Electrostatics

(Electric Field & Potenial)

ELECTROSTATICS / FIELD

Electrostatics is a branch of physics which deals with charge at rest charge is the origin of electromagnetic force. When charge is at rest then region around it called electric field while the region around the moving charge called magnetic field.

PROPERTIES OF CHARGE :

1. For a given closed system the total charge remains conserved.
2. Charge is always quantised i.e. $Q = \pm nc$; $n \in I$

   Charge on any body is always in the integral multiple of one electronic charge ($e = 1.6 \times 10^{-19}$ coul).
3. Charge is relativistically invariant.

TYPES MATERIAL

There are three kinds of material on the basis of conductivity.

(a) Conductor : Having large number of mobile electrons. It is approximately 10^{21} electrons/c.c.

(b) Bad conductor : Having very small number of free electrons, it is approximately 10^7 electrons c.c

(c) Semi conductor : Conductivity lies between conductor and insulator. Number of free electrons is approximately 10^4 electron/c.c.

COULOMB'S LAW :

When two point charges q_1 and q_2 are separated by a distance r then force of mutual intraction F is given by

$$F \propto q_1 q_2 \text{ when } r = \text{constant.}$$

and $F \propto \frac{1}{r^2}$ when q_1q_2 = constant

Hence $F \propto \frac{q_1 q_2}{r^2}$ or $F = k.\frac{q_1 q_2}{r^2}$; K is proportionality constant

In SI units, $F = \frac{1}{4\pi \in_0}.\frac{q_1 q_2}{r^2}$; $\in_0 = 8.85 \times 10^{-12} F_m$ and $\frac{1}{4\pi \in_0} = 9 \times 10^9$ nt m^2 coul $^{-2}$

In cgs units, $F = \frac{q_1 q_2}{r^2}$; $\in_0$ is per mittivity of the vacuum.

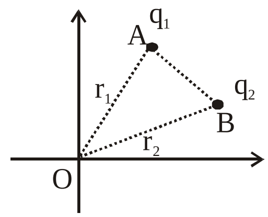
When there is a medium in the intervining region of two charges then

$$F = \frac{1}{4\pi \in_0 \in_r} \cdot \frac{q_1 q_2}{r^2}$$ ($\hat{I}_r$ is the relative permitivity of medium)

$\hat{I}_r$ is the relative permitivity which is the dimensionless qunatity that gives the factor by which force is reduced compared to vaccum.

$\in_r = \dfrac{F_0}{F_m}$, F_0 is the force in vacuum and F_m is force in medium.

$\in = \in_0 \in_r =$ Absolute permittivity of the medium.



$$\vec{F}_{AB} = \frac{1}{4\pi\in_0}\cdot\frac{q_1q_2}{|\vec{BA}|^2}\cdot\frac{\vec{BA}}{|\vec{BA}|}$$

$$\vec{F}_{AB} = \frac{q_1q_2}{4\pi\in_0|\vec{BA}|^3}\cdot\vec{BA}$$

where $\vec{BA} = \vec{OA} - \vec{OB} = \vec{r}_1 - \vec{r}_2$

$$\vec{F}_{AB} = \frac{q_1q_2}{4\pi\in_0|\vec{r}_1 - \vec{r}_2|^3}\cdot(\vec{r}_1 - \vec{r}_2)$$

when $q_1q_2 > 0$, Force is attractive

and $q_1q_2 < 0$, Force is repulsive

PROCESS OF CHARGING :

(1) By rubbing or friction - when two bodies are rubbed together there is transfer of electrons from body, which is surplus in electron to another body which is surplus in electrons and get positive charge of equal amount to negative charge.

(2) By conduction - when a charged body is in contact with another uncharged one threre is redistribution of charges on entire are is of both bodies followed by mechanical separation. The amount of charge redistribution on body depends on surface area.

(3) By induction - when an uncharged body is brought near charged on the charge opposite nature induced over the uncharged one.

The induced charge is always less than or equal to inducing charge. Induction is always followed by attraction, but attraction is not the surest test of induction.

If q be inducing charge, then charge induced on a body having dielectric constant K is given by

$q' = -q\left(1 - \dfrac{1}{K}\right)$, if charge is induced on the surface of a conductor then induced charge is

$q' = -q$ (As k is infinity for a conductor)

DISTRIBUTION OF CHARGES :

(a) Linear charge distribution : - If charge gets appeared on a body of linear dimension.

