Class 8 Maths Rational Numbers Introduction

Introduction to Rational Numbers

A number which can be written in the form of p/q, where p and q are integers and q ≠0 is called **rational number**. Numbers written in fraction are rational numbers.

- -2/3
- 6/7
- -3/1
- 4/0 is an undefined number and hence not a rational number.

For example, if there are 10 chocolates to be divided into 4 children, it is not possible to give 3 complete chocolates to each one of them. But, if 2 chocolatesare halved, then there will be 4half pieces and 8 full pieces of the chocolates. So each child would get 2 full pieces and 1 half piece

10 chocolates - 4 children

Each one is given 2 and a half or 2.5 or $\frac{5}{2}$ chocolates each. This $\frac{5}{2}$ is a rational number.

Types of numbers

To solve different types of equations, following types of numbers are usually used. The scope of these numbers may overlap. Like,

- All natural numbers are whole numbers too. Whole numbers have a single extra number zero which is not a natural number.
  - For example, 5, 50, 23643 etc all are natural numbers as well as whole numbers.
- All positive integers are natural as well as whole numbers.
  - For example, integers like 5, 50, 23643 etc all are natural numbers as well as whole numbers.

Type of Number	Range of numbers	Equation	Value of x to solve the equation
Natural Numbers	1 to ∞	x + 2 = 13	11
Whole Numbers	0 to ∞	x + 5 = 5	0
Integer (Positive)	1 to ∞	x + 18 = 22	4
Integer (Negative)	-∞ to -1	x + 18 = 5	-13
Rational Numbers (Positive)	p/q; q ≠0	2x = 3	3/2
Rational Numbers (Negative)	-p/q; q ≠0	5x + 7 = 0	-7/5

Properties of the types of numbers - Closure

A set of numbers is said to be**closed** for a specific mathematical operation if the result obtained when an operation is performed on any two numbers in the set, is itself a member of the set. If a set of numbers is closed for a particular operation then it is said to possess the **closure** property for that operation.

Whole Numbers

- **Addition -** Adding two whole numbers results in another whole number. Hence, whole numbers under addition are**closed**.
  - 2+3 = 5
  - 0 + 6 = 6
- **Subtraction** - Subtracting two whole numbers may result in a negative number which is not a whole number.Hence, whole numbers under subtraction are **notclosed**.
  - 5-3 = 2 (whole)
  - 0 - 6 = -6 (not whole)
- **Multiplication -** Multiplying two whole numbers results in another whole number. Hence, whole numbers under multiplication are**closed**.
  - 5 x 3 = 15
  - 2 x 0 = 0
- **Division** - Dividing two whole numbers may result in a fraction or a number with decimal point which is not a whole number.Hence, whole numbers under subtraction are **notclosed**.
  - 4 ÷ 2 = 2 (whole)
  - 10 ÷ 4 = 10/4 = 2.5 (not whole)

Integers

- **Addition -** Adding two integers results in another integer. Hence, integers under addition are**closed**.
  - (-2) +3 = 1
  - (-7) + (-5) = -12
- **Subtraction** - Subtracting two integers results in another integer.Hence, integersunder subtraction are**closed**.
  - 5-(-3) = 8
  - (-3) - 6 = -9
- **Multiplication -** Multiplyingtwo integers results in another integer. Hence, integersunder multiplication are**closed**.
  - 5 x (-3) = -15
  - (-2) x (-5) = 10
- **Division** - Dividing two integersmay result in a fraction or a number with decimal point which is not an integer.Hence, integers under division are

notclosed.

