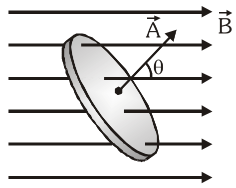
ELECTROMAGNETIC INDUCTION & ALTERNATING CURRENT

KEY CONCEPT

MAGNETIC FLUX



The magnetic flux (ϕ) linked with a surface held in a magnetic field (B) is defined as the number of magnetic lines of force crossing that area (A). If θ is the angle between the direction of the field and normal to the area, (area vector) then $\phi = \vec{B}.\vec{A} = BA\cos\theta$

FLUX LINKAGE

If a coil has more than one turn, then the flux through the whole coil is the sum of the fluxes through the individual turns. If the magnetic field is uniform, the flux through one turn is $\phi = BA\cos\theta$ If the coil has N turns, the total flux linkage $\phi = NBA\cos\theta$

- Magnetic lines of force are imaginary, magnetic flux is a real scalar physical quantity with dimensions

$$[\phi] = B \times \text{area} = \left[\frac{F}{IL}\right][L^2] \qquad \because\ B = \frac{F}{IL\sin\theta} \qquad [\because F = BIL\sin\theta]$$

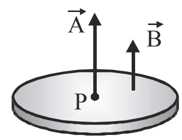
$$\therefore \quad [\phi] = \left[\frac{MLT^2}{AL}\right][L^2] = [ML^2T^{-2}A^{-1}]$$

SI UNIT of magnetic flux :

$\because$ $[ML^2T^{-2}]$ corresponds to energy

$$\frac{\text{joule}}{\text{ampere}} = \frac{\text{joule} \times \text{second}}{\text{coulomb}} = \text{ weber (Wb)}$$

or $T\text{–}m^2$ (as tesla = Wb/m^2) $\left[\text{ampere} = \frac{\text{coulomb}}{\text{sec ond}}\right]$



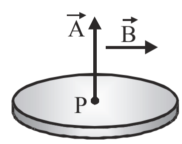
$\phi_{max} = BA$

- For a given area flux will be maximum :

when magnetic field $\vec{B}$ is normal to the area

$\theta = 0°$ $\Rightarrow$ $\cos\theta$ = maximum = 1 $\phi_{max} = BA$

For a given area flux will be minimum :

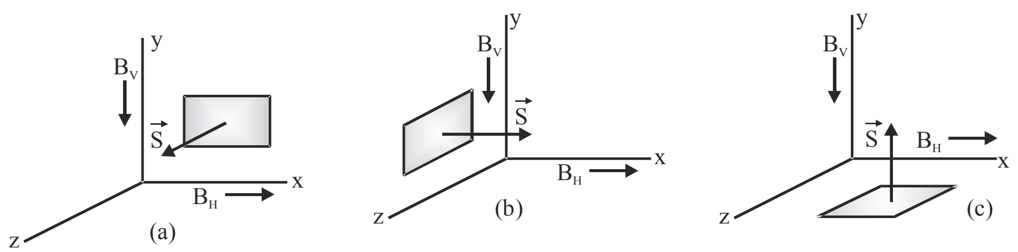


$\phi_{min} = 0$

when magnetic field $\vec{B}$ is parallel to the area

$\theta = 90°$ $\Rightarrow \cos\theta$ = minimum = 0 $\phi_{min} = 0$

Ex. At a given plane, horizontal and vertical components of earth's magnetic field B_H and B_V are along x and y axes respectively as shown in figure. What is the total flux of earth's magnetic field associated with an area S, if the area S is in (a) x-y plane (b) y-z plane and (c) z-x plane ?



Sol. $\vec{B} = \hat{i}B_H - \hat{j}B_V = \text{constant}$, so $\phi = \vec{B}.\vec{S}$ [$\vec{B}$ = constant]

(a) For area in x-y plane $\vec{S} = S\hat{k}$, $\phi_{xy} = (\hat{i}B_H - \hat{j}B_V).(\hat{k}S) = 0$

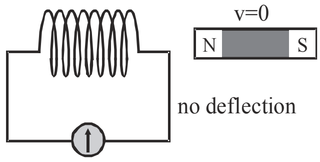
(b) For area S in y-z plane $\vec{S} = S\hat{i}$, $\phi_{yz} = (\hat{i}B_H - \hat{j}B_V).(\hat{i}S) = B_H S$

(c) For area S in z-x plane $\vec{S} = S\hat{j}$, $\phi_{zx} = (\hat{i}B_H - \hat{j}B_V).(\hat{j}S) = -B_V S$

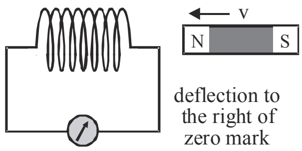
Negative sign implies that flux is directed vertically downwards.

FARADAY'S EXPERIMENT

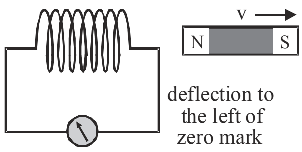
Faraday performed various experiments to discover and understand the phenomenon of electromagnetic induction. Some of them are :



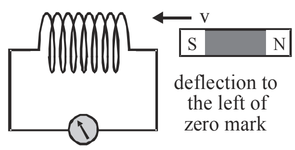
- When the magnet is held stationary anywhere near or inside the coil, the galvanometer does not show any deflection.



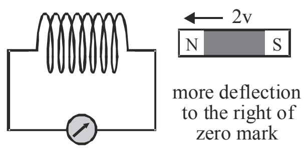
- When the N-pole of a strong bar magnet is moved towards the coil, the galvanometer shows a deflection right to the zero mark.



- When the N-pole of a strong bar magnet is moved away from the coil, the galvanometer shows a deflection left to the zero mark.



- If the above experiments are repeated by bringing the S-pole of the, magnet towards or away from the coil, the direction of current in the coil is opposite to that obtained in the case of N-pole.



- The deflection in galvanometer is more when the magnet moves faster and less when the magnet moves slow.

CONCLUSIONS

Whenever there is a relative motion between the source of magnetic field (magnet) and the coil, an emf is induced in the coil. When the magnet and coil move towards each other then the flux linked with the coil increases and emf is induced. When the magnet and coil move away from each other the magnetic flux linked with the coil decreases, again an emf is induced. This emf lasts so long the flux is changing.

Due to this emf an electric current start to flow and the galvanometer shows deflection.

The deflection in galvanometer last as long the relative motion between the magnet and coil continues.

Whenever relative motion between coil and magnet takes place an induced emf produced in coil. If coil is in closed circuit then current and charge is also induced in the circuit. This phenomenon is called electro magnetic induction.

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

Faraday's law of induction may be stated as follows:

The induced emf ε in a coil is proportional to the negative of the rate of change of magnetic flux:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

For a coil that consists of *N* loops, the total induced emf would be *N* times as large:

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

Thus, we see that an emf may be induced in the following ways:

(i) by varying the magnitude of $\vec{B}$ with time (illustrated in Figure)

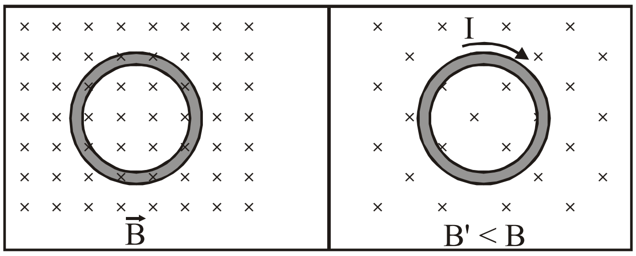


Figure : Inducing emf by varying the magnetic field strength

(ii) by varying the magnitude of $\vec{A}$, i.e., the area enclosed by the loop with time (illustrated in Figure)

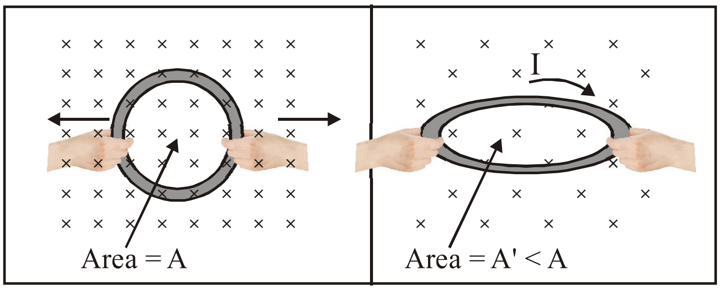


Figure : Inducing emf by changing the area of the loop

(iii) varying the angle between $\vec{B}$ and the area vector $\vec{A}$ with time (illustrated in Figure)

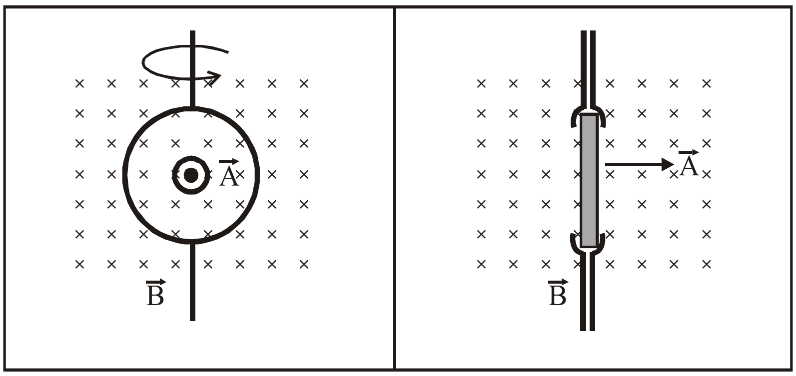


Figure : Inducing emf by varying the angle between **B** and **A**

LENZ'S LAW

Lenz's Law :

The direction of the induced current is determined by Lenz's law:

The induced current produces magnetic fields which tend to oppose the change in magnetic flux that induces such currents.

To illustrate how Lenz's law works, let's consider a conducting loop placed in a magnetic field. We follow the procedure below:

1. Define a positive direction for the area vector $\vec{A}$.

2. Assuming that $\vec{B}$ is uniform, take the dot product of $\vec{B}$ and $\vec{A}$. This allows for the determination of the sign of the magnetic flux Φ_B.

3. Obtain the rate of flux change $d\Phi_B/dt$ by differentiation. There are three possibilities:

$$\frac{d\Phi_B}{dt} : \begin{cases} > 0 \Rightarrow \text{induced emf } \varepsilon < 0 \\ < 0 \Rightarrow \text{induced emf } \varepsilon > 0 \\ = 0 \Rightarrow \text{induced emf } \varepsilon = 0 \end{cases}$$

4. Determine the direction of the induced current using the right-hand rule. With your thumb pointing in the direction of $\vec{A}$, curl the fingers around the closed loop. The induced current flows in the same direction as the way your fingers curl if $\varepsilon > 0$, and the opposite direction if $\varepsilon < 0$, as shown in Figure.

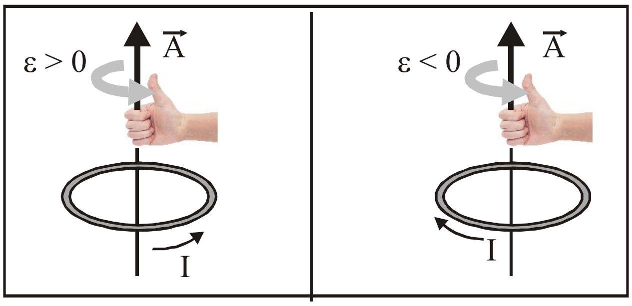


Figure : Determination of the direction of induced current by the right-hand rule

In Figure we illustrate the four possible scenarios of time-varying magnetic flux and show how Lenz's law is used to determine the direction of the induced current *I*.

