Chapter 1

Electric Charges and Fields

Solutions (Set-1)

Very Short Answer Type Questions :

1. An electrostatic field line cannot be discontinuous. Why?

Sol. An electrostatic line is a continuous curve, because tangent to it at any point represents the direction in which a test charge kept at that point will experience force. It cannot have sudden breaks, because no abrupt force acts on a test charge.

2. The distance of the point on the equatorial plane of a small electric dipole is halved. By what factor will the electric field due to the dipole at the point change?

Sol. $E \propto \frac{1}{r^3}$. Since distance is halved so electric field E will become 8 times.

3. In an electric field an electron is kept freely. If the electron is replaced by a proton, what will be the relationship between the forces experienced by them?

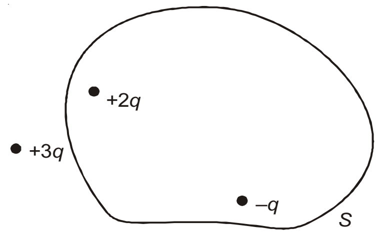
Sol. $F = qE$, so force on electron $\vec{F}_e = -e\vec{E}$ and force on proton $\vec{F}_p = e\vec{E}$

$\therefore\ \vec{F}_e = -\vec{F}_p$

4. Which orientation of an electric dipole in a uniform electric field would correspond to stable equilibrium?

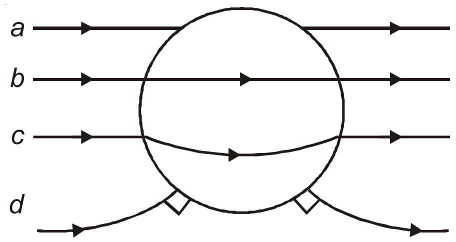
Sol. When $\vec{p}$ and $\vec{E}$ are parallel, dipole remains at stable equilibrium.

5. Figure shows three point charges, $+2q$, $-q$ and $+3q$. Two charges $+2q$ and $-q$ are enclosed within a surface S. What is the electric flux due to this configuration through the surface 'S'?



Sol. Electric flux = (charge enclosed)/ε_0 = $\frac{q}{\varepsilon_0}$

6. A metallic sphere is placed in uniform electric field as shown in the figure. Which path is followed by electric field lines and why?



Sol. Line *d* is correctly drawn, because it is not passing through conductor and it is perpendicular to the surface from where it starts or meets the conductor.

7. When two electrically charged particles having charges of different magnitude are placed at a distance *d* from each other, they experience a force of attraction *F*. These two particles are put in contact and again placed at the same distance from each other. What is the nature of new force between them?

Sol. In such cases final force will always be repulsive.

8. Can a charged body attract an uncharged body?

Sol. Yes. By induction.

9. An electric dipole of dipole moment $\vec{p}$ is placed in a uniform electrostatic field $\vec{E}$. For what angle between $\vec{p}$ and $\vec{E}$ will the potential energy of the electric dipole be half of its maximum value?

Sol. $\frac{pE}{2} = -pE\cos\theta$ $\therefore$ $\theta = 120^\circ$

10. What is the line of symmetry of a dipole field?

Sol. About axial line.

11. Find the value of electric field that would exactly balance the weight of electron.

Sol. $E = \frac{mg}{e}$ = 5.67×10^{-11} N/C ($m = 9.1 \times 10^{-31}$ kg and $e = 1.6 \times 10^{-19}$ C)

12. A small test charge is released at rest at a point in an electrostatic field. Will it travel along the field line passing through that point?

Sol. If electric line is straight, then it will move on the line. If line is curved then charge will move tangential to it.

13. If Coulomb's law involved $\frac{1}{r^3}$ dependence $\left(\text{instead of } \frac{1}{r^2}\right)$, where *r* is distance between two point changes would Gauss's law still be true?

Sol. No

14. A glass rod is rubbed with silk. Will its mass increase or decrease?

Sol. Mass will decrease because it loses electrons.

15. Repulsion is the sure test of electric charge. Explain

Sol. Because if two bodies repel, then they surely have charges and that too of similar nature.

Short Answer Type Questions :

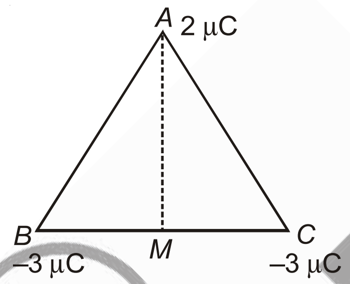
16. A system has two electric charges $q_A = 2.5 \times 10^{-7}$C and $q_B = -2.5 \times 10^{-7}$ C located at points $A(0, 0, -15$ cm) and $B(0, 0, +15$ cm) respectively. Calculate the electric dipole moment of the system. What is its direction?

Sol. Magnitude of dipole moment $p = ql = (2.5 \times 10^{-7}\text{C})(30 \times 10^{-2}\text{ m})$.

$$= 7.5 \times 10^{-8} \text{ Cm}$$

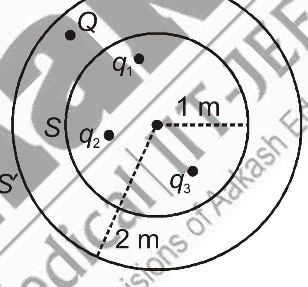
Its direction is from –ve to +ve charge *i.e.*, along positive z-axis.

17. Three point charges of +2 μC, –3 μC and –3 μC are kept at the vertices A, B and C respectively of an equilateral triangle of side 20 cm as shown in figure. What should be the sign and magnitude of the charge to be placed at the mid-point (M) of side BC so that the charge at A remains in equilibrium?



Sol. It should be positive. Resultant force on A by charges at B and C are along $\overrightarrow{AM}$. Force by charge at M should have a repulsive force on A along MA.

18. The flux of the electrostatic field through the closed spherical surface S' is found to be four times that through the closed spherical surface S. Find magnitude of charge Q. Given : $q_1 = 1\mu$C, $q_2 = -2$ μC and $q_3 = 9.854$ μC



Sol. $\dfrac{q_1 + q_2 + q_3}{Q + q_1 + q_2 + q_3} = \dfrac{1}{4}$

$\Rightarrow Q = 3(q_1 + q_2 + q_3)$

19. A charge of 17.7×10^{-4}C is distributed uniformly over a large sheet of area 200 m². Calculate the electric field intensity at a distance 20 cm from it in air.

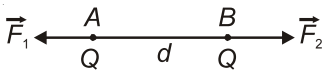
Sol. $E = \dfrac{\sigma}{2\varepsilon_0} = \dfrac{Q}{2A\varepsilon_0}$

$$= \frac{17.7 \times 10^{-4}}{2 \times 200 \times 8.85 \times 10^{-12}}$$

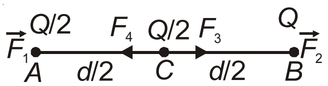
$$= 5 \times 10^5 \text{ N/C}$$

20. Two similar and equally charged identical metal spheres A and B repel each other with a force of 2×10^{-5} N. A third identical uncharged sphere C is touched with A and then placed at the mid-point between A and B. Calculate the net electric force on C.

Sol. $\left|\vec{F}_1\right| = \left|\vec{F}_2\right| \Rightarrow F = \dfrac{Q^2}{4\pi\varepsilon_0 d^2}$

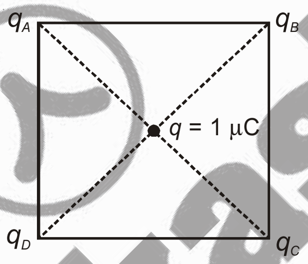


$$\vec{F}' = \vec{F}_3 + \vec{F}_4 = \frac{1}{4\pi\varepsilon_0}\left[\frac{\left(\frac{Q}{2}\right)^2}{\left(\frac{d}{2}\right)^2} - \frac{Q\cdot\frac{Q}{2}}{\left(\frac{d}{2}\right)^2}\right] = \frac{-Q^2}{4\pi\varepsilon_0 d^2}$$



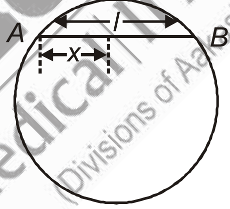
$= |\vec{F}'| = F = 2 \times 10^{-5}$N

21. Four point charges of $q_A = 2\ \mu$C, $q_B = -5\ \mu$C, $q_C = 2\ \mu$C and $q_D = -5\mu$C are located at the corners of a square $ABCD$ of side 10 cm. Find the force on a charge of 1 μC placed at the centre of the square.



Sol. By symmetry, net force on central charge will become zero.

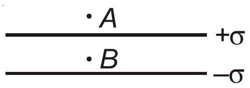
22. A charged wire AB of length l is placed inside a sphere, as shown in figure. Linear charge density of wire is $\lambda = kx$, where x is the distance measured along the wire from end A and K is constant. Calculate the flux through the sphere.



Sol. $\phi = \dfrac{Q}{\varepsilon_0}$ where $Q = \int_0^l \lambda dx = \int_0^l kx\, dx = \dfrac{kl^2}{2}$

$\therefore\ \phi = \dfrac{Q}{\varepsilon_0} = \dfrac{kl^2}{2\varepsilon_0}$

23. Two plane sheets of charge densities $+\sigma$ and $-\sigma$ are kept in air, as shown in figure. What are the electric field intensities at points A and B?



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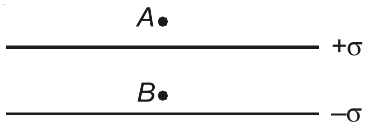
Sol. Consider vertically upward direction as positive y-axis



$$\vec{E} \text{ at } A = \frac{\sigma}{2\varepsilon_0}\hat{j} - \frac{\sigma}{2\varepsilon_0}\hat{j} = 0$$

$$\vec{E} \text{ at } B = -\frac{\sigma}{2\varepsilon_0}\hat{j} + \frac{\sigma}{2\varepsilon_0}(-\hat{j}) = \frac{\sigma}{\varepsilon_0}\left(-\hat{j}\right)$$

24. A charge particle of charge $+q$, mass m and moving with a velocity of $u\hat{i}$ enters a uniform electric field of strength $\vec{E} = E\hat{j}$. Find magnitude of velocity and magnitude of displacement of the particle after time t.