  - 4 ÷ (-2) = -2 (Integer)
  - (-10) ÷ 4 = -10/4 = -2.5 (not an integer)

Rational Numbers

- **Addition -** Adding two rational numbers results in another rational number. Hence, rational numbersunder addition are**closed**.
- 8/5 + (-2)/5 = 6/5
- 3/8 + (-5)/7 = (21 + (-40))/56 = -19/56
- **Subtraction -** Subtracting two rational numbers results in another rational number. Hence, rational numbersunder subtraction are**closed**.
- 8/5 - (-2)/5 = 10/5 = 2 or 2/1
- 3/8 -5/7 = (21 - 40)/56 = -19/56
- **Multiplication -** Multiplying two rational numbers results in another rational number. Hence, rational numbersunder multiplication are**closed**.
- 8/5 x (-2)/5 = -16/25
- (-3)/8 x(-5)/7 = 15/56
- **Division** - Dividing two rational numbersmay result in an undefined number with which is not a rational number.Hence, rational numbersunder division are **notclosed**.
  - 4/3 ÷ (-2)/7 = -28/6 = -14/3 (rational number)
  - (-10)/3 ÷ 0/1 = -10/0 = undefined (not a rational number)

Properties of the types of numbers - Commutativity

A set of numbers is said to be **commutative** for a specific mathematical operation if the result obtained when changing order of the operands does not change the result.

Whole Numbers

- **Addition–** Changing the order of operands in addition of whole numbers does not change the result. Hence, whole numbers under addition are **commutative**.
  - 2+3 = 3 + 2
  - 0 + 6 = 6 + 0
- **Subtraction** - Changing the order of operands in subtraction of whole numbers changes the result. Hence, whole numbers under subtraction are **not commutative**.
  - 5-3 ≠ 3 - 5
  - 0 - 6 ≠ 6 - 0
- **Multiplication -** Changing the order of operands in multiplication of whole numbers does not change the result. Hence, whole numbers under multiplication are **commutative**.
  - 5 x 3 = 3 x 5
  - 2 x 0 = 0 x 2
- **Division** - Changing the order of operands in division of whole numbers changes the result. Hence, whole numbers under division are **not commutative.**
  - 4 ÷ 2 ≠ 2÷ 4
  - 10 ÷ 4 ≠ 4 ÷ 10

Integers

- **Addition –** Changing the order of operands in addition of integers does not change the result. Hence, integers under addition are **commutative**.
  - 2 + (-3) = (-3) + 2
  - (-1) + 6 = 6 + (-1)
- **Subtraction** - Changing the order of operands in subtraction of integers changes the result. Hence, integers under subtraction are **not commutative**.
  - 5 -(-3) ≠ (-3) - 5
  - (-1) - 6 ≠ 6 - (-1)
- **Multiplication -** Changing the order of operands in multiplication of integers does not change the result. Hence, integers under multiplication are **commutative**.
  - 5 x (-3) = (-3) x 5
  - (-2) x 0 = 0 x (-2)

- **Division** - Changing the order of operands in division of integers changes the result. Hence, integers under division are **not commutative.**
    - 4 ÷ (-2)≠(-2)÷ 4
    - (-10) ÷ 4 ≠4 ÷ (-10)

Rational Numbers

- **Addition -** Changing the order of operands in addition of rational numbers does not change the result. Hence, rational numbers under addition are **commutative**.
- 8/5 + (-2)/5 = (-2)/5 + 8/5
- 3/8 + (-5)/7 = (-5)/7 + 3/8
- **Subtraction -** Changing the order of operands in subtractionof rational numbers changes the result. Hence, rational numbers under subtraction are **not commutative**.
- 8/5 - (-2)/5 ≠(-2)/5 - 8/5
- 3/8 -5/7 ≠5/7 - 3/8
- **Multiplication -** Changing the order of operands in multiplication of rational numbers does not change the result. Hence, rational numbers under multiplication are **commutative**.
- 8/5 x (-2)/5 = (-2)/5 x 8/5
- (-3)/8 x(-5)/7 = (-5)/7 x (-3)/8
- **Division** - Changing the order of operands in divisionof rational numbers changes the result. Hence, rational numbers under division are **not commutative**.
    - 4/3 ÷ (-2)/7≠-(-2)/7 ÷ 4/3
    - (-10)/3 ÷ 4/1≠4/1 ÷(-10)/3

Properties of the types of numbers - Associativity

A set of numbers is said to be**associative** for a specific mathematical operation if the result obtained when changing grouping (parenthesizing) of the operands does not change the result.

