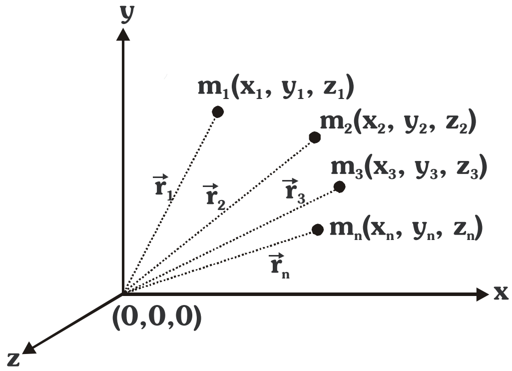
CENTER OF MASS, MOMENTUM & COLLISION

KEY CONCEPTS

CENTRE OF MASS :

For a system of particles centre of mass is that point at which its total mass is supposed to be concentrated. The centre of mass of an object is a point that represents the entire body and moves in the same way as a point mass having mass equal to that of the object, when subjected to the same external forces that act on the object.

Centre of mass of system of discrete particles



Total mass of the body : $M = m_1 + m_2 + ... + m_n$

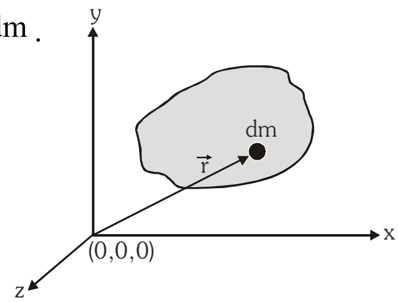
Then $\vec{R}_{CM} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...}{m_1 + m_2 + m_3 + ...} = \dfrac{1}{M}\Sigma m_i\vec{r}_i$

co-ordinates of centre of mass :

$$x_{cm} = \frac{1}{M}\Sigma m_i x_i,\ y_{cm} = \frac{1}{M}\Sigma m_i y_i \text{ and } z_{cm} = \frac{1}{M}\Sigma m_i z_i$$

Centre of mass of continuous distribution of particles

If the system has continuous distribution of mass, treating the mass element dm at position $\vec{r}$ as a point mass and replacing summation by integration. $\vec{R}_{CM} = \dfrac{1}{M}\int \vec{r}dm$.



So that $x_{cm} = \dfrac{1}{M}\int x\,dm$, $y_{cm} = \dfrac{1}{M}\int y\,dm$ and $z_{cm} = \dfrac{1}{M}\int z\,dm$

If co-ordinates of particles of mass m_1, m_2, ... are

(x_1, y_1, z_1), (x_2, y_2, z_2)....

then position vector of their centre of mass is

$$\vec{R}_{CM} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k}$$

$$= \frac{m_1(x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + m_2(x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) + m_3\left(x_3\hat{i} + y_3\hat{j} + z_3\hat{k}\right) + ...}{m_1 + m_2 + m_3 + ...}$$

$$= \frac{(m_1x_1 + m_2x_2 + ...)\hat{i} + (m_1y_1 + m_2y_2...)\hat{j} + (m_1z_1 + m_2z_2 + ..)\hat{k}}{m_1 + m_2 + m_3 + ..}$$

So, $x_{cm} = \left(\dfrac{m_1x_1 + m_2x_2 + ...}{m_1 + m_2 + m_3 + ...}\right)$, $y_{cm} = \left(\dfrac{m_1y_1 + m_2y_2 + ...}{m_1 + m_2 + ...}\right)$, $z_{cm} = \left(\dfrac{m_1z_1 + m_2z_2 + ...}{m_1 + m_2 + ...}\right)$

The centre of mass after removal of a part of a body

If a portion of a body is taken out, the remaining portion may be considered as,

Original mass (M) – mass of the removed part (m) = {original mass (M)} + {– mass of the removed part (m)}

The formula changes to : $x_{CM} = \dfrac{Mx - mx'}{M - m}$; $y_{CM} = \dfrac{My - my'}{M - m}$; $z_{CM} = \dfrac{Mz - mz'}{M - m}$

Where x', y' and z' represent the coordinates of the centre of mass of the removed part.

MOTION OF CENTRE OF MASS

As for a system of particles, position of centre of mass is $\vec{R}_{CM} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...}{m_1 + m_2 + m_3 + ...}$

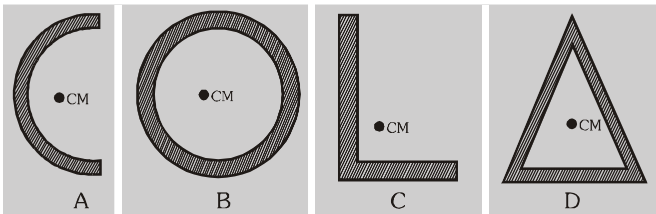
So $\dfrac{d}{dt}\left(\vec{R}_{CM}\right) = \dfrac{m_1\dfrac{d\vec{r}_1}{dt} + m_2\dfrac{d\vec{r}_2}{dt} + m_3\dfrac{d\vec{r}_3}{dt} + ...}{m_1 + m_2 + m_3 + ...} \Rightarrow$

Similarly acceleration $\vec{a}_{CM} = \dfrac{d}{dt}\left(\vec{v}_{CM}\right) = \dfrac{m_1\vec{a}_1 + m_2\vec{a}_2 + ...}{m_1 + m_2 + ...}$

We can write $M\vec{v}_{CM} = m_1\vec{v}_1 + m_2\vec{v}_2 + ... = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ...$ [$\because \vec{p} = m\vec{v}$]

$M\vec{v}_{CM} = \vec{p}_{CM}$ [$\because \Sigma\vec{p}_i = \vec{p}_{CM}$]

IMPORTANT POINTS



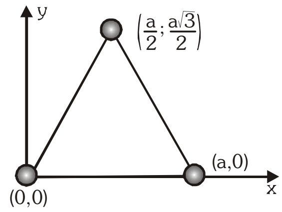
- There may or may not be any mass present physically at centre of mass (See Figure A, B, C)
- Centre of mass may be inside or outside of the body (See figure A, B, C)
- Position of centre of mass depends on the shape of the body. (See figure A, B, C)
- For a given shape it depends on the distribution of mass of within the body and is closer to massive part. (See figure A,C)
- For symmetrical bodies having homogeneous distribution of mass it coincides with centre of symmetry of geometrical centre. (See figure B,D).
- If we know the centre of mass of parts of the system and their masses, we can get the combined centre of mass by treating the parts as point particles placed at their respective centre of masses.
- It is independent of the co-ordinate system, e.g., the centre of mass of a ring is at its centre whatever be the co-ordinate system.
- If the origin of co-ordinate system is at centre of mass, i.e., $\vec{R}_{CM} = 0$, then by definition.

$\dfrac{1}{M}\Sigma m_i\vec{r}_i = 0 \Rightarrow \Sigma m_i\vec{r}_i = 0$

The sum of the moments of the masses of a system about its centre of mass is always zero.

Ex. Three bodies of equal masses are placed at (0, 0), (a, 0) and at $\left(\frac{a}{2},\frac{a\sqrt{3}}{2}\right)$.

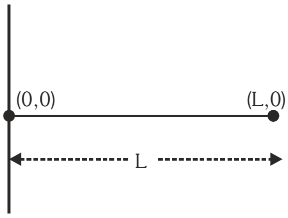
Find out the co-ordinates of centre of mass.



Sol. $x_{CM}=\dfrac{0\times m+a\times m+\frac{a}{2}\times m}{m+m+m}=\dfrac{a}{2}$, $y_{CM}=\dfrac{0\times m+0\times m+\frac{a\sqrt{3}}{2}\times m}{m+m+m}=\dfrac{a\sqrt{3}}{6}$

Ex. Calculate the position of the centre of mass of a system consisting of two particles of masses m_1 and m_2 separated by a distance L apart, from m_1.

Sol. Treating the line joining the two particles as x axis



$$x_{CM}=\frac{m_1\times 0+m_2\times L}{m_1+m_2}=\frac{m_2L}{m_1+m_2},\ y_{CM}=0 \qquad z_{CM}=0$$

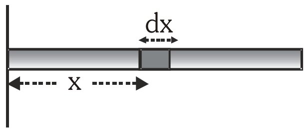
Ex. If the linear density of a rod of length L varies as $\lambda = A + Bx$, compute position of its centre of mass.

Sol. Let the x–axis be along the length of the rod and origin at one of its end as shown in figure. As rod is along x–axis, for all points on it y and z will be zero so, $y_{CM}=0$ and $z_{CM}=0$ i.e., centre of mass will be on the rod.

Now consider an element of rod of length dx at a distance x from the origin, mass of this element

$dm=\lambda dx=(A+Bx)dx$

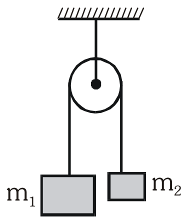
$$\text{so, } x_{CM}=\frac{\int_0^L x\,dm}{\int_0^L dm}=\frac{\int_0^L x(A+Bx)dx}{\int_0^L (A+Bx)dx}=\frac{\frac{AL^2}{2}+\frac{BL^3}{3}}{AL+\frac{BL^2}{2}}=\frac{L(3A+2BL)}{3(2A+BL)}$$



Note : (i) If the rod is of uniform density then $\lambda = A$ = constant & $B = 0$ then $X_{CM} = L/2$

(ii) If the density of rod varies linearly with x, then $\lambda = Bx$ and $A = 0$ then $X_{CM} = 2L/3$

Ex. Two bodies of masses m_1 and m_2 ($<m_1$) are connected to the ends of a massless cord and allowed to move as shown in. The pulley is both massless and frictionless. Calculate the acceleration of the centre of mass.



Sol. If $\vec{a}$ is the acceleration of m_1,then $-\vec{a}$ is the acceleration of m_2 then

acceleration of each body $a=\dfrac{\text{Net force}}{\text{Net mass}}=\left(\dfrac{m_1-m_2}{m_1+m_2}\right)g$

$$\vec{a}_{cm}=\frac{m_1\vec{a}+m_2(-\vec{a})}{m_1+m_2}=\left(\frac{m_1-m_2}{m_1+m_2}\right)\vec{a}$$

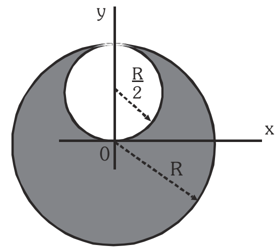
But $\vec{a}=\left(\dfrac{m_1-m_2}{m_1+m_2}\right)\vec{g}$ so $\vec{a}_{cm}=\left(\dfrac{m_1-m_2}{m_1+m_2}\right)^2\vec{g}$

Ex. A circle of radius R is cut from a uniform thin sheet of metal. A circular hole of radius $\frac{R}{2}$ is now cut out of the circle, with the hole tangent to the rim. Find the distance of centre of mass from the centre of the original uncut circle to the centre of mass.

Sol. We treat the hole as a 'negative mass' object that is combined with the original uncut circle. (When the two are added together, the hole region then has zero mass). By symmetry, the CM lies along the +y–axis in figure, so $x_{CM} = 0$. With the origin at the centre of the original circle whose mass is assumed to be m.

Mass of original uncut circle $m_1 = m$ & (0, 0)

Mass of hole of negative mass : $m_2 = \frac{m}{4}$; Location of CM $\left(0, \frac{R}{2}\right)$



Thus $$y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{m(0) + \left(-\frac{m}{4}\right)\frac{R}{2}}{m + \left(-\frac{m}{4}\right)} = \frac{R}{6}$$

So the centre of mass is at the point $\left(0, -\frac{R}{6}\right)$

Ex. Two particles of mass 1 kg and 0.5 kg are moving in the same direction with speeds of 2m/s and 6m/s, respectively, on a smooth horizontal surface. Find the speed of centre of mass of the system.

Sol. Velocity of centre of mass of the system $\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$ Since the particles m_1 and m_2 are moving in same direction, $m_1\vec{v}_1$ and $m_2\vec{v}_2$ are parallel. $\Rightarrow$ $|m_1\vec{v}_1 + m_2\vec{v}_2| = m_1v_1 + m_2v_2$

Therefore, $$v_{cm} = \frac{|m_1\vec{v}_1 + m_2\vec{v}_2|}{m_1 + m_2} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{(1)(2) + \left(\frac{1}{2}\right)(6)}{\left(1 + \frac{1}{2}\right)} = 3.33 \text{ ms}^{-1}$$

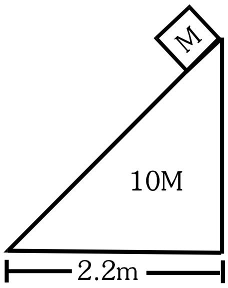
Ex. Two particles of masses 2 kg and 4 kg are approaching towards each other with acceleration 1 m/s^2 and 2 m/s^2, respectively, on a smooth horizontal surface. Find the acceleration of centre of mass of the system.

Sol. The acceleration of centre of mass of the system $\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$ $\Rightarrow$ $a_{cm} = \frac{|m_1\vec{a}_1 + m_2a_2|}{m_1 + m_2}$

Since $\vec{a}_1$ and $\vec{a}_2$ are anti–parallel, so $a_{cm} = \frac{|m_1a_1 - m_2a_2|}{m_1 + m_2} = \frac{|(2)(1) - (4)(2)|}{2 + 4} = 1 \text{ ms}^{-2}$

Since $m_2a_2 > m_1a_1$ so the direction of acceleration of centre of mass will be directed in the direction of a_2.

Ex. A block of mass M is placed on the top of a bigger block of mass 10 M as shown in figure. All the surfaces are frictionless. The system is released from rest. Find the distance moved by the bigger block at the instant the smaller block reaches the ground.



Sol. If the bigger block moves towards right by a distance (X), the smaller block will move towards left by a distance (2.2 –X) (taking the two blocks together as the system). The horizontal position of CM remains same $\Rightarrow$ M (2.2 – X) = 10 MX $\Rightarrow$ X = 0.2 m.

MOMENTUM

The total quantity of motion possessed by a moving body is known as the momentum of the body. It is the product of the mass and velocity of a body i.e. momentum $\vec{p} = m\vec{v}$

IMPULSE

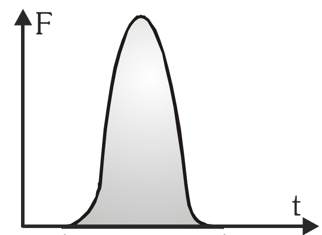
When a large force act for an extremely short duration, neither the magnitude of the force nor the time for which it acts is important. In such a case, the total effect of force is measured. The total effect of force is called impulse (measure of the action of force). This type of force is generally variable in magnitude and is sometimes called impulsive force.

If a large force act on a body or particle for a small time then

Impulse = product of force with time.

Suppose a force $\vec{F}$ acts for a short time dt then impulse = $\vec{F}dt$

For a finite interval of time from t_1 to t_2 then the impulse = $\int_{t_1}^{t_2} \vec{F}dt$

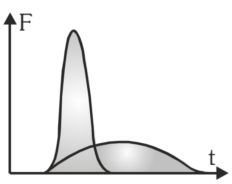


impulse = area under curve

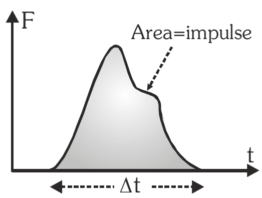
If constant force $\vec{F}$ acts for an interval Δt then Impulse = $\vec{F}\Delta t$

Impulse – Momentum theorem :

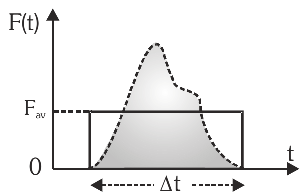
Impulse of a force is equal to the change of momentum $\boxed{\vec{F}\Delta t = \Delta\vec{p}}$



A large force acting for a short time and a small force acting for a long time may result in the same impulse, as indicated by the equal areas under the force-versus time curves.



The force F(t) involved in the collision between a ball and bat. The interaction occurs during the time Δt. The impulse is equal in magnitude to the shaded area under the curve.

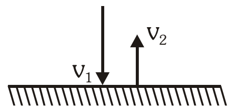


The constant force F_{av} acting for the same time interval Δt produces the same impulse as in figure since the areas under the two graphs are the same.

Ex. A ball of mass 50 g is dropped from a height h = 10 m. It rebounds losing 75 percent of its total mechanical energy. If it remains in contact with the ground for 0.01s, find the impulse of the impact force.

Sol. Impulse = change in momentum = $m(v_1+v_2)$

Here $v_1 = \sqrt{2gh}$ and for v_2, $\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2\left(1-\frac{75}{100}\right) \Rightarrow v_2 = \frac{v_1}{2}$



So impulse $= m\left(v_1+\frac{v_1}{2}\right) = \frac{3mv_1}{2} = \frac{3}{2}m \times \sqrt{2gh} = \frac{3}{2} \times 50 \times 10^{-3} \times \sqrt{2\times 9.8\times 10} = 1.05$ N–s

LAW OF CONSERVATION OF LINEAR MOMENTUM

According to Newton's Second law of motion the rate of change of momentum is equal to the applied force.

