7 Polynomial, Quadratic Equations and Inequalities

Polynomial, Solution of Quadratic Equations, Nature of Roots and Relation between Roots and Coefficient

1. If α and β are the roots of the equation $ax^2 + bx + c = 0$, where $a \neq 0$, then $(a\alpha + b)(a\beta + b)$ is equal to: **[2014-I]**

(a) ab (b) bc (c) ca (d) abc

2. The roots of the equation $2a^2x^2 - 2abx + b^2 = 0$ when $a < 0$ and $b > 0$ are: **[2014-I]**

(a) Sometimes complex (b) Always irrational
(c) Always complex (d) Always real

3. Every quadratic equation $ax^2 + bx + c = 0$ where $a, b, c, \in R, a \neq 0$ has **[2014-II]**

(a) exactly one real root (b) at least one real root.
(c) at least two real roots (d) at most two real roots.

4. If α, β are the roots of $ax^2 + bx + c = 0$ and $\alpha + h, \beta + h$ are the roots of $px^2 + qx + r = 0$, then what is h equal to? **[2014-II]**

(a) $\frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$ (b) $\frac{1}{2}\left(-\frac{b}{a} + \frac{q}{p}\right)$

(c) $\frac{1}{2}\left(\frac{b}{p} + \frac{q}{a}\right)$ (d) $\frac{1}{2}\left(-\frac{b}{p} + \frac{q}{a}\right)$

5. In solving a problem that reduces to a quadratic equation, one student makes a mistake in the constant term and obtains 8 and 2 for roots. Another student makes a mistake only in the coefficient of first-degree term and finds –9 and –1 for roots. The correct equation is **[2015-I]**

(a) $x^2 - 10x + 9 = 0$ (b) $x^2 - 10x + 9 = 0$
(c) $x^2 - 10x + 16 = 0$ (d) $x^2 - 8x - 9 = 0$

6. If m and n are the roots of the equation $(x + p)(x + q) - k = 0$, then the roots of the equation $(x - m)(x - n) + k = 0$ are **[2015-I]**

(a) p and q (b) $\frac{1}{p}$ and $\frac{1}{q}$
(c) $-p$ and $-q$ (d) $p + q$ and $p - q$

7. If $2p + 3q = 18$ and $4p^2 + 4pq - 3q^2 - 36 = 0$, then what is $(2p + q)$ equal to? **[2015-I]**

(a) 6 (b) 7 (c) 10 (d) 20

8. If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of their squares, then **[2015-II]**

(a) $a^2 + b^2 = c^2$ (b) $a^2 + b^2 = a + b$
(c) $ab + b^2 = 2ac$ (d) $ab - b^2 = 2ac$

9. If the roots of the equation $x^2 - nx + m = 0$ differ by 1, then **[2015-II]**

(a) $n^2 - 4m - 1 = 0$ (b) $n^2 + 4m - 1 = 0$
(c) $m^2 + 4n + 1 = 0$ (d) $m^2 - 4n - 1 = 0$

Consider the following for the next two (02) items that follow:

Let α and β $(\alpha < \beta)$ be the roots of the equation $x^2 + bx + c = 0$, where $b > 0$ and $c < 0$. **[2016-I]**

10. Consider the following:

1. $\beta < -\alpha$
2. $\beta < |\alpha|$

Which of the above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

11. Consider the following:

1. $\alpha + \beta + \alpha\beta > 0$
2. $\alpha^2\beta + \beta^2\alpha > 0$

Which of the above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

12. If one root of the equation $(l - m)x^2 + lx + 1 = 0$ is double the other and l is real, then what is the greatest value of m? **[2016-I]**

(a) $-\frac{9}{8}$ (b) $\frac{9}{8}$ (c) $-\frac{8}{9}$ (d) $\frac{8}{9}$

Consider the following for the next two (02) items that follow:

Let α and β be the roots of the equation $x^2 - (1 - 2a^2)x + (1 - 2a^2) = 0$

13. Under what condition does the above equation have real roots? **[2016-II]**

(a) $a^2 < \frac{1}{2}$ (b) $a^2 > \frac{1}{2}$ (c) $a^2 \leq \frac{1}{2}$ (d) $a^2 \geq \frac{1}{2}$

14. Under what condition is $\frac{1}{\alpha^2} + \frac{1}{\beta^2} < 1$? **[2016-II]**

(a) $a^2 < \frac{1}{2}$ (b) $a^2 > \frac{1}{2}$
(c) $a^2 > 1$ (d) $a^2 \in \left(\frac{1}{3}, \frac{1}{2}\right)$ only

Consider the following for the next two (02) items that follow:

$2x^2 + 3x - \alpha = 0$ has roots –2 and β while the equation $x^2 - 3mx + 2m^2 = 0$ has both roots positive, where $\alpha > 0$ and $\beta > 0$.

