OSCILLATIONS

Oscillatory Motion

When a body moves back and forth about the mean or equilibrium position in equal interval of time, it is said to be executing oscillatory or vibratory motion. It is a particular type of periodic motion in which body or particle moves back and forth about a mean or equilibrium position.

Periodic Motion : Periodic motion is one in which a particle executing motion repeats itself over and over again after an equal interval of time, e.g., motion of planets around the sun.

It should be noted that all oscillatory motions are necessarily periodic, however, every periodic motion is not oscillatory.

Simple Harmonic motion (SHM) : An oscillating body is said to be executing SHM, when its acceleration at any instant is directly proportional to its displacement from a fixed point (or mean position) and is always directed towards that point (or mean position).

Geometrically, the SHM can be regarded as the projection of uniform circular motion upon any diameter of the circle. The displacement of a particle executing SHM is represented by

$$\xi = a \cos \omega t$$

Some Important Deduction About SHM :

(a) The motion is oscillatory, as the body moves to and fro repeatedly about its mean position.

(b) The motion is periodic, as the motion is repeated after equal intervals of time.

(c) The restoring force acting on the body is directly proportional to its displacement, and

(d) This restoring force is always directed towards its mean position.

Displacement : At any instant, the distance of a particle from its mean position (or a fixed point) is called its displacement. It is represented by ξ. Its unit is metre.

Amplitude : The maximum displacement of the vibrating particle on either side of its mean position is called the amplitude of the harmonic oscillations. It is represented by 'a'. Its unit is metre.

Time-period: Time taken by a vibrating particle to make one complete vibration is called as the time-period of the simple harmonic motion. It's represented by T. Its unit is second.

Frequency : The number of oscillations completed by the particle in one second is called the frequency of simple harmonic motion. It is represented by ν. Its unit is Hz.

Angular frequency : Angle (in radians) described per second by the oscillating particle is called its angular frequency. It is represented by ω. Its unit is $rads^{-1}$.

Phase : Phase of a vibrating particle is defined as a fraction of the time period which has elapsed since the point last crossed through its mean position in the positive direction.

In other words, phase is defined as the angle swept by the radius vector since the vibrating particle last crossed its mean position.

Initial phase or Epoch : The value of phase at time $t = 0$, is called initial phase or epoch. It is represented by ϕ_0.

The particles whose phase angles differ by an even multiple of π are said in the same phase, while the particles whose phase angles differ by an odd-multiple of π are said in the opposite phase.

All above terms can be summarised as follows

Term	Symb	Relation	Unit
Displacement	ξ	$\xi = a \cos \omega t$	m
Amplitude	a	$\xi = a \cos \omega t$	m
Time period	T	$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$	s
Frequency	ν	$\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$	Hz
Angular frequency	a	$\omega = \frac{2\pi}{T} = 2\pi\nu$	$rads^{-1}$
Phase angle	$(\omega t \pm \phi_0)$	$\xi = a \cos(\omega t \pm \phi_0)$	rad

where m = mass of vibrating particle, called as inertia factor

K = force constant, called as spring factor

ϕ_0 = initial phase or epoch.

Differential equation of SHM

$$\frac{d^2\xi}{dt^2} = -\omega^2\xi$$

where $\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{\text{spring factor}}{\text{inertia factor}}}$

Energy of a particle executing SHM :

(i) Kinetic energy, $E_k = \frac{1}{2} m\omega^2 (a^2 - \xi^2)$

At mean position (i.e., $\xi = 0$), the K.E. is maximum and at extreme positions (i.e., $\xi = \pm a$), the K.E. is minimum (zero) and varies in between these two positions.

(ii) Potential energy, $E_p = \frac{1}{2} m\omega^2\xi^2$

At mean position (i.e., $\xi = 0$), the P.E. is minimum (zero) and at extreme positions (i.e., $\xi = \pm a$), the P.E. is maximum and varies in between these two positions.

(iii) Total energy, $E = E_k + E_p = \frac{1}{2} m\omega^2 a^2$

Which is a constant. Hence total energy in a SHM remains conserved.

