AVERAGE AND AGE RELATED PROBLEMS

An average or more accurately an arithmetic mean is, in crude terms, the sum of n different data divided by n :

Example : If one earns ₹ 40 on Monday, ₹ 50 on Tuesday and ₹ 60 on Wednesday, then his average income for 3 days is equal to :

(*a*) ₹ 50 (*b*) ₹ 100
(*c*) ₹ 30 (*d*) ₹ 40
(*e*) None of these

Solution. (*a*) : $\frac{40 + 50 + 60}{3} = ₹ 50$

The two formulae used in this chapter are :

$$\text{Average} = \frac{\text{Sum of observations}}{\text{Number of observations}}$$

Sum of observations
= Average × Number of observations

Example : If the marks obtained by Mohan in History, Sanskrit and English are 62, 73 and 69 respectively, then his average marks are :

(*a*) 73 (*b*) 66.2
(*c*) 68 (*d*) 63.9
(*e*) None of these

Solution. (*c*) : Average marks $= \frac{62 + 73 + 69}{3}$

$$= \frac{204}{3} = 68$$

Type 1 (Average Speed)

I. $\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$

Example : A man walks 2000 metres in 30 minutes, 1500 metres in 40 minutes and 500 metres in 10 minutes. Then what is the average speed for whole walking distance or journey?

(*a*) 50 metres / minute
(*b*) 55 metres/ minute
(*c*) 60.5 metres/minute
(*d*) 50.5 metres/minute
(*e*) None of these

Solution. (*a*) : Average speed

$$= \frac{(2000 + 1500 + 500) \text{ metres}}{(30 + 40 + 10) \text{ minutes}}$$

$$= \frac{4000}{80} = 50 \text{ m/minute}$$

II. If equal distances are travelled at the rate x and y, then

$$\text{Average speed} = \frac{2xy}{x + y}$$

Example : Madhu goes to his school at 5 km per hour and returns at 8 km per hour crossing same route. Then her average speed is :

(*a*) $7\frac{1}{3}$ km/hr (*b*) $5\frac{3}{4}$ km/hr
(*c*) $6\frac{2}{3}$ km/hr (*d*) $6\frac{2}{13}$ km/hr
(*e*) None of these

Solution. (*d*) :

$$\text{Average speed} = \frac{2 \times 5 \times 8}{5 + 8} \text{ km/hr}$$

$$= \frac{80}{13} \text{ km/hr} = 6\frac{2}{13} \text{ km/hour.}$$

Type 2 (Average Age)

Conditional Trick

A. When a person leaves a group and another person joins the group in the place of person left, then

I. In the case of increasing of average age, Age of the new comer = Age of person left + no. of persons in the group x increase in average age.

II. In the case of decreasing of average age, Age of the new comer = Age of person left – no. of persons in the group x decrease in average age.

Example : The average age of 8 men is increased by 4 years when one of them whose age is 30 years is replaced by a new man. What is the age of new man?

(*a*) 55 years (*b*) 62 years
(*c*) 42 years (*d*) 69 years
(*e*) None of these

Solution. (*b*) : The age of new man = $30 + 8 \times 4$
= 62 years

Example : The average age of 45 persons is decreased by 1/9 years when one of them whose age is 60 years replaced by new comer. What is the age of new comer?

(*a*) 40 years (*b*) 62 years
(*c*) 55 years (*d*) 59 years
(*e*) None of these

Solution. (*c*) : Age of new comer = $60 - 45 \times 1/9$
$= 60 - 5$
= 55 years.

B. When a person joins a group without replacing any previous person from that group, then

I. In the case of increasing of average age
Age of the new comer = Previous average age + no. of all persons (including new comer) × increase in average age.

II. In the case of decreasing of average age,
Age of the new comer = Previous average age – no. of all persons (including new comer) × decrease in average age.

Example : The average age of 6 women is 32 years which is increased by 1 year when a new woman joins the group. Then what is the age of new woman?

