CBSE Board Class 10 Maths Chapter 7-Coordinate Geometry
Objective Questions

Basics revisited

1. What will be the reflection of the point (4, 5) about the X-axis, in the fourth quadrant?

(A) (4, 5)
(B) (4, -5)
(C) (-4, -5)
(D) (-4, 5)

Answer: (B) (4,-5)

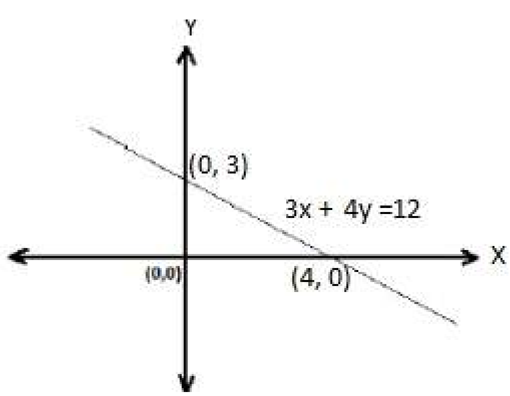
Solution: X - axis will act as a plane mirror and this point will form an image, following the sign convention, at (4, -5) in the fourth quadrant.

2. Point P lies on the line 3x+ 4y - 12 = 0. If X- coordinate of P is a, then its y-coordinate is ______.

(A) (12-3a) / 4
(B) (12-4a)/ 3
(C) (12+3a)/4
(D) (3a-12)/4

Answer: (A) (12-3a) / 4

Solution:



The points on the line should satisfy the equation of the line.
So, the point P (a, y) satisfies the equation. Hence,

$3x + 4y - 12 = 0$
$3(a) + 4(y) - 12 = 0$
$4y = 12 - 3a$
$Y = (12 - 3a) / 4$

3. If point P lies on the line y = -1, find the following
a) Its Y-coordinate
b) Its X-coordinate

(A) y = -1, x can be any real number
(B) y = 1, x = -1
(C) x = -2, y = -1
(D) x = -1, y = -1

Answer: (A) y = -1, x can be any real number

Solution: Since it is given that the point P lies on the line y = -1, its y coordinate will be -1.
Its x coordinate can be any real number.

4. Find the values of k, if the points A (k+1, 2k), B (3k, 2k+3) and C (5k-1,5k) are collinear.

(A) k = 5, 1/5
(B) k = 4, 1/4
(C) k = 3, 1/3
(D) k = 2, 1/2

Answer: (D) k = 2, 1/2

Solution: We know that, if three points are collinear, then the area of the triangle formed by these points is zero.
Since, the points A (k+1,2k), B (3k, 2k+3) and C (5k-1,5k) are collinear.
Then, area of Δ ABC = 0.
$\Rightarrow \frac{1}{2} [x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)] = 0$
Multiplying above expression by 2, we get
$[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)] = 0$
Here, $x_1=k+1$, $x_2=3k$, $x_3=5k-1$
and $y_1=2k$, $y_2=2k+3$, $y_3=5k$
$\Rightarrow [(k+1)(2k+3-5k)+3k(5k-2k)+(5k-1)(2k-(2k+3))]=0 \Rightarrow [(k+1)(3-3k)+3k(3k)+(5k-1)(2k-2k-3)]=0$
$\Rightarrow [-3k^2+3k-3k+3+9k^2-15k+3] = 0$
$\Rightarrow (6k^2-15k+6) = 0$
$\Rightarrow 6k^2-15k+6=0$
Dividing the equation by 3, we get
$\Rightarrow 2k^2-5k+2=0$

$\Rightarrow 2k^2 - 4k - k + 2 = 0$ [by factorization method]
$\Rightarrow 2k(k-2) - 1(k-2) = 0$
$\Rightarrow (k-2)(2k-1) = 0$
If $k - 2 = 0$, then $k = 2$
If $2k - 1$, then $k = 1/2$
$\therefore k = 2, 1/2$
Hence, the required values of k are 2 and 1/2.

