Cylinder, Cone & Sphere (Surface Area & Volume)

Exercise 20A

Question 1.
The height of a circular cylinder is 20 cm and the radius of its base is 7 cm. Find :
(i) the volume
(ii) the total surface area.

Solution:

For a circular cylinder,
Height = h = 20 cm
Radius of the base = r = 7 cm
(i) Volume of a cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 20 \quad cm^3$$

$$= 3080\ cm^3$$

(ii) Total surface area of a cylinder = $2\pi r(h+r)$

$$= 2 \times \frac{22}{7} \times 7(20+7)\ cm^2$$

$$= 2 \times 22 \times 27\ cm^2$$

$$= 1188\ cm^2$$

Question 2.
The inner radius of a pipe is 2.1 cm. How much water can 12 m of this pipe hold?

Solution:

Inner radius of pipe = 2.1 cm

Length of the pipe = 12 m = 1200 cm

$\therefore$ Volume = $\pi r^2 h$

$$= \frac{22}{7} \times 2.1 \times 2.1 \times 1200\ cm^3$$

$$= 16632\ cm^3$$

Question 3.

A cylinder of circumference 8 cm and length 21 cm rolls without sliding for $4\frac{1}{2}$ seconds at the rate of 9 complete rounds per second. Find

(i) distance travelled by the cylinder in $4\frac{1}{2}$ seconds, and

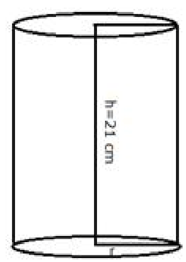
(ii) the area covered by the cylinder in $4\frac{1}{2}$ seconds

Solution:

Circumference of cylinder = 8 cm

Therefore, radius = $\frac{c}{2\pi} = \frac{8 \times 7}{2 \times 22} = \frac{14}{11}$ cm

Length of the cylinder (h)= 21 cm



(i) If distance covered in one revolution is 8 cm, then distance covered in 9 revolutions = $9 \times 8 = 72$ cm or distance covered in 1 second = 72 cm.

Therefore, distance covered in $4\frac{1}{2}$ seconds = $72 \times \frac{9}{2}$ cm = 324 cm

(ii) Curved surface area = $2\pi rh$

$= 2 \times \frac{22}{7} \times \frac{14}{11} \times 21$

$= 168\ cm^2$

Area covered in one revolution = $168\ cm^2$

Area covered in 9 revolutions = $168\ cm^2 \times 9 = 1512\ cm^2$

Therefore, area covered in 1 second = $1512\ cm^2$

Hence, area covered in $4\frac{1}{2}$ seconds = $1512\ cm^2 \times \frac{9}{2} = 6804\ cm^2$

Question 4.

How many cubic meters of earth must be dug out to make a well 28 m deep and 2.8 m in diameter? Also, find the cost of plastering its inner surface at Rs 4.50 per sq meter.

Solution:

Radius of the well = $\frac{2.8}{2} = 1.4$ m

Depth of the well = 28 m

Therefore, volume of earth dug out = $\pi r^2 h$

$= \frac{22}{7} \times 1.4 \times 1.4 \times 28$

$= \frac{17248}{100}$

$= 172.48 \text{ m}^3$

Area of curved surface = $2\pi rh$

$= 2 \times \frac{22}{7} \times 1.4 \times 28$

$= 246.40 \text{ m}^2$

Cost of plastering at the rate of Rs 4.50 per sq m

= Rs 246.40 $\times$ 4.50

= Rs 1108.80

Question 5.
What length of solid cylinder 2 cm in diameter must be taken to recast into a hollow cylinder of external diameter 20 cm, 0.25 cm thick and 15 cm long?

Solution:

External diameter of hollow cylinder = 20 cm

Therefore, radius = 10 cm

Thickness = 0.25 cm

Hence, internal radius = (10 - 0.25) = 9.75 cm

Length of cylinder (h) = 15 cm

$\therefore$ Volume $= \pi h(R^2 - r^2) = \pi \times 15(10^2 - 9.75^2)$
$= 15\pi(100 - 95.0625)cm^3$
$= 15\pi \times 4.9375\ cm^3$

Diameter = 2 cm

Therefore, radius (r) = 1 cm

Let h be the length

then, volume $= \pi r^2 h = \pi(1 \times 1)h = \pi h$

Now, according to given condition:

$\pi h = 15\pi \times 4.9375$
$\Rightarrow h = 15 \times 4.9375$
$\Rightarrow h = 74.0625$

Length of cylinder = 74.0625 cm

Question 6.
A cylinder has a diameter of 20 cm. The area of curved surface is 100 sq. cm. Find:
(i) the height of the cylinder correct to one decimal place.
(ii) the volume of the cylinder correct to one decimal place.

Solution:

Diameter of the cylinder = 20 cm

Hence, Radius (r) = 10 cm

Height = h cm

(i) Curved surface area = $2\pi rh$

$\therefore 2\pi rh = 100\ cm^2$

$\Rightarrow 2 \times \frac{22}{7} \times 10 \times h = 100$

$\Rightarrow h = \frac{100 \times 7}{22 \times 10 \times 2} = \frac{35}{22}$

$\Rightarrow h = 1.6\ cm$

(ii) Volume of the cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 10 \times 10 \times 1.6$

$= 502.9\ cm^3$

or

$= \frac{22}{7} \times 10 \times 10 \times \frac{35}{22}$

$= 500\ cm^3$

Question 7.

A metal pipe has a bore (inner diameter) of 5 cm. The pipe is 5 mm thick all round. Find the weight, in kilogram, of 2 metres of the pipe if 1 cm3 of the metal weights 7.7 g.

Solution:

Inner radius of the pipe = r = $\frac{5}{2}$ = 2.5 cm

External radius of the pipe = R = Inner radius of the pipe + Thickness of the pipe

= 2.5 cm + 0.5 cm

= 3 cm

Length of the pipe = h = 2 m= 200 cm

Volume of the pipe = External Volume – Internal Volume

$= \pi R^2 h - \pi r^2 h$

$= \pi (R^2 - r^2) h$

$= \pi (R - r)(R + r) h$

$= \frac{22}{7}(3 - 2.5)(3 + 2.5) \times 200$

$= \frac{22}{7} \times 0.5 \times 5.5 \times 200$

$= 1728.6\ cm^3$

Since 1cm^3 of the metal weights 7.7 9,

∴ Weight of the pipe = (1728.6 × 7.7)g = $\left(\frac{1728.6 \times 7.7}{1000}\right)$ kg = 13.31 kg

Question 8.
A cylindrical container with diameter of base 42 cm contains sufficient water to submerge a rectangular solid of iron with dimensions 22 cm x 14 cm 10.5 cm. Find the rise in level of the water when the solid is submerged.

Solution:

Diameter of cylindrical container = 42 cm

Therefore, radius (r) = 21 cm

Dimensions of rectangular solid = 22cm × 14cm × 10.5cm

Volume of solid = $= 22 \times 14 \times 10.5\ cm^3$......(i)

Let height of water = h

Therefore, volume of water in the container = $\pi r^2 h$

$$= \frac{22}{7} \times 21 \times 21 \times h\ cm^3 = 22 \times 63h\ cm^3 ......(ii)$$

From (i) and (ii)

$$22 \times 63h = 22 \times 14 \times 10.5$$

$$\Rightarrow h = \frac{22 \times 14 \times 10.5}{22 \times 63}$$

$$\Rightarrow h = \frac{7}{3}$$

$$\Rightarrow h = 2\frac{1}{3} \text{ or } 2.33\ cm$$

Question 9.
A cylindrical container with internal radius of its base 10 cm, contains water up to a height of 7 cm. Find the area of wetted surface of the cylinder.

Solution:

Internal radius of the cylindrical container = 10 cm

Height of water = 7 cm

Therefore, surface area of the wetted surface =

$$2\pi rh + \pi r^2$$
$$= \pi r(2h + r)$$
$$= \frac{22}{7} \times 10 \times (2 \times 7 + 10)$$
$$= \frac{220}{7} \times 24$$
$$= 754.29 \text{ cm}^2$$

Question 10.
Find the total surface area of an open pipe of length 50 cm, external diameter 20 cm and internal diameter 6 cm.

Solution:

Length of an open pipe = 50 cm

External diameter = 20 cm $\Rightarrow$ External radius (R) = 10 cm

Internal diameter = 6 cm $\Rightarrow$ Internal radius (r) = 3 cm

Surface area of pipe open from both sides =

$$= 2\pi Rh + 2\pi rh$$
$$= 2\pi h(R + r)$$
$$= 2 \times \frac{22}{7} \times 50 \times (10 + 3)$$
$$= 4085.71 \text{ cm}^2$$

Area of upper and lower part =

$$= 2\pi(R^2 - r^2)$$
$$= 2 \times \frac{22}{7} \times (10^2 - 3^2)$$
$$= 2 \times \frac{22}{7} \times 91$$
$$= 572 \text{ cm}^2$$

Total surface area = $4085.71 + 572 = 4657.71 \text{ cm}^2$

Question 11.

The height and radius of base of a cylinder are in the ratio 3:1. If its volume is 1029π cm^3; find its total surfac

Solution:

Ratio between height and radius of a cylinder = 3:1

Volume = $1029\pi\ cm^3$ ....(i)

Let radius of the base = r

then height = 3r

$\therefore$ Volume $= \pi r^2 h = \pi \times r^2 \times 3r = 3\pi r^3$......(ii)

from (i) and (ii)

$3\pi r^3 = 1029\pi$

$r^3 = \frac{1029\pi}{3\pi} = 343$

$r = 7$

Therefore, radius = 7 cm and height = 3 x 7 = 21 cm

Now, total surface area =

$2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 7 \times (21 + 7)$

$= 2 \times \frac{22}{7} \times 7 \times 28$

$= 1232\ cm^2$

Question 12.

The radius of a solid right circular cylinder increases by 20% and its height decreases by 20%. Find the percentage change in its volume.

Solution:

Let the radius of a solid right circular cylinder be r = 100 cm
And, let the height of a solid right circular cylinder be h = 100 cm
$\therefore$ Volume (original) of a solid right circular cylinder = $\pi r^2 h$

$$= \pi \times (100)^2 \times 100$$
$$= 1000000\,\pi\ cm^3$$

New radius = r' = 120 cm
New height = h' = 80 cm
$\therefore$ Volume (New) of a solid right circular cylinder = $\pi r'^2 h'$

$$= \pi \times (120)^2 \times 80$$
$$= 1152000\,\pi\ cm^3$$

$\therefore$ Increase in Volume = New Volume – Original Volume

$$= 1152000\,\pi\ cm^3 - 1000000\,\pi\ cm^3$$
$$= 152000\,\pi\ cm^3$$

Thus, Percentage change in volume = $\frac{\text{Increase in Volume}}{\text{Original Volume}} \times 100\%$

$$= \frac{152000\,\pi\ cm^3}{1000000\,\pi\ cm^3} \times 100\%$$
$$= 15.2\%$$

Question 13.
The radius of a solid right circular cylinder decreases by 20% and its height increases by 10%. Find the percentage change in its :
(i) volume
(ii) curved surface area

Solution:

Let the radius of a solid right circular cylinder be r = 100 cm
And, let the height of a solid right circular cylinder be h = 100 cm
$\therefore$ Volume (original) of a solid right circular cylinder $= \pi r^2 h$

$$= \pi \times (100)^2 \times 100$$
$$= 1000000\,\pi\ \text{cm}^3$$

New radius = r' = 80 cm
New height = h' = 110 cm
$\therefore$ Volume (New) of a solid right circular cylinder $= \pi r'^2 h'$

$$= \pi \times (80)^2 \times 110$$
$$= 704000\,\pi\ \text{cm}^3$$

$$\therefore \text{Decrease in Volume} = \text{Original Volume} - \text{New Volume}$$
$$= 1000000\,\pi\ \text{cm}^3 - 704000\,\pi\ \text{cm}^3$$
$$= 296000\,\pi\ \text{cm}^3$$

$$\text{(i) Percentage change in volume} = \frac{\text{Decrease in Volume}}{\text{Original Volume}} \times 100\%$$
$$= \frac{296000\,\pi\ \text{cm}^3}{1000000\,\pi\ \text{cm}^3} \times 100\%$$
$$= 29.6\%$$

(ii) Curved surface area (Original) of a solid right circular cylinder

$$= 2\pi rh$$
$$= 2\pi \times 100 \times \times 100$$
$$= 20000\pi\ \text{cm}^2$$

Curved surface area (New) of a solid right circular cylinder

$$= 2\pi r'h'$$
$$= 2\pi \times 80 \times \times 110$$
$$= 17600\pi\ \text{cm}^2$$

$$\text{Decrease in curved surface area} = \text{Original CSA} - \text{New CSA} = (20000\pi - 17600\pi)\ \text{cm}^2 = 2400\pi\ \text{cm}^2$$

$$\text{Percentage change in curved surface area} = \frac{\text{Decrease in curved surface area}}{\text{Original curved surface area}} \times 100\%$$
$$= \frac{2400\pi\ \text{cm}^2}{20000\pi\ \text{cm}^2} \times 100\%$$
$$= 12\%$$

Question 14.
Find the minimum length in cm and correct to nearest whole number of the thin metal sheet required to make a hollow and closed cylindrical box of diameter 20 cm and

height 35 cm. Given that the width of the metal sheet is 1 m. Also, find the cost of the sheet at the rate of Rs. 56 per m.
Find the area of metal sheet required, if 10% of it is wasted in cutting, overlapping, etc.

Solution:

Height of the cylindrical box = h = 35 cm
Base radius of the cylindrical box = r = 10 cm
Width of metal sheet = 1 m = 100 cm
Area of metal sheet required= Total surface area of the box

$\Rightarrow \text{Length} \times \text{Width} = 2\pi r(r + h)$

$\Rightarrow \text{Length} \times 100 = 2 \times \frac{22}{7} \times 10(10 + 35)$

$\Rightarrow \text{Length} \times 100 = 2 \times \frac{22}{7} \times 10 \times 45$

$\Rightarrow \text{Length} = \frac{2 \times 22 \times 10 \times 45}{100 \times 7} = 28.28 \text{ cm} = 28 \text{ cm}$

$\therefore$ Area of metal sheet = Length x Width = 28 x 100 = 2800 cm^2 = 0.28 m^2

$\therefore$ Cost of the sheet at the rate of Rs.56 per m^2 = Rs. (56 x 0.28) = Rs. 15.68

Let the total sheet required be x.

Then, x − 10% of x = 2800 cm^2

$\Rightarrow x - \frac{10}{100} \times x = 2800$

$\Rightarrow 9x = 2800$

$\Rightarrow x = 3111 \text{ cm}^2$

Question 15.
3080 cm3 of water is required to fill a cylindrical vessel completely and 2310 cm^3 of water is required to fill it upto 5 cm below the top. Find :
(i) radius of the vessel.
(ii) height of the vessel.
(iii) wetted surface area of the vessel when it is half-filled with water.

Solution:

Let r be the radius of the cylindrical vessel and
h be its height.
Now, volume of cylindrical vessel = Volume of water filled in it

$\Rightarrow \pi r^2 h = 3080$

$\Rightarrow \frac{22}{7} \times r^2 \times h = 3080$

$\Rightarrow r^2 \times h = 980 \quad ....(i)$

Volume of cylindrical vessel of height 5 cm = $(3080 - 2310)\ cm^3$

$\Rightarrow \pi r^2 \times 5 = 770$

$\Rightarrow \frac{22}{7} \times r^2 \times 5 = 770$

$\Rightarrow r^2 = 49$

$\Rightarrow r = 7$ cm

Substituting $r^2 = 49$ in (i), we get

$49 \times h = 980$

$\Rightarrow h = 20$ cm

Wetted surface area of the vessel when it is half-filled with water

$= 2\pi rh + \pi r^2$

$= \pi r(2h + r)$

$= \frac{22}{7} \times 7(2 \times 10 + 7) \quad .....\left[\text{Half-filled} \Rightarrow \text{Height} = \frac{20}{2} = 10 \text{ cm}\right]$

$= 22 \times 27$

$= 594\ cm^2$

Question 16.
Find the volume of the largest cylinder formed when a rectangular piece of paper 44 cm by 33 cm is rolled along it :
(i) shorter side.
(ii) longer side.

