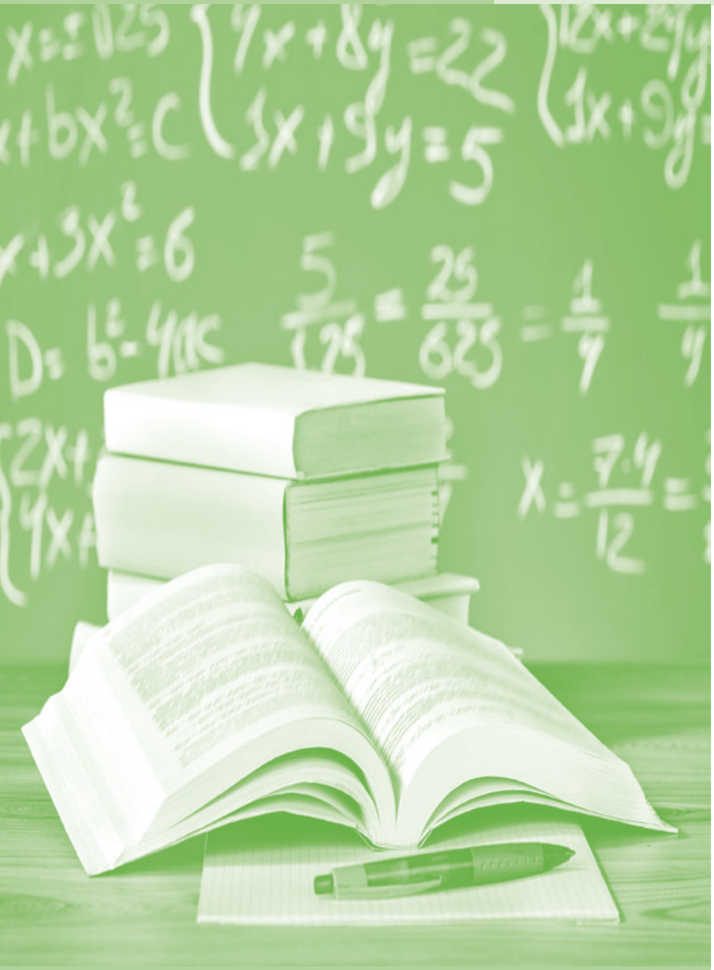
Chapter 14

Coordinate Geometry



REMEMBER

Before beginning this chapter, you should be able to:

- Know planes, lines and angles
- Remember different types of triangles and polygons

KEY IDEAS

After completing this chapter, you should be able to:

- Find the coordinates of a point and conversion of signs
- Study about points on a plane and the distance between points
- Know the applications of distance formula, mid-point of a line segment and centroid of a triangle
- Learn about equation of some standard lines

INTRODUCTION

Let $X'OX$ and YOY' be two mutually perpendicular lines intersecting at point O in a plane.

These two lines are called reference lines or coordinate axes. The horizontal reference line $X'OX$ is called X-axis and the vertical reference line YOY' is called Y-axis.

The point of intersection of these two axes, i.e., O is called the origin. The plane containing the coordinate axes is called coordinate plane or XY-plane.

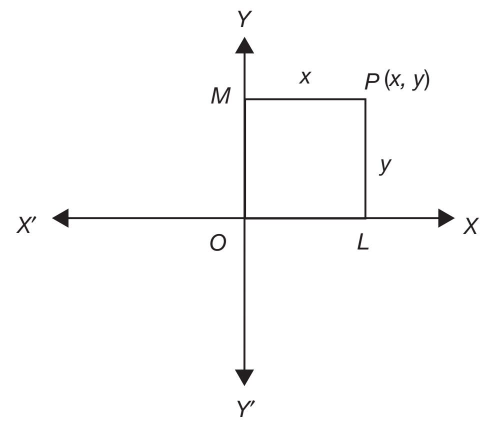


Figure 14.1

COORDINATES OF A POINT

Let P be a point in the XY-plane. Draw perpendiculars PL and PM to X-axis and Y-axis respectively (see Fig. 14.1).

Let $PL = y$ and $PM = x$. The point P is taken as (x, y). Here, x and y are called the rectangular Cartesian coordinates or coordinates of the point P. x is called x-coordinate or abscissa and y is called y-coordinate or ordinate of the point P.

Convention of Signs

1. Towards the right side of the Y-axis, x-coordinate of any point on the graph paper is taken positive and towards the left side of the Y-axis, x-coordinate is taken negative.
2. Above the X-axis, the y-coordinate of any point on the graph paper is taken positive and below the X-axis, y-coordinate is taken negative.

If (x, y) is a point in the plane and Q_1, Q_2, Q_3 and Q_4 are the four quadrants of rectangular coordinate system, then:

1. If $x > 0$ and $y > 0$, then $(x, y) \in Q_1$.
2. If $x < 0$ and $y > 0$, then $(x, y) \in Q_2$.
3. If $x < 0$ and $y < 0$, then $(x, y) \in Q_3$.
4. If $x > 0$ and $y < 0$, then $(x, y) \in Q_4$.

EXAMPLE 14.1

If $x > 0$ and $y < 0$, then $(x, -y)$ lies in which quadrant?

SOLUTION

$y < 0$

$\Rightarrow -y > 0$

$\therefore$ The point $(x, -y)$ lies in first quadrant, i.e., Q_1.

EXAMPLE 14.2

If $(x, -y) \in Q_2$, then (x, y) belongs to which quadrant?

SOLUTION

Given, $(x, -y) \in Q_2 \Rightarrow x < 0, y < 0$.

$\therefore$ (x, y) belongs to third quadrant, i.e., Q_3.

EXAMPLE 14.3

Plot the points $A(2, 3)$, $B(-1, 2)$, $C(-3, -2)$ and $D(4, -2)$ in the XY-plane.

SOLUTION

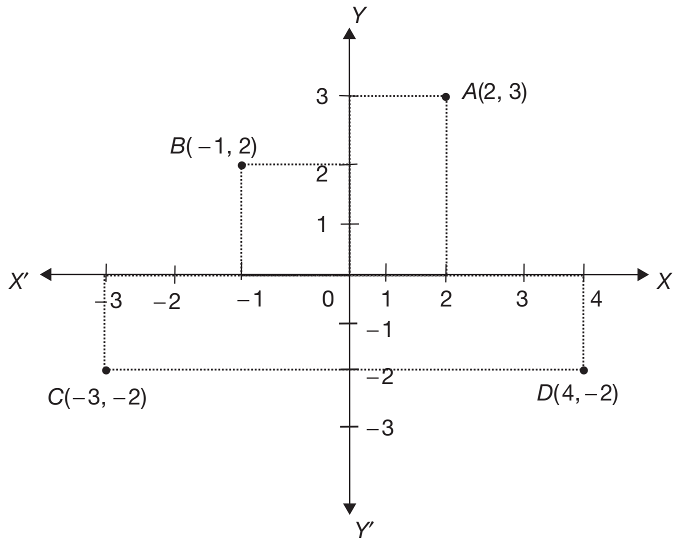


Figure 14.2

POINTS ON THE PLANE

Point on X-axis and Y-axis

Let P be a point on X-axis, so that its distance from X-axis is zero. Hence, point P can be taken as $(x, 0)$.

Let P' be a point on Y-axis, so that its distance from Y-axis is zero. Hence, point P' can be taken as $(0, y)$ (see Fig. 14.3).

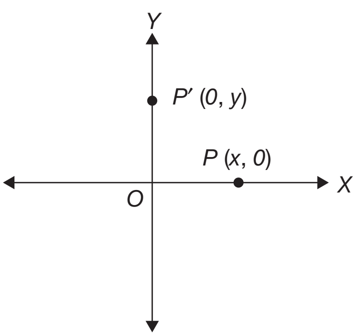


Figure 14.3

Distance Between Two Points

Consider two points $A(x_1, y_1)$ and $B(x_2, y_2)$. Draw perpendiculars AL and BM from A and B to X-axis. AN is the perpendicular drawn from A on to BM (see Fig. 14.4).

From right triangle ABN, $AB = \sqrt{AN^2 + BN^2}$, (we have $AB^2 = AN^2 + BN^2$).
Here, $AN = x_2 - x_1$ and $BN = y_2 - y_1$.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Hence, the distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ units.}$$

Note The distance of a point $A(x_1, y_1)$ from origin $O(0, 0)$ is $OA = \sqrt{x_1^2 + y_1^2}$.

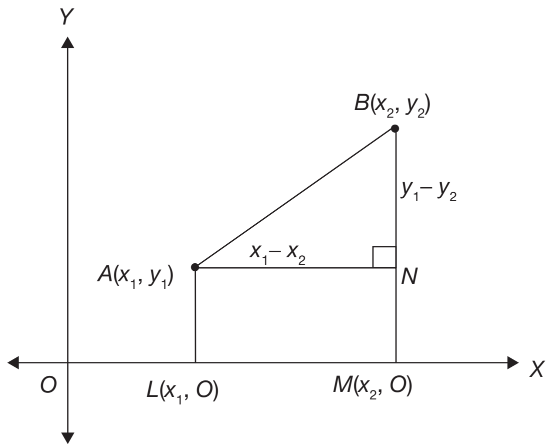


Figure 14.4

EXAMPLE 14.4

Find the distance between points (–4, 5) and (2, –3).

SOLUTION

Let the given points be $A(-4, 5)$ and $B(2, -3)$

$$AB = \sqrt{(2-(-4))^2 + (-3-5)^2}$$
$$= \sqrt{36+64} = 10 \text{ units.}$$

EXAMPLE 14.5

Find a, if the distance between points $A(8, -7)$ and $B(-4, a)$ is 13 units.

SOLUTION

Given, $AB = 13$

$\Rightarrow \sqrt{(-4-8)^2 + (a+7)^2} = 13$

Taking squares on both sides, we get

$$(a+7)^2 = 169 - 144 = 25$$
$$a + 7 = \sqrt{25}$$
$$a + 7 = \pm 5$$
$$\therefore\ a = -2 \text{ or } -12.$$

EXAMPLE 14.6

Find the coordinates of a point on Y-axis which is equidistant from points (13, 2) and (12, –3).