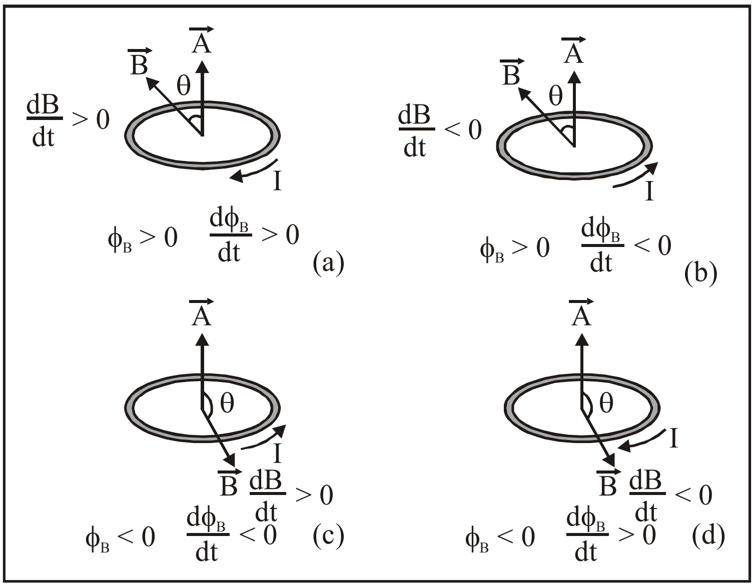


Figure : Direction of the induced current using Lenz's law

The above situations can be summarized with the following sign convention:

Φ_B	$d\Phi_B/dt$	ε	I
+	+	−	−
	−	+	+
−	+	−	−
	−	+	+

The positive and negative signs of *I* correspond to a counter clockwise and clockwise currents, respectively.

Ex. The radius of a coil decreases steadily at the rate of 10^{-2} m/s. A constant and uniform magnetic field of induction 10^{-3} Wb/m^2 acts perpendicular to the plane of the coil. What will be the radius of the coil when the induced e.m.f. in the 1μV

Sol. Induced emf $e = \frac{d(BA)}{dt} = \frac{Bd(\pi r^2)}{dt} = 2B\pi r\frac{dr}{dt}$ radius of coil $r = \frac{e}{2B\pi\left(\frac{dr}{dt}\right)} = \frac{1\times10^{-6}}{2\times10^{-3}\times\pi\times10^{-2}} = \frac{5}{\pi}$ cm

Ex. The ends of a search coil having 20 turns, area of cross-section 1 cm^2 and resistance 2 ohms are connected to a ballistic galvanometer of resistance 40 ohms. If the plane of search coil is inclined at 30° to the direction of a magnetic field of intensity 1.5 Wb/m^2, coil is quickly pulled out of the field to a region of zero magnetic field, calculate the charge passed through the galvanometer.

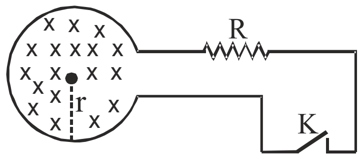
Sol. The total flux linked with the coil having turns N and area A is

$$\phi_1 = N(\vec{B}.\vec{A}) = NBA\cos\theta = NBA\cos(90° - 30°) = \frac{NBA}{2}$$

when the coil is pulled out, the flux becomes zero, $\phi_2 = 0$ so change in flux is $\Delta\phi = \frac{NBA}{2}$

the charge flowed through the circuit is $q = \frac{\Delta\phi}{R} = \frac{NBA}{2R} = \frac{20\times1.5\times10^{-4}}{2\times42} = 0.357\times10^{-4}$ C

Ex. Shown in the figure is a circular loop of radius r and resistance R. A variable magnetic field of induction $B = B_0 e^{-t}$ is established inside the coil. If the key (K) is closed. Then calculate the electrical power developed right after closing the key.



Sol. Induced emf $e = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = A\frac{dB}{dt} = \pi r^2 B_0\frac{d}{dt}(e^{-t}) = -\pi r^2 B_0 e^{-t}$

At t = 0, $e_0 = B_0 e^{-0} . \pi r^2 = B_0 \pi r^2$

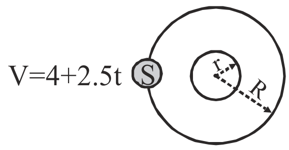
The electric power developed in the resistor R just at the instant of closing the key is

$$P = \frac{e_0^2}{R} = \frac{B_0^2\pi^2 r^4}{R}$$

Ex. Two concentric coplanar circular loops made of wire, resistance per unit length 10^{-4} Ωm^{-1}, have diameters 0.2 m and 2 m. A time-varying potential difference (4 + 2.5 t) volt is applied to the larger loop. Calculate the current in the smaller loop.

Sol. The magnetic field at the centre O due to the current in the larger loop is $B=\frac{\mu_0 I}{2R}$

If ρ is the resistance per unit length, then



$$I=\frac{\text{potential difference}}{\text{resistance}}=\frac{4+2.5t}{2\pi R\cdot\rho}$$

$\therefore$ $$B=\frac{\mu_0}{2R}\cdot\frac{4+2.5t}{2\pi R\rho}$$

$\because$ r << R, so the field B can be taken almost constant over the entire area of the smaller loop.

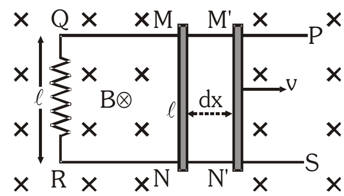
$\therefore$ the flux linked with the smaller loop is $\phi=B\times\pi r^2=\frac{\mu_0}{2R}\cdot\frac{4+2.5t}{2\pi R\rho}\cdot\pi r^2$

Induced emf $e=\frac{d\phi}{dt}=\frac{\mu_0 r^2}{4R^2\rho}\times 2.5$

The corresponding current in the smaller loop is I' then

$$I'=\frac{e}{R}=\frac{\mu_0 r^2}{4R^2\rho}\times 2.5\times\frac{1}{2\pi r\rho}=\frac{2.5\mu_0 r}{8\pi R^2\rho^2}=\frac{2.5\times 4\pi\times 10^{-7}\times 0.1}{8\pi\times(1)^2\times(10^{-4})^2}=1.25A$$

Induced emf by changing the area of the coil



A U shaped frame of wire, PQRS is placed in a uniform magnetic field B perpendicular to the plane and vertically inward. A wire MN of length ℓ is placed on this frame. The wire MN moves with a speed v in the direction shown. After time dt the wire reaches to the position M'N' and distance covered = dx.

The change in area ΔA = Length × area = ℓdx

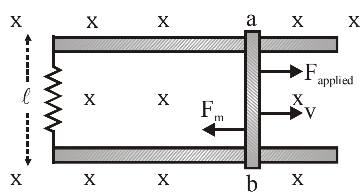
Change in the magnetic flux linked with the loop in the dt is $d\phi=B\times\Delta A=B\times\ell dx$

induced emf $e=\frac{d\phi}{dt}=B\ell\frac{dx}{dt}=B\ell v$ $\because\left[v=\frac{dx}{dt}\right]$

If the resistance of circuit is R and the circuit is closed then the current through the circuit

$$I=\frac{e}{R}\quad\Rightarrow I=\frac{Bv\ell}{R}$$

A magnetic force acts on the conductor in opposite direction of velocity is



$$F_m=i\ell B=\frac{B^2\ell^2 v}{R}\ .$$

So, to move the conductor with a constant velocity v an

equal and opposite force F has to be applied in the conductor.

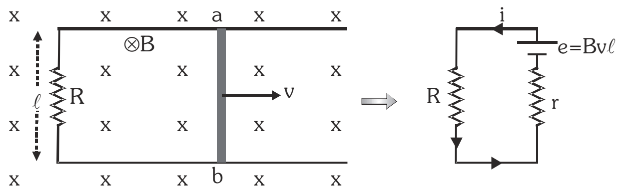
$$F = F_m = \frac{B^2 \ell^2 v}{R}$$

The rate at which work is done by the applied force is, $P_{applied} = Fv = \frac{B^2 \ell^2 v^2}{R}$

and the rate at which energy is dissipated in the circuit is, $P_{dissipated} = i^2 R = \left[\frac{Bv\ell}{R}\right]^2 R = \frac{B^2 \ell^2 v^2}{R}$

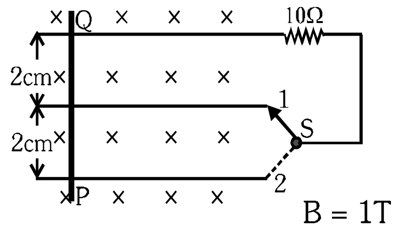
This is just equal to the rate at which work is done by the applied force.

- In the figure shown, we can replaced the moving rod ab by a battery of emf $Bv\ell$ with the positive terminal at a and the negative terminal at b. The resistance r of the rod ab may be treated as the internal resistance of the battery.



Hence, the current in the ciruit is $i = \frac{e}{R+r} = \frac{Bv\ell}{R+r}$

Ex. Wire PQ with negligible resistance slides on the three rails with 5 cm/sec. Calculate current in 10 Ω resistance when switch S is connected to (a) position 1 (b) position 2



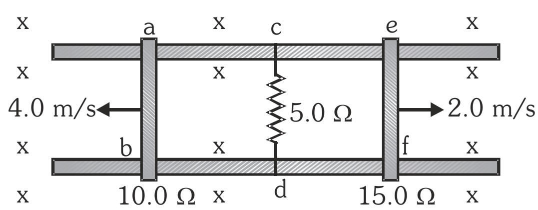
Sol. (a) For position 1

$$\text{Induced current } I = \frac{e}{R} = \frac{Bv\ell}{R} = \frac{1 \times 5 \times 10^{-2} \times 2 \times 10^{-2}}{10} = 0.1 \text{ mA}$$

(b) For position 2

$$\text{Induced current } I = \frac{e}{R} = \frac{Bv(2\ell)}{R} = \frac{1 \times 5 \times 10^{-2} \times 4 \times 10^{-2}}{10} = 0.2 \text{ mA}$$

Ex. Two parallel rails with negligible resistance are 10.0 cm apart. They are connected by a 5.0 Ω resistor. The circuit also contains two metal rods having resistance of 10.0Ω and 15.0 Ω along the rails (fig). The rods are pulled away from the resistor at constantspeeds 4.00 m/s and 2.00 m/s respectively. A uniform magnetic field of magnitude 0.01 T is applied perpendicular to the plane of the rails. Determine the current in the 5.0 Ω resistor.



Sol. Two conductors are moving in uniform magnetic field, so motional emf will induced across them.