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \text{if } \vec{F} = \vec{0} \text{ then } \frac{d\vec{p}}{dt} = \vec{0} \text{ i.e. } \vec{p} = \text{constant}$$

This leads to the law of conservation of momentum which is" In the absence of external forces, the total momentum of the system is conserved."

IMPORTANT POINTS

- For an isolated system, the initial momentum of the system is equal to the final momentum of the system. If the system consists of n bodies having momentum

  $\vec{p}_1, \vec{p}_2, \vec{p}_3, ...\vec{p}_n$, then $\vec{p}_1+\vec{p}_2+\vec{p}_3+..........+\vec{p}_n = \text{constant}$
- As linear momentum depends on frame of reference. Observers in different frames would find different values of linear momentum of a given system but each would agree that his own value of linear momentum does not change with time provided. But the system should be isolated and closed, i.e., law of conservation of linear momentum is independent of frame of reference though linear momentum depends on frame of reference.
- Conservation of linear momentum is equivalent to Newton's III law of motion for a system of two particles in absence of external force by law of conservation of linear momentum.

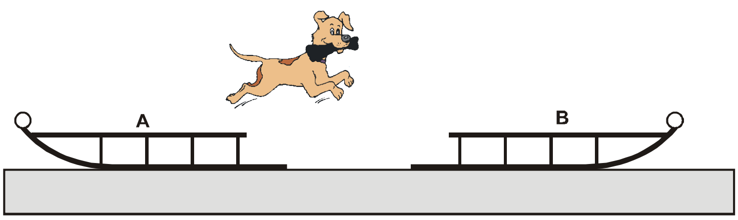
  $\Rightarrow \quad \vec{p}_1+\vec{p}_2 = \text{constant} \qquad \text{i.e. } m_1\vec{v}_1+m_2\vec{v}_2 = \text{constant}$

  Differentiating above with respect to time $m_1\frac{d\vec{v}_1}{dt}+m_2\frac{d\vec{v}_2}{dt} = \vec{0}$ [as m is constant]

  $\Rightarrow m_1\vec{a}_1+m_2\vec{a}_2 = \vec{0}$ $\left[\because \frac{d\vec{v}_1}{dt} = \vec{a}\right] \Rightarrow \vec{F}_1+\vec{F}_2 = \vec{0}$ $[\because \vec{F} = m\vec{a}] \Rightarrow \vec{F}_1 = -\vec{F}_2$

  i.e., for every action there is equal and opposite reaction which is Newton's III law of motion.
- This law is universal, i.e., it applies to body macroscopic as well as microscopic systems.

Ex. Two 22.7 kg ice sleds A and B are placed a short distance apart, one directly behind the other, as shown in figure. A 3.63 kg dog, standing on one sled, jumps across to the other and immediately back to the first. Both jumps are made at a speed of 3.05 ms^{-1} relative to the ice. Find the final speeds of the two sleds.



Sol. Total momentum imparted to B $p_B = 2 \times 3.63 \times 3.05$ kg ms^{-1}.

Velocity of $B = \frac{p_B}{m_B} = \frac{2\times3.63\times3.05}{22.7} =$ 0.975 ms^{-1}.

Velocity of A when the dog jumps away from $A = \frac{p_A}{m_A} = \frac{3.63\times3.05}{22.7}$ = 0.4877 ms^{-1}.

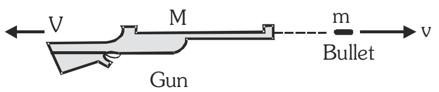
When the dog comes back to A, Velocity of $A = \frac{22.7\times0.4877+3.63\times3.05}{22.7+3.63}$ = 0.841 ms^{-1}.

APPLICATIONS OF CONSERVATION OF LINEAR MOMENTUM

Firing a Bullet from a Gun :

- If the bullet is the system, the force exerted by trigger will be external and so the linear momentum of the bullet will change from 0 to mv. This is not the violation of law of conservation of linear momentum as linear momentum is conserved only in absence of external force .
- If the bullet and gun is the system, the force exerted by trigger will be internal so.

  Total momentum of the system $\vec{p}_s = \vec{p}_B + \vec{p}_G$ = constant.



Now as initially both bullet and gun are at rest so $\vec{p}_B + \vec{p}_G = \vec{0}$ From this it is evident that :

- $\vec{p}_G = -\vec{p}_B$, i.e., if bullet acquires forward momentum, the gun will acquire equal and opposite (backward) momentum.
- As $\vec{p} = m\vec{v},\ m\vec{v} + M\vec{V} = \vec{0}$, i.e, $\vec{V} = -\frac{m}{M}\vec{v}$ i.e, if the bullet moves forward, gun 'recoils' or 'kicks' backward. Heavier the gun lesser will be the recoil velocity V.
- Kinetic energy $K = \frac{p^2}{2m}$ and $|\vec{p}_B| = |\vec{p}_G| = p$ Kinetic energy of gun $K_G = \frac{p^2}{2M}$,

  Kinetic energy of bullet $K_B = \frac{p^2}{2m}$ $\therefore \frac{K_G}{K_B} = \frac{m}{M} < 1$ ($\because$ M $\gg$ m) Thus kinetic energy of gun is smaller than bullet i.e., kinetic energy of bullet and gun will not be equal.
- Initial kinetic energy of the system is zero as both are at rest initially.

  Final kinetic energy of the system $[(1/2)(mv^2 + MV^2)] > 0$.

  So, here kinetic energy of the system is not constant but increases. If PE is assumed to be constant then Mechanical energy = (kinetic energy + potential energy) will also increase. However, energy is always conserved. Here chemical energy of gun powder is converted into KE.

Ex. A bullet of mass 100g is fired by a gun of 10kg with a speed 2000 m/s. Find recoil velocity of gun.

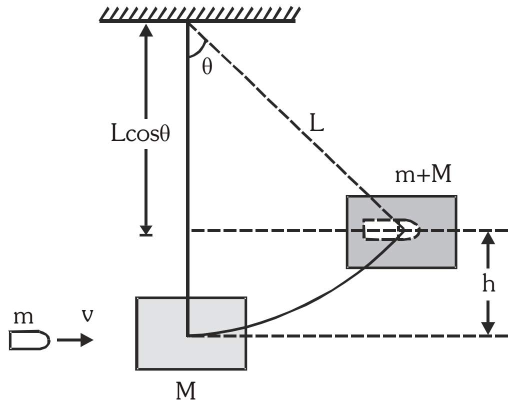
Sol. According to conservation of momentum mv + MV = 0.

Velocity of gun $V = -\frac{mv}{M} = -\frac{0.1\times2000}{10} = -20\text{m/s}$

Block Bullet System :

(a) When bullet remains in the block

Conserving momentum of bullet and block mv + 0 = (M+m) V



Velocity of block $V=\dfrac{mv}{M+m}$ ...(i)

By conservation of mechanical energy

$\dfrac{1}{2}(M+m)V^2=(M+m)gh \Rightarrow V=\sqrt{2gh}$ ....(ii)

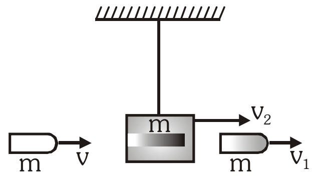
From eq^n. (i) and eq^n. (ii) $\dfrac{mv}{M+m}=\sqrt{2gh}$;

Speed of bullet $v=\dfrac{(M+m)\sqrt{2gh}}{m}$,

Maximum height gained by block $h=\dfrac{V^2}{2g}=\dfrac{m^2v^2}{2g(M+m)^2}$

$h = L - L\cos\theta$ $\therefore \cos\theta = 1-\dfrac{h}{L} \Rightarrow \theta=\cos^{-1}\left(1-\dfrac{h}{L}\right)$

(b) If bullet moves out of the block



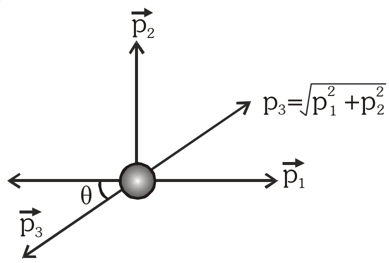
Conserving momentum $mv + 0 = mv_1 + Mv_2$

$m(v - v_1) = Mv_2$ ....(i)

Conserving energy $\dfrac{1}{2}Mv_2^2 = Mgh \Rightarrow v_2=\sqrt{2gh}$ ....(ii)

From eq^n. (i) & eq^n. (ii) $m(v - v_1) = M\sqrt{2gh} \Rightarrow h=\dfrac{m^2(v-v_1)^2}{2gM^2}$

Explosion of a Bomb at rest



Conserving momentum

$\vec{p}_1+\vec{p}_2+\vec{p}_3=\vec{0} \Rightarrow \vec{p}_3=-(\vec{p}_1+\vec{p}_2) \Rightarrow p_3=\sqrt{p_1^2+p_2^2}$ as $\vec{p}_1 \perp \vec{p}_2$

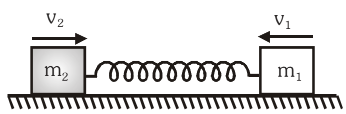
Angle made by $\vec{p}_3$ from $\vec{p}_1 = \pi + \theta$

Angle made by $\vec{p}_3$ from $\vec{p}_2 = \dfrac{\pi}{2}+\theta$

Energy released in explosion $= K_f - K_i = \dfrac{p_1^2}{2m_1}+\dfrac{p_2^2}{2m_2}+\dfrac{p_3^2}{2m_3} - 0 = \dfrac{p_1^2}{2m_1}+\dfrac{p_2^2}{2m_2}+\dfrac{p_3^2}{2m_3}$

Motion of Two Masses Connected to a Spring

Consider two blocks, resting on a frictionless surface and connected by a massless spring as shown in figure. If the spring is stretched (or compressed) and then released from rest,



Then $F_{ext}=0$ so $\vec{p}_s=\vec{p}_1+\vec{p}_2$ = constant

However, initially both the blocks were at rest so, $\vec{p}_1+\vec{p}_2=\vec{0}$

It is clear that :

- $\vec{p}_2 = -\vec{p}_1$, i.e., at any instant the two blocks will have momentum equal in magnitude but opposite in direction (Though they have different values of momentum at different positions).
- As momentum $\vec{p} = m\vec{v}$, $m_1\vec{v}_1 + m_2\vec{v}_2 = \vec{0} \Rightarrow \vec{v}_2 = -\left(\frac{m_1}{m_2}\right)\vec{v}_1$

  The two blocks always move in opposite directions with lighter block moving faster.
- Kinetic energy $KE = \frac{p^2}{2m}$ and $|\vec{p}_1| = |\vec{p}_2|$, $\frac{KE_1}{KE_2} = \frac{m_2}{m_1}$ or the kinetic energy of two blocks will not be equal but in the inverse ratio of their masses and so lighter block will have greater kinetic energy.
- Initially kinetic energy of the blocks is zero (as both are at rest) but after some time kinetic energy of the blocks is not zero (as both are in motion). So, kinetic energy is not constant but changes. Here during motion of blocks KE is converted into elastic potential energy of the spring and vice–versa but total mechanical energy of the system remain constant.

  Kinetic energy + Potential energy = Mechanical Energy = Constant

Note – If $\vec{F}$ is the average of the time varying force during collision and Δt is the duration of collision then impulse $\vec{I} = \vec{F}\Delta t$.

Conservation of Linear Momentum During Impact :

If two bodies of masses m_1 and m_2 collide in air, the total external force acting on the system of bodies $(m_1 + m_2)$ is equal to $\vec{F}_1 + m_1\vec{g} + \vec{F}_2 + m_2\vec{g} \Rightarrow F_{total} = m_1\vec{g} + m_2\vec{g} + \vec{F}_1 + \vec{F}_2$

During collision the impact forces $\vec{F}_1$ and $\vec{F}_2$ are equal in magnitude and opposite in direction.

According to Newton's 3rd law of motion, $\vec{F}_1 + \vec{F}_2 = \vec{0} \Rightarrow \vec{F}_{net} = m_1\vec{g} + m_2\vec{g}$

So Impulse = $\vec{F}_{net}\Delta t = (m_1\vec{g} + m_2\vec{g})\,\Delta t$

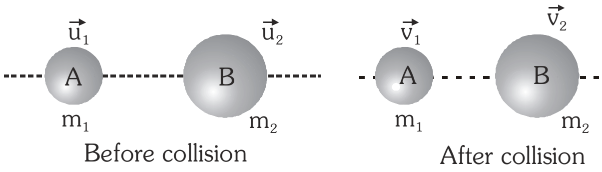
Since Δt is a very small time interval, the impulse $F(\Delta t)$ will be negligibly small. As impulse is equal to change in momentum of the system, a negligible impulse means negligible change of momentum. Let the change of momentum of 1 & 2 be $\Delta\vec{p}_1 \& \Delta\vec{p}_2$, respectively then the total change in momentum of the system $\Rightarrow \Delta\vec{p} = \Delta\vec{p}_1 + \Delta p_2 = \vec{F}_{net}.dt \approx \vec{0} \Rightarrow \Delta(\vec{p}_1 + \vec{p}_2) = 0 \Rightarrow \vec{p}_1 + \vec{p}_2 =$ constant.

Therefore, the net or total momentum of the colliding bodies remains practically unchanged along the line of action (impact) during the collision. In other words, the momentum of the system remains constant or conserved during the period of impact. Therefore, we can conveniently equate the net momentum of the colliding bodies at the beginning and at the end of the collision (or just before and just after the impact).

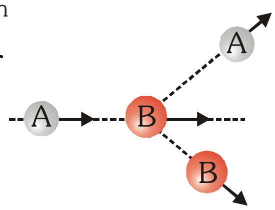
Note : Remember that the impact force F is not an external force for the system of colliding bodies. If no external force acts on the system, its momentum remains constant for all the times including the time of collision. Even if some external forces like gravitation and friction (known as non–impulsive forces in general) are present, we can conserved the momentum of the system during the impact, because the finite external forces cannot change the momentum of the system significantly in very short time. Therefore, the change in position of the system during infinitesimal time of impact can also be neglected.

- Types of collision according to the direction of collision :

(a) **Head on collision** : Direction of velocities of bodies is similar to the direction of collision.



(b) **Oblique collision** : Direction of velocities of bodies is not similar to the direction of collision.



- Types of collision according to the conservation law of kinetic energy :

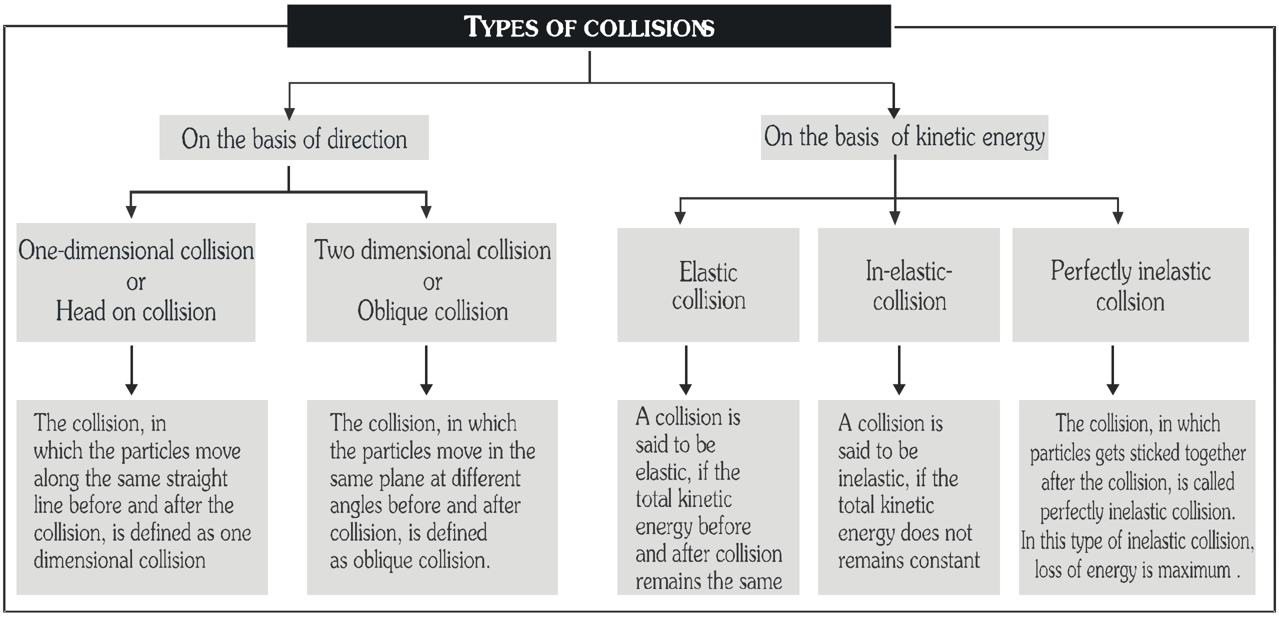
(a) **Elastic collision** : $KE_{\text{before collision}} = KE_{\text{after collision}}$

(b) **Inelastic collision** : kinetic energy is not conserved.