SOLUTION

Let $P(0, y)$ be the required point and the given points be $A(12, -3)$ and $B(13, 2)$.

Then, $PA = PB$ (given)

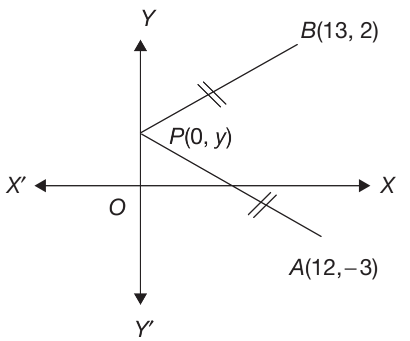


Figure 14.5

$$\sqrt{(12-0)^2+(-3-y)^2} = \sqrt{(13-0)^2+(2-y)^2}$$
$$\Rightarrow \sqrt{144+(y+3)^2} = \sqrt{169+(2-y)^2}$$

Taking squares on both sides, we get

$$169 + 4 + y^2 - 4y = 144 + 9 + y^2 + 6y$$
$$\Rightarrow 10y = 20 \Rightarrow y = 2$$

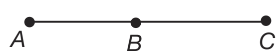
∴ The required point on Y-axis is (0, 2).

Collinearity of Three Points

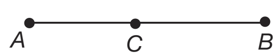
Let A, B and C be three given points. The distances AB, BC and CA can be calculated using distance formula. If the sum of any two of these distances is found to be equal to the third distance, then points A, B and C are said to be collinear.

Notes

1. If $AB + BC = AC$, then points A, B and C are collinear.



2. If $AC + CB = AB$, then points A, C and B are collinear.



3. $BA + AC = BC$, then points B, A and C are collinear.

By Notes (1), (2) and (3), we can find the position of the points in collinearity.

Applications of Distance Formula

EXAMPLE 14.7

Show that points $P(5, 6)$, $Q(4, 5)$ and $R(3, 4)$ are collinear.

SOLUTION

Given, $P = (5, 6)$, $Q = (4, 5)$ and $R = (3, 4)$.

$PQ = \sqrt{(4-5)^2+(5-6)^2} = \sqrt{2}$ units.

$QR = \sqrt{(3-4)^2 + (4-5)^2} = \sqrt{2}$ units.

$PR = \sqrt{(3-5)^2 + (4-6)^2} = \sqrt{8} = 2\sqrt{2}$ units.

Now, $PQ + QR = \sqrt{2} + \sqrt{2} = 2\sqrt{2} = PR$.

That is, $PQ + QR = PR$.

Hence, points P, Q and R are collinear.

EXAMPLE 14.8

Show that points $A(3, -1)$, $B(-1, 2)$ and $C(6, 3)$ form an isosceles right-angled triangle when joined.

SOLUTION

Given, $A = (3, -1)$, $B = (-1, 2)$ and $C = (6, 3)$.

$AB = \sqrt{(-1-3)^2 + (2+1)^2} = 5$ units

$BC = \sqrt{(6-(-1))^2 + (3-2)^2} = \sqrt{50}$ units

$AC = \sqrt{(6-3)^2 + (3-(-1))^2} = 5$ units

Clearly,

$BC^2 = AB^2 + AC^2$.

Also, $AB = AC$.

Hence, the given points form the vertices of a right-angled isosceles triangle.

EXAMPLE 14.9

Show that points $(2-\sqrt{3}, \sqrt{3}+1)$, $(1, 0)$ and $(3, 2)$ form an equilateral triangle.

SOLUTION

Let $A(2-\sqrt{3}, \sqrt{3}+1)$, $B(1, 0)$ and $C(3, 2)$ be the given points.

$$AB = \sqrt{(1-2+\sqrt{3})^2 + \left(0-(\sqrt{3}+1)\right)^2}$$

$$= \sqrt{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2}.$$

$AB = \sqrt{8}$ units.

$$BC = \sqrt{(3-1)^2 + (2-0)^2} = \sqrt{8} \text{ units}$$

$$AC = \sqrt{\left(3-(2-\sqrt{3})\right)^2 + \left(2-(\sqrt{3}+1)\right)^2}$$

$$= \sqrt{(1+\sqrt{3})^2 + (1-\sqrt{3})^2} = \sqrt{8} \text{ units.}$$

$\therefore AB = BC = AC = \sqrt{8}$ units.

Hence, the given points form an equilateral triangle.

EXAMPLE 14.10

Show that points $A(-1, 0)$, $B(-2, 1)$, $C(1, 3)$ and $D(2, 2)$ form a parallelogram.

SOLUTION

Given, $A(-1, 0)$, $B(-2, 1)$, $C(1, 3)$ and $D(2, 2)$.

$AB = \sqrt{(-2+1)^2 + (1-0)^2} = \sqrt{2}$ units

$BC = \sqrt{(1-(-2))^2 + (3-1)^2} = \sqrt{13}$ units

$CD = \sqrt{(2-1)^2 + (2-3)^2} = \sqrt{2}$ units

$DA = \sqrt{(2-(-1))^2 + (2-0)^2} = \sqrt{13}$ units

$AC = \sqrt{(1-(-1))^2 + (3-0)^2} = \sqrt{13}$ units

$BD = \sqrt{(2-(-2))^2 + (2-1)^2} = \sqrt{17}$ units

Clearly,

$AB = CD$, $BC = DA$ and $AC \neq BD$.

That is, the opposite sides of the quadrilateral are equal and diagonals are not equal.

Hence, the given points form a parallelogram.

EXAMPLE 14.11

Find the circum-centre and the circum-radius of a triangle ABC formed by the vertices $A(2, -2)$, $B(-1, 1)$ and $C(3, 1)$.

SOLUTION

Let $S(x, y)$ be the circum-centre of ΔABC.

$\therefore SA^2 = SB^2 = SC^2$

Consider, $SA^2 = SB^2$

$$\Rightarrow (x-2)^2 + (y+2)^2 = (x+1)^2 + (y-1)^2$$
$$x^2 - 4x + 4 + y^2 + 4y + 4 = x^2 + 2x + 1 + y^2 - 2y + 1$$
$$-4x + 4y + 8 = 2x - 2y + 2$$
$$6x - 6y - 6 = 0$$
$$x - y - 1 = 0 \qquad (1)$$

$SB^2 = SC^2$

$$\Rightarrow (x+1)^2 + (y-1)^2 = (x-3)^2 + (y-1)^2$$
$$x^2 + 2x + 1 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 2y + 1$$
$$2x - 2y + 2 = -6x - 2y + 10$$
$$8x - 8 = 0$$
$$\Rightarrow x = 1.$$

Substituting $x = 1$ in Eq. (1), we get $y = 0$.

$\therefore$ The required circum-centre of ΔABC is $(1, 0)$.

Circum-radius, $SA = \sqrt{(2-1)^2 + (-2-0)^2} = \sqrt{5}$ units.

EXAMPLE 14.12

Find the area of the circle whose centre is (–1, –2), and (3, 4) is a point on the circle.

SOLUTION

Let the centre of the circle be $A(-1, -2)$, and the point on the circumference be $B(3, 4)$.

Radius of circle = AB

$$= \sqrt{(3-(-1))^2 + (4-(-2))^2} = \sqrt{52} \text{ units.}$$

$\therefore$ The area of the circle $= \pi r^2$

$$= \pi(\sqrt{52})^2 = 52\pi \text{ sq. units.}$$

EXAMPLE 14.13

Find the area of the square whose one pair of opposite vertices are (2, –3) and (4, 5).

SOLUTION

Let the given vertices be $A(2, -3)$ and $C(4, 5)$.

Length of $AC = \sqrt{(4-2)^2 + (5+3)^2}$

$= \sqrt{68}$ units.

$\therefore$ Area of the square $= \dfrac{AC^2}{2} = \dfrac{(\sqrt{68})^2}{2} = 34$ sq. units.

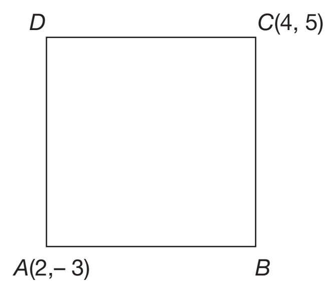


Figure 14.6

STRAIGHT LINES

Inclination of a Line

The angle made by a straight line with positive direction of X-axis in the anti-clockwise direction is called its inclination.

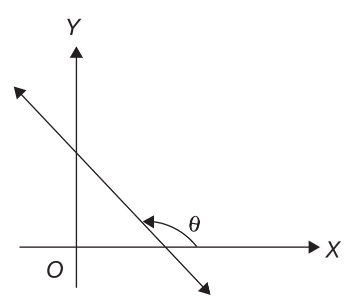


Figure 14.7

Slope or Gradient of a Line

If θ is the inclination of a line L, then its slope is denoted by m and is given by $m = \tan\theta$ (see Fig. 14.7).

Example: The inclination of the line l in adjacent Fig. 14.8 is 45°.

$\therefore$ The slope of the line is $m = \tan 45° = 1$.

Example: The line L in Fig. 14.9 makes an angle of 45° in clockwise direction with X-axis. So, the inclination of the line L is 180° – 45° = 135°.

$\therefore$ The slope of the line L is $m = \tan 135° = -1$.

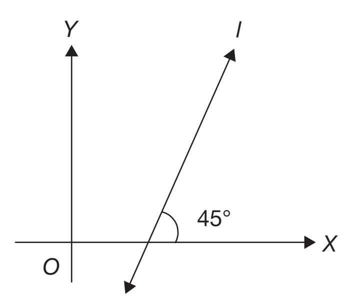


Figure 14.8

Some Results on the Slope of a Line

1. The slope of a horizontal line is zero. Hence,
   - (i) Slope of X-axis is zero.
   - (ii) Slope of any line parallel to X-axis is also zero.