5. Name the type of triangle formed by the points A (-5, 6), B (-4,-2) and C (7, 5).

(A) Equilateral triangle
(B) Scalene triangle
(C) Isosceles triangle
(D) Right-angled triangle

Answer: (B) Scalene triangle

Solution: To find the type of triangle, first, we determine the length of all the three sides and see if the condition of the triangle is satisfied by these sides.
Now, using distance formula between two points,

$$AB = \sqrt{(-4+5)^2 + (-2-6)^2}$$

$$= \sqrt{(-1)^2 + (-8)^2}$$

$$= \sqrt{1+64} \quad = \sqrt{65} \qquad [\because d = \sqrt{(X_2 - X_1) + (Y_2 - Y_1)^2}]$$

$$BC = \sqrt{(7+4)^2 + (5+2)^2} \quad = \sqrt{(11)^2 + (7)^2}$$

$$= \sqrt{121+49} \quad = \sqrt{170}$$

And $$CA = \sqrt{(-5-7)^2 + (6+5)^2} = \sqrt{(-12)^2 + (1)^2}$$

$$\sqrt{144+1} = \sqrt{145}$$

We see that, $AB \neq BC \neq CA$
And Δ ABC does not satisfy Pythagoras theorem
Hence, the required triangle is scalene because all sides are of different length.

Distance formula

6. The distance of the point (–2, –2) from the origin is

(A) $\sqrt{2}$

(B) 8

(C) $2\sqrt{2}$

(D) $\sqrt{9}$

Answer: (C) $2\sqrt{2}$

Solution: Let the origin be O and the point A be (-2, -2)

Using distance formula,

$OA^2 = (2^2 + 2^2)$
$OA^2 = 8$

$$OA = \sqrt{8} = 2\sqrt{2}$$

7. The points on X-axis at a distance of 10 units from (11, –8) are

(A) (5, 2) (17, 0)
(B) (5, 0) (17, 0)
(C) (6, 0) (17, 0)
(D) (5, 0) (16, 0)

Answer: (B) (5, 0) (17, 0)

Solution: Any point on the x-axis is of the form (x,0)
Let the point be (x,0)
Using distance formula

$(x-11)^2+8^2=10^2$
$x^2-22x+121+64=100$
$x^2-22x+85=0$,
Factorising we get,
$x^2-17x-5x+85=0$

$(x-17)(x-5)=0$
$x=17,5$
Hence the points are (17,0) and (5,0)

8. The point on the x-axis which is equidistant from (2, –5) and (–2, 9) is

(A) (7, 0)
(B) (-7, 0)
(C) (2, 0)
(D) (-2, 0)

Answer: (B) (-7, 0)

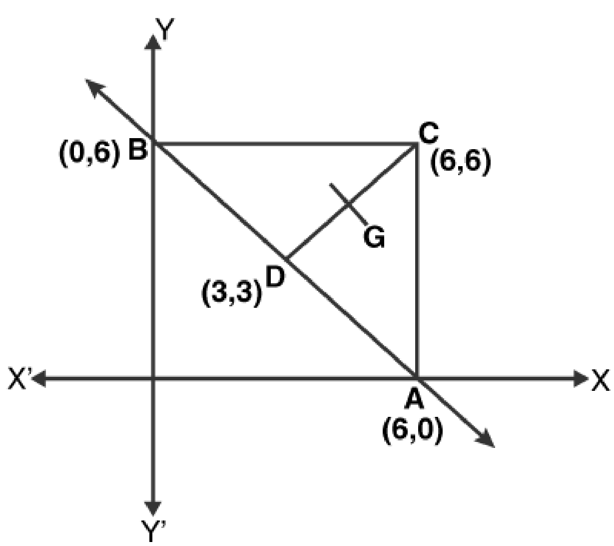
Solution: We know that a point on the x-axis is of form (x, 0). Let the point on the x-axis be P(x, 0) and the given points are A (2, -5) and B (-2, 9)

Now, $PA = \sqrt{(2-X)^2 + (-5-0)^2}, PB = \sqrt{(-2-X)^2 + (9-0)^2}$

Since PA = PB, $\sqrt{(2-X)^2 + (-5-0)^2} = \sqrt{(-2-X)^2 + (9-0)^2}$

$\Rightarrow (2-x)^2 + (-5-0)^2 = (-2-x)^2 + (9-0)^2$
$\Rightarrow 4 - 4x + x^2 + 25 = 4 + 4x + x^2 + 81$
$\Rightarrow -8x = 56$
$\Rightarrow x = -7$
Hence, the required point is (-7, 0)

9. The points (6. 6), (0, 6) and (6, 0) are the vertices of a right triangle as shown in the figure. Find the distance between its centroid and circumcentre.