15. What is the value of α? ***[2016-II]***

(*a*) 1/2 (*b*) 1 (*c*) 2 (*d*) 4

16. If β, 2, $2m$ are in GP, then what is the value of $\beta\sqrt{m}$? ***[2016-II]***

(*a*) 1 (*b*) 2 (*c*) 4 (*d*) 6

17. If the roots of the equation $x^2 + px + q = 0$ are in the same ratio as those of the equation $x^2 + lx + m = 0$, then which one of the following is correct? ***[2017-I]***

(*a*) $p^2m = l^2q$ (*b*) $m^2p = l^2p$ (*c*) $m^2p = q^2l$ (*d*) $m^2p^2 = l^2q$

18. If the graph of a quadratic polynomial lies entirely above *x*-axis, then which one of the following is correct? ***[2017-I]***

(*a*) Both the roots are real

(*b*) One root is real and the other is complex

(*c*) Both the roots are complex

(*d*) Cannot say

19. If cot α and cot β are the roots of the equation $x^2 + bx + c = 0$ with $b \neq 0$, then the value of cot $(\alpha + \beta)$ is ***[2017-I]***

(*a*) $\frac{c-1}{b}$ (*b*) $\frac{1-c}{b}$ (*c*) $\frac{b}{c-1}$ (*d*) $\frac{b}{1-c}$

20. The roots of the equation ***[2017-II]***

$(q - r)x^2 + (r - p)x + (p - q) = 0$ are

(*a*) $(r - p)/(q - r)$, 1/2 (*b*) $(p - q)/(q - r)$, 1

(*c*) $(q - r)/(p - q)$, 1 (*d*) $(r - p)/(p - q)$, 1/2

21. It is given that the roots of the equation $x^2 - 4x - \log_3 P = 0$ are real. For this, the minimum value of *P* is ***[2017-II]***

(*a*) $\frac{1}{27}$ (*b*) $\frac{1}{64}$ (*c*) $\frac{1}{81}$ (*d*) 1

22. If α and β are the roots of the equation $3x^2 + 2x + 1 = 0$, then the equation whose roots are $\alpha + \beta^{-1}$ and $\beta + \alpha^{-1}$ is ***[2017-II]***

(*a*) $3x^2 + 8x + 16 = 0$ (*b*) $3x^2 - 8x - 16 = 0$

(*c*) $3x^2 + 8x - 16 = 0$ (*d*) $x^2 + 8x + 16 = 0$

23. In ΔPQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$, then which one of the following is correct? ***[2017-II]***

(*a*) $a = b + c$ (*b*) $b = c + a$ (*c*) $c = a + b$ (*d*) $b = c$

24. If α and β $(\neq 0)$ are the roots of the quadratic equation $x^2 + \alpha x - \beta = 0$, then the quadratic expression $-x^2 + \alpha x + \beta$ where $x \in R$ has ***[2018-II]***

(*a*) Least value $-\frac{1}{4}$ (*b*) Least value $-\frac{9}{4}$

(*c*) Greatest value $\frac{1}{4}$ (*d*) Greatest value $\frac{9}{4}$

25. The ratio of roots of the equations $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ are equal. If D_1 and D_2 are respective discriminants, then what is $\frac{D_1}{D_2}$ equal to? ***[2018-II]***

(*a*) $\frac{a^2}{p^2}$ (*b*) $\frac{b^2}{q^2}$

(*c*) $\frac{c^2}{r^2}$ (*d*) None of these

26. The equation $px^2 + qx + r = 0$ (where *p, q, r*, all are positive) has distinct real roots *a* and *b*. Which one of the following is correct? ***[2019-I]***

(*a*) $a > 0, b > 0$ (*b*) $a < 0, b < 0$ (*c*) $a > 0, b < 0$ (*d*) $a < 0, b > 0$

27. If the roots of the equation $x^2 + px + q = 0$ are tan 19° and tan 26°, then which one of the following is correct? ***[2019-I]***

(*a*) $q - p = 1$ (*b*) $p - q = 1$ (*c*) $p + q = 2$ (*d*) $p + q = 3$

28. If both *p* and *q* belong to the set {1,2,3,4}, then how many equations of the form $px^2 + qx + 1 = 0$ will have real roots? ***[2019-II]***

(*a*) 12 (*b*) 10 (*c*) 7 (*d*) 6

29. What is the value of *k* for which the sum of the squares of the roots of $2x^2 - 2(k - 2)x - (k + 1) = 0$ is minimum? ***[2019-II]***

(*a*) –1 (*b*) 1 (*c*) $\frac{3}{2}$ (*d*) 2

30. If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, then which one of the following is correct? ***[2019-II]***

(*a*) *a, b* and *c* are in AP

(*b*) a, b and c are in GP

(*c*) *a, b* and *c* are in HP

(*d*) *a, b* and *c* do not follow any regular pattern

31. Under which one of the following conditions will the quadratic equation $x^2 + mx + 2 = 0$ always have real roots? ***[2019-II]***

(*a*) $2\sqrt{3} \le m^2 < 8$ (*b*) $\sqrt{3} \le m^2 < 4$

(*c*) $m^2 \ge 8$ (*d*) $m^2 \le \sqrt{3}$

32. What is the value of $2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \ldots\infty}}}$? ***[2019-II]***

(*a*) $\sqrt{2} - 1$ (*b*) $\sqrt{2} + 1$ (*c*) 3 (*d*) 4

33. Consider the following statements in respect of the quadratic equation ***[2019-II]***

$4(x - p)(x - q) - r^2 = 0$, where *p, q* and *r* are real numbers:

1. The roots are real
2. The roots are equal if $p = q$ and $r = 0$

Which of the above statements is/are correct?