2. The slope of a vertical line is not defined. Hence,
   (i) Slope of Y-axis is undefined.
   (ii) Slope of any line parallel to Y-axis is also undefined.

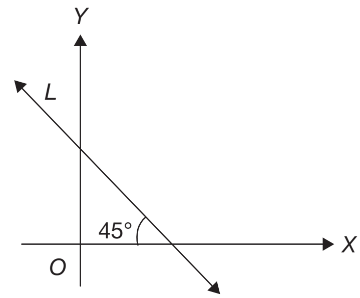


Figure 14.9

Theorem 1 Two non-vertical lines are parallel, if and only if, their slopes are equal.

Proof: Let L_1 and L_2 be two non-vertical lines with slopes m_1 and m_2 respectively. If θ_1 and θ_2 are the inclinations of the lines, L_1 and L_2 respectively, then $m_1 = \tan\theta_1$ and $m_2 = \tan\theta_2$, since $L_1 \parallel L_2$ .Then, $\theta_1 = \theta_2$ (see Fig. 14.10).

($\because$ They form a pair of corresponding angles)

$$\Rightarrow \tan\theta_1 = \tan\theta_2$$
$$\Rightarrow m_1 = m_2$$

Conversely: Let $m_1 = m_2$

$$\Rightarrow \tan\theta_1 = \tan\theta_2$$
$$\Rightarrow \theta_1 = \theta_2$$
$$\Rightarrow L_1 \parallel L_2$$

($\because$ θ_1 and θ_2 form a pair of corresponding angles.)

Hence, two non-vertical lines are parallel, if and only if, their slopes are equal.

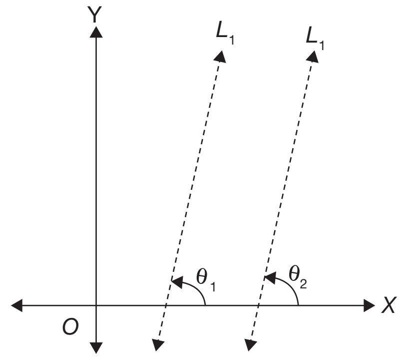


Figure 14.10

Theorem 2 Two non-vertical lines are perpendicular to each other, if and only if, the product of their slopes is −1.

Proof: Let L_1 and L_2 be two non-vertical lines with slopes m_1 and m_2. If θ_1 and θ_2 are the inclinations of the lines L_1 and L_2 respectively, then $m_1 = \tan\theta_1$ and $m_2 = \tan\theta_2$ (see Fig. 14.11).

If $L_1 \perp L_2$, then

$$\theta_2 = 90° + \theta_1$$

($\because$ The exterior angle of a triangle is equal to the sum of two opposite interior angles.)

$$\Rightarrow \tan\theta_2 = \tan(90° + \theta_1)$$
$$\Rightarrow \tan\theta_2 = -\cot\theta_1$$
$$\Rightarrow \tan\theta_2 = \frac{-1}{\tan\theta_1} \quad [\because \theta_1 \neq 0]$$
$$\Rightarrow \tan\theta_1 \times \tan\theta_2 = -1$$
$$\therefore m_1 m_2 = -1.$$

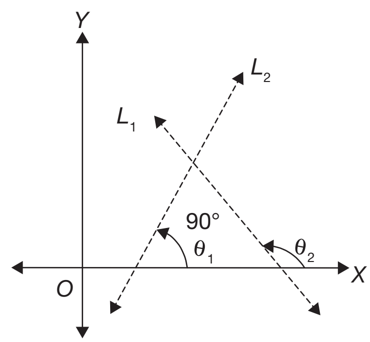


Figure 14.11

Conversely: Let $m_1 m_2 = -1$

$$\Rightarrow \tan\theta_1 \tan\theta_2 = -1$$
$$\Rightarrow \tan\theta_2 = \frac{-1}{\tan\theta_1} \quad [\because \theta_1 \neq 0]$$
$$\Rightarrow \tan\theta_2 = -\cot\theta_1$$
$$\Rightarrow \tan\theta_2 = \tan(90° + \theta_1)$$
$$\Rightarrow \theta_2 = 90° + \theta_1$$
$$\Rightarrow L_1 \perp L_2$$

Hence, two non-vertical lines are perpendicular to each other, if and only if, the product of their slopes is −1.

The Slope of a Line Passing through Points (x_1, y_1) and (x_2, y_2)

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two given points.

Let AB be the straight line passing through points A and B.

Let θ be the inclination of line $\overleftrightarrow{AB}$.

Draw perpendiculars AL and BM on to X-axis from A and B respectively. Also, draw $AN \perp BM$ (see Fig. 14.12).

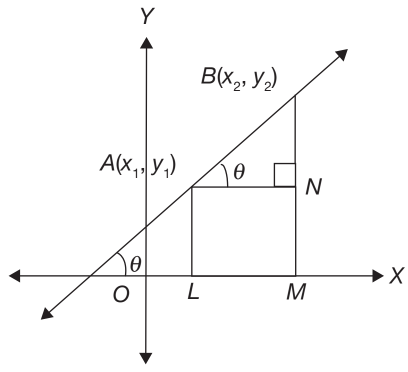


Figure 14.12

Then, let $\angle NAB = \theta$.

Here, $BN = BM - MN = BM - AL = y_2 - y_1$

$AN = LM = OM - OL = x_2 - x_1$

∴ The slope of the line L is

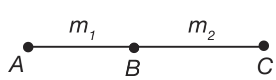
$$m = \tan\theta = \frac{BN}{AN} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Hence, the slope of a line passing through points (x_1, y_1) and (x_2, y_2) is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

The following table gives inclination (θ) of the line and its corresponding slope (m) for some particular values of θ.

θ	0°	30°	45°	60°	90°	120°	135°	150°
$m = \tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	−1	$-\frac{1}{\sqrt{3}}$

Note If points A, B and C are collinear, then the slope (m_1) of AB = the slope (m_2) of BC.



EXAMPLE 14.14

Find the slope of the line joining points (3, 8) and (−9, 6).

SOLUTION

Let $A(3, 8)$ and $B(-9, 6)$ be the given points.

Then, the slope of $\overleftrightarrow{AB} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{6-8}{-9-3} = \frac{1}{6}.$$

EXAMPLE 14.15

Find the value of p if the slope of the line joining points $(5, -p)$ and $(2, -3)$ is $\frac{-1}{3}$.

SOLUTION

Let the given points be $A(5, -p)$ and $B(2, -3)$.

Given, the slope of $\overleftrightarrow{AB} = \frac{-1}{3}$.

That is, $\frac{-3-(-p)}{2-5} = \frac{-1}{3}$

$$\Rightarrow \frac{p-3}{-3} = \frac{-1}{3}$$

$$\Rightarrow p-3 = 1$$

$$\Rightarrow p = 4.$$

EXAMPLE 14.16

Find the value of k, if lines AB and CD are perpendicular, where $A = (4, 5)$, $B = (k + 2, -3)$, $C = (-3, 2)$ and $D = (2, 4)$.

SOLUTION

The slope of $\overleftrightarrow{AB}$ $(m_1) = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-3-5}{(k+2)-4} = \frac{-8}{k-2}.$$

Slope of $\overleftrightarrow{CD}$ $(m_2) = \frac{4-2}{2-(-3)} = \frac{2}{5}.$

Since, $AB \perp PQ \Rightarrow m_1 m_2 = -1$

That is, $\frac{-8}{k-2} \times \left(\frac{2}{5}\right) = -1$

$$\Rightarrow \frac{16}{5k-10} = 1$$

$$\Rightarrow 16 = -10 + 5k$$

$$\Rightarrow k = \frac{26}{5}.$$

EXAMPLE 14.17

Find the value of k, if points $(-2, -4)$, $(k, -2)$ and $(3, 4)$ are collinear.

SOLUTION

Let the given points be $A(-2, -4)$, $B(k, -2)$ and $C(3, 4)$.

The slope of $AB = \frac{-2+4}{k+2} = \frac{2}{k+2}.$

The slope of $BC = \frac{4+2}{3-k} = \frac{6}{3-k}$.

Since the points A, B and C are collinear,

The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$

$$\Rightarrow \frac{2}{k+2} = \frac{6}{3-k}$$

$$\Rightarrow 2(3-k) = 6(k+2)$$

$$\Rightarrow 3-k = 3k+6$$

$$\Rightarrow 4k = -3$$

$$\Rightarrow k = \frac{-3}{4}.$$

EXAMPLE 14.18

Find the ortho-centre of the ΔABC formed by vertices $A(1, 6)$, $B(5, 2)$ and $C(12, 9)$.

SOLUTION

The given vertices of ΔABC are $A(1, 6)$, $B(5, 2)$ and $C(12, 9)$.

Slope of $AB = \frac{2-6}{5-1} = \frac{-4}{4} = -1$

Slope of $BC = \frac{9-2}{12-5} = \frac{7}{7} = 1$

Slope of $AC = \frac{9-6}{12-1} = \frac{3}{11}$

Slope of $AB \times$ Slope of $BC = -1$

$\therefore AB \perp BC$

Hence, ABC is a right triangle, right angle at B.

Hence, ortho-centre is the vertex containing right angle, i.e., $B(5, 2)$.

Intercepts of a Straight Line

Say a straight line L meets X-axis in A and Y-axis in B.

Then, OA is called the x-intercept and OB is called the y-intercept (see Fig. 14.13).

Note OA and OB are taken as positive or negative, based on whether the line meets positive or negative axes.

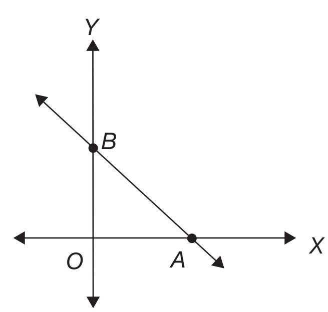


Figure 14.13

EXAMPLE 14.19

The line l in Fig. 14.14 meets X-axis at $A(-5, 0)$ and Y-axis at $B(0, -3)$.

