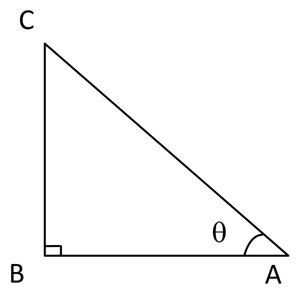
Trigonometry

In a right triangle ABC as show in the figure

AC is called hypotenuse.

BC is called "Opposite side of ∠A"

AB is called "Adjacent side of the ∠A".



Ratios in A Right Angle Triangle:

$$\sin A = \frac{\text{opposite side of } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\cos A = \frac{\text{adjacent side of } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$Tan\ A = \frac{\text{opposite side of } \angle A}{\text{Adjacent side of } \angle A} = \frac{BC}{AB}$$

$$Sec\ A = \frac{\text{hypotenuse}}{\text{adjacent side of } \angle A} = \frac{AC}{AB} = \frac{1}{\cos A}$$

$$cosec\ A = \frac{\text{hypotenuse}}{\text{opposite side of } \angle A} = \frac{AC}{BC} = \frac{1}{\sin A}$$

$$\cot A = \frac{\text{Adjacent side of } \angle A}{\text{opposite side of } \angle A} = \frac{AB}{BC} = \frac{1}{Tan\ A}$$

Note: We observe that $TanA = \frac{\sin A}{\cos A}$ and $\cot A = \frac{\cos A}{\sin A}$

Trigonometric ratios of some specific angles:

∠A	0°	30°	45°	60°	90
Sin A	**0**	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	**1**
Cos A	**1**	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	**0**
Tan A	**0**	$\frac{1}{\sqrt{3}}$	**1**	**√3**	**Not defined**
Cot A	**Not defined**	**√3**	**1**	$\frac{1}{\sqrt{3}}$	**0**
Sec A	**1**	$\frac{2}{\sqrt{3}}$	**√2**	**2**	**Not defined**
Cosec A	**Not defined**	**2**	**√2**	$\frac{2}{\sqrt{3}}$	**1**

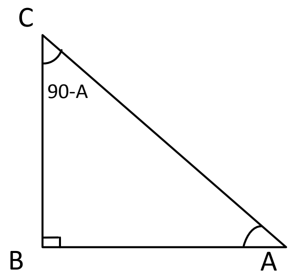
- The value of sinA will be increased from 0^0 to 90^0
- The value of cosA will decreased from 0^0 to 90^0
- If one of the trigonometric ratios of an acute angle is known, the remaining trigonometric ratios of the angle can be determined
- The value of sinA or cosA never exceeds, where as the value of sec A or CosecA is always greater than or equal to 1.

Trigonometric ratios of complementary angles:

Sin $(90^0 – A)$ = CosA   Cosec $(90^0 - A)$ = SecA

Cos $(90^0 – A)$ = SinA   Sec $(90^0 - A)$ = CosecA

Tan $(90^0 – A)$ = CotA   Cot $(90^0 - A)$ = TanA



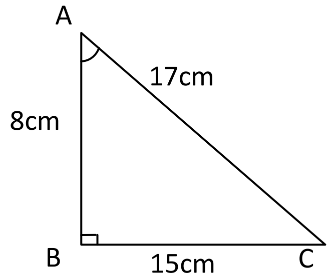
Exercise 11.1

1.In a right angle triangle ABC, 8cm, 15cm and 17cm are the lengths of AB, BC and CA respectively. Then, find out sinA, cosA and tanA.

A. In ΔABC, AB = 8cm

BC = 15 cm

AC = 17 cm



$$SinA = \frac{opposite\, side}{Hypotenuse} = \frac{BC}{AC} = \frac{15}{17}$$

$$CosA = \frac{Adjacent\, side}{Hypotenuse} = \frac{AB}{AC} = \frac{8}{17}$$

$$\tan A = \frac{oppsite\, side}{Hypotenuse} = \frac{BC}{AB} = \frac{15}{8}$$

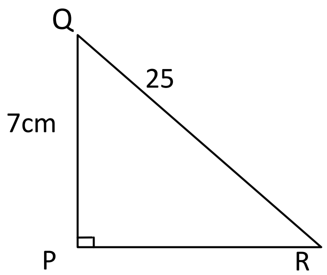
2. The sides of a right angle triangle PQR are PQ = 7cm, QR = 25cm ∠P = 90° respectively then find tanQ – tanR.

Sol: Given that in PQR,

PQ = 7cm and QR = 25 cm.

ΔPQR is a right angle triangle

By using Pythagoras theorem



$$PR = \sqrt{QR^2 - PQ^2}$$

$$= \sqrt{(25)^2 - (7)^2} = \sqrt{625 - 49}$$

$$= \sqrt{576} = 24cm.$$

$$\therefore \tan Q = \frac{PR}{PQ} = \frac{24}{7}$$

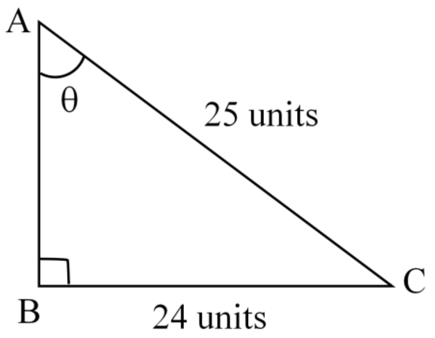
$$\tan R = \frac{PQ}{PR} = \frac{7}{24}$$

$$\therefore \tan Q - \tan R = \frac{24}{7} - \frac{7}{24} = \frac{576-49}{168} = \frac{527}{168}.$$

3. In a right angle triangle ABC with right angle at B, in which a = 24 units, b = 25 units and ∠BAC =θ. Then find cos θ and tan θ.