Some energy is lost in collision $KE_{\text{before collision}} > KE_{\text{after collision}}$

(c) **Perfect inelastic collision :** Two bodies stick together after the collision.

momentum remains conserved in all types of collisions.



Coefficient of restitution (e)

The coefficient of restitution is defined as the ratio of the impulses of recovery and deformation of either body.

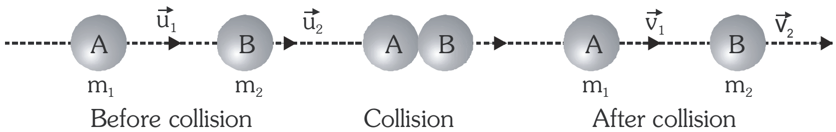
$$e = \frac{\text{impulse of recovery}}{\text{impulse of deformation}}$$

$$e = \frac{\text{velocity of separation along line of impact}}{\text{velocity of approach along line of impact}}$$

Value of e is 1 for elastic collision, 0 for perfectly inelastic collision and $0 < e < 1$ for inelastic collision.

HEAD ON ELASTIC COLLISION

The elastic collision in which the colliding bodies move along the same straight line path before and after the collision.



Assuming initial direction of motion to be positive and $u_1 > u_2$ (so that collision may take place) and applying law of conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \Rightarrow m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad ...(i)$$

For elastic collision, kinetic energy before collision must be equal to kinetic energy after collision, i.e.,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \Rightarrow m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \quad ...(ii)$$

Dividing equation (ii) by (i) $u_1 + v_1 = v_2 + u_2 \Rightarrow (u_1 - u_2) = (v_2 - v_1)$ ...(iii)

In 1–D elastic collision 'velocity of approach' before collision is equal to the 'velocity of recession' after collision, no matter what the masses of the colliding particles be.

This law is called **Newton's law for elastic collision**

Now if we multiply equation (iii) by m_2 and subtracting it from (i)

$$(m_1 - m_2)\,u_1 + 2m_2u_2 = (m_1 + m_2)v_1 \Rightarrow v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2}{m_1 + m_2}u_2 \quad ...(iv)$$

Similarly multiplying equation (iii) by m_1 and adding it to equation (i)

$$2m_1u_1 + (m_2 - m_1)u_2 = (m_2 + m_1)v_2 \Rightarrow v_2 = \frac{2m_1}{m_1 + m_2}u_1 + \frac{m_2 - m_1}{m_1 + m_2}u_2 \quad ...(v)$$

IMPORTANT POINTS

- **If the two bodies are of equal masses :** $m_1 = m_2 = m$, $v_1 = u_2$ and $v_2 = u_1$

  Thus, if two bodies of equal masses undergo elastic collision in one dimension, then after the collision, the bodies will exchange their velocities.
- **If two bodies are of equal masses and second body is at rest.**

  $m_1 = m_2$ and initial velocity of second body $u_2 = 0$, $v_1 = 0$, $v_2 = u_1$

  When body A collides against body B of equal mass at rest, the body A comes to rest and the body B moves on with the velocity of the body A. In this case transfer of energy is hundred percent

  e.g.. Billiard's Ball, Nuclear moderation.
- **If the mass of a body is negligible as compared to other.**

  If $m_1 >> m_2$ and $u_2 = 0$ then $v_1 = u_1$, $v_2 = 2u_1$

  When a heavy body A collides against a light body B at rest, the body A should keep on moving with same velocity and the body B will move with velocity double that of A.

  If $m_2 >> m_1$ and $u_2 = 0$ then $v_2 = 0$, $v_1 = -u_1$

  When light body A collides against a heavy body B at rest, the body A should start moving with same velocity just in opposite direction while the body B should practically remains at rest.

Ex. Two ball of mass 5kg each is moving in opposite directions with equal speed 5m/s. collides head on with each other. Find out the final velocities of the balls if collision is elastic.

Sol. Here $m_1 = m_2 = 5kg$, $u_1 = 5$ m/s, $u_2 = -5$ m/s

In such type of condition velocity get interchange so $v_2 = u_1 = 5$ m/s & $v_1 = u_2 = -5$ m/s

Ex. A ball of 0.1 kg makes an elastic head on collision with a ball of unknown mass that is initially at rest. If the 0.1kg ball rebounds at one third of its original speed. What is the mass of other ball ?

Sol. Here $m_1 = 0.1$ kg, $m_2 = ?$, $u_2 = 0$, $u_1 = u$, $v_1 = -u/3$

As $$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u + \frac{2m_2u_2}{m_1+m_2} \Rightarrow -\frac{u}{3} = \left(\frac{0.1 - m_2}{0.1 + m_2}\right)u \Rightarrow m_2 = 0.2 \text{ kg}$$

HEAD ON INELASTIC COLLISION OF TWO PARTICLES

Let the coefficient of restitution for collision is e

(i) Momentum is conserved $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ ...(i)

(ii) Kinetic energy is not conserved.

(iii) According to Newton's law $\frac{v_2 - v_1}{u_2 - u_1} = -e$ ...(ii)

By solving eq. (i) and (ii)

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)u_2 \text{ , } v_2 = \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2 + \left(\frac{(1+e)m_1}{m_1 + m_2}\right)u_1$$

PERFECT INELASTIC COLLISION

In case of inelastic collision, after collision two bodies move with same velocity (or stick together).

If two particles of masses m_1 and m_2, moving with velocity u_1 and u_2 ($u_2 < u_1$) respectively along the same line collide 'head on ' and after collision they have same common velocity v, then by conservation of linear momentum,

$$m_1u_1 + m_2u_2 = m_1v + m_2v \Rightarrow v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)} \quad ...(i)$$

Kinetic energy of the system before collision is $KE_I = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

And after collision is $KE_f = \frac{1}{2}(m_1+m_2)v^2$

Loss in KE during collision

$$\Delta KE = KE_i - KE_f = \left[\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right] - \frac{1}{2}(m_1 + m_2)v^2 \quad ...(ii)$$

Substituting the value of v from eq. (i),

$$\Delta KE = \frac{1}{2}[(m_1u_1^2 + m_2u_2^2) - \frac{(m_1u_1 + m_2u_2)^2}{(m_1 + m_2)}]$$

$$\Rightarrow \Delta KE = \frac{1}{2}\left[\frac{m_1m_2(u_1^2 + u_2^2 - 2u_1u_2)}{(m_1 + m_2)}\right] \Rightarrow \boxed{\Delta KE = \frac{1}{2}\frac{m_1m_2}{(m_1 + m_2)}(u_1 - u_2)^2}$$

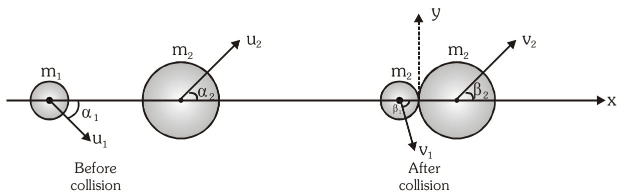
If the target is initially at rest $u_2 = 0$ and $u_1 = u$

$$\Delta KE = \frac{m_1m_2}{2(m_1 + m_2)}u^2, \quad \frac{\Delta KE}{KE_i} = \frac{m_2}{(m_1 + m_2)} \quad [\because KE_i = \frac{1}{2}m_1u_1^2]$$

Now if target is massive, i.e., $m_2 >> m_1$ then $\frac{\Delta KE}{KE_i} \approx 1$ so percentage loss in KE = 100%

i.e., if a light moving body strikes a heavy target at rest and sticks to it, practically all its KE is lost.

Oblique Collision



In oblique impact the relative velocity of approach of the bodies doesn't coincide with the line of impact. Conserving the momentum of system in directions along normal (x axis in our case) and tangential (y axis in our case)

$m_1u_1\cos\alpha_1 + m_2u_2\cos\alpha_2 = m_1v_1\cos\beta_1 + m_2v_2\cos\beta_2$ and $m_2u_2\sin\alpha_2 - m_1u_1\sin\alpha_1 = m_2v_2\sin\beta_2 - m_1v_1\sin\beta_1$

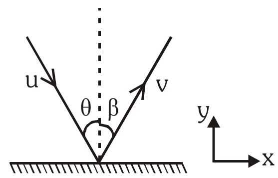
Since no force is acting on m_1 and m_2 along the tangent (i.e. y–axis) the individual momentum of m_1 and m_2 remains conserved. $m_1u_1\sin\alpha_1 = m_1v_1\sin\beta_1$ & $m_2u_2\sin\alpha_2 = m_2v_2\sin\beta_2$

By using Newton's experimental law along the line of impact $e = \dfrac{v_2\cos\beta_2 - v_1\cos\beta_1}{u_1\cos\alpha_1 - u_2\cos\alpha_2}$

Oblique Impact on a Fixed Plane

Let a small ball collides with a smooth horizontal floor with a speed u at an angle θ to the vertical as shown in the figure. Just after the collision, let the ball leaves the floor with a speed v at angle β to vertical.

It is quite clear that the line of action is perpendicular to the floor. Therefore, the impact takes place along the (normal) vertical. Now we can use Newton's experimental law as



$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}}$$

⇒ e [velocity of approach] = velocity of separation

⇒ $e[u\cos\theta(-\hat{j})] = -[v\cos\beta(+\hat{j})] \Rightarrow v\cos\beta = e\,u\cos\theta$ ...(i)

Since impulsive force N acts on the body along the normal, we cannot conserve its momentum. Since along horizontal the component of N is zero, therefore we can conserve the horizontal momentum of the body.

Momentum $(p_x)_{body}$ = constant ⇒ $(p_x)_{initial} = (p_x)_{final}$

⇒ $m\,u\sin\theta = mv\sin\beta \Rightarrow v\sin\beta = u\sin\theta$ ..(ii)

Squaring equations(i) and (ii) and adding, $v^2\cos^2\beta + v^2\sin^2\beta = e^2u^2\cos^2\theta + u^2\sin^2\theta$

⇒ $v^2 = u^2[e^2\cos^2\theta + \sin^2\theta]$ ⇒ $v = u\sqrt{\sin^2\theta + e^2\cos^2\theta}$

Dividing equation (i) by (ii)

⇒ $\dfrac{v\cos\beta}{v\sin\beta} = \dfrac{eu\cos\theta}{u\sin\theta} \Rightarrow \cot\beta = e\cot\theta \Rightarrow \beta = \cot^{-1}(e\cot\theta)$

Impulse of the blow = change of momentum of the body

$= \{(mv\sin\beta)\hat{i} + (mv\cos\beta)\hat{j}\} - \{(mu\sin\theta)\hat{i} - (mu\cos\theta)\hat{j}\}$

$= (mv\sin\beta - mu\sin\theta)\hat{i} + (mv\cos\beta + mu\cos\theta)\hat{j}$

Since $v\sin\beta = u\sin\theta$ ⇒ Impulse $= m(v\cos\beta + u\cos\theta)\hat{j}$

Putting $v\cos\beta = eu\cos\theta$ from eq. (i),

Impulse $= m(1+e)u\cos\theta\hat{j}$ ∴ Magnitude of the impulse $= m(1+e)u\cos\theta$

Change in Kinetic energy: $\Delta K.E. = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

Putting the value of v we obtain

$$\Delta KE = \frac{1}{2}m\left[\left[\sqrt{u\left(\sin^2\theta + e^2\cos^2\theta\right)}\right]^2 - u^2\right] = \frac{1}{2}mu^2\,[\sin^2\theta + e^2\cos^2\theta - 1]$$

$$= -\frac{1}{2}mu^2\,[\cos^2\theta - e^2\cos^2\theta] = -\frac{1}{2}(1-e^2)\,mu^2\cos^2\theta$$

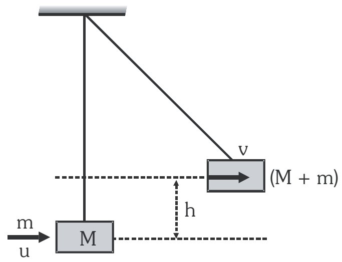
Negative sign indicates the loss of kinetic energy

IMPORTANT POINTS

- Momentum remains conserved in all types of collisions.
- Total energy remains conserved in all types of collisions.
- Only conservative forces works in elastic collisions.
- In inelastic collisions all the forces are not conservative.

Ex. A simple pendulum of length 1m has a wooden bob of mass 1kg. It is struck by a bullet of mass 10^{-2}kg moving with a speed of 2×10^2 m/s. The bullet gets embedded into the bob. Obtain the height to which the bob rises before swinging back.

Sol. Applying principle of conservation of linear momentum



$$mu = (M+m)v \Rightarrow 10^{-2} \times (2 \times 10^2) = (1 + .01)v \Rightarrow v = \frac{2}{1.01}$$

KE_i of the block with bullet in it, is converted into P.E. as it rises through a height h

$$\frac{1}{2}(M+m)v^2 = (M+m)gh \Rightarrow v^2 = 2gh \Rightarrow h = \frac{v^2}{2g} = \left(\frac{2}{1.01}\right)^2 \times \frac{1}{2\times 9.8} = 0.2 \text{ m}$$

Ex. A body falling on the ground from a height of 10m, rebounds to a height 2.5m calculate

(i) The percentage loss in K.E.

(ii) Ratio of the velocities of the body just before and just after the collision.

Sol. Let v_1 and v_2 be the velocity of the body just before and just after the collision

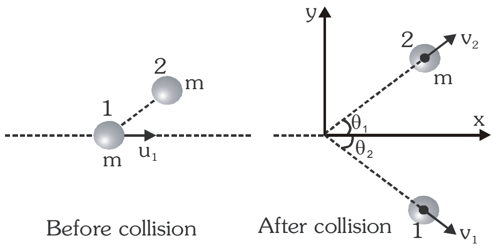
$$KE_1 = \frac{1}{2}mv_1^2 = mgh_1 ...(i) \text{ and } KE_2 = \frac{1}{2}mv_2^2 = mgh_2 ...(ii)$$

$$\Rightarrow \frac{v_1^2}{v_2^2} = \frac{h_1}{h_2} = \frac{10}{2.5} = 4 \Rightarrow \frac{v_1}{v_2} = 2$$

$$\text{Percentage loss in KE} = \frac{mg(h_1 - h_2)}{mgh_1} \times 100 = \frac{10-2.5}{10} \times 100 = 75\%$$

Ex. A body strikes obliquely with another identical stationary rest body elastically. Prove that they will move perpendicular to each other after collision.

Sol.



Conservation of linear momentum in x–direction gives

$mu_1 = mv_1\cos\theta_1 + mv_2\cos\theta_2 \Rightarrow u_1 = v_1\cos\theta_1 + v_2\cos\theta_2$ ... (i)

Conservation of linear momentum in y–direction gives

$0 = mv_1\sin\theta_1 - mv_2\sin\theta_2 \Rightarrow 0 = v_1\sin\theta_1 - v_2\sin\theta_2$ ...(ii)

Conservation of kinetic energy

$$\frac{1}{2}mu_1^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \Rightarrow u_1^2 = v_1^2 + v_2^2 \quad ...(iii)$$

$(i)^2 + (ii)^2$

$\Rightarrow u_1^2 + 0 = v_1^2\cos^2\theta_1 + v_2^2\cos^2\theta_2 + 2v_1v_2\cos\theta_1\cos\theta_2 + v_1^2\sin^2\theta_1 + v_2^2\sin^2\theta_2 - 2v_1v_2\sin\theta_1\sin\theta_2$

$\Rightarrow u_1^2 = v_1^2(\cos^2\theta_1 + \sin^2\theta_1) + v_2^2(\cos^2\theta_2 + \sin^2\theta_2) + 2v_1v_2(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2)$

$\Rightarrow u_1^2 = v_1^2 + v_2^2 + 2v_1v_2\cos(\theta_1 + \theta_2) \quad \{\because u_1^2 = v_1^2 + v_2^2\}$

$\Rightarrow \cos(\theta_1 + \theta_2) = 0 \Rightarrow \theta_1 + \theta_2 = 90°$

Ex. A steel ball is dropped on a smooth horizontal plane from certain height h. Assuming coefficient of restitution of impact as e, find the average speed of the ball till it stops.