SOLUTION

Hence, x-intercept $= -5$ and y-intercept $= -3$.

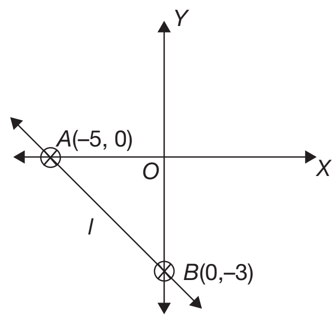


Figure 14.14

Equation of a Line in General Form

An equation of the form, $ax + by + c = 0$ (where $|a| + |b| \neq 0$, i.e., a and b are not simultaneously equal to zero), which is satisfied by every point on a line is called the equation of a line.

Equations of Some Standard Lines

Equation of X-axis

We know that the y-coordinate of every point on X-axis is zero. So, if $P(x, y)$ is any point on X-axis, then $y = 0$.

Hence, the equation of X-axis is $y = 0$.

Equation of Y-axis

We know that the x-coordinate of every point on Y-axis is zero. So, if $P(x, y)$ is any point on Y-axis, then $x = 0$.

Hence, the equation of Y-axis is $x = 0$.

Equation of a Line Parallel to X-axis

Let L be a line parallel to X-axis and at a distance of k units away from X-axis.

Then the y-coordinate of every point on the line L is k.

So, if $P(x, y)$ is any point on the line L, then $y = k$.

Hence, the equation of a line parallel to X-axis at a distance of k units from it, is $y = k$ (see Fig. 14.15).

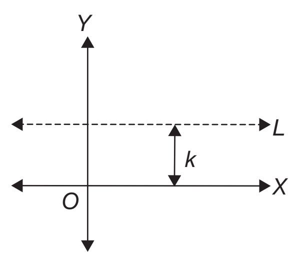


Figure 14.15

Note For the lines lying below X-axis, k is taken as negative.

Equation of a Line Parallel to Y-axis

Let L' be a line parallel to Y-axis and at a distance of k units away from it. Then the x-coordinate of every point on the line L' is k.

So, if $P(x, y)$ is any point on the line L', then $x = k$.

Hence, the equation of a line parallel to Y-axis and at a distance of k units from it, is $x = k$ (see Fig. 14.16).

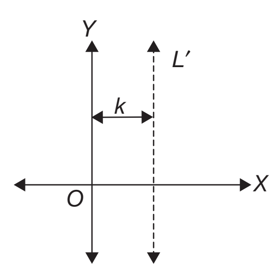


Figure 14.16

Note For the lines lying on the left side of Y-axis, k is taken as negative.

Oblique Line

A straight line which is neither parallel to X-axis nor parallel to Y-axis is called an oblique line or an inclined line.

Different Forms of Equations of Oblique Lines

Gradient Form (or) Slope Form The equation of a straight line with slope m and passing through the origin is given by $y = mx$.

Point–Slope Form The equation of a straight line passing through point (x_1, y_1) and with slope m is given by $y - y_1 = m(x - x_1)$.

Slope–intercept Form The equation of a straight line with slope m and having y-intercept as c is given by $y = mx + c$.

Note Area of triangle formed by the line $y = mx + c$ is $\frac{1}{2}\left|\frac{c^2}{m}\right|$ sq. units.

Two-point Form The equation of a straight line passing through points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \qquad \text{or} \qquad \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}.$$

Intercept Form The equation of a straight line with x-intercept as a and y-intercept as b is given by $\frac{x}{a} + \frac{y}{b} = 1$.

Note Area of triangle formed by line $\frac{x}{a} + \frac{y}{b} = 1$ with the coordinate axes is $\frac{1}{2}|ab|$ sq. units.

EXAMPLE 14.20

Find the equation of the line parallel to Y-axis and passing though point (5, –7).

SOLUTION

The equation of a line parallel to Y-axis is $x = k$.

Given, the line passes through point (5, –7)

$\Rightarrow k = 5$.

Hence, the equation of the required line is $x = 5$.

That is, $x - 5 = 0$.

EXAMPLE 14.21

Find the equation of the line passing through (3, 4) and having a slope $\frac{4}{5}$.

SOLUTION

The equation of the line passing through (x_1, y_1) and having slope m is given by $y - y_1 = m(x - x_1)$.

Hence, the equation of the required line is

$$y - 4 = \frac{4}{5}(x - 3)$$
$$5y - 20 = 4x - 12$$
$$4x - 5y + 8 = 0.$$

EXAMPLE 14.22

Find the equation of a line making intercepts 4 and 5 on the coordinate axes.

SOLUTION

Given, x-intercept $(a) = 4$ and y-intercept $(b) = 5$.

$\therefore$ The equation of the required line is $\frac{x}{a}+\frac{y}{b}=1$.

That is, $\frac{x}{4}+\frac{y}{5}=1$

$$\Rightarrow 5x + 4y - 20 = 0.$$

Equation of a Line Parallel or Perpendicular to the Given Line

Let $ax + by + c = 0$ be the equation of a straight line, then:

1. The equation of a line passing through point (x_1, y_1) and parallel to the given line:

   The slope of the required line (m) = The slope of $ax + by + c = 0$

   $$= \frac{-a}{b} \quad \text{(Since the lines are parallel)}$$

   $\therefore$ The required line is

   $$y - y_1 = m(x - x_1)$$
   $$\Rightarrow y - y_1 = \frac{-a}{b}(x - x_1)$$
   $$\Rightarrow b(y - y_1) = -a(x - x_1)$$
   $$\Rightarrow a(x - x_1) + b(y - y_1) = 0.$$

2. The equation of a line passing through point (x_1, y_1) and perpendicular to the given line:

   The slope of $ax + by + c = 0$ is $\frac{-a}{b}$.

   $\therefore$ The slope of the required line is $\frac{b}{a}$. (Since the line are perpendicular.)

   $\therefore$ The required line is

   $$(y - y_1) = m(x - x_1)$$
   $$\Rightarrow (y - y_1) = \frac{b}{a}(x - x_1)$$
   $$\Rightarrow b(x - x_1) - a(y - y_1) = 0.$$

EXAMPLE 14.23

Find the equation of a line passing through the point $P(-3, 2)$ and parallel to line $4x - 3y - 7 = 0$.

SOLUTION

Here, $(x_1, y_1) = (-3, 2)$, $a = 4$ and $b = 2$.

$\therefore$ Equation of the line passing through $P(-3, 2)$ and parallel to $4x - 3y - 7 = 0$.

$$a(x - x_1) + b(y - y_1) = 0$$
$$\Rightarrow 4(x + 3) - 3(y - 2) = 0$$
$$\Rightarrow 4x - 3y + 18 = 0.$$

Hence, the equation of the required line is $4x - 3y + 18 = 0$.

EXAMPLE 14.24

Find the equation of a line passing through point (–2, 3) and perpendicular to $7x + 2y + 3 = 0$.

SOLUTION

Here, $(x_1, y_1) = (-2, 3)$, $a = 7$ and $b = 2$.

∴ Equation of the line perpendicular to $7x + 2y + 3 = 0$ and passing through (–2, 3) is $b(x - x_1) - a(y - y_1) = 0$.

That is, $2(x + 2) - 7(y - 3) = 0$

$$\Rightarrow 2x - 7y + 25 = 0.$$

Hence, the required equation of the line is $2x - 7y + 25 = 0$.

EXAMPLE 14.25

The line $(8x + 3y - 15) + \lambda(3x - 8y + 2) = 0$ is parallel to X-axis. Find λ.

SOLUTION

The given line is $(8x + 3y - 15) + \lambda(3x - 8y + 2) = 0$.

That is, $x(8 + 3\lambda) + y(3 - 8\lambda) + (2\lambda - 15) = 0$.

Since the given line is parallel to X-axis, its slope = 0.

$$\frac{-(8+3\lambda)}{3-8\lambda} = 0$$
$$\Rightarrow 8 + 3\lambda = 0$$

Hence, $\lambda = \frac{-8}{3}$.

EXAMPLE 14.26

The equation of the line passing through the point of intersection of lines $2x - y + 3 = 0$ and $3x + y + 7 = 0$ and perpendicular to $2x - 3y + 4 = 0$, is _________.

(a) $3x + 2y - 7 = 0$ **(b)** $3x + 2y + 8 = 0$ **(c)** $3x + 2y - 8 = 0$ **(d)** $3x - 2y + 1 = 0$

HINTS

(i) Find m and the intersection point. Then use slope-point form.

(ii) Find the common point (x_1, y_1) of first two equations.

(iii) Find the slope (m) of third line.

(iv) Find the equation of the line passing through (x_1, y_1) and having slope $\left(-\frac{1}{m}\right)$.

EXAMPLE 14.27

The area of the figure formed by $|x| + |y| = 2$ is______. (in sq. units)

(a) 2 **(b)** 4 **(c)** 6 **(d)** 8

HINTS

(i) Plot the figure.

(ii) Find the intercepts made by given line.

(iii) If the intercepts are a and b, then the area of the triangle is $\frac{|ab|}{2}$.

EXAMPLE 14.28

The sum of the reciprocals of the intercepts of a line is $\frac{1}{2}$, then the line passes through the point is______.

(a) (1, 1) **(b)** (2, 1) **(c)** $\left(\frac{1}{4}, \frac{1}{4}\right)$ **(d)** (2, 2)

HINTS

(i) Use $\frac{x}{a} + \frac{y}{b} = 1$.

(ii) Solve, $\frac{1}{a} + \frac{1}{b} = \frac{1}{2}$ and get the relation between a and b.

(iii) Use the formula $\frac{x}{a} + \frac{y}{b} = 1$.

Mid-point

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two given points and M be the mid-point of AB.

