NDA/NA

National Defence Academy/Naval Academy

SOLVED PAPER 2021 (II)

PAPER I : Mathematics

1. If $x^2 + x + 1 = 0$, then what is the value of $x^{199} + x^{200} + x^{201}$?

(a) -1 (b) 0
(c) 1   (d) 3

⊙ (**b**) Given that,

$$x^2 + x + 1 = 0 \qquad \text{...(i)}$$
$$\therefore x^{199} + x^{200} + x^{201} = x^{199}(1 + x + x^2)$$
$$= x^{199} \times 0$$
$$= 0$$

2. If x, y, z are in GP, then which of the following is/are correct?

1. $\ln(3x), \ln(3y), \ln(3z)$ are in AP.
2. $xyz + \ln(x), xyz + \ln(y),$
   $xyz + \ln(z)$ are in HP.

Select the correct answer using the code given below.
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

⊙ (**a**) Given that x, y, z are in GP.
$$\Rightarrow \qquad y^2 = xz \qquad \text{...(i)}$$

(1) If $\log(3x), \log(3y), \log(3z)$ are in AP

Then, $\quad 2\log(3y) = \log(3x) + \log(3z)$
$$9y^2 = (9xz)$$
$$9y^2 = (9xz)$$
$$y^2 = xz$$

Hence, statement (1) is correct.
Hence, we can say if x, y, z are in GP.
$\therefore \log x, \log y, \log z$ are in AP.
$\Rightarrow xyz + \log x, xyz + \log y, xyz + \log z$ are in AP.
Hence, Statement (2) is wrong.
$\therefore$ Option (a) is correct.

3. If $\log_{10} 2, \log_{10}(2^x - 1), \log_{10}(2^x + 3)$ are in AP, then what is x equal to?

(a) 0   (b) 1
(c) $\log_2 5$ (d) $\log_5 2$

⊙ (**c**) Given that, $\log_{10} 2, \log_{10}(2^x - 1),$ $\log_{10}(2^x + 3)$ are in AP.

$\therefore 2\log_{10}(2^x - 1) = \log_{10} 2 + \log_{10}(2^x + 3)$
$\log_{10}(2^x - 1)^2 = \log_{10} 2(2^x + 3)$
$\Rightarrow \quad 2^{2x} + 1 - 2 \cdot 2^x = 2 \cdot 2^x + 6$
$\Rightarrow (2^x)^2 - 4(2^x) - 5 = 0$
Let $\qquad\qquad 2^x = y$
$\Rightarrow \qquad y^2 - 4y - 5 = 0$
$\qquad (y - 5)(y + 1) = 0$
$\Rightarrow \qquad\qquad y = 5 \text{ or } y = -1$
(Ignore because 2^x cannot be negative)
$\Rightarrow \quad y = 5 \Rightarrow 2^x = 5$
$$x = \log_2 5$$
Hence, option (c) is correct.

4. Let $S = \{2, 3, 4, 5, 6, 7, 9\}$. How many different 3-digit numbers (with all digits different) from S can be made which are less than 500?

(a) 30   (b) 49
(c) 90   (d) 147

⊙ (**c**) Let $\quad S = \{2, 3, 4, 5, 6, 7, 9\}$
$\Rightarrow \qquad n(S) = 7$
Three digit number less than

$500 = $ ☐ ☐ ☐
         3   6   5

$= 3 \times 6 \times 5 = 90$
$\therefore$ Option (c) is correct.

Note Hundreds digit can be filled with 3 choices that are 2, 3, 4.
Similarly, tens digit can be filled with 6 ways and unit digit can be filled with 5 ways.

5. If $p = (1111 \ldots \text{up to } n \text{ digits})$, then what is the value of $9p^2 + p$?

(a) $10^n p$ (b) $2p \cdot 10^n$
(c) $10^n p - 1$ (d) $10^n p + 1$

⊙ (**a**) Given that,
$$p = (1111\ldots \text{upto } n \text{ digits})$$
$$= 1 + 10 + 10^2 + \ldots + 10^{n-1}$$
$$= \frac{1(10^n - 1)}{10 - 1}$$

$$\left[\because a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}\right]$$

$$\Rightarrow \qquad p = \frac{10^n - 1}{9}$$
$$\Rightarrow \qquad 9p = 10^n - 1$$
$$\Rightarrow \qquad 9p + 1 = 10^n$$
$$\Rightarrow \qquad 9p^2 + p = 10^n \cdot p$$
$\therefore$ Hence, option (a) is correct.

