒ 368 = $\frac{n}{2} \times (120 - 4(n-1))$

⇒ 736 = n (124 – 4n)

⇒ $4n^2$ – 124n + 736 = 0

⇒ $4n^2$ – 92n–32n + 736 = 0

⇒ 4n(n–23) –32(n–23) = 0

⇒ (4n–32)(n–23) = 0

⇒ 4n = 32 or n = 23

⇒ n = 8 or n = 23

Therefore, no. of terms can be 8 or 23

8. Question

Find the sum of all 3 digit natural numbers, which are divisible by 9.

Answer

Series of three digit numbers divisible by 9 is:

108, 117,...................................999

In the A.P.

First term = 108

Last term = 999

Common difference = 9

Nth term = a + (n–1) d

⇒ 999 = 108 + (n–1)9

⇒ 891 = (n–1)9

⇒ 99 = (n–1)

⇒ n = 100

Sum of terms = $\frac{n}{2}(a + l)$

⇒ Sum of terms = $\frac{100}{2}(108 + 999)$

⇒ Sum of terms = $\frac{100}{2} \times (1107)$

⇒ Sum of terms = 55350

9. Question

Find the sum of first 20 terms of the arithmetic series in which 3rd term is 7 and 7th term is 2 more than three times its 3rd term.

Answer

First term = a

Common difference = d

3^{rd} term = a + (3–1)d = a + 2d

7^{th} term = a + (7–1)d = a + 6d

Now, 3^{rd} term = 7

⇒ a + 2d = 7 ....(1)

And, 7^{th} term = 3 × (3^{rd} term) + 2

⇒ a + 6d = 3(a + 2d) + 2

⇒ a + 6d = 3a + 6d + 2

⇒ 2a = –2

⇒ a = –1

Putting value of a in (1)

(⇒ –1) + 2d = 7

⇒ 2d = 8

⇒ d = 4

Sum of terms = $\frac{n}{2}(2a + (n-1)d)$

⇒ Sum of terms = $\frac{20}{2}(2 \times (-1) + (20-1) \times 4)$

⇒ Sum of terms = $\frac{20}{2} \times (74)$

⇒ Sum of terms = 740

10. Question

Find the sum of all natural numbers between 300 and 500 which are divisible by 11.

Answer

Series of all natural numbers divisible by 11 between 300and 500 is:

308, 319,...................................495

In the A.P.

First term = 308

Last term = 495

Common difference = 11

Nth term = a + (n–1) d

⇒ 495 = 308 + (n–1)11

⇒ 187 = (n–1)11

⇒ 17 = (n–1)

⇒ n = 18

Sum of terms = $\frac{n}{2}(a + l)$

⇒ Sum of terms = $\frac{18}{2}(308 + 495)$

⇒ Sum of terms = $\frac{18}{2} \times (803)$

⇒ Sum of terms = 7227

11. Question

Solve: 1 + 6 + 11 + 16 + ... +x = 148.

Answer

In the A.P.

First term = 1

Common difference = 6 – 1 = 5

Sum of terms = $\frac{n}{2}(2a + (n-1)d)$

⇒ 148 = $\frac{n}{2}(2 \times 1 + (n-1) \times 5)$

⇒ 148 = $\frac{n}{2} \times (2 + 5(n-1))$

⇒ 296 = n (5n–3)

⇒ $5n^2$ – 3n –296 = 0

⇒ $5n^2$ –40n + 37n –296 = 0

⇒ 5n(n–8) + 37(n–8) = 0

⇒ (n–8)(5n + 37) = 0

⇒ n = 8 or 5n = –37

As negative value of n is not possible, n = 8

⇒ x = 8^{th} term of the series

⇒ x = 1 + (8–1)5

⇒ x = 1 + 35 = 36

12. Question

Find the sum of all numbers between 100 and 200 which are not divisible by 5.

Answer

We first find the sum of all numbers divisible by 5

Series of all natural numbers divisible by 5 between 100and 200 is:

105, 110,...................................195

In the A.P.

First term = 105

Last term = 195

Common difference = 5

Nth term = a + (n–1) d

⇒ 195 = 105 + (n–1)5

⇒ 90 = (n–1)5

⇒ 18 = (n–1)

⇒ n = 19

Sum of terms = $\frac{n}{2}(a + l)$

⇒ Sum of terms = $\frac{19}{2}(105 + 195)$

⇒ Sum of terms = $\frac{19}{2} \times (300)$

⇒ Sum of terms = 2850

Sum of 101,102....199 = Sum of 199 natural numbers – Sum of 100 natural numbers

⇒ Sum(101, 102,.... 199) = $\frac{199(199+1)}{2} - \frac{100(100+1)}{2}$

⇒ Sum(101, 102,.... 199) = 19900–5050 = 14850

Sum of all numbers between 100 and 200 not divisible by 5 = 14850–2850 = 12000

13. Question

A construction company will be penalised each day for delay in construction of a bridge. The penalty will be ₹4000 for the first day and will increase by ₹1000 for each following day. Based on its budget, the company can afford to pay a maximum of ₹1,65,000 towards penalty. Find the maximum number of days by which the completion of work can be delayed

Answer

The amount to be paid first paid = ₹4000

It increases each day by = ₹1000

Total amount that can be paid = ₹165000

a = 4000, d = 1000, Sum = 165000

Sum of terms = $\frac{n}{2}(2a + (n-1)d)$

⇒ 165000 = $\frac{n}{2}(2 \times 4000 + (n-1) \times 1000)$

⇒ 165 = $\frac{n}{2} \times (8 + 1(n-1))$

⇒ 330 = n (n + 7)

⇒ n^2 + 7n –330 = 0

⇒ n^2 –15n + 22n –330 = 0

⇒ n(n–15) + 22(n–15) = 0

⇒ (n–15)(n + 22) = 0

⇒ n = 15 or n = –22

As negative value of n is not possible, n = 15

Therefore, number of days that construction can be delayed = 15days.

14. Question

A sum of ₹1000 is deposited every year at 8% simple interest. Calculate the interest at the end of each year.

Do these interest amounts form an A.P.? If so, find the total interest at the end of 30 years.

Answer

Simple interest = P* R * T /100

P = ₹1000

R = 8

T = 1

Simple interest = 1000*8*1/100 = 80

Interest at the end of first year = ₹80

For second year,

P = 1000 + 1000 = 2000

R = 8

T = 1

Simple interest = 2000*8*1/100 = 160

Interest at the end of second year = ₹160

Similarly, for third year, P = 1000 + 1000 + 1000 = 3000

SI = 240.