Then, $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Hence, the coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) are given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

EXAMPLE 14.29

Find the mid-point of the line segment joining the points (2, –6) and (6, –4).

SOLUTION

Let $A(2, -6)$ and $B(6, -4)$ be the given points and M be the mid-point of AB.

Then,

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{2 + 6}{2}, \frac{-6 + (-4)}{2}\right) = (4, -5).$$

Hence, the mid-point of AB is (4, –5).

Centroid

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC, and G be its centroid. Then, the coordinates of G are given by, $G = \left(\frac{x_1 + x_2 + x_3}{2}, \frac{y_1 + y_2 + y_3}{2}\right)$.

EXAMPLE 14.30

Find the centroid of ΔABC whose vertices are $A(2, -3)$, $B(4, 2)$ and $C(-3, -2)$.

SOLUTION

Given, $A(2, -3)$, $B(4, 2)$ and $C(-3, -2)$.

So, centroid of ΔABC

$$\left(\frac{x_1 + x_2 + x_3}{2}, \frac{y_1 + y_2 + y_3}{2}\right) = \left(\frac{2 + 4 - 3}{3}, \frac{-3 + 2 - 2}{3}\right) = (1, -1).$$

Hence, $(1, -1)$ is the centroid of ΔABC.

EXAMPLE 14.31

Find the third vertex of ΔABC, if two of its vertices are $A(-2, 3)$, $B(4, 5)$ and its centroid is $G(1, 2)$.

SOLUTION

Let $C(x, y)$ be the third vertex.

Given, centroid of $\Delta ABC = (1, 2)$

$$\Rightarrow \left(\frac{-2 + 4 + x}{3}, \frac{3 + 5 + y}{3}\right) = (1, 2)$$

$$\Rightarrow \left(\frac{x + 2}{3}, \frac{y + 8}{3}\right) = (1, 2)$$

$$\Rightarrow \frac{x + 2}{3} = 1, \frac{y + 8}{3} = 2$$

$$\Rightarrow x = 1, \ y = -2.$$

$\therefore$ The third vertex is $(1, -2)$.

Notes

1. If the mid-points of the sides BC, AC and AB of ΔABC, respectively, are $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$, then its vertices are $A(-x_1 + x_2 + x_3, -y_1 + y_2 + y_3)$, $B(x_1 - x_2 + x_3, y_1 - y_2 + y_3)$ and $C(x_1 + x_2 - x_3, y_1 + y_2 - y_3)$.
2. The fourth vertex of a parallelogram whose three consecutive vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) when taken in order is $(x_1 - x_2 + x_3, y_1 - y_2 + y_3)$.

EXAMPLE 14.32

Find the fourth vertex of the parallelogram whose three consecutive vertices are (8, 8), (6, 1) and (–1, 1).

SOLUTION

Let the three vertices of the parallelogram be $A(8, 8)$, $B(6, 1)$ and $C(-1, 1)$, then fourth vertex $D(x, y)$ is given by

$$D(x, y) = (x_1 - x_2 + x_3, y_1 - y_2 + y_3)$$
$$= (8 - 6 - 1, 8 - 1 + 1)$$
$$= (1, 8).$$

Hence, the fourth vertex is $D(1, 8)$.

EXAMPLE 14.33

If the centroid of a triangle is (6, 6) and its ortho-centre is (0, 0), then find its circum-centre.

(a) (3, 3) **(b)** (6, 6) **(c)** (9, 9) **(d)** (12, 12)

SOLUTION

Ortho-centre, centroid and circum-centre are collinear.

We know that centroid divides the line segment joining the ortho-centre, centroid and circum-centre $(\overline{OGS})$ in the ratio 2 : 1 from the ortho-centre (O).

Let $S(x, y)$, $G(6, 6)$ and $O(0, 0)$

$$(6, 6) = \left(\frac{2x \times 1 \times 0}{2+1}, \frac{2 \times y \times 0}{2+1}\right)$$

$$(6, 6) = \left(\frac{2x}{3}, \frac{2y}{3}\right)$$

$$\Rightarrow \frac{2x}{3} = 6, \frac{2y}{3} = 6$$

$$\Rightarrow x = 6 \text{ and } y = 9.$$

∴ The circum-centre is (9, 9).

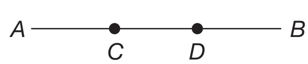
EXAMPLE 14.34

$C(3, 0)$ and $D(3, 1)$ are the points of trisection of a line segment AB. Find the respective coordinates of A and B.

(a) (3, 2), (3, 0) **(b)** (3, –1), (3, 2) **(c)** (–3, 1), (3, 2) **(d)** None of these

SOLUTION

Let A and B be (a_1, b_1) and (a_2, b_2). Given, $C(3, 0)$ and $D(3, 1)$ are the points of trisection of AB.



$\Rightarrow$ C is the mid-points of AD and D is the mid-points of CB.

$$\Rightarrow (3, 0) = \left(\frac{a_1 + 3}{2}, \frac{b_1 + 1}{2}\right)$$

$\Rightarrow a_1 = 3$ and $b_2 = -1$.

$$\text{Also, } (3, 1) = \left(\frac{3 + a_2}{2}, \frac{0 + b_2}{2}\right)$$

$\Rightarrow a_2 = 3$ and $b_2 = 2$.

$\therefore$ The coordinates of A and B are (3, −1) and (3, 2).

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. If $x > 0$ and $y < 0$, then the point $(x, -y)$ lies in _______ quadrant.
2. Which point among (2, 3), (–3, –4) and (1, –7) is nearest to the origin?
3. The lines $2y + 3 = 0$ and $x = 3$ intersect at _______.
4. The points (0, 0), (0, 4) and (4, 0) form a/an _______ triangle.
5. A linear equation in two variables is always a _______.
6. The slope of the line $ax + by + c = 0$ is ________.
7. If (x, y) represents a point and $|x| > 0$ and $y < 0$, then in which quadrant(s) can the point lie?
8. The equation of a line parallel to Y-axis and passing through (–3, –4) is _______.
9. The slope of line perpendicular to the line joining points (2, 3) and (–2, 5) is _______.
10. The slope of altitude from A to BC of triangle $A(2, 3)$, $B(-3, 2)$ and $C(3, 5)$ is _______.
11. If the line $\frac{x}{a} + \frac{y}{b} = m$ passes through origin, then the value of m is ________.
12. If (x, y) represents a point and $xy > 0$, then the point may lie in ________ or _______quadrant.
13. The slope-intercept form of the line $2x + 3y + 5 = 0$ is _______.
14. The lines $3x + 2y + 7 = 0$ and $6x + 4y + 9 = 0$ are _________to each other.
15. The points $(p, q + r)$, $(q, r + p)$ and $(r, q + p)$ are _______.
16. The area of triangle formed by the line $y = mx + c$ with the coordinate axes is _______.
17. The points (2, 3), (–1, 5) and $(x, -2)$ form a straight line, then x is ______.
18. If the point (x, y) lies in the second quadrant, then x is _______ and y is _______.
19. The angle between lines $x = 5$ and $x = 7$ is _______.
20. The point of intersection of X-axis and $3x + 2y - 5 = 0$ is _______.
21. If $a = 0$, then the line $ax + by + c = 0$ is parallel to _______.
22. The lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular to each other, then ________.
23. The point of intersection of X-axis and Y-axis is ________.
24. The line $y = k$ is parallel to ______ axis.
25. A, B and C are three points such that $AB = AC + CB$, then A, B and C are ________.
26. The line $ax + by + c = 0$ meets Y-axis at ________ point.
27. If slope of a line (l) is $\tan\theta$, then slope of a line perpendicular to (l) is _______.
28. The lines $x = 2$ and $y = -3$ intersect in _______ quadrant.
29. The slope of a line which is parallel to the line making an inclination of 45° with positive X-axis is ______.
30. If the slope of two lines are equal, then the lines are _______.

Short Answer Type Questions

31. Find the equation of a line passing through points $A(-2, 3)$ and $B(4, 7)$.
32. Find the area of the circle passing through (–2, 3) with centre (5, 2).
33. If $(2x + 3y + 1) + \lambda(x - 2y - 3) = 0$ represents the equation of a horizontal line, then find the value of λ.
34. Let $A(-3, 2)$, $B(4, 1)$ and $C(-2, k)$ be three points such that $AC = BC$. Find the value of k.
35. Find the distance between points (2, –3) and (4, 6).
36. If the line $2x - ky + 6 = 0$ passes through the point (2, –8), then find the value of k.

PRACTICE QUESTIONS

37. Find the area of square, whose diagonally opposite vertices are (−2, 3) and (4, 5).

38. If $A(a + b, a - b)$ and $B(-a + b, -a - b)$, then find the distance AB.

39. Find the intercepts made by the line $3x - 2y - 6 = 0$ on the coordinate axes.

40. Find the inclination of the line $\sqrt{3}x - 3y + 6 = 0$.

41. Find the circum-centre of the triangle whose vertices are $A(-3, -1)$, $B(1, 2)$ and $C(0, -4)$.

42. Find the equation of a line having inclination 60° and making an intercept of $\frac{-1}{3}$ on Y-axis.

43. Find the point on X-axis, which is equidistant from $A(6, 3)$ and $B(-1, 4)$.

44. Show that the points (−1, −1), (6, 1), (8, 8) and (1, 6), when joined in the given order form a rhombus.

45. Find the equation of a line, whose y-intercept is −5 and passes through point $A(-3, 2)$.

Essay Type Questions

46. Find the equations of the lines whose intercepts are the roots of the equation $4x^2 - 3x - 1 = 0$.

47. The equation of one of the diagonals of a rhombus is $3x + 4y - 7 = 0$. Find the equation of the other diagonal passing through (−1, −2).

48. Find the equation of the line passing through (−5, 11) and making equal intercepts, but opposite in magnitude on the coordinate axes.

49. Find the equations of a line which forms area 5 sq. units with the coordinate axes and having sum of intercepts is 7.

