Chapter 8

Gravitation

NEWTON'S UNIVERSAL LAW OF GRAVITATION

Gravitational force is an attractive force between any two point masses M_1 and M_2 separated by any distance r.

It is given by $F = G\left(\frac{M_1M_2}{r^2}\right)$

where G is the **universal gravitational constant**.

Its value, $G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$

Dimensions of G are $[M^{-1}\ L^3\ T^{-2}]$

(a) The gravitational force is the central force and follows inverse square law. It acts along the line joining the particles.

(b) Since the work done by the gravitational force is independent of the path followed and hence it is a conservative force.

(c) It is the weakest force in nature. It is 10^{38} times smaller than nuclear force and 10^{36} times smaller than electric force. Strongest force being nuclear force (for small range) followed by electric force.

(d) Gravitation is independent of the presence of other bodies around it.

ACCELERATION DUE TO GRAVITY (g)

The force of attraction exerted by earth on a body of mass m is the force of gravity.

So the force of gravity from Newton's gravitational law is

$$F = \frac{GM_em}{r^2} \qquad \text{……(i)}$$

where M_e = mass of earth

r = distance of the body from the centre of earth.

The force of gravity can be written as F = mg ……(ii)

where g is called the **acceleration due to gravity.**

From the expression (i) and (ii), we get

$$g = \frac{GM_e}{r^2} \qquad \text{……(iii)}$$

If body is located at the surface of earth i.e., $r = R_e$ (radius of earth) then

$$g = \frac{GM_e}{R_e^2} = \frac{G(4/3\pi R_e^3\rho)}{R_e^2} = \frac{4}{3}\pi\rho GR_e \qquad \text{……(iv)}$$

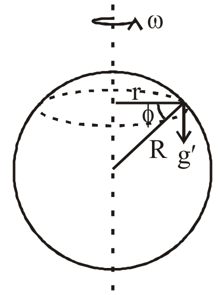
where ρ is the density of earth.

This is the relation between universal gravitational constant 'G' and acceleration due to gravity 'g'.

Change in the Value of Acceleration due to Gravity (g)

(i) ***Due to rotation or latitude of earth :*** Let us consider a particle P at rest on the surface of earth at a latitude φ.

$$g(\phi) = g - R_e\omega^2\cos^2\phi$$



At poles $\phi = 90°$, so $g(\phi) = g$ ...(v)

At equator $\phi = 0°$ so $g(\phi) = g - \omega^2R_e$ ...(vi)

(ii) ***Due to shape of earth :*** The shape of the earth is not perfectly spherical. It is flattened at poles. The polar radius is 21 km less than the equatorial radius. Hence acceleration due to gravity at poles is greater than at the equator.

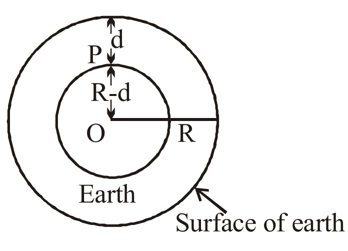
$g_p = 9.83\ ms^{-2}$ = *value of g at poles*

$g_e = 9.78\ ms^{-2}$ = *values of g at equator*

i.e. $g_p > g_e$

(iii) ***At a depth 'd' below the earth surface :*** The acceleration due to gravity at the surface of earth.

$$g = \frac{GM_e}{R_e^2} = \left[\left(\frac{4}{3}\right)\pi R_e\rho\right]G \text{ (from eq}^n\text{. iv)} \qquad \text{...(vii)}$$



If a body is taken at a depth d below the earth surface, then the body is attracted by inner core of mass M' of earth radius (R_e– d). Then acceleration due to gravity at point P is

$$g' = \frac{GM'}{(R_e - d)^2} = \frac{G\left[(4/3)\pi(R_e - d)^3\rho\right]}{(R_e - d)^2}$$

$$= G((4/3)\pi(R_e - d)\rho) \qquad ...(viii)$$

From eq^{ns}. (vii) and (viii) we have

$$g' = g\left(\frac{R_e - d}{R_e}\right) = g\left(1 - \frac{d}{R_e}\right) \qquad ...(ix)$$

The value of acceleration due to gravity at the centre of the earth is zero.

(iv) ***At a height 'h' above the earth surface :*** The acceleration due to gravity at the surface of earth from expression (iv) is defined as

$$g = \frac{GM_e}{R_e{}^2}$$

At a height h above the earth surface, the acceleration due to gravity is

$$g' = \frac{GM_e}{(R_e + h)^2} = \frac{GM_e}{r^2} \qquad ...(x)$$

$$\Rightarrow \quad g' = g\left(\frac{R_e + h}{R_e}\right)^{-2} \qquad \text{[from eqs. (ix) \& (x)]}$$

If h << R_e

$$g' \approx g\left(1 - \frac{2h}{R_e}\right) \qquad ...(xi)$$

Keep in Memory

1. The value of the acceleration due to gravity on the moon is about one sixth of that on the earth and on the sun is about 27 times that on the earth.

$$\left(g_s = 27g_E \text{ and } g_M = \frac{g_E}{6}\right)$$

2. The value of g is minimum on the mercury, among all planets.
3. For h<<R, the rate of decrease of the acceleration due to gravity with height is twice as compared to that with depth.
4. The value of g increases with the increase in latitude. Its value at latitude θ is given by : $g_\theta = g - R\omega^2 \cos^2\theta$.
5. Rotation of the earth about its own axis is responsible for decrease in the value of g with latitude.
6. The weight of the body varies along with the value of g (i.e. W = mg)
7. ***Inertial mass and gravitational mass-***

   (a) Inertial mass (M_i) defined by Newton's law of motion

$$M_i = F/a = \frac{\text{External force applied on the body}}{\text{Acceleration produced in it}}$$

$$\frac{(M_i)_1}{(M_i)_2} = \frac{a_2}{a_1}$$

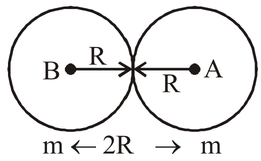
(ff applied force on $(M_i)_1$ and $(M_i)_2$ is same.)

   (b) Gravitational mass M_g defined by Newton's law of gravitation

$$M_g = \frac{F_g}{g} = \frac{W}{g} = \frac{\text{Weight of body}}{\text{Accelerati on due to gravity}}$$

8. If two spheres of same material, mass and radius are put in contact, the gravitational attraction between them is directly proportional to the fourth power of the radius.

$$F = \frac{1}{4}G\left(\frac{4}{3}\pi\sigma\right)^2 R^4$$



This system may be considered to be made up of two mass each separated by 2R distance.

9. If the earth stops rotating about its axis, the value of g at the equator will increase by about 0.35%, but that at the poles will remain unchanged.
   If the earth starts rotating at the angular speed of about 17 times the present value, there will be weightlessness on equator, but g at the poles will remain unchanged. In such a case, the duration of the day will be about 84 min.
10. If the radius of planet decreases by n%, keeping the mass unchanged, the acceleration due to gravity on its surface increases by 2n%. i.e., $\frac{\Delta g}{g} = \frac{-2\Delta R}{R}$
11. If the mass of the planet increases by m% keeping the radius constant, the acceleration due to gravity on its surface increases by m% i.e., $\frac{\Delta g}{g} = \frac{\Delta M}{M}$ where R = constant.
12. If the density of planet decreases by ρ % keeping the radius constant, the acceleration due to gravity decreases by ρ%.
13. If the radius of the planet decreases by r% keeping the density constant, the acceleration due to gravity decreases by r%.

Example 1.

What will be the acceleration due to gravity on the surface of the moon with its radius as 1/4th the radius of the earth and its mass as 1/80th the mass of earth?

Solution :

$$g = \frac{GM_e}{R_e{}^2} \text{ and } g' = \frac{GM_m}{R_m{}^2};$$

$$\therefore \quad \frac{g'}{g} = \frac{M_m}{M_e}\left(\frac{R_e}{R_m}\right)^2 = \frac{1}{80} \times \left(\frac{4}{1}\right)^2 = \frac{1}{5}; \therefore \quad g' = g/5$$

Example 2.

A mass M splits into two parts m and (M–m), which are then separated by a certain distance. What will be the ratio m/M which maximizes the gravitational force between the two parts?

Solution :

$$F = \frac{Gm\,(M-m)}{x^2}\text{; For maxima,}$$

$$\frac{dF}{dm} = \frac{G}{x^2}(M-2m) = 0 \quad \text{or} \quad \frac{m}{M} = \frac{1}{2}$$

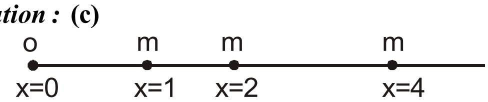
Example 3.

Identical point masses each equal to m are placed at x = 0, x = 1, x = 2, x = 4, ... The total gravitational force on mass m at x = 0 due to all other masses is

(a) infinite (b) $(4/3)\,Gm^2$

(c) $(4/3\,GM^2)$ (d) zero

Solution : **(c)**



$$F_0 = \frac{Gm^2}{1^2} + \frac{Gm^2}{2^2} + \frac{Gm^2}{4^2} + ...$$

$$F_0 = Gm^2\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + ...\right]$$

It is a G.P. with common ratio 1/4.

$$\therefore \quad F_0 = Gm^2\left[\frac{1}{1-1/4}\right] \qquad F_0 = \frac{4Gm^2}{3}$$

Example 4.

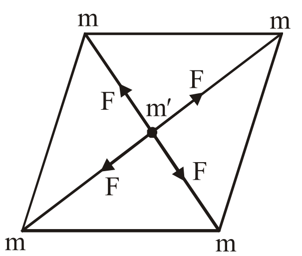
Four identical point masses each equal to m are placed at the corners of a square of side a. The force on a point mass m' placed at the point of intersection of the two diagonals is

(a) $(4Gmm')/a^2$ (b) $(2Gmm')/a^2$

(c) $(Gmm')/a^2$ (d) zero

Solution : **(d)**

The forces acting on the mass m′ placed at the point of intersection of diagonals are balanced by each other as shown in figure. Therefore net force is zero.

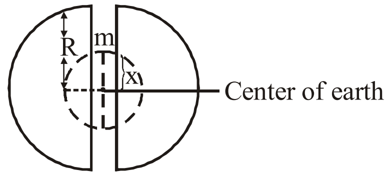


Example 5.

A tunnel is dug along the diameter of the earth. Determine the force on a particle of mass m placed in the tunnel at a distance x from the centre.

Solution :

Force on a mass m placed at a distance x from the centre is equal to the force of gravity due to a mass of spherical volume of earth of radius x.



$$\therefore F = \frac{G\left(\frac{4\pi x^3}{3}d\right)m}{x^2}$$

where d = density of earth $= \frac{3M}{4\pi R^3}$;

$$\therefore F = \frac{GMm}{R^3}.x \quad \text{[M is total mass of earth]}$$

Example 6.

If the radius of the earth were to shrink by two per cent, its mass remaining the same, the acceleration due to gravity on the earth's surface would

(a) decrease by 2% (b) increase by 2%

(c) increase by 4% (d) decrease by 4%

Solution : (c)

As $g = \frac{GM}{R^2}$; So, if R decreases then g increases.

Taking logarithm of both the sides;

log g = log G + log M – 2 log R

Differentiating it, we get $\frac{dg}{g} = 0 + 0 - \frac{2\,dR}{R}$;

$$\therefore \frac{dg}{g} = -2\left(\frac{-2}{100}\right) = \frac{4}{100}$$

$$\therefore \text{ \% increase in } g = \frac{dg}{g} \times 100 = \frac{4}{100} \times 100 = 4\%$$

Example 7.

An apple of mass 0.25 kg falls from a tree. What is the acceleration of the apple towards the earth? Also calculate the acceleration of the earth towards the apple.

Given : Mass of earth = 5.983×10^{24} kg,

Radius of earth = 6.378×10^6 *metre*,

$G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$.

Solution :

Mass of apple, m = 0.25 kg,

Mass of earth, M = 5.983×10^{24} kg,

Radius of earth, R = 6.378×10^6 metre

Gravitational constant, G = $6.67 \times 10^{-11}\ Nm^2kg^{-2}$

Let F be the gravitational force of attraction between the apple and earth.

Then, $F = \frac{GmM}{R^2}$

Let a be the acceleration of apple towards the earth.

$$a = \frac{F}{m} = \frac{GmM}{mR^2} = \frac{GM}{R^2}$$

$$\Rightarrow a = \frac{6.67 \times 10^{-11} \times 5.983 \times 10^{24}}{(6.378 \times 10^6)^2}\,ms^{-2} = 9.810\,ms^{-2}$$

Let a′ be the acceleration of the earth towards the apple.

$$a' = \frac{F}{M} = \frac{Gm}{R^2}$$

$$= \frac{6.67\times10^{-11}\times0.25}{(6.378\times10^{6})^{2}}\,ms^{-2}$$

$$= 4.099\times10^{-25}\,ms^{-2}.$$

Example 8.

The acceleration due to gravity at the moon's surface is $1.67\ ms^{-2}$. If the radius of the moon is 1.74×10^6 m, calculate the mass of the moon. Use the known value of G.

Solution :

$$g = \frac{GM}{R^2} \text{ or } M = \frac{gR^2}{G}$$

This relation is true not only for earth but for any heavenly body which is assumed to be spherical.

Here, $g = 1.67ms^{-2}$, $R = 1.74\times10^6$ m and

$$G = 6.67\times10^{-11}\,Nm^2kg^{-2}$$

∴ Mass of moon,

$$M = \frac{1.67\times(1.74\times10^6)^2}{6.67\times10^{-11}}\,kg = 7.58\times10^{22}\,kg$$

GRAVITATIONAL FIELD

(*or Gravitational Field Intensity*)

The region or space around a body in which its gravitational influence is experienced by other bodies is the gravitational field of that body.

The gravitational field strength E_g, produced by a mass M at any point P is defined as the force exerted on the unit mass placed at that point P. Then

$$E_g = \frac{F}{m} = \left(\frac{GMm}{r^2}\right)\Big/ m = \frac{GM}{r^2} \qquad ...(i)$$

where m = test mass

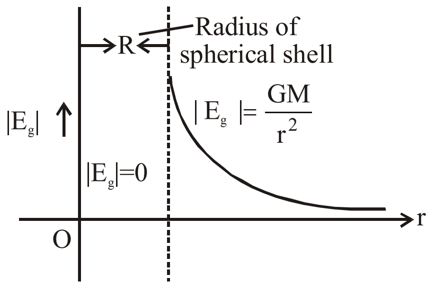
r = distance between M and m

(i) The direction of E_g always points towards the mass producing it.

(ii) The gravitational field can be represented by gravitational lines of force.

(iii) The S.I **unit** of E_g is newton/kg. Its **dimensions** are $[M^0LT^{-2}]$.

Gravitational intensity (E_g) for spherical shell



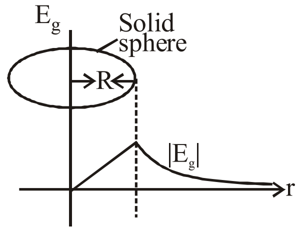
(i) $|E_g| = 0$ at points inside the spherical shell (i.e. r < R)

(ii) $|E_g| = \left|\frac{-GM}{R^2}\right|$ at the surface of shell (i.e. r = R)

(iii) $|E_g| = \left|\frac{-GM}{r^2}\right|$ for outside the spherical shell (r > R).

It is clear that as r increases, E_g decreases.