Yes the simple interests form an A.P.

a = 80

d = 160–80 = 80

n = 30

Sum of terms = $\frac{n}{2}(2a + (n-1)d)$

⇒ Sum of terms = $\frac{30}{2}(2 \times 80 + (30-1) \times 80)$

⇒ Sum of terms = $\frac{30}{2} \times (2480)$

⇒ Sum of terms = 37200

15. Question

The sum of first n terms of a certain series is given as $3n^2 - 2n$. Show that the series is an arithmetic series.

Answer

For n = 1,

Sum = $3(1)^2 - 2(1) = 1$

Therefore, first term = 1

For n = 2,

Sum = $3(2)^2 - 2(2) = 12 - 4 = 8$

Second term = 8–1 = 7

For n = 3,

Sum = $3(3)^2 - 2(3) = 21$

Third term = 21– 8 = 13

Series : 1, 7, 13.....

This is an arithmetic progression as the difference between two terms is constant.

Common difference = 7–1 = 13–7 = 6

16. Question

If a clock strikes once at 1 o'clock, twice at 2 o'clock and so on, how many times will it strike in a day?

Answer

The clock strikes once at 1, twice at 2 ....so it strikes 12times at 12

In a day this striking from 1 to 12 happens twice.

No. of strikes in one turn from 1 to 12 = 1 + 2 + 3 + ...12

⇒ No. of strikes in one turn from 1 to 12 = $\frac{12(12+1)}{2} = 6 \times 13 = 78$

As the striking happens twice in a day,

Total number of strikes in a day = 78 × 2 = 156

17. Question

Show that the sum of an arithmetic series whose first term is a, second term b and the last is c is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$

Answer

First term = a

Common difference = (b–a)

Last term = c

⇒ c = a + (n–1)(b–a)

⇒ (n–1) = (c–a) / (b–a)

⇒ n = (b + c – 2a) /(b – a)

Sum of terms = $\frac{n}{2}(a + l)$

⇒ Sum of terms = $\frac{(b+c-2a)}{2(b-a)}(a + c)$

Hence proved.

18. Question

If there are (2n + 1) terms in an arithmetic series, then prove that the ratio of the sum of odd terms to the sum of even terms is (n + 1) : n.

Answer

In the A.P, let

First term = a

Common difference = d

Number of terms = (2n + 1)

Series: a,a + d,a + 2d......a + 2nd

For Odd terms

: a, a + 2d,...a + 2nd

First term = a

Common difference = 2d

Number of terms = n + 1

Sum of terms = $\frac{n}{2}(a + l)$

Sum of odd terms = $\frac{n+1}{2}\left(a + (a + 2nd)\right)$

⇒ Sum of odd terms = $\frac{n+1}{2}(2a + 2nd)$

For Even terms

: a + d, a + 3d,...a + (2n–1)d

First term = a + d

Common difference = 2d

Number of terms = n

Sum of terms = $\frac{n}{2}(a + l)$

Sum of even terms = $\frac{n}{2}\left((a + d) + (a + (2n-1)d)\right)$

⇒ Sum of even terms = $\frac{n}{2}(2a + 2nd)$

Sum of odd terms : Sum of even terms = $\frac{n+1}{2}(2a + 2nd) : = \frac{n}{2}(2a + 2nd)$

∴ Sum of odd terms : Sum of even terms = (n + 1) : n

19. Question

The ratio of the sums of first m and first n terms of an arithmetic series is m^2: n^2show that the ratio of the mth and nth terms is (2m – 1) : (2n – 1)

Answer

Sum of m terms = $\frac{m}{2}(2a + (m-1)d)$

Sum of n terms = $\frac{n}{2}(2a + (n-1)d)$

Sum of m terms : Sum of n terms = $m^2 : n^2$

⇒ $\frac{m}{2}(2a + (m-1)d) : \frac{n}{2}(2a + (n-1)d) = m^2 : n^2$

⇒ $n^2m(2a + (m-1)d) = nm^2(2a + (n-1)d)$

⇒ $2an^2m + n^2m^2d - n^2md = 2anm^2 + n^2m^2d - nm^2d$

⇒ 2anm(n–m) = nmd(n–m)

⇒ 2a = d

mth term : nth term = a + (m–1)d : a + (n–1)d

⇒ mth term : nth term = a + (m–1)2a : a + (n–1) 2a

⇒ mth term : nth term = (2m–1) : (2n–1)

20. Question

A gardener plans to construct a trapezoidal shaped structure in his garden. The longer side of trapezoid needs to start with a row of 97 bricks. Each row must be decreased by 2 bricks on each end and the construction should stop at 25th row. How many bricks does he need to buy?

Answer

Longest side = 97

Each row decreases by = 2×2 = 4

Number of rows = 25

a = 97, d = –4, n = 25

Sum of n terms = $\frac{n}{2}(2a + (n-1)d)$

⇒ Sum of terms = $\frac{25}{2}(2 \times 97 + (25-1) \times (-4))$

⇒ Sum of terms = $\frac{25}{2} \times (98)$

⇒ Sum of terms = 1225

Exercise 2.5

1. Question

Find the sum of the first 20 terms of the geometric series $\frac{5}{2} + \frac{5}{6} + \frac{5}{18} + \ldots$.

Answer

In the G.P.,

First term = 5/2

Common ratio = $\frac{5/6}{5/2} = \frac{1}{3}$

Sum of n terms = $\frac{a(1-r^n)}{(1-r)}$

⇒ Sum of 20 terms = $\frac{5}{2}\frac{\left(1-\frac{1}{3}^{20}\right)}{\left(1-\frac{1}{3}\right)}$

⇒ Sum of 20 terms = $\frac{15}{4}\left[1-\left(\frac{1}{3}\right)^{20}\right]$

2. Question

Find the sum of the first 27 terms of the geometric series $\frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \ldots$.

Answer

In the G.P.,

First term = 1/9

Common ratio = $\frac{1/27}{1/9} = \frac{1}{3}$

Sum of n terms = $\frac{a(1-r^n)}{(1-r)}$

⇒ Sum of 27 terms = $\frac{1}{9}\frac{\left(1-\frac{1}{3}^{27}\right)}{\left(1-\frac{1}{3}\right)}$

⇒ Sum of 27 terms = $\frac{1}{6}\left[1-\left(\frac{1}{3}\right)^{27}\right]$

3. Question

Find S_n for each of the geometric series described below.

(i) a = 3, t_8 = 384, n = 8. (ii) a = 5, r = 3, n = 12.