50. If points $A(1, 6)$, $B(5, 2)$ and $C(12, 9)$ are three consecutive vertices of a parallelogram, then find the equation of the diagonal BD.

CONCEPT APPLICATION

Level 1

1. If (1, −3), (−2, −3) and (−2, 2) are the three vertices of a parallelogram taken in that order, then the fourth vertex is ______.

(a) (−1, −2) (b) (1, 2)
(c) (−1, 2) (d) (1, −2)

2. Find the equation of the line that passes through point (5, −3) and makes an intercept 4 on the X-axis.

(a) $3x - y + 12 = 0$ (b) $3x + y + 12 = 0$
(c) $3x - y - 12 = 0$ (d) $3x + y - 12 = 0$

3. The inclination of line $x - \sqrt{3}y + 1 = 0$ with the positive X-axis is ______.

(a) 60° (b) 30°
(c) 45° (d) 90°

4. The equation of the line perpendicular to Y-axis and passing through point (−5, 7) is ______.

(a) $y = -5$ (b) $x = 7$
(c) $x = -5$ (d) $y = 7$

5. If (2, 0) and (−2, 0) are the two vertices of an equilateral triangle, then the third vertex can be ______.

(a) (0, 0) (b) (2, −2)
(c) $(0, 2\sqrt{3})$ (d) $(\sqrt{3}, \sqrt{3})$

6. The points $(a, b + c)$, $(b, c + a)$ and $(c, a + b)$

(a) are collinear.
(b) form a scalene triangle.
(c) form an equilateral triangle.
(d) None of the above.

7. The equation of the line making equal intercepts and passing through the point (−1, 4) is ______.

(a) $x - y = 3$ (b) $x + y + 3 = 0$
(c) $x + y = 3$ (d) $x - y + 3 = 0$

PRACTICE QUESTIONS

8. The endpoints of the longest chord of a circle are (–4, 2) and (–6, –8). Find its centre.

(a) $\left(-\frac{10}{3}, -2\right)$ (b)(–5, –2)

(c) (–5, –4) (d) (–5, –3)

9. The equation of the line passing through point (–3, –7) and making an intercept of 10 units on X-axis can be ______.

(a) $4x + 3y = -9$ (b) $8x - 3y = 80$

(c) $7x - 13y - 70 = 0$ (d) $7x + 3y - 70 = 0$

10. The points on the Y-axis which are at a distance of 5 units from (4, –1) are ______.

(a) (0, –2), (0, 4) (b) (0, 2), (0, –4)

(c) (0, 2) (0, 4) (d) (0, –2) (0, –4)

11. If the slope and the y-intercept of a line are the roots of the equation $x^2 - 7x - 18 = 0$, then the equation of the line can be ______.

(a) $2x + y - 9 = 0$ (b)$2x - y + 9 = 0$

(c) $9x + y + 2 = 0$ (d) $9x + 2y - 2 = 0$

12. If the points $(k, k - 1)$, $(k + 2, k + 1)$ and $(k, k + 3)$ are three consecutive vertices of a square, then its area (in square units) is ______.

(a) 2 (b) 4

(c) 8 (d) 6

13. The equation of the line making intercepts of equal magnitude and opposite signs, and passing through the point (–3, –5) is _______.

(a) $x - y = 2$ (b) $2x + y = -4$

(c) $3x + 3y = 6$ (d) $x - y = -10$

14. If the endpoints of the diameter of a circle are (–2, 3) and (6, –3), then the area of the circle (in square units) is ______.

(a) $\frac{550}{3}$ (b) $\frac{540}{7}$

(c) $\frac{560}{7}$ (d) $\frac{550}{7}$

15. The inclination of the line $\sqrt{3}x - y + 3 = 0$ with the positive X-axis is _______.

(a) 30° (b)45°

(c) 60° (d) 90°

16. The two lines $3x + 4y - 6 = 0$ and $6x + ky - 7 = 0$ are such that any line which is perpendicular to the first line is also perpendicular to the second line. Then, $k =$ ____.

(a) –8 (b) –6

(c) 6 (d) 8

17. The line $x = my$, where $m < 0$, lies in the quadrants.

(a) 1st, 2nd (b) 2nd, 4th

(c) 3rd, 4th (d) 3rd, 1st

18. Find the area in square units, of the rhombus with vertices (2, 1), (–5, 2), (–4, –5) and (3, –6), taken in that order.

(a) 24 (b) 48

(c) 36 (d) 50

19. The radius of a circle with centre (–2, 3) is 5 units, then the point (2, 5) lies ______.

(a) on the circle

(b) inside the circle

(c) outside the circle

(d) None of the above

20. One end of the diameter of a circle with the centre as origin is (–2, 10). Find the other end of the diameter.

(a) (–2, –10) (b)(0, 0)

(c) (2, –10) (d) (2, 10)

21. If the roots of the quadratic equation $x^2 - 7x + 12 = 0$ are intercepts of a line, then the equation of the line can be ______.

(a) $2x + 3y = 6$ (b) $4x + 3y = 12$

(c) $4x + 3y = 6$ (d) $3x + 4y = 6$

22. Find the value of λ, if the line $x - 3y + 4 + \lambda(8x - 3y + 2) = 0$ is parallel to the X-axis.

(a) $\frac{1}{5}$ (b) $\frac{5}{8}$

(c) $-\frac{3}{8}$ (d) $-\frac{1}{8}$

23. The slope of the line joining the points $(2, k - 3)$ and (4, –7) is 3. Find k.

(a) –10 (b) –6

(c) –2 (d) 10

PRACTICE QUESTIONS

24. The angle between the lines $x = 10$ and $y = 10$ is ______.

(a) 0° (b) 90°

(c) 180° (d) None of these

25. The two lines $5x + 3y + 7 = 0$ and $kx - 4y + 3 = 0$ are perpendicular to the same line. Find the value of k.

(a) $-\frac{20}{7}$ (b) $-\frac{20}{3}$

(c) $\frac{20}{9}$ (d) $\frac{12}{5}$

26. The lines $x - 2y + 3 = 0$, $3x - y = 1$ and $kx - y + 1 = 0$ are concurrent. Find k.

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{3}{2}$ (d) $\frac{5}{2}$

27. Find the quadrant in which the lines $2x + 3y - 1 = 0$ and $3x + y - 5 = 0$ intersect each other.

(a) 1st quadrant (b) 2nd quadrant

(c) 3rd quadrant (d) 4th quadrant

28. The circum-centre of the triangle formed by points $O(0, 0)$, $A(6, 0)$ and $B(0, 6)$ is ______.

(a) (3, 3) (b) (2, 2)

(c) (1, 1) (d) (0, 0)

29. The lines $3x - y + 2 = 0$ and $x + 3y + 4 = 0$ intersect each other in the ______.

(a) 1st quadrant (b) 4th quadrant

(c) 3rd quadrant (d) 2nd quadrant

30. Centre of the circle is (a, b). If (0, 3) and (2, 0) are two points on a circle, then find the relation between a and b.

(a) $4a - 6b - 5 = 0$

(b) $4a + 6b - 5 = 0$

(c) $-4a + 5 = 0$

(d) $4a - 6b + 5 = 0$

Level 2

31. The equation of a line passing through $P(3, 4)$, such that P bisects the part of it intercepted between the coordinate axes is ______.

(a) $3x + 4y = 25$ (b) $4x + 3y = 24$

(c) $x - y = -1$ (d) $x + y = 7$

32. The line $7x + 4y = 28$ cuts the coordinate axes at A and B. If O is the origin, then the ortho-centre of ΔOAB is ______.

(a) (4, 0) (b) (0, 7)

(c) (0, 0) (d) None of these

33. If the roots of the quadratic equation $x^2 - 5x + 6 = 0$ are the intercepts of a line, then the equation of the line can be ______.

(a) $2x + 3y = 6$

(b) $3x + 2y = 6$

(c) Either (a) or (b)

(d) None of these

34. The equation of the line whose x-intercept is 5, and which is parallel to the line joining the points (3, 2) and (−4, −1) is ______.

(a) $4x + 7y - 20 = 0$

(b) $3x - 7y + 3 = 0$

(c) $3x + 2y + 15 = 0$

(d) $3x - 7y - 15 = 0$

35. Find the area of the triangle formed by the line $3x - 4y + 12 = 0$ with the coordinate axes.

(a) 6 units^2 (b) 12 units^2

(c) 1 units^2 (d) 36 units^2

36. The line joining the points $(2m + 2, 2m)$ and $(2m + 1, 3)$ passes through $(m + 1, 1)$, if the values of m are ______.

(a) $5, -\frac{1}{5}$ (b) 1, −1

(c) $2, -\frac{1}{2}$ (d) $3, -\frac{1}{3}$

37. The length (in units) of the line joining the points (4, 3) and (−4, 9) intercepted between the coordinate axes is ______.

(a) 10 (b) 8

(c) 6 (d) 4

PRACTICE QUESTIONS

38. The equation of a line parallel to $8x - 3y + 15 = 0$ and passing through the point $(-1, 4)$ is ______.

(a) $8x - 3y - 4 = 0$
(b) $8x - 3y - 20 = 0$
(c) $8x - 3y + 4 = 0$
(d) $8x - 3y + 20 = 0$

39. $(0, 0), (3, \sqrt{3})$ and $(0, 2\sqrt{3})$ are the three vertices of a triangle. The distance between the ortho-centre and the cirum-centre of the triangle is _______. (in units)

(a) $\sqrt{3}$ (b) $\sqrt{5}$
(c) $\sqrt{6}$ (d) 0

40. In a parallelogram $PQRS$, $P(15, 9)$, $Q(7, 10)$, $R(-5, -4)$, then the fourth vertex S is _______.

(a) $(3, -2)$ (b) $(3, -4)$
(c) $(9, -5)$ (d) $(3, -5)$

41. If the roots of the quadratic equation $3x^2 - 2x - 1 = 0$ are the intercepts of a line, then the line can be ______.