Solution:

Length of a rectangular paper = 44 cm
Breadth of a rectangular paper = 33 cm
(i) When the paper is rolled along its shorter side, i.e. breadth, we have
Height of cylinder = h = 44cm
Circumference of crss-section = $2\pi r$ = 33 cm

$$\Rightarrow 2 \times \frac{22}{7} \times r = 33$$

$$\Rightarrow r = \frac{33 \times 7}{2 \times 22} = 5.25 \text{ cm}$$

$$\therefore \text{Volume of cylinder} = \pi r^2 h = \frac{22}{7} \times 5.25 \times 5.25 \times 44 = 3811.5 \text{ cm}^3$$

(ii) When the paper is rolled along its longer side, i.e. length, we have
Height of cylinder = h = 33cm
Circumference of crss-section = $2\pi r$ = 44 cm

$$\Rightarrow 2 \times \frac{22}{7} \times r = 44$$

$$\Rightarrow r = \frac{44 \times 7}{2 \times 22} = 7 \text{ cm}$$

$$\therefore \text{Volume of cylinder} = \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 33 = 5082 \text{ cm}^3$$

Question 17.
A metal cube of side 11 cm is completely submerged in water contained in a cylindrical vessel with diameter 28 cm. Find the rise in the level of water.

Solution:

Clearly,
Volume of cube of side 11 cm = Volume of water displaced in the cylinder

$$\Rightarrow (11)^3 = \pi r^2 h$$

$$\Rightarrow 11 \times 11 \times 11 = \frac{22}{7} \times \frac{28}{2} \times \frac{28}{2} \times h$$

$$\Rightarrow h = \frac{11 \times 11 \times 11 \times 7 \times 2 \times 2}{22 \times 28 \times 28} = \frac{121}{56} = 2.16 \text{ cm}$$

Question 18.
A circular tank of diameter 2 m is dug and the earth removed is spread uniformly all around the tank to form an embankment 2 m in width and 1.6 m in height. Find the

depth of the circular tank.

Solution:

Let the depth of the circular tank be h m.

Radius of the tank $= \frac{2}{2}$ m $= 1$ m $= r$

$\therefore$ Volume of the tank $= \pi r^2 h = \pi \times 1 \times h = \pi h$ m^3

Now, volume of the embankment

= Volume of hollow cylinder having height 1.6 m

$= \pi(R^2 - r^2)H$

$= \pi[(1+2)^2 - (1)^2] \times 1.6$

$= \pi(9-1) \times 1.6$

$= 12.8\pi$ m^3

Now,

Volume of tank = Volume of embankment

$\Rightarrow \pi h = 12.8\pi$

$\Rightarrow h = 12.8$ m

Question 19.

The sum of the inner and the outer curved surfaces of a hollow metallic cylinder is 1056 cm^2 and the volume of material in it is 1056 cm^3. Find its internal and external radii. Given that the height of the cylinder is 21 cm.

Solution:

Let R and r be the outer and inner radii of hollow metallic cylinder.

Let h be the height of the metallic cylinder.

It is given that

Outer curved surface area + Inner curved surface area = 1056

$\Rightarrow 2\pi Rh + 2\pi rh = 1056$

$\Rightarrow 2\pi h(R + r) = 1056$

$\Rightarrow 2 \times \frac{22}{7} \times 21(R + r) = 1056$

$\Rightarrow R + r = \frac{1056 \times 7}{2 \times 22 \times 21}$

$\Rightarrow R + r = 8 \quad \ldots.(i)$

Volume of material in it = 1056 cm^3

$\Rightarrow \pi R^2 h - \pi r^2 h = 1056$

$\Rightarrow \pi h(R^2 - r^2) = 1056$

$\Rightarrow \frac{22}{7} \times 21(R^2 - r^2) = 1056$

$\Rightarrow R^2 - r^2 = \frac{1056 \times 7}{22 \times 21}$

$\Rightarrow (R + r)(R - r) = 16$

$\Rightarrow 8 \times (R - r) = 16$

$\Rightarrow R - r = 2 \quad ....(ii)$

Adding (i) and (ii), we get

$2R = 10 \Rightarrow R = 5$ cm

$\Rightarrow 5 - r = 2$

$\Rightarrow r = 3$ cm

$\therefore$ Internal radius = 3 cm and External radius = 5 cm

Question 20.

The difference between the outer curved surface area and the inner curved surface area of a hollow cylinder is 352 cm^2. If its height is 28 cm and the volume of material in it is 704 cm^3; find its external curved surface area.

Solution:

Let R and r be the outer and inner radii of hollow metallic cylinder.

Let h be the height of the metallic cylinder.

It is given that

Outer curved surface area – Inner curved surface area = 352

$\Rightarrow 2\pi Rh - 2\pi rh = 352$

$\Rightarrow 2\pi h(R - r) = 352$

$\Rightarrow 2 \times \frac{22}{7} \times 28(R - r) = 352$

$\Rightarrow R - r = \frac{352 \times 7}{2 \times 22 \times 28}$

$\Rightarrow R - r = 2 \quad ....(i)$

Volume of material in it = 704 cm^3

$\Rightarrow \pi R^2h - \pi r^2h = 704$

$\Rightarrow \pi h(R^2 - r^2) = 704$

$\Rightarrow \frac{22}{7} \times 28(R^2 - r^2) = 704$

$\Rightarrow R^2 - r^2 = \frac{704 \times 7}{22 \times 28}$

$\Rightarrow (R + r)(R - r) = 8$

$\Rightarrow (R + r) \times 2 = 8$

$\Rightarrow R + r = 4 \qquad ....(ii)$

Adding (i) and (ii), we get

$2R = 6 \Rightarrow R = 3$ cm

$\therefore$ External curved surface area $= 2\pi Rh = 2 \times \frac{22}{7} \times 3 \times 28 = 528\ cm^2$

Question 21.
The sum of the heights and the radius of a solid cylinder is 35 cm and its total surface area is 3080 cm^2, find the volume of the cylinder.

Solution:

Let r and h be the radius and height of a solid cylinder.

Then, $r + h = 35$ cm

Total surface area of a cylinder = 3080 cm^2

$\Rightarrow 2\pi r(h + r) = 3080$

$\Rightarrow 2 \times \frac{22}{7} \times r \times 35 = 3080$

$\Rightarrow 2 \times 22 \times r \times 5 = 3080$

$\Rightarrow r = \frac{3080}{2 \times 22 \times 5} = 14$ cm

$\Rightarrow h = 35 - r = 35 - 14 = 21$ cm

$\therefore$ Volume of cylinder $= \pi r^2h = \frac{22}{7} \times 14 \times 14 \times 21 = 12936\ cm^3$

Question 22.
The total surface area of a solid cylinder is 616 cm^2. If the ratio between its curved surface area and total surface area is 1 : 2; find the volume of the cylinder.

Solution:

Let r and h be the radius and height of a solid cylinder.

Total surface area of a cylinder = 616 cm^2

$\Rightarrow 2\pi r(h+r) = 616 \quad ....(i)$

Curved surface area of a cylinder = $2\pi rh$

Now,

$$\frac{\text{Curved surface area of a cylinder}}{\text{Total surface area of a cylinder}} = \frac{1}{2}$$

$$\Rightarrow \frac{2\pi rh}{2\pi r(h+r)} = \frac{1}{2}$$

$$\Rightarrow \frac{h}{h+r} = \frac{1}{2}$$

$\Rightarrow 2h = h + r$

$\Rightarrow h = r$

Substituting h = r in (i), we get

$2\pi r(r+r) = 616$

$$\Rightarrow 2 \times \frac{22}{7} \times 2r^2 = 616$$

$$\Rightarrow r^2 = \frac{616 \times 7}{2 \times 22 \times 2} = 49$$

$\Rightarrow r = 7 = h$

$$\therefore \text{Volume of cylinder} = \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 7 = 1078 \text{ cm}^3$$

Question 23.

A cylindrical vessel of height 24 cm and diameter 40 cm is full of water. Find the exact number of small cylindrical bottles, each of height 10 cm and diameter 8 cm, which can be filled with this water.

Solution:

For a large cylindrical vessel,

Height = H = 24 cm

Radius = R = $\frac{40}{2}$ = 20 cm

$\therefore$ Volume of large cylindrical vessel = $\pi R^2 H = (\pi \times 20 \times 20 \times 24)\ cm^3$

For each small cylindrical bottle,

Height = h = 10 cm

Radius = r = $\frac{8}{2}$ = 4 cm

$\therefore$ Volume of each small cylindrical bottle = $\pi r^2 h = (\pi \times 4 \times 4 \times 10)\ cm^3$

Now, number of small cylindrical bottles which can be filled

$= \frac{\text{Volume of large cylindrical vessel}}{\text{Volume of each small cylindrical bottle}}$

$= \frac{\pi \times 20 \times 20 \times 24}{\pi \times 4 \times 4 \times 10}$

= 60

Question 24.

Two solid cylinders, one with diameter 60 cm and height 30 cm and the other with radius 30 cm and height 60 cm, are melted and recanted into a third solid cylinder of height 10 cm. Find the diameter of the cylinder formed.

Solution:

For cylinder 1,

Height = h_1 = 30cm

Radius = $r_1 = \frac{60}{2}$ = 30 cm

Volume = $V_1 = \pi r_1^2 h_1 = \pi \times 30 \times 30 \times 30 = 27000\pi\ cm^3$

For cylinder 2,

Height = h_2 = 60cm

Radius = r_2 = 30 cm

Volume = $V_2 = \pi r_2^2 h_2 = \pi \times 30 \times 30 \times 60 = 54000\pi\ cm^3$

Let r be the radius of the third cylinder.

Height = h = 10 cm

Volume = $V = \pi r^2 h = \pi r^2 \times 10$

Now,

$V = V_1 + V_2$

$\Rightarrow \pi r^2 \times 10 = 27000\pi + 54000\pi$

$\Rightarrow \pi r^2 \times 10 = 81000\pi$

$\Rightarrow r^2 = 8100$

$\Rightarrow r = 90$

$\Rightarrow$ Diameter = 2r = 180 cm

Question 25.

The total surface area of a hollow cylinder, which is open from both sides, is 3575 cm^2; area of the base ring is 357.5 cm^2 and height is 14 cm. Find the thickness of the cylinder.

Solution:

Total surface area of a hollow cylinder = 3575 cm^2

Area of the base ring = 357.5 cm^2

Height = 14 cm

Let external radius = R and internal radius = r

Let thickness of the cylinder = d = (R-r)

Therefore, Total surface area = $2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$

$= 2\pi h(R + r) + 2\pi(R + r)(R - r)$

$= 2\pi(R + r)[h + R - r]$

$= 2\pi(R + r)(h + d)$

$= 2\pi(R + r)(14 + d)$

But,

$2\pi(R + r)(14 + d) = 3575$........(i)

and Area of base =

$\pi\left(R^2 - r^2\right) = 357.5$

$\Rightarrow \pi(R + r)(R - r) = 357.5$

$\Rightarrow \pi(R + r)d = 357.5 \ldots\ldots\ldots (ii)$

Dividing (i) by (ii)

$$\frac{2\pi((R+r)(14+d)}{\pi(R+r)d} = \frac{3575}{357.5}$$

$$\frac{2(14+d)}{d} = 10$$

$28 + 2d = 10d$

$8d = 28$

$d = \frac{28}{8} = 3.5 \text{ cm}$

Hence, thickness of the cylinder = 3.5 cm

Question 26.

The given figure shows a solid formed of a solid cube of side 40cm and a solid cylinder of radius 20 cm and height 50 cm attached to the cube as shown.

Find the volume and the total surface area of the whole solid (Take π = 3.14)

Solution:

Edge of a cube = l = 40 cm

$\therefore$ Volume of a cube = $l^3 = (40)^3 = 64000 \text{ cm}^3$

Radius of a solid cylinder = r = 20 cm

Height of a solid cylinder = h = 50 cm

$\therefore$ Volume of cylinder = $\pi r^2 h = 3.14 \times 20 \times 20 \times 50 = 62800 \text{ cm}^3$

$\therefore$ Volume of whole solid = Volume of cube + Volume of cylinder

$= (64000 + 62800)\ cm^3$

$= 126800\ cm^3$

Total surface area of the whole solid

= Total surface area of a cube + Curved surface area of a cylinder

$= 6l^2 + 2\pi rh$

$= 6 \times (40)^2 + 2 \times 3.14 \times 20 \times 50$

$= 9600 + 6280$

$= 15880\ cm^2$

Question 27.

Two right circular solid cylinders have radii in the ratio 3 : 5 and heights in the ratio 2 : 3, Find the ratio between their :

(i) curved surface areas.

(ii) volumes.

Solution:

Let the radii and height of two right circular cylinders be r_1, r_2 and h_1, h_2 respectively.

Itis given that,

$$\frac{r_1}{r_2} = \frac{3}{5} \quad \text{and} \quad \frac{h_1}{h_2} = \frac{2}{3}$$

(i) $$\frac{\text{Curved surface area of cylinder 1}}{\text{Curved surface area of cylinder 2}} = \frac{2\pi r_1 h_1}{2\pi r_2 h_2} = \left(\frac{r_1}{r_2}\right) \times \left(\frac{h_1}{h_2}\right) = \frac{3}{5} \times \frac{2}{3} = \frac{2}{5}$$

$\therefore$ Ratio between their curved surface areas is 2 : 5.

(ii) $$\frac{\text{Volume of cylinder 1}}{\text{Volume of cylinder 2}} = \frac{\pi (r_1)^2 h_1}{\pi (r_2)^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{h_1}{h_2}\right) = \left(\frac{3}{5}\right)^2 \times \frac{2}{3} = \frac{9}{25} \times \frac{2}{3} = \frac{18}{75} = \frac{6}{25}$$

$\therefore$ Ratio between their volumes is 6 : 25.

Question 28.
A dosed cylindrical tank, made of thin iron sheet, has diameter = 8.4 m and height 5.4 m. How much metal sheet, to the nearest m^2, is used in making this tank, if $\frac{1}{15}$ of the sheet actually used was wasted in making the tank?

Solution:

Radius of the cylindrical tank = $r = \frac{8.4}{2} = 4.2$ m

Height of the cylindrical tank = h = 5.4 m

$\therefore$ Total surface area of the cylindrical tank = $2\pi r(h + r)$

$$= 2 \times \frac{22}{7} \times 4.2(5.4 + 4.2)$$
$$= 2 \times 22 \times 0.6 \times 9.6$$
$$= 253.44 \text{ m}^2$$

Area of sheet wasted in making the tank = $\frac{1}{15} \times 253.44 = 16.896 \text{ m}^2$

Hence, total sheet required = $253.44 + 16.896 = 270.34 \text{ m}^2$

Exercise 20 B

Question 1.
Find the volume of a cone whose slant height is 17 cm and radius of base is 8 cm.