Gravitational intensity for solid sphere :



(i) $|E_g| = \left|-\frac{GM.r}{R^3}\right|$ for points inside the solid sphere (r<R)

(ii) At the surface of solid sphere (r = R) $|E_g| = \left|-\frac{GM}{R^2}\right|$

(iii) Outside the solid sphere $|E_g| = \left|-\frac{GM}{r^2}\right|$ $(r > R)$

GRAVITATIONAL POTENTIAL

Gravitational potential at any point is the work done by gravitational force in carrying a body of unit mass from infinity to that point in gravitational influence of source.

i.e., $V_A = \frac{W}{m_o} = \frac{-GM}{r}$

where r is the distance of point from source mass M.

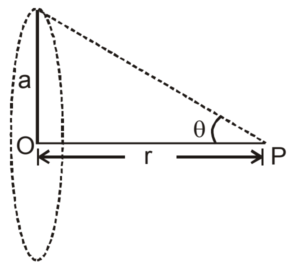
The **S.I. unit** of V is joule/kg. Its **dimensions** are $[M^0L^2T^{-2}]$. It is a scalar quantity.

Gravitational potential at the earth surface is

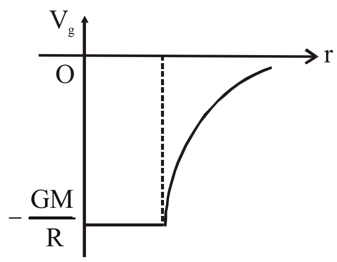
$$V_A = \frac{-GM_e}{R_e} = -gR_e$$

Gravitational potential due to a uniform ring at a point on its axis

$$V_g = -\frac{GM}{\sqrt{a^2+r^2}}$$



Gravitational potential due to a uniform thin spherical shell



(i) at a point outside the shell, $V_g = \frac{-GM}{r}$ (r > R)

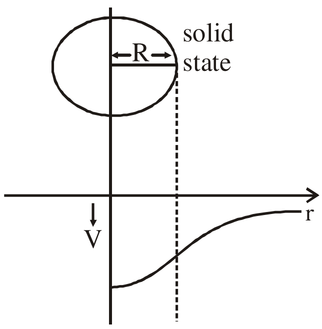
(ii) at a point inside the shell, $V_g = -\frac{GM}{R}$ (r < R)

where R = radius of spherical shell.

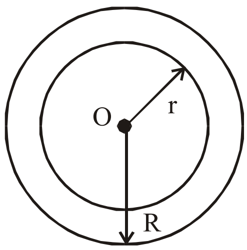
Potential due to a uniform spheral shell is constant throughout the cavity of the shell.

Gravitational potential due to a uniform solid sphere

(i) at an external point $V_g = -\frac{GM}{r}$ $(r \geq R)$



(ii) at an internal point $V_g = -\frac{GM}{2R^3}(3R^2 - r^2)$ $(r < R)$



(iii) *Potential at the centre of solid sphere is* $V = \frac{-3GM}{2R}$

GRAVITATIONAL POTENTIAL ENERGY

Let V_g be the gravitational potential at a point. If we place a mass m at that point, then we say that the gravitational potential energy possessed by the mass is

$$U_g = V_g \times m \Rightarrow U_g = -\frac{GM}{r} \times m \quad \left[\therefore V_g = -\frac{GM}{r}\right]$$

We can also therefore say that the gravitational potential energy of a system containing two masses m_1 and m_2 placed such that the centre to centre distance between them is r then

$$U_g = -\frac{Gm_1m_2}{r}$$

This formula is valid taking into consideration the fact that we have taken the gravitational potential of few at infinity. Please remember that unless otherwise stated it is understood that the gravitational potential is taken to be zero at infinity.

Relation between gravitational potential energy and gravitational potential.

Gravitational potential energy = gravitational potential × mass

Keep in Memory

1. The gravitational potential energy of a mass m at a point above the surface of the earth at a height h is given by $U_g = \frac{-GMm}{R+h}$. The –ve sign shows that if h increases, the gravitational PE decreases and becomes zero at infinity.
2. (a) If we take reference level to be at the surface of earth (not at infinity) i.e., we assume that the gravitational P.E of a mass m is zero at the surface of earth, then the gravitational potential energy at a height h above the surface of earth is (mgh), where h << R_e (radius of earth)

   (b) The gravitational P.E of mass m on the earth's surface is

   $$U_g = \frac{-GM_em}{R_e} = -mgR$$

   If we assume that gravitational P.E of mass m is zero at infinity i.e., we take reference level at infinity then P.E.of any mass m at a height h above the earth surface

   $$U_h = \frac{-GM_em}{(R_e + h)}$$

   So work done against the gravitational force, when the particle is taken from surface of earth to a height h above the earth surface

   Work done = change in potential energy

   $$\Rightarrow \quad U_h – U_g = \frac{-GM_em}{R_e + h} + \frac{GM_em}{R_e} \cong \frac{GM_em \times h}{R_e(R_e + h)}$$

   $$\approx \frac{GM_em}{R_e}\left(\frac{h}{R_e}\right) \quad \text{(if h<<R)}$$

   $$= m\left(\frac{GM_e}{R_e^2}\right)h$$

   So, work done = mgh is stored in the particle as particle potenital energy, when kept at height 'h'.

Example 9.

Four particles each of mass 1 kg are at the four corners of a square of side 1m. Find the work done to remove one of the particles to infinity.

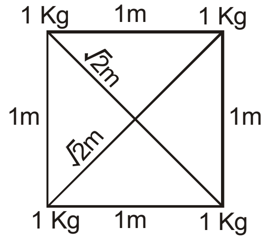
Solution :

The gravitational potential energy associated with any pair of particle of mass m_1 and m_2 separated by a distance r is

$$U = \frac{-Gm_1m_2}{r}$$

Initial P.E. of system, $-U_1 = 4.\frac{G \times 1 \times 1}{1} + \frac{2G \times 1 \times 1}{\sqrt{2}}$

or $U_1 = -G\left[\frac{4\sqrt{2} + 2}{\sqrt{2}}\right]$



When one mass is removed, then

$$\text{P.E.} = U_2 = -\left[2.\frac{G\times1\times1}{\sqrt{1}}+\frac{G\times1\times1}{\sqrt{2}}\right]$$

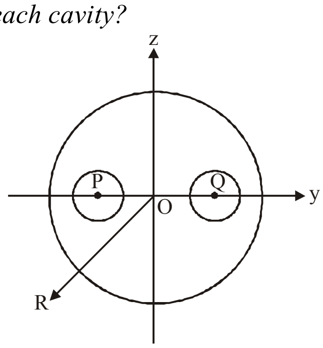
$$U_2 = -\left[\frac{2\sqrt{2}+1}{\sqrt{2}}\right]G\ ;$$

$$\therefore \text{ Work done} = G\left[\frac{4\sqrt{2}+2-2\sqrt{2}-1}{\sqrt{2}}\right] = G\left[\frac{2\sqrt{2}+1}{\sqrt{2}}\right]$$

[$\because$ Work done = change in P.E. = $-U_2-[-U_1] = U_1-U_2$]

Example 10.

A solid sphere of uniform density and mass M has a radius of 4m. Its centre is at the origin of coordinate system. Two spheres of radii 1m are taken out so that their centres are at P(0, –2, 0) and Q(0, 2, 0) respectively. This leaves two spherical cavities. What is the gravitational field at the centre of each cavity?



Solution :

If the cavities are not made, then the intensity at the point P (or Q), due to whole sphere

$$I = \frac{GM}{R^3}.x = \frac{GM}{64}\times2 = \frac{GM}{32} \quad \text{where } \vec{I} = \vec{I}_P + \vec{I}_Q + \vec{I}_R$$

$$\therefore \vec{I}_R = \vec{I} - \vec{I}_P - \vec{I}_Q \qquad \ldots\ldots(i)$$

Where I_R is the intensity of gravitational field at a point, distance x from O due to remainder sphere (exclude cavity)

Mass of big sphere $M = \dfrac{4\pi\times(4)^3 d}{3}$

Mass of small sphere P or Q, $m = \dfrac{4\pi\times(1)^3 d}{3}$ $\therefore$ $m = \dfrac{M}{64}$

At P, $I_p = 0$, $I_Q = \dfrac{Gm}{r^2} = \dfrac{G}{4^2}.\left(\dfrac{M}{64}\right)$

$$\therefore I_R = \frac{GM}{32} - \frac{GM}{1024} \text{ [From eq (i)]}$$

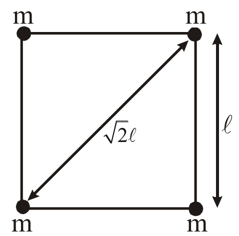
$$\therefore I_R = \frac{31GM}{1024}$$

Example 11.

Find the gravitational potential energy of a system of four particles, each having mass m, placed at the vertices of a square of side l. Also obtain the gravitational potential at the centre of the square.

Solution :

The system has four pairs with distance ℓ and two diagonal pairs with distance $\sqrt{2}\,\ell$.



$$\therefore \quad U = -4\frac{Gm^2}{\ell} - 2\frac{Gm^2}{\sqrt{2}\ell} = -\frac{2Gm^2}{\ell}\left(2+\frac{1}{\sqrt{2}}\right) = -5.41\frac{Gm^2}{\ell}$$

The gravitational potential at the centre of the square is $\left(r = \sqrt{2}\ell/2\right)$

V = Algebraic sum of potential due to each particle

$$\Rightarrow \quad V = -\frac{4\sqrt{2}Gm}{\ell}$$

Example 12.

The radius of earth is R. Find the work done in raising a body of mass m from the surface of earth to a height R/2.

Solution :

If a body of mass m is placed at a distance x from the centre of earth, the gravitational force of attraction F between the body and the earth is $F = \dfrac{GM\,m}{x^2}$

Small amount of work done in raising a body through a small distance dx is given by

$$dW = F\,dx = \frac{GM\,m}{x^2}dx$$

Total work done in raising the body from the surface of earth to a height R/2 is given by

$$W = \int_R^{R+R/2} \frac{GM\,m}{x^2}dx$$

$$= GMm\int_R^{3R/2} x^{-2}\,dx = -GMm\left(\frac{1}{x}\right)_R^{3R/2}$$

$$= -GMm\left[\frac{2}{3R}-\frac{1}{R}\right] = \frac{GM\,m}{3R} = \frac{gR^2m}{3R}$$

$$= \frac{1}{3}mg\,R \quad [\because GM = gR^2]$$

SATELLITES

A **satellite** *is any body revolving around a large body under the gravitational influence of the latter.*

The period of motion T of an artificial satellite of earth at a distance h above the surface of the earth is given by,

$$T = 2\pi\sqrt{\frac{(R_e+h)^3}{gR_e^2}}$$

where, R_e = radius of the earth

g = acceleration due to gravity on the surface of the earth.

If $R_e >> h$, then $T = 2\pi\sqrt{\frac{R_e}{g}}$; $R_e = 6.4 \times 10^6$ m ;

$g = 9.8\ ms^{-2}$

i.e. $T \simeq 84.58$ minutes ≈ 5075 sec

Assuming $R_e + h = r$, the distance of the satellite from the centre of the earth, $T \propto (r)^{3/2}$

Orbital velocity (v_0)

Let a satellite of mass m revolve around the earth in circular orbit of radius r with speed v_0. The gravitational pull between satellite and earth provides the necessary centripetal force.

Centripetal force required for the motion = $\frac{mv_0^2}{r}$

Gravitational force = $\frac{GMm}{r^2}$

$$\frac{mv_0^2}{r} = \frac{GMm}{r^2} \quad \text{or} \quad v_0^2 = \frac{GM}{r}$$

$$\text{or } v_0 = \sqrt{\frac{GM}{r}} \quad \text{..........(1)}$$

$$\text{or} \quad v_0 = R\sqrt{\frac{g}{R+h}} \quad \left[\because g = \frac{GM}{R^2} \text{ and } r = (R+h)\right]$$

Orbital velocity, $Vo = \sqrt{\frac{GM_e}{R_e}} = \sqrt{gR_e}$

(i) Value of orbital velocity does not depend on the mass of the satellite.
(ii) Around the earth the value of orbital velocity is **7.92 km/sec**.
(iii) Greater is the height of the satellite, smaller is the orbital velocity.
(iv) The direction of orbital velocity is along the tangent to the path.
(v) The work done by the satellite in a complete orbit is zero.

Angular momentum (L) : For satellite motion, angular momentum will be given by

$$L = mvr = mr\sqrt{\frac{GM}{r}} \quad \text{i.e., } L = [m^2GMr]^{1/2}$$

Angular momentum of a satellite depends on both, the mass of orbiting and central body. It also depends on the radius of the orbit.

Energy of a satellite

Total energy of satellite revolving in an orbit of radius r around the earth can be calculated as follows :

(i) The **gravitational potential energy** of a satellite of mass m is $U_g = -\frac{GMm}{r}$, where r is the radius of the orbit.
(ii) **Kinetic energy** of the satellite is $E_k = \frac{1}{2}mv_0^2 = \frac{GMm}{2r}$
(iii) So, **total energy** of the satellite $E = U_g + E_k = -\frac{GMm}{2r}$
The negative sign shows that satellite cannot leave the orbit itself. It requires an energy equal to $\frac{GMm}{2R}$, which is called **Binding Energy (B.E.)** of satellite.
(iv) Total energy of a satellite at a height equal to the radius of the earth
$$= -\frac{GMm}{2(R+R)} = -\frac{GMm}{4R} = \frac{1}{4}mgR$$

GEO-STATIONARY SATELLITE

A satelite which appears to be stationary for a person on the surface of the earth is called geostationary satellite.

It is also known as **parking satellite** or **synchronous satellite.**

(i) The orbit of the satellite must be circular and in the equatorial plane of the earth.
(ii) The angular velocity of the satellite must be in the same direction as the angular velocity of rotation of the earth i.e., from west to east.
(iii) The period of revolution of the satellite must be equal to the period of rotation of earth about its axis.
i.e. 24 hours = 24 × 60 × 60 = 86400 sec.

$$T^2 = \frac{4\pi^2}{GM}r^3 \quad \text{or} \quad r = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}} = \left(\frac{GM}{R^2}\times\frac{R^2T^2}{4\pi^2}\right)^{\frac{1}{3}}$$

$$= \left[9.8\times(6.38\times10^6)^2\times\frac{(86400)^2}{4\pi^2}\right]^{\frac{1}{3}}$$

$= 42237 \times 10^3$ m = 42,237 km. ≈ 42000 km.

h = r – R = 42000 – 6400 = 35600 km.

(a) Height of geostationary satellite from the surface of the earth is nearly 35600 km.
(b) The orbital velocity of this satellite is nearly 3.08 km/sec.
(c) The relative velocity of geostationary satellite with respect to earth is zero.

This type of satellite is used for communication purposes. The orbit of a geostationary satellite is called 'Parking Orbit'.

Polar Satellite :

Polar satellites travel around the earth in an orbit that travels around the earth over the poles. The earth rotates on its axis as the satellite goes around the earth. Thus over a period of many orbits it looks down on every part of the earth.

Different orbital shapes corresponding to different velocities of a satellite :

[1] When $v < v_0$

(i) The path is spiral. The satellite finally falls on the earth

(ii) Kinetic energy is less then potential energy

(iii) Total energy is negative

[2] When $v = v_0$

(i) The path is circular

(ii) Eccentricity is zero

(iii) Kinetic energy is less than potential energy

(iv) Total energy is negative

[3] When $v_0 < v < v_e$

(i) The path is elliptical

(ii) Eccentricity < 1

(iii) Kinetic energy is less than potential energy

(iv) Total energy is negative

[4] When $v = v_e$

(i) The path is a parabola

(ii) Eccentricity = 1

(iii) Kinetic energy is equal to potential energy

(iv) Total energy is zero

[5] When $v > v_e$

(i) The path is a hyperbola

(ii) Eccentricity > 1

(iii) Kinetic energy is greater than potential energy

(iv) Total energy is positive

If the orbit of a satellite is elliptical

(1) The energy $E = -\frac{GMm}{2a}$ = const. with 'a' as semi-major axis;

(2) KE will be maximum when the satellite is closest to the central body (at perigee) and minimum when it is farthest from the central body (at apogee) [as for a given orbit L = const., i.e., mvr = const., i.e., $v \propto 1/r$]

(3) PE = (E – KE) will be minimum when KE = max, i.e., the satellite is closest to the central body (at perigee) and maximum when KE = min, i.e., the satellite is farthest from the central body (at apogee).