Ans. 765

Answer

(i) $t_8 = 384$

$\Rightarrow ar^{8-1} = 384$

$\Rightarrow (3)r^7 = 384$

$\Rightarrow r^7 = 128$

$\Rightarrow r = 2$

Sum of n terms = $\frac{a(1-r^n)}{(1-r)}$

$\Rightarrow$ Sum of 8 terms = $3\frac{(1-2^8)}{(1-2)}$

$\Rightarrow$ Sum of 8 terms = 3(255) = 765

(ii) Sum of n terms = $\frac{a(1-r^n)}{(1-r)}$

$\Rightarrow$ Sum of 12 terms = $5\frac{(1-3^{12})}{(1-3)}$

$\Rightarrow$ Sum of 12 terms = $5\frac{(3^{12}-1)}{2}$

4. Question

Find the sum of the following finite series

(i) 1 + 0.1 + 0.01 + 0.001 + ... + (0.1)9 (ii) 1 + 11 + 111 + ... to 20 terms.

Answer

(i) In the G.P.,

First term = 1

Common ratio = $\frac{0.1}{1}$ = 0.1

Sum of n terms = $\frac{a(1-r^n)}{(1-r)}$

$\Rightarrow$ Sum of 10 terms = $1\frac{(1-(0.1)^{10})}{(1-0.1)}$

$\Rightarrow$ Sum of 10 terms = $\frac{1}{0.9}[1-(0.1)^{10}]$

(ii) Series = 1 + 11 + 111 + …..20 terms

Series = $\frac{1}{9}$[9×(1 + 11 + 111 + …..)] (Multiplying and dividing by 9)

$\Rightarrow$ Series = $\frac{1}{9}$[9 + 99 + 999 + …..]

$\Rightarrow$ Series = $\frac{1}{9}$[(10–1) + (100–1) + (1000–1) + …..]

$\Rightarrow$ Series = $\frac{1}{9}$[10 + 100 + 1000 + ….. – (20×1)]

⇒ Series = $\frac{1}{9}$[10 + 100 + 1000 + ...] - $\frac{20}{9}$

We find the sum of 10 + 100 + 1000.....20terms as:

First term = 10

Common ratio = $\frac{100}{10}$ = 10

Sum of n terms = $\frac{a(1-r^n)}{(1-r)}$

⇒ Sum of 20 terms = $10\frac{(1-(10)^{20})}{(1-10)}$

⇒ Sum of 20 terms = $\frac{10}{9}[(10)^{20}-1]$

⇒ Series = $\frac{1}{9}[\frac{10}{9}[(10)^{20}-1]]$ - $\frac{20}{9}$

⇒ Series = $\frac{10}{81}[(10)^{20}-1]$ - $\frac{20}{9}$

5. Question

How many consecutive terms starting from the first term of the series

(i) 3 + 9 + 27 + … would sum to 1092 ? (ii) 2 + 6 + 18 + … would sum to 728 ?

Answer

(i) First term = 3

Common ratio = $\frac{9}{3}$ = 3

Sum of n terms = $\frac{a(1-r^n)}{(1-r)}$

⇒ Sum of n terms = $3\frac{(1-(3)^n)}{(1-3)}$

⇒ 1092 = $\frac{3}{2}[(3)^n-1]$

⇒ 728 = $[(3)^n-1]$

⇒ 729 = 3^n

⇒ n = 6

(ii) First term = 2

Common ratio = $\frac{6}{2}$ = 3

Sum of n terms = $\frac{a(1-r^n)}{(1-r)}$

⇒ 728 = $2\frac{(1-(3)^n)}{(1-3)}$

⇒ 728 = $[(3)^n-1]$

⇒ 729 = 3^n

⇒ n = 6

6. Question

The second term of a geometric series is 3 and the common ratio is $\frac{4}{5}$. Find the sum of first 23 consecutive terms in the given geometric series.

Answer

$\frac{\text{Second term}}{\text{First term}} = \frac{4}{5}$

$\Rightarrow \frac{3}{\text{First term}} = \frac{4}{5}$

$\Rightarrow \text{First term} = \frac{15}{4}$

Sum of n terms = $\frac{a(1-r^n)}{(1-r)}$

$\Rightarrow$ Sum of 23 terms = $\frac{15}{4}\frac{\left(1-\frac{4}{5}^{23}\right)}{\left(1-\frac{4}{5}\right)}$

$\Rightarrow$ Sum of 23 terms = $\frac{75}{4}\left[1-\left(\frac{4}{5}\right)^{23}\right]$

7. Question

A geometric series consists of four terms and has a positive common ratio. The sum of the first two terms is 9 and sum of the last two terms is 36. Find the series.

Answer

Let the first term = a

And common ratio = r

Series : a, ar, ar^2, ar^3

$\Rightarrow$ a + ar = 9

$\Rightarrow$ a(1 + r) = 9 ....(1)

$\Rightarrow ar^2 + ar^3 = 36$

$\Rightarrow ar^2(1 + r) = 36$

$\Rightarrow 9r^2 = 36$ (From Equation (1))

$\Rightarrow r^2 = 4$

$\Rightarrow$ r = 2

Putting the value of r in (1),

$\Rightarrow$ a(1 + 2) = 9

$\Rightarrow$ 3a = 9

$\Rightarrow$ a = 3

∴ Series : 3 + 6 + 12 + 24

8. Question

Find the sum of first n terms of the series

(i) 7 + 77 + 777 + ... (ii) 0.4 + 0.94 + 0.994 + ...

Answer

(i) Series = 7 + 77 + 777 + ...n terms

Series = $\frac{7}{9}$[9×(1 + 11 + 111 + …..)] (Multiplying and dividing by 9)

⇒ Series = $\frac{7}{9}$[9 + 99 + 999 + …..]

⇒ Series = $\frac{7}{9}$[(10–1) + (100–1) + (1000–1) + …..]

⇒ Series = $\frac{7}{9}$[10 + 100 + 1000 + ….. – (n×1)]

⇒ Series = $\frac{7}{9}$[10 + 100 + 1000 + …..] – $\frac{7n}{9}$

We find the sum of 10 + 100 + 1000…..n terms as:

First term = 10

Common ratio = $\frac{100}{10}$ = 10

Sum of n terms = $\frac{a(1-r^n)}{(1-r)}$

⇒ Sum of n terms = $10\frac{(1-(10)^n)}{(1-10)}$

⇒ Sum of n terms = $\frac{10}{9}[(10)^n - 1]$

⇒ Series = $\frac{7}{9}[\frac{10}{9}[(10)^n - 1]]$ – $\frac{7n}{9}$

⇒ Series = $\frac{70}{81}[(10)^n - 1]$ – $\frac{7n}{9}$

(ii) Series = 0.4 + 0.94 + 0.994 + …..n terms

Series = (1–0.6) + (1–0.06) + (1–0.006) + ……..