Sol. Since the ball falls through a height h, just before the first impact its speed v will be given as $v = \sqrt{2gh}$. Let its speed the v_1 just after the first impact.Then, Newton's experimental formula yields,

$$\frac{0 - v_1}{v} = e \Rightarrow v_1 = ev$$

Similarly, its speed just before 2nd impact, $v_1 = ev = e\sqrt{2gh}$

Speed just after n^{th} impact, $v_n = e^n v = e^n\sqrt{2gh}$

The maximum height attained after 1^{st} impact $= h_1 = \dfrac{v^2}{2g} = (e\sqrt{2gh})^2 = e^2h$. Similarly, the maximum height attained after 2nd impact, $h_2 = e^4h$. Hence, the maximum height attained after n^{th} impact $= e^{2n}h$

The ball experiences infinite impacts till it becomes stationary. $\Rightarrow$ The total distance covered,

$d = h + 2h_1 + 2h_2 + \ldots d = h + 2e^2h + 2e^4h + \ldots = h[1 + 2(e^2 + e^4 + e^6 + \ldots)]$

$$= \left[1 + 2\left(\frac{e^2}{1 - e^2}\right)\right]h = \left(\frac{1 + e^2}{1 - e^2}\right)h.$$

The total time taken by the ball till it stops bouncing $T=\sqrt{\frac{2h}{g}}+2\sqrt{\frac{2h_1}{g}}+2\sqrt{\frac{2h_2}{g}}+...$

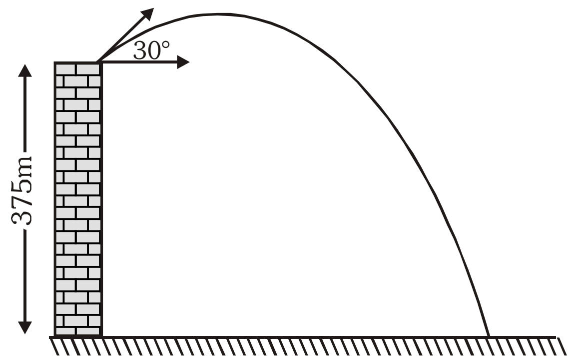
Putting $h_1 = e^2h$, $h_2 = e^4h$, $T=\sqrt{\frac{2h}{g}}+2\sqrt{\frac{2e^2h}{g}}+2\sqrt{\frac{2e^4h}{g}}+....$

$\Rightarrow\ T=\sqrt{\frac{2h}{g}}\ [1+2(e+e^2+...)]=\sqrt{\frac{2h}{g}}\left[1+\frac{2e}{1-e}\right]=\frac{1+e}{1-e}\sqrt{\frac{2h}{g}}$

Therefore, average speed of the ball for its total time of motion, $\vec{v}=\frac{\text{total distance}}{\text{total time}}=\frac{d}{T}$

Putting the values of d and T, we obtain $\vec{v}=\frac{1+e^2}{(1+e)^2}\sqrt{\frac{gh}{2}}$

Ex. A particle of mass 1 kg is projected from a tower of height 375m with initial velocity 100 ms^{-1} at an angle 30° with the horizontal. Find out its kinetic energy in joule just after collision with ground if collision is inelastic with $e=\frac{1}{2}$ $(g = 10\ ms^{-2})$



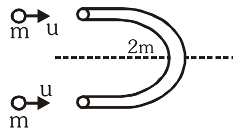
Sol. $v_y^2=u_y^2+2gh \Rightarrow v_y=\sqrt{(50)^2+2\times10\times375}=100ms^{-}$

Horizontal velocity just after collision = $50\sqrt{3}\ ms^{-1}$

Vertical velocity just after collision = $100\times\frac{1}{2}=50\ ms^{-1}$

Kinetic energy just after collision = $\frac{1}{2}\times1\times\left[(50\sqrt{3})^2+(50)^2\right]=5000J$

Ex A U shaped tube of mass 2m is placed on a horizontal surface. Two spheres each of diameter d (just less than the inner diameter of tube) and mass m enter into the tube with a velocity u as shown in figure. Taking all collisions to be elastic and all surfaces smooth. Match the following-



Column-I	**Column-II**
(A) The speed of the tube with respect to ground, when spheres are just about to collide inside the tube.	(p) u
(B) The speed of spheres when spheres are just about to collide.	(q) u/2
(C) The speed of the spheres when they comes out the tube.	(r) $\frac{\sqrt{3}}{2}u$
(D) The speed of the tube when spheres comes out the	(s) zero

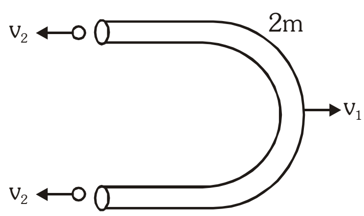
Sol. **For (A)** From conservation of linear momentum $2mu = (m+m)v \Rightarrow v = \frac{u}{2}$

For (B) Let v_1 be the velocity of spheres w.r.t. tube when they are just about to collide then by using conservation of kinetic energy $\frac{1}{2}(2m)u^2 = \frac{1}{2}(4m)\left(\frac{u}{2}\right)^2 + 2\,\frac{1}{2}mv_1^2$

$\Rightarrow v_1 = \frac{u}{\sqrt{2}}$

$\Rightarrow$ Required speed of spheres $= \sqrt{\left(\frac{u}{2}\right)^2 + \left(\frac{u}{\sqrt{2}}\right)^2} = \sqrt{\frac{u^2}{4} + \frac{u^2}{2}} = \frac{\sqrt{3}u}{2}$

For (C) $2mu = 2mv_1 - 2mv_2$



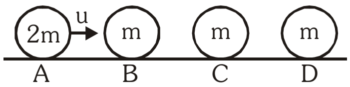
$2 \times \frac{1}{2}mu^2 = 2 \times \frac{1}{2}mv_2^2 + \frac{1}{2}(2m)v_1^2$

$\Rightarrow u^2 = v_1^2 + v_2^2 - 2v_1v_2$ & $u^2 = v_1^2 + v_2^2$

$\Rightarrow v_1v_2 = 0$ but $v_1 \neq 0$ so $v_2 = 0$

For (D) Speed of tube $v_1 = u$

Ex. Four balls A,B,C and D are kept on a smooth horizontal surface as shown in figure. Ball A is given velocity u towards B-

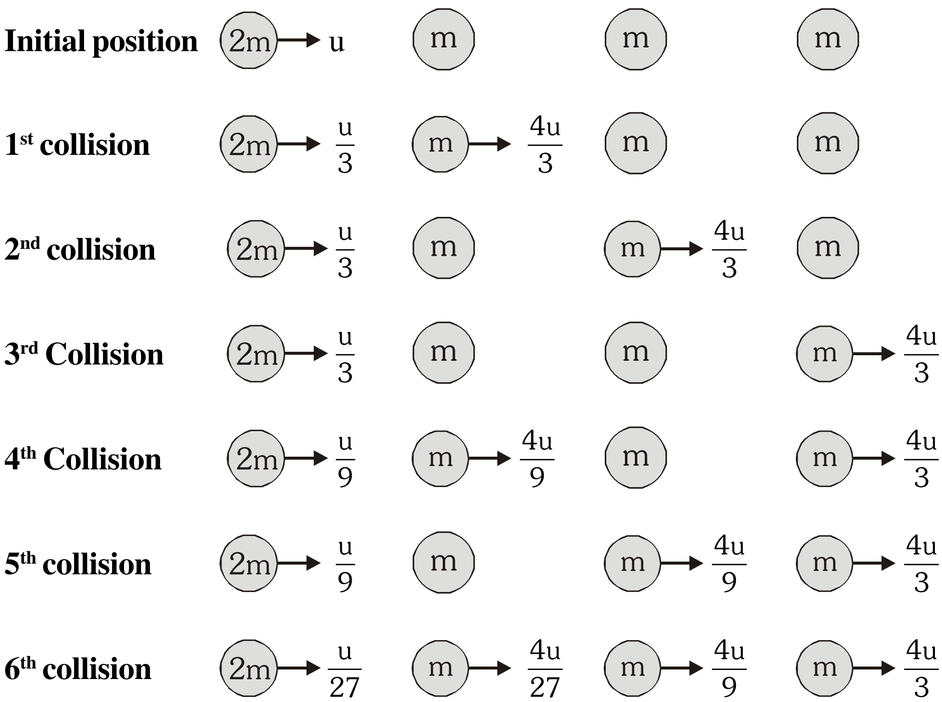


Column-I	**Column-II**
(A) Total impulse of all collisions on A	(p) $\frac{4mu}{9}$
(B) Total impulse of all collisions on B	(q) $\frac{4mu}{27}$
(C) Total impulse of all collision on C	(r) $\frac{4mu}{3}$
(D) Total impulse of all collisions on D	(s) $\frac{52}{27}mu$

Sol. In 1st collision between A & B

$$2mu = 2mv_A + 2mv_B \;\&\; e = 1 = \frac{v_B - v_A}{u} \Rightarrow v_A = \frac{u}{3}, v_B = \frac{4u}{3}$$

Situation of all collisions is shown in figure.



For (A) Total impulse on A $= 2m\left(u - \frac{u}{27}\right) = \frac{52}{27}mu$

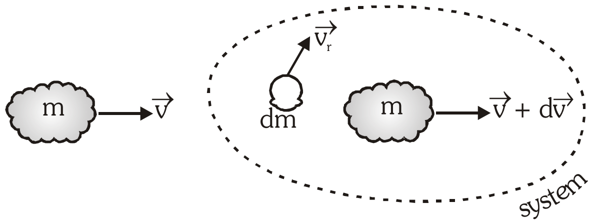
For (B) Total impulse on B $= m\left(\frac{4u}{27}-0\right)=\frac{4}{27}mu$

For (C) Total impulse on C $= m\left(\frac{4u}{9}-0\right)=\frac{4}{9}mu$

For (D) Total impulse on D $= m\left(\frac{4u}{3}-0\right)=\frac{4}{3}mu$

Variable mass system:

In previous discussion of the conservation of linear momentum, we assume that system's mass remains constant. Now we are consider those system whose mass is variable i.e. those in which mass enters or leaves the system. Suppose at some moment $t = t$ mass of a body is m and its velocity is $\vec{v}$. After some time at $t=t + dt$ its mass becomes (m–dm) and velocity becomes $\vec{v}+d\vec{v}$. The mass dm is ejected with relative velocity $\vec{v}_r$.



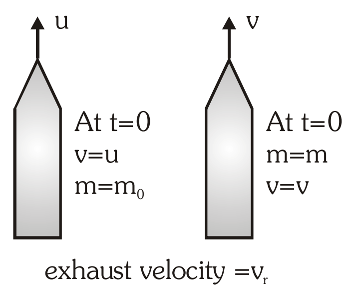
If no forces are acting on the system then the linear momentum of the system will remain conserved.

$\Rightarrow \quad \vec{F}_{ex}\,dt = (m-dm)(\vec{v}+d\vec{v}) + dm\,(\vec{v}_r+\vec{v}+d\vec{v}) - m\vec{v}$

$\because \quad F_{ex}=0 \Rightarrow m d\vec{v} = -\vec{v}_r dm \Rightarrow m\left(\frac{d\vec{v}}{dt}\right) = \vec{v}_r\left(-\frac{dm}{dt}\right)$

ocket propulsion :

Thrust force on the rocket $= v_r\left(-\frac{dm}{dt}\right)$



So for motion of rocket $m\frac{dv}{dt} = v_r\left(-\frac{dm}{dt}\right) - mg$

$\Rightarrow dv = v_r\left(-\frac{dm}{m}\right) - gdt \Rightarrow \int_{u_0}^{v} dv = -v_r\int_{m_0}^{m}\frac{dm}{m} - g\int_0^t dt$

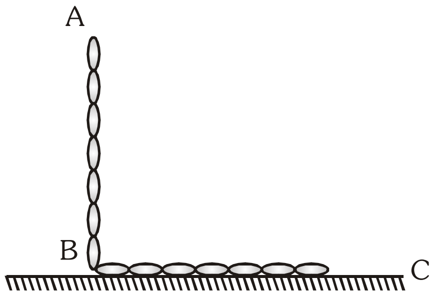
$\Rightarrow v-u = v_r \ell n\left(\frac{m_0}{m}\right) - gt \Rightarrow v = u - gt + v_r \ell n\left(\frac{m_0}{m}\right)$

Ex An open topped rail road car of mass M has an initial velocity v_0 along a straight horizontal frictionless track. It suddenly starts raining at time t=0. The rain drops fall vertically with velocity u and add a mass m kg/sec of water. Find the velocity of car after t second (assuming that it is not completely filled with water).

Sol. According to law of conservation of momentum, $Mv_0 = (M + m \times t)\,v$. Where m is the mass of water added per second and v is the velocity of the car after t second. $\therefore\ v = \frac{Mv_0}{M+mt}$

Ex. A uniform chain of mass m and length ℓ hangs on a thread and touches the surface of a table by its lower end. Find the force exerted by the chain on the table when half of its length has fallen on the table. The fallen part does not form heap.

Sol. At given condition force exerted by the chain on the table consists of two parts



(i) Weight of portion BC = $\frac{mg}{2}$

(ii) Thrust force = $v_r\left(-\frac{dm}{dt}\right) = v\left(\frac{m}{\ell}v\right) = \frac{m}{\ell}v^2$

but $v = \sqrt{2g\left(\frac{\ell}{2}\right)} = \sqrt{g\ell}$

$\Rightarrow$ Thrust force = $\frac{m}{\ell}(g\ell) = mg$

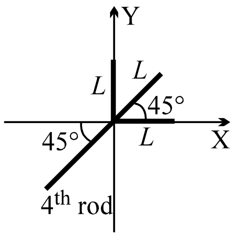
$\therefore$ Net force exerted by falling chain = $\frac{mg}{2} + mg = \frac{3mg}{2}$

EXERCISE (S-1)

1. Four particles of mass 5, 3, 2, 4 kg are at the points (1, 6), (–1, 5), (2, –3), (–1, –4). Find the coordinates of their centre of mass. **CM0001**

2. A rigid body consists of a 3 kg mass connected to a 2 kg mass by a massless rod. The 3kg mass is located at $\vec{r}_1 = (2\hat{i} + 5\hat{j})$ m and the 2 kg mass at $\vec{r}_2 = (4\hat{i} + 2\hat{j})$ m. Find the length of rod and the coordinates of the centre of mass. **CM0002**

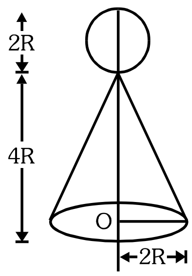
3. Three identical uniform rods of the same mass *M* and length *L* are arranged in xy plane as shown in the figure. A fourth uniform rod of mass *3M* has been placed as shown in the xy plane. What should be the value of the length of the fourth rod such that the center of mass of all the four rods lie at the origin?



CM0003

4. From a circle of radius a, an isosceles right angled triangle with the hypotenuse as the diameter of the circle is removed. The distance of the centre of gravity of the remaining position from the centre of the circle is **CM0004**

5. A man has constructed a toy as shown in figure. If density of the material of the sphere is 12 times of the cone compute the position of the centre of mass. [Centre of mass of a cone of height h is at height of h/4 from its base.]



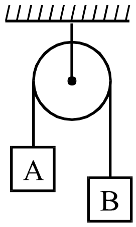
CM0005

6. The figure shows the positions and velocities of two particles. If the particles move under the mutual attraction of each other, then find the position of centre of mass at t = 1 s.

CM0006

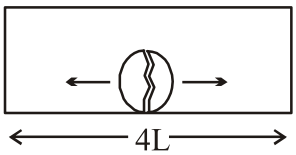
7. Mass centers of a system of three particles of masses 1, 2, 3 kg is at the point (1 m, 2 m, 3 m) and mass center of another group of two particles of masses 2 kg and 3 kg is at point (–1 m, 3 m, –2 m). Where a 5 kg particle should be placed, so that mass center of the system of all these six particles shifts to mass center of the first system? **CM0007**

8. In the arrangement shown in the figure, $m_A = 2$ kg and $m_B = 1$ kg. String is light and inextensible. Find the acceleration of centre of mass of both the blocks. Neglect friction everywhere.



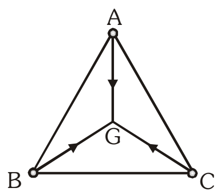
CM0008

9. A bomb of mass 3m is kept inside a closed box of mass 3m and length 4L at it's centre. It explodes in two parts of mass m & 2m. The two parts move in opposite direction and stick to the opposite side of the walls of box. Box is kept on a smooth horizontal surface. What is the distance moved by the box during this time interval.



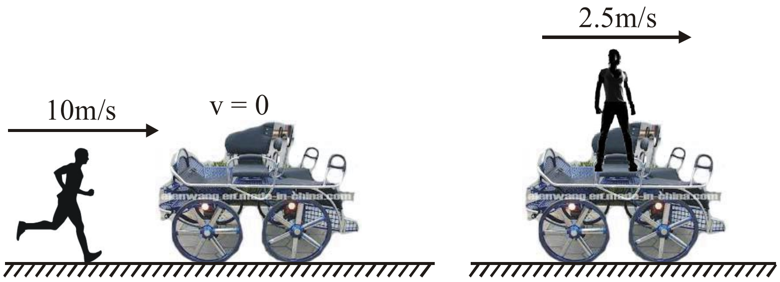
CM0009

10. Three particles A, B and C of equal mass move with equal speed v along the medians of an equilateral triangle as shown in fig. They collide at the centroid G of the triangle. After the collision, A comes to rest, B retraces its path with the speed v. What is the velocity of C ?