(*a*) 1 only (*b*) 2 only

(*c*) Both 1 and 2 (*d*) Neither 1 nor 2

34. Which one of the following is the second degree polynomial function $f(x)$ where $f(0) = 5$, $f(-1) = 10$ and $f(1) = 6$? ***[2019-II]***

(*a*) $5x^2 - 2x + 5$ (*b*) $3x^2 - 2x - 5$

(*c*) $3x^2 - 2x + 5$ (*d*) $3x^2 - 10x + 5$

35. If *p* and *q* are the roots of the equation $x^2 - 30x + 221 = 0$, what is the value of $p^3 + q^3$? ***[2019-II]***

(*a*) 7010 (*b*) 7110 (*c*) 7210 (*d*) 7240

36. If cotα and cotβ are the roots of the equation $x^2 - 3x + 2 = 0$ then what is cot$(\alpha + \beta)$ equal to? ***[2020-I & II]***

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$ (*c*) 2 (*d*) 3

37. The roots α and β of a quadratic equation satisfy the relations $\alpha + \beta = \alpha^2 + \beta^2$ and $\alpha\beta = \alpha^2\beta^2$. What is the number of such quadratic equations? ***[2020-I & II]***

(*a*) 0 (*b*) 2 (*c*) 3 (*d*) 4

58. The number of real roots of the equation $x^2 - 3|x| + 2 = 0$ is **[2015-II]**

(a) 4 (b) 3 (c) 2 (d) 1

59. If $x^2 - px + 4 > 0$ for all real values of x, then which one of the following is correct? **[2016-I]**

(a) $|p| < 4$ (b) $|p| \le 4$ (c) $|p| > 4$ (d) $|p| \ge 4$

Consider the following for the next two (02) items that follow:

Consider the function $f(x) = \dfrac{27(x^{2/3} - x)}{4}$ **[2016-I]**

60. How many solutions does the function $f(x) = 1$ have?

(a) One (b) Two (c) Three (d) Four

61. How many solutions does the function $f(x) = -1$ have?

(a) One (b) Two (c) Three (d) Four

62. If $c > 0$ and $4a + c < 2b$, then $ax^2 - bx + c = 0$ has a root in which one of the following intervals? **[2016-II]**

(a) (0, 2) (b) (2, 3) (c) (3, 4) (d) (–2, 0)

63. If both the roots of the equation $x^2 - 2kx + k^2 - 4 = 0$ lie between –3 and 5, then which one of the following is correct? **[2016-II]**

(a) $-2 < k < 2$ (b) $-5 < k < 3$

(c) $-3 < k < 5$ (d) $-1 < k < 3$

64. If $|a|$ denotes the absolute value of an integer, then which of the following are correct? **[2017-II]**

1. $|ab| = |a|\,|b|$
2. $|a + b| \le |a| + |b|$
3. $|a - b| \ge ||a| - |b||$

Select the correct answer using the code given below.

(a) 1 and 2 only (b) 2 and 3 only

(c) 1 and 3 only (d) 1, 2 and 3

65. The sum of all real roots of the equation $|x - 3|^2 + |x - 3| - 2 = 0$ is **[2017-II]**

(a) 2 (b) 3 (c) 4 (d) 6

66. The equation $|1 - x| + x^2 = 5$ has **[2018-I]**

(a) a rational root and an irrational root

(b) two rational roots

(c) two irrational roots

(d) no real roots

67. Let $[x]$ denote the greatest integer function. What is the number of solutions of the equation $x^2 - 4x + [x] = 0$ in the interval [0, 2]? **[2018-I]**

(a) Zero (No solution) (b) One

(c) Two (d) Three

68. Consider the following expressions: **[2018-II]**

1. $x + x^2 - \dfrac{1}{x}$
2. $\sqrt{ax^2 + bx + x - c + \dfrac{d}{c} - \dfrac{e}{x^2}}$
3. $3x^2 - 5x + ab$
4. $\dfrac{2}{x^2 - ax + b^2}$
5. $\dfrac{1}{x} - \dfrac{2}{x + 5}$

Which of the above are rational expressions?