(A) $\sqrt{3}$

(B) $\sqrt{2}$

(C) $\sqrt{6}$

(D) $\sqrt{5}$

Answer: (B) $\sqrt{2}$

Solution: Circumcentre = midpoint of AB

= [($X_1 + X_2$)/2, ($Y_1 + Y_2$)/2]
= [(6+0)/2, (6+0)/2]
= (3, 3)

Centroid (G) = [($X_1 + X_2 + X_3$)/3, ($Y_1 + Y_2 + Y_3$)/3]
= [(6+0+6)/3, (0+6+6)/3]
= (4, 4)

∵ Distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

∴ Distance between centroid and circumcentre $\sqrt{(4-3)^2 + (4-3)^2}$

$$= \sqrt{1+1}$$

$$= \sqrt{2}$$

10. The distance between the points (a, b) and (– a, – b) is:

(A) $2\sqrt{a^2 + b^2}$

(B) 0

(C) $\sqrt{a^2 + b^2}$

(D) $a^2 + b^2$

Answer: (A) $2\sqrt{a^2 + b^2}$

Solution: Let A (a, b) and B (-a,-b) be the two points and'd' be the distance between them.

By using distance formula, we get

$$d = \sqrt{(-a(-a))^2 + (b - (-b))^2}$$

$$d = \sqrt{(2a)^2 + (2b)^2}$$

$$d = 2\sqrt{a^2 + b^2}$$

Section formula

11. Mid-point of the line-segment joining the points (– 5, 4) and (9, – 8) is:

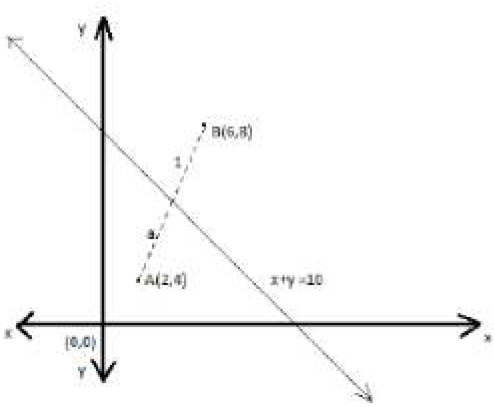
(A) (-2,2)
(B) (7,-6)
(C) (2,-2)
(D) (-7,6)

Answer: (C) (2,-2)

Solution: ∵ Midpoint of a line segment joining (x_1, y_1) and (x_2, y_2) is $[(x_1+ x_2)/2, (y_1+y_2)/2]$

∴ Mid-point of the line-segment joining the points (– 5, 4) and (9, – 8) = [(9-5)/2, (4-8)/2] = (2, -2)

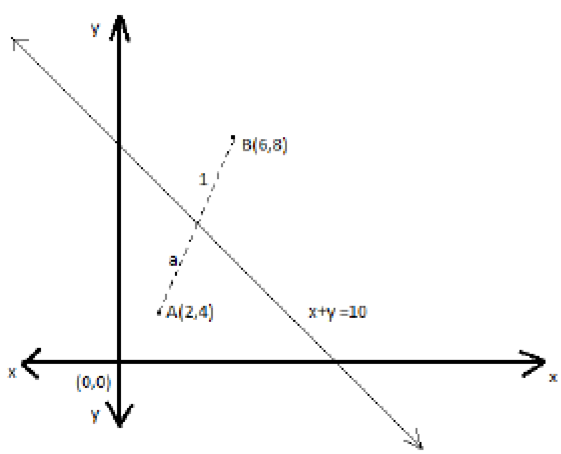
12. The line x + y = 10 divides line segment AB in the ratio a: 1. Find the value of a.



(A) ½
(B) 1
(C) 2
(D) 3

Answer: (B) 1

Solution:



The point, say P(x, y), divides the line AB into the ratio a: 1.
The equation for the point that divides a line in the ratio m: n is,

$$\left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n}\right)$$ where (x_1, y_1) and (x_2, y_2) are the coordinates of the endpoints of the line segment.