CM0010

11. A 50 kg boy runs at a speed of 10 m/s and jumps onto a cart as shown in the figure. The cart is initially at rest. If the speed of the cart with the boy on it is 2.50 m/s, what is the mass of the cart ? (Assuming friction is absent between cart and ground)



CM0011

12. Two cars initially at rest are free to move in the x direction. Car A has mass 4 kg and car B has mass 2 kg. They are tied together, compressing a spring in between them. When the spring holding them together is burned, car A moves off with a speed of 2 m/s.

(i) With what speed does car B leave.

(ii) How much energy was stored in the spring before it was burned. **CM0012**

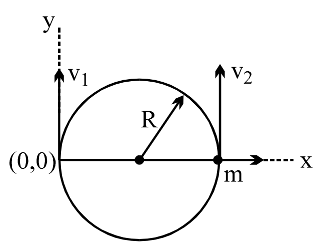
13. A 24 kg projectile is fired at an angle of 53° above the horizontal with an initial speed of 50 m/s. At the highest point in its trajectory, the projectile explodes into two fragments of equal mass, the first of which falls vertically with zero initial speed.

(i) How far from the point of firing does the second fragment strike the ground? (Assume the ground is level.)

(ii) How much energy was released during the explosion? **CM0013**

14. A particle of mass m, moving in a circular path of radius R with a constant speed v_2 is located at point (2R, 0) at time t = 0 and a man starts moving with a velocity v_1 along the +ve y-axis from origin at time t = 0. Calculate the linear momentum of the particle w.r.t. the man as a function of time.

[IIT-JEE' 2003]

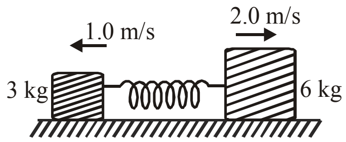


CM0014

15. A spaceship is moving with constant speed v_0 in gravity free space along +Y-axis suddenly shoots out one third of its part with speed $2v_0$ along + X-axis. Find the speed of the remaining part.

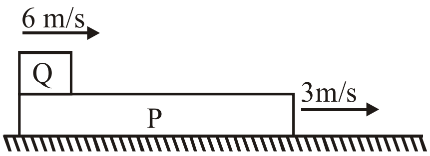
CM0015

16. Two blocks of mass 3 kg and 6 kg respectively are placed on a smooth horizontal surface. They are connected by a light spring of force constant k = 200 N/m. Initially the spring is unstretched. The indicated velocities are imparted to the blocks. The maximum extension of the spring will be :-



CM0016

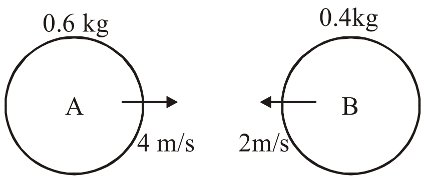
17. A plank P and block Q are arranged as shown on a smooth table top. They are given velocities 3 m/s and 6 m/s respectively. The length of plank is 1m and block is of negligible size. After some time when the block has reached the other end of plank it stops slipping on plank. Find the coefficient of friction between plank P and block Q if mass of plank is double of block).



CM0017

18. A bullet of mass m strikes an obstruction and deviates off at 60° to its original direction. If its speed is also changed from u to v, find the magnitude of the impulse acting on the bullet. **CM0018**

19. The velocities of two steel balls before impact are shown. If after head on impact the velocity of ball B is observed to be 3 m/s to the right, the coefficient of restitution is

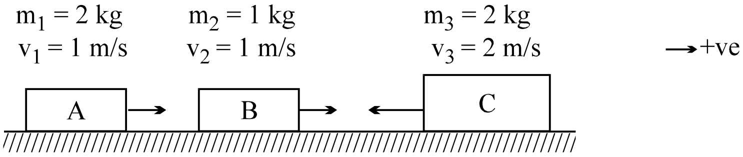


CM0019

20. Three carts move on a frictionless track with inertias and velocities as shown. The carts collide and stick together after successive collisions.

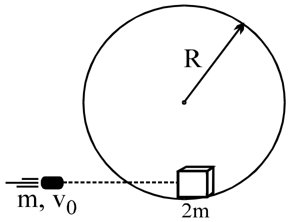
(i) Find loss of mechanical energy when B & C stick together.

(ii) Find magnitude of impulse experienced by A when it sticks to combined mass (B & C).



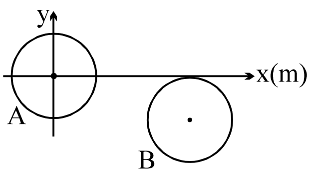
CM0020

21. A small block of mass 2m initially rests at the bottom of a fixed circular, vertical track, which has a radius of R. The contact surface between the mass and the loop is frictionless. A bullet of mass m strikes the block horizontally with initial speed v_0 and remain embedded in the block as the block and the bullet circle the loop. Determine each of the following in terms of m, v_0, R and g.
(i) The speed of the masses immediately after the impact.
(ii) The minimum initial speed of the bullet if the block and the bullet are to successfully execute a complete ride on the loop



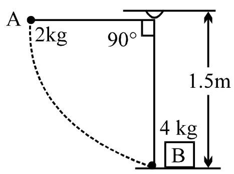
CM0021

22. Two smooth balls A and B, each of mass m and radius R, have their centres at (0,0,R) and at (5R,–R,R) respectively, in a coordinate system as shown. Ball A, moving along positive x axis, collides with ball B. Just before the collision, speed of ball A is 4 m/s and ball B is stationary. The collision between the balls is elastic. Find Velocity of the ball A just after the collision and impulse of the force exerted by A on B during the collision.



CM0022

23. A sphere A is released from rest in the position shown and strikes the block B which is at rest. If e = 0.75 between A and B and μ_k = 0.5 between B and the support, determine
(i) the velocity of A just after the impact
(ii) the maximum displacement of B after the impact.



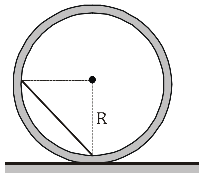
CM0023

24. Bullets of mass 10 g each are fired from a machine gun at rate of 60 bullets/minute. The muzzle velocity of bullets is 100 m/s. The thrust force due to firing bullets experienced by the person holding the gun stationary is _________. **CM0024**

EXERCISE (S-2)

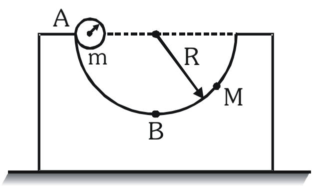
1. The linear mass density of a ladder of length ℓ increases uniformly from one end A to the other end B, (i) Form an expression for linear mass density as a function of distance x from end A where linear mass density λ_0. The density at one end being twice that of the other end. (ii) Find the position of the centre of mass from end A. **CM0025**

2. Inside a hollow uniform sphere of inner radius R a uniform rod of length $R\sqrt{2}$ is released from the state of rest as shown. The mass of the rod is same as that of the sphere. Assume friction to be absent everywhere. Horizontal displacement of sphere with respect to earth in the time in which the rod becomes horizontal, is



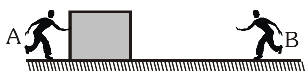
CM0026

3. A block of mass M with a semicircular track of radius R, rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest at the top point A (see Fig). The cylinder slips on the semicircular frictionless track. How far has the block moved when the cylinder reaches the bottom (point B) of the track ? How fast is the block moving when the cylinder reaches the bottom of the track?



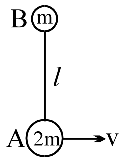
CM0027

4. Two persons A and B each of mass 100 kg are on a frictionless horizontal surface. Person A at rest is holding a block of mass 50 kg, suddenly pushes the block with some velocity (v) towards the person B approaching at a velocity of 5 m/s. B catches the block & slows down. Now the separation between A and B becomes constant. Find the speed v (in m/s).



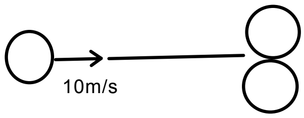
CM0028

5. Two masses A and B connected with an inextensible string of length ℓ lie on a smooth horizontal plane. A is given a velocity of v m/s along the ground perpendicular to line AB as shown in figure. Find the tension in string during their subsequent motion.



CM0029

6. A ball with initial speed of 10 m/s collides elastically with two other identical ball whose centres are on a line perpendicular to the initial velocity and which are initially in contact with each other. All the three ball are lying on a smooth horizontal table. The first ball is aimed directly at the contact point of the other two balls All the balls are smooth. Find the velocities of the three balls after the collision.



CM0030

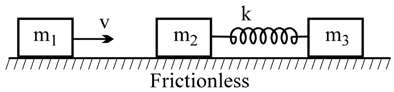
7. Mass m_1 hits & sticks with m_2 while sliding horizontally with velocity v along the common line of centres of the three equal masses ($m_1 = m_2 = m_3 = m$). Initially masses m_2 and m_3 are stationary and the spring is unstretched. Find the

(i) velocities of m_1, m_2 and m_3 immediately after impact.

(ii) maximum kinetic energy of m_3.

(iii) minimum kinetic energy of m_2.

(iv) maximum compression of the spring.



CM0031

8. A sphere of mass m is moving with a velocity $4\hat{i} - \hat{j}$ when it hits a smooth wall and rebounds with velocity $\hat{i} + 3\hat{j}$. Find the impulse it receives. Find also the coefficient of restitution between the sphere and the wall. **CM0032**

9. Two particles A and B of mass 2m and m respectively are attached to the ends of a light inextensible string of length 4a which passes over a small smooth peg at a height 3a from an inelastic table. The system is released from rest with each particle at a height a from the table. Find

(i) The speed of B when A strikes the table.

(ii) The time that elapses before A first hits the table.

(iii) The time for which A is resting on the table after the first collision & before it is first jerked off.

CM0033

10. Two particles, each of mass m, are connected by a light inextensible string of length 2ℓ. Initially they lie on a smooth horizontal table at points A and B distant ℓ apart. The particle at A is projected across the table with velocity u. Find the speed with which the second particle begins to move if the direction of u is :-

(i) along BA.

(ii) at an angle of 120° with AB.

(iii) perpendicular to AB.

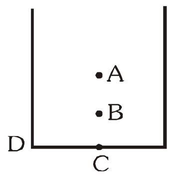
In each case calculate (in terms of m & u) the impulsive tension in the string.

CM0034

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

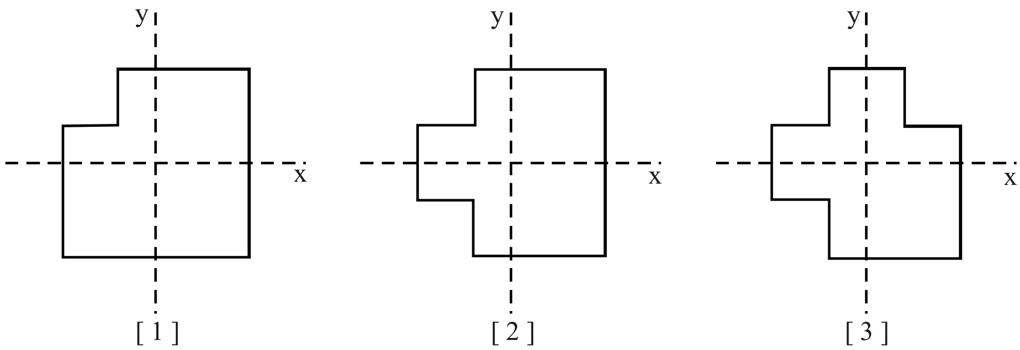
1. A thick uniform wire is bent into the shape of the letter "U" as shown. Which point indicates the location of the center of mass of this wire? A is the midpoint of the line joining mid points of two parallel sides of 'U' shaped wire.



(A) D (B) A (C) B (D) C

CM0035

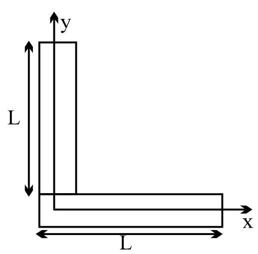
2. A machinist starts with three identical square plates but cuts one corner from one of them, two corners from the second, and three corners from the third. Rank the three plates according to the x-coordinate of their centers of mass, from smallest to largest.



(A) 3, 1, 2 (B) 1, 3, 2 (C) 3, 2, 1 (D) 1 and 3 tie, then 2

CM0036

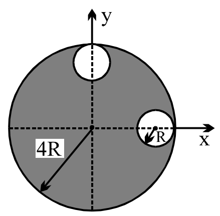
3. Centre of mass of two thin uniform rods of same length but made up of different materials & kept as shown, can be, if the meeting point is the origin of co-ordinates



(A) (L/2, L/2) (B) (2L/3, L/2) (C) (L/3, L/3) (D) (L/3, L/6)

CM0037

4. From the circular disc of radius 4R two small disc of radius R are cut off. The centre of mass of the new structure will be :



(A) $i\frac{R}{5}+j\frac{R}{5}$ (B) $-i\frac{R}{5}+j\frac{R}{5}$ (C) $\frac{-3R}{14}(\hat{i}+\hat{j})$ (D) None of these

CM0038

5. Seven identical birds are flying south together at constant velocity. A hunter shoots one of them, which immediately dies and falls to the ground. The other six continue flying south at the original velocity. After the one bird has hit the ground, the centre of mass of all seven birds

(A) continues south at the original speed, but is now located some distance behind the flying birds
(B) continues south, but at 6/7 the original velocity
(C) continues south, but at 1/7 the original velocity
(D) stops with the dead bird

CM0039

6. A man weighing 80 kg is standing at the centre of a flat boat and he is 20 m from the shore. He walks 8 m on the boat towards the shore and then halts. The boat weight 200 kg. How far is he from the shore at the end of this time ?

(A) 11.2 m (B) 13.8 m (C) 14.3 m (D) 15.4 m

CM0040

7. There are some passengers inside a stationary railway compartment. The track is frictionless. The centre of mass of the compartment itself (without the passengers) is C_1, while the centre of mass of the 'compartment plus passengers' system is C_2. If the passengers move about inside the compartment along the track.

(A) both C_1 and C_2 will move with respect to the ground.
(B) neither C_1 nor C_2 will move with respect to the ground.
(C) C_1 will move but C_2 will be stationary with respect to the ground.
(D) C_2 will move but C_1 will be stationary with respect to the ground.

CM0041

8. A non–zero external force acts on a system of particles. The velocity and acceleration of the centre of mass are found to be v_0 and a_C respectively at any instant t. It is possible that

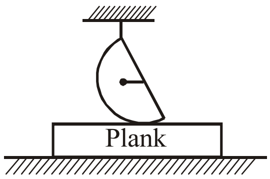
(i) $v_0 = 0, a_C = 0$ (ii) $v_0 \neq 0, a_C = 0$ (iii) $v_0 = 0, a_C \neq 0$ (iv) $v_0 \neq 0, a_C \neq 0$

Then

(A) (iii) and (iv) are true.
(B) (i) and (ii) are true.
(C) (i) and (iii) are true.
(D) (ii), (iii) and (iv) are true.

CM0042

9. Lower surface of a plank is rough and lying at rest on a rough horizontal surface. Upper surface of the plank is smooth and has a smooth hemisphere placed over it through a light string as shown in the figure. After the string is burnt, trajectory of centre of mass of the sphere is :-



(A) a circle
(B) an ellipse
(C) a straight line
(D) a parabola

CM0043

10. Three interacting particles of masses 100 g, 200 g and 400 g each have a velocity of 20 m/s magnitude along the positive direction of x-axis, y-axis and z-axis. Due to force of interaction the third particle stops moving. The velocity of the second particle is $\left(10\hat{j}+5\hat{k}\right)$. What is the velocity of the first particle?

(A) $20\hat{i}+20\hat{j}+70\hat{k}$
(B) $10\hat{i}+20\hat{j}+8\hat{k}$
(C) $30\hat{i}+10\hat{j}+7\hat{k}$
(D) $15\hat{i}+5\hat{j}+60\hat{k}$

CM0044

11. A system of N particles is free from any external forces.

(i) Which of the following is true for the magnitude of the total momentum of the system?

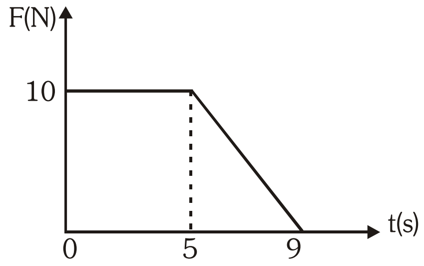
(A) It must be zero
(B) It could be non–zero, but it must be constant
(C) It could be non–zero, and it might not be constant
(D) The answer depends on the nature of the internal forces in the system

(ii) Which of the following must be true for the sum of the magnitudes of the momenta of the individual particles in the system?