A. In a right angle triangle ABC ∠B = 90^0, ∠BAC = θ.

a = BC = 24 units and b = AC = 25units.



By using Pythagoras theorem

$$AB = \sqrt{AC^2 - BC^2} = \sqrt{(25)^2 - (24)^2} = \sqrt{625-576}$$

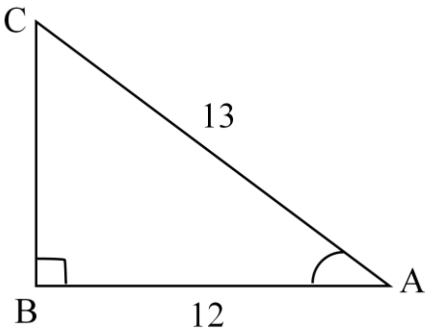
∴C = AB = √49 = 7cm.

Then $\cos\theta = \frac{AB}{AC} = \frac{7}{25}$

$$\tan\theta = \frac{BC}{AB} = \frac{24}{7}$$

4. If $\cos A = \frac{12}{13}$, then find sinA and tanA.

A. Given that cosA= $\frac{12}{13} = \frac{BA}{AC}$



$$\frac{BA}{12} = \frac{AC}{13} = K\left(say\right).$$

Where k is a positive number BA = 12k

AC = 13K

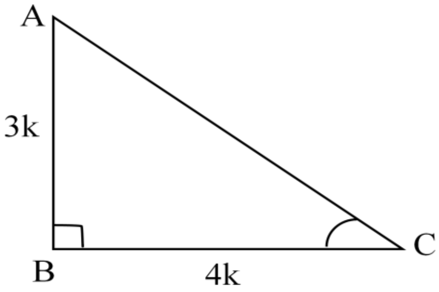
By using Pythagoras theorem $BC = \sqrt{AC^2 - BA^2}$

$$BC = \sqrt{\left(13k\right)^2 - \left(12k\right)^2} = \sqrt{169k^2 - 144k^2} = \sqrt{25k^2} = 5k$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$TanA = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$$

5. If 3 tanA = 4, Then find sinA and cosA.



A. Given that 3 tan A = 4

$$\Rightarrow \tan A = \frac{4}{3} = \frac{BC}{AB}$$

$$\Rightarrow \frac{BC}{AB} = \frac{4}{3} say.$$

$\frac{BC}{4} = \frac{AB}{3} = k$ say. for any positive integer k.

BC = 4k and AB = 3k

By using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$= (3k)^2 + (4k)^2$

$= 9k^2 + 16K^2$

$AC^2 = 25k^2$

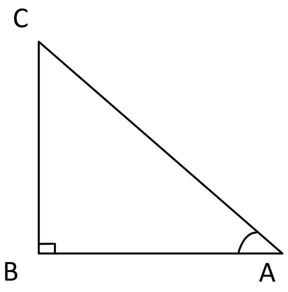
$\therefore AC = 5k.$

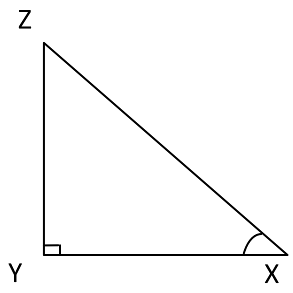
Now, $\sin A = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}.$

$\cos A = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}.$

6. If ∠A and ∠X are acute angles such that cosA= cosX, then show that ∠A= ∠X.

Sol: Let us consider two right angled triangles ΔABC and ΔXYZ and right angles at ∠B and ∠Y respectively.





Given that cosA = cosx

From ΔABC

$$\cos A = \frac{AB}{AC} \longrightarrow (1)$$

From ΔXYZ

$$\cos X = \frac{XY}{XZ} \longrightarrow (2)$$

From (1) & (2) $\frac{AB}{AC} = -\frac{XY}{XZ} \quad (\because \cos A = \cos X)$

Let $\frac{AB}{AC} = \frac{XY}{XZ} = \frac{K}{1} \Rightarrow \frac{AB}{XY} = \frac{AC}{XZ} = k \longrightarrow (3)$

$$\frac{BC}{YZ} = \frac{\sqrt{AC^2 - AB^2}}{\sqrt{XZ^2 - XY^2}} \quad (\because By\ pythagoras\ theorem)$$

$$= \frac{\sqrt{K^2 XZ^2 - K^2 XY^2}}{\sqrt{XZ^2 - XY^2}} = \frac{K\sqrt{XZ^2 - XY^2}}{\sqrt{XZ^2 - XY^2}} = K$$

$$\therefore \frac{AB}{XY} = \frac{AC}{XZ} = \frac{BC}{YZ}$$

⇒ ΔABC ~ ΔXYZ

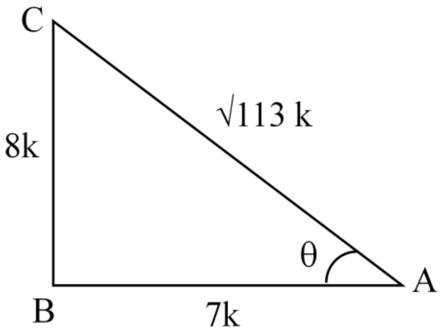
⇒∠ A = ∠X Proved .

