# DAILY PRACTICE PROBLEMS — MATHEMATICS SOLUTIONS — DPP/CM11

**1.** **(c)**

| $p$ | $q$ | $p \Rightarrow q$ | $\sim(p \Rightarrow q)$ | $\sim p$ | $\sim q$ | $\sim p \vee \sim q$ | $\sim(p \Rightarrow q) \Leftrightarrow \sim p \vee \sim q$ |
|---|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F | T |
| T | F | F | T | F | T | T | T |
| F | T | T | F | T | F | T | F |
| F | F | T | F | T | T | T | F |

Last column shows that result is neither a tautology nor a contradiction.

**2.** **(c)** The inverse of the proposition $(p \wedge \sim q) \to r$ is
$\sim (p \wedge \sim q) \to \sim r$
$\equiv \sim p \vee \sim(\sim q) \to \sim r$
$\equiv \sim p \vee q \to \sim r$

**3.** **(c)** Negation of 'f is one to one and onto' is R or not Q.

**4.** **(a)** We know that the contropositive of $p \to q$ is
$\sim q \to \sim p$. So contra positive of $p \to (\sim q \to \sim r)$ is
$\sim(\sim q \to \sim r) \to \sim p$
$\equiv \sim q \wedge [\sim(\sim r)] \sim p$
$\because \sim(p \to q) \equiv p \wedge \sim q$
$\equiv \sim q \wedge r \to \sim p$

**5.** **(c)** $S(p, q, r) = \sim p \wedge [\sim(q \vee r)]$
So, $S(\sim p, \sim q, \sim r) \equiv \sim(\sim p) \wedge [\sim(\sim q \vee \sim r)] \equiv p \wedge (q \vee r)$
$S^*(p, q, r) \equiv \sim p \vee [\sim(q \wedge r)]$
$S^*(\sim p, \sim q, \sim r) \equiv p \vee (q \vee r)$
Clearly, $S^*(\sim p, \sim q, \sim r) \equiv \sim S(p, q, r)$

**6.** **(a)**

**7.** **(b)** Let us make the truth table for the given statements, as follows :

| $p$ | $q$ | $p \vee q$ | $q \to p$ | $p \to (q \to p)$ | $p \to (p \vee q)$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | F | T | T | T | T |
| F | T | T | F | T | T |
| F | F | F | T | T | T |

From table we observe
$p \to (q \to p)$ is equivalent to $p \to (p \vee q)$

**8.** **(c)**

| p | q | $p \to q$ | $\sim p$ | $\sim p \vee q$ | $(p \to q) \leftrightarrow \sim(p \vee q)$ |
|---|---|---|---|---|---|
| T | T | T | F | T | T |
| T | F | F | F | F | T |
| F | T | T | T | T | T |
| F | F | T | T | T | T |

**9.** **(a, b, d)** Statement given in option (c) is only correct.
$\sim [p \vee (\sim q)] = (\sim p) \wedge \sim(\sim q)$
$= (\sim p) \wedge q$

**10.** **(c, d)** We know that $p \leftrightarrow q$ is true if $p$ and $q$ both are true or false.
so $p \leftrightarrow \sim q$ is true when if $p$ and $\sim q$ is true.
i.e., $p$ is true and $q$ is false.
or $p$ and $\sim q$ is false, i.e. $p$ is false and $q$ is true.
Hence, options (c) and (d) are correct

**11.** **(a, b, c)** Since $\sim (p \vee q) \equiv \sim p \wedge \sim q$
(By De-Morgans' law)
$\therefore \sim (p \vee q) \neq \sim p \vee \sim q$
$\therefore$ (d) is the false statement

**12.** **(a, b, d)** We consider following truth table.

| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \wedge q$ | $p \vee q$ | $(\sim(p \vee q))$ | $(p \wedge q) \wedge (\sim(p \vee q))$ |
|---|---|---|---|---|---|---|---|
| T | T | F | F | T | T | F | F |
| T | F | F | T | F | T | F | F |
| F | T | T | F | F | T | F | F |
| F | F | T | T | F | F | T | F |

Clearly last column of the above truth table contains only F.
Hence $(p \wedge q) \wedge (\sim(p \vee q))$ is a contradiction

**13.** **(a, b, c)** The truth value of $\sim(\sim p) \leftrightarrow p$ as follow

| $p$ | $\sim p$ | $\sim(\sim p)$ | $\sim(\sim p) \to p$ | $p \to \sim(\sim p)$ | $\sim(\sim p) \leftrightarrow p$ |
|---|---|---|---|---|---|
| T | F | T | T | T | T |
| F | T | F | T | T | T |

Since last column of above truth table contains only T.
Hence $\sim(\sim p) \to p$ is a tautology.