(a) $x - 3y - 1 = 0$
(b) $3x - y + 1 = 0$
(c) Either (a) or (b)
(d) None of these

42. The length (in units) of a line segment intercepted between the coordinate axes by the line joining the points $(1, 2)$ and $(3, 4)$ is ______.

(a) 4 (b) 6
(c) 8 (d) $\sqrt{2}$

43. If $A = (3, -4)$, $B = (7, 0)$ and $C = (14, -7)$ are the three consecutive vertices of a parallelogram $ABCD$, then find the slope of the diagonal BD. The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) $\left(\frac{x+7}{2}, \frac{y+0}{2}\right) = \left(\frac{3+14}{2}, \frac{-4-7}{2}\right)$.

(B) The slope of $BD = \frac{-11-0}{10-7} = \frac{-11}{3}$.

(C) $\frac{x+7}{2} = \frac{17}{2}$ and $\frac{y+0}{2} = \frac{-11}{2}$
$\Rightarrow x = 10, y = -11$
$\therefore D = (10, -11)$.

(D) Let the fourth vertex be $D(x, y)$. We know that the diagonals of a parallelogram bisect each other.

(a) ADCB (b) DCAB
(c) DACB (d) CDAB

44. If $A = (1, -6)$, $B = (5, -2)$ and $C = (12, -9)$ are the three consecutive vertices of a parallelogram, then find the fourth vertex. The following are the steps involved in solving the above problem. Arrange them in sequential order from beginning to end.

(A) $\frac{5+x}{2} = \frac{13}{2}, \frac{-2+y}{2} = \frac{-15}{2} \Rightarrow x = 8,$ and $y = -13$. Therefore, $D = (8, -13)$.

(B) $\therefore \left(\frac{5+x}{2}, \frac{-2+y}{2}\right) = \left(\frac{1+12}{2}, \frac{-6-9}{2}\right)$.

(C) Let the fourth vertex be $D = (x, y)$.

(D) We know that diagonals of a parallelogram bisect each other.

(a) ACBD (b) ABDC
(c) CBDA (d) CDBA

45. Find the product of intercepts made by the line $7x - 2y - 14 = 0$ with coordinate axes.

(a) -7 (b) 2
(c) 14 (d) -14

46. Find the value of k, if points $(-2, 5)$, $(-5, -10)$ and $(k, -13)$ are collinear.

(a) $\frac{5}{28}$ (b) $\frac{-28}{5}$
(c) 28 (d) 5

47. The inclination of the line $\sqrt{3}y - x + 24 = 0$, is ______.

(a) 60° (b) 30°
(c) 45° (d) 135°

48. Find the product of intercepts of the line $3x + 8y - 24 = 0$.

(a) 8 (b) 24
(c) 3 (d) 12

49. Find the value of k, if points $(10, 14)$, $(-3, 3)$ and $(k, -8)$ are collinear.

(a) 16 (b) 18
(c) -18 (d) -16

PRACTICE QUESTIONS

50. The inclination of the line $y - x + 11 = 0$, is ________.

(a) 30°
(b) 60°
(c) 0°
(d) 45°

51. The equation of a line whose x-intercept is −3 and which is parallel to $5x + 8y - 7 = 0$ is ________.

(a) $5x + 8y + 15 = 0$
(b) $5x + 8y - 15 = 0$
(c) $5x + 8y - 17 = 0$
(d) $5x - 8y - 18 = 0$

Level 3

52. The area of a square with one of its vertices as (5, −2) and the mid-point of the diagonals as (3, 2), is ______. (in sq. units)

(a) 40 (b) 20
(c) 60 (d) 70

53. The equation of the line perpendicular to the line inclined equally to the coordinate axes and passing through (2, −3) is _______.

(a) $x + y + 1 = 0$
(b) $x - y - 2 = 0$
(c) $x + y + 2 = 0$
(d) $2x + y - 1 = 0$

54. A triangle is formed by points (6, 0), (0, 0) and (0, 6). How many points with the integer coordinates are in the interior of the triangle?

(a) 7 (b) 6
(c) 8 (d) 10

55. The equation of one of the diagonals of a square is $3x - 8y + 4 = 0$. Find the equation of the other diagonal passing through the vertex (4, −6).

(a) $8x + 3y - 15 = 0$
(b) $3x - 8y - 11 = 0$
(c) $8x + 3y - 14 = 0$
(d) $8x + 3y + 15 = 0$

56. The lines $2x + 3y - 6 = 0$ and $2x + 3y - 12 = 0$ are represented on the graph. The difference between the areas of triangles formed by the lines with the coordinate axes is ______. (in sq. units)

(a) 12 (b) 9
(c) 6 (d) 3

57. The equation of a line whose x-intercept is 11 and perpendicular to $3x - 8y + 4 = 0$, is ________.

(a) $7x + 3y - 77 = 0$
(b) $8x + 3y - 88 = 0$
(c) $5x + 3y - 55 = 0$
(d) $3x + 8y - 88 = 0$

58. $A(-11, 7)$ and $B(-10, 6)$ are the points of trisection of a line segment PQ. Find the coordinates of P and Q.

(a) (−12, 8); (−9, 5)
(b) (−12, −8); (−9, 5)
(c) (12, 0); (9, −5)
(d) (12, −8); (9, −5)

59. If one of the diagonals of a rhombus is $3x - 4y + 10 = 0$, then find the equation of the other diagonal which passes through point (−2, −3).

(a) $4x + 3y + 17 = 0$
(b) $3x - 4y + 15 = 0$
(c) $4x + 3y - 15 = 0$
(d) $3x - 4y - 11 = 0$

60. The equation of the diagonal AC of a square $ABCD$ is $3x + 4y + 12 = 0$. Find the equation of BD, where D is (2, −3).

(a) $4x - 3y - 8 = 0$
(b) $4x - 3y - 17 = 0$
(c) $4x - 3y + 17 = 0$
(d) $4x + 3y - 17 = 0$

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. first
2. (2, 3)
3. $\left[3, -\frac{3}{2}\right]$
4. right-angled isosceles triangle
5. straight line
6. $\frac{-a}{b}$
7. The point may lie in Q_3 or Q_4.
8. $x + 3 = 0$
9. 2
10. –2
11. 0
12. the first or third quadrant
13. $y = \frac{-2}{3}x - \frac{5}{3}$
14. parallel
15. collinear
16. $\frac{1}{2}\left|\frac{c^2}{m}\right|$ sq. units
17. $x = \frac{19}{2}$
18. (–ve, +ve)
19. 0º
20. $\left(\frac{5}{3}, 0\right)$
21. X-axis
22. $a_1a_2 + b_1b_2 = 0$
23. origin (or) (0, 0)
24. X
25. collinear
26. $\left(0, \frac{-c}{b}\right)$
27. $-\cot\theta$
28. fourth
29. 1
30. parallel

Short Answer Type Questions

31. $2x - 3y + 13 = 0$
32. 50π sq. units
33. $\lambda = -2$
34. $k = -16$
35. $\sqrt{85}$ units
36. $k = \frac{-5}{4}$
37. 20 sq. units
38. $2\sqrt{2a}$ units
39. x-intercept $(a) = 2$

    y-intercept $(b) = -3$
40. 30°
41. $\left(\frac{1}{14}, \frac{-13}{14}\right)$
42. $3\sqrt{3}x - 3y - 1 = 0$
43. (2, 0)
45. $7x + 3y + 15 = 0$

Essay Type Questions

46. $\frac{x}{1} + \frac{y}{-\frac{1}{4}} = 1$ or $\frac{x}{-\frac{1}{4}} + \frac{y}{1} = 1$
47. $4x - 3y - 2 = 0$
48. $x - y + 16 = 0$
49. $2x + 5y = 10$ or $5x + 2y = 10$
50. $x + 3y - 8 = 0$

CONCEPT APPLICATION

Level 1

1. (b)	**2.** (d)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (a)	**7.** (c)	**8.** (d)	**9.** (c)	**10.** (b)
11. (a)	**12.** (c)	**13.** (a)	**14.** (d)	**15.** (c)	**16.** (d)	**17.** (b)	**18.** (b)	**19.** (b)	**20.** (c)
21. (b)	**22.** (d)	**23.** (a)	**24.** (b)	**25.** (b)	**26.** (a)	**27.** (d)	**28.** (a)	**29.** (c)	**30.** (d)

Level 2

31. (b)	**32.** (c)	**33.** (c)	**34.** (d)	**35.** (a)	**36.** (c)	**37.** (a)	**38.** (d)	**39.** (d)	**40.** (d)
41. (c)	**42.** (d)	**43.** (c)	**44.** (d)	**45.** (d)	**46.** (b)	**47.** (b)	**48.** (b)	**49.** (d)	**50.** (d)
51. (a)									

Level 3

52. (a)	**53.** (a)	**54.** (d)	**55.** (c)	**56.** (b)	**57.** (b)	**58.** (a)	**59.** (a)	**60.** (b)