7. Given $\cos\theta = \frac{7}{8}$**, then evaluate**

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) $\frac{1+\sin\theta}{\cos\theta}$.

Sol: let us draw a right angle triangle ABC in which ∠BAC = θ.



$\cot\theta = \frac{7}{8}$ (Given)

$\Rightarrow \frac{AB}{BC} = \frac{7}{8}$

$\Rightarrow \frac{AB}{7} = \frac{BC}{8} = k$ (Say) where k is a positive integer.

⇒AB = 7k and BC = 8k.

By using Pythagoras Theorem $AC = \sqrt{AB^2 + BC^2}$

$$= \sqrt{(7k)^2 + (8k)^2}$$

$$= \sqrt{49k^2 + 64k^2} = \sqrt{113}k.$$

$$\therefore \sin\theta = \frac{BC}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{AB}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

(i) $$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1)^2 - \sin^2\theta}{(1)^2 - \cos^2\theta} = \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2}$$

$$\Rightarrow \frac{1-\frac{64}{113}}{1-\frac{49}{113}} = \frac{113-64}{113-49} = \frac{49}{64}.$$

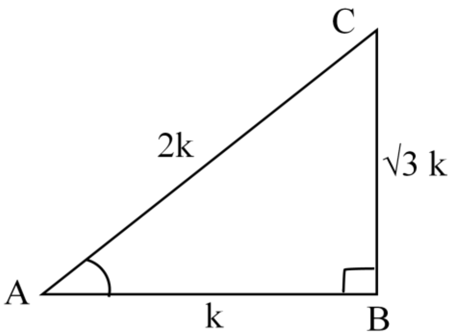
$(ii)\quad \frac{1+\sin\theta}{\cos\theta}$

$$\Rightarrow \frac{1+\frac{8}{\sqrt{113}}}{\frac{7}{\sqrt{113}}} = \frac{\sqrt{113}+8}{7}.$$

8. In a right angle triangle ABC, right angle at B, if tan A = √3. Then find the value of

(i) sinA cosC+ cosA sinC. (ii) cosA cosC – sinA sinC.

Sol: let us draw a right angled triangle ABC.



Given that Tan A = √3 = $\frac{\sqrt{3}}{1}$, B = 90^{0}.

$$\Rightarrow \frac{BC}{AB} = \frac{\sqrt{3}}{1}$$

$$\Rightarrow \frac{BC}{\sqrt{3}} = \frac{AB}{1} = k\,say.$$

Where k is any positive integer

⇒BC = √3k, AB = k.

By using Pythagoras theorem $AC = \sqrt{AB^2 + BC^2}$

$$= \sqrt{k^2 + \left(\sqrt{3}k\right)^2}$$

$$\sqrt{4k^2} = 2k.$$

$\therefore$ AC = 2k.

There fore $\sin A = \frac{BC}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$$\cos A = \frac{AB}{AC} = \frac{k}{2k} = \frac{1}{2}.$$

$$\sin C = \frac{AB}{AC} = \frac{k}{2k} = \frac{1}{2};\ \cos C = \frac{BC}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

Now (i) sinA cosC + cos A sinC

$$= \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} + \frac{1}{2}.\frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1$$

(ii) cosA cosc – sinA sinC

$$= \frac{1}{2}.\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}.\frac{1}{2} = 0.$$

Exercise - 11.2

1. Evaluate the following.

(i) $\sin 45^0 + \cos 45^0$

Sol: we know that $\sin 45^0 = \frac{1}{\sqrt{2}}$ and $\cos 45^0 = \frac{1}{\sqrt{2}}$.

$$\therefore \sin 45° + \cos 45° = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

(ii) $$\frac{\cos 45°}{\sec 30° + \cos ec60°}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}}} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2}{\sqrt{3}}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2+2}$$

$$= \frac{\sqrt{3}}{4\sqrt{2}}.$$

(iii) $$\frac{\sin 30° + \tan 45° - \cos ec60°}{\cot 45° + \cos 60° - \sec 30°}$$

We know that $\sin 30° = \frac{1}{2}, \tan 45° = 1, \cos ec\,60° = \frac{2}{\sqrt{3}}$

$\cot 45° = 1; \cos 60° = \frac{1}{2}; \sec 30° = \frac{2}{\sqrt{3}}$

$$\therefore \frac{\sin 30° + \tan 45° - \cos ec60°}{\cot 45° + \cos 60° - \sec 30°} = \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{1 + \frac{1}{2} - \frac{2}{\sqrt{3}}} = 1$$

(iv) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

A. $\tan 45° = 1$; $\cos 30° = \frac{\sqrt{3}}{2}$; $\sin 60° = \frac{\sqrt{3}}{2}$.

$\therefore$ 2 tan² 45° + cos² 30° – sin² 60°

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 2 + \frac{3}{4} - \frac{3}{4} = 2.$$