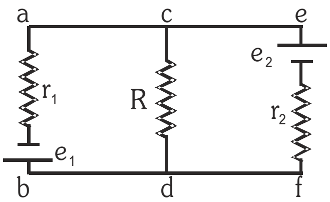
The rod ab will act as a source of emf $e_1 = Bv\ell = (0.01)(4.0)(0.1) = 4\times 10^{-3}$ V

and internal resistance $r_1 = 10.0\ \Omega$

Similarly, rod ef will also act as source of emf $e_2 = (0.01)(2.0)(0.1) = 2\times 10^{-3}$ V

and internal resistance $r_2 = 15.0\ \Omega$

From right hand rule : $V_b > V_a$ and $V_e > V_f$ Also $R = 5.0\ \Omega$,

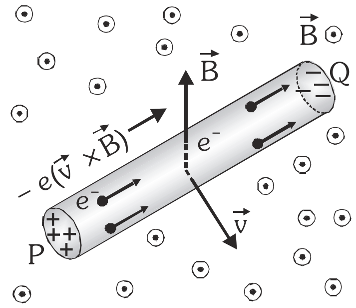


$$E_{eq} = \frac{e_1 r_2 - e_2 r_1}{r_1 + r_2} = \frac{6\times10^{-3} - 20\times10^{-3}}{15+10} = \frac{40}{25}\times10^{-3} = 1.6\times10^{-3} \text{ volt}$$

$$r_{eq} = \frac{15\times10}{15+10} = 6\Omega \qquad \text{and}$$

$$I = \frac{E_{eq}}{r_{eq}+R} = \frac{1.6\times10^{-3}}{6+6} = \frac{1.6}{11}\times10^{-3} = \frac{8}{55}\times10^{-3} \text{ amp from d to c}$$

MOTIONAL EMF FROM LORENTZ FORCE



A conductor PQ is placed in a uniform magnetic field B, directed normal to the plane of paper outwards. PQ is moved with a velocity v, the free electrons of PQ also move with the same velocity. The electrons experience a magnetic Lorentz force, $\vec{F}_m = (\vec{v}\times\vec{B})$. According to Fleming's left hand rule, this force acts in the direction PQ and hence the free electrons will move towards Q. A negative chagre accumulates at Q and a positive charge at P. An electric field E is setup in the conductor from P to Q. Force exerted by electric field on the free electrons is, $\vec{F}_e = e\vec{E}$

The accumulation of charge at the two ends continues till these two forces balance each other.

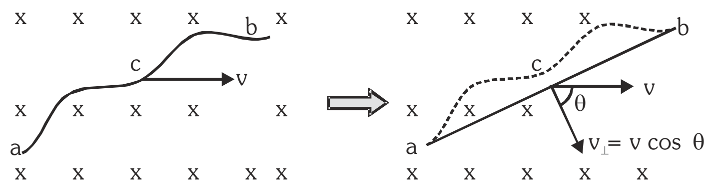
so $\vec{F}_m = -\vec{F}_e \Rightarrow e(\vec{v}\times\vec{B}) = -e\vec{E} \Rightarrow \vec{E} = -(\vec{v}\times\vec{B})$

The potential difference between the ends P and Q is $V = \vec{E}.\vec{\ell} = (\vec{v}\times\vec{B}).\vec{\ell}$ .It is the magnetic force on the moving free electrons that maintains the potential difference and produces the emf $\mathcal{E} = B\ \ell\ v$ (for $\vec{B}\perp\vec{v}\perp\vec{\ell}$)

As this emf is produced due to the motion of a conductor, so it is called a motional emf.

The concept of motional emf for a conductor can be generalized for any shape moving in any magnetic field uniform or not. For an element $\vec{d\ell}$ of conductor the contribution de to the emf is the magnitude $d\ell$ multiplied by the component of $\vec{v}\times\vec{B}$ parallel to $\vec{d\ell}$, that is $de = (\vec{v}\times\vec{B}).\vec{d\ell}$

For any two points a and b the motional emf in the direction from b to a is,

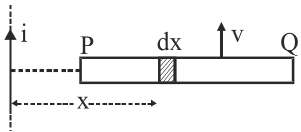


$$e = \int_b^a (\vec{v}\times\vec{B}).\vec{d\ell}$$

Motional emf in wire acb in a uniform magnetic field is the motional emf in an imaginary wire ab. Thus, $e_{acb} = e_{ab}$ = (length of ab) $(v_\perp)$ (B), $v_\perp$ = the component of velocity perpendicular to both $\vec{B}$ and ab. From right hand rule : b is at higher potential and a at lower potential. Hence, $V_{ba} = V_b - V_a = (ab)\,(v\cos\theta)\,(B)$

Ex. A rod PQ of length L moves with a uniform velocity v parallel to a long straight wire carrying a current i, the end P remaining at a distance r from the wire. Calculate the emf induced across the rod. Take v = 5.0 m/s, i = 100 amp, r = 1.0 cm and L = 19 cm.

Sol. The rod PQ is moving in the magnetic field produced by the current-carrying long wire. The field is not uniform throughout the length of the rod (changing with distance). Let us consider a small element of length dx at distance x from wire. if magnetic field at the position of dx is B then emf induced



$$d\mathcal{E} = B\,v\,dx = \frac{\mu_0}{2\pi}\frac{i}{x}v\,dx$$

$\therefore$ emf $\mathcal{E}$ is induced in the entire length of the rod PQ is $\mathcal{E} = \int d\mathcal{E} = \int_P^Q \frac{\mu_0}{2\pi}\frac{i}{x}v\,dx$

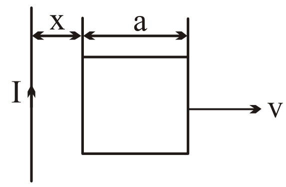
Now x = r at P, and x = r + L at Q. hence

$$\mathcal{E} = \frac{\mu_0 i v}{2\pi} = \int_r^{r+L}\frac{dx}{x} = \frac{\mu_0 i v}{2\pi}[\log_e x]_r^{r+L} = \frac{\mu_0 i v}{2\pi}[\log_e (r+L) - \log_e r] = \frac{\mu_0 i v}{2\pi}\log\frac{r+L}{r}$$

Putting the given values :

$$\mathcal{E} = (2\times 10^{-7})\,(100)\,(5.0)\log_e\frac{1.0+19}{1.0} = 10^{-4}\log_e 20\text{ Wb/s} = 3\times 10^{-4}\text{ volt}$$

Ex. A square frame with side a and a long straight wire carrying a current I are located in the same plane as shown in Fig. The frame translates to the right with a constant velocity v. Find the emf induced in the frame as a function of distance x.



Ans. $\xi_i = \frac{\mu_0}{4\pi}\;\frac{2Ia^2v}{x\;(x\;+\;a)}$

Sol. Field, due to the current carrying wire, at a perpendicular distance x from it is given by,

$$B(x) = \frac{\mu_0}{2\pi}\frac{i}{x}$$

Motional emf is given by $\left|\int -(\vec{v}\times\vec{B})\,.\,d\vec{\ell}\right|$

There will be no induced emf in the segments (2) and (4) as, $\vec{v}\uparrow\uparrow d\vec{\ell}$ and magnitude of emf induced 1 and 3, will be

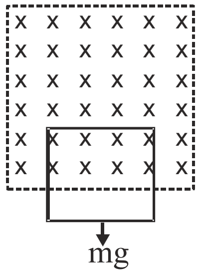
$$\xi_1 = v\left(\frac{\mu_0}{2\pi}\frac{i}{x}\right)a \text{ and } \xi_2 = v\left(\frac{\mu_0}{2\pi}\frac{i}{(a+x)}\right)a,$$

respectively, and their sense will be in the direction of $(\vec{v}\times\vec{B})$. So, emf induced in the network

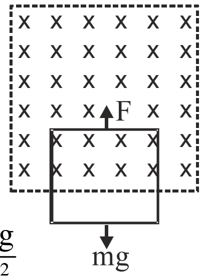
$= \xi_1 - \xi_2$ [as $\xi_1 > \xi_2$]

$$= \frac{av\mu_0 i}{2\pi}\left[\frac{1}{x} - \frac{1}{a+x}\right] = \frac{va^2\mu_0 i}{2\pi x(a+x)}$$

Ex. A horizontal magnetic field B is produced across a narrow gap between square iron pole-pieces as shown. A closed square wire loop of side ℓ, mass m and resistance R is allowed of fall with the top of the loop in the field. Show that the loop attains a terminal velocity given by $v = \dfrac{Rmg}{B^2\ell^2}$ while it is between the poles of the magnet.



Sol. As the loop falls under gravity, the flux passing through it decreases and so an induced emf is set up in it. Then a force F which opposes its fall. When this force becomes equal to the gravity force mg, the loop attains a terminal velocity v.

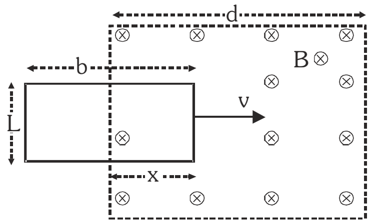


The induced emf e = B v ℓ, and the induced current is $i = \dfrac{e}{R} = \dfrac{Bv\ell}{R}$

The force experienced by the loop due to this current is $F = B\ell i = \dfrac{B^2 v \ell^2}{R}$

When v is the terminal (constant) velocity F = mg or $\dfrac{B^2 v\ell^2}{R} = mg$ or $v = \dfrac{R\,mg}{B^2\ell^2}$

Ex. Figure shows a rectangular conducting loop of resistance R, width L, and length b being pulled at constant speed v through a region of width d in which a uniform magnetic field B is set up by an electromagnet. Let L = 40 mm, b = 10 cm, d = 15 cm, R = 1.6 Ω, B = 2.0 T and v = 1.0 m/s



(i) Plot the flux ϕ through the loop as a function of the position x of the right side of the loop.

(ii) Plot the induced emf as a function of the positioin of the loop.

(iii) Plot the rate of production of thermal energy in the loop as a function of the position of the loop.

Sol. (i) When the loop is not in the field :

The flux linked with the loop $\phi = 0$

When the loop is entirely in the field :

Magnitic flux linked with the loop

$\phi = B\,L\,b$

$= 2 \times 40 \times 10^{-3} \times 10^{-1} = 8$ mWb

When the loop is entering the field :

The flux linked with the loop $\phi = B\,L\,x$

When the loop is leaving the field :

The flux $\phi = B\,L\,[b - (x - d)]$

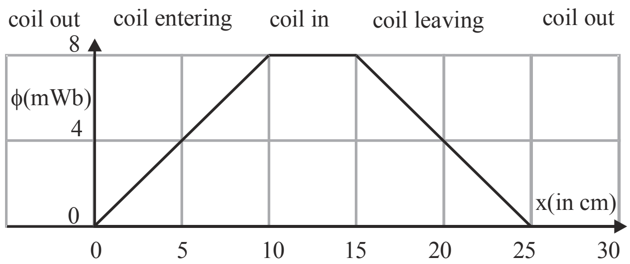


fig. (i)

(ii) Induced emf is $e = -\dfrac{d\phi}{dt} = -\dfrac{d\phi}{dx}\dfrac{dx}{dt} = -\dfrac{d\phi}{dx}v$

= – slope of the curve of figure (i) × v

The emf for 0 to 10 cm :

$$e = -\frac{(8-0)\times10^{-3}}{(10-0)\times10^{-2}}\times1 = -80\,\text{mV}$$

The emf for 10 to 15 cm : e = 0 × 1 = 0

The emf for 15 to 25 cm :

$$e = -\frac{(0-8)\times10^{-3}}{(25-15)\times10^{-2}}\times1 = +80\,\text{mV}$$

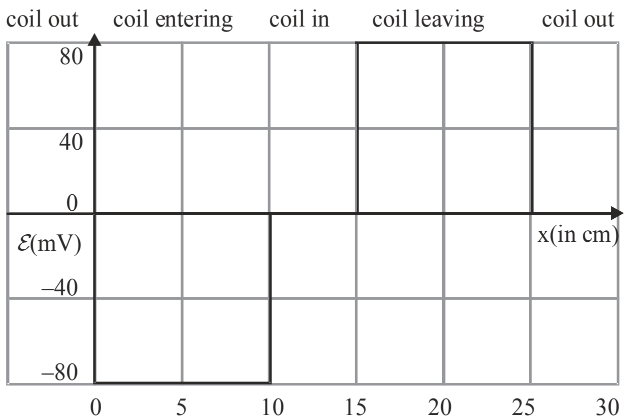


fig. (ii)