Linear charge density $(\lambda) =$ charge per unit length.

(b) Surface charge distrbution :- If charge gets appeared on a body having two dimensions.

Surface charge density $(\sigma) =$ charge per unit area

(c) Volume charge density : If charge is enclosed in a volume;

Volume charge density=charge per unit volume

Electric field : The site around the charge at rest called electric field.

Electric field strength or Electric intensity is defined as the force experienced by a unit positive charge.

$\vec{E} = \lim\limits_{q_0 \to 0} \dfrac{F}{q_0}$ where q_0 is the positive test charge.

Unit of $\vec{E}$ is newton coulmob and dimension is $\left[MLT^{-3}A^{-1}\right]$. The resultant electric field at any point is equal to vector sum of electric field at that point due to various charges i.e. $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$

Resultant of two electric fields which are at an angle θ is given by

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\theta} \text{ and } \tan\alpha = \frac{E_2\sin\theta}{E_1 + E_2\cos\theta}$$

α is the angle of $\vec{E}$ with $\vec{E}_1$

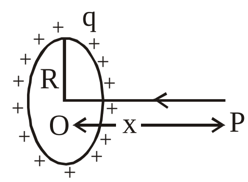
Electric field intensity $(\vec{E})$ for some body having uniformly continuous charge distribution.

1. Electric field strength due to a point charge at a distance r is given by

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}.\hat{r}$$

2. Electric field strength due to a uniformly charged ring at a distance x from centre of the axis of ring..

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{qx}{(x^2+R^2)^{3/2}}$$



E becomes max at $x = \frac{R}{\sqrt{2}}$: Direction of $\vec{E}$ is away from centre along axis.

3. Electric field strength due to uniformally charged rod of length l at a distance r along perpendicular line from centre and linear charge density is λ.

$$E \neq \frac{q}{4\pi\epsilon_0 r\sqrt{\frac{l^2}{4} + r^2}}$$

where λ = charge per unit length when $l \to \infty$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

4. Electric field strength due to uniformally charged spherical shell of radius R at a distance r from, centre of shell

E = 0, if r < R

$E = \frac{Q}{4\pi\epsilon_0 R^2} = \frac{\sigma}{\epsilon_0}$; if $r \geq R$ where Q is charged on shell or σ is surface charge density.

5. Electric field strength due to uniformally charged solid sphere of radius R at a distance r from centre of sphere.

$E = \frac{Q}{4\pi\epsilon_0 R^3}r = \frac{\rho}{3\epsilon_0}r$; r < R; where Q is charged in solid sphere enclosed ;

ρ is volume charge density and $E = \frac{Q}{4\pi\in r^2}$ where $r \geq R$

ELECTRIC DIPOLE :

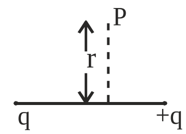
A system of two equal and opposite charges separated by a small distance called dipole. The dipole moment of dipole is defined as the product of charge and distance of separation and direction of dipole moment is from –ve charge to positive charge.

$p = q\Delta l$; Direction of dipole moment $(\vec{p})$ is from the charge to positive charge.

$\vec{E}$ due to dipole at a distance r from centre of dipole along axis of dipole, $\vec{E} = \dfrac{2\vec{P}}{4\pi \epsilon_0 r^3}$.

$\vec{E}$ (Electric field strength) due to dipole at a distance r from centre of dipole along perpendicular bisector of line (Along equational line)

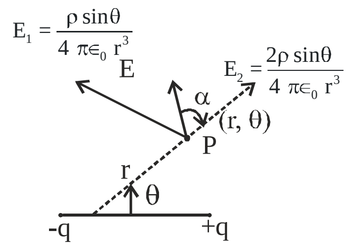
$$\vec{E} = \frac{-\vec{P}}{4\pi \epsilon_0 r^3}$$



$\vec{E}$ (Electric field strength) at a point (r, θ) from centre of dipole.

$$E = \frac{P}{4\pi \epsilon_0 r^3}\sqrt{1 + 3\cos^2\theta}$$

and α = angle made by $\vec{E}$ with $\vec{r} = \tan^{-1}\left(\frac{1}{2}\tan\theta\right)$



Dipole in an external electric field :

When dipole of dipole moment $\vec{P}$ is placed in an external electric field of field strength $\vec{E}$ then torque and potential energy at orientation θ are given by $\vec{\tau} = \vec{P} \times \vec{E}$ and $U = -\vec{P}.\vec{E}$ respectively. when dipole is placed in non uniform electric field then there no translational equilibrium and no rational equilibrium while for uniform field there is always translational equilibrium but no rational.

