13. AC Circuits

Can you recall?

1. What is Faraday's laws of induction?
2. What is induced current?
3. What is an AC generator?
4. What are alternating current and direct current?
5. How is the emf generated in an AC Generator.

13.1 Introduction:

In school you have learnt that there are two types of supplies of electricity:

(i) DC, the direct current which has fixed polarity of voltage (the positive and negative ends of the power supply are fixed).

(ii) AC, the alternating current for which the polarity of the voltage keeps changing periodically.

We have studied the generation of AC voltage in the previous chapter. Because of low cost and convenience of transport, the electricity is mostly supplied as AC. Some of the appliances that we use at home or offices like TV, computer, transistor, radio, etc. convert AC to DC by using a device like rectifier (which you will study in chapter 16) before using it. However, there are some domestic devices like fan, fridge, air conditioner, induction heater, coil heater, etc., which

Remember this

AC shock is attractive, while DC shock is repulsive, so 220V AC is more dangerous than 220V DC. Also 220 V AC has a peak value $E_0 = \pm \sqrt{2}\, E_{rms} = \pm 1.414 \times 220$ V = ± 311 V but DC has fixed value of 220 V only.

run directly on AC. Almost all these devices use components like an inductor and a capacitor. In this chapter we will study the passage of AC through resistors, inductors and capacitors.

13.2 AC Generator:

In the last chapter are have studied that the source of AC (generator) produces a time dependent emf (e) given by

$$e = e_0 \sin \omega t \qquad \text{--- (13.1)}$$

Where e_0 is the peak value of emf and ω is the angular frequency of rotation of the coil in the AC generator.

As the time variation of current is similar to that of emf, the current in a circuit connected to this generator will be of the form

$$i = i_0 \sin(\omega t + \alpha)$$

where α represent the phase difference between the current (i) and the emf, and i_0 is the peak value of current. From Eq (13.1) it can be seen that the induced emf varies sinusoidally with time as shown in Fig. 13.1 below.

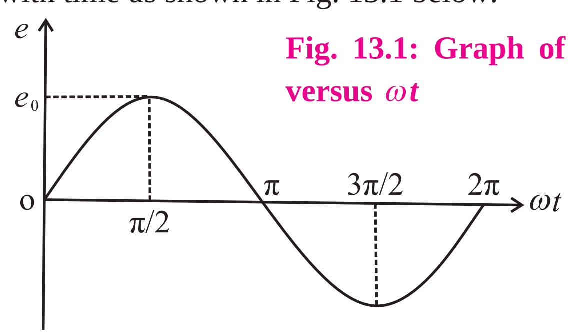


Fig. 13.1: Graph of e versus ωt

From the graph it can be seen that the direction of the emf is reversed after every half revolution of the coil. This type of emf is called the alternating emf and the corresponding current is called alternating current.

13.3. Average and RMS values:

Alternating voltages and current go through all values between zero and the peak value in one cycle.

Peak value: Peak value of an alternating current (or emf) is the maximum value of the current (or emf) in either direction.

We define some specific values which would be convenient for comparing voltage or current waveforms.

a) Average value of AC:

This is the average of all values of the voltage (or current) over one half cycle. As can be seen in Fig. 13.1, the average over a full cycle is always zero since the average value of $\sin\omega t$ over a cycle is zero. So the mean value of AC over a cycle has no significance and the mean value of AC is defined as the average over half cycle.

Average value of $\sin\theta$ for θ in the range 0^c to π^c

$$<\sin\theta> = \frac{\int_0^{\pi}\sin\theta d\theta}{\int_0^{\pi} d\theta} = \frac{[-\cos\theta]_0^{\pi}}{[\theta]_0^{\pi}}$$

$$= \frac{2}{\pi} = 0.637$$

Therefore, average value of AC current or emf = 0.637 × their peak value :

i.e., $i_{av} = 0.637\, i_0$ and $e_{av} = 0.637\, e_0$

where i_{av} and e_{av} are the average values of alternating current and emf (voltage) respectively.

b) RMS value:

A moving coil ammeter and voltmeter measure the average value of current and voltage applied across it respectively. It is obvious, therefore, that the moving coil instruments cannot be used to measure the alternating current and voltages. Hence in order to measure these quantities it is necessary to make use of a property which does not depend upon the changes in direction of alternating current or voltage. Heating effect depends upon the square of the current (the square of the current is always positive) and hence does not depend upon the direction of flow of current. Consider an alternating current of peak value i_0, flowing through a resistance R. Let H be the heat produced in time t. Now the same quantity of heat (H) can be produced in the same resistance (R) in the same time (t) by passing a steady current of constant magnitude through it. The value of such steady current is called the effective value or virtual value or rms value of the given alternating current and is denoted by i_{rms}. The relation between the rms value and peak value of alternating current is given by

$$i_{rms}^2 = \frac{\int_0^{2\pi} i^2 d\theta}{2\pi} = \frac{1}{2\pi}\int_0^{2\pi} i_0^2 \sin^2\theta d\theta$$

$$= \frac{i_0^2}{2\pi}\int_0^{2\pi}\frac{(1-\cos 2\theta)}{2} d\theta$$

$$= \frac{i_0^2}{2\times 2\pi}\left[\left(\theta - \frac{(\sin 2\theta)}{2}\right)\right]_0^{2\pi}$$

$$= \frac{i_0^2}{2}$$

$$\therefore i_{rms} = \frac{i_0}{\sqrt{2}} = 0.707\, i_0$$

Similarly it can be shown that

$$e_{rms} = \frac{e_0}{\sqrt{2}} = 0.707\, e_0$$

The heat produced by a sinusoidally varying AC over a complete cycle will be given by

$$H = \int_0^{2\pi/\omega} i^2(t) R dt$$

$$= \frac{R}{\omega}\int_0^{2\pi/\omega} i^2(\omega t)\, d(\omega t)$$

$$= \frac{2\pi R}{\omega}\frac{i_0^2}{2}$$

$$H = R(i_{rms})^2 \cdot \frac{2\pi}{\omega}\left(\because i_{rms} = \frac{i_0}{\sqrt{2}}\right)$$

It is the same as the heat produced by a DC current of magnitude i_{rms} for time $t = \frac{2\pi}{\omega}$.

Example 13.1: An alternating voltage is given by e = 6 sin (100π t). find (i) the peak value (ii) frequency (iii) time period and (iv) instantaneous value at time t = 2 ms

Solution:

$e = e_0 \sin\omega t$

e = 6 sin (100π t)

(i) Comparing the two equations, the peak value of the alternating voltage is $e_0 = 6$ V

(ii) $\omega t = 100\pi t$ ∴ $2\pi f t = 100\pi t$

$$\text{Frequency } f = \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

(iii) Time period $T = \frac{1}{f} = \frac{1}{50} = 0.02\ s$

(iv) The instantaneous value of the voltage

At $t = 2 \times 10^{-3}$ s is $e = 6 \sin 100\pi \times 2 \times 10^{-3}$
$= 3.527$ V

13.4 Phasors:

The study of AC circuits is much simplified, if we represent alternating current and alternating emf as rotating vectors with the angle between them equal to the phase difference between the current and emf. These rotating vectors are called phasors.

A rotating vector that represents a quantity varying sinusoidally with time is called a **phasor** and the diagram representing it is called **phasor** diagram.

The phasor for alternating emf and alternating current are inclined to the horizontal axis at angle ωt or ωt + α. and rotate in anticlockwise direction. The length of the arrow represents the maximum value of the quantity (i_0 and e_0).

The projection of the vector on a fixed axis gives the instantaneous value of alternating current and alternating emf. In sine form, $i = i_0 \sin \omega t$ or $e = e_0 \sin \omega t$ projection is taken on Y-axis as shown in Fig. 13.2 (a). In cosine form $i = i_0$ cost ωt or $e = e_0 \cos \omega t$ projection is taken on X-axis as shown in Fig. 13.2 (b).

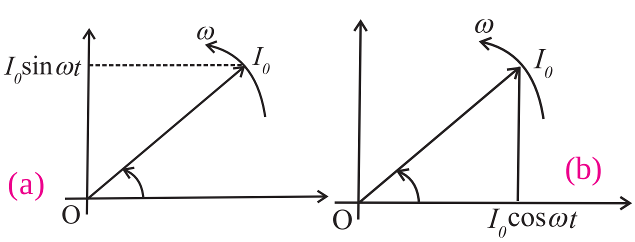


Fig. 13.2 (a) and (b): Phasor diagrams.

