Chapter-2 | Inverse Trigonometric Function
प्रतिलोम त्रिकोणमितीय फलन

MCQ:- (बहुविकल्पीय प्रश्न) -

Q 1 $\sin^{-1}x + \cos^{-1}x = \quad ; x \in [-1,1]$-

(a) $\pi/4$ (b) $\pi/2$

(c) π (d) $3\pi/2$

Q 2 $\tan^{-1}x + \cot^{-1}x = \quad ; x \in R$

(a) $3\pi/2$ (b) $\pi/4$

(c) $\pi/2$ (d) π

Q 3 $\text{cosec}^{-1}x + \sec^{-1}x = \quad ; |x| \geq 1$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) π (d) $\frac{3\pi}{2}$

Q 4 $\tan^{-1}x + \tan^{-1}y = \quad ; xy < 1$

(a) $\tan^{-1}\frac{x+y}{1-xy}$ (b) $\tan^{-1}\frac{x-y}{1+xy}$

(c) $\tan^{-1}\frac{x+y}{1+xy}$ (d) *None* / कोई नहीं

Q 5 $2\tan^{-1}x = \quad ; -1 < x < 1$

(a) $\tan^{-1}\frac{2x}{1-x^2}$ (b) $\tan^{-1}\frac{2x}{1+x^2}$

(c) $\tan^{-1}\frac{x}{1-x^2}$ (d) *None* / कोई नहीं

Q 6 $\cos(\sec^{-1}x + \text{cosec}^{-1}x) = \quad ; |x| \geq 1$

(a) $\frac{\pi}{2}$ (b) 0

(c) π (d) *None* / कोई नहीं

Q 7 $\cot(\tan^{-1}x + \cot^{-1}x) = \quad ?$

(a) 1 (b) $\frac{1}{2}$

(c) 0 (d) ∞

Q 8 $\tan^{-1}(1) + \tan^{-1}\left(\frac{1}{3}\right) = \quad ?$

(a) $\tan^{-1}\left(\frac{4}{3}\right)$ (b) $\tan^{-1}\left(\frac{2}{3}\right)$

(c) $\tan^{-1}(2)$ (d) $\tan^{-1}(3)$

Q 9 $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \quad ?$

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

Q 10 $\sin\left(\cos^{-1}\frac{3}{5}\right) = \quad ?$

(a) $\frac{3}{4}$ (b) $\frac{4}{5}$

(c) $\frac{3}{5}$ (d) *Not possible* / संभव नही हैं।

Q 11 $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \quad ?$

(a) $8\pi/3$ (b) $\pi/2$

(c) $3\pi/4$ (d) π

Q 12 If $\cot^{-1}\left(-\frac{1}{5}\right) = x$ then $\sin x = \quad ?$

यदि $\cot^{-1}\left(-\frac{1}{5}\right) = x$, तो $\sin x$ का मान क्या होगा -

(a) $\frac{1}{\sqrt{26}}$ (b) $\frac{5}{\sqrt{26}}$

(c) $\frac{1}{\sqrt{24}}$ (d) *Not possible* / संभव नही हैं।

Q 13 If $\cot^{-1}(-\sqrt{3}) = x$, where $x \in [0,\pi]$ then value of x is -

यदि $\cot^{-1}(-\sqrt{3}) = x$ जहां $x \in [0,\pi]$ तो x का मान क्या होगा -

(a) $5\pi/6$ (b) $2\pi/3$

(c) $\pi/2$ (d) $\pi/6$

Q 14 If $\cot^{-1}(-1) = x$, where $x \in [0,\pi]$ then value of x is -

यदि $\cot^{-1}(-1) = x$ जहां $x \in [0,\pi]$ तो x का मान क्या होगा -

(a) $\pi/2$ (b) $3\pi/4$

(c) π (d) $3\pi/2$

Q 15 $\sin(\tan^{-1}x + \cot^{-1}x) =$; $(x \in R)$

(a) 1 (b) 2

(c) 3 (d) 4

Very Short Question :- (अति लघु उत्तरीय प्रश्न)

2 Marks Question:-

Q 1 Write $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right), x > 1$ in the simplest form .

$\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right), x > 1$ को सरलतम रूप मे व्यक्त करें ।

Q 2 Prove that , $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

सिद्ध करें कि $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

Q 3 Prove that ,

$3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$; $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

सिद्ध करें कि,

$3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$; $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Q 4 Write in simplest form ,

$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right); 0 < x < \pi$

सरलतम रूप मे व्यक्त करें

$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right); 0 < x < \pi$

Q 5 Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$.