(v) $\dfrac{\sec^2 60° - \tan^2 60°}{\sin^2 30° + \cos^2 30°}$

A. sec 60° = 2, tan 60° = √3

$$\sin 30° = \frac{1}{2}, \cos 30° = \frac{\sqrt{3}}{2}$$

$$\therefore \frac{\sec^2 60° - \tan^2 60°}{\sin^2 30° + \cos^2 30°} = \frac{(2)^2 - (\sqrt{3})^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{4-3}{\frac{1}{4} + \frac{3}{4}} = \frac{1}{1} = 1.$$

2. Evaluate sin 60°. Cos 30° + sin 30° cos 60°. What is the value of sin (60° + 30°). What can you conclude?

A. We know that $\sin 60° = \dfrac{\sqrt{3}}{2}$ $\sin 30° = \dfrac{1}{2}$

$\cos 60° = \dfrac{1}{2}$ $\cos 30° = \dfrac{\sqrt{3}}{2}$.

$\therefore$ Sin 60°. cos 30° + sin 30° cos 60°

$$= \frac{\sqrt{3}}{2} . \frac{\sqrt{3}}{2} + \frac{1}{2} . \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1.$$

Sin (60° + 30°) = sin (90°) = 1

$\therefore$ we can conclude that

Sin (60° + 30°) = sin 60° cos 30° + sin 30°. Cos 60°

$\Rightarrow$ Sin (A + B) = sinA cosB + cosA sinB.

3. Is it right to say cos (60° + 30°) = Cos 60° cos 30° – sin 60° sin 30°.

Sol. L.H.S cos (60° + 30°) = cos 90° = 0

We know that $\sin 30° = \frac{1}{2}; \cos 30° = \frac{\sqrt{3}}{2}$

$$\sin 60° = \frac{\sqrt{3}}{2} \qquad \cos 60° = \frac{1}{2}$$

∴ R.H.S= cos 60° cos 30° – sin 60° sin 30°

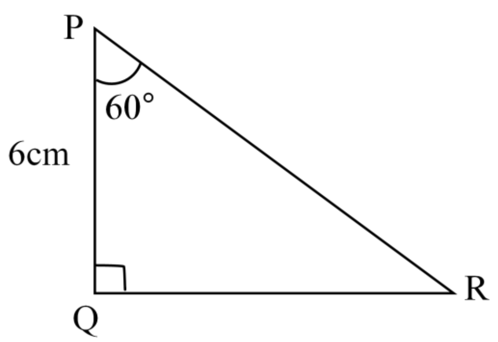
$$\frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

∴ It is right to say that

Cos (60° + 30°) = cos 60° cos 30° – sin 60° sin 30°.

4.In right angled triangle PQR, right angle is at Q and PQ = 6cm, ∠RPQ = 60°. Determine the lengths of QR and PR.

Sol: In ΔPQR ∠Q = 90°



PQ = 6cm.

Then $\cos 60° = \frac{\text{Adjacent snide}}{\textit{Hypotenuse}}$

$$\frac{1}{2} = \frac{PQ}{PR} \qquad \left(\because \cos 60° = \frac{1}{2}\right)$$

$$\therefore \frac{PQ}{PR} = \frac{1}{2} \Rightarrow PR = 2PQ = 2 \times 6cm = 12cm.$$

Similliarily

$$\sin 60^\circ = \frac{opposite\ side}{Hypotenuse} = \frac{QR}{PR}$$

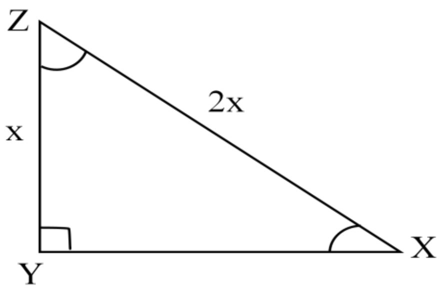
$$\frac{\sqrt{3}}{2} = \frac{QR}{PR} \Rightarrow QR = \frac{\sqrt{3}.PR}{2}$$

$$\Rightarrow QR = \frac{\sqrt{3}(12)}{2} = 6\sqrt{3}cm.$$

$$\therefore QR = 6\sqrt{3}cm, PR = 12cm.$$

5.In ΔXYZ, right angle is at y, yz = x, and XZ= 2x then determine ∠YXZ and ∠YZX.

Sol: $\sin X = \frac{oppsite\ side\ of\ \lfloor x}{hypotenuse} = \frac{x}{2x} = \frac{1}{2}$



$$\sin X = \frac{1}{2} = \sin 30^\circ$$

$\Rightarrow$X = 30° or ∠YXZ = 30°

$$\cos Z = \frac{Adjacent\ side\ of\ \lfloor z}{hypotenuse} = \frac{x}{2x} = \frac{1}{2}$$

$$\cos Z = \frac{1}{2} = \cos 60^\circ$$

Z = 60° or ∠YZX = 60°

Exercise – 11.3

1) Evaluate

(i) $\frac{\tan 36°}{\cot 54°}$ **We can write tan θ = cot (90 - θ).**

Cot θ = tan (90 - θ)

$\therefore$ tan 36° = cot (90° - 36°)

$$\Rightarrow \frac{\tan 36°}{\cot 54°} = \frac{\cot(90° - 36°)}{\cot 54°} = \frac{\cot 54°}{\cot 54°} = 1$$

ii) cos 12° – sin 78°

$\Rightarrow$ cos (90 – 78°) – sin 78° $\quad (\because \cos(90° - \theta) = \sin\theta)$