**14.** **(b, c, d)**

**15.** **(a, b, c)**

**16.** **(a, b, d)** $p \Rightarrow q \equiv \sim p \vee q \; \therefore \sim(p \Rightarrow q) \equiv p \wedge \sim q$.

**17.** **(a, b, c)** We know that $p \wedge q$ is true when both $p$ and $q$ are true.
So, option (a) is not true.
We know that $p \to q$ is false when $p$ is true and $q$ is false.
So, option (b) is not true.
We know that $p \leftrightarrow q$ is true when either both $p$ and $q$ are true or both are false. So, option (c) is not true.
If $p$ and $q$ both are false, then
$p \vee q$ is false $\Rightarrow \sim(p \vee q)$ is true.
Hence, option (d) is true.

**18.** **(a, b, c)** The truth tables of $p \to q$ and $\sim p \vee q$ are given below:

| $p$ | $q$ | $\sim p$ | $p \to q$ | $\sim(p \vee q)$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | F | F | F |
| F | T | T | T | T |
| F | F | T | T | T |

Clearly, truth tables of $p \to q$ and $\sim p \vee q$ are same.
So, $p \to q$ is logically equivalent to $\sim p \vee q$.
Hence, option (a) is correct.

If the truth value of $p, q, r$ are T, F, T respectively, then the truth values of $p \vee q$ and $q \vee r$ are each equal to T. Therefore, the truth value of $(p \vee q) \wedge (q \vee r)$ is T.
Hence, option (b) is correct.
We have, $\sim (p \vee q \vee r) \cong (\sim p \wedge \sim q \wedge \sim r)$
So, option, (c) is correct.
If $p$ is true and $q$ is false, then $p \vee q$ is true. Consequently,
$\sim (p \vee q)$ is false and hence $p \wedge \sim (p \vee q)$ is false.
Hence, option (d) is wrong.

**19.** **(a, c , d)** Since $\sim(p \vee q) \equiv (\sim p \wedge \sim q)$ and $\sim (p \wedge q) \equiv (\sim p \vee q)$
So option (b) and (d) are not true.
$(p \to q) \equiv p \wedge \sim q)$, so option (c) is not true.
Now $p \to q \sim p \vee q$
$\sim q \to \sim p \equiv [\sim (\sim q) \vee \sim p] \equiv q \vee \sim p \equiv \sim p \vee q$
$p \to q \equiv \sim q \to \sim p$

**20.** **(A) → (s); (B) → (p); (C) → (q); (D) → (r)**
(A) Dual of statement $[(p \vee q) \wedge (\sim q)] \vee (\sim p)$ is $[(p \wedge q) \vee (\sim q)] \wedge (\sim p)$
(B) Logically equivalent of $[(p \vee q) \wedge (\sim q)] \vee \sim p$ is $[(p \wedge \sim q) \vee (q \wedge \sim q)] \vee \sim p$ or $[p \wedge \sim q] \vee \sim p$
(C) Negation of $[(p \vee q) \wedge (\sim q)] \vee (\sim p)$ is $\sim [(p \vee q) \wedge (\sim q)] \wedge \sim (\sim p)$ or $[(\sim p \wedge \sim q) \vee q)] \wedge p$
(D) Contrapositive of $[(p \vee q) \wedge (\sim q)] \to (\sim p)$ is $\sim (\sim p) \to \sim [(p \vee q) \wedge (\sim q)]$ or $p \to [\sim (p \vee q) \vee q]$ or $(\sim p) \vee [(\sim p \wedge \sim q) \vee q]$