The representation of the harmonically varying quantities as rotating vectors enables us to use the laws of vector addition for adding these quantities.

13.5 Different Types of AC Circuits:

In this section we will derive voltage current relations for individual as well as combined circuit elements like resistors, inductors and capacitors, carrying a sinusoidal current. We assume the capacitor and inductor to be ideal unless otherwise specified.

(a) AC voltage applied to a resistor:

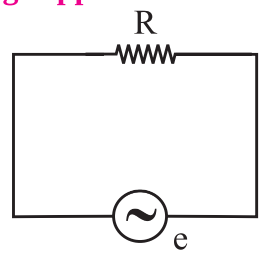


Fig.13.3 An AC voltage applied to a resistor.

Suppose a resistor of resistance R is connected to an AC source of emf with instantaneous value e given by

$$e = e_0 \sin \omega t \qquad \text{--- (13.2)}$$

Where e_0 is the peak value of the voltage and ω is its angular frequency. Let e be the potential drop across the resistance.

$$\therefore e = iR \qquad \text{--- (13.3)}$$

∵ instantaneous emf = instantaneous value of potential drop

From Eq (13.2) and Eq (13.3) we have,

$$iR = e = e_0 \sin \omega t$$

$$\therefore i = \frac{e}{R} = \frac{e_0 \sin \omega t}{R}$$

$$\therefore i = i_0 \sin \omega t \quad \left(\because i_0 = \frac{e_0}{R}\right) \qquad \text{--- (13.4)}$$

Comparing $i_0 = \frac{e_0}{R}$ with Ohm's law, we find that resistors behave similarly for both AC and DC voltage. Hence the behaviour of R in DC and AC circuits is the same. R can reduce DC as well as AC equally effectively.

From Eq (13.2) and Eq (13.4) we know that for a resistor there is zero phase difference between instantaneous alternating current and instantaneous alternating emf, i.e., they are in phase. Both e and i reach zero, minimum and maximum values at the same time as shown in Fig. 13.4.

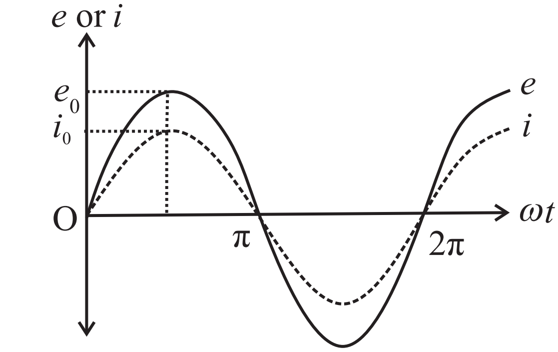


Fig. 13.4 Graph of *e* and *i* versus ωt.

Phasor diagram:

In the AC circuit containing *R* only, current and voltage are in the same phase, hence both phasors for *i* and for *e* are in the same direction making an angle ωt with OX. Their projections on vertical axis give their instantaneous values. The phase angle between alternating current and alternating voltage through *R* is zero as shown in Fig. 13.5.

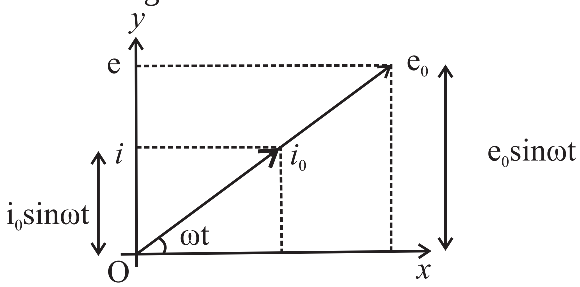


Fig. 13.5 Phasor diagram for a purely resistive load.

Example 13.2: An alternating voltage given by e = 140 sin 3142 t is connected across a pure resistor of 50 Ω. Find (i) the frequency of the source (ii) the rms current through the resistor.

Solution: Given

$e = 140 \sin 3142\, t$

$R = 50\ \Omega$

i) On comparing with $e = e_0 \sin \omega t$

We get

$\omega = 3142,\ e_0 = 140$ V

$\omega = 2\pi f \ \therefore f = \dfrac{\omega}{2\pi} = \dfrac{3142}{2\times3.142} = 500$ Hz

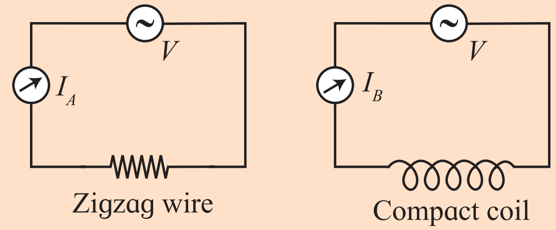
(ii) $e_0 = 140$ V

$e_{rms} = \dfrac{e_0}{\sqrt{2}} = \dfrac{140}{\sqrt{2}} = 98.99$ V

$\therefore i_{rms} = \dfrac{e_{rms}}{R} = \dfrac{99.29}{50} = 1.98$ A

Activity

Take 2 identical thin insulated copper wires about 10 cm long, imagine one of them in a zigzag form (called A) and the other in the form of a compact coil of average diameter not more than 5 cm (called B). Connect the two independently to 1.5 V cell or to a similar DC voltage and record the respective current passing through them as I_A and I_B. You will notice that the two are the same.



(b) AC voltage applied to an Inductor:

Let us now connect the source of alternating emf to a circuit containing pure inductor (*L*) only as shown in Fig. 13.6. Let us assume that the inductor has negligible resistance. The circuit is therefore a purely inductive circuit. Suppose the alternating emf supplied is represented by

$e = e_0 \sin \omega t$ --- (13.5)

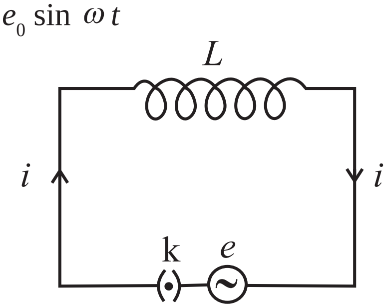


Fig. 13.6: An AC source connected to an Inductor.

When the key k is closed, current *i* begins to grow in the inductor because magnetic flux linked with it changes and induced emf is produced which opposes the applied emf (Faraday's law).

According to Lenz's law

$e = -L\dfrac{di}{dt}$ --- (13.6)

Where e is the induced emf and $\frac{di}{dt}$ is the rate of change of current.

To maintain the flow of current in the circuit, applied emf (e) must be equal and opposite to the induced emf (e'). According to Kirchhoff's voltage law, as there is no resistance in the circuit,

$e = -e'$

$$\therefore e = -\left(-L\frac{di}{dt}\right) = L\frac{di}{dt} \quad \text{(from Eq. (13.6))}$$

$$\therefore di = \frac{e}{L}dt$$

Integrating the above equation on both the sides, we get,

$$\int di = \int \frac{e}{L}dt$$

$$i = \int \frac{e_0 \sin \omega t}{L}dt \qquad (\because e = e_0 \sin \omega t)$$

$$i = \frac{e_0}{L}\left[\frac{-\cos \omega t}{\omega}\right] + \text{constant}$$

Constant of integration is time independent and has the dimensions of i. As the emf oscillates about zero, i also oscillates about zero so that there cannot be any component of current which is time independent.

Thus, the integration constant is zero

$$\therefore i = \frac{-e_0}{\omega L}\sin\left(\frac{\pi}{2} - \omega t\right) \quad \left(\because \sin\left(\frac{\pi}{2} - \omega t\right) = \cos \omega t\right)$$

$$\therefore i = \frac{e_0}{\omega L}\sin\left[\omega t - \frac{\pi}{2}\right]$$

$$i = i_0 \sin\left[\omega t - \frac{\pi}{2}\right] \qquad \text{--- (13.7)}$$

where $i_0 = \frac{e_0}{\omega L}$ --- (13.8)

where i_0 is the peak value of current. Eq. (13.7) gives the alternating current developed in a purely inductive circuit when connected to a source of alternating emf.