(a) 1, 4 and 5 only (b) 1, 3, 4 and 5 only

(c) 2, 4 and 5 only (d) 1 and 2 only

69. What are the roots of the equation $|x^2 - x - 6| = x + 2$? **[2019-I]**

(a) –2, 1, 4 (b) 0, 2, 4 (c) 0, 1, 4 (d) –2, 2, 4

70. The number of real roots for the equation $x^2 + 9|x| + 20 = 0$ is **[2019-I]**

(a) Zero (b) One (c) Two (d) Three

71. If $|x^2 - 3x + 2| > x^2 - 3x + 2$, then which one of the following is correct? **[2019-II]**

(a) $x \le 1$ or $x \ge 2$

(b) $1 \le x \le 2$

(c) $1 < x < 2$

(d) x is any real value except 3 and 4

72. What is the solution of $x \le 4, y \ge 0$ and $x \le -4, y \le 0$? **[2019-II]**

(a) $x \ge -4, y \le 0$ (b) $x \le 4, y \ge 0$

(c) $x \le -4, y = 0$ (d) $x \ge -4, y = 0$

73. How many real roots does the equation $x^2 + 3|x| + 2 = 0$ have? **[2019-II]**

(a) Zero (b) One (c) Two (d) Four

74. If, $1.5 \le x \le 4.5$, then which one of the following is correct? **[2020-I & II]**

(a) $(2x - 3)(2x - 9) > 0$ (b) $(2x - 3)(2x - 9) < 0$

(c) $(2x - 3)(2x - 9) \ge 0$ (d) $(2x - 3)(2x - 9) \le 0$

75. If $f(x) = 3x^2 - 5x + p$ and $f(0)$ and $f(1)$ are opposite in sign, then which of the following is correct? **[2020-I & II]**

(a) $-2 < p < 0$ (b) $-2 < p < 2$

(c) $0 < p < 2$ (d) $3 < p < 5$

76. Consider all the real roots of the equation $x^4 - 10x^2 + 9 = 0$. What is the sum of the absolute values of the roots? **[2021-II]**

(a) 4 (b) 6 (c) 8 (d) 10

77. Consider the inequations $5x - 4y + 12 < 0, x + y < 2, x < 0$ and $y > 0$. Which one of the following points lies in the common region? **[2022-I]**

(a) (0, 0) (b) (–2, 4) (c) (–1, 4) (d) (–1, 2)

78. How many real numbers satisfy the equation $|x - 4| + |x - 7| = 15$? **[2023-I]**

(a) Only one (b) Only two

(c) Only three (d) Infinitely many

79. For how many integral values of k, the equation $x^2 - 4x + k = 0$, where k is an integer has real roots and both of them lie in the interval (0, 5)? **[2023-I]**

(a) 3 (b) 4 (c) 5 (d) 6

Consider the following for the next two (02) items that follow:

Consider the equation $(1 - x)^4 + (5 - x)^4 = 82$.

80. What is the number of real roots of the equation? **[2023-II]**

(a) 0 (b) 2 (c) 4 (d) 8

81. What is the sum of all the roots of the equation? **[2023-II]**

(a) 24 (b) 12 (c) 10 (d) 6

82. If $-\sqrt{2}$ and $\sqrt{3}$ are roots of the equation $a_0 + a_1x + a_2x^2 + a_3x^3 + x^4 = 0$ where a_0, a_1, a_2, a_3 are integers, then which one of the following is correct? **[2024-I]**

(a) $a_2 = a_3 = 0$ (b) $a_2 = 0$ and $a_3 = -5$

(c) $a_0 = 6, a_3 = 0$ (d) $a_1 = 0$ and $a_2 = 5$

$\alpha = \frac{-b-\sqrt{b^2-4c}}{2}$ $(\because \alpha < \beta)$

$\Rightarrow -\alpha = \frac{b+\sqrt{b^2-4c}}{2}$

and $|\alpha| = \frac{b+\sqrt{b^2-4c}}{2}$

$\because \beta < -\alpha$ and $\beta < |\alpha|$ both are correct.

11. (*b*) Sum of roots $= \alpha + \beta = -b$

Multiplication of roots $= \alpha\beta = c$

Hence

$\alpha + \beta + \alpha\beta = -b + c$

$\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha+\beta) = -bc$

$\because b > 0$ & $c < 0$

$\therefore -b + c < 0$ & $-bc > 0$

12. (*b*) Given equation is

$(l-m)x^2 + lx + 1 = 0$

Let α and 2α be the roots of given equation

$\therefore 3\alpha = \frac{-l}{l-m}$ and $\alpha(2\alpha) = \frac{1}{(l-m)}$

$\Rightarrow \alpha^2 = \frac{l^2}{9(l-m)^2}$ and $2\alpha^2 = \frac{1}{l-m}$

$\Rightarrow 2\frac{l^2}{9(l-m)^2} = \frac{1}{(l-m)}$

$\Rightarrow \frac{2l^2}{9(l-m)} = 1$

$\Rightarrow 2l^2 - 9l + 9m = 0$

For l to be real discriminant should be $b^2 - 4ac \geq 0$

$\Rightarrow 81 - 4 \times 2 \times 9m \geq 0$

$m \leq \frac{9}{8}$.

13. (*d*) Using $ax^2 + bx + c = 0$

$a = 1, b = -(1-2a^2)$ & $c = (1-2a^2)$

For roots to be real,

$b^2 - 4ac \geq 0$

$\Rightarrow [-(1-2a^2)]^2 - 4(1)(1-2a^2) \geq 0$

$\Rightarrow 1 + 4a^4 - 4a^2 - 4 + 8a^2 \geq 0$

$\Rightarrow 4a^4 + 6a^2 - 2a^2 - 3 \geq 0$

$\Rightarrow 4a^4 + 4a^2 - 3 \geq 0$

$\Rightarrow (2a^2-1)(2a^2+3) \geq 0$

$\Rightarrow a^2 \geq \frac{1}{2}$ or $a^2 \leq -\frac{3}{2}$

14. (*a*) $\alpha + \beta = \frac{-b}{a} = (1-2a^2)$ &

$\alpha\beta = \frac{c}{a} = (1-2a^2)$

Now, $\frac{1}{\alpha^2} + \frac{1}{\beta^2} < 1 \Rightarrow \frac{\alpha^2+\beta^2}{(\alpha\beta)^2} < 1$

$\Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} < 1$

On solving: $\frac{4a^4-1}{4a^4-4a^2+1} < 1$

$\Rightarrow 4a^4 - 1 < 4a^4 - 4a^2 + 1$

$\Rightarrow 4a^2 < 2 \Rightarrow a^2 < \frac{1}{2}$

15. (*c*) $2x^2 + 3x - \alpha = 0$

Its roots are: -2 & β.