Electric flux :

In the region around the charge at rest there exists hypothetical electric lines, which measures the electric field strength at a point around charge at rest.

Electric flux density :

It is the number of electric lines cross through unit area held normally to the electric lines.

$$\phi_E = \int \vec{E}.\vec{ds} = \int E\cos\theta\, ds = \text{Electric flux through the area ds}$$

θ is the angle between $\vec{E}$ and area vector.

Gauss' law : The total electric flux through a closed loop is always equal to $\dfrac{1}{\epsilon_0}$ lines the total charge enclosed.

$$\phi_\epsilon = \oint \vec{E}\,\vec{ds} = \frac{1}{\epsilon_0}(\text{charge enclosed})$$

Application of Gaus's law :

(a) The charge given to the conductor always get apeared on outside of the conductor.

(b) The electric field strength at a perpendicular bisector on uniformally charged rod of infinite length at a distance r from centre of rod

$$E = \frac{\lambda}{2\pi \epsilon_0 r}.$$

(c) The electrostatic potential energy per unit volume (Energy density) = $\frac{1}{2}\epsilon_0 E^2$ in air or vacuum.

$$= \frac{1}{2}\epsilon_0 \epsilon_r E^2 \text{ in medium of relative}$$

permitivity or dielectric constant is ϵ_r or k.

(d) The electric field strength $\vec{E}$ near the surface of uniformly conducting sheet of surface charge density

$$E = \frac{\sigma}{\epsilon_0}$$

(e) The electric field strength E near the surface of infinite long plane thin non-conducting sheet of charge of surface charge density σ is $E = \frac{\sigma}{2\epsilon_0}$

Properties of electric lines :

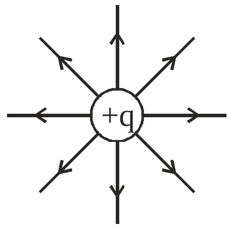
1. Electric lines are continuous curves that emanate from positive charge and terminate to negative charge.
2. Number of electric lines per unit area at a point gives the magnitudeof electric field strength while direction of electric field strength is given by the tangent drawn at the point along the direction of electric lines.
3. Direction and magnitude $\vec{E}$ at a point are unique.
4. No two electriclines intersect at a point.

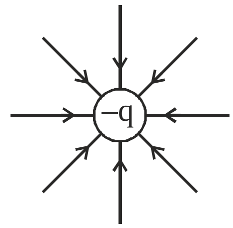
Dipole Interaction

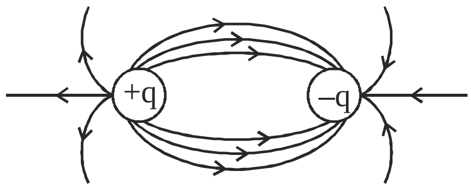
When two dipoles are placed along axial line having same direction of dipole moments then force of mutual attraction is $\frac{1}{4\pi}\frac{6P_1P_2}{r^4}$ where P_1 and P_2 are respective dipole moments while mutual potential energy is $\frac{1}{4\pi}\frac{2P_1P_2}{r^3}$; r = distance between centre S of dipole S when two dipoles are held parallel at a distance r where direction of dipole moments are same then mutual force (repulsion) and potential energy are given by $\frac{1}{4\pi}\frac{3P_1P_2}{r^4}$ and $\frac{1}{4\pi}\frac{P_1P_2}{r^3}$ respectively.

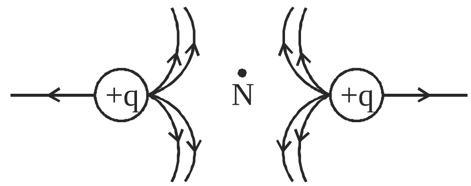
Electric lines

The region around charge at rest in which there hypothetically exists continuous lines called electric lines Electric line is an imaginary line along which a positive charge will move if left free.