6. The quadratic equation $3x^2 - (k^2 + 5k)x + 3k^2 - 5k = 0$ has real roots of equal magnitude and opposite sign. Which one of the following is correct?

(a) $0 < k < \dfrac{5}{3}$

(b) $0 < k < \dfrac{3}{5}$ only

(c) $\dfrac{3}{5} < k < \dfrac{5}{3}$

(d) No such value of k exists.

⊙ (**d**) Since, we know that if a quadratic equation $ax^2 + bx + c = 0$ has real roots of equal magnitude and opposite sign.
Then, $\qquad\qquad b = 0 \qquad \text{...(i)}$
and product of roots $< 0 \qquad \text{...(ii)}$

In the given quadratic equation,
$3x^2 - (k^2 - 5k)x + 3k^2 - 5k = 0$
$a = 3, b = -(k^2 + 5k), c = 3k^2 - 5k$
By Eq. (i), $b = 0$
$\Rightarrow$ $-(k^2 + 5k) = 0$
$\Rightarrow$ $k(k + 5) = 0$
$\therefore$ $k = 0, -5$
By Eq. (ii), Product of roots < 0
$$\frac{c}{a} < 0$$
$\Rightarrow$ $\dfrac{3k^2 - 5k}{3} < 0$
$\Rightarrow$ $k(3k - 5) < 0$
$\therefore$ $0 < k < \dfrac{5}{3}$

From (i) and (ii) no such values of k exists.
Hence, option (d) is correct.

7. If $a_n = n(n!)$, then what is
$a_1 + a_2 + a_3 + \ldots + a_{10}$ equal to?
(a) $10! - 1$ (b) $11! + 1$
(c) $10! + 1$ (d) $11! - 1$

⟩ (**d**) Given, $a_n = n(n!)$
$= (n + 1 - 1)(n!)$
$= (n + 1)n! - n!$
$= (n + 1)! - n!$
$\therefore$ $a_1 = 2! - 1!$
$a_2 = 3! - 2!$
... ...
$a_{10} = 11! - 10!$
$\therefore a_1 + a_2 + a_3 + \ldots + a_{10}$
$= 2! + 1! + 3! - 2! + 4! - 3! + \ldots + 11! - 10!$
$= 11! - 1!$
$= 11! - 1$
$\therefore$ Option (d) is correct.

8. If p and q are the non-zero roots of the equation $x^2 + px + q = 0$, then how many possible values can q have?
(a) Nil   (b) One
(c) Two   (d) Three

⟩ (**b**) Given quadratic equation
$$x^2 + px + q = 0$$
and roots are p and q (non zero)
$\therefore$ Sum of roots $= \dfrac{-\text{coefficient of } x}{\text{coefficient of } x^2}$
$p + q = -p$ ...(i)
$\therefore$ Product of roots $= \dfrac{\text{constant term}}{\text{coefficient of } x^2}$
$pq = q$ ...(ii)
$\Rightarrow$ $pq - q = 0$
$q(p - 1) = 0$
$\because$ $q \neq 0 \Rightarrow p - 1 = 0$
$p = 1$
From Eq. (i)
$p + q = -p$

$q = -2p = -2(1)$
$q = -2$
$\therefore$ Option (b) is correct.

9. If $\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ then what is
$\begin{vmatrix} 3d + 5g & 4a + 7g & 6g \\ 3e + 5h & 4b + 7h & 6h \\ 3f + 5i & 4c + 7i & 6i \end{vmatrix}$ equal to?
(a) Δ (b) 7Δ
(c) 72Δ (d) -72Δ

⟩ (**d**) Given, $\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

$= \begin{vmatrix} 3d + 5g & 4a + 7g & 6g \\ 3e + 5h & 4b + 7h & 6h \\ 3f + 5i & 4c + 7i & 6i \end{vmatrix}$

$= 6\begin{vmatrix} 3d + 5g & 4a + 7g & g \\ 3e + 5h & 4b + 7h & h \\ 3f + 5i & 4c + 7i & i \end{vmatrix}$

By $C_1 \to C_1 - 5C_3, C_2 \to C_2 - 7C_3$

$= 6\begin{vmatrix} 3d & 4a & g \\ 3e & 4b & h \\ 3f & 4c & i \end{vmatrix}$

$= 6 \times 3 \times 4 \begin{vmatrix} d & a & g \\ e & b & h \\ f & c & i \end{vmatrix}$ By $C_2 \leftrightarrow C_1$

$= -72\begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix}$ By $R \leftrightarrow C$

$= -72\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -72\Delta$

Hence, option (d) is correct.