Sol. Initial velocity of the charge

$$\vec{u} = u\hat{i}$$

acceleration of the charge

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m} = \frac{qE}{m}\hat{j}$$

velocity at time t

$$\vec{v} = \vec{u} + \vec{a}t = u\hat{i} + \frac{qEt}{m}\hat{j}$$

$\therefore$ Magnitude of velocity $= V = \sqrt{u^2 + \left(\frac{qEt}{m}\right)^2}$

Displacement at time t,

$$\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2 = (ut)\hat{i} + \frac{1}{2}\frac{qE}{m}t^2\hat{j}$$

$\therefore$ Magnitude of displacement $= S = \sqrt{(ut)^2 + \left(\frac{1}{2}\frac{qEt^2}{m}\right)^2}$

25. A uniformly charged conducting sphere of 2.5 m in diameter has a surface charge density of 100μ C/m^2. Calculate the

(i) Charge on the sphere.

(ii) Total electric flux coming out a Gaussian surface just enclosing the outer surface of the sphere.

Sol. (i) Surface charge density $\sigma = \frac{Q}{4\pi R^2} \Rightarrow Q = \sigma \cdot 4\pi R^2$

$$= 100 \times 10^{-6}\frac{\text{C}}{\text{m}^2} \times 4 \times \frac{22}{7} \times (2.5)^2$$

$= 7.9 \times 10^{-3}$C

$= 7.9$ mC

(ii) Net flux coming out of the sphere $= \frac{\text{charge enclosed}}{\varepsilon_0} \Rightarrow \phi = \frac{Q}{\varepsilon_0}$

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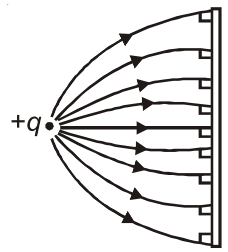
26. A positive point charge ($+q$) is kept in the vicinity of an uncharged conducting plate. Sketch electric field lines originating from the point on to the surface of the plate.

Sol.



27. A spherical conducing shell of inner radius r_1 and outer radius r_2 has a charge Q. Another charge q is placed at the centre of the shell.

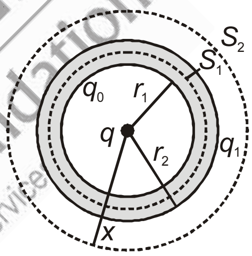
(a) What is the surface charge density on the

(i) Inner surface?

(ii) Outer surface of the shell?

(b) Derive the expression for the electric field at a point $x > r_2$ from the centre of the shell.

Sol. (a) (i) Let the charge on the inner surface of spherical shell is q_0. Applying Gauss law to Gaussian surface S_1, intermediate to outer and inner surface.



$$\oint_{S_1} \vec{E}\cdot\overrightarrow{ds} = \frac{q_0 + q}{\varepsilon_0}$$

$$\Rightarrow\ 0 = \frac{q_0 + q}{\varepsilon_0} \qquad [\vec{E} = 0 \text{ because } S_1 \text{ is inside metal}]$$

$$\Rightarrow q_0 = -q$$

$\therefore$ Surface charge density on inner surface $\sigma_1 = \dfrac{-q}{4\pi r_1^2}$

(ii) Let the charge on the outer surface is q_1.

Applying conservation of charge $q_1 + q_0 = Q$

$\Rightarrow q_1 = Q - q_0 = Q + q$

$\therefore$ Surface charge density on outer surface $\sigma_2 = \dfrac{Q+q}{4\pi r_2^2}$

(b) Applying Gauss law to the Gaussian surface S_2 at distance x from centre $\oint_{S_2} \vec{E}\cdot\overrightarrow{ds} = \dfrac{Q+q}{\varepsilon_0}$

$$\Rightarrow E\cdot 4\pi x^2 = \frac{Q+q}{\varepsilon_0}$$

$$\Rightarrow\ E = \frac{Q+q}{4\pi\varepsilon_0 x^2}$$

28. Show that the electric field at the surface of a charged conducting sphere is given by $\vec{E} = \dfrac{\sigma}{\varepsilon_0}\hat{n}$, where σ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction.

Sol. If σ is the surface charge density on the sphere, then charge on the sphere,

$Q = \sigma \times 4\pi R^2 = 4\pi R^2\sigma$

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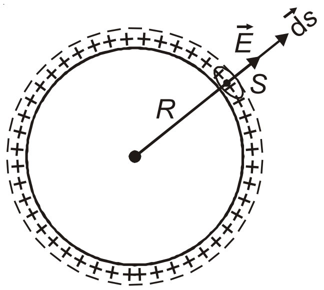
Consider a Gaussian surface S coincident with the outer surface of metal sphere.

Applying Gauss law to the surface



$$\oint_S \vec{E}\cdot\vec{ds} = \frac{Q_{in}}{\varepsilon_0}$$

$$\Rightarrow \oint_S E\, ds\cos 0 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E\oint_S ds = \frac{Q}{\varepsilon_0}$$

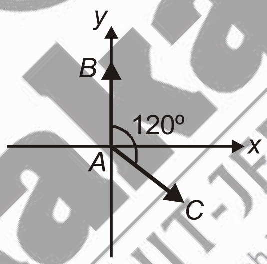
$$\Rightarrow E\,4\pi R^2 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{Q}{4\pi R^2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

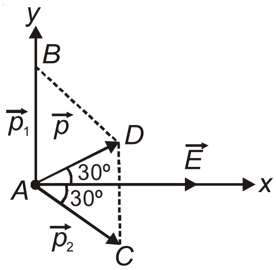
Electric field is directed radially outward so

$$\vec{E} = \frac{\sigma\hat{n}}{\varepsilon_0}$$

29. Two small identical electric dipoles AB and AC, each of dipole moment p are kept at an angle of 120º as shown in figure. What is the resultant dipole moment of this combination? If this system is subjected to electric field $(\vec{E})$ directed along $+x$ direction, what will be the magnitude and direction of torque acting on this?



Sol. Resultant dipole moment



$\vec{p} = \vec{p}_1 + \vec{p}_2$

Using parallelogram law of vector addition, resultant dipole moment

$\vec{p}$ is directed along 30º with positive x-axis ($\because$ $\angle BAD = \angle DAC = 60º$)

Magnitude of $|\vec{p}| = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos 120}$ = $\sqrt{p^2 + p^2 + 2p^2\left(-\frac{1}{2}\right)}$ = p

Electric field is acting along positive x-axis

So angle between $\vec{E}$ and $\vec{p}$ = 30º

Torque on the dipole, $\vec{\tau} = \vec{p}\times\vec{E}$ = $pE\sin\theta$ (Normally into the paper (i.e., along negative z-axes))

= $pE\sin 30$

= $\frac{pE}{2}$ normally into the paper or along negative z-axies.

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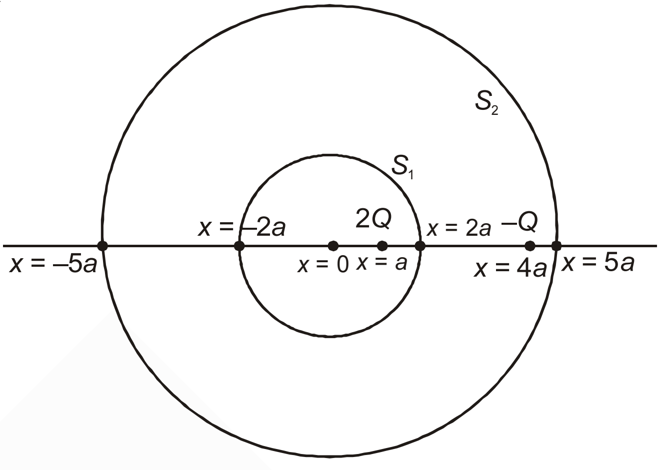
30. Charges of magnitudes $2Q$ and $-Q$ are located at points (a, 0,0) and ($4a$, 0, 0). Find the ratio of the electric flux due to these charges through concentric spheres of radii $2a$ and $5a$ centered at the origin.

Sol. Electric flux through sphere S_1 of radus $2a$



$$\phi_1 = \oint_{S_1} \vec{E}\cdot\vec{ds}$$

$$= \frac{Q_{in}}{\varepsilon_0} = \frac{2Q}{\varepsilon_0}$$

Electric flux through sphere S_2 of radius $5a$

$$\phi_2 = \oint_{S_1} \vec{E}\cdot\vec{ds}$$

$$= \frac{Q_{in}}{\varepsilon_0} = \frac{2Q-Q}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

Long Answer Type Questions :

31. (i) State Coulomb's law and write the formula of electrostatic force between two charges, separated by certain distance. Under which condition this formula is applicable?

(ii) Two positive point charges which are 0.1m apart repel each other with a force of 18 N. If the sum of the charges be 9 μC, calculate their separate values of charges.

Sol. (i) Scalar form:

Coulomb's law is a quantitative statement about the force between two point charges. Coulomb measured the force between two point charges and found that it varied inversely as the square of the distance between the charges and was directly proportional to the product of magnitude of the two charges and acted along the line joining the two charges.