ESCAPE SPEED (V_e)

It is the minimum speed with which a body should be projected from the surface of a planet so as to reach at infinity i.e., beyond the gravitational field of the planet.

If a body of mass m is projected with speed v from the surface of a planet of mass M and radius R, then

$$\text{as } K = \frac{1}{2}mv^2 \text{ and } U = -\frac{GMm}{R},\ E_S = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

Now if v' is the speed of body at ∞, then

$$E_\infty = \frac{1}{2}m(v')^2 + 0 = \frac{1}{2}m(v')^2 \quad [\text{as } U_\infty = 0]$$

So by conservation of energy

$$= \frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}m(v')^2$$

$$\text{i.e. } \frac{1}{2}mv^2 = \frac{GMm}{R} + \frac{1}{2}m(v')^2$$

so v will be minimum when $v' \to 0$,

$$\text{i.e. } v_e = v_{\min} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} \qquad \left[\text{as } g = \frac{GM}{R^2}\right]$$

- The value of escape velocity does not depend upon the mass of the projected body, instead it depends on the mass and radius of the planet from which it is being projected.
- The value of escape velocity does not depend on the angle and direction of projection.
- The value of escape velocity from the surface of the earth is **11.2** km/sec.
- The minimum energy needed for escape is = GMm/R.
- If the velocity of a satellite orbiting near the surface of the earth is increased by 41.4%, then it will escape away from the gravitational field of the earth.
- If a body falls freely from infinite distance, then it will reach the surface of earth with a velocity of 11.2 km/sec.

Relation between orbital velocity (V_0) and escape speed (V_e)

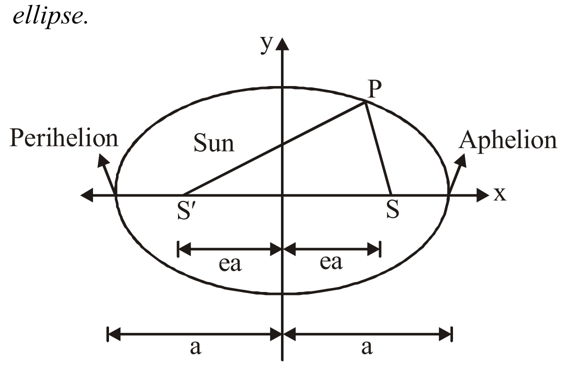
$$V_e = \sqrt{2gR} = \sqrt{2}\, V_0$$

Keep in Memory

1. The escape velocity on moon is low $\left(as\ g_m = \frac{g_E}{6}\right)$ hence there is no atmosphere on moon.
2. If the orbital radius of the earth around the sun be one fourth of the present value, then the duration of the year will be one eighth of the present value.
3. The satellites revolve around the earth in a plane that coincides with the great circle around the earth.

KEPLER'S LAWS OF PLANETARY MOTION

1. **The law of orbits :** *Each planet revolves about the sun in an elliptical orbit with the sun at one of the foci of the ellipse.*



a = Semi-major axis

A planet of mass m moving in an elliptical orbit around the sun. The sun of mass M, is at one focus S′ of the ellipse. The other focus is S, which is located in empty space. Each focus is at distance 'ea' from the ellipse's centre, with 'e' being the **eccentricity** of the ellipse and 'a' semimajor axis of the ellipse, the perihelion (nearest to the sun) distance $r_{min.}$, and the aphelion (farthest from the sun) distance $r_{max.}$ are also shown.

$r_{max} = a + e\,a = (1 + e)\,a$

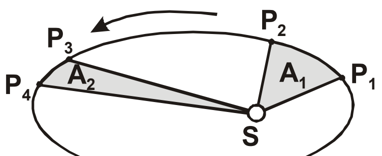
$r_{min} = a - e\,a = (1 - e)\,a$

The distance of each focus from the centre of ellipse is ea, where e is the dimensionless number between 0 to 1 called the eccentricity. If e = 0, the ellipse is a circle.

For earth e = 0.017.

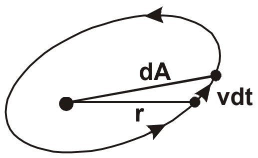
2. ***Law of areas :*** *An imaginary line that connects a planet to the sun sweeps out equal areas in the plane of the planet's orbit in equal times;*

i.e., the rate dA/dt at which it sweeps out area A is constant.



If $t_{P_1P_2} = t_{P_3P_4}$ then $A_1 = A_2$

$$\frac{dA}{dt} = \frac{1}{2}\frac{(r)(v\ dt)}{dt} = \frac{1}{2}rv \quad \text{and as } L = mvr$$



so $\dfrac{dA}{dt} = \dfrac{L}{2m}$ ....(1)

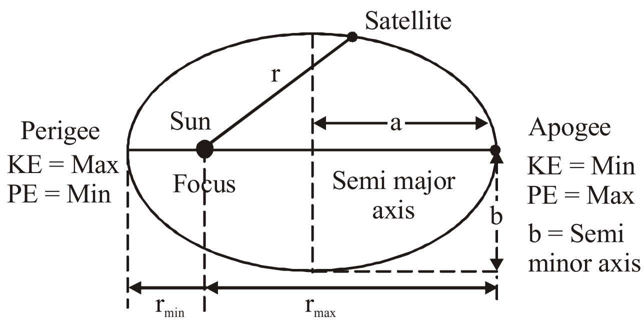
But as L = constt., (force is central, so torque = 0 and hence angular momentum of the planet is conserved)

areal velocity (dA/dt) = constant which is Kepler's IInd law, i.e., Kepler's IInd law or constancy of areal velocity is a consequence of conservation of angular momentum.

3. ***Law of periods :*** *The square of the period of revolution of any planet is proportional to the cube of the semi-major axis of the orbit,*

i.e., $T^2 \propto r^3$.

or, $\dfrac{T_1^2}{T_2^2} = \dfrac{r_1^3}{r_2^3}$



WEIGHTLESSNESS

The "weightlessness" you may feel in an aircraft occurs any time the aircraft is accelerating downward with acceleration g. It is possible to experience weightlessness for a considerable length of time by turning the nose of the craft upward and cutting power so that it travels in a ballistic trajectory. A ballistic trajectory is the common type of trajectory you get by throwing a rock or a baseball, neglecting air friction. At every point on the trajectory, the acceleration is equal to g downward since there is no support. A considerable amount of experimentation has been done with such ballistic trajectories to practice for orbital missions where you experience weightlessness all the time.

The satellite is moving in a circular orbit, it has a radial acceleration

$$a = \frac{v_0^2}{r} = \frac{GM}{r^2} \qquad \left[\text{as } v_0 = \sqrt{\left(\frac{GM}{r}\right)}\right]$$

i.e., it is falling towards earth's centre with acceleration a,

so apparent weight of the body in it $W_{ap} = m(g' - a)$

where g′ is the acceleration due to gravity of earth at the position (height) of satellite, i.e. $g' = (GM/r^2)$, so that

$$W_{ap} = m\left[\frac{GM}{r^2} - \frac{GM}{r^2}\right] = 0$$

i.e., the apparent weight of a body in a satellite is zero and is independent of the radius of the orbit.

Keep in Memory

1. The moon takes 27.3 days to revolve around the earth. The radius of its orbit is 3.85×10^5 km.
2. Kepler's second law is based on conservation of angular momentum.
3. Perihelion distance is the shortest distance between the sun and the planet.
4. Aphelion distance is the largest distance between the Sun and the planet.

$$\frac{V_{aphelion}}{V_{perihelion}} = \frac{r_{perihelion}}{r_{aphelion}}$$

5. If e is the eccentricity of the orbit

then $\dfrac{1+e}{1-e} = \dfrac{r_{aphelion}}{r_{perihelion}}$

$r_{aphelion} + r_{perihelion} = 2r$

6. If e > 1 and total energy (KE + PE) > 0, the path of the satellite is hyperbolic and it escapes from its orbit.
7. If e < 1 and total energy is negative it moves in elliptical path.
8. If e = 0 and total energy is negative it moves in circular path.
9. If e = 0 and total energy is zero it will take parabolic path.
10. The path of the projectiles thrown to lower heights is parabolic and thrown to greater heights is elliptical.
11. Kepler's laws may be applied to natural and artificial satellites as well.
12. Gravitational force does not depend upon medium so no medium can shield it or block it.
13. The escape velocity and the orbital velocity are independent of the mass of the body being escaped or put into the orbit.

Example 13.

If the earth is at 1/4 of its present distance from the sun, what would be the duration of the year?

Solution :

We know that, $\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$

Substituting the given values, we get

$$\frac{(1)^2}{T_2^2} = \frac{R^3}{\left(\frac{R}{4}\right)^3} \text{ or } \frac{1}{T_2^2} = (4)^3 = 64$$

$$T_2^2 = \frac{1}{64} \text{ or } T_2 = \frac{1}{8} \text{ year.}$$

Example 14.

Two satellites A and B go round a planet P in a circular orbits having radii 4R and R respectively. If the speed of satellite A is 3v, what will be the speed of satellite B?

Solution :

We know that $v = \sqrt{(GM/r)}$

Here $3v = \sqrt{\left(\frac{GM}{4R}\right)}$ and $v' = \sqrt{\left(\frac{GM}{R}\right)}$;

$$\therefore \frac{v'}{3v} = \sqrt{\left(\frac{GM}{R}\right)} \times \sqrt{\left(\frac{4R}{GM}\right)} = 2\,;\ v' = 6v$$

Example 15.

A spaceship is launched into a circular orbit close to earth's surface. What additional velocity has now to be imparted to the spaceship in the orbit to overcome the gravitational pull? Radius of earth = 6400 km and g = 9.8 m/s^2.

Solution :

The orbital velocity of spaceship in circular orbit

$$v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R}}$$

($\because$ spaceship is very close to earth, r = R)

As $g = \frac{GM}{R^2}$ hence $GM = gR^2$

$$\therefore v_0 = \sqrt{(Rg)} = \sqrt{(6.4 \times 10^6) \times (9.8)} = 7.9195 \text{ km/s}$$

Further, $v_e = \sqrt{2Rg} = \sqrt{(2 \times 7.9195)} = 11.2$ km/s

(where v_e is escape velocity)

Additional velocity required = 11.2000 – 7.0195
= 3.2805 km/s

So the velocity 3.2805 km/s must be added to orbital velocity of spaceship.

CONCEPT MAP

GRAVITATION
Force of attraction between any two bodies

Kepler's laws of planetary motion
(i) **Law of orbit :**
Every planet revolve around the Sun in an elliptical orbit and Sun is at its one focus
(ii) **Law of area :** The radius vector drawn from the Sun to a planet sweeps out equal areas in equal intervals of time
i.e., $\frac{dA}{dt} = \frac{L}{2m}$ = Constant
(iii) **Law of periods :**
Time period, $T^2 \propto a^3$
a = semi-major axis

Gravitational field intensity
$I = \frac{GM}{r^2}$
Gravitational potential
$V_g = \frac{\text{work done}}{\text{mass}} = \frac{-GM}{r}$
Gravitational Potential energy
$U = \frac{-GMm}{r}$

Characteristics of Gravitational force
(i) Weakest force in nature
(ii) Central as well as conservative
(iii) Always attractive nature
(iv) Applicable for all bodies irrespective of their size, shape and position

Newton's universal law of Gravitation
$F = \frac{Gm_1m_2}{r^2}, G = 6.67 \times 10^{-11} Nm^2kg^{-2}$

Satellite : A heavenly object revolves around a planet

Geostationary satellite :
Height from earth's surface = 36000 km
Radius of orbit = 42,400 km
Time period = 24 hours
Orbital velocity = 3.1 km/s

Polar satellite
Height from earth's surface = 880 km
Time period T = 84 min
Orbital velocity = 7.92 km/s

Variation in acceleration due to gravity (g)
(i) with height $g' = g\left(1 - \frac{2h}{R_E}\right)$
i.e., g decreases with height
(ii) with depth $g' = g\left(1 - \frac{d}{R_E}\right)$
i.e., g decreases with depth
(iii) With rotation of earth about its own axis
$g' = g - R\omega^2 \cos^2 \lambda$
At poles $\lambda = 90°$, g maximum
At equator $\lambda = 0°$, g minimum

Acceleration due to gravity (g)
Acceleration produced in a freely falling object due to earth's gravitational pull. $g = \frac{gM_E}{R_E^2}$

Escape speed $V_{esc} = \sqrt{2GM/R} = \sqrt{2gR}$
At earth $V_{esc} = 11.2$ km/s
Orbital speed $V_{orb} = \sqrt{gR} = \frac{V_e}{\sqrt{2}}$
Total energy of a satellite
E = KE + PE
$= \left(\frac{GMm}{2r}\right) + \left(\frac{-GMm}{r}\right)$

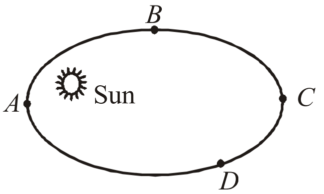
Weightlessness
During freely falling under gravity inside a spacecraft or satellite a body is weightless i.e., effective weight becomes zero

EXERCISE - 1
Conceptual Questions

1. A satellite is orbiting around the earth near its surface. If its kinetic energy is doubled, then
(a) it will remain in the same orbit.
(b) it will fall on the earth.
(c) it will revolve with greater speed.
(d) it will escape out of the gravitational field of the earth.

2. There is no atmosphere on the moon because
(a) it is closer to the earth and also it has the inactive inert gases in it.
(b) it is too for from the sun and has very low pressure in its outer surface.
(c) escape velocity of gas molecules is greater than their root mean square velocity.
(d) escape velocity of gas molecules is less than their root mean square velocity.

3. The maximum kinetic energy of a planet moving around the sun is at a position

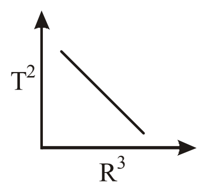


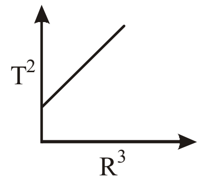
(a) A (b) B
(c) C (d) D

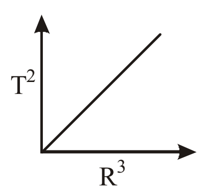
4. A man waves his arms while walking. This is to
(a) keep constant velocity
(b) ease the tension
(c) increase the velocity
(d) balance the effect of earth's gravity

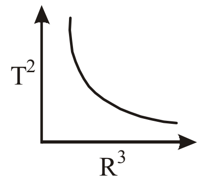
5. A missile is launched with a velocity less than escape velocity. The sum of its kinetic and potential energies is
(a) zero
(b) negative
(c) positive
(d) may be positive, negative or zero.