Whole Numbers

- **Addition –** Changing the grouping of operands in addition of whole numbers does not change the result. Hence, whole numbers under addition are **associative**.
  - 2 + (3 + 6) = (2 + 3) + 6
  - (0 + 6)+ 8 = 0 + (6 + 8)
- **Subtraction** - Changing the groupingof operands in subtractionof whole numbers changes the result. Hence, whole numbers under subtraction are **not associative**.
  - 5 - (3 - 4) ≠ (5 - 3) - 4
  - (2 - 0) - 6 ≠ 2 - (0 - 6)
- **Multiplication -** Changing the groupingof operands in multiplication of whole numbers does not change the result. Hence, whole numbers under multiplication are **associative**.
  - 5 x (3 x 6) = (5 x 3) x 6
  - (2 x 0)x 9 = 2 x (0 x 9)
- **Division** - Changing the groupingof operands in division of whole numbers changes the result. Hence, whole numbers under division are **not associative.**
  - 4 ÷ (2 ÷ 6) ≠(4 ÷ 2) ÷ 6
  - (10 ÷ 4)÷ 7 ≠10 ÷ (4 ÷ 7)

Integers

- **Addition –** Changing the grouping of operands in addition of integers does not change the result. Hence, integers under addition are **associative**.
  - 2 + (3 + (-6)) = (2 + 3) + (-6)
  - (0 + (-6)) + (-8) = 0 + ((-6) + (-8))
- **Subtraction** - Changing the grouping of operands in subtractionof integers changes the result. Hence, integers under subtraction are **not associative**.
  - 5 - (3 - (-4)) ≠ (5 - 3) - (-4)
  - ((-2) - 0) - 6 ≠ (-2) - (0 - 6)
- **Multiplication -** Changing the grouping of operands in multiplication of integers does not change the result. Hence, integers under multiplication are **associative**.
  - 5 x ((-3) x 6) = (5 x (-3)) x 6

- ((-2) x 0)x (-9)= (-2) x (0 x (-9))

- **Division** - Changing the grouping of operands in division of integers changes the result. Hence, integers under division are **not associative.**
  - 4 ÷ (2 ÷(-6)) ≠(4 ÷ 2) ÷(-6)
  - ((-10) ÷ 4) ÷ 7 ≠(-10) ÷ (4 ÷ 7)

Rational Numbers

- **Addition -** Changing the grouping of operands in addition of rational numbers does not change the result. Hence, rational numbers under addition are **associative**.
- 2/3 + (3/2 + (-6)/7) = (2/3 + 3/2) + (-6)/7
- (0/1 + (-6)/5) + (-8)/3 = 0/1 + ((-6)/5 + (-8)/3)
- **Subtraction -** Changing the grouping of operands in subtractionof rational numbers changes the result. Hence, rational numbers under subtraction are **not associative**.
- 5/4 - (3/11 - (-4)/7) ≠ (5/4 - 3/11) - (-4)/7
- ((-2)/7 - 0/3) - 6/5 ≠ (-2)/7 - (0/3 - 6/5)
- **Multiplication -** Changing the grouping of operands in multiplication of rational numbers does not change the result. Hence, rational numbers under multiplication are **associative**.
- 5/3 x ((-3)/4 x 6/11) = (5/3 x (-3)/4) x 6/11
- ((-2)/5 x 0/1)x (-9)/2 = (-2)/5 x (0/1 x (-9)/2)
- **Division** - Changing the grouping of operands in division of rational numbers changes the result. Hence, rational numbers under division are **not associative**.
  - 4/3 ÷ (2/3 ÷ (-6)/7) ≠(4/3 ÷ 2/3) ÷ (-6)/7
  - ((-10)/3 ÷ 4/7) ÷ 7/6 ≠(-10)/3 ÷ (4/7 ÷ 7/6)

Additive and Multiplicative identity of Rational numbers

- **Additive identity: Zero** is the additive identity for Rational, natural, whole numbers and integers, since adding it to them does not change the result.
  - 3 + 0 = 3
  - -4/5 + 0 = -4/5

Hence, 0 + a = a + 0 = a, where a can be rational number or natural number or whole number of integer.