⇒ Series = n×1 – (0.6 + 0.06 + 0.006 + ….)

We find the sum of 0.6 + 0.06 + 0.006…..n terms as:

First term = 0.6

Common ratio = $\frac{0.06}{0.6} = \frac{1}{10}$

Sum of n terms = $\frac{a(1-r^n)}{(1-r)}$

⇒ Sum of n terms = $0.6\frac{(1-(0.1)^n)}{(1-0.1)}$

⇒ Sum of n terms = $\frac{2}{3}[1 - (0.1)^n]$

⇒ Series = n – $\frac{2}{3}\left[1 - \left(\frac{1}{10}\right)^n\right]$

9. Question

Suppose that five people are ill during the first week of an epidemic and each sick person

spreads the contagious disease to four other people by the end of the second week and so on. By the end of 15th week, how many people will be affected by the epidemic

Answer

People infected in first week = 5

More people infected by each person = 4

Number of weeks = 15

a = 5, r = 4, n = 15

Sum of n terms = $\frac{a(1-r^n)}{(1-r)}$

⇒ Sum of 15 terms = $5\frac{(1-4^{15})}{(1-4)}$

⇒ Sum of 15 terms = $\frac{5}{3}[(4)^{15}-1]$

So, Number of people infected by 15th week = $\frac{5}{3}[(4)^{15}-1]$

10. Question

A gardener wanted to reward a boy for his good deeds by giving some mangoes. He gave the boy two choices. He could either have 1000 mangoes at once or he could get 1 mango on the first day, 2 on the second day, 4 on the third day, 8 mangoes on the fourth day and so on for ten days. Which option should the boy choose to get the maximum number of mangoes?

Answer

In the second option,

Number of mangoes on first day = 1

Number of times of mangoes on subsequent day = 2

Number of days = 10

a = 1, r = 2, n = 10

Sum of n terms = $\frac{a(1-r^n)}{(1-r)}$

⇒ Sum of 10 terms = $1\frac{(1-2^{10})}{(1-2)}$

⇒ Sum of 10 terms = $[(2)^{10}-1]$ = 1023

Therefore, the boy will get more mangoes in 2nd case as there were only 1000 mangoes in the 1st case.

11. Question

A geometric series consists of even number of terms. The sum of all terms is 3 times the sum of odd terms. Find the common ratio.

Ans. r = 2

Answer

In the G.P.,

Let First term = a,

Common ratio = r

Series: a, ar, ar^2,.......ar^{n-1}

Sum of all terms = $\frac{a(1-r^n)}{(1-r)}$

For odd terms,

a, ar^2,.........ar^{n-2}

First term = a

Common ratio = r^2

Number of terms = n/2

Sum of odd terms = $\frac{a\left(1-(r^2)^{\frac{n}{2}}\right)}{(1-r^2)}$

⇒ Sum of odd terms = $\frac{a(1-r^n)}{(1-r^2)}$

Now,

Sum of all terms = 3× Sum of odd terms

⇒ $\frac{a(1-r^n)}{(1-r)}$ = 3 × $\frac{a(1-r^n)}{(1-r^2)}$

⇒ (1–r2) = 3(1–r)

⇒ r2–3r + 2 = 0

⇒ r2–2r–r + 2 = 0

⇒ r(r–2)–1(r–2) = 0

⇒ (r–1) (r–2) = 0

r = 1 or r = 2

But r = 1 is not possible, So r = 2.

12. Question

If S_1,$S_2$and S_3 are the sum of first n, 2n and 3n terms of a geometric series respectively,

then prove that $S_1(S_3 - S_2) = (S_2 - S_1)^2$.

Answer

Sum of n terms = $\frac{a(1-r^n)}{(1-r)}$

$S_1 = \frac{a(1-r^n)}{(1-r)}$

$S_2 = \frac{a(1-r^{2n})}{(1-r)}$

$S_3 = \frac{a(1-r^{3n})}{(1-r)}$

Putting value of S_1, S_2 and S_3 on the left side, we get:

$S_1(S_3 - S_2) = \frac{a(1-r^n)}{(1-r)}\left[\frac{a(1-r^{3n})}{(1-r)} - \frac{a(1-r^{2n})}{(1-r)}\right]$

⇒ $S_1(S_3 - S_2) = \frac{a(1-r^n)}{(1-r)}\left[\frac{a(1-r^{3n})-a(1-r^{2n})}{(1-r)}\right]$

⇒ $S_1(S_3 - S_2) = \frac{a(1-r^n)}{(1-r)}\left[\frac{ar^{2n}(1-r^n)}{(1-r)}\right]$

⇒ $S_1(S_3 - S_2) = \frac{a^2r^{2n}(1-r^n)^2}{(1-r)^2}$

⇒ $S_1(S_3 - S_2) = \left[\frac{ar^n(1-r^n)}{(1-r)}\right]^2$ ....(1)

Now, we solve the right side by putting S_1, S_2 and S_3 :$(S_2 - S_1)^2 = \left[\frac{a(1-r^{2n})}{(1-r)} - \frac{a(1-r^n)}{(1-r)}\right]^2$

⇒ $(S_2 - S_1)^2 = \left[\frac{ar^n(1-r^n)}{(1-r)}\right]^2$ ....(2)

From (1) and (2), we have:

Left hand side = Right Hand side

Hence Proved.

Exercise 2.6

1 A. Question

Find the sum of the following series.

1 + 2 + 3 + ... + 45

Answer

Given that series S = 1 + 2 + 3 + 4 + 5... + 45, it has n = 45 terms and to find the sum S.

Formula for sum of first n numbers is

$S = \frac{n(n+1)}{2}$

$= \frac{45(45+1)}{2}$

$= \frac{45(46)}{2}$

$=$ 45x23

= 1035

∴The sum S = 1 + 2 + 3 + ... + 45 = 1035

1 B. Question

Find the sum of the following series.