$\sin^{-1}\left(-\frac{1}{2}\right)$ का मुख्य मान ज्ञात करें ?

Q 6 Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$. का मुख्य मान ज्ञात करें ?

Q 7 If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$, then find the value of x .

यदि $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$ तो x का मान ज्ञात करें ।

Q 8 Find the value of , $\tan\left[2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right]$

$\tan\left[2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right]$ का मान निकाले।

Q 9 If $\tan^{-1}x + \tan^{-1}3 = \tan^{-1}8$, then find the value of x .

यदि $\tan^{-1}x + \tan^{-1}3 = \tan^{-1}8$ तो x का मान ज्ञात करें ।

Short Question : - (लघु उत्तरीय प्रश्न)

3 Marks Question:-

Q 1 Show that , $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

दिखाएँ कि $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

Q 2 Find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ का मुख्य मान निकाले।

Q 3 Find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$ का मान ज्ञात करें।

Q 4 Evaluate , $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

हल करें , $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

Q 5. Evaluate , $\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]$

हल करें , $\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]$

Q 6. If $\tan^{-1}\frac{4}{3} = \theta$, then find the value of $\cos\theta$.

यदि $\tan^{-1}\frac{4}{3} = \theta$ तो $\cos\theta$. का मान ज्ञात करें ?

Q 7 Prove that , $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right) = 3\tan^{-1}\frac{x}{a}$

सिद्ध करें $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right) = 3\tan^{-1}\frac{x}{a}$

Long Question ; - (दीर्घ उत्तरीय प्रश्न)

5 Marks Question:-

Q 1 Express $\tan^{-1}\frac{\cos x}{1 - \sin x}$; $-\frac{3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form .

सरल रूप में व्यक्त करें,

$\tan^{-1}\frac{\cos x}{1 - \sin x}$; $-\frac{3\pi}{2} < x < \frac{\pi}{2}$

Q 2 Solve $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$

हल करें, $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$

Q 3 Show that , $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$.

दिखाएं कि, $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$.

Q 4 Evaluate ,

$\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

हल करें, $\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

Q 5 Evaluate ,

$\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

हल करें, $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

Q 6 Find the value of ,

$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ का मान निकालें ।

Q 7 Prove that,

$\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \left(\frac{\pi}{4} - x\right); x < \pi$

सिद्ध करें, $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \left(\frac{\pi}{4} - x\right); x < \pi$

Q 8 Prove that, $\tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right) = \left(\frac{\pi}{4} - \frac{x}{2}\right)$

सिद्ध करें, $\tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right) = \left(\frac{\pi}{4} - \frac{x}{2}\right)$

Q 9 prove that

$\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$

[Hint : Let x= cosθ]

सिद्ध करें, $\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$

Answer : "Inverse Trigonometric Function Solution"

MCQ 1 - Marks Solution

1 - (b) 2 - (c) 3 - (a) 4 - (a) 5 - (a) 6 - (b) 7 - (c) 8 - (c)
9 - (b) 10 - (b) 11 - (d) 12 - (b) 13 - (a) 14 - (b) 15 - (a)

2- Marks Solution

1 Ans : -

$\because \cot^{-1}\left(\frac{1}{\sqrt{x^2 - 1}}\right) \quad : x > 1$

let us suppose,

$x = \sec\theta \quad \Rightarrow \theta = \sec^{-1}x$

$\therefore \cot^{-1}\left(\frac{1}{\sqrt{x^2 - 1}}\right) = \cot^{-1}\left(\frac{1}{\sqrt{\sec^2\theta - 1}}\right)$

$\Rightarrow \cot^{-1}\left(\frac{1}{\sqrt{x^2 - 1}}\right) = \cot^{-1}\left(\frac{1}{\tan\theta}\right)$

$= \cot^{-1}(\cot\theta)$

$= \theta$

$\Rightarrow \cot^{-1}\left(\frac{1}{\sqrt{x^2 - 1}}\right) = \theta = \sec^{-1}x \quad , \ x > 1$

2 Ans : L.H.S $= \tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24}$

$= \tan^{-1}\left(\frac{\frac{2}{11} + \frac{7}{24}}{1 - \frac{2}{11} \times \frac{7}{24}}\right)$