If two point charges Q_1 and Q_2 at rest are separated by a distance r in vacuum, the magnitude of force between them is given by $F = \frac{k|Q_1Q_2|}{r^2}$. The constant k is usually put as $k = \frac{1}{4\pi\varepsilon_0}$, where ε_0 is called the permittivity of free space and has the value $\varepsilon_0 = 8.854 \times 10^{-12}$ C^2/Nm^2. For all practical purposes we will take $\frac{1}{4\pi\varepsilon_0} \approx 9\times10^9\ \text{Nm}^2/\text{C}^2$. The choice of k determines the size of the unit of charge. SI unit of charge is defined to be 1 C. So 1 C is the charge that when placed at a distance of 1 m from another charge of the same magnitude in vacuum experience an electrical force of repulsion of magnitude 9×10^9 N.

Coulomb's law is strictly applicable for point charges.

(ii) Using Coulomb's law,

$$F = \frac{kQ_1Q_2}{r^2}$$

or, $$18 = \frac{9\times10^9\times Q_1Q_2\times10^{-12}}{(0.1)^2}$$

$\Rightarrow Q_1Q_2 = 20$, again $Q_1 + Q_2 = 9$

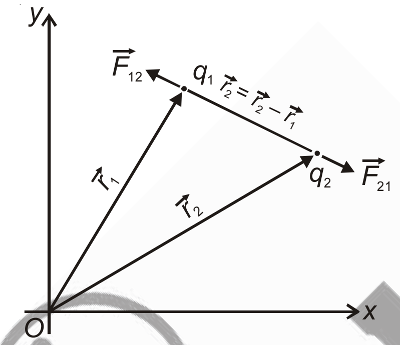
$\therefore Q_1 = 4\mu$C and $Q_2 = 5\mu$C

32. (i) Express Coulomb's law in vector form.

(ii) Two point charges q_1 = 5 × 10^{-6} C and q_2 = 3 × 10^{-6} C are located at (3, 5, 1) m and (1, 3, 2) m. Find $\overrightarrow{F_{12}}$ and $\overrightarrow{F_{21}}$ using vector form of Coulomb's law.

Sol. (i) **Coulomb's law in vector form**

Since force is a vector, Coulomb's law in the vector notation will be written as follows. Let the position vector of charges q_1 and q_2 be $\vec{r_1}$ and $\vec{r_2}$ respectively (figure). We denote force on q_1 due to q_2 by $\overrightarrow{F_{12}}$ and force or q_2 due to q_1 by $\overrightarrow{F_{21}}$.



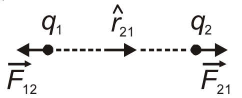
The two point charges q_1 and q_2 have been numbered 1 and 2 and the vector leading from 1 to 2 is denoted by $\overrightarrow{r_{21}}$, *i.e.*, $\vec{r_2} - \vec{r_1}$. In the same way, the vector leading from 2 to 1 is denoted by $\overrightarrow{r_{12}}$. So, $\overrightarrow{r_{12}} = \vec{r}_1 - \vec{r_2} = -\overrightarrow{r_{21}}$. The magnitude of the vectors $\overrightarrow{r_{21}}$ and $\overrightarrow{r_{12}}$ is denoted by r_{21} and r_{12}, respectively and they are equal, *i.e.*, $r_{12} = r_{21}$. To denote the direction from 1 to 2 (or from 2 to 1), we define the unit vectors:

$$\hat{r}_{21} = \frac{\overrightarrow{r_{21}}}{r_{21}} \text{ and } \hat{r}_{12} = \frac{\overrightarrow{r_{12}}}{r_{12}} \qquad \text{where } \hat{r}_{21} = -\hat{r}_{12}$$

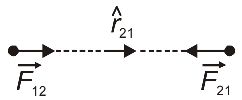
So Coulomb's force law between two point charges q_1 and q_2 located at $\vec{r_1}$ and $\vec{r_2}$ is then expressed as

$$\overrightarrow{F_{21}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21}.$$

The equation above is valid for any sign of q_1 and q_2 whether positive or negative. If q_1 and q_2 are of the same sign (either both positive or both negative), $\overrightarrow{F_{21}}$ is along $\hat{r}_{21}$, which denotes repulsion, as it should be for like charges. If q_1 and q_2 are of opposite signs, $\overrightarrow{F_{21}}$ is along $-\hat{r}_{21}\left(=\hat{r}_{12}\right)$, which denotes attraction, as expected for unlike charges. Thus, we do not have to write separate equations for the cases of like and unlike charges. Above equation takes care of both cases correctly



(Like charges *i.e.*, $q_1q_2 > 0$)



(Unlike charges *i.e.*, $q_1q_2 < 0$)

Also note that the force $\overrightarrow{F_{12}}$ on charge q_1 due to charge q_2 is obtained from above equation by simply interchanging 1 or 2, *i.e.*,

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$$\overrightarrow{F_{12}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}^2}\hat{r}_{12} = -\overrightarrow{F_{21}}$$

Thus, Coulomb's law agrees with the Newton's third law.

(ii) $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$

$$= \left(3\hat{i}+5\hat{j}+\hat{k}\right)-\left(\hat{i}+3\hat{j}+2\hat{k}\right)$$

$$= 2\hat{i}+2\hat{j}-\hat{k}$$

$$\overrightarrow{F_{12}} = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12} = \frac{kq_1q_2}{|\vec{r}_{12}|^3}(\vec{r}_{12})$$

$$= \frac{9\times10^9\times5\times10^{-6}\times3\times10^{-6}\times(2\hat{i}+2\hat{j}-\hat{k})}{(2^2+2^2+1^2)^{\frac{3}{2}}}\text{N}$$

$$= \frac{27\times5\times10^{-3}}{27}\left(2\hat{i}+2\hat{j}-\hat{k}\right)\text{N}$$

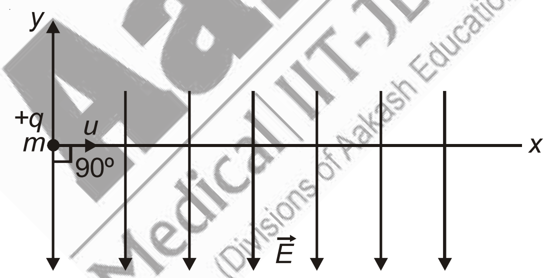
$$= \left(10\hat{i}+10\hat{j}-5\hat{k}\right)10^{-3}\text{ N}$$

Again $\vec{F}_{12} = -\vec{F}_{21}$

33. (i) Show that path of a charge particle projected normal to an uniform electrostatic field is a parabola.

(ii) A particle of mass *m* and charge +*q* is thrown at a speed *u* against a uniform electric field *E*. How much distance will it travel before coming to rest?

Sol. (i) Consider a charge +*q* of mass *m* is projected with speed *u* normal to uniform electric field as shown in figure.



Electric field, $\vec{E} = -E\hat{j}$

Initial velocity, $\vec{u} = u\hat{i}$

acceleration, $\vec{a} = \frac{-qE}{m}\hat{j}$

Instantaneous position of charge, is $\vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2 = (ut)\hat{i} + \frac{1}{2}\left(\frac{-qE}{m}\hat{j}\right)t^2 = (ut)\hat{i} - \left(\frac{qEt^2}{2m}\right)\hat{j} = x\hat{i} - y\hat{j}$

x-co-ordinate of position $x = ut \Rightarrow t = \frac{x}{u}$

y-co-ordinate of position $y = \frac{qEt^2}{2m} = \frac{qE}{2m}\left(\frac{x}{4}\right)^2$

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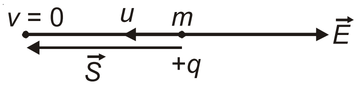
$$\Rightarrow\ y = \frac{qE}{2mu^2}x^2$$

$$\boxed{y \propto x^2}$$

Above equation represent equation of parabola, so path of the charge is a parabola.

(ii) 

Initial velocity of charge $\vec{u} = -u\hat{i}$

acceleration of charge $\vec{a} = \frac{q\vec{E}}{m} = \frac{qE\hat{i}}{m} = \frac{qE\hat{i}}{m}$, displacement $\vec{S} = -S\hat{i}$

Instantaneous velocity v at displacement $\vec{S}$

$$v^2 = u^2 + 2\vec{a}\cdot\vec{S}$$

$$\Rightarrow\ 0 = u^2 + 2\left[\frac{qE}{m}\hat{i}\cdot(-S\hat{i})\right]$$

$$\Rightarrow 0 = u^2 + 2\left(-\frac{qES}{m}\right)$$

$$\Rightarrow\ S = \frac{mu^2}{2qE}$$

34. (i) Derive an expression for electric field due to a point charge at a distance r from the charge and express it in vector form.

(ii) Four identical point charges of 4 μC each are placed at the corners of a square of each side 0.1m. Calculate the electric field at the centre of the square.

(iii) Calculate the electric field intensity at the centre, when one of the corner charges is removed.

Sol. (i) Consider a charge Q at orgin O, as shown in figure

Electric field at point P, at position vector $\vec{r}$ due to charge Q at origin.

$$\vec{E} = \frac{\vec{F}}{q_0}\ \ (q_0 = \text{test charge at } P)$$

$$= \left(\frac{\frac{Qq_0}{4\pi\varepsilon_0 r^2}\hat{r}}{q_0}\right) = \frac{Q\hat{r}}{4\pi\varepsilon_0 r^2}$$

$\hat{r}$ indicates the electric field is radially outward from charge Q.