Comparing Eq. (13.5) and (13.7) we find that the alternating current i lags behind the alternating emf e by a phase angle of $\pi/2$ radian (90°) or the voltage across L leads the current by a phase angle of $\pi/2$ radian (90°) as shown in Fig. 13.7.

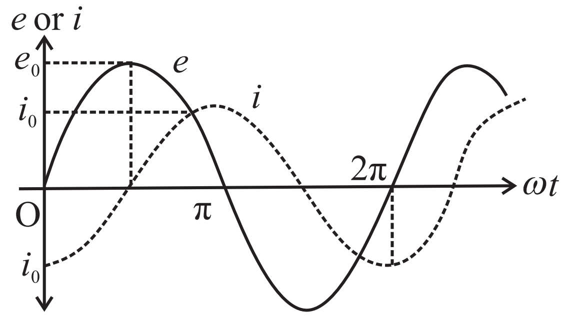


Fig 13.7: Graph of e and i versus ωt.

Phasor diagram:

The phasor representing peak emf e_0 makes an angle ωt in anticlockwise direction from horizontal axis. As current lags behind the voltage by 90°, the phasor representing i_0 is turned clockwise with the direction of e_0 as shown in Fig. 13.8.

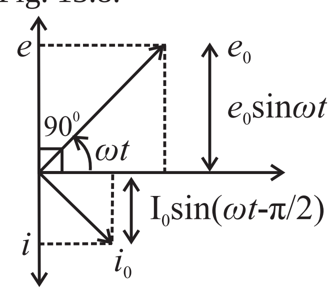


Fig. 13.8 Phasor diagram for purely inductive circuit.

Inductive Reactance (X_L):

The opposing nature of inductor to the flow of alternating current is called inductive reactance.

Comparing Eq (13.8) with Ohm's law, $i_0 = \frac{e_0}{R}$ we find that ωL represents the effective resistance offered by the inductance L, it is called the inductive reactance and denoted by X_L.

$\therefore X_L = \omega L = 2\pi f L$. ($\because \omega = 2\pi/T = 2\pi f$)

where f is the frequency of the AC supply.

The function of the inductive reactance is similar to that of the resistance in a purely resistive circuit. It is directly proportional to the inductance (L) and the frequency (f) of the alternating current.

The dimensions of inductive reactance is the same as those of resistance and its SI unit is ohm (Ω).

In DC circuits $f = 0 \therefore X_L = 0$

It implies that a pure inductor offers zero resistance to DC, i.e., it cannot reduce DC. Thus, its passes DC and blocks AC of very high frequency.

In an inductive circuit, the self induced emf opposes the growth as well as decay of current.

Example 13.3: An inductor of inductance 200 mH is connected to an AC source of peak emf 210 V and frequency 50 Hz. Calculate the peak current. What is the instantaneous voltage of the source when the current is at its peak value?

Solution: Given

L = 200 mH =0.2 H

e_0 = 210 V

f = 50 Hz

Peak Current $i_0 = \frac{e_0}{X_L} = \frac{e_0}{2\pi fL}$

$= \frac{210}{2 \times 3.142 \times 50 \times 0.2}$

$\therefore i_0$ = 3.342 A

As in an inductive AC circuit, current lags behind the emf by $\frac{\pi}{2}$, so the voltage is zero when the current is at its peak value.

c) AC voltage applied to a capacitor:

Let us consider a capacitor with capacitance C connected to an AC source with an emf having instantaneous value

$$e = e_0 \sin \omega t \quad \text{--- (13.9)}$$

This is shown in Fig. 13.9

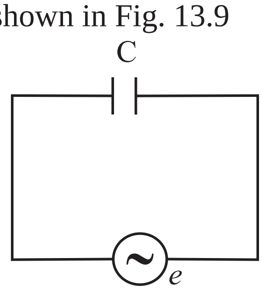


Fig: 13.9 An AC source connected to a capacitor.

The current flowing in the circuit transfers charge to the plates of the capacitor which produces a potential difference between the plates. As the current reverses its direction in each half cycle, the capacitor is alternately charged and discharged.

Suppose q is the charge on the capacitor at any given instant t. The potential difference across the plates of the capacitor is

$$V = \frac{q}{C} \text{ or } q = CV \quad \text{--- (13.10)}$$

The instantaneous value of current i in the circuit is

$$i = \frac{dq}{dt} = \frac{d}{dt}(Ce) \quad (\because V = e \text{ at every instant})$$

$$= \frac{d}{dt}(Ce_0 \sin \omega t) \quad (\because e = e_0 \sin \omega t)$$

$$= Ce_0 \cos \omega t \cdot \omega$$

$$= \frac{e_0}{1/\omega C} \cos \omega t$$

$$\therefore i = \frac{e_0}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right) \quad \left(\because \cos \omega t = \sin\left(\frac{\pi}{2} + \omega t\right)\right)$$

--- (13.11)

The current will be maximum when sin ($\omega t + \pi/2$) = 1, so that $i = i_0$ where, peak value of current is

$$i_0 = \frac{e_0}{1/\omega C} \quad \text{--- (13.12)}$$

$$\therefore i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{--- (13.13)}$$

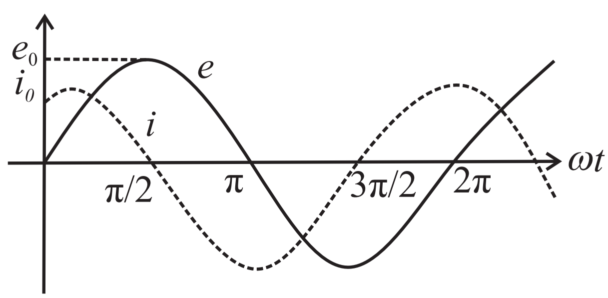


Fig. 13.10 Graph of e and i versus ωt.

From Eq. (13.9) and Eq. (13.13) we find that in an AC circuit containing a capacitor only, the alternating current i leads the alternating emf e by phase angle of π/2 radian as shown in Fig. 13.10.

Phasor diagram:

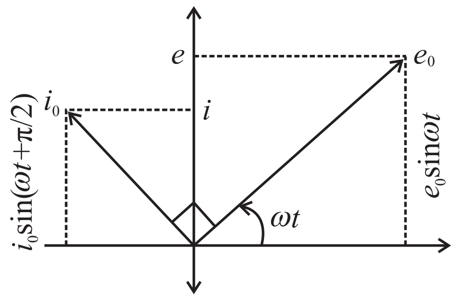


Fig.13.11: Phasor diagram for purely capacitive circuit.

The phasor representing peak emf makes an angle ωt in anticlockwise direction with respect to horizontal axis. As current leads the voltage by 90°, the phasor representing i_0 current is turned 90° anticlockwise with respect to the phasor representing emf e_0. The projections of these phasors on the vertical axis gives instantaneous values of *e* and *i*.

Capacitive Reactance: The instantaneous value of alternating current through a capacitor is given by

$$i = \frac{e_0}{(1/\omega C)}\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Comparing Eq. (13.12) with Ohm's law, $i_0 = \frac{e_0}{R}$ we find that $(1/\omega C)$ represents effective resistance offered by the capacitor called the capacitive reactance denoted by X_C.

$\therefore X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$ where f is the frequency of AC supply.

The function of capacitive reactance in a purely capacitive circuit is to limit the amplitude of the current similar to the resistance in a purely resistive circuit.

X_C varies inversely as the frequency of AC and also as the capacitance of the condenser.

In a DC circuit, $f = 0 \therefore X_C = \infty$

Thus, capacitor blocks DC and acts as open circuit while it passes AC of high frequency.

The dimensions of capacitive reactance are the same as that of resistance and its SI unit is ohm (Ω).

Table 13.1: Comparison between resistance and reactance.

Resistance	Reactance
Equally effective for AC and DC	Current is affected (reduced) but energy is not consumed (heat is not generated). The energy consumption by a coil is due to its resistive component.
Its value is independent of frequency of the AC	Inductive reactance ($X_L = 2\pi fL$) is directly proportional and capacitive reactance $\left(Xc = \frac{1}{2\pi fC}\right)$ is inversely proportional to the frequency of the AC.
Current opposed by a resistor is in phase with the voltage.	Current opposed by a pure inductor lags in phase while that opposed by a pure capacitor leads is phase by π^c over the voltage.