(A) It must be zero
(B) It could be non–zero, but it must be constant
(C) It could be non–zero, and it might not be constant
(D) It could be zero, even if the magnitude of the total momentum is not zero

CM0045

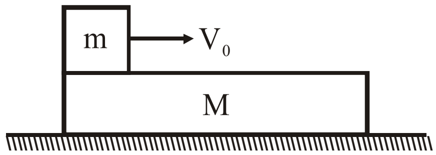
12. A body of mass 4 kg is acted on by a force which varies as shown in the graph below. The momentum acquired is



(A) 280 N–s (B) 140 N–s (C) 70 N–s (D) 210 N–s

CM0046

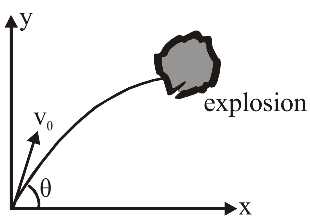
13. The coefficient of friction between the block and plank is μ and ground is smooth. The value of μ is such that block becomes stationary with respect to plank before it reaches the other end. Then which of the following statement is **incorrect.**



(A) The work done by friction on the block is negative.
(B) The work done by friction on the plank is positive .
(C) The net work done by friction is negative.
(D) Net work done by the friction is zero.

CM0047

14. A projectile is projected in x-y plane with velocity v_0. At top most point of its trajectory projectile explodes into two identical fragments. Both the fragments land simultaneously on ground and stick there. Taking point of projection as origin and R as range of projectile if explosion had not taken place. Which of the following can not be position vectors of two pieces, when they land on ground.



(A) $\frac{R}{2}\hat{i}, \frac{3R}{2}\hat{i}$ (B) $0\hat{i}, 2R\hat{i}$ (C) $R\hat{i} - R\hat{k}, R\hat{i} + R\hat{k}$ (D) $2R\hat{i} + \frac{R}{2}\hat{k}, R\hat{i} - -k$

CM0048

15. A boy hits a baseball with a bat and imparts an impulse J to the ball. The boy hits the ball again with the same force, except that the ball and the bat are in contact for twice the amount of time as in the first hit. The new impulse equals:

(A) half the original impulse (B) the original impulse
(C) twice the original impulse (D) four times the original impulse

CM0049

16. Two balls of same mass are dropped from the same height h, on to the floor. The first ball bounces to a height h/4 ,after the collision & the second ball to a height h/16. The impulse applied by the first & second ball on the floor are I_1 and I_2 respectively. Then

(A) $5I_1 = 6I_2$ (B) $6I_1 = 5I_2$ (C) $I_1 = 2I_2$ (D) $2I_1 = I_2$

CM0050

17. Ball A of mass 5.0 kilograms moving at 20 m/s collides with ball B of unknown mass moving at 10m/s in the same direction. After the collision, ball A moves at 10 m/s and ball B at 15 m/s, both still in the same direction. What is the mass of ball B?

(A) 6.0 kg (B) 10. kg (C) 2.0 kg (D) 12 kg

CM0051

18. A smooth small spherical ball of mass m, moving with velocity u collides head on with another small spherical ball of mass 3m, which was initially at rest. Two-third of the initial kinetic energy of the system is lost. The coefficient of restitution between the spheres is

(A) $\frac{1}{3}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{2}$ (D) zero

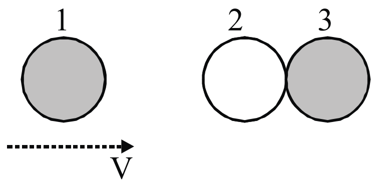
CM0052

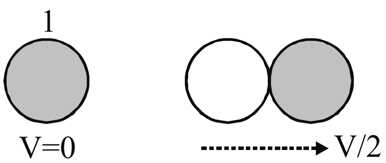
19. A ball strikes a smooth horizontal ground at an angle of 45° with the vertical. What cannot be the possible angle of its velocity with the vertical after the collision. (Assume $e \le 1$).

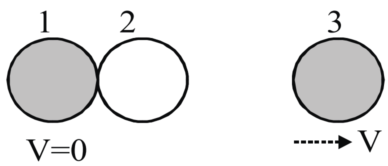
(A) 45° (B) 30° (C) 53° (D) 60°

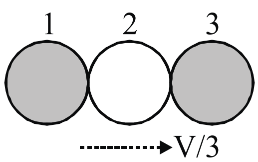
CM0053

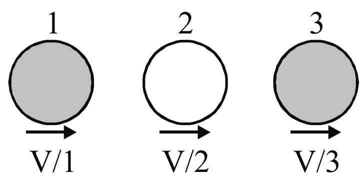
20. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed *V* as shown in figure (i). If the collision is elastic, which of the following is a possible result after collision?

fig. (i)


(A)


(B)


(C)


(D)


CM0054

21. A ball is projected from ground with a velocity V at an angle θ to the vertical. On its path it makes an elastic collison with a vertical wall and returns to ground. The total time of flight of the ball is

(A) $\frac{2v\sin\theta}{g}$ (B) $\frac{2v\cos\theta}{g}$ (C) $\frac{v\sin 2\theta}{g}$ (D) $\frac{v\cos\theta}{g}$

CM0055

22. A ball is thrown downwards with initial speed = 6 m/s, from a point at height = 3.2 m above a horizontalfloor. If the ball rebounds back to the same height then coefficient of restitution equals to

(A) 1/2 (B) 0.75 (C) 0.8 (D) None

CM0056

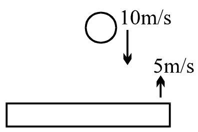
23. A particle is projected from a smooth horizontal surface with velocity v at an angle θ from horizontal. Coefficient of restitution between the surface and ball is e. The distance of the point where ball strikes the surface second time from the point of projection is

(A) $\frac{v^2\sin 2\theta(1+e^2)}{g}$ (B) $\frac{v^2\sin 2\theta(1+e^4)}{g}$

(C) $\frac{v^2\sin 2\theta(1+e^3)}{g}$ (D) $\frac{v^2\sin 2\theta(1+e)}{g}$

CM0057

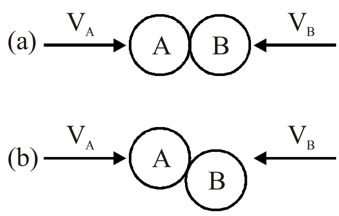
24. A ball of mass 1kg strikes a heavy platform, elastically, moving upwards with a velocity of 5m/s. The speed of the ball just before the collision is 10m/s downwards. Then the impulse imparted by the platform on the ball is :-



(A) 15 N – s (B) 10 N – s (C) 20 N – s (D) 30 N – s

CM0058

25. Two bodies, A and B, collide as shown in figures a and b below. Circle the true statement :



(A) They exert equal and opposite forces on each other in (a) but not in (b)
(B) They exert equal and opposite force on each other in (b) but not in (a)
(C) They exert equal and opposite force on each other in both (a) and (b)
(D) The forces are equal and opposite to each other in (a), but only the components of the forces parallel to the velocities are equal in (b).

CM0059

26. A mass 'm' moves with a velocity 'v' and collides inelastically with another identical mass at rest. After collision the 1st mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision :-

(A) $\frac{2v}{\sqrt{3}}$

(B) $\frac{v}{\sqrt{3}}$

(C) $v\sqrt{\frac{2}{3}}$

(D) the situation of the problem is not possible without external impulse

CM0060

MULTIPLE CORRECT TYPE QUESTIONS

27. Two charges moving under their only own mutual attraction separated by large distance initially. Then choose the correct statement(s)

(A) If both are free, mechanical energy is conserved.

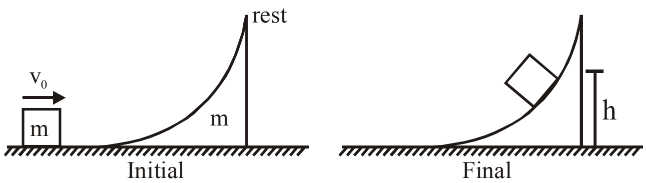
(B) If one is fixed and other is free, mechanical energy is conserved.

(C) If one is fixed and other is free, momentum is conserved.

(D) If both are free momentum is conserved.

CM0061

28. In the arrangement shown, horizontal surface is smooth, but friction is present between the block and the surface of the wedge. Block is given velocity v_0 at $t = 0$. After achieving height 'h' on the wedge, block comes to rest with respect to wedge at $t = t_0$. Then from $t = 0$ to $t = t_0$:-



(A) Work done by friction on the block is negative

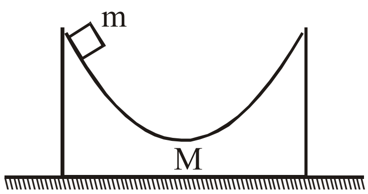
(B) Work done by friction on the wedge is negative

(C) Work done by block on the wedge is positive

(D) Work done by wedge on the block is positive

CM0062

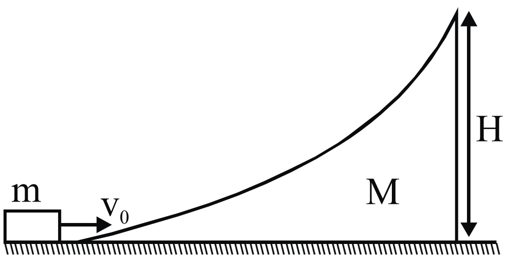
29. Figure shows a wedge on which a small block is released from rest. All the surfaces are smooth system comprises of wedge and blocks. Mark the correct statement(s) regarding motion of block on wedge till block attains maximum height on wedge.



(A) Acceleration of centre of mass of system is initially vertically down then vertically up.
(B) Initially centre of mass moves down and then up.
(C) At the maximum height block and wedge move with common velocity.
(D) Centre of mass of wedge moves towards left then right

CM0063

30. Figure shows a block of mass m projected with velocity v_0 towards a wedge. Consider all the surfaces to be smooth. Block does not have sufficient energy to negotiate (over come) wedge. Mark the correct option(s)



(A) when block is at the maximum height on wedge, block and wedge have velocity equal to velocity of centre of mass of block wedge system
(B) wedge acquires maximum speed with respect to ground when block returns to lowest point on wedge.
(C) momentum of wedge and block is conserved at all times
(D) centre of mass of wedge and block remains stationary

CM0064

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 31 and 32

A projectile of mass "m" is projected from ground with a speed of 50 m/s at an angle of 53° with the horizontal. It breaks up into two equal parts at the highest point of the trajectory. One particle coming to rest immediately after the explosion.

31. The ratio of the radii of curvatures of the moving particle just before and just after the explosion are:

(A) 1 : 4 (B) 1 : 3 (C) 2 : 3 (D) 4 : 9

CM0065

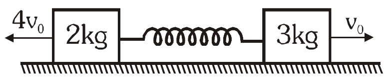
32. The distance between the pieces of the projectile when they reach the ground are:

(A) 240 (B) 360 (C) 120 (D) none

CM0065

Paragraph for Question 33 to 35

2 kg and 3 kg blocks are placed on a smooth horizontal surface and connected by spring which is unstretched initially. The blocks are imparted velocities as shown in the figure.



33. The maximum energy stored in the spring in the subsequent motion will be

(A) $5v_0^2$ (B) $15v_0^2$ (C) zero (D) $10v_0^2$

CM0066

34. Maximum speed of 3 kg block in the subsequent motion will be

(A) v_0 (B) $2v_0$ (C) $3v_0$ (D) $4v_0$

CM0066

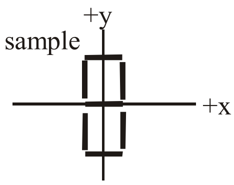
35. Maximum speed of 2 kg block in the subsequent motion will be

(A) v_0 (B) $2v_0$ (C) $3v_0$ (D) $4v_0$

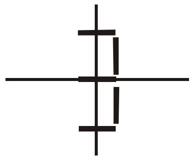
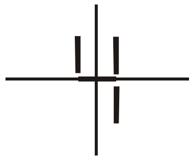
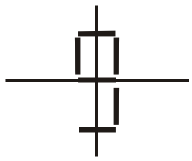
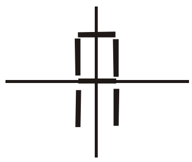
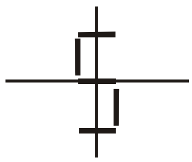
CM0066

MATRIX MATCH TYPE QUESTIONS

36. On the left are statements about the location of the center of mass of the objects depicted on the right. The objects on the right are symbols constructed out of sticks of equal length and mass. The location of the center of mass is described using the coordinate system depicted in the sample.

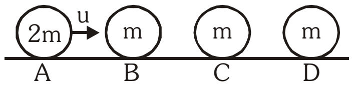


The centre of mass lies at $x = 0$, $y = 0$

Column I	**Column II**	
(A) The center of mass is at $x > 0$ and $y = 0$	(P)	
(B) The center of mass is at $x = 0$ and $y > 0$	(Q)	
(C) The center of mass is at $x > 0$ and $y > 0$	(R)	
(D) The center of mass is at $x = 0$ and $y = 0$	(S)	
	(T)	

CM0067

37. Four balls A,B,C and D are kept on a smooth horizontal surface as shown in figure. Ball A is given velocity u towards B- (Assume each collision to be elastic)



	Column-I		Column-II
(A)	Total impulse of all collisions on A	(p)	$\frac{4mu}{9}$
(B)	Total impulse of all collisions on B	(q)	$\frac{4mu}{27}$
(C)	Total impulse of all collision on C	(r)	$\frac{4mu}{3}$
(D)	Total impulse of all collisions on D	(s)	$\frac{52}{27}mu$

CM0068

38. In Column-I, 4 situations are depicted and in column-II, 4 possible kinds of collision are listed. Match the situation with type of collision.

	Column-I Before		Column-II After
(A)		(P)	Elastic
(B)		(Q)	Perfectly Inelastic
(C)		(R)	Partially elastic
		(S)	Collision is not possible

CM0069

39. A particle of mass m, kinetic energy K and momentum p collides head on elastically with another particle of mass 2 m at rest. After collision :

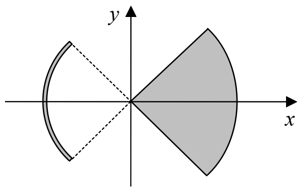
	Column I		Column II
(A)	Momentum of first particle	(P)	3/4 p
(B)	Momentum of second particle	(Q)	– K/9
(C)	Kinetic energy of first particle	(R)	– p/3
(D)	Kinetic energy of second particle	(S)	$\frac{8K}{9}$
		(T)	None

CM0070

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

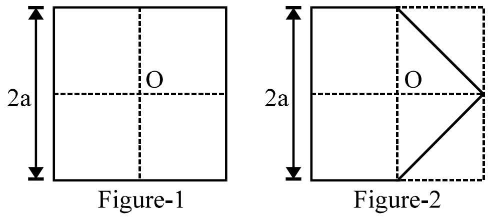
1. A sector cut from a uniform disk of radius 12 cm and a uniform rod of the same mass bent into shape of an arc are arranged facing each other as shown in the figure. If center of mass of the combination is at the origin, what is the radius of the arc?



(A) 8 cm (B) 9 cm (C) 12 cm (D) 18 cm

CM0071

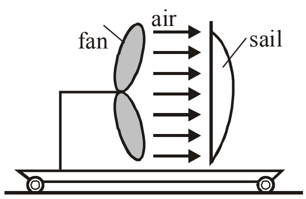
2. A piece of paper (shown in figure-1) is in form of a square. Two corners of this square are folded to make it appear like figure-2. Both corners are put together at centre of square 'O'. If O is taken to be (0, 0), the centre of mass of new system will be at



(A) $\left(\frac{-a}{8}, 0\right)$ (B) $\left(\frac{-a}{6}, 0\right)$ (C) $\left(\frac{a}{12}, 0\right)$ (D) $\left(\frac{-a}{12}, 0\right)$

CM0072

3. A fan and a sail are mounted vertically on a cart that is initially at rest on a horizontal table as shown in the diagram. When the fan is turned on, an air stream is blown towards the right and is incident on the sail. The cart is free to move with negligible resistance forces. After the fan has been turned on the cart will



(A) move to the right and then to the left (B) remain at rest
(C) move towards the right (D) move towards the left

CM0073

4. Two identical carts constrained to move on a straight line, on which sit two twins of same mass, are moving with same velocity. At some time snow begins to drop uniformly vertically downward. Ram, sitting on one of the trolleys, throws off the falling snow sideways with respect to himself and in the second cart shyam is asleep. (Assume that friction is absent)

(A) Cart carrying Ram will speed up while cart carrying shyam will slow down

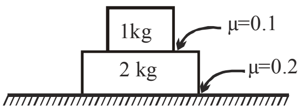
(B) Cart carrying Ram will remain at the same speed while cart carrying shyam will slow down

(C) Cart carrying Ram will speed up while cart carrying shyam will remain at the same speed

(D) Cart carrying Ram as well as shyam will slow down

CM0074

5. If both the blocks as shown in the given arrangement are given together a horizontal velocity towards right. If a_{cm} be the subsequent acceleration of the centre of mass of the system of blocks then a_{cm} equals

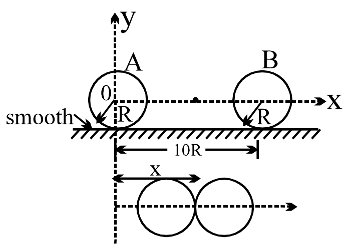


(A) 0 m/s^2 (B) $\frac{5}{3}$ m/s^2

(C) $\frac{7}{3}$ m/s^2 (D) 2 m/s^2

CM0075

6. Two uniform non conducting balls A & B have identical size having radius R but made of different density material (density of A = 2 density of B). The ball A is +vely charged & ball B is –vely charged. The balls are released on the horizontal smooth surface at the separation 10R as shown in figure. Because of mutual attraction the balls start moving towards each other. They will collide at a point.