(ii) $\vec{E}_A$, $\vec{E}_B$, $\vec{E}_C$ and $\vec{E}_D$ are electric fields by charges at corners A, B, C and D respectively. O is equidistant from all charges

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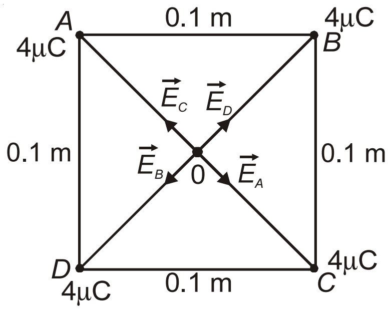


So $E_A = E_B = E_C = E_D$

and $\vec{E}_A = -\vec{E}_C$ and $\vec{E}_B = -\vec{E}_D$

Net electric field at O,

$$\vec{E} = \vec{E}_A + \vec{E}_B + \vec{E}_C + \vec{E}_D = \vec{0}$$

(iii) If all charges are present

$$\vec{E}_A + \vec{E}_B + \vec{E}_C + \vec{E}_D = \vec{0}$$

$$\Rightarrow \vec{E}_A + \vec{E}_B + \vec{E}_C = -\vec{E}_D$$

If charge at D is absent, then electric field will be

$$\vec{E}_A + \vec{E}_B + \vec{E}_C = -\vec{E}_D$$

$$\Rightarrow \vec{E}_B = \frac{Q}{4\pi\varepsilon_0 r^2} \text{ along } \overrightarrow{OD}$$

$$= \frac{9\times10^9 \times 4\times10^{-6}}{\left(\frac{0.1}{\sqrt{2}}\right)}$$

$$= 72\times10^5 \text{ N}$$

35. (i) Two charges of a dipole –4 μC and +4 μC are placed at the points $A(1, 0, 4)$m and $B(2, -1, 5)$m located in an electric field $\vec{E} = 0.20\hat{i}$ V/cm. Calculate the torque acting on the dipole.

(ii) A Gaussian surface is shown in figure. Charges q_1 and q_2 are inside and charge q_3 is outside the surface. Indicate the charges which contributes the electric field appearing in the formula $\oint \vec{E}\cdot\overrightarrow{ds} = \frac{q_{in}}{\varepsilon_0}$.

(iii) The electric field in a region is given by $\vec{E} = \frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}$ with $E_0 = 2\times10^3 \text{NC}^{-1}$. Find the flux of this field through a rectangular surface of area 0.2 m^2 parallel to y-z plane.

Sol. (i) Dipolemoment

$$\vec{p} = q\times 2\vec{l}$$

$$= 4\times10^{-6}\text{C}\times\left(\overline{AB}\right)$$

$$= 4\times10^{-6}\text{C}\left[(2-1)\hat{i} + (-1-0)\hat{j} + (5-4)\hat{k}\right]$$

$$= 4\times10^{-6}\left[\hat{i} - \hat{j} + \hat{k}\right]\text{Cm}$$

$$\Rightarrow \vec{p} = 4\left[\hat{i} - \hat{j} + \hat{k}\right]\times10^{-6} \text{ Cm}$$

Electric field $\vec{E} = 0.2\hat{i}$ V/Cm, $0.2\hat{i}\times 10^2$ V/m

$\therefore\ \vec{\tau} = \vec{p}\times\vec{E} = 4\times10^{-6}\left(\hat{i}-\hat{j}+\hat{k}\right)\times(0.2\times10^2)\hat{i}$ Nm

$= 8\times10^{-5}\left[\hat{k}+\hat{j}\right]$ Nm

(ii) E is due to all the charges q_1, q_2 and q_3.

(iii) $\vec{E} = \frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}$

$\vec{A} = 0.2m^2\hat{i}$ (Area vector is normal to surface)

$\phi = \vec{E}\cdot\vec{A}$

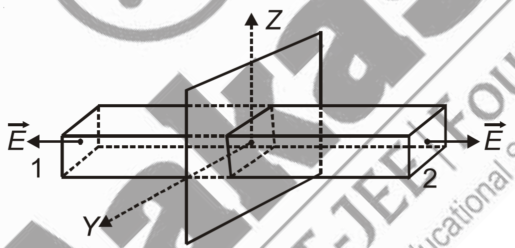
$= \frac{3}{5}E_0\times 0.2$

$= \frac{3}{5}\times 0.2\times 2\times 10^3$ Vm

= 240 Vm

36. Derive an expression of electric field due to uniformly charged infinite plane sheet.

Sol. Let σ be the uniform surface charge density on the sheet. We take the X axis normal to the given plane.



By symmetry the electric field will not depend on Y and Z coordinates and its direction is always parallel to the X-coordinate. We can take the Gaussian surface to be a rectangular parallelopiped of cross-section area A. The electric flux is non zero for surface 1 and 2 as shown in figure and for other surfaces the flux is zero.

Flux = $\frac{q_{in}}{\varepsilon_0}$, by Gauss's law

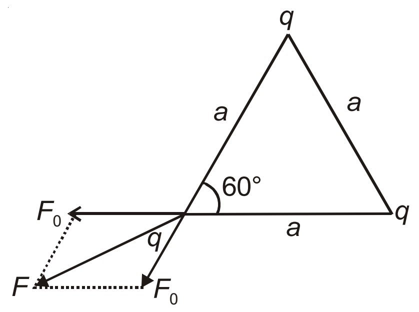
So $2EA = \frac{\sigma A}{\varepsilon_0}$

$E = \frac{\sigma}{2\varepsilon_0}$

37. Three charges of same magnitude q are placed at the corners of an equilateral triangle of side length a. Find the net force on any one of charges.

Sol. Net force on charge is = $\sqrt{F_0^2 + F_0^2 + 2F_0^2\cos 60^\circ}$

where $F_0 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}$

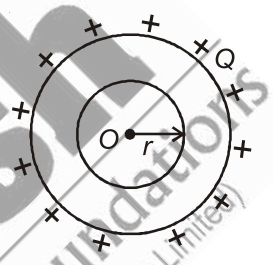


Net force $F = \sqrt{3}\,F_0 = \dfrac{\sqrt{3}}{4\pi\varepsilon_0}\dfrac{q^2}{a^2}$

38. Find electric field inside a uniformly charged thin spherical shell having charge Q. If a charge q is placed at the centre of this shell, how the electric field will change?

Sol. For thin spherical shell charge is always on the outer surface.

Consider a Gaussian spherical surface at distance r



$$\oint \vec{E}\cdot d\vec{S} = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow \quad E4\pi r^2 = \frac{q_{in}}{\varepsilon_0}$$

but $q_{in} = 0$

$E = 0$

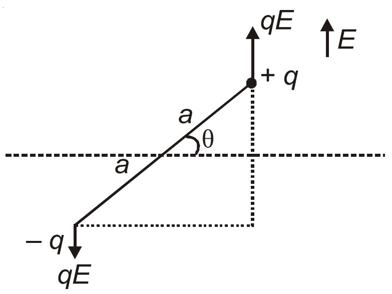
but when we put a charge at the centre

$q_{in} = q$

$$\therefore \quad E = \frac{q_{in}}{4\pi r^2 \varepsilon_0}$$

39. Derive an expression for torque acting on a dipole of dipole moment $\vec{p}$, when it is placed in uniform external field $\vec{E}$.

Sol.



Electric field is in upward direction as shown in figure force on positive charge is in upward direction and that on negative charge is in downward direction.

Magnitude of torque $= qE \times 2a \sin\theta$

$= 2qaE \sin\theta$

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Its direction is normal to the plane of the paper coming out of it.

The magnitude of $\vec{p} \times \vec{E}$ is also $pE\sin\theta$ and its direction is normal to the paper, coming out of it

Therefore, in vector form the torque can be given as

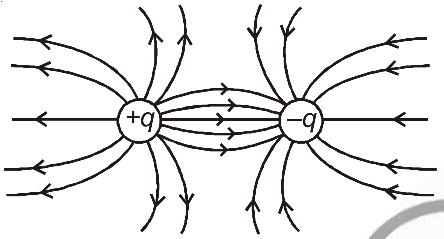
$\therefore\ \vec{\tau} = \vec{p} \times \vec{E}$

40. (a) Explain the invariant property of charge.

(b) Draw the electric field lines for a dipole.

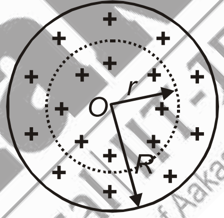
Sol. (a) The magntidue of charge on a body is independent of its velocity, it is invariant for all frames of reference in relative motion. This is not always true for every scalar.

(b)



41. Derive an expression for electric field inside a solid non-conducting sphere of charge Q, when the charge is distributed uniformly. Also calculate electric field at the centre of sphere and electric field on the surface of sphere.

Sol. *R* is radius of sphere and we want to calculate field at *r*. Consider a spherical Gaussian surface at distance *r* from the centre. By symmetry electric field at this closed surface is always radially outwards.



Flux through this surface is

$$E4\pi r^2 = \frac{q_{in}}{\varepsilon_0} \qquad \text{...(1)}$$

charge Q is distributed uniformly

charge per unit volume = $\dfrac{Q}{\frac{4}{3}\pi R^3}$

charge in sphere of radius r is

$$q_{in} = \frac{Q}{\frac{4}{3}\pi R^3}\ \frac{4}{3}\pi r^3$$

$$q_{in} = \frac{Qr^3}{R^3}$$

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Put value of q_{in} in equation 1

$$E4\pi r^2 = \frac{Qr^3}{\varepsilon_0 R^3}$$

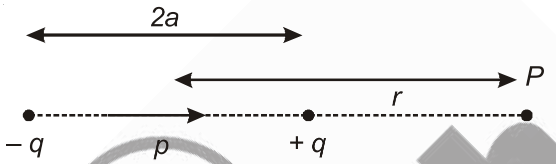
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}$$

At the centre $r = 0$ $\quad\therefore E = 0$

On the surface, $r = R$ $\quad\therefore E = \frac{Q}{4\pi\varepsilon_0 R^2}$

42. Derive the expression for electric field on the axis of an electric dipole.

Sol. Let the point P be at distance r from the centre of the dipole on the side of the charge q as shown in figure. Then



$$\vec{E}_{-q} = \frac{-q}{4\pi\varepsilon_0 (r+a)^2}\hat{p}$$

where $\hat{p}$ is the unit vector along the dipole axis (from $-q$ to $+q$).