Electric lines never formed closed loop and meet the equipotential surface or conducting surface perpendicularly.

ELECTROSTATIC POTENTIAL ENERGY :

When there is a system of charges (assembing by two or more charges) there exists potential energy in the system. Electrostatic potential energy is the work done against electrostatic force to create a system of charges. When two charges q_1, q_2 are separated by a distance r then mutual potential energy

$U = \frac{1}{4\pi \in_o} \frac{q_1 q_2}{r}$ Electric potential at a point.

ELECTRIC POTENTIAL :

Electric potential at a point defined as the work required by an external agent to displace a unit positive charge from infinity to the point or also defined as work done by electrostatic force to displace from point to infinity.

$$V(r) = -\int_{\infty}^{r} \vec{E} \cdot \vec{dr}$$

ELECTRIC POTENTIAL AND ELECTRIC FIELD STRENGTH :

Negative rate of change of electric potential is the electric field strength along the line there is change in potential

$$-\frac{dv}{dr} = E \text{ or } \vec{E} = E_x\hat{i} + Ey\,\hat{j} + E_z\hat{k}$$

where $\quad E_x = -\frac{\partial v}{\partial x};\ Ey = -\frac{\partial v}{\partial y};\ E_z = -\frac{\partial v}{\partial z}$

Negative sign gives the direction of $\vec{E}$ along the line in which increase in distance causes the decrease in potential.

EQUIPOTENTIAL SURFACE :

Every point on a surface in at same potential called equipotential surface. Electric lines always join the equipotential surface perpendicularly.

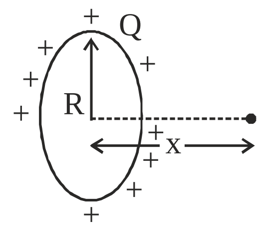
ELECTRIC POTENTIAL DUE TO SOME CHARGE DISTRIBUTIONS :

1. Electric Potential at a distance r due to a point charge q

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

2. Electric potential due to uniformally charged circular ring at a distance x from centre of ring along axis of ring

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{\sqrt{x^2+R^2}}$$

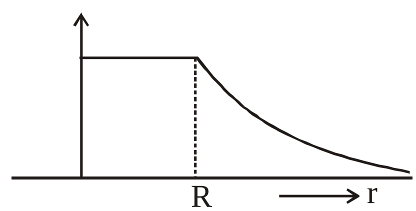


3. Electric potential due to uniformally charged hollow sphere or shell at a distance r

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R};\ r \le R$$

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2};\ r > R$$

R = Radius of the hollow sphere



4. Electric potential due to uniformally charged sphere of radius R and volume charge density (ρ)

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r} = \frac{\rho R^3}{3\varepsilon_o r};\left(\rho = \frac{Q}{\frac{4}{3}\pi R^3}\right) \qquad ;\ r \ge R$$

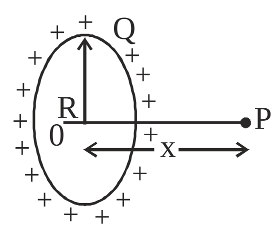
$$V = \frac{Q}{4\pi\varepsilon_o}\left[\frac{3R^2 - r^2}{2R^3}\right] = \frac{\rho\left(3R^2 - r^2\right)}{6\varepsilon_o} \qquad ;\ r < R$$

at centre of informally charged solid sphere,

$$V = \frac{3}{2}\times\frac{1}{4\pi\varepsilon_o}\frac{Q}{R} = \frac{3Q}{8\pi\varepsilon_o R}$$

5. Electrical potential at a distance x from uniformally charged disc

$$V = \frac{\sigma}{2\varepsilon_o}\left[\sqrt{x^2+R^2} - x\right]$$



σ = surface charge density.

6. Electric Potential due to dipole

At axial point

$$V = \frac{1}{4\pi\varepsilon_o}\frac{P}{r^2};\ P = \text{Dipole moment},$$

At equatorial point

$$V = 0$$

At general point (r, θ)

$$V = \frac{1}{4\pi\varepsilon_o}\frac{P\cos\theta}{r^2}$$