$= \tan^{-1}\left(\frac{\frac{48 + 77}{11 \times 24}}{\frac{264 - 14}{11 \times 24}}\right)$

$= \tan^{-1}\left(\frac{125}{250}\right)$

L.H.S $= \tan^{-1}\left(\frac{1}{2}\right) =$ R.H.S

3 Ans : -

Let us suppose

$x = \sin\theta \quad \Rightarrow \theta = \sin^{-1}x \quad ...(1)$

$\because$ L.H.S $= 3\sin^{-1}x$

$= 3\theta \quad ....(2)$

and ,

R.H.S $= \sin^{-1}(3x - 4x^3)$

$= \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$= \sin^{-1}(\sin 3\theta)$

R.H.S $= 3\theta \quad ....(3)$

From equation (2) and (3) we get,

$3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$

4 Ans : -

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right) \quad ;0 < x < \pi$$

$$= \tan^{-1}\left(\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}\right)$$

$$= \tan^{-1}\left(\sqrt{\tan^2\frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\tan\frac{x}{2}\right)$$

$$= \frac{x}{2}$$

5 Ans : -

$$\sin^{-1}\left(-\frac{1}{2}\right) = \sin^{-1}\left(-\sin\frac{\pi}{6}\right)$$

$$= \sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$$

$$= -\frac{\pi}{6}$$

$-\frac{\pi}{6}$ **be the required principal value as** $-\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

6 Ans : -

$$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \cos^{-1}\left[-\cos\left(\frac{\pi}{4}\right)\right]$$

$$= \cos^{-1}\left[\cos\left(\pi - \frac{\pi}{4}\right)\right]$$

$$= \pi - \frac{\pi}{4}$$

$$= \frac{3\pi}{4} \in [0, \pi]$$

$\Rightarrow \frac{3\pi}{4}$ **be the required principal value .**

7 Ans : -

$$\because \sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$$

$$\Rightarrow \sin^{-1}\frac{1}{5} + \cos^{-1}x = \sin^{-1}1$$

$$\Rightarrow \cos^{-1}x = \sin^{-1}(1) - \sin^{-1}\left(\frac{1}{5}\right)$$

$$= \sin^{-1}\left(\sin\frac{\pi}{2}\right) - \sin^{-1}\frac{1}{5}$$

$$\Rightarrow \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}\frac{1}{5}$$

$$\Rightarrow \cos^{-1}x = \cos^{-1}\frac{1}{5}$$

$$\Rightarrow x = \frac{1}{5}$$

8 Ans : -

$$\tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$$

Let $\quad 2\tan^{-1}\frac{1}{5} = \theta \quad ....(1)$

$$\Rightarrow \frac{1}{5} = \tan\frac{\theta}{2}$$

$$\Rightarrow \sin\frac{\theta}{2} = \frac{1}{\sqrt{26}} \ \& \cos\frac{\theta}{2} = \frac{5}{\sqrt{26}}$$

$$\therefore \tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$$

$$= \tan\left(\theta - \frac{\pi}{4}\right)$$

$$= \frac{\tan\theta - \tan\frac{\pi}{4}}{1 + \tan\theta \times \tan\frac{\pi}{4}}$$

$$= \frac{\tan\theta - 1}{1 + \tan\theta} = \frac{\sin\theta - \cos\theta}{\cos\theta + \sin\theta}$$

$$\Rightarrow \tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2} - \left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)}$$

$$= \frac{2 \times \frac{1}{\sqrt{26}} \times \frac{5}{\sqrt{26}} - \left(\frac{25}{26} - \frac{1}{26}\right)}{2 \times \frac{1}{\sqrt{26}} \times \frac{5}{\sqrt{26}} + \left(\frac{25}{26} - \frac{1}{26}\right)}$$

$$= \frac{\frac{5}{13} - \frac{12}{13}}{\frac{12}{13} + \frac{5}{13}} = \frac{-\frac{7}{13}}{\frac{17}{13}}$$

$$\Rightarrow \tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right) = -\frac{7}{17}$$