6. Which of the following graphs represents the motion of a planet moving about the sun ?

(a)


(b)


(c)


(d)


7. Due to rotation of the earth the acceleration due to gravity g is
(a) maximum at the equator and minimum at the poles
(b) minimum at the equator and maximum at the poles
(c) same at both places
(d) None of these

8. A planet moves around the sun. At a point P it is closest from the sun at a distance d_1 and has a speed v_1. At another point Q, when it is farthest from the sun at a distnace d_2 its speed will be
(a) $d_1^2 v_1 / d_2^2$ (b) $d_2 v_1 / d_1$
(c) $d_1 v_1 / d_2$ (d) $d_2^2 v_1 / d_1^2$

9. The weight of an object in the coal mine, sea level and at the top of the mountain, are respectively W_1, W_2 and W_3 then
(a) $W_1 < W_2 > W_3$ (b) $W_1 = W_2 = W_3$
(c) $W_1 < W_2 < W_3$ (d) $W_1 > W_2 > W_3$

10. Two planets of radii r_1 and r_2 are made from the same material. The ratio of the acceleration due to gravity g_1/g_2 at the surfaces of the two planets is
(a) r_1/r_2 (b) r_2/r_1
(c) $(r_1/r_2)^2$ (d) $(r_2/r_1)^2$

11. What would be the length of a sec. pendulum at a planet (where acc. due to gravity is g/4) if it's length on earth is ℓ
(a) $\ell/2$ (b) 2ℓ
(c) $\ell/4$ (d) 4ℓ

12. Time period of a simple pendulum inside a satellite orbiting earth is
(a) zero (b) ∞
(c) T (d) 2T

13. The ratio of the radii of the planets R_1 and R_2 is k. The ratio of the acceleration due to gravity is r. The ratio of the escape velocities from them will be
(a) kr (b) $\sqrt{kr}$
(c) $\sqrt{(k/r)}$ (d) $\sqrt{(r/k)}$

14. If v_e and v_0 represent the escape velocity and orbital velocity of a satellite corresponding to a circular orbit of radius R, then
(a) $v_e = v_o$ (b) $v_e = \sqrt{2}\, v_o$
(c) $v_e = (1/\sqrt{2})\, v_o$ (d) v_e and v_o are not related

15. The kinetic energy needed to project a body of mass *m* from the earth surface (radius *R*) to infinity is
(a) mgR/2 (b) 2mgR
(c) mgR (d) mgR/4.

16. The escape velocity of a body depends upon mass as
(a) m^0 (b) m^1
(c) m^2 (d) m^3.

17. The radius of a planet is $1/4^{th}$ of R_e and its acc. due to gravity is 2g. What would be the value of escape velocity on the planet, if escape velocity on earth is v_e.
(a) $\frac{v_e}{\sqrt{2}}$ (b) $v_e\sqrt{2}$
(c) $2\,v_e$ (d) $\frac{v_e}{2}$

18. If the gravitational force had varied as $r^{-5/2}$ instead of r^{-2}; the potential energy of a particle at a distance 'r' from the centre of the earth would be proportional to
(a) r^{-1} (b) r^{-2}
(c) $r^{-3/2}$ (d) $r^{-5/2}$

19. Two satellites revolve round the earth with orbital radii 4R and 16R, if the time period of first satellite is T then that of the other is
(a) 4 T (b) $4^{2/3}$ T
(c) 8 T (d) None of these

20. A planet revolves in an elliptical orbit around the sun. The semi-major and semi-minor axes are a and b. Then the square of time period, T is directly proportional to
(a) a^3 (b) b^3
(c) $\left(\frac{a+b}{2}\right)^3$ (d) $\left(\frac{a-b}{2}\right)^3$

21. Which of the following quantities do not depend upon the orbital radius of the satellite ?
(a) $\frac{T}{R}$ (b) $\frac{T^2}{R}$
(b) $\frac{T^2}{R^2}$ (d) $\frac{T^2}{R^3}$

22. The orbital velocity of an artificial satellite in a circular orbit very close to Earth is v. The velocity of a geosynchronous satellite orbiting in a circular orbit at an altitude of 6R from Earth's surface will be
(a) $\frac{v}{\sqrt{7}}$ (b) $\frac{v}{\sqrt{6}}$
(c) v (d) $\sqrt{6}v$

23. Escape velocity when a body of mass m is thrown vertically from the surface of the earth is v, what will be the escape velocity of another body of mass 4 m is thrown vertically
(a) v (b) 2v
(c) 4v (d) None of these

24. The potential energy of a satellite of mass m and revolving at a height R_e above the surface of earth where R_e = radius of earth, is
(a) $-m\,g\,R_e$ (b) $\frac{-m\,g\,R_e}{2}$
(c) $\frac{-m\,g\,R_e}{3}$ (d) $\frac{-m\,g\,R_e}{4}$

25. Energy required to move a body of mass m from an orbit of radius 2R to 3R is
(a) $GMm/12R^2$ (b) $GMm/3R^2$
(c) $GMm/8R$ (d) $GMm/6R$

EXERCISE - 2
Applied Questions

1. The escape velocity from the earth's surface is 11 km/s. The escape velocity from a planet having twice the radius and same mean density as that of earth is
(a) 5.5 km/s (b) 11 km/s
(c) 22 km/s (d) None of these

2. Two point masses each equal to 1 kg attract one another with a force of 10^{-10} N. The distance between the two point masses is (G = 6.6×10^{-11} MKS units)
(a) 8 cm (b) 0.8 cm
(c) 80 cm (d) 0.08 cm

3. There are two bodies of masses 10^3 kg and 10^5 kg separated by a distance of 1 km. At what distance from the smaller body, the intensity of gravitational field will be zero
(a) 1/9 km (b) 1/10 km
(c) 1/11 km (d) 10/11 km

4. Taking the gravitational potential at a point infinte distance away as zero, the gravitational potential at a point A is –5 unit. If the gravitational potential at point infinite distance away is taken as + 10 units, the potential at point A is
(a) – 5 unit (b) + 5 unit
(c) + 10 unit (d) + 15 unit

5. A planet of mass 3×10^{29} gm moves around a star with a constant speed of 2×10^6 ms^{-1} in a circle of radii 1.5×10^{14} cm. The gravitational force exerted on the planet by the star is
(a) 6.67×10^{22} dyne (d) 8×10^{27} Newton
(c) 8×10^{26} N (d) 6.67×10^{19} dyne

6. The mass of the moon is 1/81 of earth's mass and its radius 1/4 that of the earth. If the escape velocity from the earth's surface is 11.2 km/sec, its value from the surface of the moon will be
(a) 0.14 kms^{-1} (b) 0.5 kms^{-1}
(c) 2.5 kms^{-1} (d) 5.0 kms^{-1}

7. If the mass of earth is eighty times the mass of a planet and diameter of the planet is one fourth that of earth, then acceleration due to gravity on the planet would be
(a) 7.8 m/s^2 (b) 9.8 m/s^2
(c) 6.8 m/s^2 (d) 2.0 m/s^2

8. Two bodies of masses 10 kg and 100 kg are separated by a distance of 2m ($G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}). The gravitational potential at the mid point on the line joining the two is

(a) 7.3×10^{-7} J/kg (b) 7.3×10^{-9} J/kg
(c) -7.3×10^{-9} J/kg (d) 7.3×10^{-6} J/kg

9. The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become

(a) 10 hours (b) 80 hours
(c) 40 hours (d) 20 hours

10. At sea level, a body will have minimum weight at

(a) pole (b) equator
(c) 42° south latitude (d) 37° north latitude

11. The distance of neptune and saturn from the sun is nearly 10^{13} and 10^{12} meter respectively. Assuming that they move in circular orbits, their periodic times will be in the ratio

(a) 10 (b) 100
(c) $10\sqrt{10}$ (d) 1000

12. A geostationary satellite is orbiting the earth at a height of 5*R* above that surface of the earth, *R* being the radius of the earth. The time period of another satellite in hours at a height of 2*R* from the surface of the earth is

(a) 5 (b) 10
(c) $6\sqrt{2}$ (d) $\frac{6}{\sqrt{2}}$

13. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45º with the vertical, the escape velocity will be

(a) 22 km/s (b) 11 km/s
(c) $\frac{11}{\sqrt{2}}$ km/s (d) $11\sqrt{2}$ km/s

14. If the length of a simple pendulum is increased by 2%, then the time period

(a) increases by 2% (b) decreases by 2%
(c) increases by 1% (d) decreases by 1%

15. The kinetic energy of a satellite in its orbit around the earth is E. What should be the kinetic energy of the satellite so as to enable it to escape from the gravitational pull of the earth?

(a) 4E (b) 2E
(c) $\sqrt{2}$ E (d) E

16. The time period of a satellite in a circular orbit of radius R is T, the period of another satellite in a circular orbit of radius 4 R is

(a) 4 T (b) T/4
(c) 8 T (d) T/8

17. If the change in the value of g at the height h above the surface of the earth is the same as at a depth 'x' below it, then (both x and h being much smaller than the radius of the earth)

(a) x = h (b) x = 2 h
(c) x = h/2 (d) $x = h^2$

18. A satellite of mass m revolves around the earth of radius R at a height 'x' from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is

(a) $\frac{gR^2}{R+x}$ (b) $\frac{gR}{R-x}$
(c) gx (d) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

19. The time period of an earth satellite in circular orbit is independent of

(a) both the mass and radius of the orbit
(b) radius of its orbit
(c) the mass of the satellite
(d) neither the mass of the satellite nor the radius of its orbit.

20. Suppose the gravitational force varies inversely as the nth power of distance. Then the time period of a planet in circular orbit of radius 'R' around the sun will be proportional to

(a) R^n (b) $R^{\left(\frac{n-1}{2}\right)}$
(c) $R^{\left(\frac{n+1}{2}\right)}$ (d) $R^{\left(\frac{n-2}{2}\right)}$

21. An earth satellite of mass m revolves in a circular orbit at a height h from the surface of the earth. R is the radius of the earth and g is acceleration due to gravity at the surface of the earth. The velocity of the satellite in the orbit is given by

(a) $g R^2/(R+h)$
(b) g R
(c) g R/(R – h)
(d) $\sqrt{[gR^2/(R+h)]}$

22. Mass of the Earth has been determined through

(a) use of Kepler's $\frac{T^2}{R^3}$ constancy law and Moon's period
(b) sampling the density of Earth's crust and using Earth's radius
(c) Cavendish's determination of G and using Earth radius and g at its surface
(d) use of periods of satellites at different heights above Earth's surface and known radius of Earth

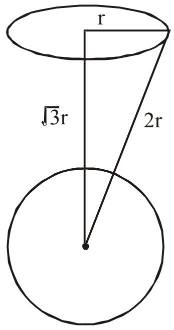
23. Consider Earth to be a homogeneous sphere. Scientist A goes deep down in a mine and scientist B goes high up in a balloon. The gravitational field measured by

(a) A goes on decreasing and that by B goes on increasing
(b) B goes on decreasing and that by A goes on increasing
(c) each decreases at the same rate
(d) each decreases at different rates

24. There are _______ gravitational lines of force inside a spherically symmetric shell.
(a) infinitely many
(b) zero
(c) varying number depending upon surface area
(d) varying number depending upon volume

25. A uniform ring of mass m and radius r is placed directly above a uniform sphere of mass M and of equal radius. The centre of the ring is directly above the centre of the sphere at a distance $r\sqrt{3}$ as shown in the figure.

The gravitational force exerted by the sphere on the ring will be



(a) $\frac{GMm}{8r^2}$
(b) $\frac{GMm}{4r^2}$
(c) $\sqrt{3}\frac{GMm}{8r^2}$
(d) $\frac{GMm}{8r^3\sqrt{3}}$

26. The gravitational potential difference between the surface of a planet and a point 20 m above the surface is 2 joule/kg. If the gravitational field is uniform, then the work done in carrying a 5 kg body to a height of 4 m above the surface is
(a) 2 J (b) 20 J
(c) 40 J (d) 10 J

27. The ratio of the kinetic energy required to be given to a satellite so that it escapes the gravitational field of Earth to the kinetic energy required to put the satellite in a circular orbit just above the free surface of Earth is
(a) 1 (b) 2
(c) 3 (d) 9

28. The radii of two planets are respectively R_1 and R_2 and their densities are respectively ρ_1 and ρ_2. The ratio of the accelerations due to gravity at their surfaces is
(a) $g_1 : g_2 = \frac{\rho_1}{R_1^2} : \frac{\rho_2}{R_2^2}$
(b) $g_1 : g_2 = R_1R_2 : \rho_1\rho_2$
(c) $g_1 : g_2 = R_1\rho_2 : R_2\rho_1$
(d) $g_1 : g_2 = R_1\rho_1 : R_2\rho_2$

29. An artificial satellite moving in a circular orbit around the earth has a total (K.E. + P.E.) is E_0. Its potential energy is –
(a) $-E_0$ (b) $1.5\,E_0$
(c) $2E_0$ (d) E_0

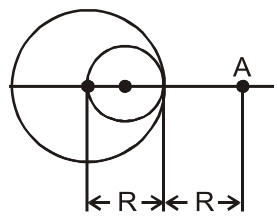
30. If value of acceleration due to gravity changes from one place to another, which of the following forces will undergo a change ?
(a) Viscous force (b) Buoyant force
(c) Magnetic force (d) All of the above

31. The amount of work done in lifting a mass 'm' from the surface of the earth to a height 2R is
(a) 2mgR (b) 3mgR
(c) $\frac{3}{2}$ mgR (d) $\frac{2}{3}$ mgR

32. The radius of the earth is 4 times that of the moon and its mass is 80 times that of the moon. If the acceleration due to gravity on the surface of the earth is 10 m/s^2, then on the surface of the moon its value will be
(a) 1 ms^{-2} (b) 2 ms^{-2}
(c) 3 ms^{-2} (d) 4 ms^{-2}

33. A satellite of mass 'm', moving around the earth in a circular orbit of radius R, has angular momentum L. The areal velocity of satellite is (M_e = mass of earth)
(a) L/2m (b) L/m
(c) 2L/m (d) $2L/M_e$

34. A solid sphere of uniform density and radius R applies a gravitational force of attraction equal to F_1 on a particle placed at A, distance 2R from the centre of the sphere.