Solution:

Slant height (ℓ) = 17 cm

Radius (r) = 8 cm

But,

$\ell^2 = r^2 + h^2$

$\Rightarrow h^2 = \ell^2 - r^2$

$\Rightarrow h^2 = 17^2 - 8^2$

$\Rightarrow h^2 = 289 - 64 = 225 = (15)^2$

$\therefore h = 15$

Now, volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 8 \times 8 \times 15 \text{ cm}^3$

$= \frac{7040}{7} \text{cm}^3$

$= 1005.71 \text{ cm}^3$

Question 2.

The curved surface area of a cone is $12320\ cm^2$. If the radius of its base is 56 cm, find its height.

Solution:

Curved surface area = $12320\ cm^2$.

Radius of base (r) = 56 cm

Let slant height = ℓ

$\therefore\ \pi r\ell = 12320$

$\Rightarrow \frac{22}{7} \times 56 \times \ell = 12320$

$\Rightarrow \ell = \frac{12320 \times 7}{56 \times 22}$

$\Rightarrow \ell = 70cm$

Height of the cone =

$\sqrt{\ell^2 - r^2}$

$= \sqrt{(70)^2 - (56)^2}$

$= \sqrt{4900 - 3136}$

$= \sqrt{1764}$

$= 42\ cm$

Question 3.

The circumference of the base of a 12 m high conical tent is 66 m. Find the volume of the air contained in it.

Solution:

Circumference of the conical tent = 66 m

and height (h) = 12 m

$\therefore$ Radius $= \frac{c}{2\pi} = \frac{66 \times 7}{2 \times 22} = 10.5m$

Therefore, volume of air contained in it = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 12m^3$

$= 1386\ m^3$

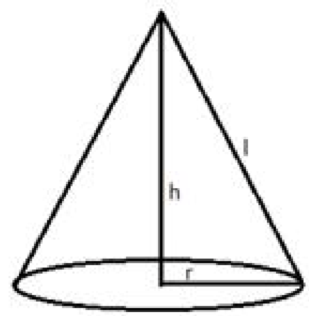
Question 4.
The radius and height of a right circular cone are in the ratio 5:12 and its volume is 2512 cubic cm. Find the radius and slant height of the cone. (Take π = 3.14)

Solution:

The ratio between radius and height = 5:12

Volume = 5212 cubic cm

Let radius (r) = 5x, height (h) = 12x and slant height = ℓ



$\therefore \ell^2 = r^2 + h^2$
$\Rightarrow \ell^2 = (5x)^2 + (12x)^2$
$\Rightarrow \ell^2 = 25x^2 + 144x^2$
$\Rightarrow \ell^2 = 169x^2$
$\Rightarrow \ell = 13x$

Now Volume = $\frac{1}{3}\pi r^2 h$

$\Rightarrow \frac{1}{3}\pi r^2 h = 2512$

$\Rightarrow \frac{1}{3}(3.14)(5x)^2(12x) = 2512$

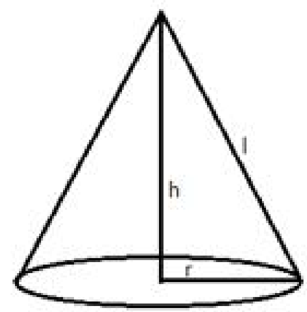
$\Rightarrow \frac{1}{3}(3.14)(300x^3) = 2512$

$\therefore x^3 = \frac{2512 \times 3}{3.14 \times 300} = \frac{2512 \times 3 \times 100}{314 \times 300} = 8$
$\Rightarrow x = 2$
$\therefore$ Radius = 5x = 5 x 2 = 10 cm
Height = 12x = 12 x 2 = 24 cm
Slant height = 13x = 13 x 2 = 26 cm

Question 5.
Two right circular cones x and y are made, x having three times the radius of y and y having half the volume of x. Calculate the ratio between the heights of x and y.

Solution:

Let radius of cone y = r
Therefore, radius of cone x = 3r
Let volume of cone y = V
then volume of cone x = 2V
Let h_1 be the height of x and h2 be the height of y.



Therefore, Volume of cone = $\frac{1}{3}\pi r^2 h$

Volume of cone x = $\frac{1}{3}\pi(3r)^2 h_1 = \frac{1}{3}\pi 9r^2 h_1 = 3\pi r^2 h_1$

Volume of cone y = $\frac{1}{3}\pi r^2 h_2$

$$\therefore \frac{2V}{V} = \frac{3\pi r^2 h_1}{\frac{1}{3}\pi r^2 h_2}$$

$$\Rightarrow \frac{2}{1} = \frac{3h_1 \times 3}{h_2} = \frac{9h_1}{h_2}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{2}{1} \times \frac{1}{9} = \frac{2}{9}$$

$$\therefore h_1 : h_2 = 2 : 9$$

Question 6.
The diameters of two cones are equal. If their slant heights are in the ratio 5:4, find the ratio of their curved surface areas.

Solution:

Let radius of each cone = r

Ratio between their slant heights = 5:4

Let slant height of the first cone = 5x

and slant height of second cone = 4x

Therefore, curved surface area of the first cone =

$\pi r \ell = \pi r \times (5x) = 5\pi r x$

curved surface area of the second cone = $\pi r \ell = \pi r \times (4x) = 4\pi r x$

Hence, ratio between them = $5\pi r x : 4\pi r x = 5 : 4$

Question 7.
There are two cones. The curved surface area of one is twice that of the other. The slant height of the latter is twice that of the former. Find the ratio of their radii.

Solution:

Let slant height of the first cone = ℓ

then slant height of the second cone = 2ℓ

Radius of the first cone = r_1

Radius of the second cone = r_2

Then, curved surface area of first cone = $\pi r_1 \ell$

curved surface area of second cone = $\pi r_2(2\ell) = 2\pi r_2 \ell$

According to given condition:

$$\pi r_1 \ell = 2(2\pi r_2 \ell)$$
$$\pi r_1 \ell = 4\pi r_2 \ell$$
$$r_1 = 4r_2$$
$$\frac{r_1}{r_2} = \frac{4}{1}$$
$$\therefore r_1 : r_2 = 4 : 1$$

Question 8.
A heap of wheat is in the form of a cone of diameter 16.8 m and height 3.5 m. Find its volume. How much cloth is required to just cover the heap?

Solution:

Diameter of the cone = 16.8 m

Therefore, radius (r) = 8.4 m

Height (h) = 3.5 m

(i) Volume of heap of wheat = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 8.4 \times 8.4 \times 3.5$

$= 258.72\ m^3$

(ii) Slant height (ℓ) = $\sqrt{r^2 + h^2}$

$= \sqrt{(8.4)^2 + (3.5)^2}$

$= \sqrt{70.56 + 12.25}$

$= \sqrt{82.81}$

$= 9.1\ m$

Therefore, cloth required or curved surface area = $\pi r \ell$

$= \frac{22}{7} \times 8.4 \times 9.1$

$= 240.24\ m^2$

Question 9.
Find what length of canvas, 1.5 m in width, is required to make a conical tent 48 m in diameter and 7 m in height. Given that 10% of the canvas is used in folds and stitching. Also, find the cost of the canvas at the rate of Rs. 24 per meter.

Solution:

Diameter of the tent = 48 m

Therefore, radius (r) = 24 m

Height (h) = 7 m

Slant height (ℓ) = $\sqrt{r^2 + h^2}$

$= \sqrt{(24)^2 + (7)^2}$

$= \sqrt{576 + 49}$

$= \sqrt{625}$

$= 25 \text{ m}$

Curved surface area = $\pi r \ell$

$= \frac{22}{7} \times 24 \times 25$

$= \frac{13200}{7} \text{ m}^2$

Canvas required for stitching and folding

$= \frac{13200}{7} \times \frac{10}{100}$

$= \frac{1320}{7} \text{ m}^2$

Total canvas required (area)

$= \frac{13200}{7} + \frac{1320}{7}$

$= \frac{14520}{7} \text{ m}^2$

Length of canvas

$$= \frac{\frac{14520}{7}}{\frac{3}{2}}$$

$$= \frac{14520}{7} \times \frac{2}{3}$$

$$= \frac{9680}{7}$$

$$= 1382.86 \text{ m}$$

Rate = Rs 24 per meter

Total cost = $\frac{9680}{7} \times \text{Rs } 24 = \text{Rs } 33,188.64$

Question 10.
A solid cone of height 8 cm and base radius 6 cm is melted and re-casted into identical cones, each of height 2 cm and diameter 1 cm. Find the number of cones formed.

Solution:

Height of solid cone (h) = 8 cm

Radius (r) = 6 cm

Volume of solid cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \pi \times 6 \times 6 \times 8$$

$$= 96\pi \text{ cm}^3$$

Height of smaller cone = 2 cm

and radius = $\frac{1}{2}$ cm

Volume of smaller cone

$$= \frac{1}{3} \times \pi \times \frac{1}{2} \times \frac{1}{2} \times 2$$

$$= \frac{1}{6}\pi \text{ cm}^3$$

Number of cones so formed

$$= \frac{96\pi}{\frac{1}{6}\pi}$$

$$= 96\pi \times \frac{6}{\pi}$$

$$= 576$$

Question 11.

The total surface area of a right circular cone of slant height 13 cm is $90\pi\ cm^2$. Calculate:

(i) its radius in cm

(ii) its volume in cm^3. Take $\pi = 3.14$

Solution:

Total surface area of cone = $90\pi\ cm^2$

slant height (l) = 13 cm

(i) Let r be its radius, then

Total surface area = $\pi r\ell + \pi r^2 = \pi r(\ell + r)$

$$\therefore \pi r(\ell + r) = 90\pi$$
$$\Rightarrow r(13 + r) = 90$$
$$\Rightarrow r^2 + 13r - 90 = 0$$
$$\Rightarrow r^2 + 18r - 5r - 90 = 0$$
$$\Rightarrow r(r + 18) - 5(r + 18) = 0$$
$$\Rightarrow (r + 18)(r - 5) = 0$$

Either r+18 = 0, then r = -18 which is not possible

or r-5=0, then r = 5

Therefore, radius = 5 cm

(ii) Now

$h = \sqrt{\ell^2 - r^2}$

$= \sqrt{13^2 - 5^2}$

$= \sqrt{169 - 25}$

$= \sqrt{144}$

$h = 12 \text{ cm}$

$\text{Volume} = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3.14 \times 5 \times 5 \times 12$

$= 314 \text{ cm}^3$

Question 12.

The area of the base of a conical solid is 38.5 cm^2 and its volume is 154 cm^3. Find the curved surface area of the solid.

Solution:

Area of the base, $\pi r^2 = 38.5 \text{ cm}^2$

Volume of the solid, $V = 154 \text{ cm}^3$

Curved surface area of the solid $= \pi r^2 h$

Volume, $V = \frac{1}{3}\pi r^2 h$

$\Rightarrow 154 = \frac{1}{3}\pi r^2 h$

$\Rightarrow h = \frac{154 \times 3}{\pi r^2}$

$\Rightarrow h = \frac{154 \times 3}{38.5} = 12 \text{ cm}$

Area = 38.5

$\pi r^2 = 38.5$

$\Rightarrow r^2 = \frac{38.5}{3.14}$

$\Rightarrow r = \sqrt{\frac{38.5}{3.14}} = 3.5$

Curved surface area of solid $= \pi r l$

$= \pi r \sqrt{r^2 + h^2}$

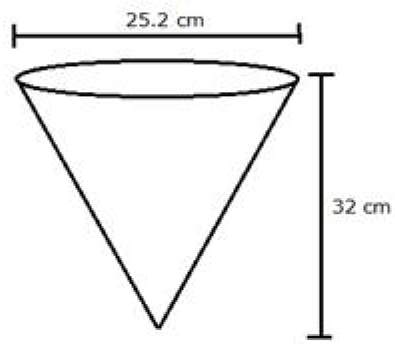
$= \pi \times 3.5 \times \sqrt{3.5^2 + 12^2}$

$= \pi \times 3.5 \times 12.5$

$= 137.44 \text{ cm}^2$

Question 13.
A vessel, in the form of an inverted cone, is filled with water to the brim. Its height is 32 cm and diameter of the base is 25.2 cm. Six equal solid cones are dropped in it, so that they are fully submerged. As a result, one-fourth of water in the original cone overflows. What is the volume of each of the solid cones submerged?

Solution:



Volume of vessel = volume of water = $\frac{1}{3}\pi r^2 h$

diameter = 25.2 cm, therefore radius = 12.6 cm

height = 32 cm

Volume of water in the vessel = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 12.6 \times 12.6 \times 32$

$= 5322.24 \text{ cm}^3$

On submerging six equal solid cones into it, one-fourth of the water overflows.

Therefore, volume of the equal solid cones submerged

= Volume of water that overflows

$= \frac{1}{4} \times 5322.24$

$= 1330.56 \text{ cm}^3$

Now, volume of each cone submerged

$= \frac{1330.56}{6} = 221.76 \text{ cm}^3$

Question 14.

The volume of a conical tent is 1232 m^3 and the area of the base floor is 154 m^2. Calculate the:

(i) radius of the floor

(ii) height of the tent

(iii) length of the canvas required to cover this conical tent if its width is 2 m.

Solution:

(i) Let r be the radius of the base of the conical tent, then area of the base floor = πr^2 m^2

$$\Rightarrow \pi r^2 = 154$$
$$\Rightarrow \frac{22}{7} \times r^2 = 154$$
$$\Rightarrow r^2 = \frac{154 \times 7}{22} = 49$$
$$\Rightarrow r = 7$$

Hence, radius of the base of the conical tent i.e. the floor = 7 m

(ii) Let h be the height of the conical tent, then the volume =

$$\frac{1}{3}\pi r^2 h \text{ m}^3$$
$$\therefore \frac{1}{3}\pi r^2 h = 1232$$
$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times h = 1232$$
$$\Rightarrow h = \frac{1232 \times 3}{22 \times 7} = 24$$

Hence, the height of the tent = 24 m

(iii) Let l be the slant height of the conical tent, then $\ell = \sqrt{h^2 + r^2}$ m

$$\therefore \ell = \sqrt{h^2 + r^2} = \sqrt{(24)^2 + (7)^2} = \sqrt{576 + 49} = \sqrt{625} = 25 \text{ m}$$

The area of the canvas required to make the tent = $\pi r \ell$ m^2

$$\therefore \pi r \ell = \frac{22}{7} \times 7 \times 25 \text{ m}^2 = 550 \text{ m}^2$$

Length of the canvas required to cover the conical tent of its width 2 m = $\frac{550}{2} = 275$ m

Exercise 20 C

Question 1.

The surface area of a sphere is 2464 cm^2, find its volume.

Solution:

Surface area of the sphere = 2464 cm^2

Let radius = r, then

$4\pi r^2 = 2464$

$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 2464$

$\Rightarrow r^2 = \frac{2464 \times 7}{4 \times 22} = 196$

$\Rightarrow r = 14$ cm

Volume = $\frac{4}{3}\pi r^3$

$\therefore \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14 = 11498.67\ cm^3$

Question 2.

The volume of a sphere is 38808 cm^3; find its diameter and the surface area.

Solution:

Volume of the sphere = 38808 cm^3

Let radius of sphere = r

$\therefore \frac{4}{3}\pi r^3 = 38808$

$\Rightarrow \frac{4}{3} \times \frac{22}{7} \times r^3 = 38808$

$\Rightarrow r^3 = \frac{38808 \times 7 \times 3}{4 \times 22} = 9261$

$\Rightarrow r = 21$ cm

$\therefore$ Diameter = $2r = 21 \times 2$ cm = 42 cm

Surface area = $4\pi r^2 = 4 \times \frac{22}{7} \times 21 \times 21\ cm^2 = 5544\ cm^2$

Question 3.
A spherical ball of lead has been melted and made into identical smaller balls with radius equal to half the radius of the original one. How many such balls can be made?