9 Ans : -

$$\because \tan^{-1}x + \tan^{-1}3 = \tan^{-1}8$$

$$\Rightarrow \tan^{-1}x = \tan^{-1}8 - \tan^{-1}3$$

$$= \tan^{-1}\left(\frac{8-3}{1+8\times 3}\right)$$

$$= \tan^{-1}\left(\frac{5}{1+24}\right) = \tan^{-1}\left(\frac{5}{25}\right)$$

$$\Rightarrow \tan^{-1}x = \tan^{-1}\frac{1}{5}$$

$$\therefore x = \frac{1}{5}$$

3 Marks Solution

1 Ans : -

$$\text{L.H.S} = 2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{1}{2} + \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{7}}{1-\frac{1}{2}\times\frac{1}{7}}\right)$$

$$= \tan^{-1}\frac{1}{2} + \tan^{-1}\left(\frac{\frac{9}{14}}{\frac{13}{14}}\right)$$

$$= \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{9}{13}$$

$$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{9}{13}}{1-\frac{1}{2}\times\frac{9}{13}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{31}{26}}{\frac{17}{26}}\right)$$

$$\Rightarrow \text{L.H.S} = \tan^{-1}\left(\frac{31}{17}\right) = \text{R.H.S}$$

2 Ans ; -

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \sin^{-1}\left(\sin\frac{\pi}{4}\right)$$

$$= \frac{\pi}{4}$$

$\because$ **Principal interval of sin is** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

and $\frac{\pi}{4} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow \frac{\pi}{4}$ **is the principal value of** $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

3 Ans : -

$$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$$

$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \sec^{-1}\left(-\sec\frac{\pi}{3}\right)$$

$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \sec^{-1}\left(\sec\left(\pi-\frac{\pi}{3}\right)\right)$$

$$= \frac{\pi}{3} - \left(\pi - \frac{\pi}{3}\right)$$

$$= \frac{\pi}{3} - \pi + \frac{\pi}{3}$$

$$= \frac{2\pi}{3} - \pi = \frac{2\pi - 3\pi}{3} = -\frac{\pi}{3}$$

$$\Rightarrow \tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = -\frac{\pi}{3}$$

4 Ans : -

$$\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$$

$$= \sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\sin\frac{\pi}{6}\right)\right)$$

$$= \sin\left[\frac{\pi}{3} - \sin^{-1}\left\{\sin\left(-\frac{\pi}{6}\right)\right\}\right]$$

$$\Rightarrow \sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$$

$$= \sin\left(\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right)$$

$$= \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$$

$$= \sin\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right) = 1$$

5 Ans : -

$$\text{Let } \cos^{-1}\left(-\frac{3}{5}\right) = \theta \quad ....(1)$$

$$\Rightarrow \cos\theta = -\frac{3}{5}$$

$$\therefore \sin\theta = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}}$$

$$\Rightarrow \sin\theta = \frac{4}{5}$$

$$\therefore \sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right] = \sin(2\theta) \qquad [\text{from } (1)]$$

$$= 2\sin\theta\cdot\cos\theta$$

$$= 2\times\left(\frac{4}{5}\right)\times\left(-\frac{3}{5}\right)$$

$$= -\frac{24}{25}$$

$$\Rightarrow \sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right] = -\frac{24}{25}$$

6 Ans : -

$$\because \tan^{-1}\frac{4}{3}=\theta$$

$$\Rightarrow \tan\theta=\frac{4}{3} \text{ ....(1)}$$

as $\quad 1+\tan^2\theta=\sec^2\theta$

$$\Rightarrow 1+\left(\frac{4}{3}\right)^2=\sec^2\theta$$

$$\Rightarrow 1+\frac{16}{9}=\sec^2\theta$$

$$\Rightarrow \frac{25}{9}=\sec^2\theta$$

$$\therefore \sec\theta=\pm\frac{5}{3}$$

$$\Rightarrow \cos\theta=\frac{1}{\sec\theta}=\frac{1}{\pm\left(5/3\right)}$$

$$\Rightarrow \cos\theta=\pm\frac{3}{5}$$

7 Ans : -

Let $x = a\tan\theta$ ....(1)

$$\therefore \text{L.H.S} = \tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$$

$$=\tan^{-1}\left[\frac{3a^3\tan\theta-a^3\tan^3\theta}{a^3-3a^3\tan^2\theta}\right]$$

$$=\tan^{-1}\left[\frac{a^3(3\tan\theta-\tan^3\theta)}{a^3(1-3\tan^2\theta)}\right]$$

$$=\tan^{-1}\left[\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right]$$

$$=\tan^{-1}[\tan 3\theta]$$

$$=3\theta$$

$$\text{L.H.S}=3\theta=3\cdot\tan^{-1}\frac{x}{a}=\text{R.H.S}$$

5 Marks Solutions

1 Ans :

$$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) \quad ; -\frac{3\pi}{2}<x<\frac{\pi}{2}$$