Also, $\vec{E}_{+q} = \frac{q}{4\pi\varepsilon_0 (r-a)^2}\hat{p}$

Total field

$$\vec{E} = E_{+q} + E_{-q}$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{4ar}{(r^2-a^2)^2}\hat{p}$$

For $r >> a$,

$$\vec{E} = \frac{4qa}{4\pi\varepsilon_0 r^3}\hat{p} = \left(\frac{2p}{4\pi\varepsilon_0 r^3}\right)\hat{p}$$

43. What is meant by the statement that the electric field of a point charge has spherical symmetry whereas that of a linear charge of large length is cylindrically symmetrical?

Sol. Consider a charge q at the centre of a sphere of radius r. The magnitude of electric field at all points on the surface of the sphere is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.$$

So the electric field due to a point charge is spherically symmetric.

In the case of a line charge of large length the magnitude of electric field at a distance r from the line is given by

$E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$, where λ is linear charge density.

Now, imagine a cylinder of radius r drawn with the line charge as axis. The electric field, due to the line charge, at all points on the surface of the cylinder will be the same. So the electric field due to the linear charge has cylindrical symmetry.

44. (a) A copper sphere of mass 2 g contains nearly 2×10^{22} atoms. The charge on the nucleus of each atom is 29 e. What fraction of the electrons must be removed from the sphere to give it a charge of +2.9 μC?

(b) What is an electric line of force? What is its importance?

Sol. (a) Total number of electrons = $29 \times 2 \times 10^{22}$

The number of electrons removed = $\dfrac{2.9 \times 10^{-6}}{1.6 \times 10^{-19}}$

$\therefore$ Fraction of electrons removed $= \dfrac{2.9 \times 10^{-6}}{1.6 \times 10^{-19} \times 29 \times 2 \times 10^{22}}$

$= 3.125 \times 10^{-11}$

(b) An electric line of force is an imaginary straight or curved line drawn in such a way that the tangent on it gives the direction of electric force experienced by a positive charge at all points.

The tangent at a point on an electric line of force also gives the direction of the electric field at that point.

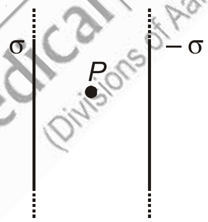
The relative closeness of electric lines of force in a certain region provides us an estimate of the electric field strength in that region.

45. (a) Two infinite parallel planes have uniform charge densities $+\sigma$ and $-\sigma$. What is the electric field between the planes?

(b) How can you charge an 'uncharged insulated conductor' negatively by electrostatic induction?

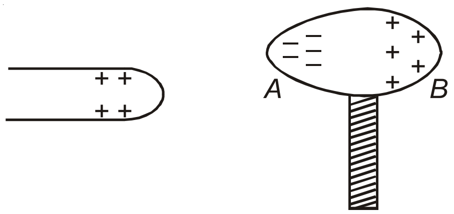
Sol. (a) At a point P between the two planes, the electric fields due +ve and –ve charged planes are respectively

$E_+ = \dfrac{\sigma}{2\varepsilon_0}$; $E_- = \dfrac{\sigma}{2\varepsilon_0}$ both are in the same direction.

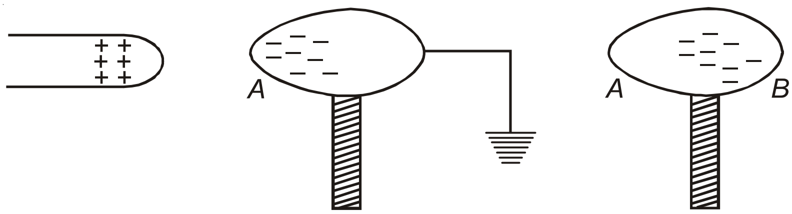


$\therefore$ Net field $= \dfrac{\sigma}{2\varepsilon_0} + \dfrac{\sigma}{2\varepsilon_0} = \dfrac{\sigma}{\varepsilon_0}$.

(b) First of all, bring a positively charged glass rod near the given conductor AB. The end A of the conductor will be charged negatively while the end B will be charged positively as shown in figure.



Keeping the glass rod near the conductor, the conductor is earthed as shown in figure.



Finally, the glass rod and earth-connection are removed. The conductor AB acquires a negative charge.

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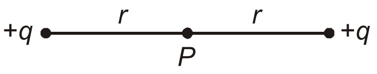
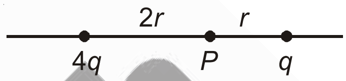
Chapter 1

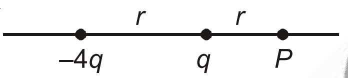
Electric Charges and Fields

Solutions (Set-2)

1. In which of the following cases electric field at point P is non-zero?

(1) 

(2) 

(3) 

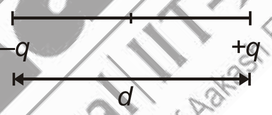
(4) 

Sol. Answer (4)

$$E = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q}{r^2}\right)$$

$$E = 0 \text{ if } \frac{q_1}{r_1^2} = \frac{q_2}{r_2^2}$$

2. For two equal and opposite charges placed at distance d, then electric field will be zero at

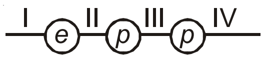


(1) Mid way
(2) Distance d from $(-q)$ and to the left
(3) Distance d from $(+q)$ and to the right
(4) No finite distance

Sol. Answer (4)

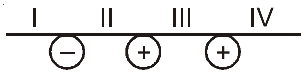
Electric field will be zero at mid point.

3. In which regions, electric field can be zero for given arrangement of an electron and two protons?



(1) I & II
(2) I & III
(3) I, III & IV
(4) Only IV

Sol. Answer (2)



Only in region I and III field can be zero.

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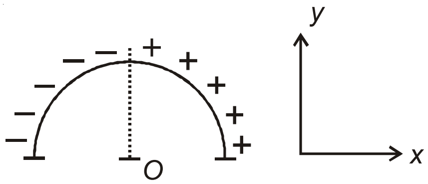
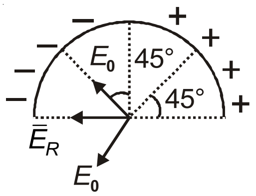
4. The linear charge density upon the semi-circular ring, on both side is same in magnitude, the electric field at *O* is along



(1) $\hat{i}$ (2) $-\hat{i}$

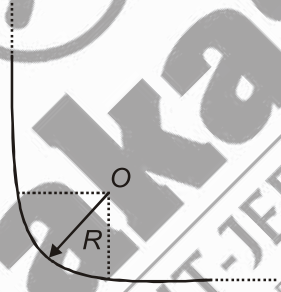
(3) $\hat{j}$ (4) $-\hat{j}$

Sol. Answer (2)



$\bar{E}_R$ will be along $-x$ axis.

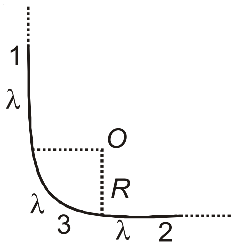
5. An infinite long thread carries a charge uniformly spread over its length. It is bent in the form as shown. If the charge per unit length is λ, the electric field at point *O* is



(1) $\dfrac{\lambda}{2\pi\varepsilon_0 r}$ (2) $\dfrac{\lambda}{\sqrt{2}\pi\varepsilon_0 R}$

(3) $\dfrac{\lambda}{2\sqrt{2}\pi\varepsilon_0 R}$ (4) Zero

Sol. Answer (3)



$$\vec{E}_0 = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$\left|\vec{E}_0\right| = \frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 R}$$

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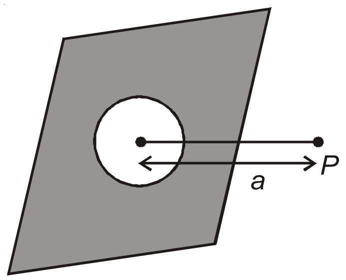
6. An infinitely large non-conducting plane sheet of charge density σ has a circular aperture of certain radius carved out of it. The electric field at a point which is at a distance 'a' from the centre of aperture is $\dfrac{\sigma}{2\sqrt{2}\varepsilon_0}$. The radius of aperture is



(1) a (2) $\dfrac{a}{\sqrt{2}}$

(3) $\sqrt{2}a$ (4) $\dfrac{a}{2}$

Sol. Answer (1)

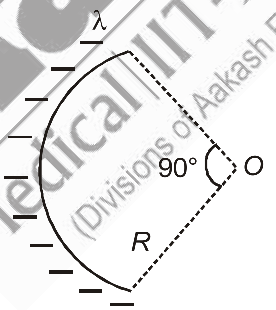
$$E_P = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{a}{\sqrt{R^2 + a^2}}\right)$$

Given, $E_P = \dfrac{\sigma}{2\sqrt{2}\varepsilon_0}$

$$\Rightarrow \left(\frac{\sigma}{2\sqrt{2}\varepsilon_0}\right) = \frac{\sigma}{2\varepsilon_0}\left(1 - 1 + \frac{a}{\sqrt{R^2 + a^2}}\right)$$

$$\frac{1}{\sqrt{2}} = \frac{a}{\sqrt{R^2 + a^2}} \quad \Rightarrow R = a$$

7. The electric field due to charged circular arc shown in following figure, at centre O is



(1) $\dfrac{k\lambda}{R}$ (2) $\sqrt{2}\dfrac{k\lambda}{R}$

(3) $\dfrac{k\lambda}{R^2}$ (4) $\sqrt{2}\dfrac{k\lambda}{R^2}$

Sol. Answer (2)

$$E_0 = \frac{\lambda}{2\pi\varepsilon_0 R}\sin\left(\frac{\theta}{2}\right)$$

Here, $\theta = 90°$

$$E_0 = \frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 R}$$

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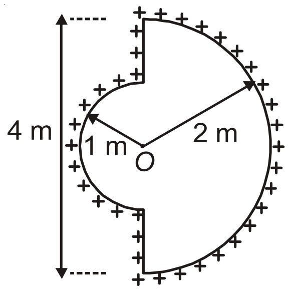
8. If $\lambda = 1\ \mu$ C/m, then electric field intensity at O is



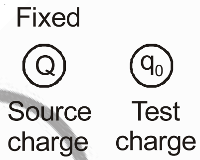
(1) 9 N/C (2) 900 N/C

(3) 9000 N/C (4) 9×10^9 N/C

Sol. Answer (3)

Find electric field due to each sagment and add them in vector form.