$\Rightarrow$ sin 78° – sin 78° = 0.

iii) cosec 31°- sec 59°

$\Rightarrow$ cosec (90° - 59°) – sec 59°

$\Rightarrow$ sec 59° – sec 59° = 0. (sec θ = cosec (90° - θ))

iv) sin 15° sec 75°

sin 15° . sec (90 - 15°) $\quad (\because \sec(90° - \theta) = \cos ec\theta)$

= sin 15°. Cosec 15° $\quad \left(\because \sin\theta = \frac{1}{\cos ec\theta}\right)$

$$= \sin 15°.\frac{1}{\sin 15°} = 1$$

v) tan 26° . Tan 64°

tan 26° . tan 64° = tan 26° .tan (90° - 26°) $\left(\begin{array}{l}\because \tan(90°-\theta)=\cot\theta \\ =\cot\theta.\dfrac{1}{\tan\theta}\end{array}\right)$

= tan 26°. Cot 26°

$$=\tan 26°.\frac{1}{\tan 26°}=1$$

2. Show that

(i) tan 48°. tan 16° . tan 42°. tan 74° = 1

(ii) cos 36°. cos 54° - sin 36°.sin54° = 0

A. (i) tan 48° . tan 16° tan 42° . tan 74°

We know that tan (90 - θ) = cot θ.

Re write tan 48° = tan (90° - 42°) and tan 16° = tan (90 - 74°)

$\Rightarrow$ tan 48° . tan 16°. tan 42°. tan 74°

= tan (90 - 42°). tan (90 - 74°) .tan 42°. tan 74°

= cot 42°. Cot 74°. tan 42°. tan 74°.

= (cot 42°.tan42°) (cot 74°. tan 74°)

= 1 × 1 $\left(\begin{array}{l}\because \cot\theta.\tan\theta=1 \\ or=\tan\theta.\dfrac{1}{\cot\theta}\end{array}\right)$

= 1

(ii) cos 36° . cos 54° - sin 36° sin 54° = 0.

A. take L.H.S. cos 36°. cos 54° - sin 36°sin 54°

= cos 36°.cos (90 - 36°) – sin 36° (90 - 36°)

= cos 36°. sin36° - sin 36°. cos 36°.

$$\begin{pmatrix} \because (\cos(90° - \theta) = \sin\theta \\ \text{Sin}(90° - \theta) = \cos\theta \end{pmatrix}$$

= 0. R.H.S.

3. If tan 2A = cot (A - 18°), where 2A is an acute angle. Find the value of A.

A. Given that tan 2A = cot (A - 18°)

$\Rightarrow$ cot (90° - 2A) = cot (A - 18°) $(\because \cot(90° - 2A) = \tan 2A)$

$\Rightarrow$ 90° - 2A = A - 18° $\begin{pmatrix} \because 90° - 2A \text{ and } A - 18° \\ \text{both are acute angles} \end{pmatrix}$

$\Rightarrow$ - 2A – A = - 18 - 90°

- 3A = - 108°

$\Rightarrow A = \frac{-108}{-3} = 36°$ ➔ A = 36°.

4. If tan A = cot B, where A & B are acute angles, prove that A + B = 90°.

Sol: Given that tan A = cot B.

$\Rightarrow$ tan A = tan (90° - B) $(\because \tan(90° - \theta) = \cot\theta)$

$\Rightarrow$ A = 90° - B. $\begin{pmatrix} \because A \text{ and } (90° - B) \\ \text{both are acute angle.} \end{pmatrix}$

$\Rightarrow$ A + B = 90°.

5. If A, B and C are interior angle of a triangle ABC, then show that

$$\tan\left(\frac{A+B}{2}\right)=\cot\frac{C}{2}.$$

Sol: The sum of the interior angles in ABC is 180

$$\Rightarrow A + B + C = 180$$

$$\frac{A+B}{2}+\frac{C}{2}=\frac{180^\circ}{2}=90^\circ$$

$$\Rightarrow \frac{A+B}{2}=90^\circ-\frac{C}{2}$$

$$\Rightarrow \tan\left(\frac{A+B}{2}\right)=\tan\left(90^\circ-\frac{C}{2}\right) \qquad \left(\because \tan\left(90^\circ-\theta\right)=\cot\theta\right)$$

$$\therefore \tan\left(\frac{A+B}{2}\right)=\cot\frac{C}{2}.$$

6. Express sin 75° + cos 65° in terms of trigonometric ratios of angles between o° and 45°.

A. sin 75° = sin (90° - 15°) = cos 15° $\quad \left(\because \sin\left(90^\circ-\theta\right)=\cos\theta\right)$

cos 65° = cos (90° - 25°) = sin 25°. $\quad \left(\because \cos\left(90^\circ-\theta\right)=\sin\theta\right)$

$\therefore$ sin 75° + cos 65° = cos 15° + sin 25°.

Exercise – 11.4

1. Evaluating the following.

(i) (1 + tan θ + sec θ) (1 + cot θ - cosec θ).