-

$$=\tan^{-1}\left(\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{1-2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}\right)$$

$$=\tan^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}\right]$$

$$=\tan^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}\right]$$

$$=\tan^{-1}\left[\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right]$$

$$=\tan^{-1}\left[\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right]$$

$$=\tan^{-1}\left[\frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\cdot\tan\frac{x}{2}}\right]$$

$$=\tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$$

$$\Rightarrow \tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)=\frac{\pi}{4}+\frac{x}{2} \quad ; -\frac{3\pi}{2}<x<\frac{\pi}{2}$$

2 Ans : -

$$\because \tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{2x+3x}{1-2x\cdot 3x}\right)=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{5x}{1-6x^2}\right)=\frac{\pi}{4}$$

$$\Rightarrow \frac{5x}{1-6x^2}=\tan\frac{\pi}{4}$$

$$\Rightarrow \frac{5x}{1-6x^2}=1$$

$$\Rightarrow 1-6x^2=5x$$

$$\Rightarrow 6x^2+5x-1=0$$

$$\Rightarrow 6x^2+6x-x-1=0$$

$$\Rightarrow 6x(x+1)-(x+1)=0$$

$$\Rightarrow (x+1)(6x-1)=0$$

$\therefore$ either $x+1=0$ or $6x-1=0$

$$\Rightarrow x=-1 \quad \text{or} \quad x=\frac{1}{6}$$

3 Ans : -

let $\sin^{-1}\frac{3}{5}=x$ ....(1)

$$\Rightarrow \sin x=\frac{3}{5}$$

$$\therefore \cos x=\sqrt{1-\sin^2 x}$$

$$= \sqrt{1 - \frac{9}{25}}$$

$$= \sqrt{\frac{16}{25}}$$

$$\Rightarrow \cos x = \frac{4}{5}$$

and $\sin^{-1}\frac{8}{17} = y$ ....(2)

$$\Rightarrow \sin y = \frac{8}{17}$$

$$\therefore \cos y = \sqrt{1 - \left(\frac{8}{17}\right)^2}$$

$$= \sqrt{\frac{289 - 64}{289}}$$

$$= \sqrt{\frac{225}{289}}$$

$$\Rightarrow \cos y = \frac{15}{17}$$

as , $\cos(x - y) = \cos x \cos y + \sin x \sin y$

$$= \frac{4}{5} \times \frac{15}{17} + \frac{3}{5} \times \frac{8}{17}$$

$$= \frac{60}{85} + \frac{24}{85}$$

$$= \frac{60 + 24}{85}$$

$$\Rightarrow \cos(x - y) = \frac{84}{85}$$

$$\Rightarrow x - y = \cos^{-1}\frac{84}{85}$$

[by using equation (1) and (2)]

$$\Rightarrow \sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$$

4 Ans :

$$\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$$

we know that principal interval of cos is $[0 , \pi]$

$$\therefore \cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$$

$$= \cos\left[\cos^{-1}\left(-\cos\frac{\pi}{6}\right) + \frac{\pi}{6}\right]$$

$$= \cos\left[\cos^{-1}\left(\cos\left(\pi - \frac{\pi}{6}\right)\right) + \frac{\pi}{6}\right]$$

$$= \cos\left[\cos^{-1}\left(\cos\frac{5\pi}{6}\right) + \frac{\pi}{6}\right]$$

$$= \cos\left[\frac{5\pi}{6} + \frac{\pi}{6}\right] \quad ; \text{as } \frac{5\pi}{6} \in [0 , \pi]$$

$$\Rightarrow \therefore \cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$$

$$= \cos\left[\frac{5\pi + \pi}{6}\right]$$

$$= \cos\left(\frac{6\pi}{6}\right)$$

$$= \cos(\pi)$$

$$= -1$$

$$\therefore \cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right] = -1$$