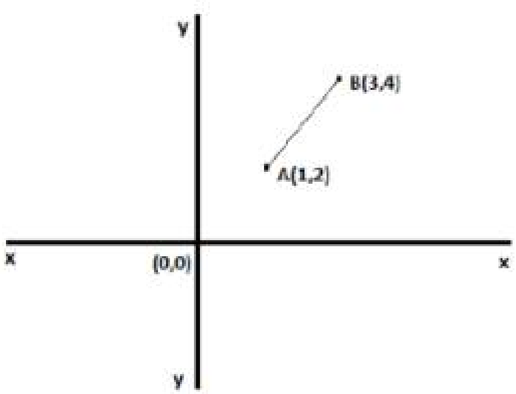
Applying the formula, we get $$\left(\frac{a \times 6 + 1 \times 2}{a+1}, \frac{a \times 8 + 1 \times 4}{a+1}\right)$$

This point lies on the line x + y = 10, so substitute the points in the equation for the line

$$\frac{a \times 6 + 1 \times 2}{a+1} + \frac{a \times 8 + 1 \times 4}{a+1} = 10$$

6a + 2 + 8a + 4 = 10(a+1)
4a = 4
a = 1

13. Point A (1, 2) and B (3, 4) are two ends of a line segment. Find the point which divides AB in the ratio 3:4



(A) (4,3)
(B) (2,3)
(C) 15/7, 22/7
(D) 13/7, 20/7

Answer: (D) 13/7, 20/7

Solution: The coordinates for the point that divides a line in the ratio m:n is,

$$\frac{n \times x_1 + m \times x_2}{m+n}, \frac{n + y_1 + m \times y_2}{n+m}$$

Substituting in the equation, we get

$$= \frac{3 \times 3 + 4 \times 1}{3+4}, \frac{3 \times 4 + 4 \times 2}{3+4}$$

= 13/7, 20/7

The point which divides the line segment in the ratio 3:4 is,

13/7, 20/7

14. Find the point (x,y) that divides the join of A(3,6) and B(7,10) in the ratio 3:1

(A) None of these
(B) (6,9)
(C) (4,5)
(D) (8,9)

Answer: (B) (6, 9)

Solution: If (x, y) divides the join of A(x_1,y_1) and (x_2,y_2) in the ratio m: n

Then, $x = \frac{mx_2 + nx_1}{m+n}$ and $y = \frac{my_2 + ny_1}{m+n}$

Here, x_1=3, x_2=7, y_1=6, y_2=10, m = 3 and n = 1

$x = \frac{3 \times 7 + 1 \times 3}{3+1}$ and $y = \frac{3 \times 10 + 1 \times 6}{3+1}$

x = 6 and y = 9

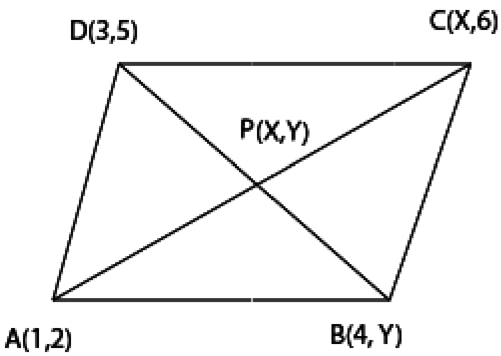
Therefore the point is (6, 9)

15. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

(A) (2,3)
(B) (4,3)
(C) (6,3)
(D) (2,5)

Answer: (C) (6, 3)

Solution:



The given points are : A(1,2), B(4, y), C(x, 6), and D(3,5)
Since, the diagonals of a parallelogram bisect each other.
∴ The coordinates of P are:

X= (x+1)/ 2 = (3+4)/2

⇒x+1=7⇒x=6
Y = (5+Y)/2= (6+2)/2
⇒5+y=8⇒y=3
∴ The required values of x and y are:
x=6,y=3.

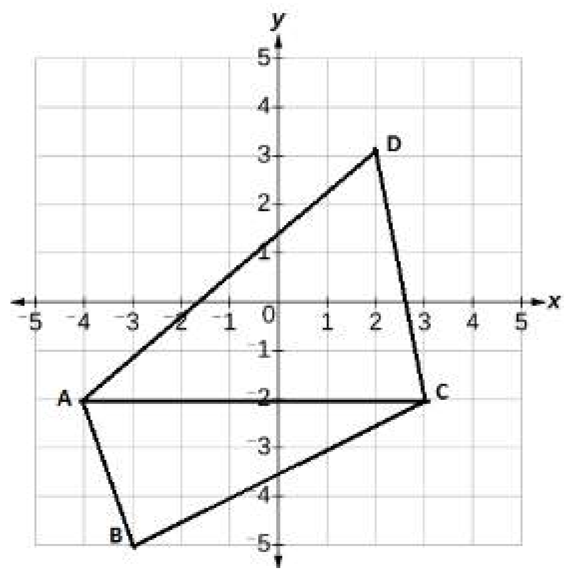
Area from coordinates

16. The area of a quadrilateral whose vertices taken in order are (–4, –2), (–3, –5), (3, –2) and (2, 3) is _______.

(A) 26 sq. units
(B) 28 sq. units
(C) 30 sq. units
(D) 27 sq. units

Answer: (B) 28 sq. units

Solution:



Consider the points A(−4,−2),B(−3,−5),C(3,−2) and D(2,3).
Area of a triangle having coordinates (x_1,y_1), (x_2,y_2), (x_3,y_3) is given by

$\frac{1}{2} \times |x_1 (y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|$

∴ Area of Δ ABC

$= 1/2 \mid (-4)(-5 + 2) - 3(-2 + 2) + 3(-2 + 5) \mid$

$= 1/2|20-8-6+15| = 21/2 = 10.5$ sq. units

Similarly, area of ΔACD

$= 1/2|(-4)(-2 - 3) + 3(3 + 2) + 2(-2 + 2)|$

$= 1/2|20+15| = 35/2 = 17.5$ sq. units

Now, area of quadrilateral ABCD = Area (ΔABC) + Area (ΔACD) = (10.5 + 17.5) sq. units = 28 sq. units.

17. If the points (a, 0), (0, b) and (1, 1) are collinear, then which of the following is true?

(A) (1/a) + (1/b)= 2
(B) (1/a) + (1/b)= 1
(C) (1/a) + (1/b)= 0
(D) (1/a) + (1/b)= 4

Answer: (B) (1/a) + (1/b) = 1

Solution: Area of a triangle formed by (x_1, y_1), (x_2, y_2), (x_3, y_3) = $1/2\ [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

For 3 points to be collinear the area of the triangle should be zero:

$\Rightarrow 1/2[a\ (b-1) + 0(1-0) + 1(0-b)] = 0$

$\Rightarrow 1/2[a\ (b-1) + 1(0-b)] = 0$

$\Rightarrow ab = a + b$

$\Rightarrow (1/a) + (1/b) = 1$

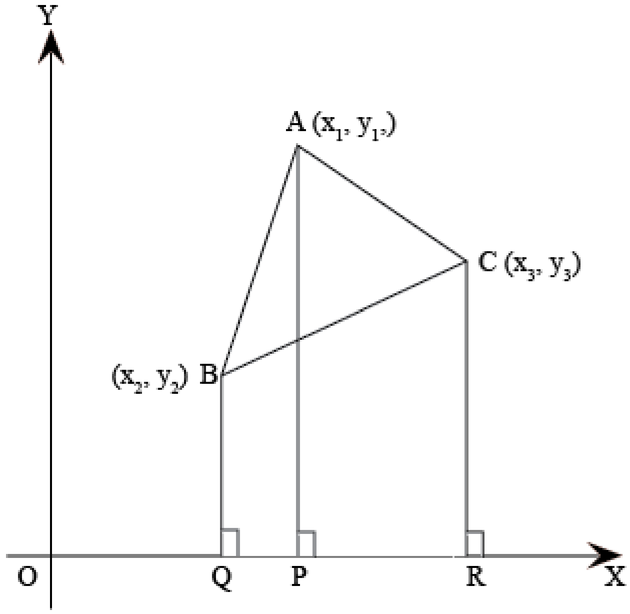
18. If 2 triangles have the same height, the ratio of their areas is equal to the

(A) Ratio of any 2 sides
(B) Ratio of their corresponding bases
(C) The ratio of their heights
(D) 1

Answer: (B) Ratio of their corresponding bases

Solution: Area of triangle 1 / Area of triangle 2= (½ × Base 1× height) / (½ × Base 2× height) = base 1/ base 2