Example 13.4: 4. A Capacitor of 2 μF is connected to an AC source of emf $e = 250 \sin 100\pi t$. Write an equation for instantaneous current through the circuit and give reading of AC ammeter connected in the circuit.

Solution: Given

$C = 2\mu F = 2 \times 10^{-6} F$

$e_0 = 250$ V

$\omega = 100\pi$ rad/sec

The instantaneous current through the circuit

$i = i_0 \sin(\omega t + \frac{\pi}{2})$

$= \omega C e_0 \sin(\omega t + \frac{\pi}{2})$

$= 3.142 \times 2 \times 10^{-4} \times 250 \sin(100\pi t + \frac{\pi}{2})$

$= 0.1571 \sin(100\pi t + \frac{\pi}{2})$

Reading of the AC ammeter is

$i_{rms} = 0.707\ i_0$

$= 0.707 \times 0.1571$

$i_{ms} = 0.111A$

(d) AC circuit containing resistance inductance and capacitance in series (LCR circuit):

Above we have studied the opposition offered by a resistor, pure inductor and capacitor to the flow of AC current independently.

Now let us consider the total opposition offered by a resistor, pure inductor and capacitor connected in series with the alternating source of emf as shown in Fig. 13.12.

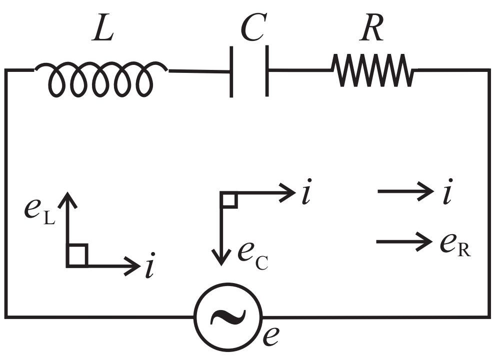


Fig. 13.12: Series LCR circuit.

Let a pure resistor *R*, a pure inductance *L* and an ideal capacitor of capactance *C* be connected in series to a source of alternative emf. As *R*, *L* and *C* are in series, the current at any instant through the three elements has the same amplitude and phase. Let it be represented by

$i = i_0 \sin \omega t.$

The voltage across each element bears a different phase relationship with the current. The voltages e_L, e_C and e_R are given by

$e_R = iR$, $e_L = iX_L$ and $e_C = iX_C$

As the voltage across the capacitor lags behind the alternating current by 90°, it is represented by $\overrightarrow{OC}$, rotated clockwise through 90° from the direction of $\vec{i}_0$. $\overrightarrow{OC}$ is along OY′ in the phasor diagram shown in the phasor diagrams in Fig. 13.13.

As e_R is in phase with current i_0 the vector e_R is drawn in the same direction as that of *i*, along the positive direction of *X*-axis represented by $\overrightarrow{OA}$. The voltage across *L* and *C* have a phase different of 180° hence the net reactive voltage is $(e_L - e_C)$.

Assuming $e_L > e_C$ represented by OB′ in the figure.

The resultant of $\overrightarrow{OA}$ and $\overrightarrow{OB'}$ is the diagonal OK of the rectangle OAKB'

$$\therefore OK = \sqrt{OA^2 + OB^2}$$

$$e_0 = \sqrt{e_R^{\ 2} + (e_L - e_C)^2}$$

$$= \sqrt{(i_0 R)^2 + (i_0 X_L - i_0 X_C)^2}$$

$$e_0 = i_0\sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore \frac{e_0}{i_0} = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\frac{e_0}{i_0} = Z$$

Comparing the above equation with the relation $\frac{V}{i} = R$, the quantity $\sqrt{R^2 + (X_L - X_C)}$ represents the effective opposition offered by the inductor, capacitor and resistor connected in series to the flow of AC current. This total effective resistance of LCR circuit is called the impedance of the circuit and is represented by *Z*. The reciprocal of impedance of an AC circuit is called admittance. Its SI unit is ohm^{-1} or siemens.

It can be defined as the ratio of rms voltage to the rms value of current Impedance is expressed in ohm (Ω).

Phasor diagram:

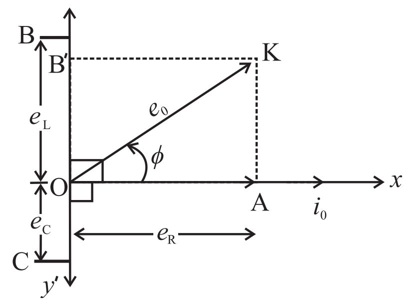


Fig. 13.13: Phasor diagram for an LCR circuit.

From the phasor diagram (Fig. 13.13) it can be seen that in an AC circuit containing L, C and R, the voltage leads the current by a phase angle ϕ,

$$\tan\phi = \frac{AK}{OA} = \frac{OB'}{OA} = \frac{e_L - e_C}{e_R} = \frac{i_o X_L - i_o X_C}{i_o R}$$

$$\tan\phi = \frac{X_L - X_C}{R} \quad \therefore \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$\therefore$ The alternating current in LCR circuit would be represented by

$i = i_0 \sin(\omega t + \phi)$

and $e = e_0 \sin(\omega t + \phi)$

We can now discuss three cases based on the above discussion.

(i) When $X_L = X_C$ then tan ϕ = 0.
Hence voltage and current are in phase. Thus the AC circuit is non inductive.

(ii) When $X_L > X_C$, tan ϕ is positive $\therefore$ ϕ is positive.
Hence voltage leads the current by a phase angle ϕ The AC circuit is inductance dominated circuit.

(iii) When $X_L < X_C$, tan ϕ is negative $\therefore$ ϕ is negative.
Hence voltage lags the current by a phase angle ϕ. The AC circuit is capacitance dominated circuit.

Impedance triangle:

From the three phasors

$\vec{e}_R = \vec{i}_0 R,\ \vec{e}_L = \vec{i}_0 X_L,\ \vec{e}_C = \vec{i}_0 X_C$

we obtain the impedance triangle as shown in Fig 13.14.

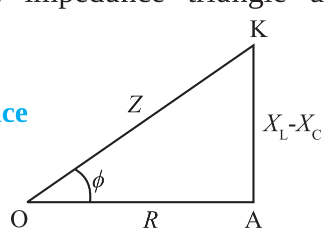


Fig. 13.14: Impedance triangle.

The diagonal OK represents the impedance Z of the AC circuit.

$Z = \sqrt{R^2 + (X_L - X_C)^2}$, the base OA represents the Ohmic resistance R and the perpendicular AK represents reactance (X_L-X_C). $\angle$AOK = ϕ, is the phase angle by which the voltage leads the current is the circuit, where $\tan\phi = \frac{X_L - X_C}{R}$

Example 13.5: A 100mH inductor, a 25 µF capacitor and a 15 Ω resistor are connected in series to a 120 V, 50 Hz AC source. Calculate

(i) impedance of the circuit at resonance
(ii) current at resonance
(iii) resonant frequency

Solution: Given

L = 100 mH = 10^{-1}H

C = 25 µF = 25 x 10^{-6}F

R = 15Ω

e_{rms}=120 V

f = 50 Hz

(i) At resonance, $Z = R$ = 15Ω

(ii) $i_{rms} = \frac{e_{rms}}{R} = \frac{120}{15} = 8A$

(iii) $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times3.142\sqrt{10^{-1}\times25\times10^{-6}}}$

$= \frac{1}{9.9356\times10^{-3}}$

$\therefore f$ = 100.6 Hz

Example 13.6: A coil of 0.01H inductance and 1Ω resistance is connected to 200V, 50Hz AC supply. Find the impedance of the circuit and time lag between maximum alternating voltage and current.