5 Ans ; -

$$\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$$

we know that principal interval of sin is $\left[-\frac{\pi}{2} , \frac{\pi}{2}\right]$

$$\therefore \sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$$

$$= \sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\sin\frac{\pi}{3}\right)\right]$$

$$= \sin\left\{\frac{\pi}{2} - \sin^{-1}\left[\sin\left(-\frac{\pi}{3}\right)\right]\right\}$$

$$= \sin\left[\frac{\pi}{2} - \left(-\frac{\pi}{3}\right)\right]$$

$$= \sin\left[\frac{\pi}{2} + \frac{\pi}{3}\right]$$

$$= \sin\left[\frac{3\pi + 2\pi}{6}\right]$$

$$= \sin\left[\frac{5\pi}{6}\right]$$

$$= \cos\frac{\pi}{3}$$

$$= \frac{1}{2}$$

$$\therefore \sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] = \frac{1}{2}$$

6 Ans : -

$$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right) \quad ....(1)$$

we know that principal interval of sin is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, cos is $[0 , \pi]$ and tan is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{2}\right\}$

$$\therefore \tan^{-1}(1) = \tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$\Rightarrow \tan^{-1} 1 = \frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \quad ....(2)$$

$$\cos^{-1}\left(-\frac{1}{2}\right) = \cos^{-1}\left[-\cos\frac{\pi}{3}\right]$$

$$= \cos^{-1}\left[\cos\left(\pi - \frac{\pi}{3}\right)\right]$$

$$= \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3} \in [0, \pi] \text{ ....(3)}$$

And $\sin^{-1}\left(-\frac{1}{2}\right) = \sin^{-1}\left(-\sin\frac{\pi}{6}\right)$

$$= \sin^{-1}\left[\sin\left(-\frac{\pi}{6}\right)\right]$$

$$= -\frac{\pi}{6} \text{ .... (4)}$$

by using eq^{ns} (2),(3) and (4)

$$eq^{n}(1) \Rightarrow \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$$

$$\Rightarrow \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= \frac{3\pi + 8\pi - 2\pi}{12}$$

$$= \frac{9\pi}{12}$$

$$= \frac{3\pi}{4}$$

7 Ans : -

$$\text{L.H.S} = \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$$

$$= \tan^{-1}\left[\frac{\cos x(1 - \tan x)}{\cos x(1 + \tan x)}\right]$$

$$= \tan^{-1}\left[\frac{1 - \tan x}{1 + \tan x}\right]$$

$$= \tan^{-1}\left[\frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4} \times \tan x}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - x\right)\right]$$

$$= \left(\frac{\pi}{4} - x\right) = \text{R.H.S}$$

hence ,

$$\therefore \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \left(\frac{\pi}{4} - x\right)$$

8 Ans: -

$$\text{L.H.S} = \tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$$

$$= \tan^{-1}\left[\frac{\cos 2\frac{x}{2}}{1 + \sin 2\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{1 + 2\sin\frac{x}{2}\cos\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}\right]$$

$$= \tan^{-1}\left[\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}\right]$$

$$= \tan^{-1}\left[\frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\tan\frac{\pi}{4} - \tan\frac{x}{2}}{1 + \tan\frac{\pi}{4} \times \tan\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]$$

$$\Rightarrow \text{L.H.S} = \left(\frac{\pi}{4} - \frac{x}{2}\right) = \text{R.H.S}$$

9 Ans :

$$\text{L.H.S} = \tan^{-1}\left\{\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right\}$$

Let $x = \cos\theta$ ....(1)

$$= \tan^{-1}\left\{\frac{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}\right\}$$

$$= \tan^{-1}\left\{\frac{\sqrt{2\cos^2\frac{\theta}{2}} - \sqrt{2\sin^2\frac{\theta}{2}}}{\sqrt{2\cos^2\frac{\theta}{2}} + \sqrt{2\sin^2\frac{\theta}{2}}}\right\}$$

$$= \tan^{-1}\left\{\frac{\sqrt{2}\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)}{\sqrt{2}\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)}\right\}$$

$$= \tan^{-1}\left\{\frac{1 - \tan\frac{\theta}{2}}{1 + \tan\frac{\theta}{2}}\right\}$$

$$= \tan^{-1}\left\{\frac{\tan\frac{\pi}{4} - \tan\frac{\theta}{2}}{1 + \tan\frac{\pi}{4} \times \tan\frac{\theta}{2}}\right\}$$

$$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)\right\}$$

$$= \frac{\pi}{4} - \frac{\theta}{2}$$

$$= \frac{\pi}{4} - \frac{1}{2} \times \cos^{-1}x \text{ [by using eqn (1)]}$$

$$= \text{R.H.S}$$

hence ,

$$\tan^{-1}\left\{\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right\} = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$$