- **Multiplicative identity: One** is the additive identity for Rational, natural, whole numbers and integers, since multiplying it to them does not change the result.
  - 3 x 1 = 3
  - -4/5 x 1 = -4/5

Hence, 1x a = ax1 = a, where a can be rational number or natural number or whole number of integer.

Additive inverse and Multiplicative inverse of Rational numbers

- **Negative or Additive inverse: - or minus** is the additive inverse for Rational, natural, whole numbers and integers, since adding its additive inverse to a number results in zero.
  - 3 + (-3) = 0
  - (-4/5) + (-(-4/5)) = (-4/5) + 4/5 = 0

Hence, a + (-a) = (-a) + a = 0, where a can be rational number or natural number or whole number of integer.

Example ExamFear.com

Problem: Verify that –(-x) = x for:

(i) $x = \frac{11}{15}$

Solution:

The additive inverse of x is –x. Hence,

$-x = -\frac{11}{15}$

$x + (-x) = \frac{11}{15} + (-\frac{11}{15}) = 0$

This shows $(-\frac{11}{15})$ is additive inverse of $\frac{11}{15}$ and vice versa.

Hence, additive inverse of $(-\frac{11}{15})$,

$= -(-\frac{11}{15}) = \frac{11}{15}$

(ii) $x = -\frac{13}{17}$

Solution:

The additive inverse of x is –x. Hence,

$-x = -(-\frac{13}{17})$

$x + (-x) = -\frac{13}{17} + (-(-\frac{13}{17})) = -\frac{13}{17} + \frac{13}{17} = 0$

This shows $\frac{13}{17}$ is additive inverse of $-\frac{13}{17}$ and vice versa.

Hence, additive inverse of $\frac{13}{17}$,

$= -(\frac{13}{17}) = -\frac{13}{17}$

- **Reciprocal or Multiplicative inverse: Dividing a number by 1** is the multiplicative inverse for Rational, natural, whole numbers and integers, since multiplying it to the original number always results in 1.
  - 3 x 1/3 = 1
  - (-4/5) x (1/(-4/5) = (-4/5) x (-5/4) = 1

Hence, ax 1/a = 1/a x a = 1, where a can be rational number or natural number or integer.

Example ExamFear.com

Problem: Is $\frac{8}{9}$ the multiplicative inverse of $-1\frac{1}{8}$? Also, is 0.3 multiplicative inverse of $3\frac{1}{3}$?

Solution: For multiplicative inverse, the product of the two numbers should be 1.

$\frac{8}{9} \times (-1\frac{1}{8}) = \frac{8}{9} \times (-\frac{9}{8}) = -1 \neq 1$

Since the product is not equal to 1, $\frac{8}{9}$ is not the multiplicative inverse of $-1\frac{1}{8}$.

$0.3 \times (3\frac{1}{3}) = \frac{3}{10} \times \frac{10}{3} = 1$

Since the product is equal to 1, 0.3 is multiplicative inverse of $3\frac{1}{3}$.