Solution: Given

Inductance L = 0.01H

Resistance R = 1Ω

e_0 = 200 V

Frequency f = 50 Hz

Impedance of the circuit $Z = \sqrt{R^2 + X_L^2}$

$= \sqrt{R^2 + (2\pi fL)^2}$

$= \sqrt{(1)^2 + (2\times3.142\times50\times0.01)^2}$

$= \sqrt{10.872}$ = 3.297Ω

$\tan\phi = \frac{\omega L}{R} = \frac{2\pi fL}{R} = \frac{2\times3.142\times50\times0.01}{1}$

= 3.142

$\phi = tan^{-1}(3.142) = 72.35°$

Phase difference, $\phi = \frac{72.35\times\pi}{180}$ rad

Time lag between maximum alternating voltage and current

$\Delta t = \frac{\phi}{\omega} = \frac{72.35\times\pi}{180\times2\pi\times50} = 4.019\ s$

13.6 Power in AC circuit:

We know that power is defined as the rate of doing work. For a DC circuit, power is measured as a product of voltage and current. But since in an AC circuit the values of current and voltage change at every instant the power in an AC circuit at a given instant is the product of instantaneous voltage and instantaneous current.

a) Average power associated with resistance (power in AC circuit with resistance).

In a pure resistor, the alternating current developed is in phase with the alternating voltage applied i.e. when $e = e_0 \sin \omega t$

then $i = i_0 \sin \omega t$

Now instantaneous power $P = ei$.

$P = (e_0 \sin \omega t)(i_0 \sin \omega t)$

$= e_0 i_0 \sin^2 \omega t$ --- (13.14)

The instantaneous power varies with time, hence we consider the average power for a complete cycle by integrating Eq. (13.14).

$$\therefore P_{av} = \frac{\left[\begin{array}{c}\text{work done by the emf on the} \\ \text{charges in one cycle}\end{array}\right]}{\text{time for one cycle}}$$

$$= \frac{\int_0^T P dt}{T} = \frac{\int_0^T e_0 i_0 \sin^2 \omega t \, dt}{T}$$

$$= \frac{e_0 i_0}{T} \int_0^T \sin^2 \omega t \, dt$$

$$= \frac{e_0 i_0}{T}\left(\frac{T}{2}\right) \quad \left[\because \int_0^T \sin^2 \omega t \, dt = \frac{T}{2}\right]$$

$$= \frac{e_0}{\sqrt{2}} \frac{i_0}{\sqrt{2}} \therefore P = P_{av} = e_{rms} \times i_{rms} \quad \text{--- (13.15)}$$

P is also called as apparent power.

Example 13.7: A 100Ω resistor is connected to a 220V rms, 50Hz supply

(i) What is the rms value of current in the circuit?

(ii) What is the net power consumed over a full cycle?

Solution: Given

$R = 100\Omega$, $e_{rms} = 220V$, $f = 50$ Hz

(i) $i_{rms} = \frac{e_{rms}}{R} = \frac{220}{100} = 2.2 A$

(ii) Net Power Consumed

$P_{av} = e_{rms} . i_{rms}$

$= 220 \times 2.2 = 484 \ W$

b) Average power associated with an inductor:

In an purely inductive circuit, the current lags behind the voltage by a phase angle of π/2. i.e., when $e = e_0 \sin \omega t$ then

$i = i_0 \sin(\omega t - \pi/2)$.

Now, instantaneous power $P = ei$

$P = (e_0 \sin \omega t)(i_0 \sin(\omega t - \pi/2))$

$= - e_0 i_0 \sin \omega t \cos \omega t$

$= - e_0 i_0 \sin \omega t \cos \omega t$

$$\therefore P_{av} = \frac{\text{work done in one cycle}}{\text{time for one cycle}}$$

$$= \frac{\int_0^T P dt}{T} = \frac{\int_0^T -e_0 i_0 \sin \omega t \cos \omega t \, dt}{T}$$

$$= \frac{-\frac{e_0 i_0}{2} \int_0^T 2 \sin \omega t \cos \omega t \, dt}{T}$$

$$= -\frac{e_0 i_0}{2} \int_0^T \sin 2\omega t \, dt$$

$$= -\frac{e_0 i_0}{2T}\left[\left(-\frac{\cos 2\omega t}{2\omega}\right)\right]_0^T$$

$$P_{av} = 0$$

∴ average power over a complete cycle of AC through an ideal inductor is zero.

c) Average power associated with a capacitor:

In a purely capacitive circuit the current leads the emf by a phase angle of π/2 ie when $e = e_0 \sin \omega t$ then $i = i_0 \sin(\omega t + \pi/2)$

$i = i_0 \cos \omega t$

Now, instantaneous power P = ei

$= (e_0 \sin \omega t)(i_0 \cos \omega t)$

$= e_0 i_0 \sin \omega t \cos \omega t$.

$$\therefore P_{av} = \frac{\text{work done in one cycle}}{\text{time for one cycle}}$$

$$P_{av} = \frac{\int_0^T P dt}{T} = \frac{\int_0^T e_0 i_0 \sin\omega t \cos\omega t}{T}$$

$$P_{av} = 0 \left(\text{as shown above}\right)$$

Average power supplied to an ideal capacitor by the source over a complete cycle of AC is also zero.

d) Average power in LCR Circuit:

Let $e = e_0 \sin \omega t$ be the alternating emf applied across the series combination of pure inductor, capacitor and resistor as shown in Fig. 13.15.

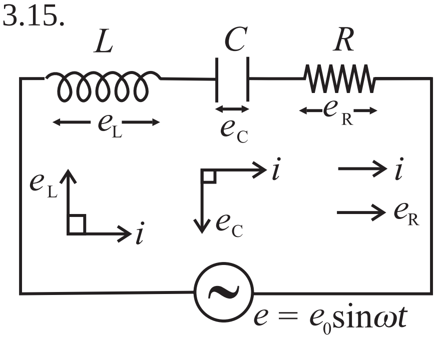


Fig. 13.15: LCR series circuit.

There is a phase difference ϕ between the applied emf and current given by

$i = i_0 \sin(\omega t \pm \phi)$

Instantaneous power is given by

$P = ei$

$= (e_0 \sin \omega t)\, i_0 \sin(\omega t \pm \phi)$

$= e_0\, i_0 [\sin \omega t \cos \phi \pm \cos\omega t \sin\phi] \sin \omega t$

$= e_0\, i_0 [\sin^2 \omega t \cos \phi \pm \cos \omega t \sin\phi \sin \omega]$

$\therefore$ Average power

$$P_{av} = \frac{\text{work done in one cycle}}{\text{time for one cycle}}$$

$$= \frac{\int_0^T P dt}{T}$$

$$= \frac{\int_0^T e_0 i_0 \left[\sin^2 \omega t \cos\phi \pm \cos\omega t \sin\omega t \sin\phi\right] dt}{T}$$

$$= \frac{e_0 i_0}{T}\left[\cos\phi dt \int_0^T \sin^2 \omega t\right] \pm \left(\sin\phi \int_0^T \cos\omega t \sin\omega t\ dt\right) \quad \text{--- (13.16)}$$

As seen above,

$$= \int_0^T \sin^2 \omega t\ dt = T/2$$

$$\text{and} \int_0^T \cos\omega t \sin\omega t\ dt = 0 \quad \text{--- (13.17)}$$

substituting (13.17) in (13.16)

$$P_{av} = \frac{e_0 i_0}{T}\left[\left(\cos\phi . \frac{T}{2}\right) \pm \left(\sin\phi\,(0)\right)\right]$$

$$= \frac{e_0 i_0}{T} \cdot \cos\phi . \frac{T}{2}$$

$$P_{av} = e_{rms}\, i_{rms} \cos\phi \quad \text{---(13.18)}$$

This power (P_{av}) is also called as true power. The average power dissipated in the AC circuit of inductor. Capacitor and resistor connected in series not only depends on rms values of current and emf but also on the phase difference ϕ between them.

The factor $\cos\phi$ is called as power factor

$$\therefore \text{Power factor } (\cos\phi) = \frac{\text{true power } (P)}{\text{apparent power } P_{av}}$$

$$= \frac{R}{\sqrt{R^2 + \left(X_L - X_C\right)^2}} \left[\begin{array}{l}\text{from} \\ \text{impedance} \\ \text{triangle}\end{array}\right]$$

$$\therefore \text{Power factor } \cos\phi = \frac{R}{Z} = \frac{\text{resistance}}{\text{impedance}}$$

In a non inductive circuit $X_L = X_C$

$$\therefore \text{Power factor } (\cos\phi) = \frac{R}{\sqrt{R^2}}$$

$$= \frac{R}{R} = 1 \therefore \phi = 0$$

In a purely inductive and capacitive circuit; $\phi = 90°$

$\therefore$ Power factor = 0

Average power consumed in a pure inductor or ideal capacitor $P_{av} = e_{rms}\ i_{rms} \cos 90° =$ zero.