(*a*) 42 years (*b*) 35 years
(*c*) 45 years (*d*) 39 years
(*e*) None of these

Solution. (*d*) : Age of new woman = $32 + (6 + 1) \times 1$
$= 32 + 7$
= 39 years.

Example: The average age of 20 teachers is 45 years which is decreased by 6/7 years when a student joins this group. Then what is the age of that student?

(*a*) 15 years (*b*) 27 years
(*c*) 18 years (*d*) 25 years
(*e*) None of these

Solution. (*b*) : Age of the student
$= 45 - (20 + 1) \times 6/7$
$= 45 - 18 = 27$ years

C. When a person leaves the group but nobody joins this group, then

I. In the case of increasing of average age, Age of man left = Previous average age – no. of present persons × increase in the average age.

II. In the case of decreasing of average age, Age of man left = Previous average age + no. of present persons × decrease in average age.

Example : The average age of 10 girls in a hostel is 19 years. But one girl left the hostel and average age is increased by 1/2 year. Then how many years old is she?

(*a*) $14\frac{1}{2}$ years (*b*) 15 years
(*c*) $15\frac{1}{2}$ years (*d*) 18 years
(*e*) None of these

Solution. (*a*) : Age of the girl left

$$= 19 - (10 - 1) \times \frac{1}{2}$$

$$= 19 - 9 \times \frac{1}{2} = 14\frac{1}{2} \text{ years.}$$

Example : The average age of 26 labours is 30 years. It is decreased by 1/5 years, when a labour went home. Then the age of that labour is

(*a*) 30 years (*b*) 32 years
(*c*) 24 years (*d*) 35 years
(*e*) None of these

Solution : Age of the labour left

$$= 30 + (26 - 1) \times \frac{1}{5}$$

$$= 30 + 25 \times \frac{1}{5} = 35 \text{ years.}$$

Type 3 (Average of Numbers)

A. Average related to Natural Numbers

I. Average of consecutive n natural numbers $= \frac{(n+1)}{2}$

For example, Average of consecutive natural numbers till $7 = \frac{7+1}{2} = 4$.

II. Average of squares of n natural numbers $= \frac{(n+1)(2n+1)}{6}$

For example, Average of square of numbers till 11

$$= \frac{(11+1)(2\times 11+1)}{6} = \frac{12\times 23}{6} = 46.$$

III. Average of cubes of n consecutive natural numbers $= \frac{n(n+1)^2}{4}$

For example, Average of $1^3, 2^3, 3^3, 4^3 = \frac{4(4+1)^2}{4} = 25$

B. Average related to Even Numbers

I. Average of n consecutive even numbers $= n+1$

For example, Average of 4 consecutive even numbers $= 4 + 1 = 5$.

II. Average of consecutive even numbers till $n = (n/2+1)$

Note : When n is even.

For example, Average of consecutive even numbers till $10 = (10/2 + 1) = 6$.

III. Average of squares of n consecutive even numbers $= \frac{2(n+1)(2n+1)}{3}$

For example, Average of 2, 4, 6, 8

$$= \frac{2(4+1)(2\times 4+1)}{3} = \frac{2\times 5\times 9}{3} = 30$$

IV. Average of squares of consecutive even numbers till $n = \frac{(n+1)(n+2)}{3}$

For example, Average of squares of consecutive even numbers till 16

$$= \frac{(16+1)(16+2)}{3} = 102.$$

C. Average related to Odd numbers

I. Average of n consecutive odd Numbers $= n$

For example, Average of 7 consecutive odd numbers $= 7$.

II. Average of consecutive odd numbers till $n = \frac{(n+1)}{2}$

Note : When n is odd.

For example, Average of consecutive odd numbers till 13

$$= \frac{13+1}{2} = 7.$$

III. Average of squares of consecutive odd numbers till $n = \frac{n(n+2)}{3}$

For example, Average of squares of consecutive odd numbers till 9

$$= \text{average of } 1^2, 3^2, 5^2, 7^2, 9^2 = \frac{9(9+2)}{3} = 33$$

EXERCISE

1. A person reached Delhi from Jaipur by his car at a speed of 60 km per hour and returned to Jaipur along the same route at a speed of 40 km per hour. What is his average speed?