Solution:

Let the radius of spherical ball = r

$\therefore$ Volume $= \frac{4}{3}\pi r^3$

Radius of smaller ball $= \frac{r}{2}$

$\therefore$ Volume of smaller ball $= \frac{4}{3}\pi\left(\frac{r}{2}\right)^3 = \frac{4}{3}\pi\frac{r^3}{8} = \frac{\pi r^3}{6}$

Therefore, number of smaller balls made out of the given ball =

$$\frac{\frac{4}{3}\pi r^3}{\frac{\pi r^3}{6}} = \frac{4}{3} \times 6 = 8$$

Question 4.
How many balls each of radius 1 cm can be made by melting a bigger ball whose diameter is 8 cm.

Solution:

Diameter of bigger ball = 8 cm

Therefore, Radius of bigger ball = 4 cm

$$\text{Volume} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 4 \times 4 \times 4 = \frac{265\pi}{3}\text{cm}^3$$

Radius of small ball = 1 cm

$$\text{Volume} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 1 \times 1 \times 1 = \frac{4\pi}{3}\text{cm}^3$$

$$\text{Number of balls} = \frac{\frac{256\pi}{3}}{\frac{4\pi}{3}} = \frac{256\pi}{3} \times \frac{3}{4\pi} = 64$$

Question 5.
8 metallic sphere; each of radius 2 mm, are melted and cast into a single sphere. Calculate the radius of the new sphere.

Solution:

Radius of metallic sphere = 2 mm = $\frac{1}{5}$cm

$$\text{Volume} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{88}{21 \times 125}\text{cm}^3$$

$$\text{Volume of 8 spheres} = \frac{88 \times 8}{21 \times 125} = \frac{704}{21 \times 125}\text{cm}^3 \ldots\ldots(i)$$

Let radius of new sphere = R

$$\therefore \text{Volume} = \frac{4}{3}\pi R^3 = \frac{4}{3} \times \frac{22}{7}R^3 = \frac{88}{21}R^3 \ldots\ldots(ii)$$

From (i) and (ii)

$$\frac{88}{21}R^3 = \frac{704}{21 \times 125}$$

$$\Rightarrow R^3 = \frac{704}{21 \times 125} \times \frac{21}{88} = \frac{8}{125}$$

$$\Rightarrow R = \frac{2}{5} = 0.4 \text{ cm} = 4 \text{ mm}$$

Question 6.
The volume of one sphere is 27 times that of another sphere. Calculate the ratio of their:
(i) radii
(ii) surface areas

Solution:

Volume of first sphere = 27 $\times$ volume of second sphere

Let radius of first sphere = r_1

and radius of second sphere = r_2

Therefore, volume of first sphere = $\frac{4}{3}\pi r_1^3$

and volume of second sphere = $\frac{4}{3}\pi r_2^3$

(i) Now, according to the question

$$\frac{4}{3}\pi r_1^3 = 27 \times \frac{4}{3}\pi r_2^3$$
$$r_1^3 = 27r_2^3 = (3r_2)^3$$
$$\Rightarrow r_1 = 3r_2$$
$$\Rightarrow \frac{r_1}{r_2} = \frac{3}{1}$$
$$\therefore r_1 : r_2 = 3 : 1$$

(ii) Surface area of first sphere = $4\pi r_1^2$

and surface area of second sphere = $4\pi r_2^2$

Ratio in surface area = $\frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \frac{3^2}{1^2} = \frac{9}{1} = 9 : 1$

Question 7.
If the number of square centimeters on the surface of a sphere is equal to the number of cubic centimeters in the volume, what is the diameter of the sphere?

Solution:

Let r be the radius of the sphere.

Surface area = $4\pi r^2$ and volume = $\frac{4}{3}\pi r^3$

According to the condition:

$$4\pi r^2 = \frac{4}{3}\pi r^3$$
$$\Rightarrow \frac{r^3}{r^2} = 4\pi \times \frac{3}{4\pi}$$
$$\Rightarrow r = 3 \text{ cm}$$

Diameter of sphere = 2×3 cm = 6 cm

Question 8.

A solid metal sphere is cut through its centre into 2 equal parts. If the diameter of the sphere is $3\frac{1}{2}$ cm, find the total surface area of each part correct to 2 decimal places.

Solution:

Diameter of sphere = $3\frac{1}{2}$ cm = $\frac{7}{2}$ cm

Therefore, radius of sphere = $\frac{7}{4}$ cm

Total curved surface area of each hemispheres =

$2\pi r^2 + \pi r^2 = 3\pi r^2$

$= 3 \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4}$

$= 28.88 \text{ cm}^2$

Question 9.

The internal and external diameters of a hollow hemi-spherical vessel are 21 cm and 28 cm respectively. Find:

(i) internal curved surface area
(ii) external curved surface area
(iii) total surface area
(iv) volume of material of the vessel.

Solution:

External radius (R) = 14 cm

Internal radius (r) = $\frac{21}{2}$ cm

(i) Internal curved surface area =

$2\pi r^2$

$= 2 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$

$= 693 \text{ cm}^2$

(ii) External curved surface area =

$2\pi R^2$

$$= 2 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$$
$$= 693 \text{ cm}^2$$

(ii) External curved surface area =

$$2\pi R^2$$
$$= 2 \times \frac{22}{7} \times 14 \times 14$$
$$= 1232 \text{ cm}^2$$

(iii) Total surface area =

$$2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$$
$$= 693 + 1232 + \frac{22}{7}\left((14)^2 - \left(\frac{21}{2}\right)^2\right)$$
$$= 1925 + \frac{22}{7}(196 - \frac{441}{4})$$
$$= 1925 + \frac{22}{7} \times \frac{343}{4}$$
$$= 1925 + 269.5$$
$$= 2194.5 \text{ cm}^2$$

(iv) Volume of material used =

$$\frac{2}{3}\pi(R^3 - r^3)$$
$$= \frac{2}{3} \times \frac{22}{7}\left((14)^3 - \left(\frac{21}{2}\right)^3\right)$$
$$= \frac{44}{21}(2744 - 1157.625)$$
$$= \frac{44}{21} \times 1586.375$$
$$= 3323.83 \text{ cm}^3$$

Question 10.
A solid sphere and a solid hemi-sphere have the same total surface area. Find the ratio between their volumes.

Solution:

Let the radius of the sphere be 'r_1'.
Let the radius of the hemisphere be 'r_2'
TSA of sphere = $4\pi r_1^2$
TSA of hemisphere = $3\pi r_2^2$
TSA of sphere = TSA of hemi-sphere

$$4\pi r_1^2 = 3\pi r_2^2$$

$$\Rightarrow r_2^2 = \frac{4}{3} r_1^2$$

$$\Rightarrow r_2 = \frac{2}{\sqrt{3}} r_1$$

Volume of sphere, $V_1 = \frac{4}{3}\pi r_1^3$

Volume of hemisphere, $V_2 = \frac{2}{3}\pi r_2^3$

$$V_2 = \frac{2}{3}\pi r_2^3$$

$$\Rightarrow V_2 = \frac{2}{3}\pi\left(\frac{r_1 2}{\sqrt{3}}\right)^3$$

$$\Rightarrow V_2 = \frac{2}{3}\pi\frac{r_2^3 8}{3\sqrt{3}}$$

Dividing V_1 by V_2,

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{2}{3}\pi\frac{8}{3\sqrt{3}}r_1^3}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{\frac{4}{3}}{\frac{2}{3}\frac{8}{3\sqrt{3}}}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{4}{3} \times \frac{9\sqrt{3}}{16}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{3\sqrt{3}}{4}$$

Question 11.
Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively are melted and recasted

into a single solid sphere. Taking ∏ = 3.1, find the surface area of solid sphere formed.

Solution:

Let radius of the larger sphere be 'R'

Volume of single sphere

= Vol. of sphere 1 + Vol. of sphere 2 + Vol. of sphere 3

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$$

$$\Rightarrow \frac{4}{3}\pi R^3 = \frac{4}{3}\pi 6^3 + \frac{4}{3}\pi 8^3 + \frac{4}{3}\pi 10^3$$

$$\Rightarrow R^3 = [6^3 + 8^3 + 10^3]$$

$$\Rightarrow R^3 = 1728$$

$$\Rightarrow R = 12$$

Surface area of the sphere

$$= 4\pi R^2$$

$$= 4\pi 12^2$$

$$= 1785.6 \text{ cm}^2$$

Question 12.

The surface area of a solid sphere is increased by 21% without changing its shape. Find the percentage increase in its:

(i) radius
(ii) volume

Solution:

Let the radius of the sphere be 'r'.
Total surface area the sphere, S $= 4\pi r^2$

New surface area of the sphere, S'

$= 4\pi r^2 + \frac{21}{100} \times 4\pi r^2$

$= \frac{121}{100} 4\pi r^2$

(i) Let the new radius be r_1

$S' = 4pr_1^2$

$S' = \frac{121}{100} 4\pi r^2$

$\Rightarrow 4\pi r_1^2 = \frac{121}{100} 4\pi r^2$

$\Rightarrow r_1^2 = \frac{121}{100} r^2$

$\Rightarrow r_1 = \frac{11}{10} r$

$\Rightarrow r_1 = r + \frac{r}{10}$

$\Rightarrow r_1 - r = \frac{r}{10}$

$\Rightarrow \text{Change in radius} = \frac{r}{10}$

Percentage change in radius

$= \frac{\text{Change in radius}}{\text{Original radius}} \times 100$

$= \frac{r/10}{r} \times 100$

$= 10$

Percentage change in radius = 10%

(ii) Let the volume of the sphere be V
Let the new volume of the sphere be V'.

$$V = \frac{4}{3}\pi r^3$$

$$V' = \frac{4}{3}\pi r_1^3$$

$$\Rightarrow V' = \frac{4}{3}\pi\left(\frac{11r}{10}\right)^3$$

$$\Rightarrow V' = \frac{4}{3}\pi\frac{1331}{1000}r^3$$

$$\Rightarrow V' = \frac{4}{3}\pi r^3\frac{1331}{1000}$$

$$\Rightarrow V' = \frac{1331}{1000}V$$

$$\Rightarrow V' = V + \frac{331}{1000}V$$

$$\Rightarrow V' - V = \frac{331}{1000}V$$

$$\therefore \text{Change in volume} = \frac{331}{1000}V$$

$$\text{Percentage change in volume} = \frac{\text{Change in volume}}{\text{Original Volume}} \times 100$$

$$= \frac{\frac{331}{1000}V}{V} \times 100$$

$$= \frac{331}{10}$$

$$= 33.1$$

Percentage change in volume =33.1%

Exercise 20 D

Question 1.
A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate the number of cones recast.

Solution:

Radius of a solid sphere = R = 15 cm

$\therefore$ Volume of sphere melted = $\frac{4}{3}\pi R^3 = \frac{4}{3} \times \pi \times 15 \times 15 \times 15$

Radius of each cone recasted = r = 2.5 cm

Height of each cone recasted = h = 8 cm

$\therefore$ Volume of each cone recasted = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 2.5 \times 2.5 \times 8$

$$\therefore \text{Number of cones recasted} = \frac{\text{Volume of sphere melted}}{\text{Volume of each cone formed}}$$

$$= \frac{\frac{4}{3} \times \pi \times 15 \times 15 \times 15}{\frac{1}{3} \times \pi \times 2.5 \times 2.5 \times 8}$$

$$= 270$$

Question 2.

A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. Find the height of the cone.

Solution:

External diameter = 8 cm

Therefore, radius (R) = 4 cm

Internal diameter = 4 cm

Therefore, radius (r) = 2 cm

Volume of metal used in hollow sphere = $\frac{4}{3}\pi(R^3 - r^3) = \frac{4}{3} \times \frac{22}{7} \times (4^3 - 2^3) = \frac{88}{21}(64 - 8) = \frac{88}{21} \times 56\ cm^3 \ldots.(i)$

Diameter of cone = 8 cm

Therefore, radius = 4 cm

Let height of cone = h

$\therefore$ Volume = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 4 \times 4 \times h = \frac{352}{21} h \ldots\ldots(ii)$

From (i) and (ii)

$$\frac{352}{21}h = \frac{88}{21} \times 56$$

$$\Rightarrow h = \frac{88 \times 56 \times 21}{21 \times 352} = 14 \text{ cm}$$

Height of the cone = 14 cm

Question 3.
The radii of the internal and external surfaces of a metallic spherical shell are 3 cm and 5 cm respectively. It is melted and recast into a solid right circular cone of height 32 cm. find the diameter of the base of the cone.

Solution:

Internal radius = 3cm
External radius = 5 cm
Volume of spherical shell

$$= \frac{4}{3}\pi(5^3 - 3^3)$$

$$= \frac{4}{3} \times \frac{22}{7}(125 - 27)$$

$$= \frac{4}{3} \times \frac{22}{7} \times 98$$

Volume of solid circular cone

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times r^2 \times 32$$

Vol. of Cone = Vol. of sphere

$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times r^2 \times 32 = \frac{4}{3} \times \frac{22}{7} \times 98$$

$$\Rightarrow r^2 = \frac{4 \times 98}{32}$$

$$\therefore r = \frac{7}{2} = 3.5\text{cm}$$

Hence, diameter = 2r = 7 cm

Question 4.
Total volume of three identical cones is the same as that of a bigger cone whose height is 9 cm and diameter 40 cm. find the radius of the base of each smaller cone, if height of each is 108 cm.

Solution:

Let the radius of the smaller cone be 'r' cm.
Volume of larger cone

$$= \frac{1}{3}\pi \times 20^2 \times 9$$

Volume of smaller cone

$$= \frac{1}{3}\pi \times r^2 \times 108$$

Volume of larger cone=3× Volume of smaller cone

$$\frac{1}{3}\pi \times 20^2 \times 9 = \frac{1}{3}\pi \times r^2 \times 108 \times 3$$

$$\Rightarrow r^2 = \frac{20^2 \times 9}{108 \times 3}$$

$$\Rightarrow r = \frac{20}{6} = \frac{10}{3}$$

$$\Rightarrow r = 3\frac{1}{3} \text{ cm}$$

Question 5.
A solid rectangular block of metal 49 cm by 44 cm by 18 cm is melted and formed into a solid sphere. Calculate the radius of the sphere.

Solution:

Volume of rectangular block = $49 \times 44 \times 18 \text{ cm}^3 = 38808 \text{ cm}^3$.......(i)

Let r be the radius of sphere

$\therefore$ Volume = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times r^3 = \frac{88}{21} r^3$......(ii)

From (i) and (ii)

$\frac{88}{21} r^3 = 38808$

$\Rightarrow r^3 = 38808 \times \frac{21}{88} = 441 \times 21$

$\Rightarrow r^3 = 9261$

$\Rightarrow r = 21 \text{ cm}$

Radius of sphere = 21 cm

Question 6.
A hemi-spherical bowl of internal radius 9 cm is full of liquid. This liquid is to be filled into conical shaped small containers each of diameter 3 cm and height 4 cm. How many containers are necessary to empty the bowl?

Solution:

Radius of hemispherical bowl = 9 cm

Volume = $\frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi 9^3 = \frac{2}{3}\pi \times 729 = 486\pi \text{cm}^2$

Diameter each of cylindrical bottle = 3 cm
Radius = $\frac{3}{2}$cm, and height = 4 cm

∴ Volume of bottle = $\frac{1}{3}\pi \pi^2 n = \frac{1}{3}\pi \times \left(\frac{3}{2}\right)^2 \times 4 = 3\pi$
∴ No. of bottles = $\frac{486\pi}{3\pi} = 162$

Question 7.
A hemispherical bowl of diameter 7.2 cm is filled completely with chocolate sauce. This sauce is poured into an inverted cone of radius 4.8 cm. Find the height of the cone if it is completely filled.