A spherical cavity of radius R/2 is now made in the sphere as shown in the figure. The sphere with cavity now applies a gravitational force F_2 on the same particle placed at A. The ratio F_2/F_1 will be
(a) 1/2 (b) 3
(c) 7 (d) 1/9

35. A body starts from rest from a point distance R_0 from the centre of the earth. The velocity acquired by the body when it reaches the surface of the earth will be (R represents radius of the earth).
(a) $2GM\left(\frac{1}{R}-\frac{1}{R_0}\right)$ (b) $\sqrt{2GM\left(\frac{1}{R_0}-\frac{1}{R}\right)}$
(c) $GM\left(\frac{1}{R}-\frac{1}{R_0}\right)$ (d) $2GM\sqrt{\left(\frac{1}{R}-\frac{1}{R_0}\right)}$

36. The largest and the shortest distance of the earth from the sun are r_1 and r_2. Its distance from the sun when it is at perpendicular to the major-axis of the orbit drawn from the sun
(a) $(r_1 + r_2)/4$ (b) $(r_1 + r_2)/(r_1 - r_2)$
(c) $2r_1r_2/(r_1 + r_2)$ (d) $(r_1 + r_2)/3$

37. The orbital velocity of an artificial satellite in a circular orbit just above the centre's surface is υ. For a satellite orbiting at an altitude of half of the earth's radius, the orbital velocity is

(a) $\left(\sqrt{\left(\frac{2}{3}\right)}\right)v_0$ (b) $\frac{2}{3}v_0$

(c) $\frac{3}{2}v_0$ (d) $\sqrt{\left(\frac{3}{2}\right)}v_0$

38. A planet is revolving around the sun in an elliptical orbit. Its closests distance from the sun is r_{min}. The farthest distance from the sun is r_{max}. If the orbital angular velocity of the planet when it is nearest to the sun is ω, then the orbital angular velocity at the point when it is at the farthest distance from the sun is

(a) $\sqrt{(r_{min}/r_{max})}\,\omega$ (b) $\sqrt{(r_{max}/r_{min})}\,\omega$

(c) $(r_{max}^2/r_{min}^2)\omega$ (d) $(r_{min}^2/r_{max}^2)\omega$

39. Two spherical bodies of mass M and 5M and radii R and 2R respectively are released in free space with initial separation between their centres equal to 12 R. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is

(a) 2.5 R (b) 4.5 R
(c) 7.5 R (d) 1.5 R

40. If the radius of the earth were to shrink by one per cent, its mass remaining the same, the acceleration due to gravity on the earth's surface would

(a) decrease (b) remain unchanged
(c) increase (d) None of these

41. If earth is supposed to be a sphere of radius R, if g_{30} is value of acceleration due to gravity at lattitude of 30° and g at the equator, the value of $g - g_{30}$ is

(a) $\frac{1}{4}\omega^2 R$ (b) $\frac{3}{4}\omega^2 R$

(c) $\omega^2 R$ (d) $\frac{1}{2}\omega^2 R$

42. A ball is dropped from a satellite revolving around the earth at height of 120 km. The ball will

(a) continue to move with same speed along a straight line tangentially to the satellite at that time
(b) continue to move with same speed along the original orbit of satellite.
(c) fall down to earth gradually
(d) go far away in space

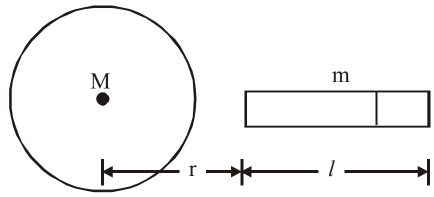
43. Two identical geostationary satellites are moving with equal speeds in the same orbit but their sense of rotation brings them on a collision course. The debris will

(a) fall down
(b) move up
(c) begin to move from east to west in the same orbit
(d) begin to move from west to east in the same orbit

44. If there were a small gravitational effect, then which of the following forces will undergo a change?

(a) Viscous force (b) Electrostatic force
(c) Magnetic force (d) Archimedes' uplift

45. The gravitational force of attraction between a uniform sphere of mass M and a uniform rod of length l and mass m oriented as shown is



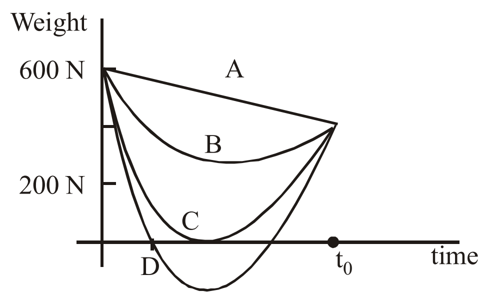
(a) $\frac{GMm}{r(r+l)}$ (b) $\frac{GM}{r^2}$

(c) $Mmr^2 + l$ (d) $(r^2 + l)mM$

46. Explorer 38, a radio-astronomy satellite of mass 200 kg, circles the Earth in an orbit of average radius $\frac{3R}{2}$ where R is the radius of the Earth. Assuming the gravitational pull on a mass of 1 kg at the earth's surface to be 10 N, calculate the pull on the satellite

(a) 889 N (b) 89 N
(c) 8889 N (d) 8.9 N

47. Suppose, the acceleration due to gravity at the Earth's surface is 10 m s^{-2} and at the surface of Mars it is 4.0 m s^{-2}. A 60 kg pasenger goes from the Earth to the Mars in a spaceship moving with a constant velocity. Neglect all other objects in the sky. Which part of figure best represents the weight (net gravitational force) of the passenger as a function of time?



(a) A (b) B
(c) C (d) D

48. A projectile is fired vertically from the Earth with a velocity kv_e where v_e is the escape velocity and k is a constant less than unity. The maximum height to which projectile rises, as measured from the centre of Earth, is

(a) $\frac{R}{k}$ (b) $\frac{R}{k-1}$

(c) $\frac{R}{1-k^2}$ (d) $\frac{R}{1+k^2}$

49. A diametrical tunnel is dug across the Earth. A ball is dropped into the tunnel from one side. The velocity of the ball when it reaches the centre of the Earth is .... (Given : gravitational potential at the centre of Earth = $-\frac{3}{2}\frac{GM}{R}$)

(a) $\sqrt{R}$ (b) $\sqrt{gR}$

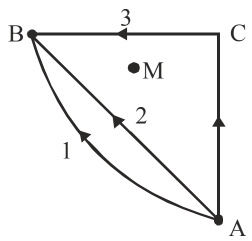
(c) $\sqrt{2.5gR}$ (d) $\sqrt{7.1gR}$

50. A man of mass m starts falling towards a planet of mass M and radius R. As he reaches near to the surface, he realizes that he will pass through a small hole in the planet. As he enters the hole, he sees that the planet is really made of two pieces a spherical shell of negligible thickness of mass 2M/3 and a point mass M/3 at the centre. Change in the force of gravity experienced by the man is

(a) $\frac{2}{3}\frac{GMm}{R^2}$ (b) 0

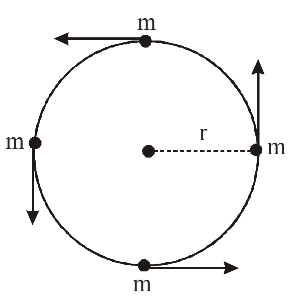
(c) $\frac{1}{3}\frac{GMm}{R^2}$ (d) $\frac{4}{3}\frac{GMm}{R^2}$

51. In a region of only gravitational field of mass 'M' a particle is shifted from A to B via three different paths in the figure. The work done in different paths are W_1, W_2, W_3 respectively then



(a) $W_1 = W_2 = W_3$ (b) $W_1 > W_2 > W_3$
(c) $W_1 = W_2 > W_3$ (d) $W_1 < W_2 < W_3$

52. Four similar particles of mass m are orbiting in a circle of radius r in the same angular direction because of their mutual gravitational attractive force. Velocity of a particle is given by



(a) $\left[\frac{GM}{r}\left(\frac{1+2\sqrt{2}}{4}\right)\right]^{1/2}$ (b) $\sqrt[3]{\frac{GM}{r}}$

(c) $\sqrt{\frac{GM}{r}\left(1+2\sqrt{2}\right)}$ (d) $\left[\frac{1}{2}\frac{GM}{r}\left(\frac{1+\sqrt{2}}{2}\right)\right]^{1/2}$

53. The percentage change in the acceleration of the earth towards the sun from a total eclipse of the sun to the point where the moon is on a side of earth directly opposite to the sun is

(a) $\frac{M_s}{M_m}\frac{r_2}{r_1}\times 100$ (b) $\frac{M_s}{M_m}\left(\frac{r_2}{r_1}\right)^2\times 100$

(c) $2\left(\frac{r_1}{r_2}\right)^2\frac{M_m}{M_s}\times 100$ (d) $\left(\frac{r_1}{r_2}\right)^2\frac{M_s}{M_m}\times 100$

54. A satellite is revolving round the earth in an orbit of radius r with time period T. If the satellite is revolving round the earth in an orbit of radius $r + \Delta r$ ($\Delta r << r$) with time period $T + \Delta T$ then,

(a) $\frac{\Delta T}{T} = \frac{3}{2}\frac{\Delta r}{r}$ (b) $\frac{\Delta T}{T} = \frac{2}{3}\frac{\Delta r}{r}$

(c) $\frac{\Delta T}{T} = \frac{\Delta r}{r}$ (d) $\frac{\Delta T}{T} = -\frac{\Delta r}{r}$

55. A cavity of radius R/2 is made inside a solid sphere of radius R. The centre of the cavity is located at a distance R/2 from the centre of the sphere. The gravitational force on a particle of mass 'm' at a distance R/2 from the centre of the sphere on the line joining both the centres of sphere and cavity is – (opposite to the centre of gravity)

[Here $g = GM/R^2$, where M is the mass of the sphere]

(a) $\frac{mg}{2}$ (b) $\frac{3mg}{8}$

(c) $\frac{mg}{16}$ (d) None of these

56. A spherical uniform planet is rotating about its axis. The velocity of a point on its equator is V. Due to the rotation of planet about its axis the acceleration due to gravity g at equator is 1/2 of g at poles. The escape velocity of a particle on the pole of planet in terms of V is

(a) $V_e = 2V$ (b) $V_e = V$

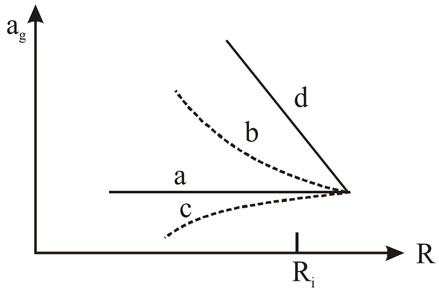
(c) $V_e = V/2$ (d) $V_e = \sqrt{3}V$

57. The escape velocity from a planet is v_e. A tunnel is dug along a diameter of the planet and a small body is dropped into it at the surface. When the body reaches the centre of the planet, its speed will be

(a) v_e (b) $v_e/\sqrt{2}$

(c) $v_e/2$ (d) zero

58. A (nonrotating) star collapses onto itself from an initial radius R_i with its mass remaining unchanged. Which curve in figure best gives the gravitational acceleration a_g on the surface of the star as a function of the radius of the star during the collapse



(a) a (b) b
(c) c (d) d

59. The earth is assumed to be sphere of radius R. A platform is arranged at a height R from the surface of Earth. The escape velocity of a body from this platform is kv, where v is its escape velocity from the surface of the earth. The value of k is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\sqrt{2}$

60. Four equal masses (each of mass M) are placed at the corners of a square of side a. The escape velocity of a body from the centre O of the square is

(a) $4\sqrt{\frac{2GM}{a}}$ (b) $\sqrt{\frac{8\sqrt{2}GM}{a}}$

(c) $\frac{4GM}{a}$ (d) $\sqrt{\frac{4\sqrt{2}GM}{a}}$

61. A planet of mass m moves around the sun of mass M in an elliptical orbit. The maximum and minimum distance of the planet from the sun are r_1 and r_2 respectively. The time period of planet is proportional to

(a) $r_1^{2/5}$ (b) $(r_1 + r_2)^{3/2}$

(c) $(r_1 - r_2)^{3/2}$ (d) $r^{3/2}$

62. The change in potential energy, when a body of mass m is raised to a height nR from the earth's surface is (R = radius of earth)

(a) $mgR\left(\frac{n}{n-1}\right)$ (b) nmgR

(c) $mgR\left(\frac{n^2}{n^2+1}\right)$ (d) $mgR\left(\frac{n}{n+1}\right)$

63. A satellite is launched into a circular orbit of radius R around the earth. A second satellite is launched into an orbit of radius 1.01 R. The period of second satellite is larger than the first one by approximately

(a) 0.5% (b) 1.0%
(c) 1.5% (d) 3.0%

64. If g_E and g_M are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio

$$\frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}} \text{ to be}$$

(a) g_M / g_E (b) 1

(c) 0 (d) g_E / g_M

65. The density of a newly discovered planet is twice that of earth. The acceleration due to gravity at the surface of the planet is equal to that at the surface of the earth. If the radius of the earth is R, the radius of the planet would be

(a) ½ R (b) 2 R
(c) 4 R (d) 1/4 R

66. Imagine a new planet having the same density as that of earth but it is 3 times bigger than the earth in size. If the acceleration due to gravity on the surface of earth is g and that on the surface of the new planet is g′, then

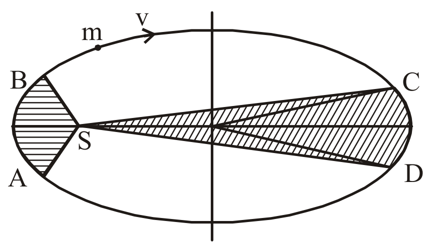
(a) g′ = g/9 (b) g′ = 27g
(c) g′=9g (d) g′=3g

67. For a satellite moving in an orbit around the earth, the ratio of kinetic energy to potential energy is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) 2 (d) $\sqrt{2}$

68. The figure shows elliptical orbit of a planet m about the sun S. The shaded area SCD is twice the shaded area SAB. If t_1 is the time for the planet to move from C to D and t_2 is the time to move from A to B then :



(a) $t_1 = 4t_2$ (b) $t_1 = 2t_2$
(c) $t_1 = t_2$ (d) $t_1 > t_2$

69. The radii of circular orbits of two satellites A and B of the earth, are 4R and R, respectively. If the speed of satellite A is 3 V, then the speed of satellite B will be

(a) 3 V/4 (b) 6V
(c) 12V (d) 3 V/2

70. A particle of mass M is situated at the centre of a spherical shell of same mass and radius a. The gravitational potential at a point situated at $\frac{a}{2}$ distance from the centre, will be

(a) $-\frac{3GM}{a}$ (b) $-\frac{2GM}{a}$

(c) $-\frac{GM}{a}$ (d) $-\frac{4GM}{a}$

71. A planet moving along an elliptical orbit is closest to the sun at a distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are the linear velocities at these points respectively, then the ratio $\frac{v_1}{v_2}$ is

(a) $(r_1/r_2)^2$ (b) r_2/r_1

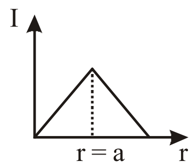
(c) $(r_2/r_1)^2$ (d) r_1/r_2

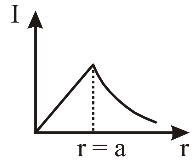
72. A particle of mass m is thrown upwards from the surface of the earth, with a velocity u. The mass and the radius of the earth are, respectively, M and R. G is gravitational constant and g is acceleration due to gravity on the surface of the earth. The minimum value of u so that the particle does not return back to earth, is

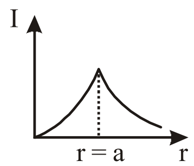
(a) $\sqrt{\frac{2GM}{R}}$ (b) $\sqrt{\frac{2GM}{R^2}}$

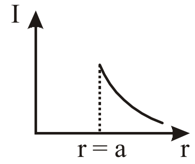
(c) $\sqrt{2gR^2}$ (d) $\sqrt{\frac{2GM}{R^2}}$

73. Which one of the following graphs represents correctly the variation of the gravitational field intensity (I) with the distance (r) from the centre of a spherical shell of mass M and radius a ?

(a)


(b)


(c)


(d)


Directions for Qs. (74 to 75) : Each question contains STATEMENT-1 and STATEMENT-2. Choose the correct answer (ONLY ONE option is correct) from the following

(a) Statement -1 is false, Statement-2 is true

(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1

(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1

(d) Statement -1 is true, Statement-2 is false

74. **Statement -1 :** For the planets orbiting around the sun, angular speed, linear speed and K.E. changes with time, but angular momentum remains constant.

Statement -2 : No torque is acting on the rotating planet. So its angular momentum is constant.

75. **Statement -1 :** Gravitational potential is maximum at infinity.

Statement -2 : Gravitational potential is the amount of work done to shift a unit mass from infinity to a given point in gravitational attraction force field.

EXERCISE - 3
Exemplar & Past Years NEET/AIPMT Questions

Exemplar Questions

1. The earth is an approximate sphere. If the interior contained matter which is not of the same density everywhere, then on the surface of the earth, the acceleration due to gravity

(a) will be directed towards the centre but not the same everywhere

(b) will have the same value everywhere but not directed towards the centre

(c) will be same everywhere in magnitude directed towards the centre

(d) cannot be zero at any point

2. As observed from the earth, the sun appears to move in an approximate circular orbit. For the motion of another planet like mercury as observed from the earth, this would

(a) be similarly true

(b) not be true because the force between the earth and mercury is not inverse square law

(c) not be true because the major gravitational force on mercury is due to the sun

(d) not be true because mercury is influenced by forces other than gravitational force

3. Different points in the earth are at slightly different distances from the sun and hence experience different forces due to gravitation. For a rigid body, we know that if various forces act at various points in it, the resultant motion is as if a net force acts on the CM (centre of mass) causing translation and a net torque at the CM causing rotation around an axis through the CM. For the earth-sun system (approximating the earth as a uniform density sphere).