9. A test charge q_0 is placed in the electric field of a source charge Q. The test charge will not follow path of electric field line if



(1) It has zero velocity when released

(2) It has non-zero velocity along outward radius, when released

(3) It has non-zero velocity along inward radius when released

(4) It has a velocity different from the above three

Sol. Answer (4)

Charge will not follow path of electric field rim if it has a velocity which is not radially.

10. The electric field in a region of space is spherically symmetric and radially outward. The flux of electric field through a sphere of radius r, centred at origin is $\phi = kr^4$. Which of the following is the correct relation between electric field strength with radial distance r from origin?

(1) $E \propto r^2$ (2) $E \propto r$

(3) $E \propto \dfrac{1}{r^2}$ (4) $E \propto r^3$

Sol. Answer (1)

$\phi = (E)\ 4\pi r^2$

$\Rightarrow kr^4 = (E)\ 4\pi r^2$

$\Rightarrow E \propto r^2$

11. If electric field in a region is given by

$$\vec{E} = \frac{E_0.x}{l}\hat{i} + \frac{E_0.y^2}{l^2}\hat{j} + \frac{E_0.z^3}{l^3}\hat{k}$$

where $E_0 = 5 \times 10^3$ N/C, $l = 2$ cm.

then electric flux passing through the plane $x = 2$ cm. The plane is a square of side 2 cm.

(1) $2\dfrac{\text{N.m}^2}{\text{C}}$ (2) $3\dfrac{\text{N.m}^2}{\text{C}}$

(3) $5\dfrac{\text{N.m}^2}{\text{C}}$ (4) None of these

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Sol. Answer (1)

$$\phi_{x=2} = \frac{(E_0)((2)/100)}{\ell}\left(\frac{2}{100}\right)^2$$

$$= (5\times10^3)\times\frac{4}{10^4} = 2\frac{\text{N.m}^2}{\text{C}}$$

12. If electric field in a region is given as $\vec{E} = \frac{E_0.x}{l}\hat{i} + \frac{E_0.y^2}{l^2}\hat{j} + \frac{E_0.z^3}{l^3}\hat{k}$ ($E_0 = 5 \times 10^3$ N/C, l = 2 cm), then calculate the flux of electric field passing through a rectangular plane (2 cm × 1 cm) at y = 2 cm

(1) $2\frac{\text{N.m}^2}{\text{C}}$

(2) $3\frac{\text{N.m}^2}{\text{C}}$

(3) $4\frac{\text{N.m}^2}{\text{C}}$

(4) 1 Nm^2/C

Sol. Answer (4)

$$\phi_{y=2} = \left(\frac{E_0y^2}{\ell^2}\right)(A)$$

$$= \left(5\times10^3\right)\left(\frac{2}{2}\right)^2\left(\frac{2}{10^4}\right)$$

$= 1\ \text{N.m}^2/\text{C}$

13. In previous problem, the flux passing through a triangular plane (Area 2 cm^2) at z = 0 will be

(1) Zero

(2) $2\frac{\text{N.m}^2}{\text{C}}$

(3) $3\frac{\text{N.m}^2}{\text{C}}$

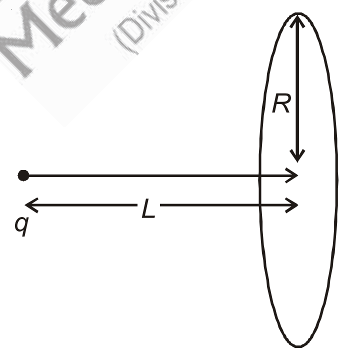
(4) None of these

Sol. Answer (1)

$$\phi_{z=0} = \frac{E_0z^3}{\ell^3}A$$

at $z = 0$, $\phi = 0$

14. The electric flux through ring shown in figure is



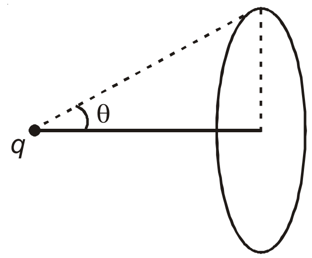
(1) $\frac{q}{2\varepsilon_0}\left[1+\frac{L}{\sqrt{R^2+L^2}}\right]$

(2) $\frac{q}{2\varepsilon_0}\left[1-\frac{L}{\sqrt{R^2+L^2}}\right]$

(3) $\frac{q}{\varepsilon_0}\left[1-\frac{L}{\sqrt{R^2+L^2}}\right]$

(4) Zero

Sol. Answer (2)



$\cos\theta = \dfrac{L}{\sqrt{R^2 + L^2}}$

$\phi = \left(\dfrac{q}{2\varepsilon_0}\right)(1 - \cos\theta)$

$= \left(\dfrac{q}{2\varepsilon_0}\right)\left(1 - \dfrac{L}{\sqrt{R^2 + L^2}}\right)$

15. A ball of radius R carries a positive charge whose volume charge density depends only on the distance r from the ball's centre as $\rho = \rho_0\left[1 - \dfrac{r}{R}\right]$ where ρ_0 is a constant.
The electric field at distance $r > R$ will be

(1) $E = \dfrac{\rho_0 R^3}{8\varepsilon_0 r^2}$ (2) $E = \dfrac{\rho_0 R^3}{12\varepsilon_0 r^2}$

(3) $E = \dfrac{\rho_0 R^3}{16\varepsilon_0 r^2}$ (4) $E = \dfrac{\rho_0 R^3}{24\varepsilon_0 r^2}$

Sol. Answer (2)
If $r > R$

$E = \dfrac{kQ}{r^2}$ ...(1)

$Q = \int_0^R \rho_0\left(1 - \dfrac{r}{R}\right)4\pi r^2 dr$

Put value of Q in equation (1) to get E.

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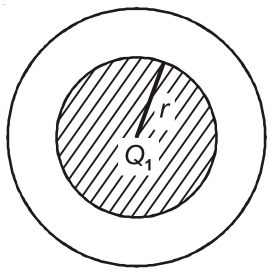
16. Inside a uniformly charged ball, at $r = r_m$, the electric field is maximum. Then (Given $\rho = \rho_0\left(1 - \dfrac{r}{R}\right)$ is volume charge density)

(1) $r_m = R/3$ (2) $r_m = 3R/2$

(3) $r_m = 2R/3$ (4) $r_m = 4R/3$

Sol. Answer (3)

If $r < R$



$E = \left(\dfrac{kQ_1}{r^2}\right)$ ...(i)

$Q_1 = \int_0^r e_0\left(1 - \dfrac{r}{R}\right)4\pi r^2 dr$

Put value of Q_1 in equation (i) we get $E = f(r)$

For maximum $\dfrac{dE}{dr} = 0$

17. The maximum electric field inside ball in previous problem is

(1) $\dfrac{\rho_0 R}{9\varepsilon_0}$

(2) $\dfrac{\rho_0 \varepsilon_0}{9R}$

(3) $\dfrac{\rho_0 R}{3\varepsilon_0}$

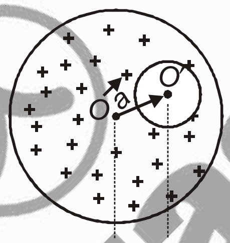
(4) $\dfrac{\rho_0 R}{6\varepsilon_0}$

Sol. Answer (1)

For maximum field.

$$\frac{dE}{dr} = 0$$

18. Charge is uniformly distributed with volume charge density ρ in a spherical volume of radius R. A cavity of radius r is made in the charge distribution such that the centre of the cavity is at position vector $\vec{a}$ from the centre of the charge distribution, then the electric field in the cavity is



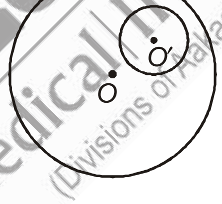
(1) Zero

(2) Non-zero & non-uniform

(3) Non-zero, uniform and equal to $\dfrac{\rho}{2\varepsilon_0} \cdot \vec{a}$

(4) Non-zero & uniform and equal to $\dfrac{\rho}{3\varepsilon_0} \vec{a}$

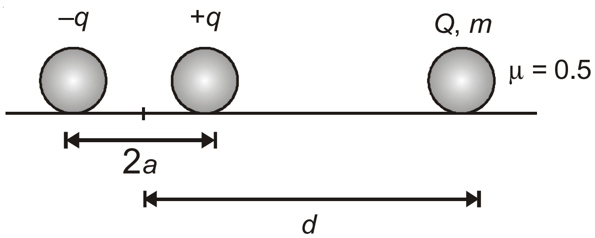
Sol. Answer (4)



$$\vec{E} = \frac{\sigma\, \overrightarrow{OO'}}{3\varepsilon_0}$$

Electric field inside the cavity is uniform.

19. A dipole having charges + q and – q is fixed on a rough surface as shown. Another charge Q of mass m is placed at distance d from this dipole. The minimum value of d for which charge Q will not move. (Given a << d, co-efficient of static friction μ_s = 0.5)



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(1) $\left(\dfrac{2aqQ}{\pi\varepsilon_0 mg}\right)^{1/3}$

(2) $\left(\dfrac{aqQ}{\pi\varepsilon_0 mg}\right)^{1/3}$

(3) $\left(\dfrac{aqQ}{2\pi\varepsilon_0 mg}\right)^{1/2}$

(4) $\left(\dfrac{2aqQ}{\pi\varepsilon_0 mg}\right)^{1/2}$

Sol. Answer (1)

We must have

$QE \le f_5^{\max}$

$\dfrac{2k(q2a)Q}{d^3} \le (0.5)\, mg$

$\Rightarrow\ d \ge \left(\dfrac{2aqQ}{\pi\varepsilon_0 mg}\right)^{1/3}$

20. Two balls of charges q_1 and q_2 initially have exactly same velocity. Both the balls are subjected to same uniform electric field for same time. As a result, the velocity of the first ball is reduced to half of its initial value and its direction changes by 60°. The direction of the velocity of second ball is found to change by 90°.