Solution:

Diameter of the hemispherical bowl = 7.2 cm

Therefore, radius = 3.6 cm

Volume of sauce in hemispherical bowl = $\frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times (3.6)^3$

Radius of the cone = 4.8 cm

Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (4.8)^2 \times h$

Now, volume of sauce in hemispherical bowl = volume of cone

$$\Rightarrow \frac{2}{3}\pi \times (3.6)^3 = \frac{1}{3}\pi \times (4.8)^2 \times h$$
$$\Rightarrow h = \frac{2 \times 3.6 \times 3.6 \times 3.6}{4.8 \times 4.8}$$
$$\Rightarrow h = 4.05 \text{ cm}$$

Height of the cone = 4.05 cm

Question 8.
A solid cone of radius 5 cm and height 8 cm is melted and made into small spheres of radius 0.5 cm. Find the number of spheres formed.

Solution:

Radius of a solid cone (r) = 5 cm

Height of the cone = 8 cm

$\Rightarrow$ Volume of a cone

$$= \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi \times 5 \times 5 \times 8 \text{ cm}^3$$
$$= \frac{200\pi}{3} \text{ cm}^3$$

Radius of each sphere = 0.5 cm

$\therefore$ Volume of one sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ cm^3

$= \frac{\pi}{6}$ cm^3

Number of spheres $= \frac{\text{Total volume}}{\text{Volume of one sphere}}$

$= \frac{\frac{200\pi}{3}}{\frac{\pi}{6}}$

$= \frac{200\pi}{3} \times \frac{6}{\pi}$

$= 400$

Question 9.

The total area of a solid metallic sphere is 1256 cm^2. It is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate:

(i) the radius of the solid sphere

(ii) the number of cones recasted [$\pi = 3.14$]

Solution:

Total area of solid metallic sphere = 1256 cm^2

(i)Let radius of the sphere is r then

$4\pi r^2 = 1256$

$4 \times \frac{22}{7} r^2 = 1256$

$\Rightarrow r^2 = \frac{1256 \times 7}{4 \times 22}$

$\Rightarrow r^2 = \frac{157 \times 7}{11}$

$\Rightarrow r^2 = \frac{1099}{11}$

$\Rightarrow r = \sqrt{99.909} = 9.995$ cm

$\Rightarrow r = 10$ cm

(ii) Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 10 \times 10 \times 10 = \frac{88000}{21}$ cm^3 Volume of right circular cone =

$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (2.5)^2 \times 8 = \frac{1100}{21}$ cm^3

Number of cones

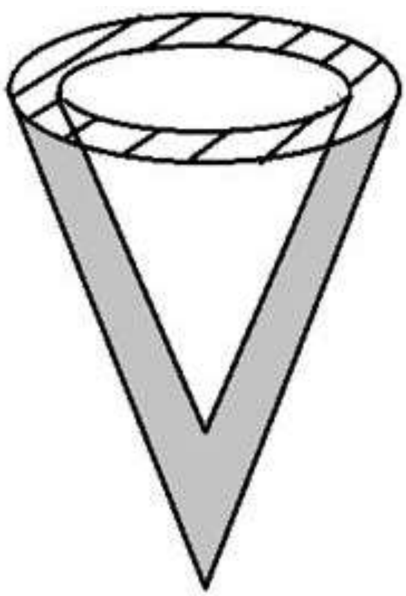
$= \frac{88000}{21} \div \frac{1100}{21}$

$= \frac{88000}{21} \times \frac{21}{1100}$

$= 80$

Question 10.
A solid metallic cone, with radius 6 cm and height 10 cm, is made of some heavy metal A. In order to reduce weight, a conical hole is made in the cone as shown and it is completely filled with a lighter metal B.

The conical hole has a diameter of 6 cm and depth 4 cm. Calculate the ratio of the volume of the metal A to the volume of metal B in the solid.



Solution:

Volume of the whole cone of metal A

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 6^2 \times 10$

$= 120\pi$

Volume of the cone with metal B

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 3^2 \times 4$

$= 12\pi$

Final Volume of cone with metal A=120 π-12 π=108 π

$\frac{\text{Volume of cone with metal A}}{\text{Volume of cone with metal B}}$

$= \frac{108\pi}{12\pi} = \frac{9}{1}$

Question 11.

A hollow sphere of internal and external radii 6 cm and 8 cm respectively is melted and recast into small cones of base radius 3 cm and height 8 cm. Find the number of cones.

Solution:

Let the number of small cones be 'n'
Volume of sphere

$$= \frac{4}{3}\pi(8^3 - 6^3)$$

$$= \frac{4}{3} \times \pi \times 2 \times 148$$

Volume of small spheres

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \times 2^2 \times 8$$

Volume of sphere = n× Volume of small sphere

$$\Rightarrow \frac{4}{3} \times \pi \times 2 \times 148 = n \times \frac{1}{3} \times \pi \times 2^2 \times 8$$

$$\Rightarrow n = \frac{4 \times 2 \times 148 \times 3}{4 \times 8 \times 3}$$

$$\Rightarrow n = 37$$

The number of cones = 37

Question 12.
The surface area of a solid metallic sphere is 2464 cm^2. It is melted and recast into solid right circular cones of radius 3.5 cm and height 7 cm. Calculate :
(i) the radius of the sphere.
(ii) the number of cones recast. (Take $\pi = \frac{22}{7}$)

Solution:

(i) Let R be the radius of a solid metallic sphere.

Surface area of a solid metallic sphere = 2464 cm^2

$\Rightarrow 4\pi R^2 = 2464$

$\Rightarrow 4 \times \frac{22}{7} \times R^2 = 2464$

$\Rightarrow R^2 = \frac{2464 \times 7}{4 \times 22} = 196$

$\Rightarrow R = 14$ cm

(ii) Volume of sphere melted = $\frac{4}{3}\pi R^3 = \frac{4}{3} \times \pi \times 14 \times 14 \times 14$

Radius of each cone recasted = r = 3.5 cm

Height of each cone recasted = h = 7 cm

$\therefore$ Volume of each cone recasted = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 3.5 \times 3.5 \times 7$

$\therefore$ Number of cones recasted = $\frac{\text{Volume of sphere melted}}{\text{Volume of each cone formed}}$

$$= \frac{\frac{4}{3} \times \pi \times 14 \times 14 \times 14}{\frac{1}{3} \times \pi \times 3.5 \times 3.5 \times 7}$$

$= 128$

Exercise 20 E

Question 1.

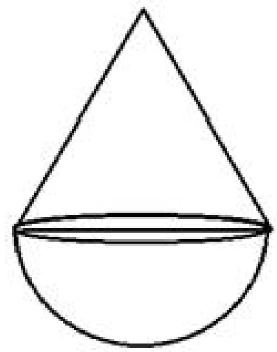
A cone of height 15 cm and diameter 7 cm is mounted on a hemisphere of same diameter. Determine the volume of the solid thus formed.

Solution:

Height of cone = 15 cm

and radius of the base = $\frac{7}{2}$ cm

Therefore, volume of the solid = volume of the conical part + volume of hemispherical part.



$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2(h + 2r)$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}(15 + 2 \times \frac{7}{2})$$

$$= \frac{847}{3}$$

$$= 282.33 \text{ cm}^3$$

Question 2.
A buoy is made in the form of a hemisphere surmounted by a right circular cone whose circular base coincides with the plane surface of the hemisphere.

The radius of the base of the cone is 3.5 m and its volume is two-third the volume of hemisphere. Calculate the height of the cone and the surface area of the buoy, correct to two decimal places.

Solution:

Radius of hemispherical part (r) = 3.5 m = $\frac{7}{2}$m

Therefore, Volume of hemisphere = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{539}{6} m^3$

Volume of conical part = $\frac{2}{3} \times \frac{539}{6} m^3$ (2/3 of hemisphere)

Let height of the cone = h

Then,

$\frac{1}{3}\pi r^2 h = \frac{2 \times 539}{3 \times 6}$

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h = \frac{2 \times 539}{3 \times 6}$

$\Rightarrow h = \frac{2 \times 539 \times 2 \times 2 \times 7 \times 3}{3 \times 6 \times 22 \times 7 \times 7}$

$\Rightarrow h = \frac{14}{3} m = 4\frac{2}{3} m = 4.67m$

Height of the cone = 4.67 m

Surface area of buoy = $2\pi r^2 + \pi r \ell$

But $\ell = \sqrt{r^2 + h^2}$

$\ell = \sqrt{\left(\frac{7}{2}\right)^2 + \left(\frac{14}{3}\right)^2}$

$= \sqrt{\frac{49}{4} + \frac{196}{9}} = \sqrt{\frac{1225}{36}} = \frac{35}{6} m$

Therefore, Surface area =

$= \left(2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) + \left(\frac{22}{7} \times \frac{7}{2} \times \frac{35}{6}\right) m^2$

$= \frac{77}{1} + \frac{385}{6} = \frac{847}{6}$

$= 141.17 m^2$

Surface Area = 141.17 m^2

Question 3.

From a rectangular solid of metal 42 cm by 30 cm by 20 cm, a conical cavity of diameter 14 cm and depth 24 c

(i) the surface area of the remaining solid

(ii)the volume of remaining solid

(iii) the weight of the material drilled out if it weighs 7 gm per cm^3.

Solution:

(i) Total surface area of cuboid = $2(\ell b + bh + \ell h)$

$=2(42 \times 30 + 30 \times 20 + 20 \times 42)$

$=2(1260 + 600 + 840)$

$=2 \times 2700$

$=5400\ cm^2$

Diameter of the cone = 14 cm

$\Rightarrow$ Radius of the cone = $\frac{14}{2} = 7$ cm

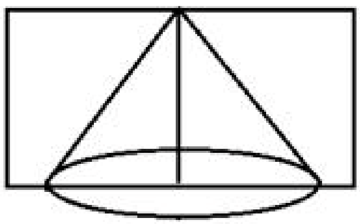
Area of circular base= $\pi r^2 = \frac{22}{7} \times 7 \times 7 = 154\ cm^2$

Area of curved surface area of cone= $\pi r \ell = \frac{22}{7} \times 7 \times \sqrt{7^2 + 24^2} = 22\sqrt{49 + 576} = 22 \times 25 = 550\ cm^2$

Surface area of remaining part $= 5400 + 550 - 154 = 5796\ cm^2$

(ii) Dimensions of rectangular solids = $(42 \times 30 \times 20)$ cm

volume = $(42 \times 30 \times 20) = 25200\ cm^3$



Radius of conical cavity (r) =7 cm

height (h) = 24 cm

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24$

$= 1232\ cm^3$

Volume of remaining solid = (25200 - 1232) = 23968 cm^3

(iii) Weight of material drilled out

$=1232 \times 7$ g = 8624g = 8.624 kg

Question 4.
The cubical block of side 7 cm is surmounted by a hemisphere of the largest size. Find the surface area of the resulting solid.

Solution:

The diameter of the largest hemisphere that can be placed on a face of a cube of side 7 cm will be 7 cm.

Therefore, radius = $r = \frac{7}{2}$ cm

Its curved surface area =

$2\pi r^2$

$= 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 77\ cm^2$..............(i)

Surface area of the top of the resulting solid = Surface area of the top face of the cube - Area of the base of the hemisphere

$= (7 \times 7) - (\frac{22}{7} \times \frac{49}{4})$

$= 49 - \frac{77}{2}$

$= \frac{98 - 77}{2}$

$= \frac{21}{2}$

$= 10.5\ cm^2$.......(ii)

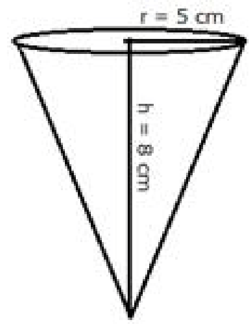
Surface area of the cube = $5 \times (side)^2 = 5 \times 49 = 245\ cm^2$..........(iii)

Total area of resulting solid = 245 + 10.5 + 77 = 332.5 cm^2

Question 5.
A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of the top which is open is 5 cm. It is filled with water.

When lead shots, each of which is a sphere of radius 0.5 cm, are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Solution:



Height of cone = 8 cm
Radius = 5 cm

Volume = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 8 cm^3$

$= \frac{4400}{21} cm^3$

Therefore, volume of water that flowed out =

$= \frac{1}{4} \times \frac{4400}{21} cm^3$

$= \frac{1100}{21} cm^3$

Radius of each ball = 0.5 cm = $\frac{1}{2}$ cm

Volume of a ball = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} cm^3$

$= \frac{11}{21} cm^3$

Therefore, No. of balls = $\frac{1100}{21} \div \frac{11}{21} = 100$

Hence, number of lead balls = 100

Question 6.
A hemispherical bowl has negligible thickness and the length of its circumference is 198 cm. find the capacity of the bowl.

Solution:

Let r be the radius of the bowl.

$\therefore 2\pi r = 198$

$\Rightarrow r = \frac{198 \times 7}{2 \times 22}$

$\Rightarrow r = 31.5 \text{ cm}$

Capacity of the bowl =

$\frac{2}{3}\pi r^3$

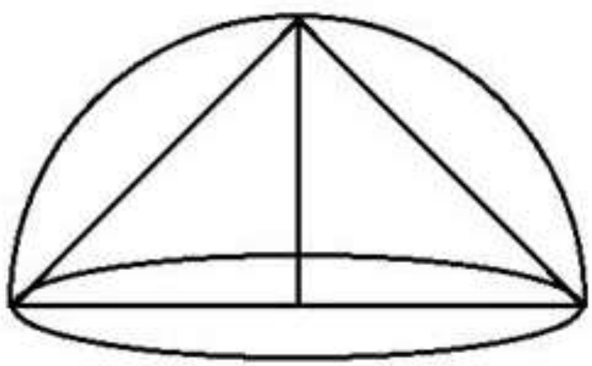
$= \frac{2}{3} \times \frac{22}{7} \times (31.5)^3$

$= 65488.5 \text{ cm}^3$

Question 7.

Find the maximum volume of a cone that can be carved out of a solid hemisphere of radius r cm.

Solution:



For the volume of cone to be largest, h = r cm

Volume of the cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times r^2 \times r$

$= \frac{1}{3}\pi r^3$

Question 8.

The radii of the bases of two solid right circular cones of same height are r_1 and r_2 respectively. The cones are melted and recast into a solid sphere of radius R. Find the height of each cone in terms r_1, r_2 and R.

Solution:

Let the height of the solid cones be 'h'

Volume of solid circular cones

$$V_1 = \frac{1}{3}\pi r_1^2 h$$

$$V_2 = \frac{1}{3}\pi r_2^2 h$$

Volume of sphere

$$= \frac{4}{3}\pi R^3$$

Volume of sphere = Volume of cone 1 + volume of cone 2

$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi r_1^2 h + \frac{1}{3}\pi r_2^2 h$$

$$\Rightarrow 4R^3 = r_1^2 h + r_2^2 h$$

$$\Rightarrow h(r_1^2 + r_2^2) = 4R^3$$

$$\Rightarrow h = \frac{R^3}{(r_1^2 + r_2^2)}$$

Question 9.

A solid hemisphere of diameter 28 cm is melted and recast into a number of identical solid cones, each of diameter 14 cm and height 8 cm. Find the number of cones so formed.

Solution:

Volume of the solid hemisphere

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi 14^3$

$= \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14$

Volume of 1 cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 8$

No. of cones formed

$= \frac{\text{Volume of sphere}}{\text{Volume of 1 cone}}$

$= \frac{\frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14}{\frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 8}$

$= 28$

Question 10.
A cone and a hemisphere have the same base and same height. Find the ratio between their volumes.