(*a*) 50 km per hour (*b*) 45 km per hour
(*c*) 48 km per hour (*d*) 55.5 km per hour
(*e*) None of these

2. The average weight of 8 persons is increased by 2.5 kg When one of them whose weight is 56 kg is replaced by a new man. The weight of new man is :

(*a*) 66 kg. (*b*) 75 kg.
(*c*) 67.6 kg. (*d*) 76 kg.
(*e*) None of these

3. The average of consecutive natural numbers from 1 to 49 is :
(*a*) 28 (*b*) 50
(*c*) 48 (*d*) 49
(*e*) None of these

4. Raju's average daily expenditure is ₹ 15 during September, ₹ 20 during October and ₹ 13 during November. What is the average daily expenditure for the three months?
(*a*) ₹ 16 approximately
(*b*) ₹ 18 approximately
(*c*) ₹ 19.50 approximately
(*d*) ₹ 17 approximately
(*e*) None of these

5. The average of first five multiples of 3 is :
(*a*) 15 (*b*) 9
(*c*) 12 (*d*) 3
(*e*) None of these

6. If difference between age of Ritu and Ram is 15 years and ratio of their ages is 3 : 2, then the age of Ram is :
(*a*) 25 years (*b*) 20 years
(*c*) 28 years (*d*) 30 years
(*e*) None of these

7. The sum of the ages of husband and wife is 70 years and ratio of their ages is 3 : 2. The age of the wife is :
(*a*) 32 years (*b*) 25 years
(*c*) 28 years (*d*) 27 years
(*e*) None of these

8. The average score of a cricket for 10 matches is 49.9 runs. If the average for the first six matches is 49, then what is average score for the last 4 matches?
(*a*) 48.7 (*b*) 49.8
(*c*) 46.4 (*d*) 50
(*e*) None of these

9. The present age difference between father and son is 14 years. The ratio of their age will be 4 : 3 after 11 years. How old is son now?
(*a*) 25 years (*b*) 31 years
(*c*) 30 years (*d*) 28 years
(*e*) None of these

EXPLANATORY ANSWERS

1. (*c*) : Average speed $= \frac{2xy}{x+y} = \frac{2 \times 60 \times 40}{60+40}$
$= 48$ kmph

2. (*d*) : Weight of new man $= 56 + 8 \times 2.5$ kg
$= (56 + 20)$ kg $= 76$ kg

3. (*e*) : Average $= \frac{(n+1)}{2} = \frac{49+1}{2} = 25.$

4. (*a*) : Average daily expenditure
$= \frac{\text{Total expenditure}}{\text{Total days}}$
$= \frac{15 \times 30 + 20 \times 31 + 13 \times 30}{30 + 31 + 30}$
$= \frac{1460}{91} =$ ₹ 16 approximately

5. (*b*) : The average of first five multiples of 3
$= \frac{3+6+9+12+15}{5} = \frac{45}{5} = 9$

6. (*d*) : Let Ritu's age $= 3x$ years
Ram's age $= 2x$ years
$3x - 2x = 15 \Rightarrow x = 15$
Ram's age $= 2 \times 15 = 30$ years.

7. (*c*) : The age of the wife $= \frac{2}{(3+2)} \times 70$
$= 28$ years

8. (*e*) : Average score $= \frac{(10 \times 49.9 - 6 \times 49)}{4}$
$= \frac{499 - 294}{4} = \frac{205}{4}$
$= 51.25$ runs.

9. (*b*) : Let present age of father be x years.
Present age of son $= (x - 13)$ years
According to the question,
$\frac{x+11}{(x-13)+11} = \frac{4}{3}$
$\Rightarrow 3x + 33 = 4x - 8$
$\Rightarrow x = 41$
Thus, present age of son $= 41 - 13$
$= 28$ years.