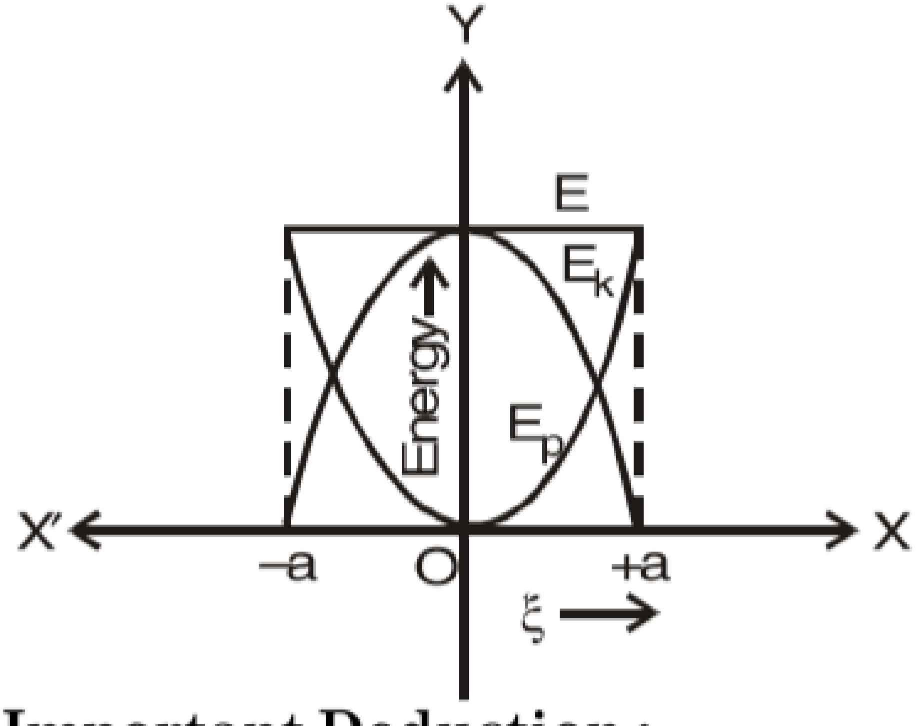
Since $\omega = \frac{2\pi}{T} = 2\pi\nu$

hence $E = \frac{1}{2} m (2\pi\nu)^2 a^2 = 2\pi^2 m \nu^2 a^2$

The above results can be summarised as follows

Energy	At mean position	At extreme positions
E_k	maximum	zero
E_p	zero	maximum
E	constant	constant

The three energies E_k, E_p and E can be graphically represented as below :



Some Important Deduction :

(i) ***Free Vibrations :*** When a body vibrates freely with its natural frequency having no external interference, then its vibrations are called as free vibrations.

(ii) ***Damped Vibrations :*** Vibrations of a body in presence of external interference or retarding force, are called as damped vibrations. The external resistance is called as damping. Here amplitude of the vibrations goes on decreasing continuously.

(iii) ***Maintained Vibrations*** : If the loss of energy of damped vibrations is fully compensated by a suitable arrangement, then amplitude for these vibrations would remain constant. These vibrations are now called as maintained vibrations.

(iv) ***Forced Vibrations*** : When a body vibrates under the action of a strong periodic force, then its vibrations are called as forced vibrations. Here the body vibrates with the frequency of the applied periodic force, irrespective of its natural frequency.

(v) ***Resonant Vibrations :*** If the frequency of the periodic force is equal to the natural frequency of the vibrating body, then these forced vibrations are called as the resonant vibrations and this phenomenon is called as *resonance.* At resonance, the amplitude of the vibrating body is the maximum.

For examples :

1. When the frequency of the water waves becomes equal to the natural frequency of the ship, it begins to swing dangerously due to resonance.
2. Sometimes windows of a house start ratting when an aeroplane passes nearby. This occurs when the frequency of sound waves from the engine of the aeroplane becomes equal to the natural frequency of the windows, causing resonance.
3. *Tuning of a radio:* Radio sounds when the frequency of its circuit is equal to the frequency of the radio station to be tuned.
4. Soldiers marching over a suspension bridge are always ordered to break their steps, because it may be possible that the frequency of the foot-march of the soldiers may become equal to the natural frequency of the bridge and cause it to vibrate dangerously because of resonance.
5. *Ratling of a bus body:* Sometimes a speeding bus starts producing ratling sound at a particular speed. It happens when the frequency from the engine becomes equal

to the natural frequency of bus body, this causing resonance.

Simple pendulum : A simple pendulum consists of a heavy point mass suspended by a weightless, inextensible and inflexible string fixed to a rigid support. It is an ideal conception and cannot be realized in actual practice.

The time period of a simple pendulum is given by,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where, l = length of the pendulum.

If the time period of a simple pendulum is 2 second, then it is known as second pendulum. Length of second's pendulum comes out to be 99.3 cm.

Drawbacks :

(i) In actual practice, we can't have an ideal pendulum, as we do not have a heavy point mass and a weightless inextensible string.

(ii) The formula, $T = 2\pi\sqrt{\frac{l}{g}}$ is true only when the amplitude of vibrations is very small, which is not the case in actual practice.

(iii) The resistance of air also effects the motion of the bob.

(iv) The bob may have side way motion, besides linear motion.

(v) The period of a second's pendulum in 2 seconds.