$16^2 + 17^2 + 18 + \ldots + 25^2$

Answer

Given the the series $S = 16^2 + 17^2 + 18 + \ldots + 25^2$

Let $S_1 = 1^2 + 2^2 + 3^2 + \ldots + 25^2$ with n = 25

Let $S_2 = 1^2 + 2^2 + 3^2 + .. + 15^2$ with n = 15

Required sum $S = S_1 - S_2$

To find the sum S_1 -

Formula to find the sum of first n squares of natural numbers is

$S = \frac{n(n+1)(2n+1)}{6}$

$\Rightarrow S_1 = \frac{25(25+1)(2 \times 25+1)}{6}$

$= \frac{25(26)(51)}{6}$

$S_1 = 5525$

To find the sum S_2 -

Formula to find the sum of first n squares of natural numbers is

$S = \frac{n(n+1)(2n+1)}{6}$

$\Rightarrow S_2 = \frac{15(15+1)(2\times 15+1)}{6}$

$S_2 = \frac{15(16)(31)}{6}$

$S_2 = 1240$

Required sum $S = S_1 - S_2$

$\Rightarrow S = 5525 - 1240 = 4285$

$\therefore$The sum $S = 16^2 + 17^2 + 18 + \ldots + 25^2 = 4285$

1 C. Question

Find the sum of the following series.

2 + 4 + 6 + … + 100

Answer

Given the series S = 2 + 4 + 6 + … + 100,

We see that there is a common term 2 in all the numbers in the series. Taking 2 common, we have

S = 2(1 + 2 + 3 + .. + 50)

Let 1 + 2 + 3 + .. + 50 be S_1, with n = 50

$\Rightarrow S = 2S_1$

To find S_1 -

Formula for sum of first n numbers is

$S_1 = \frac{n(n+1)}{2}$

$= \frac{50(50+1)}{2}$

$= \frac{50(51)}{2}$

$= 25\text{x}51$

= 1275

$S = 2S_1$

= 2 × 1275 = 2550

S = 2550

$\therefore$The sum S = 2 + 4 + 6 + … + 100 = 2550

1 D. Question

Find the sum of the following series.

7 + 14 + 21…. + 490

Answer

Given the series S = 7 + 14 + 21….+ 490,

We see that there is a common term 7 in all the numbers in the series. Taking 7 common, we have

S = 7(1 + 2 + 3 + .. + 70)

Let 1 + 2 + 3 + .. + 70 be S_1, with n = 70

So S becomes $S = 7S_1$

<u>To find S_1</u>-

Formula for sum of first n numbers is

$S_1 = \frac{n(n+1)}{2}$

$\Rightarrow \frac{70(70\ +\ 1)}{2}$

$\Rightarrow \frac{70(71)}{2}$

$\Rightarrow 35 \times 71$

= 2485

$S = 7S_1$

$S = 7 \times 2485 = 17395$

∴The sum S = 7 + 14 + 21.... + 490 = 17395

1 E. Question

Find the sum of the following series.

$5^2 + 7^2 + 9^2 + \ldots + 39^2$

Answer

Given the the series $S = 5^2 + 7^2 + 9^2 + \ldots + 39^2$

Let $S_1 = 1^2 + 3^2$

Let $S_2 = 2^2 + 4^2 + 6^2 + .. + 38^2$ with n = 19

Let $S_3 = 1^2 + 2^2 + 3^2 + 4^2 + .. + 39^2$ with n = 39

Required sum $S = S_3 - S_2 - S_1$

<u>To find the sum S_1</u> -

$S_1 = 1 + 9 = 10$

<u>To find the sum S_2</u> –

S2 can be rewritten as $S_2 = 2^2(1^2 + 2^2 + 3^2 + 4^2 + .. + 19^2)$

Formula to find the sum of first n squares of natural numbers is

$S = \frac{n(n+1)(2n+1)}{6}$

$\Rightarrow S_2 = 2^2 \frac{n(n+1)(2n+1)}{6}$

$= 4\frac{19(19+1)(2\times 19+1)}{6}$

$= 4\frac{19(20)(39)}{6}$

$S_2 = 9880$

<u>To find the sum S_3</u> -

Formula to find the sum of first n squares of natural numbers is

$S = \frac{n(n+1)(2n+1)}{6}$

$\Rightarrow S_3 = \frac{39(39+1)(2\times 39+1)}{6}$

$= \frac{39(40)(79)}{6}$

= 20540

Required sum S = S_3-S_2-S_1

S = 20540-9880-10 = 10650

∴The sum S = $5^2 + 7^2 + 9^2 + \ldots + 39^2$ = 10650

1 F. Question

Find the sum of the following series.

$16^3 + 17^3 + \ldots + 35^3$

Answer

Given the the series S = $16^3 + 17^3 + \ldots + 35^3$

Let $S_1 = 1^3 + 2^3 + 3^3 + \ldots + 35^3$ with n = 35

Let $S_2 = 1^3 + 2^3 + 3^3 + .. + 15^3$ with n = 15

Required sum S = S_1-S_2

To find the sum S_1 -

Formula to find the sum of first n cubes of natural numbers is

$S = (\frac{n(n+1)}{2})^2$

$\Rightarrow S_1 = (\frac{35(35+1)}{2})^2$

$= (\frac{35\times 36}{2})^2$

S_1 = 396900

To find the sum S_2 -

Formula to find the sum of first n cubes of natural numbers is

$S = (\frac{n(n+1)}{2})^2$

$\Rightarrow S_2 = (\frac{15(15+1)}{2})^2$

$= (\frac{15\times 16}{2})^2$

S_2 = 14400

Required sum S = S_1-S_2

S = 396900-14400 = 382500

∴The sum S = $16^3 + 17^3 + \ldots + 35^3$ = 382500

2 A. Question

Find the value of k if

$1^3 + 2^3 + 3^3 + \ldots + k^3 = 6084$

Answer

Given the first k cubes of natural numbers and that their sum is 6084,

By formula we have S = $\left(\frac{n(n+1)}{2}\right)^2$

6084 = $\left(\frac{k(k+1)}{2}\right)^2$

Taking square root on both sides, we get $\frac{k(k+1)}{2} = 78$

k(k + 1) = 156

$k^2 + k = 156$

Or $k^2 + k - 156 = 0$

Solving the quadratic using the quadratic formula

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where b = 1, a = 1, c = -156

We get,

$$k = \frac{-1 \pm \sqrt{1^2 - 4(1)(-156)}}{2}$$

$$\Rightarrow k = \frac{-1 \pm 25}{2}$$

$$= k = \frac{-1 + 25}{2} \text{ or } \frac{-1 - 25}{2}$$

k = $\frac{-1-25}{2}$ is invalid because it yields a negative k which doesn't make sense because number of terms in a series cannot be negative.

$\Rightarrow k = \frac{-1+25}{2} = 12$

∴The sum $S = 1^3 + 2^3 + 3^3 + \ldots + k^3 = 6084$ corresponds to k = 12.