A. (1 + tan θ + sec θ) (1 + cot θ - cosec θ)

$$=(1+\tan\theta+\sec\theta)\left(1+\frac{1}{\tan\theta}-\operatorname{cosec}\theta\right)$$

$$=(1+\tan\theta+\sec\theta)\left(\frac{\tan\theta+1-\tan\theta.\operatorname{cosec}\theta}{\tan\theta}\right)$$

$$\left(\frac{(1+\tan\theta+\sec\theta)(\tan\theta+1-\sec\theta)}{\tan\theta}\right)\left\{\begin{array}{l}\because \tan\theta\operatorname{cosec}\theta\\ =\frac{\sin\theta}{\cos\theta}.\frac{1}{\sin\theta}\\ \frac{1}{\cos\theta}=\sec\theta\end{array}\right\}$$

$$\frac{(1+\tan\theta)^2-\sec^2\theta}{\tan\theta}$$

$$\frac{1+\tan^2\theta+2\tan\theta-\sec^2\theta}{\tan\theta}$$

$$\frac{1+2\tan\theta-(\sec^2\theta-\tan^2\theta)}{\tan\theta}$$

$$=\frac{1+2\tan\theta-1}{\tan\theta}=\frac{2\tan\theta}{\tan\theta}=2.$$

ii) $(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2$.

A. $(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2$

$\text{Sin}^2\theta+\cos^2\theta+2\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta$

$= 2\sin^2\theta + 2\cos^2\theta = 2(\sin^2\theta + \cos^2\theta)$ $\left(\because \sin^2\theta + \cos^2\theta = 1\right)$

$= 2\,(1) = 2.$

iii) **$(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1)$**

A. $(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1)$ $\begin{pmatrix}\because \sec^2\theta - 1 = \tan^2\theta \\ \operatorname{cosec}^2\theta - 1 = \cot^2\theta\end{pmatrix}$

$= \tan^2\theta \,.\cot^2\theta$

$= 1.$

2) **Show that** $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

A. L.H.S $(\operatorname{cosec}\theta - \cot\theta)^2$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta} = R.H.S$$

3) **Show that** $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A.$

A. **L.H.S**

$$\sqrt{\frac{1+\sin A}{1-\sin A}} = \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}} = \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} \qquad \left(\because 1-\sin^2 A = \cos^2 A\right)$$

$$= \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

= sec A + tan A. R.H.S.

4. **Show that** $\frac{1-\tan^2 A}{\cot^2 A-1} = \tan^2 A.$

A. L.H.S

$$\frac{1-\tan^2 A}{\cot^2 A-1} = \frac{1-\frac{\sin^2 A}{\cos^2 A}}{\frac{\cos^2 A}{\sin^2 A}-1} = \frac{\frac{\cos^2 A-\sin^2 A}{\cos^2 A}}{\frac{\cos^2 A-\sin^2 A}{\sin^2 A}}$$

$$= \frac{\cos^2 A-\sin^2 A}{\cos^2 A} \times \frac{\sin^2 A}{\cos^2 A-\sin^2 A}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A. \qquad R.H.S$$

II Method:

$$\frac{1-\tan^2 A}{\cot^2 A-1} = \frac{1-\tan^2 A}{\frac{1}{\tan^2 A}-1} = \frac{1-\tan^2 A}{\frac{1-\tan^2 A}{\tan^2 A}}$$

= $\tan^2$A=R.H.S.

5. **Show that** $\frac{1}{\cos\theta} - \cos\theta = \tan\theta.\sin\theta.$

A. L.H.S.

$$\frac{1}{\cos\theta} - \cos\theta = \frac{1-\cos^2\theta}{\cos\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta}$$

$= \frac{\sin\theta}{\cos\theta}.\sin\theta = \tan\theta.\sin\theta. =$ R.H.S.

$\therefore$

L.H.S.=R.H.S.

6. Simplify sec A . (1 - sinA) . (sec A + tan A).

A. Sec A (1 – sinA). (sec A + tan A)

= (sec A – sec A sinA) (sec A + tan A)

$= (\sec A - \frac{1}{\cos A}.\sin A)$ (sec A + tan A) $\left(\because \sec A = \frac{1}{\cos A}\right)$

= (sec A – tan A) (sec A + tan A) $\left(\because \tan A = \frac{\sin A}{\cos A}\right)$

$= \text{Sec}^2 A - \tan^2 A$ $\left(\because \sec^2 A - \tan^2 A = 1\right)$

= 1.

7. Prove that $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A \cot^2 A$

A. L.H.S $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \text{cosec}^2 A + 2 \sin A.\text{cosec} A + \cos^2 A + \sec^2 A + 2 \sec A . \cos A$

$= (\sin^2 A + \cos^2 A) + \text{cosec}^2 A + \sec^2 A + 2\sin A \text{ cosec} A + 2\sec A \cos A$

$= 1 + (1 + \cot^2 A) + (1 + \tan^2 A) + 2\sin A.\frac{1}{\sin A} + 2.\frac{1}{\cos A}.\cos A$

$= 1 + 1 + \cot^2 A + 1 + \tan^2 A + 2 + 2$

$= 7 + \cot^2 A + \tan^2 A$

=R.H.S.

8. Simplify (1 – cos θ) (1 + cos θ) (1 + cot^2 θ).

A. (1 – cos θ) (1 + cos θ) (1 + cot^2 θ)

= (1 – cos^2 θ) (1 + cot^2 θ) $\left(\because 1+\cot^2\theta = \cos ec^2\theta\right)$

= sin^2 θ (1 + cot^2 θ) = sin^2 θ. Cosec^{2}θ.