19. The area of triangle with vertices $A(x_1,y_1)$,$B(x_2,y_2)$ and $C(x_3,y_3)$ is:



(A) $1/2[x_2(y_2-y_3) + x_3(y_3-y_1) + x_2(y_1-y_2)]$

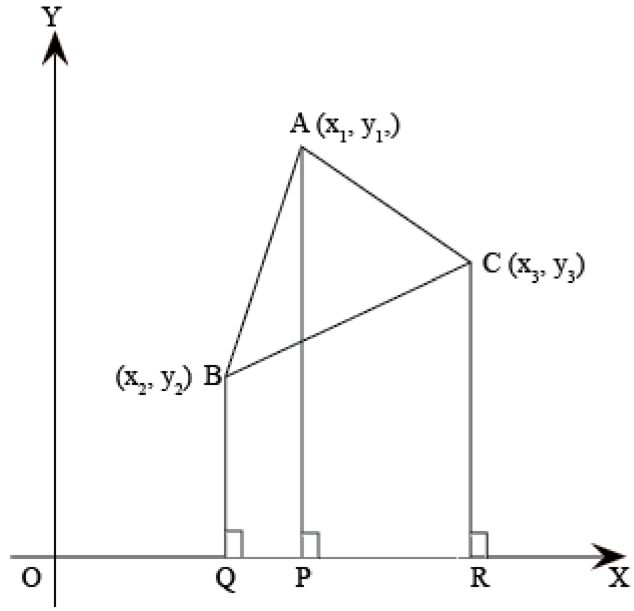
(B) $1/2[x_1(y_2-y_3) + x_3(y_3-y_1) + x_1(y_1-y_2)]$

(C) All of these

(D) $1/2[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$

Answer: (D) $1/2[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$

Solution:



Let ABC be any triangle whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$. Draw AP, QR and CR perpendiculars from A, B and C, respectively, to the x-axis. Clearly ABQP, APRC and BQRC are all trapezia.

Now from figure, it is clear that

Area of △ABC = Area of trapezium ABQP + Area of trapezium APRC - Area of trapezium BQRC.

Area of a trapezium =1/2 (Sum of parallel sides)(Distance between the parallel sides)

Therefore,

Area of ΔABC=1/2(BQ+AP)QP+1/2(AP+CR)PR−1/2(BQ+CR)QR

$=1/2(y_2+y_1)(x_1-x_2)+1/2(y_3+y_1)(x_3-x_1)-1/2(y_2+y_3)(x_3-x_2)$

$=1/2[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

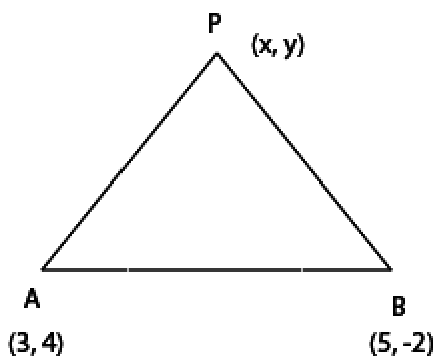
Thus, the area of ΔABC is the numerical value of the expression $1/2[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

20. P(x, y), A (3, 4) and B (5, -2) are the vertices of triangle PAB such that |PA|=|PB| and area of ΔPAB=10 sq. units, then $PA = k\sqrt{5}$ units. Find the value of k.

(A) 2
(B) 1
(C) $\sqrt{5}$
(D) 3

Answer: (A) 2

Solution:



Given, $|PA|=|PB|$
$\therefore PA^2=PB^2$

$\Rightarrow (x-3)^2+(y-4)^2=(x-5)^2+(y+2)^2$

$\Rightarrow x^2-6x+9+y^2-8y+16=x^2-10x+25+y^2+4y+4$

$\Rightarrow 4x-12y=4 \Rightarrow x-3y=1 \ldots (i)$

Area of $\Delta PAB=1/2|x(4+2)+3(-2-y)+5(y-4)|$

$\Rightarrow 10=1/2|6x-6-3y+5y-20|$

$\Rightarrow 20=6x+2y-26 \Rightarrow 6x+2y-26=20$ or $6x+2y-26=-20$

$\Rightarrow 6x+2y=46$ or $6x+2y=63x+y=23 \ldots (2)$

or $3x+y=3 \ldots (3)$

Solving (1) and (3), we get
x=1, y=0
Solving (1) and (2), we get
x=7, y=2
∴ Coordinates of P are (7, 2) and (1, 0)

Length of $PA = \sqrt{(7-3)^2 + (2-4)^2}$

$$PA = \sqrt{16+4} = 2\sqrt{5}$$

∴ $2\sqrt{5} = k\sqrt{5}$

⇒k=2

Also, Length of PA $= \sqrt{(1-3)^2 + (0-4)^2}$

$$= \sqrt{4+16} = 2\sqrt{5}$$

∴ $2\sqrt{5} = k\sqrt{5}$

⇒k=2