2 B. Question

Find the value of k if

$1^3 + 2^3 + 3^3 + \ldots + k^3 = 2025$

Answer

Given the first k cubes of natural numbers and that their sum is 2025,

By formula we have S = $\left(\frac{n(n+1)}{2}\right)^2$

$\Rightarrow$ 2025 = $\left(\frac{k(k+1)}{2}\right)^2$

Taking square root on both sides, we get $\frac{k(k+1)}{2} = 45$

k(k + 1) = 90

$k^2 + k = 90$

Or $k^2 + k - 90 = 0$

Solving the quadratic using the quadratic formula

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where b = 1, a = 1, c = -90

We get,

$$k = \frac{-1 \pm \sqrt{1^2 - 4(1)(-90)}}{2}$$

$$\Rightarrow k = \frac{-1 \pm 19}{2}$$

$$= k = \frac{-1 + 19}{2} \text{ or } \frac{-1 - 19}{2}$$

$k = \frac{-1-19}{2}$ is invalid because it yields a negative k which doesn't make sense because number of terms in a series cannot be negative.

$\Rightarrow k = \frac{-1+19}{2} = 9$

∴The sum $S = 1^3 + 2^3 + 3^3 + \ldots + k^3 = 2025$ corresponds to k = 9.

3. Question

If 1 + 2 + 3 + … + p = 171, then find $1^3 + 2^3 + 3^3 + \ldots + p^3$.

Answer

Given that the series S = 1 + 2 + 3 + … + p = 171

We have $S = \frac{p(p+1)}{2} = 171$

p(p + 1) = 342

$p^2 + p = 342$

Or $p^2 + p - 342 = 0$

Solving the quadratic using the quadratic formula

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where b = 1, a = 1, c = -342

We get,

$$p = \frac{-1 \pm \sqrt{1^2 - 4(1)(-342)}}{2}$$

$$\Rightarrow p = \frac{-1 \pm 37}{2}$$

$$\Rightarrow p = \frac{-1 + 37}{2} \text{ or } \frac{-1 - 37}{2}$$

$p = \frac{-1-37}{2}$ is invalid because it yields a negative p which doesn't make sense because number of terms in a series cannot be negative.

$p = \frac{-1+37}{2} = 18$

$S = 1^3 + 2^3 + 3^3 + \ldots + p^3$ where p = 18

Formula to find the sum of first n cubes of natural numbers is

$S = (\frac{n(n+1)}{2})^2$

$\Rightarrow S = (\frac{18(18+1)}{2})^2$

$S = (\frac{18 \times 19}{2})^2$

S = 29241

∴The sum $S = 1^3 + 2^3 + 3^3 + ... + p^3$ corresponds to p = 18 and S = 29241.

4. Question

If $1^3 + 2^3 + 3^3 + ... + k^3 = 8281$, then find 1 + 2 + 3 + … + k.

Answer

Given that the series $S = 1^3 + 2^3 + 3^3 + ... + k^3 = 8281$

Formula to find the sum of first k cubes of natural numbers is

$S = (\frac{k(k+1)}{2})^2$

$\Rightarrow 8281 = (\frac{k(k+1)}{2})^2$

Taking square root on both sides,

$91 = \frac{k(k+1)}{2}$

k(k + 1) = 182

$k^2 + k = 182$

Or $k^2 + k - 182 = 0$

Solving the quadratic using the quadratic formula

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where b = 1, a = 1, c = -182

We get,

$$k = \frac{-1 \pm \sqrt{1^2 - 4(1)(-91)}}{2}$$

$$\Rightarrow k = \frac{-1 \pm 27}{2}$$

$$\Rightarrow k = \frac{-1 + 27}{2} \text{ or } \frac{-1 - 27}{2}$$

$k = \frac{-1-27}{2}$ is invalid because it yields a negative k which doesn’t make sense because number of terms in a series cannot be negative.

$p = \frac{-1+27}{2} = 13$

The sum $S = 1^3 + 2^3 + 3^3 + ... + k^3$ corresponds to k = 13.

Given series 1 + 2 + 3…. + k, we have k = 13

We have $S = \frac{k(k+1)}{2}$

$= \frac{13(13+1)}{2}$

$= \frac{13(14)}{2}$

S = 91

∴ The sum $S = 1^3 + 2^3 + 3^3 + ... + k^3$ corresponds to k = 13 and 1 + 2 + 3.... + k = 91.

5. Question

Find the total area of 12 squares whose sides are 12 cm, 13cm, g, 23cm. respectively.

Answer

Given that there are 12 squares,

We see that their sides 12cm, 13cm..,23cm are in series.

To find the total area, we know that the area of a square is simply l^2 where l is the length of the side.

Area of first square = 12cm × 12cm = 12^2 cm^2

Area of the second square = 13cm × 13cm = 13^2 cm^2

And so on.

We observe that this is in a series.

So $S = 12^2 + 13^2 + 14^2 + ... + 23^2$

To find S,

Let S_1 be $1^2 + 2^2 + ... + 23^2$ with n = 23

Let S_2 be $1^2 + 2^2 + ... + 11^2$ with n = 11

$S = S_1 - S_2$

<u>To find S_1</u>

$1^2 + 2^2 + ... + 23^2$ with n = 23

Formula to find the sum of first n squares of natural numbers is

$S = \frac{n(n+1)(2n+1)}{6}$

$\Rightarrow S_1 = \frac{23(23+1)(2 \times 23+1)}{6}$

$= \frac{23(24)(47)}{6}$

$S_1 = 4324$

<u>To find S_2</u>

$1^2 + 2^2 + ... + 11^2$ with n = 11

Formula to find the sum of first n squares of natural numbers is

$S = \frac{n(n+1)(2n+1)}{6}$

$\Rightarrow S_2 = \frac{11(11+1)(2 \times 11+1)}{6}$

$= \frac{11(12)(23)}{6}$

$S_2 = 506$

$S = S_1 - S_2$

$S = 4324 - 506$

$S = 3818cm^2$

∴The required sum $S = 12^2 + 13^2 + 14^2 + \ldots + 23^2 = 3818cm^2$

6. Question

Find the total volume of 15 cubes whose edges are 16 cm, 17 cm, 18 cm, …., 30 cm respectively

Answer

Given that there are 15 cubes,

We see that their sides 16cm, 17cm..,30cm are in series.

To find the total volume, we know that the volume of a cube is simply l^3 where l is the length of the side.

Volume of first cube = 16cm × 16cm × 16cm = 16^3 cm^3

Volume of second cube = 17cm × 17cm × 17cm = 17^3 cm^3

And so on.

We observe that this is in a series.