$\therefore \frac{-3}{2} = \beta - 2 \Rightarrow \beta = 2 - \frac{3}{2} = \frac{1}{2}$

Now, $\frac{\alpha}{2} = 2\beta \Rightarrow \alpha = 4 \times \frac{1}{2} \Rightarrow \alpha = 2$

16. (*a*) If β, 2, $2m$ are in GP, then

$\Rightarrow \frac{2}{\beta} = \frac{2m}{2}$

$\Rightarrow m = \frac{2}{\beta} = 2 \times \frac{2}{1}$

$\Rightarrow m = 4$

$\therefore \beta\sqrt{m} = \frac{1}{2} \times \sqrt{4} = 1$

17. (*a*) Let a, b be the roots of $x^2 + px + q = 0$

So, $a + b = -p$, $ab = q$ ...(i)

Let c, d be the roots of $x^2 + lx + m = 0$

So, $c + d = -l$, $cd = m$ ...(ii)

Given that roots of both the equations are in the same ratio.

So, $\frac{a}{b} = \frac{c}{d}$ ...(iii)

$\Rightarrow \frac{b}{a} = \frac{d}{c}$ ...(iv)

On adding eq. (iii) & (iv), we get

$\frac{a}{b} + \frac{b}{a} = \frac{c}{d} + \frac{d}{c}$

$\Rightarrow \frac{a^2+b^2}{ab} = \frac{c^2+d^2}{cd}$

$\Rightarrow \frac{a^2+b^2}{ab} + 2 = \frac{c^2+d^2}{cd} + 2$

$\Rightarrow \frac{a^2+b^2+2ab}{ab} = \frac{c^2+d^2+2cd}{cd}$

$\Rightarrow \frac{(a+b)^2}{ab} = \frac{(c+d)^2}{cd}$

$\Rightarrow \frac{(-p)^2}{q} = \frac{(-l)^2}{m}$ (from (i) and (ii))

$\Rightarrow p^2m = l^2q$.

18. (*c*) Since the graph is not meeting the x-axis at all, roots are Complex numbers.

19. (*b*) Given equation, $x^2 + bx + c = 0$

Roots are $\cot\alpha$, $\cot\beta$.

Sum of roots $= \cot\alpha + \cot\beta = -b$

Product of roots $= \cot\alpha \cdot \cot\beta = c$

$\cot(\alpha+\beta) = \frac{\cot\alpha\cdot\cot\beta - 1}{\cot\beta + \cot\alpha} = \frac{c-1}{-b} = \frac{1-c}{b}$

20. (*b*) Given equation, $(q-r)x^2 + (r-p)x + (p-q) = 0$

If $x = 1$, then $q - r + r - p + p - q = 0$.

$\therefore$ 1 is one root of given equation.

Since, the given equation is quadratic equation, we know that product of roots is $\frac{c}{a}$.

Let the second root be α.

$\therefore (1)(\alpha) = \frac{p-q}{q-r} \Rightarrow \alpha = \frac{p-q}{q-r}$

21. (*c*) $x^2 - 4x - \log_3 P = 0$

For roots to be real, discriminant ≥ 0

$\Rightarrow b^2 \geq 4ac \Rightarrow (-4)^2 \geq 4(1)(-\log_3 P)$

$\Rightarrow 16 \geq -4\log_3 P$

$\Rightarrow 4 \geq -\log_3 P$

$\Rightarrow 4 \geq \log_3\left(\frac{1}{P}\right)$

$\Rightarrow 3^4 \geq \frac{1}{P} \Rightarrow P \geq \frac{1}{81}$.

$\therefore$ The minimum value of P is $\frac{1}{81}$.

22. (*a*) $3x^2 + 2x + 1 = 0$.

Sum of the roots $= \alpha + \beta = \frac{-2}{3}$ ...(i)

Product of the roots $= \alpha \cdot \beta = \frac{1}{3}$ ...(ii)

We have to find the equation with the roots $\alpha + \beta^{-1}$ and $\beta + \alpha^{-1}$.

Sum of the roots $(S) = \alpha + \beta^{-1} + \beta + \alpha^{-1}$

$= (\alpha+\beta) + \left(\frac{\alpha+\beta}{\alpha\beta}\right)$

$= \frac{-2}{3} + \left(\frac{\frac{-2}{3}}{\frac{1}{3}}\right)$ (from (i), (ii))

$= \frac{-2}{3} - 2 = \frac{-2-6}{3} = \frac{-8}{3}$.