$\therefore$ Current through pure inductor or ideal capacitor which consumes no power for its maintenance, in the circuit is called idle current or wattless current. Power dissipated in a circuit is due to resistance only.

Example 13.8: A sinusoidal voltage of peak value 283 V and frequency 50 Hz is applied to a series LCR circuit in which $R = 3\Omega$, $L = 25.48$ mH and $C = 796$ μf. Find

i) The impedance of the circuit

ii) The phase difference between the voltage across source and the currents

iii) The power factor

iv) The power dissipated in the surface

Solution: Given

$e_0 = 283$ V, $f = 50$ Hz, $R = 3\Omega$,

$L = 25.48 \times 10^{-3}$ H, $C = 796 \times 10^{-6}$ F

$X_L = 2\varpi fL = 2 \times 3.142 \times 50 \times 25.48 \times 10^{-3}$

$= 8\Omega$

$$X_C = \frac{1}{2\pi fL} = \frac{1}{2\times 3.142\times 50\times 796\times 10^{-6}}$$

$$= \frac{1}{0.2501} = 4\Omega$$

Therefore $Z = \sqrt{R^2 + (X_c - Y_c)^2}$

$$= \sqrt{3^2 + (8-4)^2} = 5\ \Omega$$

Phase difference ϕ is given by

$$\tan\phi = \frac{X_L - X_c}{R} = \frac{8-4}{3} = \frac{4}{3}$$

Therefore, $\phi = \tan^{-1}(\frac{4}{3}) = 53.1^0$

Thus the current lags behind the voltage across the source by a phase angle of 53.1^0

Power factor = $\cos\phi = 0.6$

Power dissipated in the circuit

$P_{av} = e_{rms}\, i_{rms} \cos\phi$

$$= \frac{e_0}{\sqrt{2}} \frac{e_0}{\sqrt{2}R} (0.6)$$

$$= \frac{283}{\sqrt{2}} \frac{283}{\sqrt{2}\times 3} 0.6$$

$= 8008.9\ W$

13.7 LC Oscillations:

We have seen in chapters 8,10 and 12 that capacitors and inductors store energy in their electric and magnetic fields respectively. When a capacitor is supplied with an AC current it gets charged. When such a fully charged capacitor is connected to an inductor, the charge is transferred to the inductor and current starts flowing through the inductor. Because of the increasing current there will be a change in the magnetic flux of the inductor in the circuit. Hence induced emf is produced in the circuit. This self- induced emf will try to oppose the growth of the current. Due to this the charge (energy stored in) on the capacitor decreases and an equivalent amount of energy is stored in the inductor in the form of magnetic field. When the discharging of the capacitor completes, all the energy stored in the capacitor will be stored in the inductor. The capacitor will become fully discharged whereas inductor will be storing all the energy. As a result now the inductor will start charging the capacitor. The current and magnetic flux linked with the inductor starts decreasing. Therefore an induced emf is produced which recharges the capacitor in the opposite direction. This process of charging and discharging of capacitor is repeated and energy taken from the source keeps on oscillating between the capacitor (C) and the inductor (L).

When a charged capacitor is allowed to discharge through a non-resistive inductor, electrical oscillations of constant amplitude and frequency are produced. These oscillations are called LC oscillations. This is explaind in Fig. 13. 16.

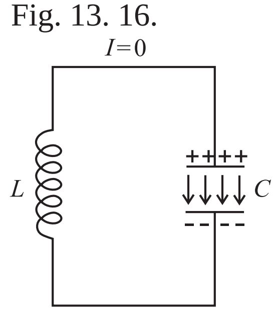


Fig. 13.16 (a) Let a capacitor with initial charge q_0 at (t = 0) be connected to an ideal inductor (zero resistance). The electrical energy stored in the dielectric medium between the plates of the capacitor is $U_E = \frac{1}{2}\frac{q_0^2}{C}$. Since there is no current in the circuit the energy stored in the magnetic field of the inductor is zero.

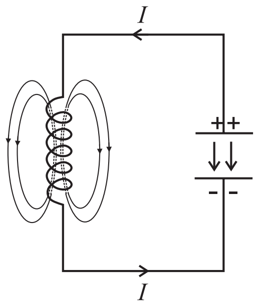


Fig. 13.16 (b) As the circuit is closed, the capacitor begins to discharge itself through the inductor giving rise to a current (I) in the circuit. As the current (I) increases, it builds up a magnetic field around the inductor. A part of the electrical energy of the capacitor gets stored in the inductor in the form of magnetic energy $U_B = \frac{1}{2}LI^2$

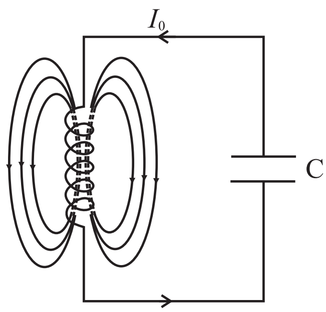


Fig. 13.16 (c) At a later instant the capacitor gets fully discharged and the potential difference across its plates becomes zero. When the current reaches its maximum value I_0, the energy in the magnetic field is energy $\frac{1}{2}LI_0^2$. Thus the entire electrostatic energy of the capacitor has been converted into the magnetic field energy of the inductor.

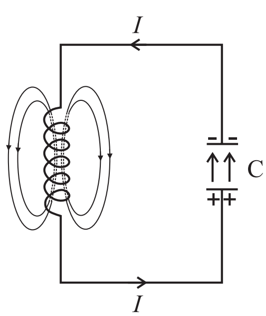


Fig. 13. 16 (d) After the discharge of the capacitor is complete, the magnetic flux linked with the inductor decreases inducing a current in the same direction (Lenz's Law) as the earlier current. The current thus persists but with decreasing magnitude and charges the capacitor in the opposite direction. The magnetic energy of the inductor begins to change into the electrostatic energy of the capacitor.

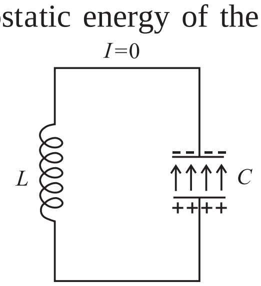


Fig. 13.16 (e) The process continues till the capacitor is fully charged with a polarity which is opposite to that in its initial state.

Thus the entire energy is again stored as $\frac{1}{2}\frac{q_0^2}{k}$ in the electric field of the capacitor.

The capacitor begins to discharge again sending current in opposite direction.

The energy is once again transferred to the magnetic field of the inductor. Thus the process repeats itself in the opposite direction.

The circuit eventually returns to the initial state.

Thus the energy of the system continuously surges back and forth between the electric field of the capacitor and magnetic field of the inductor. This produces electrical oscillations of a definite frequency . These are called LC Oscillations. If there is no loss of energy the amplitude of the oscillations remains constant and the oscillations are undamped.

However LC oscillations are usually damped due to following reasons.

1. Every inductor has some resistance. This causes energy loss as heat. The amplitude of oscillations goes on decreasing and they finally die out.
2. Even if the resistance were zero, total energy of the system would not remain constant. It is radiated away in the form of electromagnetic waves. Working of radio and TV transmitters is based on such radiations.

13.8 Electric Resonance:

Have you ever wondered how radio picks certain frequencies so you can play your favourite channel or why does a glass break down in an orchestra concert? Why do you think you encounter such situations? The answer lies in the phenomenon of resonance. The phenomenon of resonance can be observed in systems that have a tendency to oscillate at a particular frequency, which is called the natural frequency of oscillation of the system. When such a system is driven by an energy source, whose frequency is equal to the natural frequency of the system, the amplitude of oscillations become large and resonance is said to occur.

(a) Series resonance circuit:

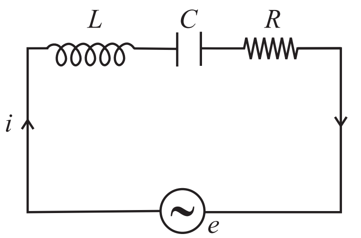


Fig. 13.17: Series resonance circuit.

A circuit in which inductance L, capacitance C and resistance R are connected in series (Fig. 13.17), and the circuit admits maximum current corresponding to a given frequency of AC, is called a series resonance circuit.