CONCEPT APPLICATION

Level 1

1. Fourth vertex of a parallelogram is $(x_1 + x_3 - x_2, y_1 + y_3 - y_2)$.
2. If x-intercept is a, then $(a, 0)$ is a point on the line. Now, use two point form.
3. Use slope $m = \tan\theta$, where θ is the inclination.
4. Equation of a line perpendicular to Y-axis is of the form x = constant.
5. (i) All the three vertices of an equilateral triangle are not rational
   (ii) Let the third vertex be (x, y).
   (iii) Sides of an equilateral triangle are equal.
6. Find slopes of AB and BC, if they are equal then the given points are collinear.
7. Equation of the line making equal intercepts is of the form, $x + y = a$.
8. The centre of a circle is the mid-point of the diameter.
9. If x-intercept is 10, then the line passing through (10, 0). Now, use the two point form.
10. Take the point on Y-axis as $(0, y)$.
11. Find the roots, and then use slope-intercept form of line.
12. Area of square = $(\text{side})^2$.
13. Use $\frac{x}{a}+\frac{y}{b}=1$ and $a=-b$.
14. (i) Find the diameter of the circle then find its area.
   (ii) The distance between (−2, 3) and (6, −3) is the length of the diameter of the circle.
   (iii) Area of circle $=\frac{\pi d^2}{4}$, where d is the length of the diameter.
15. Use $m = \tan\theta$.
16. If two lines are perpendicular to the same line, then they are parallel to each other.
17. Identify the sign of y for each sign of x.
18. Find lengths of the diagonals, then area of rhombus $=\frac{1}{2}\times d_1 \times d_2$.
19. Find the distance between the given two points and compare that distance with the radius given.
20. The mid-point of the diameter is the centre of a circle.
21. Find the roots of the given equation, then use intercepts form of line.
22. (i) Equation of a line parallel to X-axis is of the form y = constant.
   (ii) Slope of a line parallel to X-axis is zero.
   (iii) Slope of a line $=\frac{-(x\text{-coefficient})}{(y\text{-coeffcient})}$.
23. The slope of the line joining two points is $\frac{y_2 - y_1}{x_2 - x_1}$.
24. The first line is parallel to the Y-axis. The second line is parallel to the X-axis.
25. Two lines, which are perpendicular to the same line, must be parallel to each other.
26. Solve the first two equations and substitute (x, y) in the third equation and evaluate k.
27. Find the point of intersection of the given lines, then decide.
28. The circum-centre of a right-angled triangle is the mid-point of its hypotenuse.
29. Find the coordinates of the point of intersection.
30. The distance from centre of the circle to any point on the circle is its radius.

Level 2

31. (i) P is the mid-point of the line joining the intercepts. Find the intercepts using the mid-point formula.
   (ii) Let the line cut coordinate axes at $A(a, 0)$ and $B(0, b)$.
   (iii) Using the above data find a and b, then the equation of line, i.e., $\frac{x}{a}+\frac{y}{b}=1$.
32. (i) Identify the type of ΔOAB.

HINTS AND EXPLANATION

(ii) Ortho-centre of right triangle is the vertex containing right angle.

33. (i) Find the roots and take the equation as $\frac{x}{a}+\frac{y}{b}=1$.

(ii) Roots of $x^2 - 5x + 6 = 0$ are 2 and 3.

(iii) x-intercept is either 2 or 3. y-intercept is either 3 or 2.

34. (i) Find m, then use slope–point form.

(ii) Find the equation of the line passing through the given points.

(iii) Any line parallel to $ax + by + c_1 = 0$ is $ax + by + c_2 = 0$.

(iv) The required line $ax + by + c_2 = 0$ passes through (5, 0).

35. (i) Area $= \frac{1}{2}\left|\frac{c^2}{ab}\right|$, when the equation of the line is $ax + by + c = 0$.

(ii) If a and b are x- and y-intercepts, then the area of the triangle formed by the line with coordinate axes is $\left|\frac{ab}{2}\right|$.

36. (i) The three points are collinear.

(ii) Given points A, B and C are collinear.

(iii) Use, slope of AB = slope of AC and find m.

37. (i) Find the intercepts, then find the distance between them.

(ii) Find the equation of the line joining the given points.

(iii) Find the intercepts (a and b) made by the above line with coordinate axes.

(iv) The distance between $(a, 0)$ and $(0, b)$.

38. (i) Use the formula $a(x - x_1) + b(y - y_1) = 0$.

(ii) Slopes of parallel lines are equal.

(iii) Use, point-slope form, i.e., $(y - y_1) = m(x - x_1)$ and find the equation of the line.

39. (i) The given vertices form an equilateral triangle.

(ii) Given points are the vertices of an equilateral triangle.

(iii) In any equilateral triangle, geometric centres (except ex-centre) coincide.

40. If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are the successive vertices of a parallelogram, then the fourth vertex $= (x_1 - x_2 + x_3, y_1 - y_2 + y_3)$.

41. (i) Find the roots, then use $\frac{x}{a}+\frac{y}{b}=1$.

(ii) Roots of $3x^2 - 2x - 1 = 0$ are $\frac{-1}{3}$ and 1.

(iii) x-intercept is either $\frac{-1}{3}$ or 1.

y-intercept is either 1 or $\frac{-1}{3}$.

(iv) The required equation of the line is $\frac{x}{a}+\frac{y}{b}=1$.

42. (i) Find the intercepts, then find the distance between the intercepted points.

(ii) Find the equation of the line joining points (1, 2) and (3, 4).

(iii) Find the intercepts (a and b) of the line by putting $x = 0$ and $y = 0$.

(iv) Length of the required line is $\sqrt{a^2+b^2}$.

43. DACB is the required sequential order.

44. CDBA is the required sequential order.

45. $7x - 2y - 14 = 0 \Rightarrow 7x - 2y = 14$

$$\Rightarrow \frac{x}{2}-\frac{y}{7}=1 \Rightarrow \frac{x}{2}+\frac{y}{(-7)}=1.$$

∴ Intercepts are 2 and −7, and their product $= 2 \times (-7) = -14$.

46. (−2, 5), (−5, −10) and (k, −13) are collinear

$$\Rightarrow \frac{-10-5}{-5+2}=\frac{-13+10}{k+5}$$

$$\Rightarrow \frac{-15}{-3}=\frac{-3}{k+5}$$

$$\Rightarrow 5(k+5) = -3$$

$$\Rightarrow k+5=\frac{-3}{5}$$

$$\Rightarrow k=\frac{-3}{5}-5=\frac{-28}{5}$$

$$\therefore k=\frac{-28}{5}.$$

47. $\sqrt{3}y - x + 24 = 0 \Rightarrow m = \dfrac{1}{\sqrt{3}}$.

$\therefore m = \tan\theta = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30°. \quad \left(\because \tan 30° = \dfrac{1}{\sqrt{3}}\right)$

48. $3x + 8y = 24$

$\Rightarrow \dfrac{x}{8} + \dfrac{y}{3} = 1$

$\Rightarrow$ Intercepts are 8 and 3 and their product is $8 \times 3 = 24$.

49. Let the points be $A = (10, 14)$, $B = (-3, 3)$ and $C = (k, -8)$.

Given, they are collinear $\Rightarrow$ The slopes are same.

$\Rightarrow$ Slope of AB = Slope of BC

$\Rightarrow \dfrac{3 - 14}{-3 - 10} = \dfrac{-8 - 3}{k + 3}$

$\Rightarrow \dfrac{-11}{-13} = \dfrac{-11}{k + 3}$

$\Rightarrow k + 3 = -13$

$\Rightarrow k = -16$.

51. $y - x + 11 = 0 \Rightarrow y = x - 11$

$\Rightarrow m = \tan\theta = 1$

$\Rightarrow \theta = 45°$.

52. Slope of the line parallel to $5x + 8y - 7 = 0$ is $\dfrac{-5}{8}$.

Given that the x-intercept of the required line is -3.

$\therefore$ It passes through $(-3, 0)$. Hence, the required line is

$-\dfrac{5}{8} = \dfrac{y - 0}{x - (-3)}$

$y - 0 = \dfrac{-5}{8}(x + 3)$

$\Rightarrow 8y = -5x - 15 \Rightarrow 5x + 8y + 15 = 0$.

Level 3

53. (i) Required line is perpendicular to $x = y$ and passes through $(2, -3)$.

(ii) Find m, then use the slope–point form.

54. Draw the triangle and list the possible points.

55. (i) In a square, the diagonals are perpendicular to each other.

(ii) Find the slope of the second diagonal and use the slope–point form.

56. (i) Find the intercepts made by the lines on the coordinate axes by writing the equations in the intercept form.

(ii) If the intercepts made are a and b, then the area of the triangle formed is $\dfrac{ab}{2}$.

57. Slope of the line perpendicular to $3x - 8y + 4 = 0$ is $\dfrac{-1}{\text{Slope of } 3x - 8y + 4 = 0} = \dfrac{-1}{\frac{3}{8}} = -\dfrac{8}{3}$.

Given that, x-intercept of the required line is 11.

$\therefore$ It passes through $(11, 0)$.

Hence, the required line is $-\dfrac{8}{3} = \dfrac{y - 0}{x - 11}$.

That is, $y - 0 = -\dfrac{8}{3}(x - 11)$

$\Rightarrow 8x + 3y - 88 = 0$.

58. Let $P = (a_1, b_1)$, and $Q = (a_2, b_2)$. Given, $A = (-11, 7)$ and $B = (-10, 6)$.

$\Rightarrow A$ is the mid-point of the line segment PB.

$\therefore A = \left(\dfrac{a_1 - 10}{2}, \dfrac{b_1 + 6}{2}\right)$

$(-11, 7) = \left(\dfrac{a_1 - 10}{2}, \dfrac{b_1 + 6}{2}\right)$

$\Rightarrow a_1 = -12$ and $b_1 = 8$

$\Rightarrow P = (-12, 8)$. Similarly, $Q = (-9, 5)$.

59. One diagonal of the rhombus is $3x - 4y + 10 = 0$.

Slope of $3x - 4y + 10 = 0$ is $\frac{3}{4}$.

In a rhombus, diagonals bisect each other perpendicularly.

∴ The slope of the other diagonal

$= \frac{-1}{\frac{3}{4}} = -\frac{4}{3}$.

The required equation is $-\frac{4}{3} = \frac{y-(-3)}{x-(-2)}$

$\Rightarrow y + 3 = -\frac{4}{3}\ (x + 2)$

$\Rightarrow 4x + 3y + 17 = 0$.

60. The equation of diagonal AC is $3x + 4y + 12 = 0$.

Its slope $= -\frac{3}{4}$

Slope of the other diagonal BD $\frac{1}{\frac{-3}{4}} = \frac{4}{3}$.

BD passes through $(2, -3)$

$\therefore \frac{4}{3} = \frac{y-(-3)}{x-2}$

$\therefore y + 3 = \frac{4}{3}(x - 2)$

$4x - 3y - 17 = 0$.