(iii) The rate of thermal energy production is $P = \dfrac{e^2}{R}$

for 0 to 10 cm : $P = \dfrac{(80 \times 10^{-3})^2}{1.6} = 4\,\text{mW}$

for 10 to 15 cm : $P = 0$

for 15 to 25 cm : $P = \dfrac{(80 \times 10^{-3})^2}{1.6} = 4\,\text{mW}$

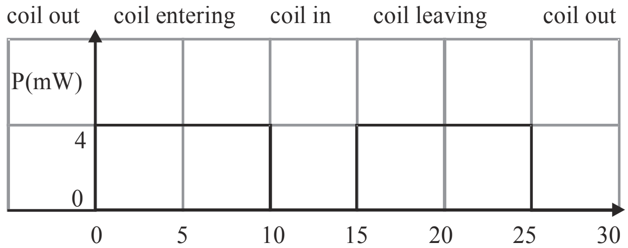
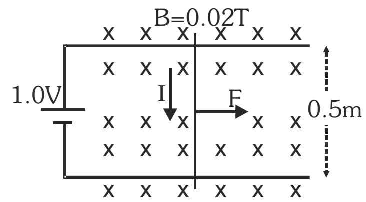


fig. (iii)

Ex. Two long parallel wires of zero resistance are connected to each other by a battery of 1.0 V. The separation between the wires is 0.5 m. A metallic bar, which is perpendicular to the wires and of resistance 10 Ω, moves on these wires. When a magneticfield of 0.02 testa is acting perpendicular to the plane containing the bar and the wires. Find the steady-state veclocity of the bar. If the mass of the bar is 0.002 kg then find its velocity as a function of time.



Sol. The current in the 10Ω bar is $I = \dfrac{1.0\,V}{10\,\Omega} = 0.1\,A$

The current carrying bar is placed in the magnetic field $\vec{B}$ (0.2 T) perpendicular to the plane of paper and directed downwards.

The magnetic force of the bar is $F = B\,I\,\ell = 0.02 \times 0.5 \times 0.10 = 1 \times 10^{-3}$ N

The moving bar cuts the lines of force of $\vec{B}$. If v be the instantaneous velocity of the bar, then the emf induced in the bar is $\mathcal{E} = B\ell v = 0.02 \times 0.5 \times v = 0.01$ v volt. By Lenz's law, $\mathcal{E}$ will oppose the motion of the bar which will ultimately attain a steady velocity. In this state, the induced emf $\mathcal{E}$ will be equal to the applied emf (1.0 volt). $\therefore$ 0.01 v = 1.0 or $v = \dfrac{1.0}{0.01} = 100\ \text{ms}^{-1}$

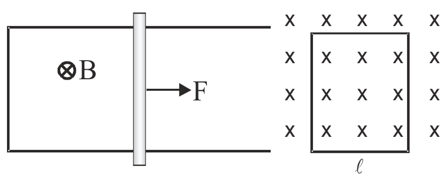
Again, a magnetic force F acts on the bar. If m be the mass of the bar, the acceleration of the rod is

$$\frac{dv}{dt} = \frac{F}{m} \Rightarrow dv = \frac{F}{m}.dt \qquad \text{Integrating, } \int dv = \int \frac{F}{m} dt \Rightarrow v = \frac{F}{m} t + C \ \text{(constant)}$$

If at t = 0, v = 0 then C = 0. $\quad \therefore \ v = \dfrac{F}{m} t$ But F = 1 × 10^{-3} N, m = 0.002 kg

$$\therefore \quad v = \frac{1 \times 10^{-3}}{0.002} t = 0.5\,t$$

Ex. In figure, a rod closing the circuit moves along a U-shaped wire at a constant speed v under the action of the force F. The circuit is in a uniform magnetic field perpendicualr to its plane. Calculate F if the rate generation of heat is P.

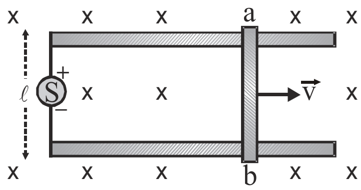


Sol. The emf induced across the ends of the rod, $\mathcal{E} = B\ell v$

Current in the circuit, $I = \dfrac{\mathcal{E}}{R} = \dfrac{B\ell v}{R}$ Magnetic force on the conductor, $F' = I\ell B$, towards left

$\because$ acceleration is zero $F' = F \Rightarrow BI\ell = F$ or $I = \dfrac{F}{B\ell}$ $\because$ $P = \mathcal{E} I = B\,\ell\,v \times \dfrac{F}{B\ell} = Fv$ $\therefore F = \dfrac{P}{v}$

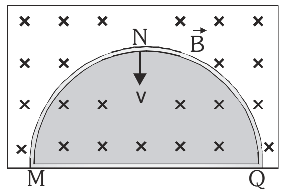
Ex. The diagram shows a wire ab of length ℓ and resistance R sliding on a smooth pair of rails with a velocity v towards right. A uniform magnetic field of induction B acts normal to the plane containing the rails and the wire inwards. S is a current source providing a constant I in the circuit. Determine the potential difference between a and b.



Sol. The wire ab which is moving with a velocity v is equivalent to an emf source of value B v ℓ with its positive terminal towards a.

$\therefore$ Potential difference $V_a - V_b = Bv\ell - IR$

Ex. A thin semicircular conduting ring of radius R is falling with its plane vertical in a horizontal magnetic induction $\vec{B}$ (fig.). At the position MNQ, the speed of the ring is v. What is the potential difference developed across the ring at the position MNQ ?

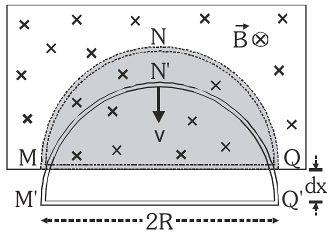


Sol. Let semiconductor ring falls through an infinitesimally small distance dx from its initial position MNQ to M'Q'N' in time dt (fig). decrease in area of the ring inside the magnetic field,

$dA = -MQQ'M' = -M'Q' \times QQ' = -2R\,dx$

$\therefore$ change in magnetic flux linked with the ring,

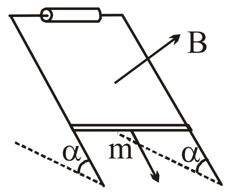
$d\phi = B \times dA = B \times (-R\,dx) = -2BR\,dx$



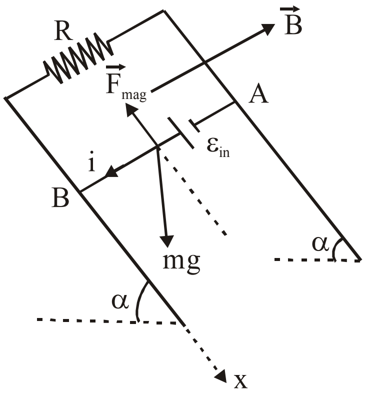
The potential difference developed across the ring, $e = -\frac{d\phi}{dt} = -\left[-2BR\frac{dx}{dt}\right] = 2BRv$

the speed with which the ring is falling $v = \frac{dx}{dt}$

Ex. A copper connector of mass m slides down two smooth copper bars, set at an angle α to the horizontal, due to gravity (Fig.). At the top the bars are interconnected through a resistance R. The separation between the bars is equal to l. The system is located in a uniform magnetic field of induction B, perpendicular to the plane in which the connector slides. The resistances of the bars, the connector and the sliding contacts, as well as the self-inductance of the loop, are assumed to be negligible. Find the steady-state velocity of the connector.



Ans. $v = \frac{mgR\sin\alpha}{B^2 l^2}$



Sol. From Lenz's law, the current through the connector is directed from A to B. Here $\xi_{in} = vB\ell$ between A and B.

where v is the velocity of the rod at any moment.

For the rod, from $F_x = mw_x$

or, $\quad mg\sin\alpha - i\ell B = mw$

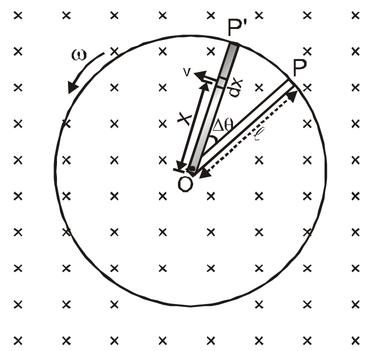
For steady state, acceleration of the rod must be equal to zero.

Hence, $mg\sin\alpha = i\ell B$ .... (1)

But, $i = \frac{\xi_{in}}{R} = \frac{vB\ell}{R}$

from (1) and (2) $v = \dfrac{mg\sin\alpha R}{B^2\ell^2}$

INDUCED E.M.F. DUE TO ROTATION OF A CONDUCTOR ROD IN A UNIFORM MAGNETIC FIELD



Let a conducting rod is rotating in a magnetic field around an axis passing through its one end, normal to its plane.
Consider an small element dx at a distance x from axis of rotation.
Suppose velocity of this small element = v
So, according to Lorent's formula induced e.m.f. across this small element

$d\varepsilon = B\, v.\, dx$

$\because$ This small element dx is at distance x from O (axis of rotation)
$\therefore$ Linear velocity of this element dx is $v = \omega x$
substitute of value of v in eq^n (i) $d\varepsilon = B\,\omega\, x\, dx$
Every element of conducting rod is normal to magnetic field and moving in perpendicular direction to the field

So, net induced e.m.f. across conducting rod $\quad \varepsilon = \int d\varepsilon = \int_0^{\ell} B\omega\ x\ dx = \omega\ B\left(\dfrac{x^2}{2}\right)_0^{\ell}$

or $\quad \varepsilon = \dfrac{1}{2}B\omega\ell^2$ $\ \varepsilon = \dfrac{1}{2}B\times 2\pi f\times \ell^2$ [f = frequency of rotation]

$= B\, f\, (\pi\ell^2)$ area traversed by the rod $A = \pi\,\ell^2$ or $\varepsilon = BAf$

Ex. A wheel with 10 metallic spokes each 0.5 m long is rotated with angular speed of 120 revolutions per minute in a plane normal to the earth's mangetic field. If the earth's magnetic field at the given place is 0.4 gauss, find the e.m.f. induced between the axle and the rim of the wheel.

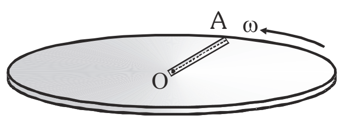
Sol. $\omega = 2\pi n = 2\pi\times\dfrac{120}{60} = 4\pi$, $\quad B = 0.4\ G = 4\times 10^{-5}$ T, length of each spoke = 0.5 m

induced emf $e = \dfrac{1}{2}B\,\omega\,\ell^2 = \dfrac{1}{2}\times 4\times 10^{-5}\times 4\pi\times(0.5)^2 = 6.28\times 10^{-5}$ volt

As all the ten spokes are connected with their one end at the axle and the other end at the rim, so they are connected in parallel and hence emf across each spoke is same. The number of spokes is immaterial.

Ex. A horizontal copper disc of diameter 20 cm, makes 10 revolutions/sec about a vertical axis passing through its centre. A uniform magnetic field of 100 gauss acts perpendicular to the plane of the disc. Calculate the potential difference its centre and rim in volts.