The impedance (Z) of an LCR circuit is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

At very low frequencies, inductive reactance $X_L = \omega L$ is negligible but capacitive reactance $X_C = \frac{1}{\omega C}$ is very high.

As we increase the applied frequency then X_L increases and X_C decreases.

At some angular frequency (ω_r), $X_L = X_C$

i.e. $\omega_r L = \frac{1}{\omega_r C}$

$\therefore (\omega_r)^2 = \frac{1}{LC}$ or $(2\pi f_r)^2 = \frac{1}{LC}$

$\therefore 2\pi f_r = \frac{1}{\sqrt{LC}}$

$\therefore f_r = \frac{1}{2\sqrt{LC}}$

Where f_r is called the resonant frequency.

At this particular frequency f_r , since $X_L = X_C$ we get $Z = \sqrt{R^2 + 0} = R$. This is the least value of Z Thus, when the impedance of on LCR circuit is minimum ,circuit is said to be purely resistive, current and voltage are in phase and hence the current $i_o = \frac{e_0}{z} = \frac{e_0}{R}$ is maximum.

This condition of the LCR circuit is called **resonance condition** and this frequency is called **series resonant frequency.**

At $\omega = \omega_r$, value of peak current (i_0) is maximum. The maximum value of peak current is inversely proportional to R $(\because i_0 = \frac{e_0}{R})$. For lower R values, i_0 is large and vice versa. The variation of rms current with frequency of AC is as shown in graph 13.18. The curve is called the series resonance curve. At resonance rms current becomes maximum. This circuit at resonant condition is very useful for radio and TV receivers for tuning the signal from a desired transmitting station or channel.

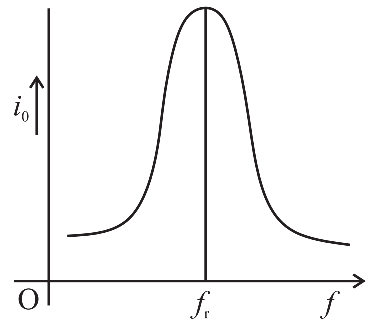


Fig. 13.18: Series resonance curve.

Characteristics of series resonance circuit

1) Resonance occurs when $X_L = X_C$
2) Resonant frequency $f_r = \frac{1}{2\pi\sqrt{LC}}$
3) Impedance is minimum and circuit is purely resistive.
4) Current has a maximum value.
5) When a number of frequencies are fed to it, it accepts only one frequency (f_r) and rejects the other frequencies. The current is maximum for this frequency. Hence it is called **acceptor circuit**.

b) Parallel resonance circuit:

A parallel resonance circuit consists of a coil of inductance L and a condenser of capacity C joined in parallel to a source of alternating emf. as shown in Fig. 13.19.

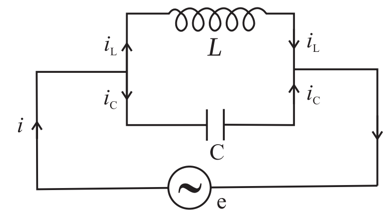


Fig. 13.19 : Parallel resonance circuit.

Let the alternating emf supplied by the source be

$$e = e_0 \sin \omega t$$

In case of an inductor, the current lags behind the applied emf by a phase angle of π/2, then the instantaneous current through L is given by

$$i_L = \frac{e_0}{X_L}\sin(\omega t - \pi/2)$$

Similarly in a capacitor ,as current leads the emf by a phase angle of π/2, we can write

$$i_c = \frac{e_0}{X_C}\sin(\omega t + \pi/2)$$

∴ The total current i in the circuit at this instant is

$$i = i_c + i_L$$

$$= \frac{e_0}{X_L}\sin(\omega t - \pi/2) + \frac{e_0}{X_C}\sin(\omega t + \pi/2)$$

$$= \frac{e_0}{X_L}(-\cos\omega t) + \frac{e_0}{X_C}\cos\omega t$$

$$= e_0 \cos\omega t\left(\frac{1}{X_c} - \frac{1}{X_L}\right)$$

$$i = e_0 \cos\omega t\left(\omega C - \frac{1}{\omega L}\right)$$

We find that,

i = minimum when $\omega C - \frac{1}{\omega L} = 0$

i.e. $\omega C = \frac{1}{\omega L}$ i.e. $\omega^2 = \frac{1}{LC}$

$$\therefore \omega = \frac{1}{\sqrt{LC}} \text{ or } 2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$\therefore f_r = \frac{1}{2\pi\sqrt{LC}}$$

Where f_r is called the resonant frequency.

Therefore at parallel resonance frequency f_r, i = minimum i.e. the circuit allows minimum current to flow through it. (as shown in the graph 13.20). Impedance is maximum at this frequency. The circuit is called parallel resonance circuit . A parallel resonant circuit is very useful in wireless transmission or radio communication and filter circuits.

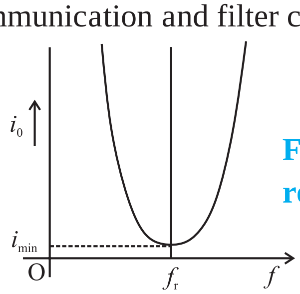


Fig. 13.20: Parallel resonant curve.

Characteristics of parallel resonance circuit

1. Resonance occurs when $X_L = X_C$.
2. Resonant frequency $f_r = \frac{1}{2\sqrt{LC}}$
3. Impedance is maximum
4. Current is minimum.
5. When alternating current of different frequencies are sent through parallel resonant circuit, it offers a very high impedance to the current of the resonant frequency (f_r) and rejects it but allows the current of the other frequencies to pass through it, hence called a **rejector circuit**.

Do you know?

Resonance occurs in a series LCR circuit when $X_L = X_C$ or $\omega = \frac{1}{\sqrt{LC}}$. For resonance to occur, the presence of both L and C elements in the circuit is essential. Only then the voltages L and C (being 180° out of phase) will cancel each other and current amplitude will be e_0/R i.e., the total source voltage will appear across R. So we cannot have resonance LR and CR circuit.

13.9 Sharpness of Resonance: Q factor

We have seen in section 13.4 (d) that the amplitude of current in the series LCR circuit is given by

$$i_0 = \frac{e_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Also if ω is varied, then at a particular frequency $\omega = \omega_r$, $X_L = X_C$ i.e. $\omega_r L = \frac{1}{\omega_r C}$. For a given resistance R, the amplitude of current is maximum when $\omega_r L - \frac{1}{\omega_r C} = 0$

$$\therefore \omega_r = \frac{1}{\sqrt{LC}}$$

For values of ω other than ω_r, the amplitude of the current is less than the maximum value i_0.

Suppose we choose a value for ω for which the amplitude is $\frac{1}{\sqrt{2}}$ times its maximum value, the power dissipated by the circuit becomes half (called half power frequency).

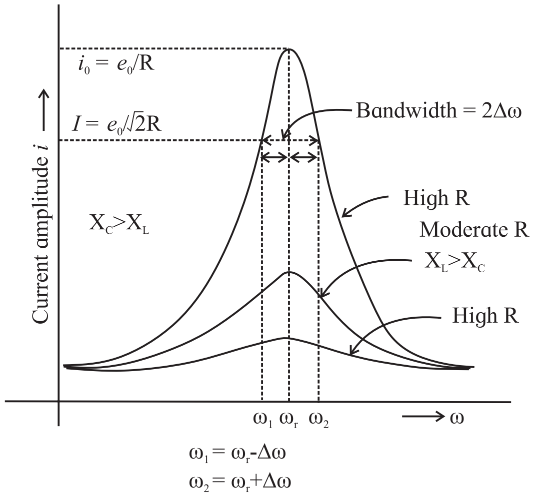


Fig. 13.21: Sharpness resonance.