Product of the roots

$(P) = (\alpha + \beta^{-1})(\beta + \alpha^{-1})$

$= \left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$

$= \alpha\beta + 2 + \frac{1}{\alpha\beta}$

$= \frac{1}{3} + 2 + \frac{1}{\frac{1}{3}} = \frac{16}{3}$

$f(-1) = a - b + 5 = 10$ ...(i)

$f(1) = a + b + 5 = 6$ ...(ii)

Adding equation (i) and (ii) we get:

$2a + 10 = 16$

$\Rightarrow 2a = 6$

$\Rightarrow a = 3$

Now, putting the value of $a = 3$ in the equation (i), we get:

$3 - b + 5 = 10$

$b = -2$

So, the value of $f(x) = 3x^2 - 2x + 5$

35. (*b*) $p + q = 30$ and $pq = 221$

$p^3 + q^3 = (p + q) \times (p^2 - pq + q^2)$

$= (p + q)((p + q)^2 - 3pq)$

$= 30 \times ((30)^2 - 3(221)) = 7110$

36. (*b*) The quadratic equation is $x^2 - 3x + 2 = 0$

Sum of roots $= \cot\alpha + \cot\beta = \frac{-(-3)}{1} = 3$

Product of roots $= \cot\alpha . \cot\beta = \frac{2}{1} = 2$

$\cot(\alpha + \beta) = \frac{\cot\alpha \cot\beta - 1}{\cot\beta + \cot\alpha}$

$\cot(\alpha + \beta) = \frac{2-1}{3} = \frac{1}{3}$

37. (*b*) $\alpha\beta = \alpha^2\beta^2$

$\Rightarrow \alpha\beta - \alpha^2\beta^2 = 0$

$\Rightarrow \alpha\beta(1 - \alpha\beta) = 0$

$\Rightarrow \alpha\beta = 0, \alpha\beta = 1$

So, $\alpha + \beta = \alpha^2 + \beta^2$

$\alpha\beta = 1, \Rightarrow \alpha = \frac{1}{\beta}$

So, $\frac{1}{\beta} + \beta = \frac{1}{\beta^2} + \beta^2$

$\Rightarrow \beta^2 - \beta = \frac{1}{\beta} - \frac{1}{\beta^2} \Rightarrow \beta(\beta - 1) = \frac{\beta - 1}{\beta^2}$

$\Rightarrow (\beta - 1)\left(\beta - \frac{1}{\beta^2}\right) = 0$

$\Rightarrow (\beta - 1)(\beta^3 - 1) = 0$

$\Rightarrow (\beta - 1)(\beta - 1)(\beta^2 + \beta + 1) = 0$

$\Rightarrow (\beta - 1)^2(\beta^2 + \beta + 1) = 0$

$\Rightarrow \beta = 1, \beta = \frac{-1 \pm \sqrt{3}i}{2}$

Now, put the value of β in $\alpha = \frac{1}{\beta}$

$\alpha = 1, \alpha = \frac{-1 \mp \sqrt{3}i}{2}$

Here two value of α and β takes place.

So, the number of quadratic equations will be two.

38. (*d*) Given quadratic equation is $x^2 + 2x + k = 0$

Since, roots are real

$\Rightarrow D \geq 0$

$\Rightarrow (2)^2 - 4(1)(k) \geq 0 \Rightarrow k \leq 1$

39. (*a*) Given quadratic equation $4x^2 + 2x - 1 = 0$ ...(i)

If α, β are the roots of Eq. (i), then these value will satisfy the given equation.

$4\alpha^2 + 2\alpha - 1 = 0$ ...(ii)

and $4\beta^2 + 2\beta - 1 = 0$ ...(iii)

From Eq. (i),

Sum of roots $= \frac{-2}{4} \Rightarrow \alpha + \beta = \frac{-1}{2}$

$\Rightarrow \beta = \frac{-1}{2} - \alpha$

On putting the value of β in Eq. (iii),

$4\left(\frac{1}{4} + \alpha^2 + \alpha\right) - 1 = -2\beta$

$\Rightarrow \beta = -2\alpha^2 - 2\alpha$

40. (*c*) Let the roots be α and $\frac{1}{a}$.

$\therefore \alpha \times \frac{1}{\alpha} = \frac{k}{5} \Rightarrow k = 5$

41. (*c*) Let other root be β

We have, $x(x + 1) + 1 = 0$

$\Rightarrow x^2 + x + 1 = 0$

As k is root of the equation.

$k^2 = -1 - k$ ...(i)

Now, sum of roots $\beta + k = -1 \Rightarrow \beta = -1 - k = k^2$

42. (*d*) Since, we know that if a quadratic equation $ax^2 + bx + c = 0$ has real roots of equal magnitude and opposite sign.

Then, $b = 0$ ...(i)

$\therefore k = 0, -5$

And product of roots < 0 ...(ii)

$\therefore 0 < k < \frac{5}{3}$

Hence, no such value exist.

43. (*b*) **Hint:** Use concept of sum of root and product of roots.