Sol. $B = 100$ gauss $= 100\times 10^{-4}\ Wb/m^2 = 10^{-2}$,
$r = 10$ cm $= 0.10$ m, frequency of rotaion = 10 rot/sec



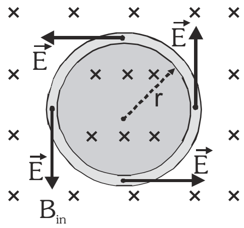
The emf induced between centre and rim $\mathcal{E} = \dfrac{1}{2}B\,\omega\,\ell^2 = \dfrac{1}{2}B\,\omega\, r^2\ (\because r = \ell)$

$\omega = 2\pi f = 2\times 3.14\times 10 = 62.8\ s^{-1}$

$\therefore\ \mathcal{E} = \dfrac{1}{2}\times 10\times 62.8\times(0.1)^2 = 3.14\times 10^{-3}\ V = 3.14$ mV.

INDUCED ELECTRIC FIELD

When the magnetic field changes with time (let it increases with time) there is an induced electric field in the conductor caused by the changing magnetic flux.



Important properties of induced electric field :

(i) It is non conservative in nature. The line integral of $\vec{E}$ around a closed path is not zero. When a charge q goes once around the loop, the total work done on it by the electric field is equal to q times the emf.

Hence $\oint \vec{E}.\vec{d\ell} = e = -\frac{d\phi}{dt}$ ...(i)

This equation is valid only if the path around which we integrate is stationary.

(ii) Due to of symmetry, the electric field $\vec{E}$ has the same magnitude at every point on the circle and it is tangential at each point (figure).

(iii) Being nonconservative field, so the concept of potential has no meaning for such a field.

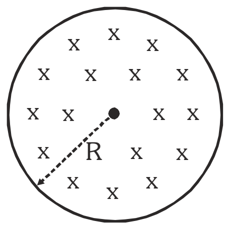
(iv) This field is different from the conservative electrostatic field produced by stationary charges.

(v) The relation $\vec{F} = q\vec{E}$ is still valid for this field. (vi) This field can vary with time.

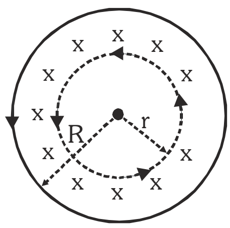
• For symmetrical situations $E\ell = \left|\frac{d\phi}{dt}\right| = A\left|\frac{dB}{dt}\right|$

ℓ = the length of closed loop in which electric field is to be calculated
A = the area in which magnetic field is changing.

Direction of induced electric field is the same as the direction of included current.



Ex. The magnetic field at all points within the cylindrical region whose cross-section is indicated in the figure start increasing at a constant rate $\alpha \frac{\text{tesla}}{\text{sec ond}}$. Find the magnitude of electric field as a function of r, the distance from the geomatric centreof the region.

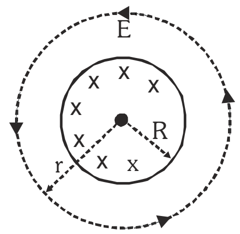


Sol. For r ≤ **R :**

$\because \quad E\ell = A\left|\frac{dB}{dt}\right|$

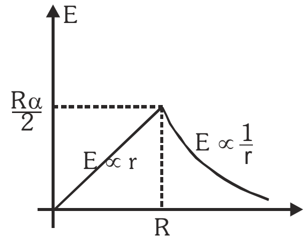
$\therefore \quad E(2\pi r) = (\pi r^2)\alpha \Rightarrow E = \frac{r\alpha}{2} \Rightarrow E \propto r$

E-r graph is straight line passing through origin.



At $r = R$, $E = \frac{R\alpha}{2}$

For r ≥ R :



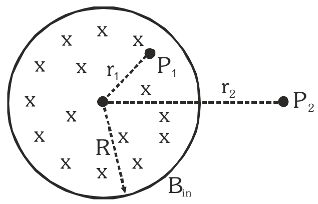
$\because \quad E\ell = A\left|\frac{dB}{dt}\right|$

$\therefore \quad E(2\pi r) = (\pi R^2)\alpha \quad \Rightarrow E = \frac{\alpha R^2}{2r} \Rightarrow E \propto \frac{1}{r}$

Ex. For the situation described in figure the magnetic field changes with time according to, $B = (2.00\,t^3 - 4.00\,t^2 + 0.8)$ T and $r_2 = 2R = 5.0$ cm

(a) Calculate the force on an electron located at P_2 at t = 2.00 s

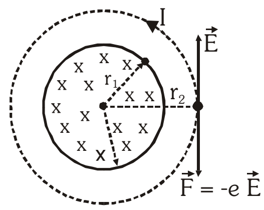
(b) What are the magnetude and direction of the electric field at P_1 when t = 3.00 s and $r_1 = 0.02$m.



Sol. $E\ell = A\left|\frac{dB}{dt}\right| \Rightarrow E = \frac{\pi R^2}{2\pi r_2}\frac{d}{dt}(2t^3 - 4t^2 + 0.8) = \frac{R^2}{2r_2}(6t^2 - 8t)$

(a) Force on electron at P_2 is F = eE

$\therefore$ at t = 2 s $F = \frac{1.6\times10^{-19}\times(2.5\times10^{-2})^2}{2\times5\times10^{-2}}\times[6(2)^2 - 8(2)]$



$= \frac{1.6}{4}\times2.5\times10^{-21}\times(24-16) = 8\times10^{-21}\,\text{N}$ at t = 2s,

$\frac{dB}{dt}$ is positive so it is increasing.

$\therefore$ direction of induced current and E are as shown in figure and hence force of electron having charge –e is right perpendicular to r_2 downwards

(b) For $r_1 = 0.02$ m and at t = 3s, $E = \frac{\pi r_1^2}{2\pi r_1}(6t^2 - 8t) = \frac{0.02}{2}\times[6(3)^2 - 8(3)]$

$= 0.3$ V/m at t = 3sec, $\frac{dB}{dt}$

is positive so B is increasing and hence direction of E is same as in case (a) and it is left perpendicular to r_1 upwards.

Generators :

One of the most important applications of Faraday's law of induction is to generators and motors. A generator converts mechanical energy into electric energy, while a motor converts electrical energy into mechanical energy.

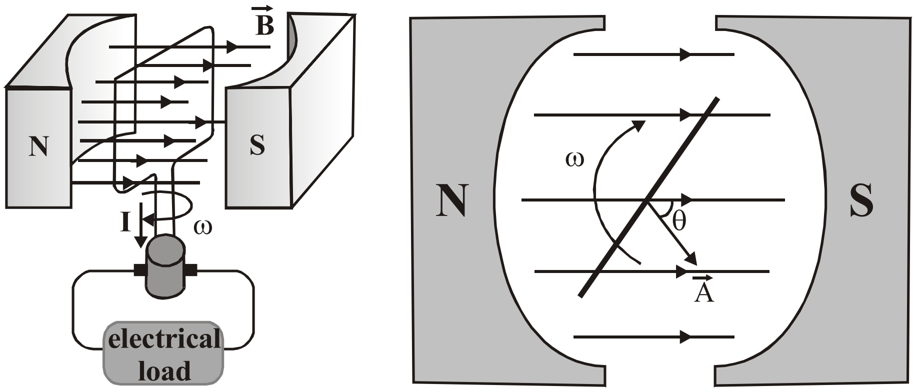


Figure : (a) A simple generator. (b) The rotating loop as seen from above.

Figure (a) is a simple illustration of a generator. It consists of an *N*-turn loop rotating in a magnetic field which is assumed to be uniform. The magnetic flux varies with time, thereby inducing an emf. From Figure (b), we see that the magnetic flux through the loop may be written as

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta = BA\cos\omega t$$

The rate of change of magnetic flux is

$$\frac{d\Phi_B}{dt} = -BA\,\omega\sin\omega t$$

Since there are N turns in the loop, the total induced emf across the two ends of the loop is

$$\varepsilon = -N\frac{d\Phi_B}{dt} = NBA\omega\sin\omega t$$

If we connect the generator to a circuit which has a resistance R, then the current generated in the circuit is given by $I = \frac{|\varepsilon|}{R} = \frac{NBA\omega}{R}\sin\omega t$

The current is an alternating current which oscillates in sign and has an amplitude $I_0 = NBA\omega / R$. The power delivered to this circuit is

$$P = I|\varepsilon| = \frac{(NBA\omega)^2}{R}\sin^2\omega t$$

On the other hand, the torque exerted on the loop is

$$\tau = \mu B\sin\theta = \mu B\sin\omega t$$

Thus, the mechanical power supplied to rotate the loop is

$$P_m = \tau\omega = \mu B\,\omega\sin\omega t$$

Since the dipole moment for the N-turn current loop is

$$\mu = NIA = \frac{N^2A^2B\omega}{R}\sin\omega t$$

the above expression becomes

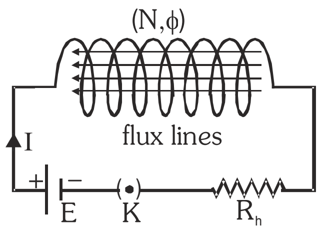
$$P_m = \left(\frac{N^2A^2B\omega}{R}\sin\omega t\right)B\omega\sin\omega t = \frac{(NAB\omega)^2}{R}\sin^2\omega t$$

As expected, the mechanical power put in is equal to the electric power output.

SELF INDUCTION

When the current through the coil changes, the magnetic flux linked with the coil also changes. Due to this change of flux a current induced in the coil itself according to lenz concept it opposes the change in magnetic flux. This phenomenon is called self induction and a factor by virtue of coil shows opposition for change in magnetic flux called cofficient of self inductance of coil.

Considering this coil circuit in two cases.



Case - I Current through the coil is constant

If $I \rightarrow B \rightarrow \phi$ (constant) $\Rightarrow$ No EMI

total flux of coil ($N\phi$) $\propto$ current through the coil

$$N\phi \propto I \quad \Rightarrow \quad N\phi = LI \qquad L = \frac{N\phi}{I} = \frac{NBA}{I} = \frac{\phi_{total}}{I}$$

where L = coefficient of self inductance of coil

S I unit of L : $1\ \frac{Wb}{amp} = 1$ Henry $= 1\frac{N.m}{A^2} = 1\frac{J}{A^2}$ **Dimensions :** $[M^1L^2T^{-2}A^{-2}]$

Note : L is constant of coil it **does not depends on current** flow through the coil.

Case - II Current through the coil changes w.r.t. time

$$\text{If } \frac{dI}{dt} \rightarrow \frac{dB}{dt} \rightarrow \frac{d\phi}{dt} \Rightarrow \text{Static EMI} \Rightarrow N\phi = LI$$

$-N\frac{d\phi}{dt} = -L\frac{dI}{dt}$, $(-N\frac{d\phi}{dt})$ called total self induced emf of coil 'e_s'

$e_s = -L\frac{dI}{dt}$ **S.I. unit of L** $\rightarrow \frac{V.s}{A}$

SELF-INDUCTANCE OF A PLANE COIL

Total magnetic flux linked with N turns,

$$\phi = NBA = N\left(\frac{\mu_0 NI}{2r}\right)A = \frac{\mu_0 N^2 I}{2r}\ A = \frac{\mu_0 N^2 I}{2r} \times \pi r^2 = \frac{\mu_0 \pi N^2 r}{2} I \text{ But } \phi = LI \therefore L = \frac{\mu_0 \pi N^2 r}{2}$$

Ex. Self-Inductance of a Solenoid :

Compute the self-inductance of a solenoid with turns, length ℓ, and radius *NR* with a current *I* flowing through each turn, as shown in Figure.