From the curve in the Fig. (13.21) we see that there are two such values of ω say ω_1 and ω_2, one greater and other smaller than ω_r and symmetrical about ω_r such that

$$\omega_1 = \omega_r + \Delta\omega$$
$$\omega_2 = \omega_r - \Delta\omega$$

The difference $\omega_1 - \omega_2 = 2\Delta\omega$ is called the bandwidth of the circuit. The quantity $\left(\frac{\omega_r}{2\Delta\omega}\right)$ is regarded as the measure of the sharpness of resonance. The sharpness of resonance is measured by a coefficient called the quality or Q factor of the circuit

The Q factor of a series resonant circuit is defined as the ratio of the resonant frequency to the difference in two frequencies taken on both sides of the resonant frequency such that at each frequency the current amplitude becomes $\frac{1}{\sqrt{2}}$ times the value at resonant frequency.

$$\therefore \text{Q} = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r}{2\Delta\omega} = \frac{\textit{Resonant frequency}}{\textit{Bandwidth}}$$

Q-factor is a dimensionless quantity. The larger the value of Q-factor, the smaller the value of $2\Delta\omega$ or the bandwidth and sharper is the peak in the current or the series resonant circuit is more selective.

Fig. (13.21) shows that the lower angular frequency side of the resonance curve is dominated by the capacitor's reactance, the high angular frequency side is dominated by the inductor's reactance and resonance occurs in the middle.

Do you know?

The tuning circuit of a radio or TV is an example of LCR resonant circuit. Signals are transmitted by different stations at different frequencies which are picked up by the antenna. Corresponding to these frequencies a number of voltages appear across the series LCR circuit. But maximum current flows through the circuit for that AC voltage which has frequency equal to $f_r = \frac{1}{2\sqrt{LC}}$. If Q-value of the circuit is large, the signals of the other stations will be very weak. By changing the value of the adjustable capacitor C, the signal from the desired station can be tuned in.

13.10 Choke Coil:

If we use a resistance to reduce the current passing through an AC circuit, there will be loss of electric energy in the form of heat ($I^2 RT$) due to Joule heating. A choke coil helps to minimise this effect.

A choke coil is an inductor, used to reduce AC passing through a circuit without much loss of energy. It is made up of thick insulated copper wires wound closely in a large number of turns over a soft iron laminated core. Choke coil offers large resistance $X_L = \omega L$ to the flow of AC and hence current is reduced. Laminated core reduces eddy current loss.

Average power dissipated in the choke is $P = I_{rms} E_{rms} \cos\phi$, where the power factor $\cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$.

For a choke coil, L is very large. Hence R is very small so $\cos\phi$ is nearly zero and power loss is very small. The only loss of energy is due hysteresis loss in the iron core, which can be reduced using a soft iron core.

Internet my friend

1. https://en.m.wikipedia.org
2. hyperphysics.phy-astr.gsu.edu
3. https://www.britannica.com/science
4. www.khanacademy.org

Exercises

1. Choose the correct option.

i) If the rms current in a 50 Hz AC circuit is 5A, the value of the current 1/300 seconds after its value becomes zero is

(A) $5\sqrt{2}$ A (B) $5\sqrt{\frac{3}{2}}$ A

(C) $\frac{5}{6}$ A D) $\frac{5}{\sqrt{2}}$ A

ii) A resistor of 500 Ω and an inductance of 0.5 H are in series with an AC source which is given by $V = 100\sqrt{2}$ sin (1000 t). The power factor of the combination is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{\sqrt{3}}$

(C) 0.5 (D) 0.6

iii) In a circuit L, C & R are connected in series with an alternating voltage of frequency f. the current leads the voltage by 45^0. The value of C is

(A) $\frac{1}{\pi f(2\pi fL - R)}$

(B) $\frac{1}{2\pi f(2\pi fL - R)}$

(C) $\frac{1}{\pi f(2\pi fL + R)}$

(D) $\frac{1}{2\pi f(2\pi fL + R)}$

iv) In an AC circuit, e and i are given by e = 150 sin (150t) V and i = 150 sin (150 t + $\frac{\pi}{3}$) A. the power dissipated in the circuit is

(A) 106W (B) 150W

(C) 5625W (D) Zero

v) In a series LCR circuit the phase difference between the voltage and the current is 45^0. Then the power factor will be

(A) 0.607 (B) 0.707

(C) 0.808 (D) 1

2. Answer in brief.

i) An electric lamp is connected in series with a capacitor and an AC source is glowing with a certain brightness. How does the brightness of the lamp change on increasing the capacitance ?

ii) The total impedance of a circuit decreases when a capacitor is added in series with L and R. Explain why ?

iii) For very high frequency AC supply, a capacitor behaves like a pure conductor. Why ?

iv) What is wattless current ?

v) What is the natural frequency of L C circuit ? What is the reactance of this circuit at this frequency

3. In a series LR circuit $X_L = R$ and power factor of the circuit is P_1. When capacitor with capacitance C such that $X_L = X_C$ is put in series, the power factor becomes P_2. Calculate P_1 / P_2 .

4. When an AC source is connected to an ideal inductor show that the average power supplied by the source over a complete cycle is zero.

5. Prove that an ideal capacitor in an AC circuit does not dissipate power

6. (a) An emf $e = e_0 \sin \omega t$ applied to a series L - C – R circuit derives a current $I = I_0 \sin \omega t$ in the circuit. Deduce the expression for the average power dissipated in the circuit.

(b) For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain

7. A device Y is connected across an AC source of emf $e = e_0 \sin \omega t$. The current through Y is given as $i = i_0 \sin(\omega t + \pi/2)$

a) Identify the device Y and write the expression for its reactance.

b) Draw graphs showing variation of emf and current with time over one cycle of AC for Y.

c) How does the reactance of the device Y vary with the frequency of the AC ? Show graphically

d) Draw the phasor diagram for the device Y.

8. Derive an expression for the impedance of an LCR circuit connected to an AC power supply.

9. Compare resistance and reactance.

10. Show that in an AC circuit containing a pure inductor, the voltage is ahead of current by $\pi/2$ in phase.

11. An AC source generating a voltage $e = e_0 \sin \omega t$ is connected to a capacitor of capacitance C. Find the expression for the current i flowing through it. Plot a graph of e and i versus ωt.

12. If the effective current in a 50 cycle AC circuit is 5 A, what is the peak value of current? What is the current 1/600 sec. after if was zero ?

[Ans: 7.07A, 3.535 A]

13. A light bulb is rated 100W for 220 V AC supply of 50 Hz. Calculate (a) resistance of the bulb. (b) the rms current through the bulb.

[Ans: 484Ω, 0.4545A]

14. A 15.0 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what will happen to the capacitive reactance and the current.

[Ans: 212.1Ω, 1.037 A, 1.465A, halved, doubled]

15. An AC circuit consists of only an inductor of inductance 2 H. If the current is represented by a sine wave of amplitude 0.25 A and frequency 60 Hz, calculate the effective potential difference across the inductor (π = 3.142)

[Ans: 133.4V]

16. Alternating emf of e = 220 sin 100 πt is applied to a circuit containing an inductance of (1/π) henry. Write an equation for instantaneous current through the circuit. What will be the reading of the AC galvanometer connected in the circuit?

[Ans: i = 2.2 sin (100π t-π/2), 1.555A]

17. A 25 μF capacitor, a 0.10 H inductor and a 25Ω resistor are connected in series with an AC source whose emf is given by e = 310 sin 314 t (volt). What is the frequency, reactance, impedance, current and phase angle of the circuit?

[Ans: 50Hz, 95.98Ω, 99.19Ω, 2.211A, 1.316 rad]

18. A capacitor of 100 μF, a coil of resistance 50Ω and an inductance 0.5 H are connected in series with a 110 V-50Hz source. Calculate the rms value of current in the circuit.

[Ans: 0.8153A]

19. Find the capacity of a capacitor which when put in series with a 10Ω resistor makes the power factor equal to 0.5. Assume an 80V-100Hz AC supply.

[Ans: 9.187×10^{-5} F]

20. Find the time required for a 50 Hz alternating current to change its value from zero to the rms value.

[Ans: 2.5×10^{-3} s]

21. Calculate the value of capacitance in picofarad, which will make 101.4 micro henry inductance to oscillate with frequency of one megahertz.

[Ans: 249.7 picofarad]

22. A 10 μF capacitor is charged to a 25 volt of potential. The battery is disconnected and a pure 100 m H coil is connected across the capacitor so that LC oscillations are set up. Calculate the maximum current in the coil.

[Ans: 0.25 A]

23. A 100 μF capacitor is charged with a 50 V source supply. Then source supply is removed and the capacitor is connected across an inductance, as a result of which 5A current flows through the inductance. Calculate the value of the inductance.

[Ans: 0.01 H]

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