The electric field and initial velocity of the charged particle are inclined at angle

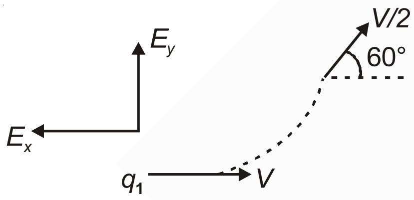
(1) 60°

(2) 30°

(3) 90°

(4) 150°

Sol. Answer (4)



For q_1

$\dfrac{V}{4} = V - \dfrac{q_1 E_x}{m}t$ ...(i)

$\dfrac{\sqrt{3}V}{4} = \dfrac{q_1 E_y}{m}t$ ...(ii)

For q_2

$0 = V - \dfrac{q_2 E_x}{m}t$ ...(iii)

From equation (i) and (ii),

$\dfrac{E_x}{E_y} = \sqrt{3}$ $\Rightarrow$ Angle will be 150°

21. In previous problem, if new velocity of second charged particle has a magnitude 'x' times the initial velocity, then x is

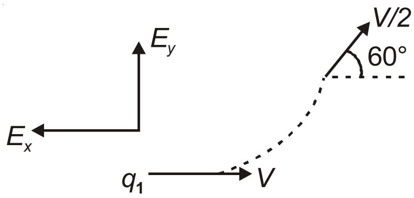
(1) $\sqrt{3}$

(2) $\dfrac{1}{\sqrt{3}}$

(3) $\dfrac{1}{2}$

(4) 2

Sol. Answer (2)



For q_1

$$\frac{V}{4} = V - \frac{q_1E_x}{m}t \qquad \text{...(i)}$$

$$\frac{\sqrt{3}V}{4} = \frac{q_1E_y}{m}t \qquad \text{...(ii)}$$

For q_2

$$0 = V - \frac{q_2E_x}{m}t \qquad \text{...(iii)}$$

From equation (i) and (ii),

$$\frac{E_x}{E_y} = \sqrt{3} \quad \Rightarrow \text{Angle will be } 150°$$

22. Consider the situation of problem 20. If the specific charge (charge to mass ratio) of first ball is K, the specific charge of second ball is

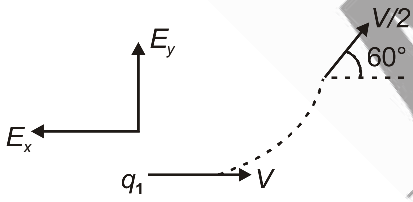
(1) $2K$

(2) $4K$

(3) $\frac{4}{3}K$

(4) $\frac{3}{4}K$

Sol. Answer (3)



For q_1

$$\frac{V}{4} = V - \frac{q_1E_x}{m}t \qquad \text{...(i)}$$

$$\frac{\sqrt{3}V}{4} = \frac{q_1E_y}{m}t \qquad \text{...(ii)}$$

For q_2

$$0 = V - \frac{q_2E_x}{m}t \qquad \text{...(iii)}$$

From equation (i) and (ii),

$$\frac{E_x}{E_y} = \sqrt{3} \quad \Rightarrow \text{Angle will be } 150°$$

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23. There is a non-conducting rod of length L and negligible mass with two small balls each of mass m and electric charge Q attached to its ends. The rod can rotate in the horizontal plane about a fixed vertical axis crossing it a distance $\frac{L}{4}$ from one of its ends.

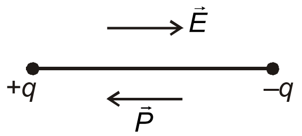
A uniform horizontal electric field E (along $+x$ axis) is established.

At first the rod is in unstable equilibrium. If it is disturbed slightly from this position, the maximum velocity attained by the ball which is closer to the axis in subsequent motion, is

(1) $\sqrt{\frac{2QEL}{m}}$ (2) $\sqrt{\frac{2QEL}{5m}}$

(3) $\sqrt{\frac{QEL}{5m}}$ (4) $\sqrt{\frac{4QEL}{m}}$

Sol. Answer (3)



$$2PE = \frac{1}{2}I\omega^2$$

$$\omega = \sqrt{\frac{4PE}{I}}, \quad P = qL$$

$$I = m\left(\frac{L}{4}\right)^2 + m\left(\frac{3L}{4}\right)^2 = \frac{5}{8}mL^2$$

24. In what position, the rod must be set so that if displaced a little from that position, it begins simple harmonic oscillations about the axis? (Consider the situation of previous problem)

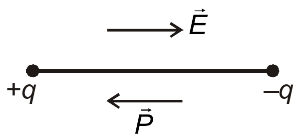
(1) 

(2) 

(3) 

(4) 

Sol. Answer (1)



$$2PE = \frac{1}{2}I\omega^2$$

$$\omega = \sqrt{\frac{4PE}{I}}, \quad P = qL$$

$$I = m\left(\frac{L}{4}\right)^2 + m\left(\frac{3L}{4}\right)^2 = \frac{5}{8}mL^2$$

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25. What would be the angular frequency of SHM in the above question (*i.e.*, Q. 24)?

(1) $\sqrt{\dfrac{8QE}{5mL}}$ (2) $\sqrt{\dfrac{5mL}{QE}}$ (3) $\sqrt{\dfrac{3QE}{2mL}}$ (4) $\sqrt{\dfrac{QE}{mL}}$

Sol. Answer (1)

$$PE\theta = \frac{5}{8}mL^2\alpha \Rightarrow \omega = \sqrt{\frac{8QE}{5mL}}$$

26. The force of repulsion between two point charges is F, when they are d distance apart. If the point charges are replaced by conducting spheres each of radius r and the charges remain same. The separation between the centre of sphere is d, then force of repulsion between them is

(1) Equal to F (2) Less than F

(3) Greater than F (4) Cannot be said

Sol. Answer (2)

The charges will move away, increasing the effective distance.

27. Two identical small bodies each of mass m and charge q are suspended from two strings each of length l from a fixed point. This whole system is taken into an orbiting artificial satellite, then find the tension in strings

(1) $\dfrac{Kq^2}{l^2} + 2mg$ (2) $\dfrac{Kq^2}{4l^2} + 2mg$

(3) $\dfrac{Kq^2}{l^2}$ (4) $\dfrac{Kq^2}{4l^2}$

Sol. Answer (4)

$T = F = Kq^2/4l^2$

28. If two identical spheres having charge 16 μC and –8 μC are kept at certain distance apart, then the force is F. They are touched and again kept at the same distance, the force becomes

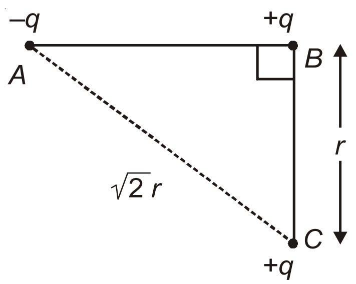
(1) F (2) $\dfrac{F}{18}$ (3) $\dfrac{F}{8}$ (4) $\dfrac{F}{2}$

Sol. Answer (3)

$F = C\,(16)\,(8)$

$$F' = C\left(\frac{16-8}{2}\right)^2 \Rightarrow F' = F/8$$

29. In the following figure, if magnitude of force between B and C is F, then magnitude of force between A and B is



(1) $\dfrac{F}{\sqrt{2}}$ (2) $\sqrt{2}F$ (3) F (4) $\dfrac{F}{2}$

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Sol. Answer (3)

$F_{BC} = Kq^2/r^2$, $F_{AB} = Kq^2/r^2$

$\Rightarrow F_{AB} = F_{BC}$

30. Two points charges +4 q and $+q$ are separated by distance r, where should a third point charge Q be placed that it remains in equilibrium

(1) $\frac{2r}{3}$ from q (2) $\frac{r}{3}$ from q (3) $\frac{r}{2}$ from q (4) $\frac{r}{3}$ from $4q$

Sol. Answer (2)

$$\frac{kQ(4q)}{x^2} = \frac{kQ(q)}{(r-x)^2} \Rightarrow x = \frac{2r}{3};$$

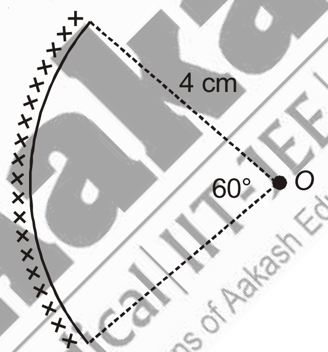
31. Five identical charges $+q$ are placed at five corner of a regular hexagon of side a. Find the magnitude of electric field at centre.

(1) $\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$ (2) $\frac{q}{2\pi\varepsilon_0 a^2}$ (3) $\frac{5q}{4\pi\varepsilon_0 a^2}$ (4) $\frac{5q}{2\pi\varepsilon_0 a^2}$

Sol. Answer (1)

Electric field due to 4 charges will cancel out. So, $E = \frac{q}{4\pi\varepsilon_0 a^2}$.