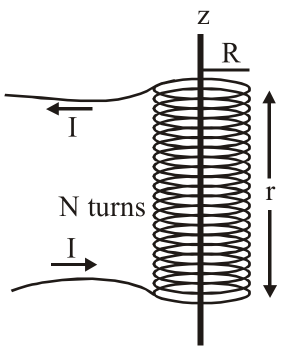


Figure : Solenoid

Solution :

Ignoring edge effects and applying Ampere's law, the magnetic field inside a solenoid is given by Eq. :

$$\vec{B} = \frac{\mu_0 NI}{\ell}\hat{k} = \mu_0 nI \quad \hat{k}$$

where $n = N/\ell$ is the number of turns per unit length. The magnetic flux through each turn is

$$\Phi_B = BA = \mu_0 nI \,.\, (\pi R_2) = \mu_0 nI\pi R^2$$

Thus, the self-inductance is $\quad L = \dfrac{N\Phi_B}{I} = \mu_0 n^2 \pi R^2 \ell$

We see that *L* depends only on the geometrical factors (*n*, *R* and ℓ) and is independent of the current *I*.

Ex. Self-Inductance of a Toroid :

Calculate the self-inductance of a toroid which consists of *N* turns and has a rectangular cross section, with inner radius *a*, outer radius *b* and height *h*, as shown in Figure (a).

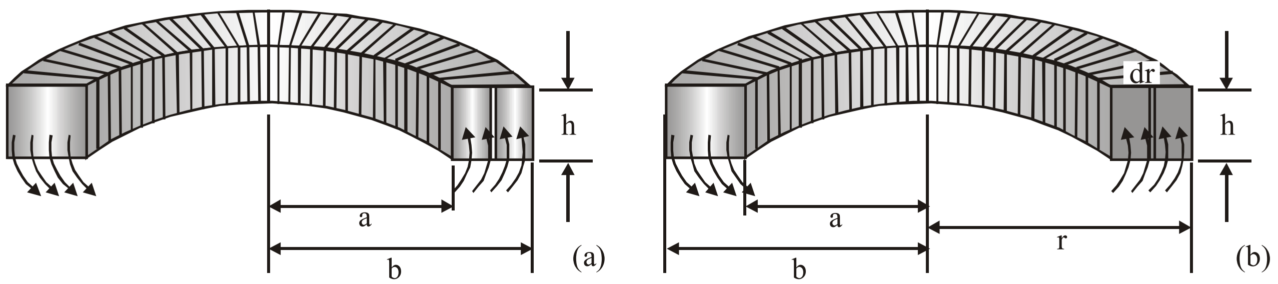


Figure : A toroid with *N* turns

Solution : According to Ampere's law discussed in section, the magnetic field is given by

$$\oint \vec{B}.d\vec{s} \oint Bds = B\oint ds = B\,(2\pi r) = \mu 0NI$$

or $\quad B = \dfrac{\mu_0 NI}{2\pi r}$

The magnetic flux through one turn of the torid may be obtained by integrating over the rectangular cross section,k with dA = hdr as the differential area element (figure-b)

$$\Phi_B = \iint \vec{B} \;.\; d\vec{A} = \int_a^b \left(\frac{\mu_0 NI}{2\pi r}\right) hdr = \frac{\mu_0 NIh}{2\pi}\ln\left(\frac{b}{a}\right)$$

The total flux is $N\Phi_B$. Therefore, the self-inductance is

$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)$$

Again, the self-inductance L depends only on the geometrical factors. Let's consider the situation where $a >> b - a$. In this limit, the logarithmic term in the equation above may be expanded as

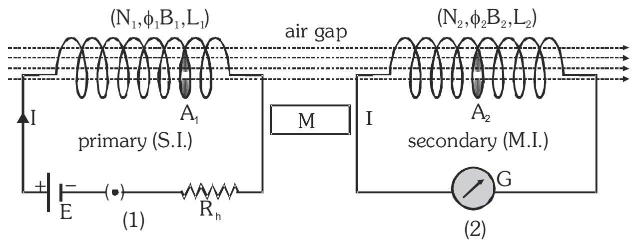
$$\ln\left(\frac{b}{a}\right) = \ln\left(1 + \frac{b-a}{a}\right) \approx \frac{b-a}{a}$$

and the self-inductance becomes

$$L \approx \frac{\mu_0 N^2 h}{2\pi} \cdot \frac{b-a}{a} = \frac{\mu_0 N^2 A}{2\pi a} = \frac{\mu_0 N^2 A}{\ell}$$

where $A = h(b-a)$ is the cross-sectional area, and $\ell = 2\pi a$. We see that the self inductance of the torid in this limit has the same form as that of a solenoid.

MUTUAL INDUCTION



Whenever the current passing through primary coil or circuit change then magnetic flux neighbouring secondary coil or circuit will also change. Acc. to Lenz for opposition of flux change, so an emf induced in the neighbouring coil or circuit.

This phenomenon called as 'Mutual induction'. In case of mutual inductance for two coils situated close to each other, flux linked with the secondary due to current in primary.

Due to Air gap always $\phi_2 < \phi_1$ and $\phi_2 = B_1 A_2$ $(\theta = 0°)$.

Case - I When current through primary is constant

Total flux of secondary is directly proportional to current flow through the primary coil

$N_2 \phi_2 \propto I_1 \Rightarrow N_2 \phi_2 = MI_1$, $M = \frac{N_2\phi_2}{I_1} = \frac{N_2 B_1 A_2}{I_1} = \frac{(\phi_T)_s}{I_p}$ where M : is coefficient of mutual induction.

Case - II When current through primary changes with respect to time

If $\frac{dI_1}{dt} \to \frac{dB_1}{dt} \to \frac{d\phi_1}{dt} \to \frac{d\phi_2}{dt}$

$\Rightarrow$ Static EMI $N_2\phi_2 = MI_1 - N_2 \frac{d\phi_2}{dt} = -M\frac{dI_1}{dt}$, $\left[-N_2 \frac{d\phi}{dt}\right]$

$$e_m = -M\left(\frac{dI_1}{dt}\right)$$

Secondary ← → Primary

called total mutual induced emf of secondary coil e_m.

- The units and dimension of M are same as 'L'.
- The mutual inductance does not depends upon current through the primary and it is constant for circuit system.

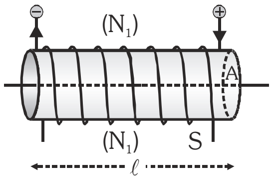
'M' depends on :

- Number of turns (N_1, N_2).
- Area of cross section.
- Distance between two coils (As $d\downarrow$ $= M\uparrow$).
- Cofficient of self inductance (L_1, L_2).
- Magnetic permeability of medium (μ_r).
- Orientation between two coils.
- **Coupling factor** 'K' between two coils.

DIFFERENT COEFFICIENT OF MUTUAL INDUCTANCE

- In terms of their number of turns
- In terms of their coefficient of self inductances
- **In terms of their nos of turns (N_1, N_2)**

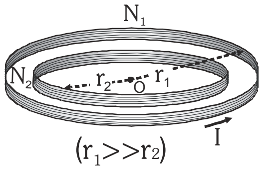
(a) Two co–axial solenoids :– ($M_{s_1s_2}$)



Coefficient of mutual inductance between two solenoids

$$M_{s_1s_2} = \frac{N_2B_1A}{I_1} = \frac{N_2}{I_1}\left[\frac{\mu_0N_1I_1}{\ell}\right]A \Rightarrow M_{s_1s_2} = \left[\frac{\mu_0N_1N_2A}{\ell}\right]$$

(b) Two plane concentric coils ($M_{c_1c_2}$)

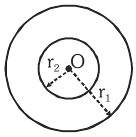


$$M_{c_1c_2} = \frac{N_2B_1A_2}{I_1} \text{ where } B_1 = \frac{\mu_0N_1I_1}{2r_1},\ A_2 = \pi r_2^2$$

$$M_{c_1c_2} = \frac{N_2}{I_1}\left[\frac{\mu_0N_1I_1}{2r_1}\right](\pi r_2^2) \Rightarrow M_{c_1c_2} = \frac{\mu_0N_1N_2\pi r_2^2}{2r_1}$$

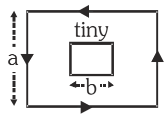
Two concentric loop :

$$M \propto \frac{r_2^2}{r_1}(r_1 >> r_2)$$



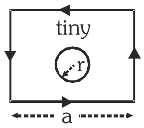
Two concentric square loops :

$$M \propto \frac{b^2}{a}$$



A square and a circular loop

$$M \propto \frac{r^2}{a}$$

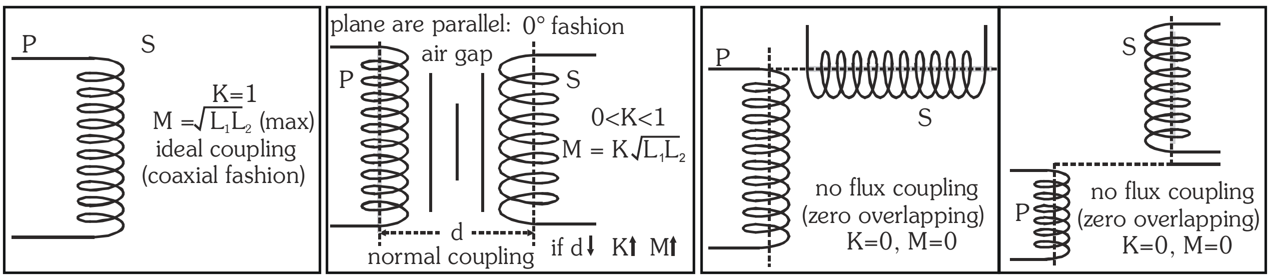


In terms of L_1 and L_2 : For two magnetically coupled coils :-

$M = K\sqrt{L_1L_2}$ here 'K' is coupling factor between two coils and its range $0 \le K \le 1$

- For ideal coupling $K_{max} = 1 \Rightarrow M_{max} = \sqrt{L_1L_2}$ (where M is geometrical mean of L_1 and L_2)
- For real coupling $(0 < K < 1)$ $M = K\sqrt{L_1L_2}$
- Value of coupling factor 'K' decided from fashion of coupling.

- **Different fashion of coupling**



'K' also defined as $K = \frac{\phi_s}{\phi_p} = \frac{\text{mag. flux linked with secondary (s)}}{\text{mag. flux linked with primary (p)}}$

INDUCTANCE IN SERIES AND PARALLEL

Two coil are connected in series : Coils are lying close together (M)

If M = 0, $L = L_1 + L_2$ If $M \neq 0$ $L = L_1 + L_2 + 2M$

(a) When current in both is in the same direction Then $L = (L_1 + M) + (L_2 + M)$

(b) When current flow in two coils are mutually in opposite directions.
$L = L_1 + L_2 - 2M$

Two coils are connected in parallel :

(a) If M = 0 then $\frac{1}{L}=\frac{1}{L_1}+\frac{1}{L_2}$ or $L=\frac{L_1L_2}{L_1+L_2}$ (b) If $M \neq 0$ then $\frac{1}{L}=\frac{1}{(L_1+M)}+\frac{1}{(L_2+M)}$

Ex. A coil is wound on an iron core and looped back on itself so that the core has two sets of closely would wires in series carrying current in the opposite sense. What do you expect about its self-inductance ? Will it be larger or small ?