So $S = 16^3 + 17^3 + 18^3 + \ldots + 30^3$

To find S,

Let S_1 be $1^3 + 2^3 + \ldots. + 30^3$ with n = 30

Let S_2 be $1^3 + 2^3 + \ldots. + 15^3$ with n = 15

$S = S_1 - S_2$

To find S_1

$1^3 + 2^3 + \ldots. + 30^3$ with n = 30

Formula to find the sum of first n cubes of natural numbers is

$S = (\frac{n(n+1)}{2})^2$

$\Rightarrow S_1 = (\frac{30(30+1)}{2})^2$

$= (\frac{30 \times 31}{2})^2$

$S_1 = 216225$

To find the sum S_2 –

$1^3 + 2^3 + \ldots. + 15^3$ with n = 15

Formula to find the sum of first n cubes of natural numbers is

$S = (\frac{n(n+1)}{2})^2$

$\Rightarrow S_2 = (\frac{15(15+1)}{2})^2$

$= (\frac{15 \times 16}{2})^2$

$S_2 = 14400$

$S = S_1 - S_2$

S = 216225-14400

$S = 201825\ cm^3$

∴The required sum $S = 16^3 + 17^3 + 18^3 + \ldots + 30^3 = 201825\ cm^3$

Exercise 2.7

1. Question

Which one of the following is not true?

A. A sequence is a real valued function defined on N.

B. Every function represents a sequence.

C. A sequence may have infinitely many terms.

D. A sequence may have a finite number of terms.

Answer

"Not true" – tells us that there is one option among A, B C or D which is false, while the rest are true.

Let us examine each option separately.

● A, is true because a sequence a is defined on a set of natural numbers.

● C, is true because an infinite sequence is possible. (ex. Infinite GP)

● D, is true because a finite sequence is possible. (ex. Finite AP, GP or HP)

● B, is false because every function is not a sequence, but every sequence is a function.

Hence, the correct option is B.

2. Question

The 8th term of the sequence 1, 1, 2, 3, 5, 8, g is

A. 25

B. 24

C. 23

D. 21

Answer

The above series is also called a Fibonacci sequence, where each term is the sum of its preceding two terms.

$\Rightarrow a_n = a_{n-1} + a_{n-2}$

We are interested in 8^{th} term so substituting n = 8 in the above equation,

$a_8 = a_7 + a_6$. Also, $a_7 = a_6 + a_5$

$\Rightarrow a_8 = (a_6 + a_5) + a_6$

$\Rightarrow a_8 = (8 + 5) + 8$

$\Rightarrow a_8 = 21$

So the correct option is D.

3. Question

The next term of $\frac{1}{20}$ in the sequence $\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \ldots$ is

A. $\frac{1}{24}$

B. $\frac{1}{22}$

C. $\frac{1}{30}$

D. $\frac{1}{18}$

Answer

The general term would be

$$\frac{1}{n(n + 1)}$$

Where n = 1,2,3….

Next term after $\frac{1}{20}$ is the fifth term so n = 5

We have

$$\frac{1}{5(5 + 1)} = \frac{1}{30}$$

Therefore, the correct option is C.

4. Question

If a, b, c, l, m are in A.P, then the value of a - 4b + 6c - 4l + m is

A. 1

B. 2

C. 3

D. 0

Answer

If a, b, c, l, m are in A.P, then

b-a = c-b = l-c = m-l.

a-4b + 6c-4l + m becomes

a-b-b-b-b + c + c + c + c + c + c-l-l-l-l + m

Grouping the terms

We have

a + 4(c-b)-2(l-c)-2l + m

⇒a + 2(c-b)-2(l-c) + 2(c-b)-2l + m

Since c-b = l-c,

⇒a + 2(c-b)-2l + m

⇒a + 2c-2b-2l + m

$\Rightarrow$-(b-a) + (c-b)-(l-c) + (m-l)

= 0

So, the correct option is D.

5. Question

If a, b, c are in A.P. then $\frac{a-b}{b-c}$ is equal to

A. $\frac{a}{b}$

B. $\frac{b}{c}$

C. $\frac{a}{c}$

D. 1

Answer

If a,b,c are in AP, then b-a = c-b

Or $\frac{b-a}{c-b} = 1$

Or $\frac{a-b}{b-c} = 1$

Therefore, the correct option is D.

6. Question

If the n^{th} term of a sequence is 100 n + 10, then the sequence is

A. an A.P.

B. a G.P.

C. a constant sequence

D. neither A.P. nor G.P.

Answer

given nth term = 100n + 10

This can be rewritten as 110 + (n-1)100

This is in the form T_n = a + (n-1)d which forms an AP.

Therefore, the correct option is A.

7. Question

If a_1, $a_{\mathbf{2}}$, a_3,...are in A.P. such that $\frac{a_4}{a_7} = \frac{3}{2}$, then the 13th term of the A.P. is

A. $\frac{3}{2}$

B. 0

C. 12_{a1}

D. 14_{a1}

Answer

Given $\frac{a_4}{a_7} = \frac{3}{2}$

Let common ratio be d

$a_4 = a_1 + 3d$

$a_7 = a_1 + 6d$

$\Rightarrow \frac{a_1 + 3d}{a_1 + 6d} = \frac{3}{2}$

$\Rightarrow 2a_1 + 6d = 3a_1 + 18d$

$\Rightarrow a_1 + 12d = 0$

Which corresponds to the 13^{th} term $(a_1 + (13-1)d) = 0$

Therefore, the correct option is B.

8. Question

If the sequence $a_1, a_2, a_3, \ldots$ is in A.P. , then the sequence $a_5, a_{10}, a_{15}, \ldots$ is

A. a G.P.

B. an A.P.

C. neither A.P nor G.P.

D. a constant sequence

Answer

Terms collected from an AP with a common interval between two terms is always in AP, $a_5, a_{10}, a_{15\ldots.}$ have a common interval of 5, so it is also in AP.

9. Question

If k + 2, 4k–6, 3k–2 are the three consecutive terms of an A.P, then the value of k is

A. 2

B. 3

C. 4

D. 5

Answer

From AP, it is clear that

(4k-6)-(k + 2) = (3k-2)-(4k-6)

$\Rightarrow$3k-8 = -k + 4

$\Rightarrow$4k = 12 or k = 3.

So the correct choice is B.

10. Question

If a, b, c, l, m. n are in A.P., then 3a + 7, 3b + 7, 3c + 7, 3l + 7, 3m + 7, 3n + 7 form

A. a G.P.

B. an A.P.

C. a constant sequence

D. neither A.P. nor G.P

Answer

Terms of an AP if multiplied with a constant will still be in AP.

The constant term is 3.

Therefore, 3a, 3b, 3c…are in AP.

Adding a constant to an AP does not change the sequence type. i.e. It will still be in AP.

The constant term added is 7.

Therefore, 3a + 7, 3b + 7….also form an AP.