44. (*c*) Given, equation $ax^2 + bx + c = 0$

$\because$ Roots are $\sin\theta$ and $\cos\theta$

$\therefore \sin\theta + \cos\theta = -\frac{b}{a}$ ... (i)

And $\sin\theta \times \cos\theta = \frac{c}{a}$ ... (ii)

Now after squaring equ (i) & putting values we get

$a^2 - b^2 + 2ac = 0$

45. (*c*) Given equation,

$4x^2 - (5k + 1)x + 5k = 0$ ... (i)

Let the roots are α and β.

$\alpha + \beta = \frac{-(-(5k+1))}{4} = \frac{5k+1}{4}$

and $\alpha \cdot \beta = \frac{5k}{4}$

Given that, $\alpha - \beta = 1 \Rightarrow (\alpha - \beta)^2 = 1$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 1$

$\Rightarrow \frac{25k^2 + 1 + 10k}{16} = 1 + 5k$

$\Rightarrow k = 3$ or $-\frac{1}{5}$

46. (*d*) $\because \alpha + \beta = -p$ and $\alpha\beta = q$

$\alpha^3 + \beta^3 = -m$ and $\alpha^3\beta^3 = n$

$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha + \beta)^2 - 3\alpha\beta) = -m$

$\Rightarrow -p(p^2 - 3q) = -m$

$\therefore m + n = p(p^2 - 3q) + q^3$

$= p^3 + q^3 - 3pq$

47. (*a*) $\alpha + \beta = a + b$ ...(*i*)

$\alpha\beta = ab - c$

$\Rightarrow ab = \alpha\beta + c$ ...(*ii*)

$\therefore$ Quadratic equation with roots a and b is,

$x^2 - (a + b)x + ab = 0$

From (i) and (ii),

$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta + c = 0$

$\Rightarrow x^2 - \alpha x - \beta x + \alpha\beta + c = 0$

48. (*c*) $x^2 - ax - bx - cx + bc + ca = 0$

$\Rightarrow x^2 - (a + b + c)x + bc + ca = 0$

For equal roots, $D = 0$

$\Rightarrow [-(a + b + c)]^2 - 4(bc + ca) = 0$

$\Rightarrow a^2 + b^2 + c^2 + 2ab - 2bc - 2ca = 0$

$\Rightarrow (a + b - c)^2 = 0$

$\therefore a + b - c = 0$

49. (*d*) $\because \alpha + \beta = 9, \alpha\beta = q$

$\therefore \alpha^2 - \beta^2 = 16 \Rightarrow (\alpha - \beta) = 2$

$\because (\alpha - \beta)^2 + 4\alpha\beta = (\alpha + \beta)^2$

$\Rightarrow 4 + 4q = 64 \Rightarrow q = 15$

50. (*d*) Let the quadratic equation be $ax^2 + bx + c = 0$ and roots be α and β.

$\alpha + \beta = \frac{-b}{a}$ and $\alpha . \beta = \frac{c}{a}$

Now, $\alpha + \beta = \alpha\beta$

$\Rightarrow -\frac{b}{a} = \frac{c}{a} \Rightarrow b + c = 0$

$\therefore$ There are infinite possible values for which $b + c = 0$. So, infinitely many quadratic equations are possible.

51. (*a*) p, q are the roots of $x^2 + bx + c = 0$

$p + q = -b$ and $pq = c$

Now, $p^2 + q^2 - 11pq = 0$

$\Rightarrow (p - q)^2 - 9pq = 0$

$\Rightarrow p - q = 3\sqrt{c}$

52. (*c*) Statement 1: $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow 2 = -2\alpha\beta$

$\Rightarrow \alpha = -1/\beta$

64. (*d*) For absolute value all the given statements are true.

65. (*d*) $|x-3|^2+|x-3|-2=0$

Let $|x-3|=t$

$\therefore t^2+t-2=0 \Rightarrow t^2+2t-t-2=0$

$\Rightarrow (t+2)(t-1)=0$

$\Rightarrow t=-2$ or $t=1$

Since t is modulus of a number, it cannot be negative.

$\therefore t=1 \Rightarrow |x-3|=1$

$\Rightarrow x-3=1$ or $x-3=-1$

$\Rightarrow x=4$ or 2

Sum of roots $= 4+2=6$.

66. (*a*) $|1-x|+x^2=5$

First case: When $x<1$, $|1-x|$ is positive.

$\therefore 1-x+x^2=5$

$\Rightarrow x^2-x-4=0$

Roots are $\frac{1\pm\sqrt{17}}{2}$

Since, $x<1$, root cannot be $\frac{1+\sqrt{17}}{2}$. So, the root is $\frac{1-\sqrt{17}}{2}$, which is irrational.

Second case: If $x>1$, $|1-x|=-1+x$

$\therefore |1-x|+x^2=5$

$\Rightarrow x-1+x^2=5$

$\Rightarrow x^2+x-6=0$

$\Rightarrow x^2+3x-2x-6=0$

$\Rightarrow x=2,-3$

Since $x>1$, root cannot be -3. So, root is 2 which is rational.

$\therefore$ Given expression has one irrational root and One rational root.