= 1.

9. If sec θ + tan θ = P; then what is the value of sec θ - tan θ?

A. secθ + tan θ = P. (Given)

We know that sec^{2}θ - tan^2 θ = 1 $\left(a^2 - b^2 = (a+b)(a-b)\right)$

⇒ (secθ + tan θ) (sec θ - tanθ) = 1

P (sec θ - tan θ) = 1

⇒ Sec θ - tan θ = $\frac{1}{p}$

∴ The value of secθ - tan θ = $\frac{1}{p}$.

10. If cosecθ + cot θ = k, then show that $\cos\theta = \frac{k^2-1}{K^2+1}$.

A. Given that cosecθ + cotθ = k → (1)

Then cosec θ - cotθ = $\frac{1}{k} \longrightarrow (2)$

$$\left[\begin{array}{l} \because \cos ec^2\theta - \cot^2\theta = 1 \\ \left(\cos ec\theta + \cot\theta\right) \\ \left(\cos ec\theta - \cot\theta\right) = 1 \\ \cos ec\theta - \cot\theta = \dfrac{1}{\cos ec\theta + \cot\theta} \end{array}\right]$$

Adding (1) & (2) we get

$$2\cos ec\theta = k + \frac{1}{k} \Rightarrow 2\cos ec\theta = \frac{k^2+1}{k} \longrightarrow (3)$$

Subtracting (2) from (1) we get

$$2\cot\theta = k - \frac{1}{k} \Rightarrow 2\cot\theta = \frac{k^2-1}{k} \longrightarrow (4)$$

Dividing (4) by (3) we get

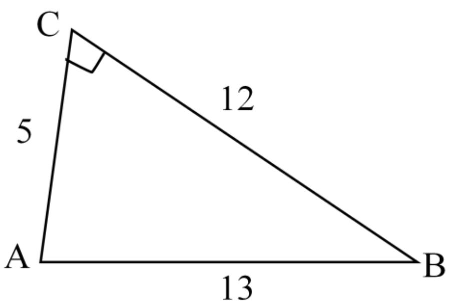
$$\frac{2\cot\theta}{2\cos ec\theta} = \frac{\frac{k^2-1}{k}}{\frac{k^2+1}{k}} = \frac{k^2-1}{k} \times \frac{k}{k^2+1}$$

$$\frac{\cos\theta}{\sin\theta}.\sin\theta = \frac{k^2-1}{k^2+1}$$

$$\therefore \cos\theta = \frac{k^2-1}{k^2+1}$$

Objective Type Questions

1. In the following figure, the value of cot A is **[ ]**



a) $\frac{12}{13}$ b) $\frac{5}{12}$ c) $\frac{5}{13}$ d) $\frac{13}{5}$

2. If in ΔABC, ∠B = 90°, AB = 12 cm and BC = 5cm then the value of cos c is. **[ ]**

a) $\frac{5}{13}$ b) $\frac{5}{12}$ c) $\frac{12}{5}$ d) $\frac{13}{5}$

3. If $\cot\theta = \frac{b}{a}$ **then the value of** $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$ **is** **[ ]**

a) $\frac{b-a}{b+a}$ b) b – a c) b + a d) $\frac{b+a}{b-a}$

4. The maximum value of sinθ is _______ **[ ]**

a) $\frac{1}{2}$ b) $\frac{\sqrt{3}}{2}$ c) 1 d) $\frac{1}{\sqrt{2}}$

5. If A is an acute angle of a ABC, right angled at B, then the value of sin A + cos A is **[ ]**

a) Equal to one b) greater than two

c) Less than one d) equal to two

6. **The value of** $\frac{2\tan 30°}{1+\tan^2 30°}$ **is** [ ]

a) sin 60° b) cos 60° c) tan 60° d) sin 30°

7. **If** $\sin\theta = \frac{1}{2}$ **, then the value of** $(\tan\theta + \cot\theta)^2$ **is** [ ]

a) $\frac{16}{3}$ b) $\frac{8}{3}$ c) $\frac{4}{3}$ d) $\frac{10}{3}$

8. **If sinθ - cosθ = 0; then the value of** $\sin^4\theta + \cos^4\theta$ **is** [ ]

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{3}{4}$ d) 1

9. **If θ = 45° then the value of** $\frac{1-\cos 2\theta}{\sin 2\theta}$ **is** [ ]

a) 0 b) 1 c) 2 d) $\propto$

10. **If tanθ = cotθ, then the value of secθ is** [ ]

a) 2 b) 1 c) $\frac{1}{\sqrt{3}}$ d) $\sqrt{2}$

11. **If A + B = 90°, cot B =** $\frac{3}{4}$ **then tan A is equal to** [ ]

a) $\frac{5}{3}$ b) $\frac{1}{3}$ c) $\frac{3}{4}$ d) $\frac{1}{4}$

12. **If sin (x - 20°) = cos (3x - 10)°. Then x is** [ ]

a) 60° b) 30° c) 45° d) 35.5

13. **The value of 1 + tan 5° cot 85° is equal to** [ ]

a) $\sin^2 5°$ b) $\cos^2 5°$ c) $\sec^2 5°$ d) $\text{cosec}^2 5°$

14. **If any triangle ABC, the value of** $Sin\frac{B+C}{2}$ **is ______** **[ ]**

a) $\cos\frac{A}{2}$ b) $\sin\frac{A}{2}$ c) $-\sin\frac{A}{2}$ d) $-\cos\frac{A}{2}$