∴The correct choice is B.

11. Question

If the third term of a G.P is 2, then the product of first 5 terms is

A. 5^2

B. 2^5

C. 10

D. 15

Answer

$2 = ar^{n-1} = ar^{3-1} = ar^2$

Product of the first five terms = $a x ar x ar^2 x ar^3 x ar^4$

This can be rewritten as a^5r^{10} or $(ar^2)^5$

So, Product of the first five terms is 2^5

Hence the correct option is B.

12. Question

If a, b, c are in G.P, then is equal to $\frac{a-b}{b-c}$ is equal to

A. $\frac{a}{b}$

B. $\frac{b}{a}$

C. $\frac{a}{c}$

D. $\frac{c}{b}$

Answer

Given that a, b, c are in GP,

$\frac{b}{a} = \frac{c}{b}$

Subtracting 1 from both sides, we get

$$\frac{b}{a} - 1 = \frac{c}{b} - 1$$

$$\Rightarrow \frac{b - a}{a} = \frac{c - b}{b}$$

$$\Rightarrow \frac{b - a}{c - b} = \frac{a}{b}$$

$$\Rightarrow \frac{a - b}{b - c} = \frac{a}{b}$$

Therefore, the correct option is A.

13. Question

If x, 2x + 2, 3x + 3 are in G.P, then 5x, 10x + 10, 15x + 15 form

A. an A.P.

B. a G.P.

C. a constant sequence

D. neither A.P. nor a G.P.

Answer

Any GP if multiplied with a constant term remains in GP.

x, 2x + 2, 3x + 3 are in GP, 5(x), 5(2x + 2), 5(3x + 3) will also form a GP because a constant term 5 is merely multiplied.

So the correct option is B.

14. Question

The sequence –3, –3, –3,... is

A. an A.P. only

B. a G.P. only

C. neither A.P. nor G.P

D. both A.P. and G.P.

Answer

The given sequence is -3, -3, -3

Test for AP

If a,b,c...are in AP the b-a = c-b...

$\Rightarrow$-3-(-3) = -3-(-3) = 0

Therefore, the series is in AP.

Test for GP

If a,b,c.. are in GP then $\frac{b}{a} = \frac{c}{b}$...

$$\frac{-3}{-3} = \frac{-3}{-3}$$

Therefore, the series is in GP.

The series is both in AP and GP.

So the correct option is D.

15. Question

If the product of the first four consecutive terms of a G.P is 256 and if the common ratio is 4 and the first term is positive, then its 3rd term is

A. 8

B. $\frac{1}{16}$

C. $\frac{1}{32}$

D. 16

Answer

Given the product of first four consecutive terms of GP = 256.

Let the first four terms be a, ar, ar^2, ar^3

Product = 256 = $a \times ar \times ar^2 \times ar^3 = a^4 r^6$

Also given r = 4,

We have 256 = $a^4 \times (4)^6$

$\Rightarrow a^4 = \frac{1}{16}$

$\Rightarrow a = \frac{1}{2}$ which is positive.

Third term is $ar^2 = \frac{1}{2} \times 4^2 = 8$

So the correct option is A.

16. Question

In a G.P, $t_2 = \frac{3}{5}$ and $t_3 = \frac{1}{5}$. Then the common ratio is

A. $\frac{1}{5}$

B. $\frac{1}{3}$

C. 1

D. 5

Answer

Common ratio = $\frac{t_3}{t_2} = \frac{\frac{1}{5}}{\frac{3}{5}} = \frac{1}{3}$

So the correct option is B.

17. Question

If $x \neq 0$, then $1 + \sec x + \sec^2 x + \sec^3 x + \sec^4 x + \sec^5 x$ is equal to

A. $(1 \sec x)(\sec^2 x + \sec^3 x + \sec^4 x)$

B. $(1 + \sec x)(1 + \sec^2 x + \sec^4 x)$

C. $(1 \sec x)(\sec x + \sec^3 x + \sec^5 x)$

D. $(1 + \sec x)(1 + \sec^3 x + \sec^4 x)$

Answer

We see that the problem under consideration is a GP with a = 1 and common ratio r = sec x.

Sum of GP S = $\frac{a(r^n - 1)}{(r-1)}$

Where n = number of terms, here n = 6.

Therefore

S = $\frac{1(\sec x)^6 - 1}{\sec x - 1}$

The numerator is of the form $a^6 - b^6$

$a^6 - b^6 = (a + b)(a-b)(a^2 + b^2 + ab)(a^2 + b^2 - ab)$

Therefore $(\sec x)^6 - 1^6 = (\sec x + 1)(\sec x - 1)(\sec^2 x + 1 + \sec x)(\sec^2 x + 1 - \sec x)$

$\Rightarrow (\sec x)^6 - 1^6 = (\sec x + 1)(\sec x - 1)(1 + \sec^2 x + \sec^4 x)$

$\Rightarrow$ S = $\frac{1(\sec x)^6 - 1}{\sec x - 1}$ = $(\sec x + 1)(1 + \sec^2 x + \sec^4 x)$

So the correct answer is option B.

18. Question

If the n^{th} term of an A.P. is $t_n = 3 - 5n$, then the sum of the first n terms is

A. $\frac{n}{2}[1 - 5n]$

B. n(1 - 5n)

C. $\frac{n}{2}[1 + 5n]$

D. $\frac{n}{2}[1 + n]$

Answer

Given $t_n = 3 - 5_n$, $t_1 = 3 - 5 = -2$

Sum of n terms of an AP = $S_n = \frac{n}{2}(a + t_n)$

$S_n = \frac{n}{2}(-2 + 3 - 5n) = \frac{n}{2}(1 - 5n)$

So the correct answer is B.

19. Question

The common ratio of the G.P. a^{m-n}, a^m, a^{m+n} is

A. a^m

B. a^{-m}

C. a^n

D. a^{-n}

Answer

If x, y, z are in GP, then Common ratio r = $\frac{y}{x}$ or $\frac{z}{y}$

Common ratio in this problem is $\frac{a^m}{a^{m-n}}$ or $\frac{a^{m+n}}{a^m}$ = a^n

Therefore, the correct option is C.

20. Question

If 1 + 2 + 3 + . . . +n = k then $1^3 + 2^3 + \ldots + n^3$ is equal to

A. k^2

B. k^3

C. $\frac{k(k+1)}{2}$

D. $(k + 1)^3$

Answer

Sum of first n natural numbers is S = $\frac{n(n+1)}{2}$ = k

Sum of first n natural squares is S = $(\frac{n(n+1)}{2})^2$

Which is k^2

So the correct option is A.