(a) the torque is zero

(b) the torque causes the earth to spin

(c) the rigid body result is not applicable since the earth is not even approximately a rigid body

(d) the torque causes the earth to move around the sun

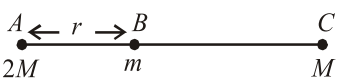
4. Satellites orbitting the earth have finite life and sometimes debris of satellites fall to the earth. This is because
(a) the solar cells and batteries in satellites run out
(b) the laws of gravitation predict a trajectory spiralling inwards
(c) of viscous forces causing the speed of satellite and hence height to gradually decrease
(d) of collisions with other satellites

5. Both the earth and the moon are subject to the gravitational force of the sun. As observed from the sun, the orbit of the moon
(a) will be elliptical
(b) will not be strictly elliptical because the total gravitational force on it is not central
(c) is not elliptical but will necessarily be a closed curve
(d) deviates considerably from being elliptical due to influence of planets other than the earth

6. In our solar system, the inter-planetary region has chunks of matter (much smaller in size compared to planets) called asteroids. They
(a) will not move around the sun, since they have very small masses compared to the sun
(b) will move in an irregular way because of their small masses and will drift away into outer space
(c) will move around the sun in closed orbits but not obey Kepler's laws
(d) will move in orbits like planets and obey Kepler's laws

7. Choose the wrong option.
(a) Inertial mass is a measure of difficulty of accelerating a body by an external force whereas the gravitational mass is relevant in determining the gravitational force on it by an external mass
(b) That the gravitational mass and inertial mass are equal is an experimental result
(c) That the acceleration due to gravity on the earth is the same for all bodies is due to the equality of gravitational mass and inertial mass
(d) Gravitational mass of a particle like proton can depend on the presence of neighbouring heavy objects but the inertial mass cannot

8. Particles of masses $2M$, m and M are respectively at points A, B and C with $AB = \frac{1}{2}(BC) \cdot m$ is much-much smaller than M and at time $t = 0$, they are all at rest as given in figure. At subsequent times before any collision takes place.



(a) m will remain at rest
(b) m will move towards M
(c) m will move towards $2M$
(d) m will have oscillatory motion

NEET/AIPMT (2013-2017) Questions

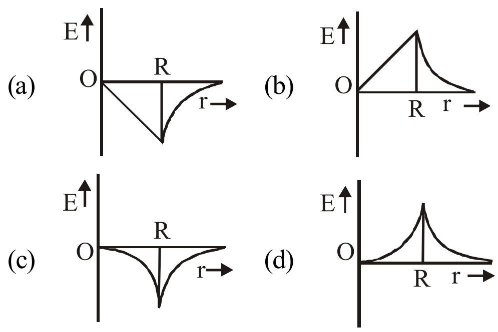
9. A body of mass 'm' is taken from the earth's surface to the height equal to twice the radius (R) of the earth. The change in potential energy of body will be ***[2013]***
(a) $\frac{2}{3}$ mgR (b) 3 mgR
(c) $\frac{1}{3}$ mgR (d) mg2R

10. Infinite number of bodies, each of mass 2 kg are situated on x-axis at distances 1m, 2m, 4m, 8m, ... respectively, from the origin. The resulting gravitational potential due to this system at the origin will be ***[2013]***
(a) $-\frac{8}{3}$ G (b) $-\frac{4}{3}$ G
(c) – 4 G (d) –G

11. The radius of a planet is twice the radius of earth. Both have almost equal average mass-densities. If V_P and V_E are escape velocities of the planet and the earth, respectively, then ***[NEET Kar. 2013]***
(a) $V_E = 1.5V_P$ (b) $V_P = 1.5V_E$
(c) $V_P = 2V_E$ (d) $V_E = 3V_P$

12. A particle of mass 'm' is kept at rest at a height $3R$ from the surface of earth, where 'R' is radius of earth and 'M' is mass of earth. The minimum speed with which it should be projected, so that it does not return back, is (g is acceleration due to gravity on the surface of earth) ***[NEET Kar. 2013]***
(a) $\left(\frac{GM}{R}\right)^{\frac{1}{2}}$ (b) $\left(\frac{GM}{2R}\right)^{\frac{1}{2}}$
(c) $\left(\frac{gR}{4}\right)^{\frac{1}{2}}$ (d) $\left(\frac{2g}{4}\right)^{\frac{1}{2}}$

13. Dependence of intensity of gravitational field (E) of earth with distance (r) from centre of earth is correctly represented by: ***[2014]***



14. A black hole is an object whose gravitational field is so strong that even light cannot escape from it. To what approximate radius would earth (mass = 5.98×10^{24} kg) have to be compressed to be a black hole? ***[2014]***
(a) 10^{-9} m (b) 10^{-6} m
(c) 10^{-2} m (d) 100 m

15. Two spherical bodies of mass M and 5 M and radii R and 2 R released in free space with initial separation between their centres equal to 12 R. If they attract each other due to gravitational force only, then the distance covered by the smaller body before collision is ***[2015]***

(a) 4.5 R (b) 7.5 R
(c) 1.5 R (d) 2.5 R

16. Kepler's third law states that square of period of revolution (T) of a planet around the sun, is proportional to third power of average distance r between sun and planet i.e. $T^2 = Kr^3$ here K is constant. If the masses of sun and planet are M and m respectively then as per Newton's law of gravitation force of attraction between them is $F = \frac{GMm}{r^2}$, here G is gravitational constant. The relation between G and K is described as ***[2015]***

(a) $GMK = 4\pi^2$ (b) $K = G$
(c) $K = \frac{1}{G}$ (d) $GK = 4\pi^2$

17. A remote - sensing satellite of earth revolves in a circular orbit at a height of 0.25×10^6 m above the surface of earth. If earth's radius is 6.38×10^6 m and g = 9.8 ms^{-2}, then the orbital speed of the satellite is: ***[2015 RS]***

(a) 8.56 km s^{-1} (b) 9.13 km s^{-1}
(c) 6.67 km s^{-1} (d) 7.76 km s^{-1}

18. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. Then, ***[2015 RS]***

(a) the total mechanical energy of S varies periodically with time.
(b) the linear momentum of S remains constant in magnitude.
(c) the acceleration of S is always directed towards the centre of the earth.
(d) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant.

19. The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p) whose radius and mean density are twice as that of earth is : ***[2016]***

(a) 1 : 2 (b) $1 : 2\sqrt{2}$
(c) 1 : 4 (d) 1 : 2

20. At what height from the surface of earth the gravitational potential and the value of g are -5.4×10^7 J kg^{-1} and 6.0 ms^{-2} respectively ?
Take the radius of earth as 6400 km : ***[2016]***

(a) 2600 km (b) 1600 km
(c) 1400 km (d) 2000 km

21. Two astronauts are floating in gravitation free space after having lost contact with their spaceship. The two will ***[2017]***

(a) move towards each other.
(b) move away from each other.
(c) become stationary
(d) keep floating at the same distance between them.

22. The acceleration due to gravity at a height 1 km above the earth is the same as at a depth d below the surface of earth. Then ***[2017]***

(a) d = 1 km (b) $d = \frac{3}{2}$ km
(c) d = 2 km (d) $d = \frac{1}{2}$ km

Hints & Solutions

EXERCISE - 1

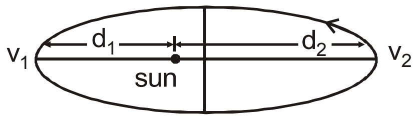
1.	(d)	2.	(d)	3.	(a)	4.	(d)
5.	(b)	6.	(c)	7.	(b)		

8. (c) In planetary motion $\vec{\tau}_{ext.} = 0 \Rightarrow \vec{L} = \text{constant}$

$\vec{L} = \vec{r} \times \vec{p} (= m\vec{v}) = mrv \; (\because \theta = 90^\circ)$

So $m_1 d_1 v_1 = m_2 d_2 v_2$ (here r = d)

$\Rightarrow v_2 = \dfrac{v_1 d_1}{d_2}$



9. (a) At the surface of earth, the value of g = 9.8m/sec^2. If we go towards the centre of earth or we go above the surface of earth, then in both the cases the value of g decreases.
Hence $W_1 = mg_{mine}$, $W_2 = mg_{sea\ level}$, $W_3 = mg_{moun}$
So $W_1 < W_2 > W_3$ (g at the sea level = g at the suface of earth)

10. (a) According to Gravitational Law

$$F = \frac{GM_1M_2}{r^2} \quad \ldots\ldots (i)$$

Where M_1 is mass of planet & m_2 is the mass of any body. Now according to Newton's second law, body of mass m_2 feels gravitational acceleration g which is
$F = m_2 g$ ……(ii)

So from (i) & (ii), we get $g = \dfrac{GM_1}{r^2}$

So the ratio of gravitational acceleration due to two planets is

$$\frac{g_1}{g_2} = \frac{M_1}{r_1^2} \times \frac{r_2^2}{M_2} = \frac{(4/3)\pi r_1^3 \times \rho}{r_1^2} \times \frac{r_2^2}{(4/3)\pi r_2^3 \times \rho}$$

$\dfrac{g_1}{g_2} = \left(\dfrac{r_1}{r_2}\right)$ (both planet have same material, so density is same)

11. (c) Since $T(\text{time period}) = 2\pi\sqrt{\dfrac{l}{g}}$

for second pendulum T=2sec
Now on the planet the value of acceleration due to gravity is g/4.
So for the planet, the length of sec. pendulam l' is

$$\frac{2\sec}{2\sec} = \sqrt{\frac{l}{g} \times \frac{g/4}{l'}} \Rightarrow l' = l/4$$

12. (b) Since $T = 2\pi\sqrt{\dfrac{l}{g}}$

but inside the satellite g = 0
So $T = \infty$

13. (b) We know that, $v_e = \sqrt{(2gR)}$

$$\therefore \frac{(v_e)_{P_1}}{(v_e)_{P_2}} = \frac{\sqrt{(2g_1R_1)}}{\sqrt{(2g_2R_2)}} = \sqrt{\left(\frac{g_1}{g_2}\right)} \times \sqrt{\left(\frac{R_1}{R_2}\right)} = \sqrt{kr}$$

14. (b) $v_e = \sqrt{2}v_o$ where v_e and v_o are the escape velocity and orbital velocity respectively.

15. (c) $KE = \dfrac{1}{2}mv_{esc}^2 = \dfrac{1}{2}m(\sqrt{2gR})^2 = mgR$.

16. (a) $v_{esc} = \sqrt{2gR}$, where R is radius of the planet.
Hence escape velocity is independent of m.

17. (a) The escape velocity on the earth is defined as

$v_e = \sqrt{2g_eR_e}$

Where R_e & g_e are the radius & acceleration due to gravity of earth.
Now for planet $g_P = 2g_e$, $R_P = R_e/4$

So $v_P = \sqrt{2g_PR_P} = \sqrt{2 \times 2g_e \times R_e/4} = \dfrac{v_e}{\sqrt{2}}$

18. (c) The potential energy for a conservative force is defined as

$$F = \frac{-dU}{dr} \text{ or } U = -\int_{\infty}^{r} \vec{F}.d\vec{r} \quad \ldots\ldots (i)$$

$$\text{or } U_r = \int_{\infty}^{r} \frac{GM_1M_2}{r^2} dr = \frac{-GM_1M_2}{r} \quad \ldots\ldots (ii)$$

$(\because U_\infty = 0)$
If we bring the mass from the infinity to the centre of earth, then we obtain work, 'so it has negative (gravitational force do work on the object) sign & potential energy decreases. But if we bring the mass from the surface of earth to infinite, then we must do work against gravitational force & potential energy of the mass increases.

Now in equation (i) if $F = \dfrac{GM_1M_2}{r^{5/2}}$ instead of

$F = \dfrac{GM_1M_2}{r^2}$ then

$$U_r = \int_{\infty}^{r} \frac{GM_1M_2}{r^{5/2}} dr = \frac{-2}{3}\frac{GM_1M_2}{r^{3/2}}$$

$$\Rightarrow U_r \propto \frac{1}{r^{+3/2}}$$

19. (c) $\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} \Rightarrow \frac{T}{T_2} = \left(\frac{4R}{16R}\right)^{3/2}$

$\Rightarrow T_2 = 8T$

20. (a) $T^2 \propto (\text{major axis})^3 \Rightarrow T^2 \alpha a^3$

21. (d) According to 3rd law of Kepler

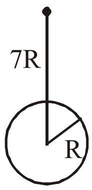
$T^2 \propto R^3$

$\Rightarrow T^2 = KR^3$

where K is a constant

Thus $\frac{T^2}{R^3}$ does not depends on radius.

22. (a) $v_1 \propto \frac{1}{\sqrt{R}}, \; v_2 \propto \frac{1}{\sqrt{7R}}$



$\frac{v_2}{v_1} = \frac{1}{\sqrt{7}} \Rightarrow v_2 = \frac{v_1}{\sqrt{7}} = \frac{v}{\sqrt{7}}$

23. (a) Escape velocity does not depend upon the mass of the body.

24. (b) At a height h above the surface of earth the gravitational potential energy of the particle of mass m is

$U_h = -\frac{GM_e m}{R_e + h}$

Where M_e & R_e are the mass & radius of earth respectively.
In this question, since $h = R_e$

So $U_{h=R_e} = -\frac{GM_e m}{2R_e} = \frac{-mgR_e}{2}$

25. (d) Energy required to move a body of mass m from an orbit of radius 2R to 3R is

$\Delta U = -\frac{GMm}{3R} - \frac{-GMm}{2R} = \frac{GMm}{2R} - \frac{GMm}{3R} = \frac{GMm}{6R}$

EXERCISE - 2

1. (c)

2. (c) According to Newton's Gravitation Law

$F_g = \frac{GM_1M_2}{r^2}$ here $F_g = 10^{-10}$ Newton,

$m_1 = m_2 = 1$kg,
$G = 6.6 \times 10^{-11}$

So $r^2 = \frac{GM_1M_2}{F_g} = \frac{6.6 \times 10^{-11} \times 1 \times 1}{10^{-10}} = 0.66$

or $r = 0.8125$ metre $= 81.25$ cm ≈ 80cm

3. (c) $\frac{G \times 10^3}{(r)^2} = \frac{G \times 10^5}{(1-r)^2}$

$\frac{1}{r^2} = \frac{10^2}{(1-r)^2}$

$\frac{1}{r} = \frac{10}{1-r} \Rightarrow 10r = 1 - r$

$\therefore r = \frac{1}{11}$ km

4. (b) The gravitational potential V at a point distant 'r' from a body of mass m is equal to the amount of work done in moving a unit mass from infinity to that point.

$V_r - V_\infty = -\int_\infty^r \vec{E}.d\vec{r} = -GM(1/r - 1/\infty)$

$= \frac{-GM}{r} \left(\text{As } \vec{E} = \frac{-dV}{dr}\right)$

(i) In the first case

when $V_\infty = 0, \; V_r = \frac{-GM}{r} = -5$ unit

(ii) In the second case $V_\infty = +10$ unit

$V_r - 10 = -5$
or $V_r = +5$ unit

5. (c) Gravitational force supplies centripetal force

$\therefore F = \frac{mv^2}{r} = \frac{3 \times 10^{29} \times (2 \times 10^8)^2}{1.5 \times 10^{14}}$ dynes

$= 8 \times 10^{31}$ dyne $= 8 \times 10^{26}$ N ($\because$ 1N $= 10^5$ dyne.)