Solution:

Let the radius of base be 'r' and the height be 'h'

Volume of cone, V_c

$= \frac{1}{3}\pi r^2 h$

Volume of hemisphere, V_h

$= \frac{2}{3}\pi r^3$

$$\frac{V_c}{V_h} = \frac{\frac{1}{3}\pi r^2 r}{\frac{2}{3}\pi r^3}$$

$$\Rightarrow \frac{V_c}{V_h} = \frac{1}{2}$$

Exercise 20 F

Question 1.

From a solid right circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and same base are removed. Find the volume of the remaining solid.

Solution:

Height of the cylinder (h) = 10 cm

and radius of the base (r) = 6 cm

Volume of the cylinder = $\pi r^2 h$

Height of the cone = 10 cm

Radius of the base of cone = 6 cm

Volume of the cone = $\frac{1}{3}\pi r^2 h$

Volume of the remaining part

$= \pi r^2 h - \frac{1}{3}\pi r^2 h$

$= \frac{2}{3}\pi r^2 h$

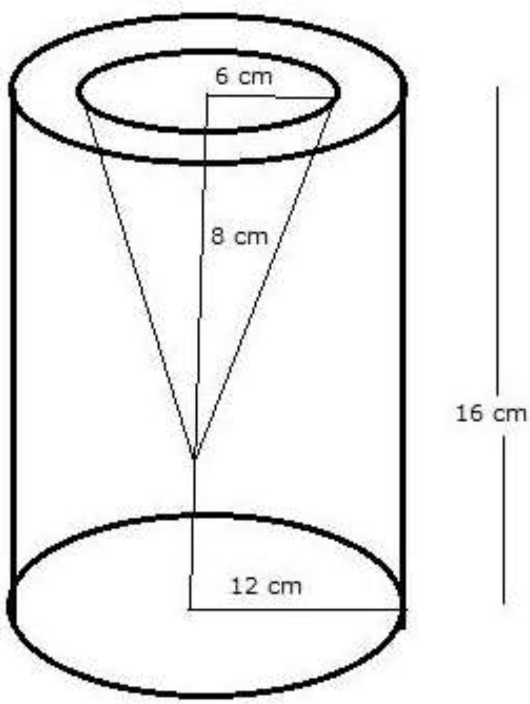
$= \frac{2}{3} \times \frac{22}{7} \times 6 \times 6 \times 10$

$= \frac{5280}{7}$

$= 754\frac{2}{7}\ cm^3$

Question 2.
From a solid cylinder whose height is 16 cm and radius is 12 cm, a conical cavity of height 8 cm and of base radius 6 cm is hollowed out. Find the volume and total surface area of the remaining solid.

Solution:



Radius of solid cylinder (R) = 12 cm
and Height (H) = 16 cm

$\therefore$ Volume = $\pi R^2 H$

$= \frac{22}{7} \times 12 \times 12 \times 16$

$= \frac{50688}{7}\text{ cm}^3$

Radius of cone (r) = 6 cm, and height (h) = 8 cm.

$\therefore$ Volume = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 8$

$= \frac{2112}{7}\text{ cm}^3$

(i) Volume of remaining solid

$= \frac{50688}{7} - \frac{2112}{7}$

$= \frac{48567}{7}$

$= 6939.43\text{ cm}^3$

(ii) Slant height of cone $\ell = \sqrt{h^2 + r^2}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10$ cm

Therefore, total surface area of remaining solid = curved surface area of cylinder + curved surface area of cone + base area of cylinder + area of circular ring on upper side of cylinder

$= 2\pi Rh + \pi r\ell + \pi R^2 + \pi(R^2 - r^2)$

$= (2 \times \frac{22}{7} \times 12 \times 16) + (\frac{22}{7} \times 6 \times 10) + (\frac{22}{7} \times 12 \times 12) + (\frac{22}{7}(12^2 - 6^2))$

$= \frac{8448}{7} + \frac{1320}{7} + \frac{3168}{7} + \frac{22}{7}(144 - 36)$

$= \frac{8448}{7} + \frac{1320}{7} + \frac{3168}{7} + \frac{2376}{7}$

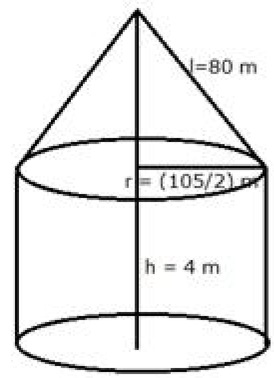
$= \frac{15312}{7}$

$= 2187.43\text{ cm}^2$

Question 3.

A circus tent is cylindrical to a height of 4 m and conical above it. If its diameter is 105 m and its slant height is 80 m, calculate the total area of canvas required. Also, find the total cost of canvas used at Rs 15 per meter if the width is 1.5 m

Solution:



Radius of the cylindrical part of the tent (r) = $\frac{105}{2}$ m

Slant height (ℓ) = 80 m

Therefore, total curved surface area of the tent = $2\pi rh + \pi r\ell$

$= (2 \times \frac{22}{7} \times \frac{105}{2} \times 4) + (\frac{22}{7} \times \frac{105}{2} \times 80)$

$= 1320 + 13200$

$= 14520 \text{ m}^2$

Width of canvas used = 1.5 m

Length of canvas = $\frac{14520}{1.5} = 9680$ m

Total cost of canvas at the rate of Rs 15 per meter

$= 9680 \times 15 = $ Rs 145200

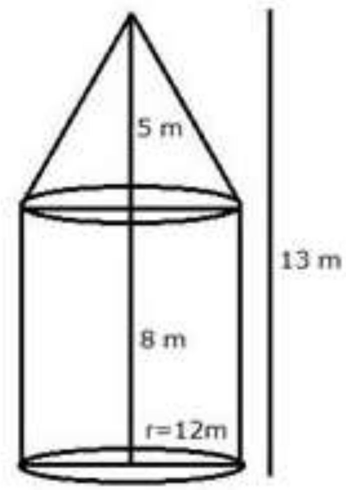
Question 4.

A circus tent is cylindrical to a height of 8 m surmounted by a conical part. If total height of the tent is 13 m and the diameter of its base is 24 m; calculate:

(i) total surface area of the tent

(ii) area of canvas, required to make this tent allowing 10% of the canvas used for folds and stitching.

Solution:



Height of the cylindrical part = H = 8 m
Height of the conical part = h = (13 – 8)m = 5 m

Diameter = 24 m → radius = r = 12 m
Slant height of the cone = l

$$\ell = \sqrt{r^2 + h^2}$$
$$\ell = \sqrt{12^2 + 5^2}$$
$$\ell = \sqrt{169} = 13m$$

Slant height of cone = 13 m

(i) Total surface area of the tent = 2πrh + πrl = πr(2h + l)

$$= \frac{22}{7} \times 12 \times (2 \times 8 + 13)$$
$$= \frac{264}{7}(16 + 13)$$
$$= \frac{264}{7} \times 29$$
$$= \frac{7656}{7} \text{ m}^2$$
$$= 1093.71 \text{ m}^2$$

(ii)Area of canvas used in stitching = total area

$$\text{Total area of canvas} = \frac{7656}{7} + \frac{\text{Total area of canvas}}{10}$$

$$\Rightarrow \text{Total area of canvas} - \frac{\text{Total area of canvas}}{10} = \frac{7656}{7}$$

$$\Rightarrow \text{Total area of canvas}\left(1 - \frac{1}{10}\right) = \frac{7656}{7}$$

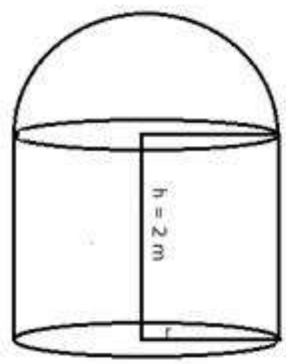
$$\Rightarrow \text{Total area of canvas} \times \frac{9}{10} = \frac{7656}{7}$$

$$\Rightarrow \text{Total area of canvas} = \frac{7656}{7} \times \frac{10}{9}$$

$$\Rightarrow \text{Total area of canvas} = \frac{76560}{63} = 1215.23 \text{ m}^2$$

Question 5.
A cylindrical boiler, 2 m high, is 3.5 m in diameter. It has a hemispherical lid. Find the volume of its interior, including the part covered by the lid.

Solution:



Diameter of cylindrical boiler = 3.5 m

$$\therefore \text{Radius}(r) = \frac{3.5}{2} = \frac{35}{20} = \frac{7}{4} \text{ m}$$

Height (h) = 2 m

Radius of hemisphere (R) = $\frac{7}{4}$ m

Total volume of the boiler = $\pi r^2 h + \frac{2}{3}\pi r^3$

$= \pi r^2(h + \frac{2}{3}r)$

$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4}(2 + \frac{2}{3} \times \frac{7}{4})$

$= \frac{77}{8}(2 + \frac{7}{6})$

$= \frac{77}{8} \times \frac{19}{6}$

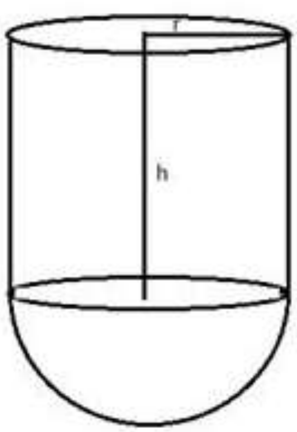
$= \frac{1463}{48}$

$= 30.48\ m^3$

Question 6.

A vessel is a hollow cylinder fitted with a hemispherical bottom of the same base. The depth of the cylindrical part is $4\frac{2}{3}m$ and the diameter of hemisphere is 3.5 m. Calculate the capacity and the internal surface area of the vessel.

Solution:



Diameter of the base = 3.5 m

Therefore, radius = $\frac{3.5}{2}$ m = 1.75 m = $\frac{7}{4}$ m

Height of cylindrical part = $4\frac{2}{3} = \frac{14}{3}$ m

(i) Capacity (volume) of the vessel = $\pi r^2 h + \frac{2}{3}\pi r^3 = \pi r^2\left(h + \frac{2}{3}r\right)$

$$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4}\left(\frac{14}{3} + \frac{2}{3} \times \frac{7}{4}\right)$$
$$= \frac{77}{8}\left(\frac{14}{3} + \frac{7}{6}\right)$$
$$= \frac{77}{8}\left(\frac{28+7}{6}\right)$$
$$= \frac{77}{8} \times \frac{35}{6}$$
$$= \frac{2695}{48}$$
$$= 56.15 \text{ m}^3$$

(ii)Internal curved surface area = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

$$= 2 \times \frac{22}{7} \times \frac{7}{4}\left(\frac{14}{3} + \frac{7}{4}\right)$$
$$= 11\left(\frac{56+21}{12}\right)$$
$$= 11 \times \frac{77}{12}$$
$$= \frac{847}{12}$$
$$= 70.58 \text{ m}^2$$

Question 7.

A wooden toy is in the shape of a cone mounted on a cylinder as shown alongside.

If the height of the cone is 24 cm, the total height of the toy is 60 cm and the radius of the base of the cone is twice the radius of the base of the cylinder = 10 cm; find the total surface area of the toy. [Take $\pi = 3.14$]

Solution:

Height of the cone = 24 cm

Height of the cylinder = 36 cm

Radius of the cone = twice the radius of the cylinder = 10 cm

Radius of the cylinder = 5 cm

Slant height of the cone = $\sqrt{r^2 + h^2}$

$= \sqrt{10^2 + 24^2}$
$= \sqrt{100 + 576}$
$= \sqrt{676}$
$= 26 \text{ cm}$

Now, the surface area of the toy = curved area of the conical point + curved area of the cylinder

$= \pi r\ell + \pi r^2 + 2\pi RH$
$= \pi[r\ell + r^2 + 2RH]$
$= 3.14[10 \times 26 + (10)^2 + 2 \times 5 \times 36]$
$= 3.14[260 + 100 + 360]$
$= 3.14[720]$
$= 2260.8 \text{ cm}^2$

Question 8.
A cylindrical container with diameter of base 42 cm contains sufficient water to submerge a rectangular solid of iron with dimensions 22 cm x 14 cm x 10.5 cm. Find the rise in level of the water when the solid is submerged.

Solution:

Diameter of cylindrical container = 42 cm

Therefore, radius (r) = 21 cm

Dimensions of rectangular solid = 22cm × 14cm × 10.5cm

Volume of solid = $= 22 \times 14 \times 10.5\ cm^3$......(i)

Let height of water = h

Therefore, volume of water in the container = $\pi r^2 h$

$$= \frac{22}{7} \times 21 \times 21 \times h\ cm^3 = 22 \times 63h\ cm^3 \text{......(ii)}$$

From (i) and (ii)

$$22 \times 63h = 22 \times 14 \times 10.5$$

$$\Rightarrow h = \frac{22 \times 14 \times 10.5}{22 \times 63}$$

$$\Rightarrow h = \frac{7}{3}$$

$$\Rightarrow h = 2\frac{1}{3} \text{ or } 2.33\text{ cm}$$

Question 9.

Spherical marbles of diameter 1.4 cm are dropped into beaker containing some water and are fully submerged. The diameter of the beaker is 7 cm. Find how many marbles have been dropped in it if the water rises by 5.6 cm.

Solution:

Diameter of spherical marble = 1.4 cm

Therefore, radius = 0.7 cm

Volume of one ball = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.7)^3$ cm^3.........(i)

Diameter of beaker = 7 cm

Therefore, radius = $\frac{7}{2}$ cm

Height of water = 5.6 cm

Volume of water = $\pi r^2 h = \pi \times \left(\frac{7}{2} \times \frac{7}{2} \times 5.6\right)$ cm^3 $= \pi \times \frac{49 \times 56}{4 \times 10}$ cm^3

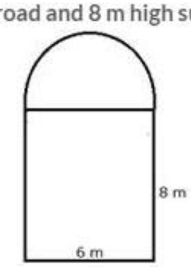
No. of balls dropped

$= \frac{\pi \times 49 \times 56 \times 3}{4 \times 10 \times 4\pi \times (0.7)^3}$

$= 150$

Question 10.

The cross-section of a railway tunnel is a rectangle 6 m broad and 8 m high surmounted by a semi-circle as shown in the figure.



The tunnel is 35 m long. Find the cost of plastering the internal surface of the tunnel (excluding the floor) at the rate of Rs 2.25 per m^2.

Solution:

Breadth of the tunnel = 6 m

Height of the tunnel = 8 m

Length of the tunnel = 35 m

Radius of the semi-circle = 3 m

Circumference of the semi-circle = $\pi r = \frac{22}{7} \times 3 = \frac{66}{7}$ m

Internal surface area of the tunnel

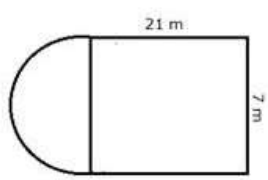
$$= 35\left(8 + 8 + \frac{66}{7}\right)$$
$$= 35\left(16 + \frac{66}{7}\right)$$
$$= 35\left(\frac{112 + 66}{7}\right)$$
$$= 35 \times \frac{178}{7}$$
$$= 890 \text{ m}^2$$

Rate of plastering the tunnel = Rs 2.25 per m^2

Therefore, total expenditure = Rs $890 \times \frac{225}{100}$ = Rs $890 \times \frac{9}{4}$ = Rs $\frac{8010}{4}$ = Rs 2002.50

Question 11.

The horizontal cross-section of a water tank is in the shape of a rectangle with semicircle at one end, as shown in the following figure.