67. (*b*) $x^2-4x+[x]=0$

Case 1: Let $0\le x<1 \quad \Rightarrow [x]=0$

$\therefore x^2-4x+0=0 \Rightarrow x(x-4)=0$

$\Rightarrow x=0, x=4$

$x=4$ can't be taken in $0\le x<1$

$\therefore x=0$

Case 2: Let $1\le x<2 \quad \Rightarrow [x]=1$

$\therefore x^2-4x+1=0$

roots are $2\pm\sqrt{3}$

In interval $1\le x<2$, $2\pm\sqrt{3}$ are not the roots.

Case 3: Let $x=2 \quad \Rightarrow [x]=2$

$\therefore x^2-4x+2=0$

roots are $\frac{4\pm\sqrt{16-8}}{2}=2\pm\sqrt{2}$

Since, $x=2$, roots can't be $2\pm\sqrt{2}$

$\therefore$ There is only one solution, i.e., $x=0$

68. (*b*) A rational expression is nothing more than a fraction in which the numerator and denominator are polynomials.

1, 3, 4 and 5 are rational expression.

69. (*d*) It is given that $|x^2-x-6|=x+2$

$|x^2-x-6|=|(x-3)(x+2)|$

$|(x-3)(x+2)|=\{(x-3)(x+2)$ if $(x-3)(x+2)\ge 0\}$

$|(x-3)(x+2)|=-(x-3)(x+2)$ if $(x-3)(x+2)<0$

Case 1: $(x-3)(x+2)\ge 0$ then,

$x\in(-\infty,-2]\cup[3,\infty)$

and $x^2-x-6=x+2$

$\Rightarrow x^2-2x-8=0$

$\Rightarrow (x-4)(x+2)=0$

$\Rightarrow x=4,-2\in(-\infty,-2][3,\infty)$

Therefore the roots of quadratic equation are 4 and -2

Case 2: $(x-3)(x+2)<0$ then $x\in[-2,3]$

and $-(x^2-x-6)=x+2$

$\Rightarrow x^2-4=0$

$\Rightarrow (x-2)(x+2)=0$

$\Rightarrow x=2,-2\in[-2,3]$

Therefore, the roots of quadratic equation are 2 and -2.

70. (*a*) Given:

$x^2+9|x|+20=0$

Case- 1: if $x>0$

$\Rightarrow x^2+9x+20=0$

$\Rightarrow (x+4)(x+5)=0$

$\Rightarrow x=-4,-5$

so, F(x) has no real roots for this case

Case-2 if $x<0$

$\Rightarrow x^2-9x+20=0$

$\Rightarrow (x-4)(x-5)=0$

$\Rightarrow x=4$ or 5

$\because x<0$

so, F(x) has no real root for this case

Therefore, no solution.

71. (*c*) $|x^2-3x+2|>x^2-3x+2$

For $x^2-3x+2\ge 0$

$x^2-3x+2>x^2-3x+2$

No real value of x satisfies

For $x^2-3x+2<0$

$-(x^2-3x+2)>x^2-3x+2$

$x^2-3x+2<0$

$(x-1)(x-2)<0$

$1<x<2$

72. (*c*) We have, $x\le 4, y\ge 0$

$x\le -4, y\le 0$

Possible value of x and y

$x=\{4,3,2,1,0,-1,-2,-3,-4,-5\ldots\}$

$y=\{0,1,2,3,4\}$ …(i)

And $x=\{-4,-5,-6,-7\ldots\}$

$y=\{0,-1,-2,-3,-4\ldots\}$ …(ii)

From intersection of (i) and (ii), we get $x\le -4, y=0$

73. (*a*) If $x<0$ then $|x|=-x$ and if $x>0$ then $|x|=x$

For $x<0$

$x^2+3(-x)+2=0$

$(x-1)(x-2)=0$

$x=1,2$

But for condition $x<0$, $x=1,2$ does not satisfy.

For $x>0$

$x^2+3(x)+2=0$

$(x+1)(x+2)=0$

$x=-1,-2$

But for condition $x>0$, $x=-1,-2$ does not satisfy.

Therefore, there are no real roots.

74. (*d*) It is given that: $1.5\le x\le 4.5$

So, the critical points will be: $x=1.5=\frac{3}{2}$ or

$x=4.5=\frac{9}{2}$

Further, we get: $2x-3=0$ or $2x-9=0$

Now, changing the inequality to equality form, we get:

+ve | $x=3/2$ | −ve | $x=9/2$ | +ve

The wavy curve of the values $(2x-3)(2x-9)$ are seen negative.

$(2x-3)(2x-9)\le 0$

75. (*c*) Given: $f(x)=3x^2-5x+p$

$\therefore f(0)=3(0)-5(0)+p=p$

Now, $f(1)=3(1)^2-5(1)+p=-2+p$

Since the numbers are opposite in sign,

$f(0)\times f(1)<0$

$p\times(-2+p)<0$

$p\times(p-2)<0$

$\therefore p\in(0,2)$

76. (*c*) **Hint:** Let $x^2=y$, After solving quadratic $x=\pm 3,\pm 1$

$\therefore$ sum $=8$

77. (*d*) $x=0, y=2 \Rightarrow (0,2)$

$x=-2, y=0 \Rightarrow (-2,0)$