6. (c) $v_e = \sqrt{\frac{2GM_e}{R_e}}; \quad v_m = \sqrt{\frac{2G\frac{M_e}{81}}{\frac{R_e}{4}}} = \frac{2}{9} v_e$

$= 2/9 \times 11.2$ kms^{-1} $= 2.5$ kms^{-1}

7. (d) Since gravitational acceleration on earth is defined as

$g_e = \frac{GM_e}{R_e^2}$ ……(i)

mass of planet is $M_P = \frac{M_e}{80}$ & radius $R_P = \frac{R_e}{4}$

So $g_P = \frac{GM_P}{R_P^2}$ ……(ii)

From (i) & (ii), we get

$g_P = g_e \frac{M_P}{R_P^2} \times \frac{R_e^2}{M_e} = \frac{g_e}{5}$ =2m/sec^2

(as g=10m/sec^2)

8. (c) $V_g = \frac{-6.67 \times 10^{-11} \times 10}{1} - \frac{6.67 \times 10^{-11} \times 100}{1}$

$= -6.67 \times 10^{-10} - 6.67 \times 10^{-9}$
$= -6.67 \times 10^{-10} \times 11 = -7.3 \times 10^{-9}$ J/kg

9. (c) $T^2 \propto R^3$

$\Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$

$\Rightarrow \left(\frac{T_1}{T_2}\right) = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{1}{4}\right)^{3/2}$

$\Rightarrow \frac{T_2}{T_1} = (4)^{3/2} = 8$

$\Rightarrow T_2 = 8 \times T_1 = 8 \times 5 = 40$ hours

10. (b) At poles, the effect of rotation is zero and also the distance from the centre of earth is least.

11. (c) $T^2 \propto R^3$ (According to Kepler's law)

$T_1^2 \propto (10^{13})^3$ and $T_2^2 \propto (10^{12})^3$

$\therefore \frac{T_1^2}{T_2^2} = (10)^3$ or $\frac{T_1}{T_2} = 10\sqrt{10}$.

12. (c) According to Kepler's law of period $T^2 \propto R^3$

$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} = \frac{(6R)^3}{(3R)^3} = 8$

$\frac{24 \times 24}{T_2^2} = 8$

$T_2^2 = \frac{24 \times 24}{8} = 72 = 36 \times 2$

$T_2 = 6\sqrt{2}$

13. (b) Since escape velocity $\left(v_e = \sqrt{2gR_e}\right)$ is independent of angle of projection, so it will not change.

14. (c) Given that $\ell_2 = 1.02\,\ell_1$

We know that $T = 2\pi\sqrt{\left(\frac{\ell}{g}\right)}$ or $T \propto \sqrt{\ell}$

$\therefore \frac{T_2}{T_1} = \sqrt{\left(\frac{\ell_2}{\ell_1}\right)} = \sqrt{\frac{(1.02\,\ell_1)}{\ell_1}} = 1.01$

Thus time period increases by 1%.

15. (b) We know that $v_e = \sqrt{2}\,v_0$, where v_0 is orbital velocity.

K.E. in the orbit, $E = \frac{1}{2} M v_0^2$

K.E. to escape $E = \frac{1}{2} M v_e^2 = \frac{1}{2} M (2 v_0^2)$

$= \frac{1}{2} M v_0^2 \times 2 = 2E$

16. (c) $T = \frac{2\pi}{R}\left[\frac{(R+h)^3}{g}\right]^{1/2}$

Here (R + h) changes from R to 4 R. Hence period of revolution changes from T to $(4^3)^{1/2}$ T = 8 T.

17. (b) $g_h = g\left(1 - \frac{2h}{R}\right)$

$g_x = g\left(1 - \frac{x}{R}\right)$

Given that, $g_h = g_x$

$\therefore x = 2h$

18. (d) $\frac{mv^2}{(R+x)} = \frac{GmM}{(R+x)^2}$ also $g = \frac{GM}{R^2}$

$\therefore \frac{mv^2}{(R+x)} = m\left(\frac{GM}{R^2}\right)\frac{R^2}{(R+x)^2}$

$\therefore \frac{mv^2}{(R+x)} = mg\frac{R^2}{(R+x)^2}$

$\therefore v^2 = \frac{gR^2}{R+x} \Rightarrow v = \left(\frac{gR^2}{R+x}\right)^{1/2}$

19. (c) $\frac{mv^2}{R+x} = \frac{GmM}{(R+x)^2}$

x = height of satellite from earth surface
m = mass of satellite

$\Rightarrow v^2 = \frac{GM}{(R+x)}$ or $v = \sqrt{\frac{GM}{R+x}}$

$T = \frac{2\pi(R+x)}{v} = \frac{2\pi(R+x)}{\sqrt{\frac{GM}{R+x}}}$

which is independent of mass of satellite

20. (c) $F = KR^{-n} = MR\omega^2 \Rightarrow \omega^2 = KR^{-(n+1)}$

or $\omega = K'R^{\frac{-(n+1)}{2}}$ [where $K' = K^{1/2}$, a constant]

$\frac{2\pi}{T}\ \alpha\ R^{\frac{-(n+1)}{2}}$

$\therefore T\ \alpha\ R^{\frac{(n+1)}{2}}$

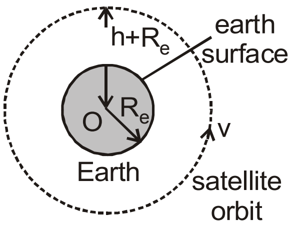
21. (d) According to Newton gravitational force

$\frac{GM_e m}{R_e^2} = mg$ & $\frac{GM_e m}{(R_e+h)^2} = mg'$

$\Rightarrow g' = g\frac{1}{(1+h/R_e)^2} = \frac{gR_e^2}{(R_e+h)^2}$

where M_e is mass of earth, G is gravitational constant, R_e is radius & earth, h is height of satellite above the surface of earth, g is value at the surface of earth, g' is value at height h above the surface of earth.

$$\text{so}\ \frac{mv^2}{(R_e+h)} = \frac{mgR_e^2}{(R_e+h)^2} = mg' \Rightarrow v = \sqrt{\frac{gR_e^2}{R_e+h}}$$



22. (c)
23. (d) Both decreases but variation are different.
24. (b) There is no gravitational field in the shell.
25. (c) The gravitational field due to the ring at a distance $\sqrt{3}r$ is given by

$$E = \frac{Gm(\sqrt{3}r)}{[r^2+(\sqrt{3}r)^2]^{3/2}} \Rightarrow E = \frac{\sqrt{3}Gm}{8r^2}$$

Attractive force $= \dfrac{\sqrt{3}GmM}{8r^2}$

26. (a) For uniform gravitational field

$$E_g = -\frac{V}{r} = -\frac{-2}{20} = \frac{1}{10}\ ms^{-2}$$

Now, $W = mgh = 5\times\frac{1}{10}\times 4 = 2J$

27. (b) $E_1 = \frac{1}{2}mv_e^2,\ E_2 = \frac{1}{2}mv_0^2$

$$\therefore \frac{E_1}{E_2} = \left(\frac{v_e}{v_0}\right)^2 = 2 \qquad \because v_e = \sqrt{2}v_0$$

28. (d) $g \propto \rho R$
29. (c)
30. (b) As buoyant force involves 'g' in it.
31. (d) Work done $= U_f - U_i$

$$= -\frac{GMm}{2R+R} - \left(-\frac{Gmm}{R}\right) = \frac{2}{3}\frac{GmM}{R} = \frac{2}{3}mgR$$

32. (b) 33. (a)
34. (d) The gravitational force due to the whole sphere at A point is

$F_1 = \dfrac{GM_e m_o}{(2R)^2}$, where m_0 is the assumed rest mass at point A.

In the second case, when we made a cavity of radius (R/2), then gravitational force at point A is

$$F_2 = \frac{GM_e m_o}{(R+R/2)^2} \qquad \therefore F_2/F_1 = 1/9$$

35. (b) $P.E. = \int_{R_0}^{R}\frac{GMm}{r^2}dr = -GMm\left[\frac{1}{R}-\frac{1}{R_0}\right]$

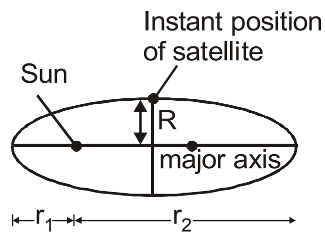
The K.E. acquired by the body at the surface $= \frac{1}{2}mv^2$

$$\therefore \frac{1}{2}mv^2 = -GMm\left[\frac{1}{R}-\frac{1}{R_0}\right]$$

$$v = \sqrt{2GM\left(\frac{1}{R_0}-\frac{1}{R}\right)}$$

36. (c) Applying the properties of ellipse, we have

$$\frac{2}{R} = \frac{1}{r_1}+\frac{1}{r_2} = \frac{r_1+r_2}{r_1 r_2}$$



$$R = \frac{2r_1 r_2}{r_1+r_2}$$

37. (a) $v = \sqrt{\left(\frac{GM}{r}\right)}$ where r is radius of the orbit of the satellite.

here $r = R_e + h = R_e + \frac{R_e}{2} = \frac{3}{2}R_e$

So, $v = \sqrt{\frac{2GM}{3R_e}} = v_0\sqrt{\frac{2}{3}}$,

where v_0 is the orbital velocity of the satellite, which is moving in circular orbit of radius, $r = R_e$

38. (d) $v_1 r_1 = v_2 r_2$ or $r_1^2\omega_1 = r_2^2\omega_2$ ($\because L = mrv = \text{constant}$)

or $r_{min}^2\,\omega = r_{max}^2\,\omega'$

$\therefore \omega' = (r_{min}^2 / r_{max}^2)\,\omega$

39. (c) Distance between the surface of the spherical bodies $= 12R - R - 2R = 9R$

Force $\propto$ Mass

Acceleration $\propto$ Mass

Distance $\propto$ Acceleration

$$\Rightarrow \frac{a_1}{a_2} = \frac{M}{5M} = \frac{1}{5} \Rightarrow \frac{S_1}{S_2} = \frac{1}{5} \Rightarrow S_2 = 5S_1$$

$S_1 + S_2 = 9$

$\Rightarrow 6S_1 = 9 \Rightarrow S_1 = \frac{9}{6} = 1.5$

$S_2 = 1.5 \times 5 = 7.5$

Note: Maximum distance will be travelled by smaller bodies due to the greater acceleration caused by the same gravitational force

40. (c) $g = \frac{GM}{R^2};\ g' = \frac{GM}{(0.99R)^2}$

$\therefore \frac{g'}{g} = \left(\frac{R^2}{0.99R^2}\right) \Rightarrow g' > g$

41. (b) Acceleration due to gravity at lattitude' λ ' is given by

$g_\lambda = g_e - R_e\omega^2 \cos^2 \lambda$

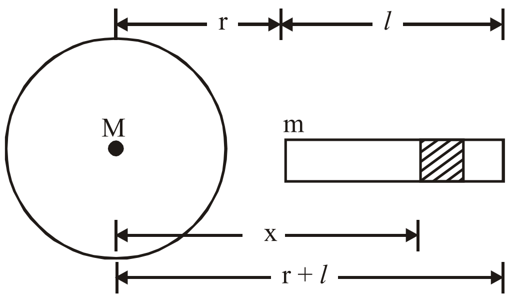
At equator, $\lambda = 90° \Rightarrow \cos\lambda = \cos 90° = 0$

or $g_\lambda = g_e = g$ (as given in question)

At 30°, $g_{30} = g - R\omega^2 \cos^2 30 = g - \frac{3}{4}R\omega^2$

or, $g - g_{30} = \frac{3}{4}R\omega^2$

42. (b) The orbital speed of satellite is independent of mass of satellite, so ball will behave as a satellite & will continue to move with same speed in original orbit.
43. (a) The total momentum will be zero and hence velocity will be zero just after collisiion. The pull of earth will make it fall down.
44. (d) Weight of liquid displaced = mg.
45. (a) The force of attraction between sphere and shaded position $dF = GM\frac{\left(\frac{m}{l}dx\right)}{x^2}$



$$F = \int_r^{r+l} \frac{GMm}{lx^2}dx = \frac{GMm}{l}\int_r^{r+l} \frac{1}{x^2}dx$$

$$= \frac{GMm}{l}\int_r^{r+l} x^{-2}dx = \frac{GMm}{l}\left[\frac{x^{-2+1}}{-2+1}\right]_r^{r+l}$$

$$= -\frac{GMm}{l}\left[x^{-1}\right]_r^{r+l} = -\frac{GMm}{l}\left[\frac{1}{x}\right]_r^{r+l} = \frac{GMm}{r(r+l)}$$

46. (a) $g_h = g\left(1 - \frac{2h}{R}\right) = \frac{4g}{h}\left(\text{since } h = R + \frac{3R}{2}\right)$

Force on the satellite = $mg_h = \frac{4}{9}mg$

$= \frac{4}{9} \times 200 \times 10 \approx 889\ N$

47. (c) $g \propto \frac{1}{R^2}$ so we will not get a straight line.

Also F = 0 at a point where Force due to Earth = Force due to mars

48. (c) Applying conservation of energy principle, we get

$\frac{1}{2}mk^2v_e^2 - \frac{GMm}{R} = -\frac{GMm}{r}$

$\Rightarrow \frac{1}{2}mk^2\frac{2GM}{R} - \frac{GMm}{R} = -\frac{GMm}{r}$

$\Rightarrow \frac{k^2}{R} - \frac{1}{R} = -\frac{1}{r} \Rightarrow \frac{1}{r} = \frac{1}{R} - \frac{k^2}{R}$

$\Rightarrow \frac{1}{r} = \frac{1}{R}(1-k^2) \Rightarrow r = \frac{R}{1-k^2}$

49. (b) Loss in potential energy = Gain in kinetic energy

$-\frac{GMm}{R} - \left(-\frac{3}{2}\frac{GMm}{R}\right) = \frac{1}{2}mv^2$

$\Rightarrow \frac{GMm}{2R} = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

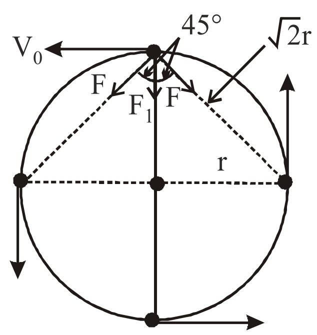
50. (a) Change in force of gravity

$= \frac{GMm}{R^2} - \frac{G\frac{M}{3}m}{R^2}$ (only due to mass M/3 due to shell gravitational field is zero (inside the shell))

$= \frac{2GMm}{3R^2}$

51. (a)
52. (a) Centripetal force = net gravitational force

$\frac{mv_0^2}{r} = 2F\cos 45° + F_1 = \frac{2GM^2}{(\sqrt{2}r)^2}\frac{1}{\sqrt{2}} + \frac{Gm^2}{4r^2}$



$\frac{mv_0^2}{r} = \frac{Gm^2}{4r^2}[2\sqrt{2}+1] \Rightarrow \left(\frac{GM(2\sqrt{2}+1)}{4r}\right)^{1/2}$

53. (c) During total eclipse :
Total attraction due to sun and moon,

$$F_1 = \frac{GM_sM_e}{r_1^2} + \frac{GM_mM_e}{r_2^2}$$

When moon goes on the opposite side of earth. Effective force of attraction,

$$F_2 = \frac{GM_sM_e}{r_1^2} - \frac{GM_mM_e}{r_2^2}$$

Change in force,

$$\Delta F = F_1 - F_2 = \frac{2GM_mM_e}{r_2^2}$$

Change in acceleration of earth

$$\Delta a = \frac{\Delta F}{M_e} = \frac{2GM_m}{r_2^2}$$

Average force on earth, $F_{av} = \frac{F_{av}}{M_e} = \frac{GM_s}{r_1^2}$

%age change in acceleration

$$= \frac{\Delta a}{a_{av}} \times 100 = \frac{2GM_m}{r_2^2} \times \frac{r_1^2}{GM_s} \times 100$$

$$= 2\left(\frac{r_1}{r_2}\right)^2 \frac{M_m}{M_s} \times 100$$

54. (a) Since, $T^2 = kr^3$
Differentiating the above equation

$$\Rightarrow 2\frac{\Delta T}{T} = 3\frac{\Delta r}{r} \Rightarrow \frac{\Delta T}{T} = \frac{3}{2}\frac{\Delta r}{r}$$