Sol. As the two sets of wire carry currents in opposite directions, their induced emf's also act in opposite directions. These induced emf's tend to cancel each other, making the self-inductance of the coil very small.

This situation is similar to two coils connected in series and producing fluxes in opposite directions. Therefore, their equivalent inductance must be $L_{eq} = L + L - 2M = L + L - 2L = 0$

Ex. A solenoid has 2000 turns wound over a length of 0.3 m. The area of cross-section is 1.2×10^{-3} m^2. Around its central section a coil of 300 turns is closely would. If an initial current of 2A is reversed in 0.25 s, find the emf induced in the coil.

Sol. $$M=\frac{\mu_0 N_1 N_2 A}{\ell}=\frac{4\pi\times10^{-7}\times2000\times300\times1.2\times10^{-3}}{0.3}=3\times10^{-3}\,H$$

$$\mathcal{E}=-M\frac{dI}{dt}=-3\times10^{-3}\left[\frac{-2-2}{0.25}\right]=48\times10^{-3}\,V = 48 \text{ mV}$$

ENERGY STORED IN INDUCTOR

The energy of a capacitor is stored in the electric field between its plates. Similary, an inductor has the capability of storing energy in its magnetic field.An increasing current in an inductor causes an emf between its terminals.

Power P = The work done per unit time $=\frac{dW}{dt}=-ei=-\left[L\frac{di}{dt}\right]i=-L\,i\frac{di}{dt}$

here i = instanteneous current and L = inductance of the coil

From dW = – dU (energy stored) so $\frac{dW}{dt}=-\frac{dU}{dt}$ $\therefore$ $\frac{dU}{dt}=Li\frac{di}{dt} \Rightarrow dU = Li\,di$

The total energy U supplied while the current increases from zero to final value i is,

$$U=L\int_0^I i\,di=\frac{1}{2}L(i^2)_0^I \quad \therefore U=\frac{1}{2}LI^2$$

the energy stored in the magnetic field of an inductor when a current I is $=\frac{1}{2}LI^2$.

The source of this energy is the external source of emf that supplies the current.

- After the current has reached its final steady state value I, $\frac{di}{dt}=0$ and no more energy is input to the inductor.
- When the current decreases from i to zero, the inductor acts as a source that supplies a total amount of energy $\frac{1}{2}Li^2$ to the external circuit. If we interrupt the circuit suddenly by opening a switch the current decreases very rapidly, the induced emf is very large and the energy may be dissipated in an arc the switch.

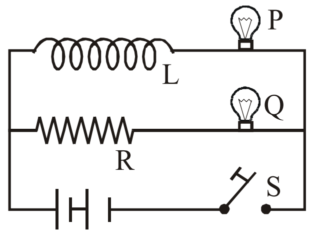
MAGNETIC ENERGY PER UNIT VOLUME OR ENERGY DENSITY

- The energy is an inductor is actually stored in the magnetic field within the coil. For a long solenoid its magnetic field can be assumed completely within the solenoid.
  The energy U stored in the solenoid when a current I is,

$$U = \frac{1}{2} L I^2 = \frac{1}{2}(\mu_0 n^2 V) I^2 \qquad (L = \mu_0 n^2 V) \qquad (V = \text{Volume} = A\ell)$$

The energy per unit volume $u = \frac{U}{V} = \frac{1}{2}\mu_0 n^2 I^2 = \frac{(\mu_0 n I)^2}{2\mu_0} = \frac{B^2}{2\mu_0}$ $(B = \mu_0 n I)$ $\therefore$ $u = \frac{1}{2}\frac{B^2}{\mu_0}$

Ex. Figure shows an inductor L a resistor R connected in paralled to a battery through a switch. The resistance of R is same as that of the coil that makes L. Two identical bulb are put in each arm of the circuit.



(a) Which of two bulbs lights up earlier when S is closed?
(b) Will the bulbs be equally bright after some time?

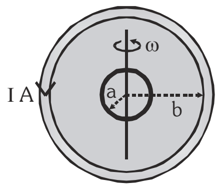
Sol. (i) When switch is closed induced e.m.f. in inductor i.e. back e.m.f. delays the glowing of lamp P so lamp Q light up earlier.
(ii) Yes. At steady state inductive effect becomes meaningless so both lamps become equally bright after some time.

Ex. A very small circular loop of area 5×10^{-4} m^2, resistance 2 ohm and negligible inductance is initially coplanar and concentric with a much larger fixed circular loop of radius 0.1 m. A constant current of 1 ampere is passed in the bigger loop and the smaller loop is rotated with angular velocity ω rad/s about a diameter. Calculate (a) the flux linked with the smaller loop (b) induced emf and induced current in the smaller loop as a function of time.

Sol. (a) The field at the centre of larger loop $B_1 = \frac{\mu_0 I}{2R} = \frac{2\pi\times10^{-7}}{0.1} = 2\pi \times 10^{-6}$ Wb/m^2

is initially along the normal to the area of smaller loop. Now as the smaller loop (and hence normal to its plane) is rotating at angular velocity ω, with respect to $\vec{B}$ so the flux linked with the smaller loop at time t is, $\phi_2 = B_1 A_2 \cos\theta = (2\pi \times 10^{-6})(5 \times 10^{-4}) \cos\omega t$
i.e., $\phi_2 = \pi \times 10^{-9} \cos\omega t$ Wb

(b) The induced emf in the smaller loop

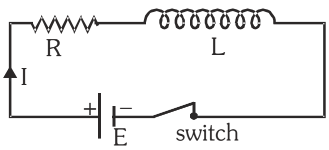


$$e_2 = -\frac{d\phi_2}{dt} = -\frac{d}{dt}(\pi\times10^{-9}\cos\omega t)$$
$$= \pi \times 10^{-9}\, \omega \sin\omega t \text{ volt}$$

(c) The induced current in the smaller loop is, $I_2 = \frac{e_2}{R} = \frac{1}{2}\pi\omega\times10^{-9}\sin\omega t$ ampere

R-L DC CIRCUIT

Current Growth



(i) emf equation $E = IR + L\frac{dI}{dt}$

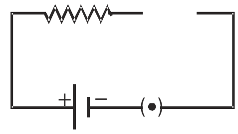
(ii) Current at any instant

When key is closed the current in circuit increases exponentially with respect to time. The current in circuit at any instant 't' given by $I = I_0\left[1 - e^{\frac{-t}{\lambda}}\right]$

t = 0 (just after the closing of key) $\Rightarrow I = 0$

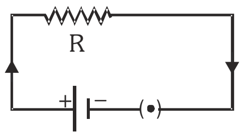
t = ∞ (some time after closing of key) $\Rightarrow I \to I_0$

(iii) Just after the closing of the key inductance behaves like open circuit and current in circuit is zero.



Open circuit, t = 0, I = 0 Inductor provide infinite resistence

(iv) Some time after closing of the key inductance behaves like simple connecting wire (short circuit) and current in circuit is constant.



Short circuit, $t \to \infty, I \to I_0$, Inductor provide zero resistence $I_0 = \frac{E}{R}$

(Final, steady, maximum or peak value of current) or ultimate current

Note : Peak value of current in circuit does not depends on self inductance of coil.

(v) Time constant of circuit (λ)

$\lambda = \frac{L}{R}$ sec. It is a time in which current increases up to 63% or 0.63 times of peak current value.

(vi) Half life (T)

It is a time in which current increases upto 50% or 0.50 times of peak current value.

$I = I_0(1 - e^{-t/\lambda}),\ t = T,\ I = \frac{I_0}{2} \Rightarrow \frac{I_0}{2} = I_0(1 - e^{-T/\lambda}) \Rightarrow e^{-T/\lambda} = \frac{1}{2} \Rightarrow e^{T/\lambda} = 2$

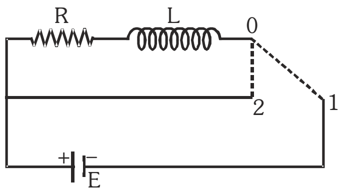
$\frac{T}{\lambda}\log_e e = \log_e 2 \quad \Rightarrow T = 0.693\lambda \quad \Rightarrow \quad T = 0.693\frac{L}{R}$ sec.

(vii) Rate of growth of current at any instant :–

$\left[\frac{dI}{dt}\right] = \frac{E}{L}(e^{-t/\lambda}) \Rightarrow \quad t = 0 \Rightarrow \quad \left[\frac{dI}{dt}\right]_{max} = \frac{E}{L} \quad t = \infty \Rightarrow \quad \left[\frac{dI}{dt}\right] \to 0$

Note : Maximum or initial value of rate of growth of current does not depends upon resistance of coil.

Current Decay



(i) Emf equation $IR + L\frac{dI}{dt} = 0$

(ii) Current at any instant

Once current acquires its final max steady value, if suddenly switch is put off then current start decreasing exponentially wrt to time. At switch put off condition t = 0, $I = I_0$, source emf E is cut off from circuit $I = I_0(e^{-t/\lambda})$

Just after opening of key t = 0 $\Rightarrow \quad I = I_0 = \frac{E}{R}$

Some time after opening of key $t \to \infty \Rightarrow I \to 0$

(iii) Time constant (λ)

It is a time in which current decreases up to 37% or 0.37 times of peak current value.

(iv) Half life (T)

It is a time in which current decreases upto 50% or 0.50 times of peak current value.

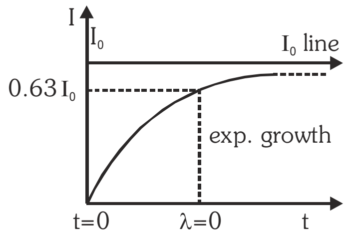
(v) Rate of decay of current at any instant

$$\left[-\frac{dI}{dt}\right]=\left[\frac{E}{L}\right]e^{-t/\lambda} \qquad t=0\ ,\ \left[-\frac{dI}{dt}\right]_{max.}=\frac{E}{L}\ t\to\infty \qquad \Rightarrow \qquad \left[-\frac{dI}{dt}\right]\to 0$$

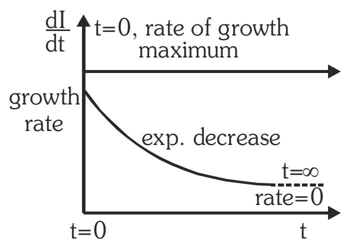
Graph for R–L circuit :–

Current Growth :–

(a)

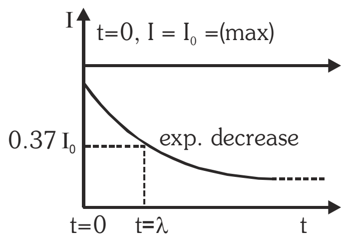


(b)

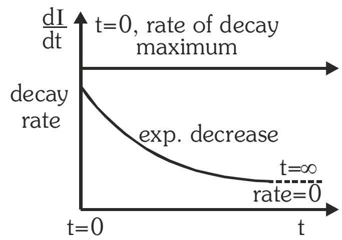


Current decay :–

(a)



(b)



LC Oscillations :

Consider an *LC* circuit in which a capacitor is connected to an inductor, as shown in Figure.