32. Find the electric field at point O, due to the segment of a ring, whose linear charge density is 8 C/cm.

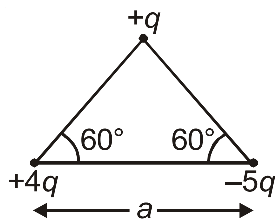


(1) 9×10^{13} V/m (2) 16×10^{13} V/m (3) 8×10^{13} V/m (4) 18×10^{13} V/m

Sol. Answer (4)

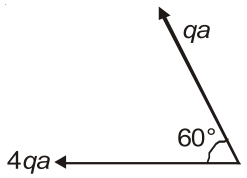
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}\sin(\theta/2)$$

33. Find the magnitude of dipole moment of the following system.



(1) $\sqrt{21}\,aq$ (2) Zero (3) 4 aq (4) 10 aq

Sol. Answer (1)



$$p = \sqrt{(qa)^2 + (4qa)^2 + 2(qa)(4qa)\cos 60^\circ}$$

34. Two short dipoles, each of dipole moment p, are placed at origin. The dipole moment of one dipole is along x-axis, while that of other is along y-axis. The electric field at a point (a, 0) is given by

(1) $\left(\dfrac{1}{4\pi\varepsilon_0}\right)\dfrac{2p}{a^3}$

(2) $\left(\dfrac{1}{4\pi\varepsilon_0}\right)\dfrac{p}{a^3}$

(3) $\left(\dfrac{1}{4\pi\varepsilon_0}\right)\dfrac{\sqrt{5}p}{a^3}$

(4) Zero

Sol. Answer (3)

$$\vec{E} = \vec{E_1} + \vec{E_2} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{a^3}\hat{i} - \frac{1}{4\pi\varepsilon_0}\frac{p}{a^3}\hat{j}$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{5}.p}{a^3}$$

35. Which of the following is true, when a dipole is placed in non-uniform electric field?

(1) Net force on it is equal to zero

(2) Torque may or may not be zero

(3) Torque must be zero

(4) All of these

Sol. Answer (2)

Torque may or may not be zero. Similarly, force may or may not be zero.

36. Which of the following is true for electric flux through a Gaussian surface?

(1) It depends on magnitude of charge enclosed by Gaussian surface

(2) Electric flux is a scalar quantity

(3) Electric flux is independent of shape of Gaussian surface enclosing the charge

(4) All are true

Sol. Answer (4)

$\phi = q_{enc}/\varepsilon_0$, it has no direction, and is independent of shape.

37. A point charge of +6 μC is placed at a distance 20 cm directly above the centre of a square of side 40 cm. The magnitude of the flux through the square is

(1) ε_0

(2) $\dfrac{1}{\varepsilon_0}$

(3) $\varepsilon_0 \times 10^{-6}$

(4) $\dfrac{1}{\varepsilon_0}\times 10^{-6}$

Sol. Answer (4)

$\phi = Q/6\varepsilon_0$

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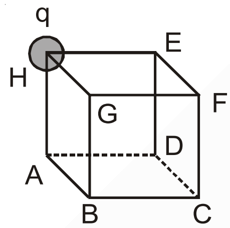
38. If the electric field is given by $3\hat{i}+2\hat{j}+6\hat{k}$. Find the electric flux through a surface area 20 unit lying in *xy* plane

(1) Zero (2) 60 unit (3) 40 unit (4) 120 unit

Sol. Answer (4)

$\phi=\vec{E}\cdot\vec{A}=(3\hat{i}+2\hat{j}+6\hat{k})\cdot 20\hat{k}=120$

39. A small spherically symmetric charge *q* is placed at one vertex of a cube as shown. The flux through the faces *ABCD* and *HGEF* are, respectively,



(1) $\frac{q}{24\varepsilon_0}, \frac{q}{24\varepsilon_0}$ (2) $0, \frac{q}{8\varepsilon_0}$ (3) $\frac{q}{8\varepsilon_0}, 0$ (4) $\frac{q}{24\varepsilon_0}, 0$

Sol. Answer (4)

By Gauss's law total flux through the cube = $\frac{q}{8\varepsilon_0}$, as $\frac{1}{8}$th of charge *q* lies inside the cube.

Flux through the three faces meeting at *q* is zero. Flux through *HGEF* = 0. Three other faces are symmetrically located with respect to *q*.

$\therefore$ Flux through $ABCD = \frac{1}{3}\times\frac{q}{8\varepsilon_0} = \frac{q}{24\varepsilon_0}$

40. A tiny spherical oil drop carrying a net charge *q* is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7}\times 10^5$ Vm^{-1}. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} ms^{-1}. Given g = 9.8 ms^{-2}, viscosity of the air = 1.8×10^{-5} Ns m^{-2} and the density of oil = 900 kg m^{-3}, the magnitude of *q* is

(1) 1.6×10^{-19} C (2) 3.2×10^{-19} C (3) 4.8×10^{-19} C (4) 8.0×10^{-19} C

Sol. Answer (4)

During equilibrium in presence of electric field $qE = mg$

$\Rightarrow qE = \frac{4}{3}\pi r^3 \rho g$ ...(i)

When the drop descends with constant velocity

$mg = 6\pi\eta rV$

$\Rightarrow \frac{4}{3}\pi r^3\rho g = 6\pi\eta rV$

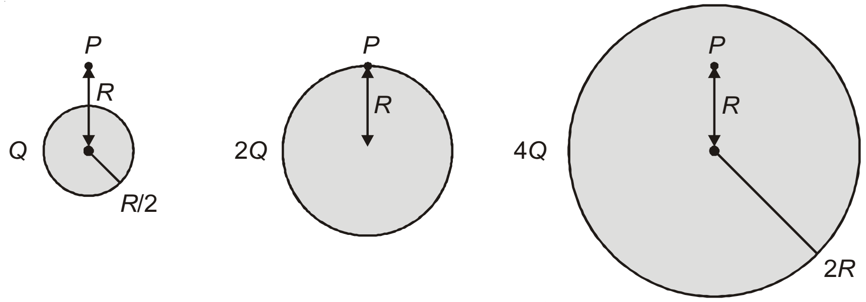
Putting the values, we obtain

$r = \frac{3}{7}\times 10^{-5}$ m

Putting back in equation (i)

$q \approx 8 \times 10^{-19}$ C

41. Charges Q, $2Q$ and $4Q$ are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii $R/2$, R and $2R$ respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of sphere 1, 2 and 3 are E_1, E_2 and E_3 respectively, then



Sphere 1 Sphere 2 Sphere 3

(1) $E_1 > E_2 > E_3$ (2) $E_3 > E_1 > E_2$ (3) $E_2 > E_1 > E_3$ (4) $E_3 > E_2 > E_1$

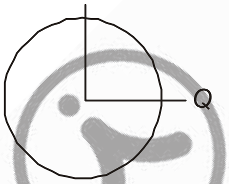
Sol. Answer (3)

$$E_1 = \frac{KQ}{R^2}$$

$$E_2 = \frac{K(2Q)}{R^2} = \frac{2KQ}{R^2}$$

$$E_3 = \frac{K(4Q)R}{8R^3} = \frac{KQ}{2R^3}$$

$$E_2 > E_1 > E_3$$



42. Under the influence of the Coulomb field of charge $+Q$, a charge $-q$ is moving around it in an elliptical orbit. Find out the correct statement(s)

(1) The angular momentum of the charge $-q$ is constant

(2) The linear momentum of the charge $-q$ is constant

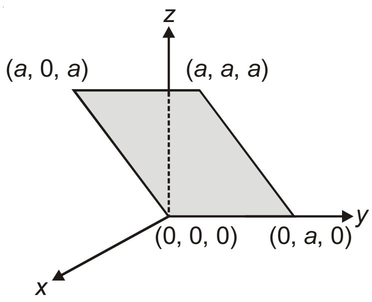
(3) The angular velocity of the charge $-q$ is constant

(4) The linear speed of the charge $-q$ is constant

Sol. Answer (1)

As τ about Q is zero, L = constant about $+Q$ (although this should have been mentioned in the problem). Linear velocity, momentum, speed and angular speed vary.

43. Consider an electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is



(1) $2E_0a^2$ (2) $\sqrt{2}E_0a^2$ (3) E_0a^2 (4) $\frac{E_0a^2}{\sqrt{2}}$

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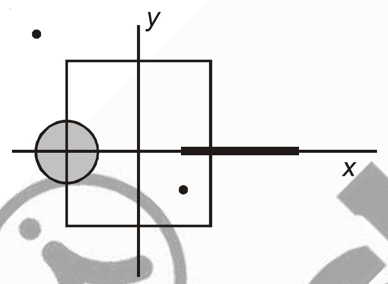
Sol. Answer (3)

$$\vec{A} = \frac{1}{2}\times\vec{d}_1\times\vec{d}_2$$

$$= \frac{1}{2}\left(a\hat{i}+a\hat{j}+a\hat{k}\right)\times\left(a\hat{i}-a\hat{j}+a\hat{k}\right)$$

$$\phi = \vec{E}\cdot\vec{A}\left\{\vec{E}\text{ is constant}\right\} = E_0a^2$$

44. A disk of radius $a/4$ having a uniformly distributed charge 6 C is placed in the x-y plane with its centre at $(-a/2, 0, 0)$. A rod of length a carrying a uniformly distributed charge 8 C is placed on the x-axis from $x = a/4$ to $x = 5a/4$. Two point charges – 7 C and 3 C are placed at $(a/4, -a/4, 0)$ and $(-3a/4\ 3a/4, 0)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm\, a/2$, $y = \pm\, a/2$, $z = \pm\, a/2$. The electric flux through this cubical surface is



(1) $\frac{-2C}{\varepsilon_0}$ (2) $\frac{2C}{\varepsilon_0}$ (3) $\frac{10C}{\varepsilon_0}$ (4) $\frac{12C}{\varepsilon_0}$

Sol. Answer (1)

$$q_{in} = \underset{(\text{Disc})}{3C} + \underset{(\text{Rod})}{2C} + \underset{(\text{Point charge})}{(-7C)}$$

$$q_{in} = -2C$$

$$\text{Flux} = \frac{-2C}{\varepsilon_0}$$

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