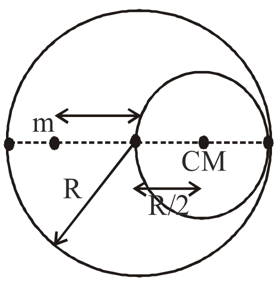
55. (b) Gravitational field at mass m due to full solid sphere

$$\vec{E}_1 = \frac{\rho \vec{r}}{3\varepsilon_0} = \frac{\rho R}{6\varepsilon_0} \ldots\ldots\left[\varepsilon_0 = \frac{1}{4\pi G}\right]$$

Gravitational field at mass m due to cavity $(-\rho)$

$$\vec{E}_2 = \frac{(-\rho)(R/2)^3 \vec{r}}{3\varepsilon_0 R^2} \ldots\ldots\left[\text{using } E = \frac{\rho a^3}{3\varepsilon_0 r^2}\right]$$

$$= -\frac{(-\rho)R^3}{24\varepsilon_0 R^2} = -\frac{-\rho R}{24\varepsilon_0}$$



Net gravitational field

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho R}{6\varepsilon_0} - \frac{\rho R}{24\varepsilon_0}$$

$$= \frac{\rho R}{8\varepsilon_0}$$

Net force on m $\rightarrow F = m\vec{E} = \frac{m\rho R}{8\varepsilon_0}$

Here, $\rho = \frac{M}{(4/3)\pi R^3}$ & $\varepsilon_0 = \frac{1}{4\pi G}$ then $F = \frac{3mg}{8}$

56. (a) $V = \omega R$

$g = g_0 - \omega^2 R$ [g = at equator, g_0 = at poles]

$$\frac{g_0}{2} = g_0 - \omega^2 R \;;\; \omega^2 R = \frac{g_0}{2} \;;\; V^2 = \frac{g_0 R}{2}$$

$$V_e = \sqrt{2g_0R} = \sqrt{4V^2} = 2V$$

57. (b) $\frac{1}{2}mv_e^2 = \frac{GMm}{R}$; $v_e = \sqrt{\frac{GMm}{R}} = \sqrt{2gR}$

In tunnel body will perform SHM at centre
$V_{max} = A\omega$ (see chapter on SHM)

$$= \frac{R\,2\pi}{2\pi\sqrt{R/g}} = \sqrt{gR} = \frac{v_e}{\sqrt{2}}$$

58. (b) $g \propto \frac{1}{R^2}$

R decreasing g increase hence, curve b represents correct variation.

59. (a) Here, $v = \sqrt{\frac{2GM}{R}}$ and $kv = \sqrt{\frac{2GM}{R+R}}$.

Solving $k = \frac{1}{\sqrt{2}}$

60. (b) Potential energy of particle at the centre of square

$$= -4\left(\frac{GMm}{\frac{a}{\sqrt{2}}}\right)$$

$$\therefore -4\left(\frac{GMm}{a/\sqrt{2}}\right) + \frac{1}{2}mv^2 = 0 \Rightarrow v^2 = \frac{8\sqrt{2}\,GM}{a}$$

61. (b) $T^2 \alpha r^3$, where r = mean radius = $\frac{r_1 + r_2}{2}$

62. (d) $\Delta U = U_f - U_i = -\frac{GMm}{nR + h} - \left(-\frac{GMm}{R}\right)$

$$= \frac{n}{n+1} \cdot \frac{GMm}{R} = \frac{n}{n+1} mgR$$

63. (c) $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$

$$T_1 = 2\pi\sqrt{\frac{R^3}{GM}},\quad T_2 = 2\pi\sqrt{\frac{(1.01R)^3}{GM}}$$

$$\frac{T_2 - T_1}{T_1} \times 100 = 1.5\%$$

64. (b) Electronic charge does not depend on acceleration due to gravity as it is a universal constant.
So, electronic charge on earth
= electronic charge on moon
$\therefore$ Required ratio = 1.

65. (a) $g = \frac{GM}{R^2}$ also $M = d \times \frac{4}{3}\pi R^3$

$\therefore g = \frac{4}{3} d\pi R$ at the surface of planet

$g_p = \frac{4}{3}(2d)\pi R'$, $g_e = \frac{4}{3}(d)\pi R$

$g_e = g_p \Rightarrow dR = 2d\,R'$
$\Rightarrow R' = R/2$

66. (d) We know that

$$g = \frac{GM}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\right)\rho}{R^2} = \frac{4}{3}\pi GR\rho$$

$$\frac{g'}{g} = \frac{R'}{R} = \frac{3R}{R} = 3 \quad \therefore g' = 3g$$

67. (a) K.E. of satellite moving in an orbit around the earth is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\sqrt{\frac{GM}{r}}\right)^2 = \frac{GMm}{2r}$$

P.E. of satellite and earth system is

$$U = \frac{GMm}{r} \Rightarrow \frac{K}{U} = \frac{\frac{GMm}{2r}}{\frac{GMm}{r}} = \frac{1}{2}$$

68. (b) According to Kepler's law, the areal velocity of a planet around the sun always remains constant.
SCD : $A_1 - t_1$ (areal velocity constant)
SAB : $A_2 - t_2$
$\frac{A_1}{t_1} = \frac{A_2}{t_2}$,
$t_1 = t_2 \cdot \frac{A_1}{A_2}$, (given $A_1 = 2A_2$)
$= t_2 \cdot \frac{2A_2}{A_2}$
$\therefore \; t_1 = 2t_2$

69. (b) Orbital velocity of a satellite in a circular orbit of radius a is given by

$$v = \sqrt{\frac{GM}{a}} \Rightarrow v \propto \sqrt{\frac{1}{a}} \Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{a_1}{a_2}}$$

$$\therefore \; v_2 = v_1\sqrt{\frac{4R}{R}} = 2\,v_1 = 6V$$

70. (a) Potential at the given point = Potential at the point due to the shell + Potential due to the particle

$$= -\frac{GM}{a} - \frac{2GM}{a} = -\frac{3GM}{a}$$

71. (b) Angular momentum is conserved
$\therefore L_1 = L_2$

$$\Rightarrow mr_1v_1 = mr_2v_2 \Rightarrow r_1v_1 = r_2v_2 \Rightarrow \frac{v_1}{v_2} = \frac{r_2}{r_1}$$

72. (a) The velocity u should be equal to the escape velocity.
That is, $u = \sqrt{2gR}$

But $g = \frac{GM}{R^2}$

$$\therefore u = \sqrt{2 \cdot \frac{GM}{R^2} \cdot R} \Rightarrow \sqrt{\frac{2GM}{R}}$$

73. (d) Intensity will be zero inside the spherical shell.

$I = 0$ upto $r = a$ and $I \propto \frac{1}{r^2}$ when $r > a$

74. (b) 75. (b)

EXERCISE - 3

Exemplar Questions

1. (d) Let the density of earth as a sphere is uniform, then it can be treated as point mass placed at its centre then acceleration due to gravity $g = 0$, at the centre. But if the density of earth is considered as a sphere of non-uniform then value of 'g' will be different at different points

$\left(\because g = \frac{4}{3}\pi\rho GR\right)$. So g cannot be zero at any point.

2. (c) Force of attraction between any two objects obeys the inverse square law.
As observed from the earth, the sun appears to move in an approximate circular orbit. The gravitational force of attraction between the earth and the sun always follows inverse square law.
Due to relative motion between the earth and mercury, the orbit of mercury, as observed from the earth will not be approximately circular, since the major gravitational force on mercury is due to the sun is very large than due to earth and due to the relative motion to sun and earth with mercury.

3. (a) As we know that, the torque on earth due to gravitational attractive force on earth is zero.
As the earth is revolving around the sun in a circular motion due to gravitational attraction. The force of attraction will be of radial nature i.e., angle between position vector r and force F is also, zero.

So, torque $= |\tau| = |r \times F| = rF\sin 0° = 0$

4. (c) As the total (P.E.) of the earth satellite orbiting in orbit is negative $\left(\frac{-GM}{2r}\right)$, where r is radius of the satellite and M is mass of the earth.
Due to the viscous force acting on satellite, energy decreases continuously and radius of the orbit or height decreases gradually.

5. (b) The major force acting on moon is due to gravitational force of attraction by sun and earth and moon is not always in line of joining sun and earth.
As observed from the sun, two types of forces are acting on the moon one is due to gravitational attraction between the sun and the moon and the other is due to gravitational attraction between the earth and the moon. So these two force have different lines of action and it will not be strictly elliptical because total force on the moon is not central.

6. (d) Asteroids are also being acted upon by central gravitational forces, hence Asteroid will move in circular orbits like planets and obey Kepler's laws.

7. (d) Gravitational mass of proton is equivalent to its inertial mass and is independent of presence of neighbouring heavy objects so verifies the option (d).

8. (c) Force of Gravitation, $F_g = \frac{GMm}{r^2}$

Let $AB = r$

So, force on B due to A

$= F_{BA} = \frac{G(2Mm)}{(AB)^2}$ towards BA.

$= \frac{G(2Mm)}{r^2} = 2F_g$

and force on B due to C

$= F_{BC} = \frac{GMm}{(BC)^2}$ towards BC

As, $(BC) = 2AB$

$\Rightarrow F_{BC} = \frac{GMm}{(2AB)^2} = \frac{GMm}{4(AB)^2} = \frac{GMm}{4r^2} = \frac{F_g}{4}$

As $F_{BA} > F_{BC}$,

hence, m will move towards (BA) i.e., $(2M)$.

NEET/AIPMT (2013-2017) Questions

9. (a) Initial P.E., $U_i = \frac{-GMm}{R}$,

Final P.E., $U_f = \frac{-GMm}{3R}$ $[\because R' = R + 2R = 3R]$

$\therefore$ Change in potential energy,

$\Delta U = \frac{-GMm}{3R} + \frac{GMm}{R}$

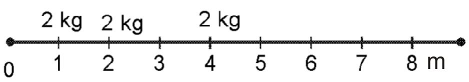
$= \frac{GMm}{R}\left(1 - \frac{1}{3}\right) = \frac{2}{3}\frac{GMm}{R} = \frac{2}{3}mgR$

$\left(\because \frac{GMm}{R} = mgR\right)$

ALTERNATE : $\Delta U = \frac{mgh}{1 + \frac{h}{R}}$

By placing the value of h = 2R we get

$\Delta U = \frac{2}{3}mgR.$

10. (c) 

Gravitational potential $V = \frac{-Gm}{r}$

$V_0 = -\frac{G \times 2}{1} - \frac{G \times 2}{2} - \frac{G \times 2}{4} - \frac{G \times 2}{8}$

$-2G\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\infty\right]$

$= -2G \times \frac{1}{1 - \frac{1}{2}} = -2G \times \frac{1}{\frac{1}{2}} = -4G.$

11. (c) Escape velocity, $V_e = R\sqrt{\frac{8}{3}\pi GP}$

$\Rightarrow V_e \propto R \Rightarrow \frac{V_P}{V_E} = \frac{R_P}{R_E} = 2$

$\Rightarrow V_P = 2V_E.$

12. (b) As we know, the minimum speed with which a body is projected so that it does not return back is called escape speed.

$V_e = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2GM}{4R}}$

$= \left(\frac{GM}{2R}\right)^{\frac{1}{2}}$ $(\because h = 3R)$

13. (b) First when (r < R) E ∝ r and then when r > R $E \propto \frac{1}{r^2}$.

Hence graph (b) correctly dipicts.

14. (c) From question,

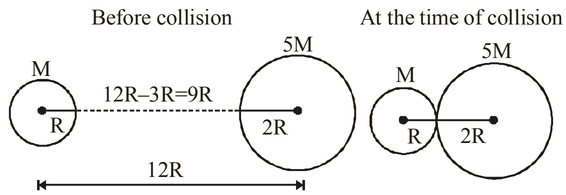
Escape velocity

$= \sqrt{\frac{2GM}{R}} = c = \text{speed of light}$

$\Rightarrow R = \frac{2GM}{c^2}$

$= \frac{2 \times 6.6 \times 10^{-11} \times 5.98 \times 10^{24}}{(3 \times 10^8)^2} m$

$= 10^{-2} m$

15. (b) 

Let the distance moved by spherical body of mass M is x_1 and by spherical body of mass 5m is x_2

As their C.M. will remain stationary

So, $(M)(x_1) = (5M)(x_2)$ or, $x_1 = 5x_2$

and for tauching $x_1 + x_2 = 9R$

So, $x_1 = 7.5R$

16. (a) As we know, orbital speed, $V_{orb} = \sqrt{\frac{GM}{r}}$

Time period $T = \frac{2\pi r}{v_{orb}} = \frac{2\pi r}{\sqrt{GM}}\sqrt{r}$

Squarring both sides,

$$T^2 = \left(\frac{2\pi r\sqrt{r}}{\sqrt{GM}}\right)^2 = \frac{4\pi^2}{GM}.r^3$$

$\Rightarrow \frac{T^2}{r^3} = \frac{4\pi^2}{GM} = K$

$\Rightarrow GMK = 4\pi^2$.

17. (d) **Given:** Height of the satellite from the earth's surface $h = 0.25 \times 10^6 m$

Radius of the earth $R = 6.38 \times 10^6 m$

Acceleration due to gravity $g = 9.8\ m/s^2$

Orbital velocity, $V_0 = ?$

$$V_0 = \sqrt{\frac{GM}{(R+h)}} = \sqrt{\frac{GM}{R^2}.\frac{R^2}{(R+h)}}$$

$$= \sqrt{\frac{9.8 \times 6.38 \times 6.38}{6.63 \times 10^6}}$$

$$= 7.76\ km/s \qquad \left[\because \frac{GM}{R^2} = g\right]$$

18. (c) The gravitational force on the satellite will be aiming towards the centre of the earth so acceleration of the satellite will also be aiming towards the centre of the earth.

19. (b) As we know, escape velocity,

$$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R}\cdot\left(\frac{4}{3}\pi R^3 \rho\right)} \propto R\sqrt{\rho}$$

$\therefore \frac{V_e}{V_p} = \frac{R_e}{R_p}\sqrt{\frac{\rho_e}{\rho_p}}$

$\Rightarrow \frac{V_e}{V_p} = \frac{R_e}{2R_e}\sqrt{\frac{\rho_e}{2\rho_e}}$

$\therefore$ Ratio $\frac{V_e}{V_p} = 1 : 2\sqrt{2}$

20. (a) As we know, gravitational potential (v) and acceleration due to gravity (g) with height

$$V = \frac{-GM}{R+h} = -5.4 \times 10^7 \quad \ldots(1)$$

$$\text{and } g = \frac{GM}{(R+h)^2} = 6 \quad \ldots(2)$$

Dividing (1) by (2)

$$\frac{\frac{-GM}{R+h}}{\frac{GM}{(R+h)^2}} = \frac{-5.4 \times 10^7}{6}$$

$\Rightarrow \frac{5.4 \times 10^7}{(R+h)} = 6$

$\Rightarrow R + h = 9000$ km so, $h = 2600$ km

21. (a) Both the astronauts are in the condition of weightlessness. Gravitational force between them pulls towards each other. Hence Astronauts move towards each other under mutual gravitional force.

22. (c) Above earth surface Below earth surface

$$g_h = g\left(1 - \frac{2h}{R_e}\right) \qquad g_d = g\left(1 - \frac{d}{R_e}\right)$$

According to question, $g_h = g_d$

$$g\left(1 - \frac{2h}{R_e}\right) = g\left(1 - \frac{d}{R_e}\right)$$

Clearly,

$d = 2h = 2$ km