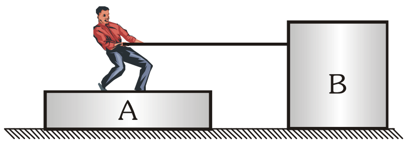


(A) x = $\frac{10R}{3}$ (B) x = $\frac{11R}{3}$

(C) x = 5R (D) x = $\frac{7R}{5}$

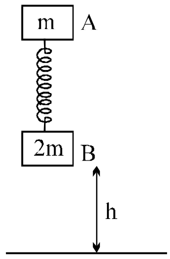
CM0076

7. In adjacent figure a boy, on a horizontal platform A, kept on a smooth horizontal surface, holds a rope attached to a box B. Boy pulls the rope with a constant force of 50N. The coefficient of friction between boy and platform is 0.5. (Mass of boy = 80 kg, mass of platform = 120kg and mass of box = 100 kg)



(A) Velocity of platform relative to box after 4 sec. is 2m/s
(B) Velocity of boy relative to platform after 4sec is 2m/s
(C) Friction force between boy and platform is 30N
(D) Friction force between boy and platform is 50N **CM0077**

8. From what minimum height h must the system be released when spring is unstretched so that after perfectly inelastic collision (e = 0) with ground, B may be lifted off the ground (Spring constant = k).



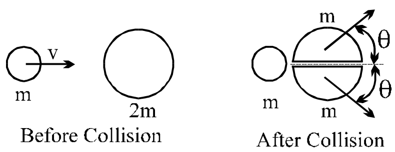
(A) mg/(4k) (B) 4mg/k (C) mg/(2k) (D) none

CM0078

9. An isolated particle of mass m is moving in horizontal plane (x–y), along the x–axis, at a certain height above the ground. It suddenly explodes into two fragment of masses $\frac{m}{4}$ and $\frac{3m}{4}$. An instant later, the smaller fragment is at y = +15 cm. The larger fragment at this instant is at :–
(A) y = –5 cm (B) y = +20 cm (C) y = +5 cm (D) y = –20 cm

CM0079

10. A particle of mass m is moving along the x-axis with speed v when it collides with a particle of mass 2m initially at rest. After the collisions, the first particle has come to rest, and the second particle has split into two equal-mass pieces that move at equal angles $\theta > 0$ with the x-axis, as shown in the figure. Which of the following statements correctly describes the speeds of the two pieces?



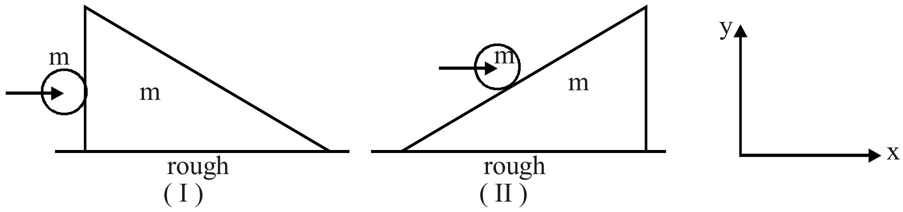
(A) Each piece moves with speed v
(B) One of the pieces moves with speed v, the other moves with speed less than v
(C) Each piece moves with speed $v/2$
(D) Each piece moves with speed greater than $v/2$

CM0080

11. A ball of mass m collides horizontally with a stationary wedge on a rough horizontal surface, in the two orientations as shown. Neglect friction between ball and wedge. Two student comment on system of ball and wedge in these situations

Saurav : Momentum of system in x-direction will change by significant amount in both cases.

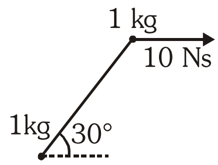
Rahul : There are no impulsive external forces in y-direction in both cases hence the total momentum of system in y-direction can be treated as conserved in both cases.



(A) **Saurav** is wrong and **Rahul** is correct (B) **Saurav** is correct and **Rahul** is wrong
(C) Both are correct (D) Both are wrong

CM0081

12. Two balls of masses 1 kg each are connected by an inextensible massless string. The system is resting on a smooth horizontal surface. An impulse of 10 Ns is applied to one of the balls at an angle 30° with the line joining two balls in horizontal direction as shown in the figure. Assuming that the string remains taut after the impulse, the magnitude of impulse of tension is :-



(A) 6Ns (B) $\frac{5}{2}\sqrt{3}$Ns (C) 5 Ns (D) $\frac{5}{\sqrt{3}}$Ns

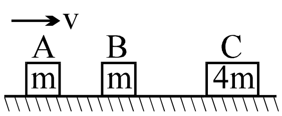
CM0082

13. A force exerts an impulse I on a particle changing its speed from u to 2u. The applied force and the initial velocity are oppositely directed along the same line. The work done by the force is

(A) $\frac{3}{2}Iu$ (B) $\frac{1}{2}Iu$ (C) I u (D) 2 I u

CM0083

14. Three blocks are initially placed as shown in the figure. Block A has mass m and initial velocity v to the right. Block B with mass m and block C with mass 4m are both initially at rest. Neglect friction. All collisions are elastic. The final velocity of block A is



(A) 0.6v to the left (B) 1.4v to the left (C) v to the left (D) 0.4v to the right

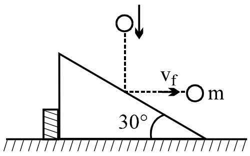
CM0084

15. Two billiard balls undergo a head-on collision. Ball 1 is twice as heavy as ball 2. Initially, ball 1 moves with a speed v towards ball 2 which is at rest. Immediately after the collision, ball 1 travels at a speed of v/3 in the same direction. What type of collision has occured?

(A) inelastic (B) elastic
(C) completely inelastic (D) cannot be determined from the information given

CM0085

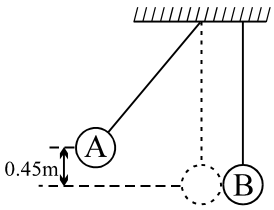
16. As shown in the figure a body of mass m moving vertically with speed 3 m/s hits a smooth fixed inclined plane and rebounds with a velocity v_f in the horizontal direction. If $\angle$ of inclined is 30°, the velocity v_f will be



(A) 3 m/s (B) $\sqrt{3}$ m/s (C) $1/\sqrt{3}$ m/s (D) this is not possible

CM0086

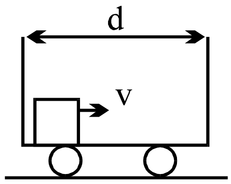
17. Two massless string of length 5 m hang from the ceiling very near to each other as shown in the figure. Two balls A and B of masses 0.25 kg and 0.5 kg are attached to the string. The ball A is released from rest at a height 0.45 m as shown in the figure. The collision between two balls is completely elastic. Immediately after the collision, the kinetic energy of ball B is 1 J. The velocity of ball A just after the collision is



(A) 5 ms^{-1} to the right (B) 5 ms^{-1} to the left (C) 1 ms^{-1} to the right (D) 1 ms^{-1} to the left

CM0087

18. In a smooth stationary cart of length d, a small block is projected along it's length with velocity v towards front. Coefficient of restitution for each collision is e. The cart rests on a smooth ground and can move freely. The time taken by block to come to rest w.r.t. cart is



(A) $\frac{ed}{(1-e)v}$ (B) $\frac{ed}{(1+e)v}$ (C) $\frac{d}{e}$ (D) infinite

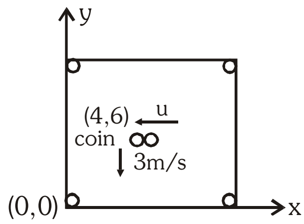
CM0088

19. A smooth sphere is moving on a horizontal surface with a velocity vector $(2\hat{i}+2\hat{j})$ m/s immediately before it hit a vertical wall. The wall is parallel to vector $\hat{j}$ and coefficient of restitution between the sphere and the wall is $e = 1/2$. The velocity of the sphere after it hits the wall is

(A) $\hat{i}-\hat{j}$ (B) $-\hat{i}+2\hat{j}$ (C) $-\hat{i}-\hat{j}$ (D) $2\hat{i}-\hat{j}$

CM0089

20. On a smooth carom board, a coin moving in negative y–direction with a speed of 3 m/s is being hit at the point (4, 6) by a striker moving along negative x–axis. The line joining centres of the coin and the striker just before the collision is parallel to x–axis. After collision the coin goes into the hole located at the origin. Masses of the striker and the coin are equal. Considering the collision to be elastic, the initial and final speeds of the striker in m/s will be



(A) (1.2, 0) (B) (2, 0) (C) (3, 0) (D) None of these

CM0090

21. Figure shows a block A of mass 5 kg kept at rest on a horizontal smooth surface. A spring (K = 200 N/m) which is compressed by 10 cm and tied with the help of a string to maintain the compression is attached to block A as shown in figure. Block B also of mass 5 kg moving with 2 m/s collides with A, as shown. During the collision the string breaks and after the collision the spring is in its natural state. Assume the bodies to be elastic and let the velocities of A and B be v_1 and v_2 respectively assuming positive direction towards right, after collision. Then

(A) $v_1 + v_2 > 2$

(B) Initial kinetic energy of system = final kinetic energy of system

(C) $v_1^2 + v_2^2 = 4.4\ (m/s)^2$

(D) $v_1 - v_2 = 2$

CM0091

22. An open water tight railway wagon of mass 5×10^3 kg coasts at an initial velocity 1.2 m/s without friction on a railway track. Rain drops fall vertically downwards into the wagon. The velocity of the wagon after it has collected 10^3 kg of water will be

(A) 0.5 m/s (B) 2 m/s (C) 1 m/s (D) 1.5 m/s

CM0092

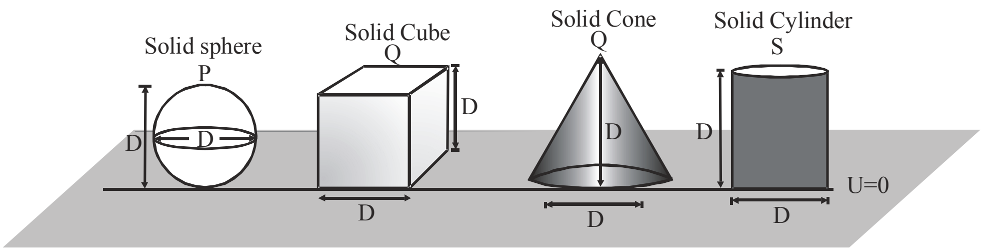
23. A rocket of mass 4000 kg is set for vertical firing. How much gas must be ejected per second so that the rocket may have initial upwards acceleration of magnitude 19.6 m/s^2. [Exhaust speed of fuel = 980 m/s.]

(A) 240 kg s^{-1} (B) 60 kg s^{-1} (C) 120 kg s^{-1} (D) None

CM0093

MULTIPLE CORRECT TYPE QUESTIONS

24. Assuming potential energy 'U' at ground level to be zero.



All objects are made up of same material.

U_P = Potential energy of solid sphere

U_Q = Potential energy of solid cube

U_R = Potential energy of solid cone

U_S = Potential energy of solid cylinder

(A) $U_S > U_P$ (B) $U_Q > U_S$ (C) $U_P > U_Q$ (D) $U_S > U_R$

CM0094

25. A blast breaks a body initially at rest of mass 0.5 kg into three pieces, two smaller pieces of equal mass and the third double the mass of either of small piece. After the blast the two smaller masses move at right angles to one another with equal speed. Find the statements that is/are true for this case assuming that the energy of blast is totally transferred to masses.

(A) All the three pieces share the energy of blast equally

(B) The speed of bigger mass is $\sqrt{2}$ times the speed of either of the smaller mass

(C) The direction of motion of bigger mass makes an angle of 135° with the direction of smaller pieces

(D) The bigger piece carries double the energy of either piece.

CM0095

26. A particle moving with kinetic energy = 3 joule makes an elastic head on collision with a stationary particle which has twice its mass during the impact.

(A) The minimum kinetic energy of the system is 1 joule.

(B) The maximum elastic potential energy of the system is 2 joule.

(C) Momentum and total kinetic energy of the system are conserved at every instant.

(D) The ratio of kinetic energy to potential energy of the system first decreases and then increases.

CM0096

27. In a one dimensional collision between two identical particles A and B, B is stationary and A has momentum p before impact. During impact, B gives impulse J to A.

(A) The total momentum of the 'A plus B' system is p before and after the impact, and (p-J) during the impact.

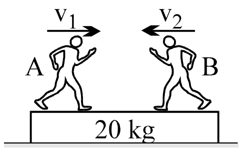
(B) During the impact A gives impulse of magnitude J to B

(C) The coefficient of restitution is $\frac{2J}{p}-1$

(D) The coefficient of restitution is $\frac{J}{p}+1$

CM0097

28. In the figure shown the system is at rest initially. Two persons 'A' and 'B' of masses 40 kg each move with speeds v_1 and v_2 respectively towards each other on a plank lying on a smooth horizontal surface as shown in figure. Plank travels a distance of 20 m towards right direction in 5 sec. (Here v_1 and v_2 are given with respect to the plank). Then the possible condition(s) can be



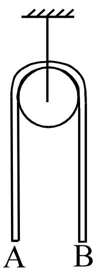
(A) $v_1 = 0$ m/s, $v_2 = 10$ m/s
(B) $v_1 = 5$ m/s, $v_2 = 15$ m/s
(C) $v_1 = 10$ m/s, $v_2 = 20$ m/s
(D) $v_1 = 2$ m/s, $v_2 = 12$ m/s

CM0098

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 29 and 30

A uniform chain of length 2L is hanging in equilibrium position, if end B is given a slightly downward displacement the imbalance causes an acceleration. Here pulley is small and smooth & string is inextensible



29. The acceleration of end B when it has been displaced by distance x, is

(A) $\frac{x}{L}g$ (B) $\frac{2x}{L}g$ (C) $\frac{x}{2}g$ (D) g

CM0099

30. The velocity v of the string when it slips out of the pulley (height of pulley from floor > 2L)

(A) $\sqrt{\frac{gL}{2}}$ (B) $\sqrt{2gL}$ (C) $\sqrt{gL}$ (D) none of these

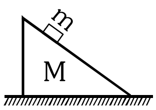
CM0099

MATRIX MATCH TYPE QUESTION

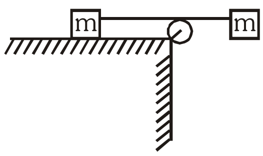
31. In each situation of column–I, a system involving two bodies is given. All strings and pulleys are light and friction is absent everywhere. Initially each body of every system is at rest. Consider the system in all situation of column I from rest till any collision occurs. Then match the statements in column–I with the corresponding results in column–II

Column I / **Column II**

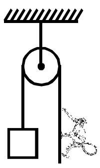
(A) The block plus wedge system is placed over smooth horizontal surface. After the system is released from rest, the centre of mass of system



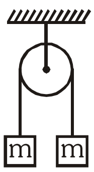
(B) The string connecting both the blocks of mass m is horizontal. Left block is placed over smooth horizontal table as shown. After the two block system is released from rest, the centre of mass of system



(C) The block and monkey have same mass. The monkey starts climbing up the rope. After the monkey starts climbing up, the centre of mass of monkey+block system



(D) Both block of mass m are initially at rest. The left block is given initial velocity u downwards. Then, the centre of mass of two block system afterwards



(P) Shifts towards right

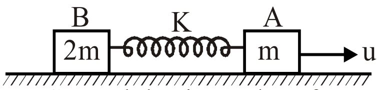
(Q) Shifts downwards

(R) Shifts upwards

(S) Does not shift

CM0100

32. Two blocks A and B of mass m and 2m respectively are connected by a massless spring of spring constant K. This system lies over a smooth horizontal surface. At t = 0 the block A has velocity u towards right as shown while the speed of block B is zero, and the length of spring is equal to its natural length at that instant.