15. **If** $\cos\theta=\frac{a}{b}$**, then cosec θ is equal to** **[ ]**

a) $\frac{b}{a}$ b) $\frac{b}{\sqrt{b^2-a^2}}$ c) $\frac{\sqrt{b^2-a^2}}{b}$ d) $\frac{a}{\sqrt{b^2-a^2}}$

16. **The value of cos 20° cos 70° - sin 20° sin 70° is equal to** **[ ]**

a) 0 b) 1 c) ∝ d) cos 50°

17. **The value of tan5° tan 25° tan45° tan 65° tan 85° is______** **[ ]**

a) 2 b) 3 c) 1 d) 4

18. **IFf tanθ + cotθ = 5 then the value of** $\tan^2\theta + \cot^2\theta$ **is ______** **[ ]**

a) 23 b) 25 c) 27 d) 15

19. **If cosecθ = 2 and cotθ = √3 p where θ is an acute angle, then the value of p is** **[ ]**

a) 2 b) 1 c) 0 d) √3

20. $\sqrt{\frac{1+\sin A}{1-\sin A}}$ **is equal to** **[ ]**

a) sin A + cos A b) sec A + tan A

c) sec A – tan A d) $\sec^2 A + \tan^2 A$

Key:

1. b; 2. a; 3. d; 4. c; 5. b; 6. a; 7. a; 8. a; 9. b; 10. d;

11. c; 12. b; 13. c; 14. a; 15. c; 16. b; 17. c; 18. a; 19. b; 20. b.

Fill in the Blanks

1. If cosecθ - cotθ $= \frac{1}{4}$ then the value of cosecθ + cotθ is ________

2. sin 45° + cos 45° = ___________

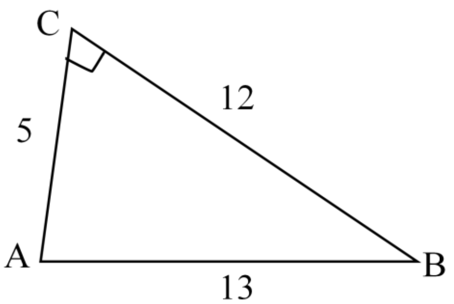
3. 2 $\tan^2 45°$ + $\cos^2 30°$ - $\sin^2 60°$ = _____________

4. sin (90° - A) = _______________

5. If sinA = cosB then, the value of A + B = ______

6. If secθ $= \frac{m+n}{2\sqrt{mn}}$ then sinθ = ______________

7. In the adjacent figure, the value of secA is _____________.



8. If sinA $= \frac{1}{2}$ $\tan^2 45°$, where A is an acute angle then the value of A is ______

9. The maximum value of $\frac{1}{\sec\theta}$, 0° < θ < 90° is _________

10. $\frac{\sin^2\theta}{1-\cos^2\theta}$ is equal to ________

11. if cotθ = 1 then $\frac{1+\sin\theta}{\cos\theta} =$ _________

12. $\sec^2$θ - 1 = ________

13. If secθ + tan θ = p, then the value of secθ - tanθ = ________.

14. The value of sinA or cosA never exceeds ________.

15. sec (90° - A) = _______

Key:

1) 4; 2) √2; 3) 2; 4) cos A; 5) 90°; 6) $\frac{m-n}{m+n}$;

7) $\frac{13}{5}$; 8) 15°; 9) 1; 10) 1; 11) √2 + 1; 12) $\tan^2\theta$;

13) $\frac{1}{p}$; 14) 1; 15) cosecA.

Trigonometric Identities

An identity equation having trigonometric ratios of an angle is called trigonometric identity. And it is true for all the values of the angles involved in it.

(1) $\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A, \cos^2 A = 1 - \sin^2 A$

$$\sin A = \sqrt{1-\cos^2 A} = \sqrt{(1+\cos A)(1-\cos A)}$$

$$\cos A = \sqrt{1-\sin^2 A} = \sqrt{(1+\sin A)(1-\sin A)}$$

(2) $1 + \operatorname{Tan}^2 A = \operatorname{Sec}^2 A$ or $\operatorname{Sec}^2 A - 1 = \operatorname{Tan}^2 A$

$\operatorname{Sec}^2 A - \operatorname{Tan}^2 A = 1$ or $\operatorname{Tan}^2 A - \operatorname{Sec}^2 A = -1$

(SecA + Tan A) (Sec A – Tan A) = 1

$$Sec A + Tan A = \frac{1}{SecA - TanA} \quad (or) \quad SecA - TanA = \frac{1}{\sec A + \tan A}$$

(3) $1 + \cot^2 A = \operatorname{cosec}^2 A$ (or) $\operatorname{cosec}^2 A - 1 = \cot^2 A$

$\operatorname{Cosec}^2 A - \cot^2 A = 1$ (or) $\cot^2 A - \operatorname{cosec}^2 A = -1$

$(\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A) = 1$

$$\operatorname{cosec} A + \cot A = \frac{1}{\operatorname{cosec} A - \cot A} \; (or) \; \operatorname{cosec} A - \cot A = \frac{1}{\operatorname{cosec} A + \cot A}$$

$(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

$(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1) = 1.$