The water is 2.4 metres deep in the tank. Calculate the volume of water in the tank in gallons. (Given: 1 gallon = 4.5 litres)

Solution:

Length = 21 m

Depth of water = 2.4 m

Breadth = 7 m

Therefore, radius of semicircle = $\frac{7}{2}$ m

Area of cross-section = area of rectangle + Area of semicircle

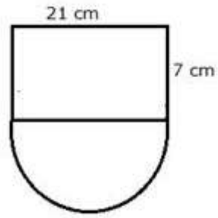
$$= l \times b + \frac{1}{2}\pi r^2$$
$$= 21 \times 7 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$
$$= 147 + \frac{77}{4}$$
$$= \frac{588 + 77}{4}$$
$$= \frac{665}{4} \text{ m}^2$$

Therefore, Volume of water filled in gallons

$$= \frac{665}{4} \times 2.4 \text{ m}^3$$
$$= 665 \times 0.6$$
$$= 399 \text{ m}^3$$
$$= 399 \times 100^3 \text{cm}^3$$
$$= \frac{399 \times 100 \times 100 \times 100}{1000} \text{litres}$$
$$= \frac{399 \times 100 \times 100 \times 100}{1000 \times 4.5} \text{gallons}$$
$$= \frac{399 \times 100 \times 100 \times 100 \times 10}{1000 \times 45} \text{gallons}$$
$$= \frac{1330000}{15} \text{gallons}$$
$$= \frac{266000}{3} \text{gallons}$$
$$= 88666.67 \text{ gallons}$$

Question 12.

The given figure shows the cross-section of a water channel consisting of a rectangle and a semicircle. Assuming that the channel is always full, find the volume of water discharged through it in one minute if water is flowing at the rate of 20 cm per second. Give your answer in cubic meters correct to one place of decimal.



Solution:

Length = 21 cm, Breadth = 7 cm

Radius of semicircle = $\frac{21}{2}$ cm

Area of cross section of the water channel = $l \times b + \frac{1}{2}\pi r^2$

$= 21 \times 7 + \frac{1}{2} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$

$= 147 + \frac{693}{4}$

$= \frac{588 + 693}{4}$

$= \frac{1281}{4}$ cm^2

Flow of water in one minute at the rate of 20 cm per second

$\Rightarrow$ Length of the water column = $20 \times 60 = 1200$ cm

Therefore, volume of water =

$= \frac{1281}{4} \times 1200$ cm^3

$= 384300$ cm^3

$= \frac{384300}{100 \times 100 \times 100}$ m^3

$= 0.3843$ m^3

$= 0.4$ m^3

Question 13.

An open cylindrical vessel of internal diameter 7 cm and height 8 cm stands on a horizontal table. Inside this is placed a solid metallic right circular cone, the diameter of whose base is $3\frac{1}{2}$ cm and height 8 cm. Find the volume of water required to fill the vessel. If this cone is replaced by another cone, whose height is $1\frac{3}{4}$ cm and radius of whose base is 2 cm, find the drop in the water level.

Solution:

Diameter of the base of the cylinder = 7 cm

Therefore, radius of the cylinder = $\frac{7}{2}$ cm

Volume of the cylinder $\pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 8 = 308 \text{ cm}^3$

Diameter of the base of the cone = $\frac{7}{2}$ cm

Therefore, radius of the cone = $\frac{7}{4}$ cm

Volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 8 = \frac{77}{3} \text{ cm}^3$

On placing the cone into the cylindrical vessel, the volume of the remaining portion where the water is to be filled

$= 308 - \frac{77}{3}$

$= \frac{924 - 77}{3}$

$= \frac{847}{3}$

$= 282.33 \text{ cm}^3$

Height of new cone = $1\frac{3}{4} = \frac{7}{4}$ cm

Radius = 2 cm

Therefore, volume of new cone

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 2 \times 2 \times \frac{7}{4} = \frac{22}{3} \text{ cm}^3$

Volume of water which comes down = $\frac{77}{3} - \frac{22}{3} \text{ cm}^3 = \frac{55}{3} \text{ cm}^3$.....(i)

Let h be the height of water which is dropped down.

Radius = $\frac{7}{2}$ cm

$\therefore$ Volume $= \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h = \frac{77}{2} h$.......(ii)

From (i) and (ii)

$$\frac{77}{2}h = \frac{55}{3}$$

$$\Rightarrow h = \frac{55}{3} \times \frac{2}{77}$$

$$\Rightarrow h = \frac{10}{21} \text{cm}$$

Drop in water level = $\frac{10}{21}$ cm

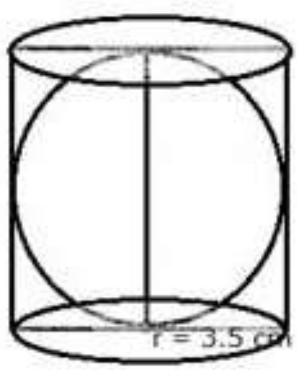
Question 14.

A cylindrical can, whose base is horizontal and of radius 3.5 cm, contains sufficient water so that when a sphere is placed in the can, the water just covers the sphere. Given that the sphere just fits into the can, calculate:

(i) the total surface area of the can in contact with water when the sphere is in it;

(ii) the depth of water in the can before the sphere was put into the can.

Solution:



Radius of the base of the cylindrical can = 3.5 cm

(i) When the sphere is in can, then total surface area of the can =

Base area + curved surface area

$$= \pi r^2 + 2\pi r h$$

$$= \left(\frac{22}{7} \times 3.5 \times 3.5\right) + \left(2 \times \frac{22}{7} \times 3.5 \times 7\right)$$

$$= \frac{77}{2} + 154$$

$$= 38.5 + 154$$

$= 192.5 \text{ cm}^2$

(ii) Let depth of water = x cm

When sphere is not in the can, then volume of the can =

volume of water + volume of sphere

$$\Rightarrow \pi r^2 h = \pi r^2 \times x + \frac{4}{3}\pi r^3$$

$$\Rightarrow \pi r^2 h = \pi r^2 (x + \frac{4}{3} r)$$

$$\Rightarrow h = x + \frac{4}{3} r$$

$$\Rightarrow x = h - \frac{4}{3} r$$

$$\Rightarrow x = 7 - \frac{4}{3} \times \frac{7}{2}$$

$$\Rightarrow x = 7 - \frac{14}{3}$$

$$\Rightarrow x = \frac{21 - 14}{3}$$

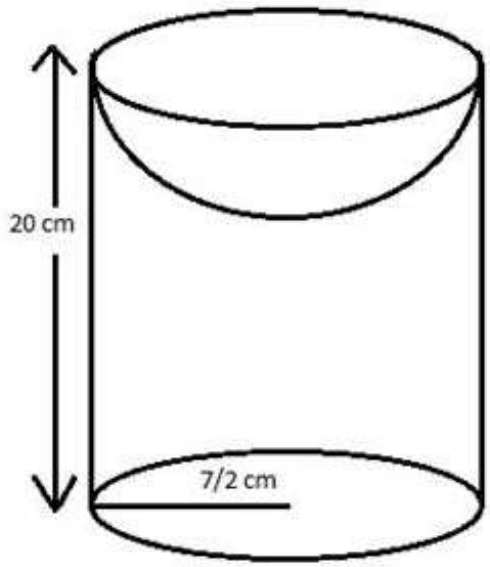
$$\Rightarrow x = \frac{7}{3}$$

$$\Rightarrow x = 2\frac{1}{3} \text{ cm}$$

Question 15.

A hollow cylinder has solid hemisphere inward at one end and on the other end it is closed with a flat circular plate. The height of water is 10 cm when flat circular surface is downward. Find the level of water, when it is inverted upside down, common diameter is 7 cm and height of the cylinder is 20 cm.

Solution:



Let the height of the water level be 'h', after the solid is turned upside down.
Volume of water in the cylinder

$= \pi\left(\frac{7}{2}\right)^2 10$

Volume of the hemisphere

$= \frac{2}{3}\pi\left(\frac{7}{2}\right)^3$

Volume of water in the cylinder
= Volume of water level - Volume of the hemisphere

$\pi\left(\frac{7}{2}\right)^2 10 = \pi\left(\frac{7}{2}\right)^2 h - \frac{2}{3}\pi\left(\frac{7}{2}\right)^3$

$\Rightarrow 10 = h - \frac{7}{3}$

$\Rightarrow h = 10 + \frac{7}{3}$

$\Rightarrow h = 12\frac{1}{3}$ cm

Exercise 20 G

Question 1.
What is the least number of solid metallic spheres, each of 6 cm diameter, that should be melted and recast to form a solid metal cone whose height is 45 cm and diameter is

12 cm?

Solution:

Let the number of solid metallic spheres be 'n'
Volume of 1 sphere

$$= \frac{4}{3}\pi(3)^3$$

Volume of metallic cone

$$= \frac{1}{3}\pi 6^2 \times 45$$

$$n = \frac{\text{Volume of metal cone}}{\text{Volume of 1 sphere}}$$

$$\Rightarrow n = \frac{\frac{1}{3}\pi 6^2 \times 45}{\frac{4}{3}\pi(3)^3}$$

$$\Rightarrow n = \frac{6 \times 6 \times 45}{4 \times 3 \times 3 \times 3}$$

$$\Rightarrow n = 15$$

The least number of spheres needed to form the cone is 15

Question 2.
A largest sphere is to be carved out of a right circular cylinder of radius 7 cm and height 14 cm. Find the volume of the sphere. (Answer correct to the nearest integer)

Solution:
Radius of largest sphere that can be formed inside the cylinder should be equal to the radius of the cylinder.

Radius of the largest sphere = 7 cm
Volume of sphere

$$= \frac{4}{3}\pi 7^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$$= \frac{4312}{3}$$

$$= 1437\ \text{cm}^3$$

Question 3.
A right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. The ice-cream is to be filled in identical cones of height 12 cm and diameter 6 cm having a semi-spherical shape on the top. Find the number of cones required.

Solution:

Let the number of cones be 'n'.
Volume of the cylinder = $\pi \times 6^2 \times 15$

Volume of 1 cone = $\frac{1}{3}\pi \times 3^2 \times 12$

$$n = \frac{\text{Volume of cylinder}}{\text{Volume of 1 cone}}$$

$$= \frac{\pi \times 6^2 \times 15}{\frac{1}{3}\pi \times 3^2 \times 12}$$

$$= 15$$

Number of cones required = 15

Question 4.
A solid is in the form of a cone standing on a hemisphere with both their radii being equal to 8 cm and the height of cone is equal to its radius. Find in terms of π, the volume of the solid.

Solution:

Volume of the solid

$$= \frac{1}{3}\pi r^2 r + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3} \times \pi \times 8^3 + \frac{2}{3} \times \pi \times 8^3$$

$$= \pi 8^3$$

$$= 512\pi \text{ cm}^3$$

Question 5.
The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 0.2 cm. Find the length of wire.

Solution:

Diameter of a sphere = 6 cm

Radius = 3 cm

$\therefore$ Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3 = \frac{792}{7} cm^3$.....(i)

Diameter of cylindrical wire = 0.2 cm

Therefore, radius of wire $= \frac{0.2}{2} = 0.1 = \frac{1}{10}$ cm

Let length of wire = h

$\therefore$ Volume $= \pi r^2 h = \frac{22}{7} \times \frac{1}{10} \times \frac{1}{10} \times h\ cm^3 = \frac{22h}{700} cm^3$.....(ii)

From (i) and (ii)

$\frac{22h}{700} = \frac{792}{7}$

$\Rightarrow h = \frac{792}{7} \times \frac{700}{22}$

$\Rightarrow h = 3600\ cm = 36\ m$

Hence, length of the wire = 36 m

Question 6.
Determine the ratio of the volume of a cube to that of a sphere which will exactly fit inside the cube.

Solution:

Let edge of the cube = a

volume of the cube = $a \times a \times a = a^3$

The sphere, which exactly fits in the cube, has radius = $\frac{a}{2}$

Therefore, volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \left(\frac{a}{2}\right)^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{a^3}{8} = \frac{11}{21}a^3$

Volume of cube : volume of sphere

$= a^3 : \frac{11}{21}a^3$

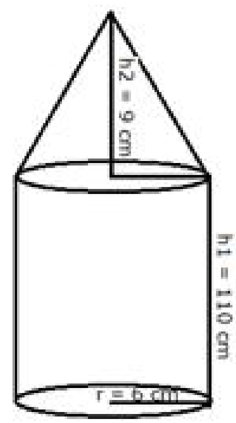
$= 1 : \frac{11}{21}$

$= 21 : 11$

Question 7.

An iron pole consisting of a cylindrical portion 110 cm high and of base diameter 12 cm is surmounted by a cone 9 cm high. Find the mass of the pole, given that 1 cm^3 of iron has 8 gm of mass (approx). (Take $\pi = \frac{355}{113}$)

Solution:



Radius of the base of poles (r) = 6 cm

Height of the cylindrical part (h_1) = 110 cm

Height of the conical part (h_2) = 9 cm

Total volume of the iron pole = $\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2 = \pi r^2(h_1 + \frac{1}{3}h_2)$

$$= \frac{355}{113} \times 6 \times 6(110 + \frac{1}{3} \times 9)$$

$$= \frac{355}{113} \times 36 \times 113$$

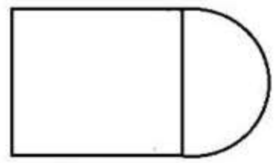
$$= 12780 \text{ cm}^3$$

Weight of 1 cm^3 = 8 gm

Therefore, total weight = 12780 × 8 = 102240 gm = 102.24 kg

Question 8.

In the following diagram a rectangular platform with a semicircular end on one side is 22 meters long from one end to the other end. If the length of the half circumference is 11 meters, find the cost of constructing the platform, 1.5 meters high at the rate of Rs 4 per cubic meters.



Solution:

Length of the platform = 22 m

Circumference of semicircle = 11 m

$\therefore$ Radius $= \frac{c \times 2}{2 \times \pi} = \frac{11 \times 7}{22} = \frac{7}{2}$ m

Therefore, breadth of the rectangular part $= \frac{7}{2} \times 2 = 7$ m

And length $= 22 - \frac{7}{2} = \frac{37}{2} = 18.5$ m

Now area of platform $= l \times b + \frac{1}{2}\pi r^2$

$= 18.5 \times 7 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$ m^2

$= 129.5 + \frac{77}{4}$ m^2

$= 148.75$ m^2

Height of the platform = 1.5 m

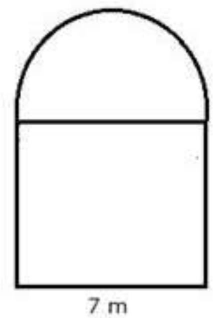
$\therefore$ Volume $= 148.75 \times 1.5 = 223.125$ m^3

Rate of construction = Rs 4 per m^3

Total expenditure = Rs 4 $\times$ 223.125 = Rs 892.50

Question 9.

The cross-section of a tunnel is a square of side 7 m surmounted by a semicircle as shown in the following figure.



The tunnel is 80 m long. Calculate:

(i) its volume

(ii) the surface area of the tunnel (excluding the floor) and

(iii) its floor area

Solution:

Side of square = 7 m

Radius of semicircle = $\frac{7}{2}$ m

Length of the tunnel = 80 m

Area of cross section of the front part = $a^2 + \frac{1}{2}\pi r^2$

$= 7 \times 7 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 49 + \frac{77}{4}\, m^2$

$= \frac{196 + 77}{4}$

$= \frac{273}{4}\, m^2$

(i) Therefore, volume of the tunnel = area x length

$= \frac{273}{4} \times 80$

$= 5460\ m^3$

(ii) Circumference of the front of tunnel

$= 2 \times 7 + \frac{1}{2} \times 2\pi r$

$= 14 + \frac{22}{7} \times \frac{7}{2}$

$= 14 + 11$

$= 25 \text{ m}$

Therefore, surface area of the inner part of the tunnel

$= 25 \times 80$

$= 2000 \text{ m}^2$

(iii) Area of floor = $l \times b = 7 \times 80 = 560 \text{ m}^2$

Question 10.
A cylindrical water tank of diameter 2.8m and height 4.2m is being fed by a pipe of diameter 7 cm through which water flows at the rate of 4m/s. Calculate, in minutes, the time it takes to fill the tank.