Example ExamFear.com

Problem: Write additive and multiplicative inverse of :

Number	Additive Inverse	Multiplicative Inverse
$\frac{2}{8}$	$-\frac{2}{8}$	$\frac{8}{2}$
$-\frac{5}{9}$	$\frac{5}{9}$	$\frac{9}{5}$
$\frac{-6}{-5}$	$\frac{6}{-5}$ or $\frac{-6}{5}$	$\frac{-5}{-6}$ or $\frac{5}{6}$
$\frac{2}{-9}$ or $\frac{-2}{9}$	$\frac{2}{9}$	$\frac{-9}{2}$
$\frac{19}{-6}$ or $\frac{-19}{6}$	$\frac{19}{6}$	$\frac{-6}{19}$

Example ExamFear.com

Problem: Find the property under multiplication used in each of the following:

Solution:

(i) $\frac{-4}{5} \times 1 = 1 \times \frac{-4}{5} = -\frac{4}{5}$ Property used – Multiplicative identity 1 and commutativity

(ii) $\frac{-13}{7} \times \frac{-2}{7} = \frac{-2}{7} \times \frac{-13}{7}$ Property used – Commutativity

(iii) $\frac{-19}{29} \times \frac{29}{-19} = 1$ Property used – Multiplicative Inverse

(iv) $\frac{1}{3} \times (6 \times \frac{4}{3}) = (\frac{1}{3} \times 6) \times \frac{4}{3}$ Property used - Associativity

Distributive property of multiplication over addition and subtraction for rational numbers

The distributive property of multiplication is:

- **Over addition**: a(b + c) = ab + ac
  - -3/4 {2/3 + (-5/6)} = -3/4 {(4+(-5))/6} = (-3/4) x (-1/6) = 3/24 = 1/8
  - -3/4 {2/3 + (-5/6)} = -3/4 x 2/3 + (-3/4) x (-5/6) = -1/2 + 5/8 = (-4+5)/8 = 1/8
- **Over subtraction**: a(b - c) = ab - ac
  - -3/4 {2/3 - 5/6} = -3/4 {(4-5)/6} = (-3/4) x (-1/6) = 3/24 = 1/8
  - -3/4 {2/3 - 5/6} = -3/4 x 2/3 - (-3/4) x 5/6 = -1/2 + 5/8 = (-4+5)/8 = 1/8

Example ExamFear.com

Problem: Using appropriate properties find:

(i) $-\frac{2}{3} \times \frac{3}{5} + \frac{5}{2} - \frac{3}{5} \times \frac{1}{6}$

Solution:

Using commutativity,

$-\frac{2}{3} \times \frac{3}{5} + \frac{5}{2} - \frac{3}{5} \times \frac{1}{6} = -\frac{2}{3} \times \frac{3}{5} - \frac{3}{5} \times \frac{1}{6} + \frac{5}{2}$

Using distributivity,

$= (-\frac{3}{5}) \times (\frac{2}{3} + \frac{1}{6}) + \frac{5}{2} = (-\frac{3}{5}) \times (\frac{2x2+1}{6}) + \frac{5}{2}$

$= (-\frac{3}{5}) \times (\frac{5}{6}) + \frac{5}{2} = (-\frac{3}{6}) + \frac{5}{2}$

$= (\frac{-3+15}{6}) = \frac{12}{6} = 2$

(ii) $\frac{2}{5} \times (-\frac{3}{7}) - \frac{1}{6} \times \frac{3}{2} + \frac{1}{14} \times \frac{2}{5}$

Solution:

Using commutativity,

$\frac{2}{5} \times (-\frac{3}{7}) - \frac{1}{6} \times \frac{3}{2} + \frac{1}{14} \times \frac{2}{5} = \frac{2}{5} \times (-\frac{3}{7}) + \frac{1}{14} \times \frac{2}{5} - \frac{1}{6} \times \frac{3}{2}$

Using distributivity,

$= \frac{2}{5} \times (-\frac{3}{7} + \frac{1}{14}) - \frac{1}{4} = \frac{2}{5} \times (\frac{-3 \times 2+1}{14}) - \frac{1}{4}$

$= \frac{2}{5} \times (\frac{-5}{14}) - \frac{1}{4} = -\frac{1}{7} - \frac{1}{4}$