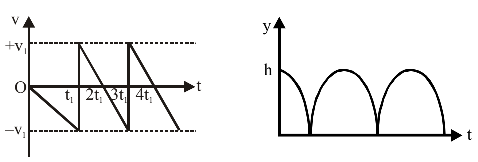


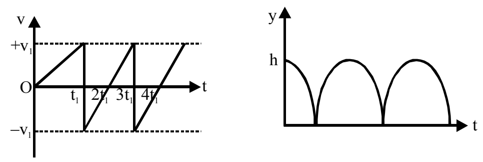
Column-I	Column-II
(A) The velocity of block A	(P) can never be zero
(B) The velocity of block B	(Q) may be zero at certain instants of time
(C) The kinetic energy of system of two block	(R) is minimum at maximum compression of spring
(D) The potential energy of spring	(S) is maximum at maximum extension of spring

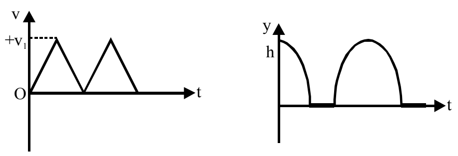
CM0101

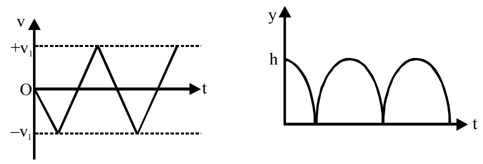
EXERCISE (J-M)

1. Consider a rubber ball freely falling from a height h = 4.9 m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic.
Then the velocity as a function of time and the height as a function of time will be :- **[AIEEE - 2009]**

(1) 

(2) 

(3) 

(4) 

CM0102

Directions : Question number 4 contain Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best discribes the two statements.

2. **Statement-1 :** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.

[AIEEE - 2010]

(1) Statement–1 is true, Statement–2 is false
(2) Statement–1 is true, Statement–2 is true; Statement–2 is the correct explanation of Statement–1
(3) Statement–1 is true, Statement–2 is true; Statement–2 is not the correct explanation of Statement–1
(4) Statement–1 is false, Statement–2 is true

CM0103

3. This question has Statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes the two Statements. **[JEE Main-2013]**

Statement - I : A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f = \left(\frac{m}{M+m}\right)$.

Statement - II : Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(1) Statement–I is true, Statement–II is true, Statement–II is a correct explanation of Statement–I.
(2) Statement–I is true, Statement–II is true, Statement–II is a not correct explanation of Statement–I.
(3) Statement–I is true, Statement–II is false.
(4) Statement–I is false, Statement–II is true

CM0104

4. A particle of mass m moving in the x direction with speed 2υ is hit by another particle of mass 2m moving in the y direction with speed υ. If the collisions perfectly inelastic , the percentage loss in the energy during the collision is close to : **[JEE Main-2015]**

(1) 56 % (2) 62% (3) 44% (4) 50%

CM0105

5. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to :- **[JEE Main-2015]**

(1) $\frac{5h}{8}$ (2) $\frac{3h^2}{8R}$ (3) $\frac{h^2}{4R}$ (4) $\frac{3h}{4}$

CM0106

6. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d ; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c. The values of p_d and p_c are respectively : **[JEE Main-2018]**

(1) (·28, ·89) (2) (0, 0) (3) (0, 1) (4) (·89, ·28)

CM0107

7. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50 % greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is : **[JEE Main-2018]**

(1) $\sqrt{2}v_0$ (2) $\frac{v_0}{2}$ (3) $\frac{v_0}{\sqrt{2}}$ (4) $\frac{v_0}{4}$

CM0108

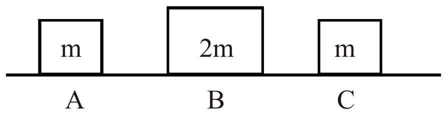
8. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly : **[JEE Main-2018]**

(1) 4.70×10^3 N/m^2 (2) 2.35×10^2 N/m^2 (3) 4.70×10^2 N/m^2 (4) 2.35×10^3 N/m^2

CM0109

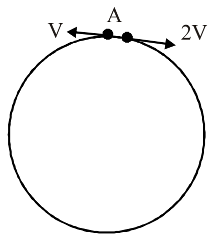
EXERCISE (J-A)

1. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, 2m and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in m/s) of the object C. **[IIT-JEE-2009]**



CM0110

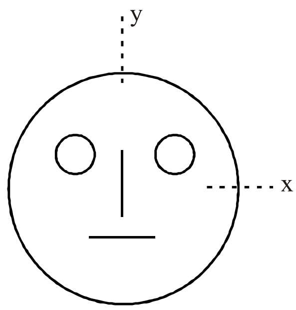
2. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and 2v, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A? **[IIT-JEE-2009]**



(A) 4 (B) 3 (C) 2 (D) 1

CM0111

3. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink used to draw the outer circle is 6m. The coordinates of the centres of the different parts are: outer circle (0, 0), left inner circle (–a, a), right inner circle (a, a), vertical line (0, 0) and horizontal line (0, –a). The y-coordinate of the centre of mass of the ink in this drawing is **[IIT-JEE-2009]**



(A) $\frac{a}{10}$ (B) $\frac{a}{8}$ (C) $\frac{a}{12}$ (D) $\frac{a}{3}$

CM0112

4. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. Aftr their collision, the 1 kg mass reverses its direction and moves with a speed of 2 m/s. Which of the following statemet(s) is (are) correct for the system of these two masses ? **[IIT-JEE 2010]**

(A) Total momentum of the system is 3 kg m/s.

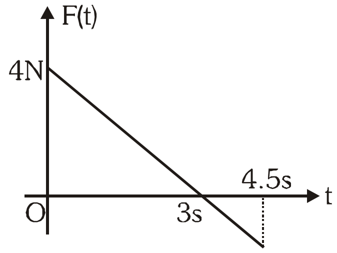
(B) Momentum of 5 kg mass after collision is 4 kg m/s.

(C) Kinetic energy of the centre of mass is 0.75 J.

(D) Total kinetic energy of the system is 4 J.

CM0113

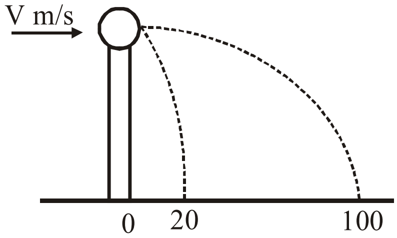
5. A block of mass 2 kg is free to move along the x–axis. It is at rest and from t=0 onwards it is subjected to a time–dependent force F(t) in the x–direction. The force F(t) varies with t as shown in the figure. The kinetic energy of the block after 4.5 second is **[IIT-JEE-2010]**



(A) 4.50 J (B) 7.50 J
(C) 5.06 J (D) 14.06 J

CM0114

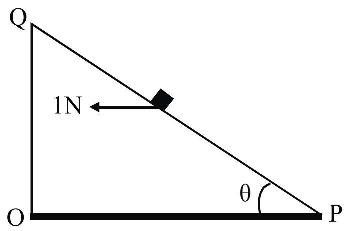
6. A ball of mass 0.2 kg rests on a vertical post of height 5m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity V of the bullet is + **[IIT-JEE 2011]**



(A) 250 m/s (B) $250\sqrt{2}$ m/s (C) 400 m/s (D) 500 m/s

CM0115

7. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its center of mass as shown in the figure. The block remains stationary if (take $g = 10\ m/s^2$) **[IIT-JEE 2012]**



(A) $\theta = 45°$

(B) $\theta > 45°$ and a frictional force acts on the block towards P

(C) $\theta > 45°$ and a frictional force acts on the block towards Q

(D) $\theta < 45°$ and a frictional force acts on the block towards Q

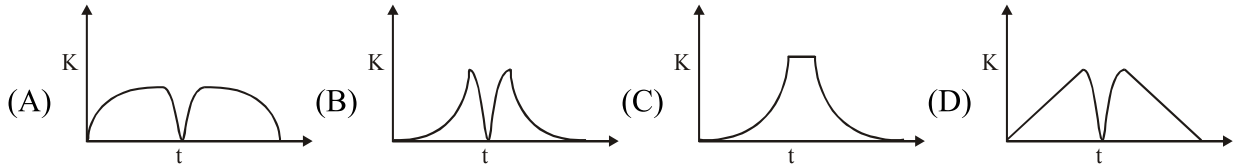
CM0116

8. A bob of mass m, suspended by a string of length ℓ_1 is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length ℓ_2, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum sped required to complete a full circle in the vertical plane, the ratio $\frac{\ell_1}{\ell_2}$ is. **[JEE Advanced-2013]**

CM0117

9. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately ? The figures are only illustrative and not to the scale.

[JEE Advanced-2014]



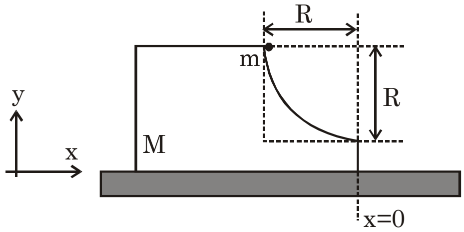
CM0118

10. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at x = 0, in a *co-ordinate system fixed to the table*. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following options is/are **correct**? **[JEE Advanced-2017]**

(A) The x component of displacement of the centre of mass of the block M is : $-\frac{mR}{M+m}$

(B) The position of the point mass is : $x = -\sqrt{2}\frac{mR}{M+m}$

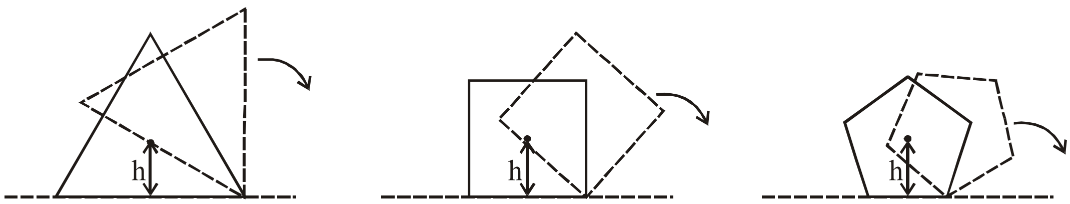
(C) The velocity of the point mass m is : $v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$



(D) The velocity of the block M is : $V = -\frac{m}{M}\sqrt{2gR}$

CM0119

11. Consider regular polygons with number of sides n = 3, 4, 5 ..... as shown in the figure. The center of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ. Then Δ depends on n and h as :**[JEE Advanced-2017]**



(A) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$

(B) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$

(C) $\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)}-1\right)$

(D) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$

12. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass $m = 0.4kg$ is at rest on this surface. An impulse of 1.0 N s is applied to the block at time to $t = 0$ so that it starts moving along the x-axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and $\tau = 4$ s. The displacement of the block, in metres, at $t = \tau$ is........... Take $e^{-1} = 0.37$? **[JEE Advanced-2018]**

CM0121

13. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V << \frac{dL}{L} v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct ? **[JEE Advanced-2019]**

(1) The rate at which the particle strikes the piston is v/L

(2) After each collision with the piston, the particle speed increases by 2V

(3) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2} L_0$

(4) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$

CM0122

ANSWER KEY

EXERCISE (S-1)

1. Ans. (1/7, 23/14) **2. Ans.** √13 m, $\left(\frac{14}{5},\frac{19}{5}\right)$ **3. Ans.** L($\sqrt{2}+1$)/3

4. Ans. $\frac{a}{3(\pi-1)}$ **5. Ans.** 4R from O **6. Ans.** x = 6m

7. Ans. (3 m, 1 m, 8 m)

$$\frac{6\left(\hat{i}+2\hat{j}+3\hat{k}\right)+5\left(-\hat{i}+3\hat{j}-2\hat{k}\right)+5\vec{r}}{16}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)$$

$$\vec{r}=\left(3\hat{i}+\hat{j}+8\hat{k}\right)$$

8. Ans. g/9 downwards **9. Ans.** $\frac{L}{3}$ **10. Ans.** $\vec{v}_C=-\vec{v}_B$ **11. Ans.** 150 kg

12. Ans. (i) 4 m/s, (ii) 24 J **13. Ans.** (i) 360 m, (ii) 10800 J

14. Ans. $\vec{P}_{PM}=m\vec{v}_{PM}$

$$=-mv_2\sin\omega t\,\hat{i}+m(v_2\cos\omega t-v_1)\,\hat{j}$$

15. Ans. $\frac{\sqrt{13}}{2}v_0$ **16. Ans.** 30 cm **17. Ans.** 0.3 **18. Ans.** $m\times\sqrt{u^2-uv+v^2}$

19. Ans. $\frac{7}{18}$ **20. Ans.** (i) 3 J, (ii) $\frac{12}{5}$ N-s

21. Ans. (i) $v_0/3$, (ii) $3\sqrt{5gR}$. **22. Ans.** $(\hat{i}+\sqrt{3}\hat{j})$ m/s, $(3m\hat{i}-\sqrt{3}m\hat{j})$ kg-m/s

23. Ans. (i) $v_A=\sqrt{g/12}$ m/s, (ii) S_{max} = 49/48 m **24. Ans.** 1 N

EXERCISE (S-2)

1. Ans. (i) $\lambda(x)=\lambda_0+\frac{\lambda_0 x}{\ell}$, (ii) $\frac{5}{9}\ell$ **2. Ans.** $\frac{R}{4}$

3. Ans. $\frac{m(R-r)}{M+m}$, $m\sqrt{\frac{2g(R-r)}{M(M+m)}}$ **4. Ans.** 4 **5. Ans.** $\frac{2mv^2}{3\ell}$

6. Ans. –2m/s, 6.93 m/s ∠ 30°

7. Ans. (i) v/2, v/2, 0; (ii) $2mv^2/9$; (iii) $mv^2/72$; (d) $x=\sqrt{m/6k}\,v$

8. Ans. $m\left(-3\hat{i}+4\hat{j}\right)$, $e=\frac{9}{16}$ **9. Ans.** (i) $\sqrt{\frac{2ag}{3}}$ (ii) $\frac{3v}{g}$ (iii) $\frac{2v}{g}$

10. Ans. (i) $\frac{u}{2}$, $\frac{mu}{2}$ (ii) $\frac{u\sqrt{13}}{8}$, $\frac{mu\sqrt{13}}{8}$ (iii) $\frac{u\sqrt{3}}{4}$, $\frac{mu\sqrt{3}}{4}$

EXERCISE (O-1)

1. Ans. (C) **2. Ans. (B)** **3. Ans. (D)** **4. Ans. (C)** **5. Ans. (B)** **6. Ans. (C)**
7. Ans. (C) **8. Ans. (A)** **9. Ans. (C)** **10. Ans. (A)** **11. Ans. (i) (B) (ii) (C)**
12. Ans. (C) **13. Ans. (D)** **14. Ans. (D)** **15. Ans. (C)** **16. Ans. (A)** **17. Ans. (B)**
18. Ans. (A) **19. Ans. (B)** **20. Ans. (B)** **21. Ans. (B)** **22. Ans. (C)** **23. Ans. (D)**
24. Ans. (D) **25. Ans. (C)** **26. Ans. (D)** **27. Ans. (A, B,D)** **28. Ans. (A, C)**
29. Ans. (B,C) **30. Ans. (AB)** **31. Ans. (A)** **32. Ans. (A)** **33. Ans. (B)**
34. Ans. (C) **35. Ans. (D)** **36. Ans. (A)-P; (B)-S; (C)-Q, R; (D)-T**
37. Ans. A-(s), B-(q), C-(p), D-(r) **38. Ans. (A)-Q (B)-S (C)-P**
39. Ans. (A) R, (B) T, (C) T, (D)S

EXERCISE (O-2)

1. Ans. (A) **2. Ans. (D)** **3. Ans. (B)** **4. Ans. (D)** **5. Ans. (D)** **6. Ans. (B)**
7. Ans. (C) **8. Ans. (B)** **9. Ans. (A)** **10. Ans. (D)** **11. Ans. (D)** **12. Ans. (B)**
13. Ans. (B) **14. Ans. (A)** **15. Ans. (B)** **16. Ans. (B)** **17. Ans. (D)** **18. Ans. (D)**
19. Ans. (B) **20. Ans. (B)** **21. Ans. (C)** **22. Ans. (C)** **23. Ans. (C)** **24. Ans. (A,B,D)**
25. Ans. (A,C) **26. Ans. (A,B,D)** **27. Ans. (B,C)**
28. Ans. (A,B,C,D) **29. Ans. (A)** **30. Ans. (C)**
31. Ans. (A)-Q; (B)-P, Q; (C)-R; (D) S **32. Ans. (A)-Q; (B)-Q; (C)-P,R;, (D)-Q,S**

EXERCISE (J-M)

1. Ans. (1) **2. Ans. (2)** **3. Ans. (4)** **4. Ans. (1)** **5. Ans. (4)** **6. Ans. (4)**
7. Ans. (1) **8. Ans. (4)**

EXERCISE (J-A)

1. Ans. 4m/s **2. Ans. (C)** **3. Ans. (A)** **4. Ans. (A,C)** **5. Ans. (C)** **6. Ans. (D)**
7. Ans. (A,C) **8. Ans. 5** **9. Ans. (B)** **10. Ans. (A,C)** **11. Ans. (C)**
12. Ans. 6.3 [6.29, 6.31] **13. Ans. (2, 3)**