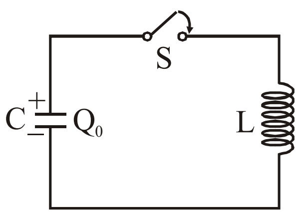


Figure *LC* Circuit

Suppose the capacitor initially has charge Q_0. When the switch is closed, the capacitor begins to discharge and the electric energy is decreased. On the other hand, the current created from the discharging process generates magnetic energy which then gets stored in the inductor. In the absence of resistance, the total energy is transformed back and forth between the electric energy in the capacitor and the magnetic energy in the inductor. This phenomenon is called electromagnetic oscillation.

The total energy in the *LC* circuit at some instant after closing the switch is

$$U=U_C+U_L=\frac{1}{2}\frac{Q^2}{C}+\frac{1}{2}LI^2$$

The fact that U remains constant implies that

$$\frac{dU}{dt}=\frac{d}{dt}\left(\frac{1}{2}\frac{Q^2}{C}+\frac{1}{2}LI^2\right)=\frac{Q}{C}\frac{dQ}{dt}+LI\frac{dI}{dt}=0$$

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

where $I = -dQ/dt$ (and $dI/dt = -d^2Q/dt^2$). Notice the sign convention we have adopted here. The negative sign implies that the current I is equal to the rate of decrease of change in the capacitor plate immediately after the switch has been closed. The same equation can be obtained by applying the modified Kirchhoff's loop rule clockwise.

$$\frac{Q}{C} - L\frac{dI}{dt} = 0$$

followed by our definition of current.

The general solution to equation is $Q(t) = Q_0 \cos(\omega_0 t + \phi)n$

where Q_0 is the amplitude of the charge and ϕ is the phase. The angular frequency ω_0 is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

The corresponding current in the inductor is

$$I(t) = -\frac{dQ}{dt} = \omega_0 Q_0 \sin(\omega_0 t + \phi) = I_0 \sin(\omega_0 t + \phi)$$

where $I_0 = \omega_0 Q_0$. From the initial conditions $Q(t = 0) = Q_0$ and $I(t = 0) = 0$, the phase ϕ can be determined to $\phi = 0$. Thus, the solutions for the charge and the current in our LC circuit are

$Q(t) = Q_0 \cos \omega_0 t$

and $I(t) = I_0 \sin w_0 t$

The time dependence of Q(t) and I(t) are depicted in figure.

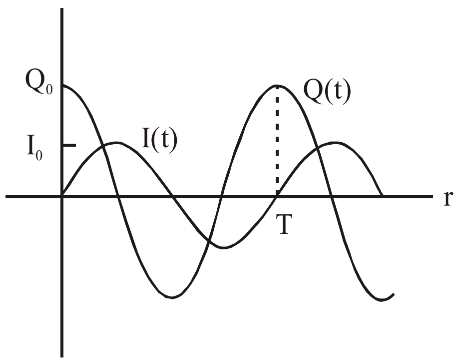


Figure : Charge and current in the *LC* circuit as a function of time

Using Eqs., we see that at any instant of time, the electric energy and the magnetic energies are given by

$$U_E = U_E = \frac{Q^2(t)}{2C} = \left(\frac{Q_0^2}{2C}\right)\cos^2 \omega_0 t$$

and $$U_B = \frac{1}{2}LI^2(t) = \frac{LI_0^2}{2}\sin^2 \omega t = \frac{L(-\omega_0 Q_0)^2}{2}\sin^2 \omega_0 t = \left(\frac{Q_0^2}{2C}\right)\sin^2 \omega_0 t = \frac{Q_0^2}{2C}$$

The electric and magnetic energy oscillation is illustrated in figure.

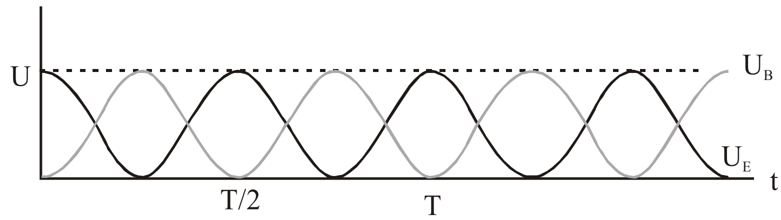


Figure : Electric and magnetic energy oscillations

The mechanical analog of the *LC* oscillations is the mass-spring system, shown in Figure.

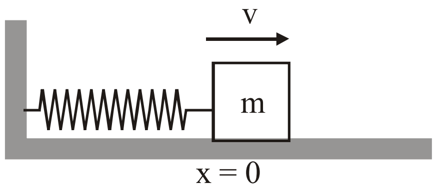


Figure : Mass-spring oscillations

If the mass is moving with a speed *v* and the spring having a spring constant *k* is displaced from its equilibrium by *x*, then the total energy of this mechanical system is

$$U = K + U_{sp} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

where K and U_{sp} are the kinetic energy of the mass and the potential energy of the spring, respectively. In the absence of friction, U is conserved and we obtain

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2 + \frac{1}{2}kx^2\right) = mv\frac{dv}{dt} + kx\frac{dx}{dt} = 0$$

Using v = dx/dt and dv/dt = d^2x/dt^2, the above equation may be rewritten as

$$m\frac{d^2x}{dt^2} + kx = 0$$

The general solution for the displacement is

$$x(t) = x_0 \cos(\omega_0 t + \phi)$$

where $\omega_0 = \sqrt{\frac{k}{m}}$

is the angular frequency and x_0 is the amplitude of the oscillations. Thus, at any instant in time, the energy of the system may be written as

$$U = \frac{1}{2}mx_0^2\omega_0^2 \sin^2(\omega_0 t + \phi) + \frac{1}{2}kx_0^2\cos^2(\omega_0 t + \phi)$$

$$= \frac{1}{2}kx_0^2[\sin^2(\omega_0 t + \phi) + \cos^2(\omega_0 t + \phi) = \frac{1}{2}kx_0^2$$

In figure we illustrate the energy oscillations in the LC circuit and the mass spring system (harmonic oscillator).

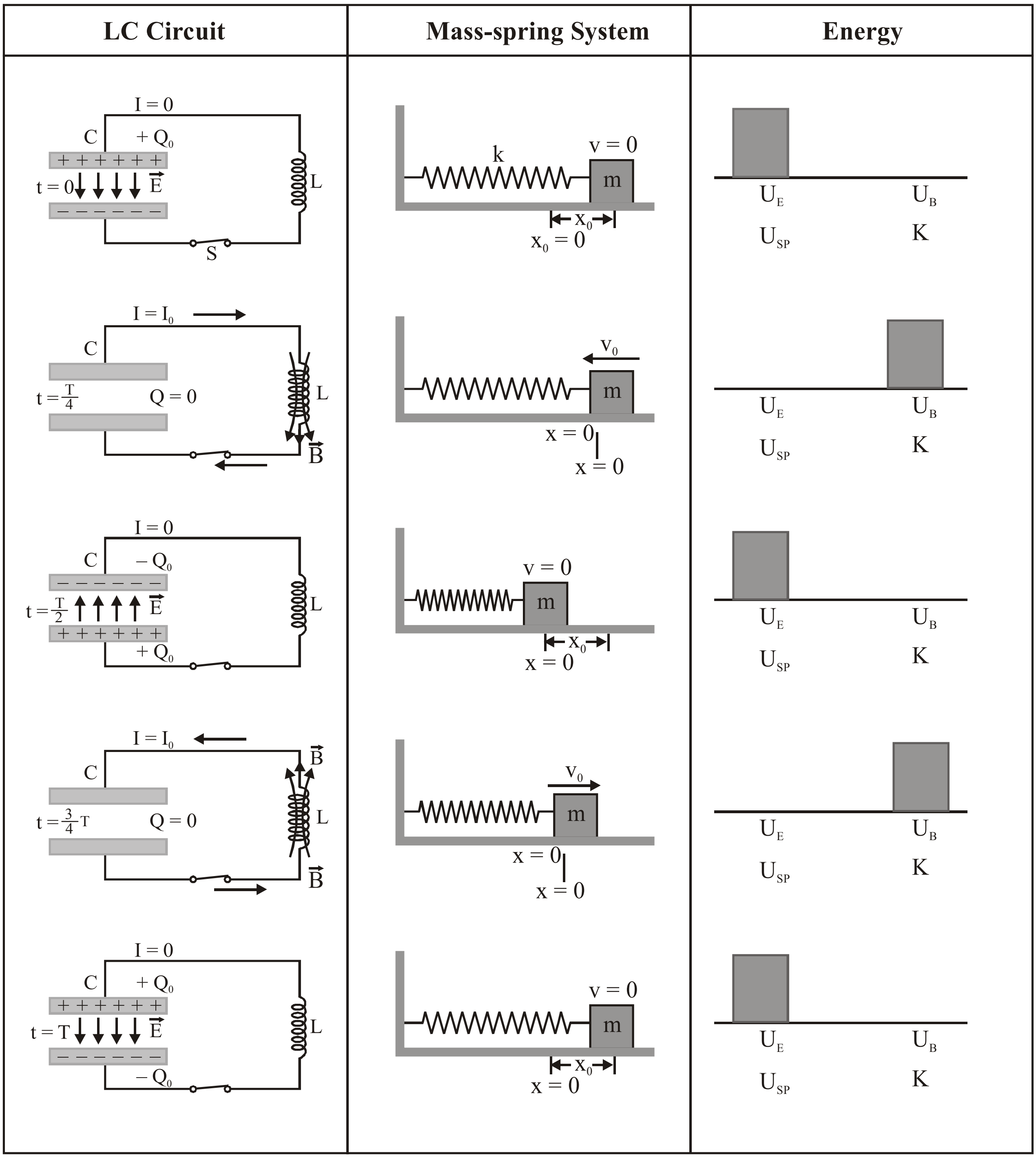


Figure : Energy oscillations in the *LC* Circuit and the mass-spring system

LC Circuit :

Ex. Consider the circuit shown in Figure. Suppose the switch which has been connected to point *a* for a long time is suddenly thrown to *b* at $t = 0$.

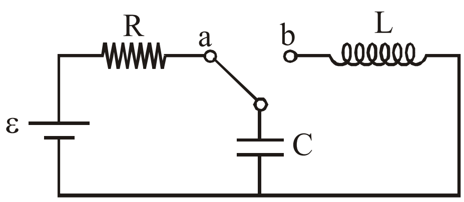


Figure : *LC* circuit

Find the following quantities :

(a) the frequency of oscillation of the *LC* circuit.

(b) the maximum charge that appears on the capacitor.

(c) the maximum current in the inductor.

(d) the total energy the circuit possesses at any time *t*.

Solution :

(a) The (angular) frequency of oscillation of the *LC* circuit is given by $\omega = 2\pi f = 1/\sqrt{LC}$. Therefore, the frequency is :

$$f = \frac{1}{2\pi\sqrt{LC}}$$

(b) The maximum charge stored in the capacitor before the switch is thrown to b is

$$Q = C\varepsilon$$

(c) The energy stored in the capacitor before the switch is thrown is :

$$U_E = \frac{1}{2}C\varepsilon^2$$

On the other hand, the magnetic energy stored in the inductor is :

$$U_B = \frac{1}{2}LI^2$$

Thus, when the current is at its maximum, all the energy originally stored in the capacitor is now in the inductor :

$$\frac{1}{2}C\varepsilon^2 = \frac{1}{2}LI_0^2$$

This implies a maximum current

$$I_0 = \varepsilon\sqrt{\frac{C}{L}}$$

(d) At any time, the total energy in the circuit would be equal to the initial energy that the capacitance stored, that is

$$U = U_E + U_B = \frac{1}{2}C\varepsilon^2$$