$= \frac{-4-7}{28} = \frac{-11}{28}$

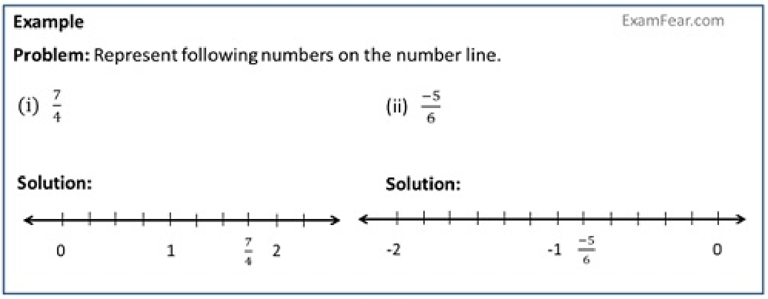
<u>Representation of number types on number line</u>

The number types are represented on number line using the below table:

Number Type	**Range of numbers**	**Number Line**
Natural Numbers	1 to ∞	
Whole Numbers	0 to ∞	
Integers	-∞ to ∞	
Rational Numbers	-∞ to ∞	

While representing rational numbers on number line,

- Between two integers, the line should be divided into number of equal parts which is same as the denominator of the rational number. For eg, to represent 1/3, three equal parts are made from 0 to 1 representing 1/3, 2/3 and 3/3 (that is 1) respectively.


Example ExamFear.com

Problem: Represent following numbers on the number line.

(i) $\frac{7}{4}$ (ii) $\frac{-5}{6}$

Solution: **Solution:**

Rational numbers between two rational numbers

There are always definite amount of numbers between two natural/whole numbers or integers. But, there can be **indefinite** amount of numbers between two rational numbers.

- Natural numbers between 4 and 22 are 17 (5, 6, …, 20, 21).
- Whole numbers between 0 and 5 are 4 (1,2,3 and 4).
- Integers between -4 and 9 are 12 (-3, -2, …., 7, 8)
- Rational numbers between 3/10 and 7/10 can be:
  - 4/10, 5/10, 6/10
  - Or, 31/100, 32/100, …., 68/100, 69/100 since 3/10 can also be written as 30/100 and 7/10 can be written as 70/100.
  - Since 3/10 can also be written as 300/1000 and 7/10 can be written as 700/1000, there can be numbers 301/1000 to 699/1000.
  - And so on.

Between any 2 numbers, it is not necessary that there will be an integer or a whole number but there is always a rational number.

Example, there are no integer or whole or natural numbers between 1 and 2, but there are rational numbers like, 1/2, 1/3 and 2/3, 1/4, 2/4, 3/4 etc.

Example

Problem: Find 10 rational numbers between $-\frac{2}{5}$ and $\frac{1}{2}$?

Solution: $-\frac{2}{5}$ and $\frac{1}{2}$ can be represented as $-\frac{8}{20}$ and $\frac{10}{20}$ respectively.

Therefore, 10 rational numbers between $-\frac{2}{5}$ and $\frac{1}{2}$ are:

$-\frac{7}{20}, -\frac{6}{20}, -\frac{5}{20}, -\frac{4}{20}, -\frac{3}{20}, -\frac{2}{20}, -\frac{1}{20}, 0, \frac{1}{20}, \frac{2}{20}$

Example

Problem: Find 5 rational numbers which are smaller than 2 ?

Solution: 2 can be represented as $\frac{14}{7}$ or $\frac{10}{5}$ etc.

Therefore, 5 rational numbers smaller than $\frac{14}{7}$ are:

$\frac{13}{7}, \frac{12}{7}, \frac{11}{7}, \frac{10}{7}, \frac{9}{7}$

Also, 5 rational numbers smaller than $\frac{10}{5}$ are:

$\frac{9}{5}, \frac{8}{5}, \frac{7}{5}, \frac{6}{5}, \frac{5}{5}$