Solution:

Diameter of cylindrical tank = 2.8 m

Therefore, radius = 1.4 m

Height = 4.2 m

Volume of water filled in it = $\pi r^2 h$

$= \frac{22}{7} \times 1.4 \times 1.4 \times 4.2 \text{ m}^3$

$= \frac{181.104}{7} \text{m}^3$

$= 25.872 \text{ m}^3$........(i)

Diameter of pipe = 7 cm

Therefore, radius (r) = $\frac{7}{2}$cm

Let length of water in the pipe = h_1

$\therefore$ Volume $= \pi r^2 h_1$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h_1$

$= \frac{77}{2} h_1 cm^3$........(ii)

From (i) and (ii)

$\frac{77}{2} h_1\ cm^3 = 25.872 \times 100^3 cm^3$

$\Rightarrow \quad h_1 = \frac{25.872 \times 100^3 \times 2}{77} cm$

$\Rightarrow \quad h_1 = \frac{25.872 \times 100^3 \times 2}{77 \times 100} m$

$\Rightarrow \quad h_1 = 0.672 \times 100^2 m$

$\Rightarrow \quad h_1 = 6720\ m$

Therefore, time taken at the speed of 4 m per second

$= \frac{6720}{4 \times 60}$ minutes $= 28$ minutes

Question 11.

Water flows, at 9 km per hour, through a cylindrical pipe of cross-sectional area 25 cm^2. If this water is collected into a rectangular cistern of dimensions 7.5m by 5m by 4 m; calculate the rise in level in the cistern in 1 hour 15 minutes.

Solution:

Rate of flow of water = 9 km/hr

Water flow in 1 hour 15 minutes

i.e. in $\frac{5}{4} hr = 9 \times \frac{5}{4} = \frac{45}{4} km = \frac{45}{4} \times 1000 = 11250\ m$

Area of cross-section = $25\ cm^2 = \frac{25}{10000} m^2 = \frac{1}{400} m^2$

Therefore, volume of water $= \frac{1}{400} \times 11250 = 28.125\ m^3$

Dimensions of water tank = 7.5m $\times$ 5m $\times$ 4m

Area of tank = $l \times b = 7.5 \times 5 = 37.5\ m^2$

Let h be the height of water then,

$37.5 \times h = 28.125$

$$h = \frac{28.125}{37.5} = 0.75 \text{ m} = 75 \text{ cm}$$

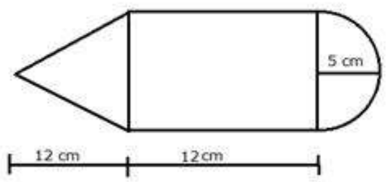
Question 12.

The given figure shows the cross-section of a cone, a cylinder and a hemisphere all with the same diameter 10 cm and the other dimensions are as shown. Calculate:

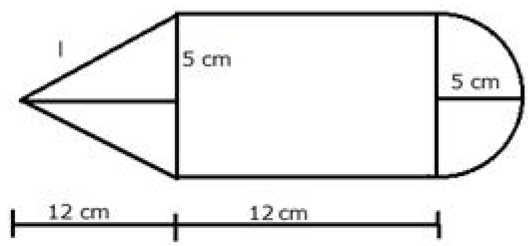
(i) the total surface area

(ii) the total volume of the solid

(iii) the density of the material if its total weight is 1.7 kg



Solution:



Diameter = 10 cm

Therefore, radius (r) = 5 cm

Height of the cone (h) = 12 cm

Height of the cylinder = 12 cm

$$\therefore \ell = \sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13 \text{ cm}$$

(i) Total surface area of the solid

$$= \pi r \ell + 2\pi r h + 2\pi r^2$$
$$= \pi r(\ell + 2h + 2r)$$
$$= \frac{22}{7} \times 5[13 + (2 \times 12) + (2 \times 5)]$$
$$= \frac{110}{7}[13 + 24 + 10]$$

$$= \frac{110}{7} \times 47$$
$$= \frac{5170}{7}$$
$$= 738.57 \text{ cm}^2$$

(ii) Total volume of the solid

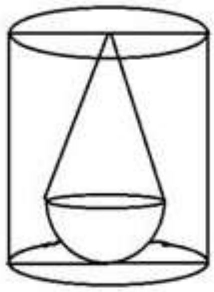
$$= \frac{1}{3}\pi r^2 h + \pi r^2 h + \frac{2}{3}\pi r^3$$
$$= \pi r^2 \left[\frac{1}{3}h + h + \frac{2}{3}r\right]$$
$$= \frac{22}{7} \times 5 \times 5 \left[\frac{1}{3} \times 12 + 12 + \frac{2}{3} \times 5\right]$$
$$= \frac{550}{7}\left[4 + 12 + \frac{10}{3}\right]$$
$$= \frac{550}{7}\left[16 + \frac{10}{3}\right]$$
$$= \frac{550}{7} \times \frac{58}{3}$$
$$= \frac{31900}{21}$$
$$= 1519.0476 \text{ cm}^3$$

(iii) Total weight of the solid = 1.7 kg

$$\therefore \text{Density} = \frac{1.7 \times 1000}{1519.0476} \text{gm / cm}^3 = 1.119 \text{ gm / cm}^3$$
$$\Rightarrow \text{Density} = 1.12 \text{ gm / cm}^3$$

Question 13.
A solid, consisting of a right circular cone, standing on a hemisphere, is placed upright, in a right circular cylinder, full of water, and touches the bottom. Find the volume of water left in the cylinder, having given that the radius of the cylinder is 3 cm and its height is 6 cm; the radius of the hemisphere is 2 cm and the height of the cone is 4 cm. Give your answer to the nearest cubic centimeter.

Solution:



Radius of cylinder = 3 cm

Height of cylinder = 6 cm

Radius of hemisphere = 2 cm

Height of cone = 4 cm

Volume of water in the cylinder when it is full =

$\pi r^2 h = \pi \times 3 \times 3 \times 6 = 54\pi \text{ cm}^3$

Volume of water displaced = volume of cone + volume of hemisphere

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2(h + 2r)$

$= \frac{1}{3}\pi \times 2 \times 2(4 + 2 \times 2)$

$= \frac{1}{3}\pi \times 4 \times 8$

$= \frac{32}{3}\pi \text{ cm}^3$

Therefore, volume of water which is left

$= 54\pi - \frac{32}{3}\pi$

$= \frac{130}{3}\pi \text{ cm}^3$

$= \frac{130}{3} \times \frac{22}{7} \text{ cm}^3$

$= \frac{2860}{21} \text{ cm}^3$

$= 136.19 \text{ cm}^3$

$= 136 \text{ cm}^3$

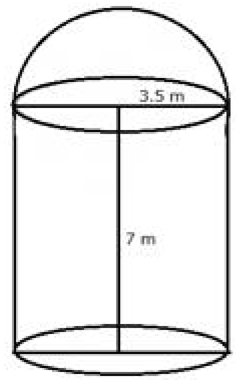
Question 14.

A metal container in the form of a cylinder is surmounted by a hemisphere of the same radius. The internal height of the cylinder is 7 m and the internal radius is 3.5 m. Calculate:

(i) the total area of the internal surface, excluding the base;

(ii) the internal volume of the container in m^3.

Solution:



Radius of the cylinder = 3.5 m

Height = 7 m

(i) Total surface area of container excluding the base = Curved

surface area of the cylinder + area of hemisphere

$$= 2\pi rh + 2\pi r^2$$
$$= \left(2 \times \frac{22}{7} \times 3.5 \times 7\right) + \left(2 \times \frac{22}{7} \times 3.5 \times 3.5\right)$$
$$= 154 + 77 \ m^2$$
$$= 231 \ m^2$$

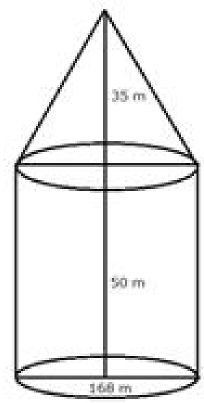
(ii) Volume of the container = $\pi r^2 h + \frac{2}{3}\pi r^3$

$$= \left(\frac{22}{7} \times 3.5 \times 3.5 \times 7\right) + \left(\frac{2}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5\right)$$
$$= \frac{539}{2} + \frac{539}{6}$$
$$= \frac{1617 + 539}{6}$$
$$= \frac{2156}{6}$$
$$= 359.33 \ m^3$$

Question 15.

An exhibition tent is in the form of a cylinder surmounted by a cone. The height of the tent above the ground is 85 m and the height of the cylindrical part is 50 m. If the diameter of the base is 168 m, find the quantity of canvas required to make the tent. Allow 20% extra for fold and for stitching. Give your answer to nearest m^2.

Solution:



Total height of the tent = 85 m

Diameter of the base = 168 m

Therefore, radius (r) = 84 m

Height of the cylindrical part = 50 m

Then height of the conical part = (85 - 50) = 35 m

Slant height (l) $= \sqrt{r^2 + h^2} = \sqrt{84^2 + 35^2} = \sqrt{7056 + 1225} = \sqrt{8281} = 91 \text{ m}$

Total surface area of the tent = $2\pi rh + \pi r\ell = \pi r(2h + \ell)$

$= \frac{22}{7} \times 84(2 \times 50 + 91)$

$= 264(100 + 91)$

$= 264 \times 191$

$= 50424 \text{ m}^2$

Since 20% extra is needed for folds and stitching,

total area of canvas needed

$= 50424 \times \frac{120}{100}$

$= 60508.8$

$= 60509 \text{ m}^2$

Question 16.

A test tube consists of a hemisphere and a cylinder of the same radius. The volume of the water required to fill the whole tube is $\frac{5159}{6}$ cm^3 and $\frac{4235}{6}$ cm^3 of water are required to fill the tube to a level which is 4 cm below the top of the tube. Find the radius of the tube and the length of its cylindrical part.

Solution:

Dividing (i) by (ii)

$$\frac{2r + 3h}{2r + 3h - 12} = \frac{5159}{4235} \text{.......(iii)}$$

Subtracting (ii) from (i)

$$\pi r^2(12) = \frac{5159}{2} - \frac{4235}{2} = \frac{924}{2}$$

$$\Rightarrow 12 \times \frac{22}{7} \times r^2 = \frac{924}{2}$$

$$\Rightarrow r^2 = \frac{924 \times 7}{2 \times 12 \times 22} = \frac{7 \times 7}{2 \times 2}$$

$$\Rightarrow r^2 = \frac{49}{4}$$

$$\Rightarrow r = \frac{7}{2} = 3.5 \text{ cm}$$

Substituting the value of r in (iii)

$$\frac{2 \times \frac{7}{2} + 3h}{2 \times \frac{7}{2} + 3h - 12} = \frac{5159}{4235}$$

$$\Rightarrow \frac{7 + 3h}{7 + 3h - 12} = \frac{5159}{4235}$$

$$\Rightarrow \frac{7 + 3h}{7 + 3h - 12} = \frac{469}{385}$$

$$\Rightarrow 2695 + 1155h = 1407h - 2345$$

$$\Rightarrow 252h = 5040$$

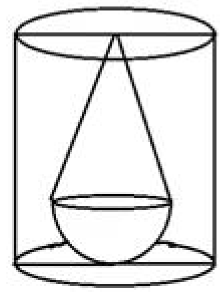
$$\Rightarrow h = 20$$

Hence, Height = 20 cm and radius = 3.5 cm

Question 17.

A solid is in the form of a right circular cone mounted on a hemisphere. The diameter of the base of the cone, which exactly coincides with hemisphere, is 7 cm and its height is 8 cm. The solid is placed in a cylindrical vessel of internal radius 7 cm and height 10 cm. How much water, in cm^3, will be required to fill the vessel completely?

Solution:



Diameter of hemisphere = 7 cm

Diameter of the base of the cone = 7 cm

Therefore, radius (r) = 3.5 cm

Height (h) = 8 cm

Volume of the solid =

$$\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2(h + 2r)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5(8 + 2 \times 3.5)$$

$$= \frac{77}{6}(8 + 7)$$

$$= \frac{385}{2}$$

$$= 192.5 \text{ cm}^3$$

Now, radius of cylindrical vessel (R) = 7 cm

Height (H) = 10 cm

$$\therefore \text{Volume} = \pi R^2 H$$

$$= \frac{22}{7} \times 7 \times 7 \times 10$$

$$= 1540 \text{ cm}^3$$

Volume of water required to fill = 1540 - 192.5 = 1347.5 cm^3

Question 18.

Two solid spheres of radii 2 cm and 4 cm are melted and recast into a cone of height 8 cm. Find the radius of the cone so formed.

Solution:

We have,

Volume of the cone = Sum of volumes of the two melted spheres

$\Rightarrow \frac{1}{3}\pi(r)^2 \times 8 = \frac{4}{3}\pi \times (2)^3 + \frac{4}{3}\pi \times (4)^3$

$\Rightarrow 8r^2 = 4 \times 8 + 4 \times 64$

$\Rightarrow 8r^2 = 32 + 256$

$\Rightarrow 8r^2 = 288$

$\Rightarrow r^2 = 36$

$\Rightarrow r = 6$

Thus, the radius of the cone so formed is 6 cm.

Question 19.

A certain number of metallic cones, each of radius 2 cm and height 3 cm, are melted and recast into a solid sphere of radius 6 cm. Find the number of cones used.

Solution:

Let the number of cones melted be n.

Let the radius of sphere be r_s = 6 cm

Radius of cone be r_c = 2 cm

And, height of the cone be h = 3 cm

Volume of sphere = n (Volume of a metallic cone)

$\Rightarrow \frac{4}{3}\pi r_s^3 = n\left(\frac{1}{3}\pi r_c^2 h\right)$

$\Rightarrow \frac{4}{\cancel{3}}\cancel{\pi} r_s^3 = n\left(\frac{1}{\cancel{3}}\cancel{\pi} r_c^2 h\right)$

$\Rightarrow \frac{4r_s^3}{r_c^2 h} = n$

$\Rightarrow n = \frac{4(6)^3}{(2)^2(3)}$

$\Rightarrow n = \frac{\cancel{4} \times 216}{\cancel{4} \times 3}$

$\Rightarrow n = 72$

Hence, the number of cones is 72.

Question 20.

A conical tent is to accommodate 77 persons. Each person must have $16m^3$ of air to breathe. Given the radius of the tent as 7m, find the height of the tent and also its curved surface area.

Solution:

According to the condition in the question,

$$77 \times 16 = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow 77 \times 16 = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times h$$

$$\Rightarrow h = \frac{77 \times 16 \times 3 \times 7}{22 \times 7 \times 7}$$

$$\Rightarrow h = \frac{11 \times 16 \times 3}{22}$$

$$\Rightarrow h = 24 \text{ m}$$

We know that,

$l^2 = r^2 + h^2$

$\Rightarrow l^2 = (7)^2 + (24)^2$

$\Rightarrow l^2 = 49 + 576$

$\Rightarrow l^2 = 625$

$\Rightarrow l = 25$ m

$\therefore$ Curved Surface Area = $\pi rl = \frac{22}{7} \times 7 \times 25 = 550 m^2$

Therefore the height of the tent is 24m and it curved surface area is $550m^2$.