10. If $\dfrac{1}{b+c}, \dfrac{1}{c+a}, \dfrac{1}{a+b}$ are in HP, then which of the following is/are correct?
1. a, b, c are in AP
2. $(b+c)^2, (c+a)^2, (a+b)^2$ are in GP.
Select the correct answer using the code given below.
(a) 1 only   (b) 2 only
(c) Both 1 and 2   (d) Neither 1 nor 2

⟩ (**a**) Given that,
$\dfrac{1}{b+c}, \dfrac{1}{c+a}, \dfrac{1}{a+b}$ are in HP.
$\Rightarrow b + c, c + a, a + b$ are in AP.
$\Rightarrow (a + b + c) - (b + c), (a + b + c)$
$- (c + a), (a + b + c) - (a + b)$ are in AP.
$\Rightarrow a, b, c$ are in AP.

2. From 1; a, b, c are in AP.
$\therefore$ $b = a + d, c = a + 2d$
where, d is common difference.
$\therefore$ $(b + c)^2 = (a + d + a + 2d)^2$
$= (2a + 3d)^2$
$(c + a)^4 = (a + 2d + a)^4 = (2a + 2d)^4$
$(a + b)^2 = (a + a + d)^2 = (2a + d)^2$
Here, $(c + a)^4 = (b + c)^2 \cdot (a + b)^2$
So, $(b+c)^2, (c+a)^2, (a+b)^2$ are not in G.P.
Hence, option (a) is correct.

11. If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$, where $a \in \mathbb{N}$, then what is $A^{100} - A^{50} - 2A^{25}$ equal to?
(a) $-2I$ (b) $-I$
(c) $2I$ (d) I
where I is the identity matrix.

⟩ (**a**) $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} a \in \mathbb{N}$

The sequence for given matrix A is
$$A^n = \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$$
$\therefore A^{100} - A^{50} - 2A^{25} = \begin{bmatrix} 1 & 100a \\ 0 & 1 \end{bmatrix}$
$- \begin{bmatrix} 1 & 50a \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 25a \\ 0 & 1 \end{bmatrix}$
$= \begin{bmatrix} 1 - 1 - 2 & 100a - 50a - 50a \\ 0 - 0 - 0 & 1 - 1 - 2 \end{bmatrix}$
$= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} = -2I$

Hence, option (a) is correct.

12. If $\begin{vmatrix} a & -b & a-b-c \\ -a & b & -a+b-c \\ -a & -b & -a-b+c \end{vmatrix} - kabc = 0$
$(a \neq 0, b \neq 0, c \neq 0)$
then what is the value of k?
(a) -4 (b) -2
(c) 2 (d) 4

⟩ (**a**) Given that,
$\begin{vmatrix} a & -b & a-b-c \\ -a & b & -a+b-c \\ -a & -b & -a-b+c \end{vmatrix} - kabc = 0$
$(a \neq 0, b \neq 0, c \neq 0)$
$R_1 \to R_1 + R_2$
$\begin{vmatrix} 0 & 0 & -2c \\ -a & b & -a+b-c \\ -a & -b & -a-b+c \end{vmatrix} - kabc = 0$
$-2c[(-a)(-b) - (-a)b] - kabc = 0$
$-2c(2ab) - kabc = 0$
$- kabc = 4abc$
$\Rightarrow$ $k = -4$
Hence, option (a) is correct.

13. What is $\displaystyle\sum_{n=1}^{8n+7} i^n$ equal to,

where $i = \sqrt{-1}$?

(a) -1　　　　(b) 1
(c) i　　　　(d) $-i$

⊙ (**a**) Let $S = \displaystyle\sum_{n=1}^{8n+7} i^n$

$S = i + i^2 + i^3 + \ldots + i^{8n+7}$

$= i\left[\dfrac{(i)^{8n+7} - 1}{i - 1}\right] = i\left[\dfrac{i^{4(2n+1)+3} - 1}{i - 1}\right]$

$= i\left(\dfrac{i^3 - 1}{i - 1}\right)$　　　$[\because i^{4n+r} = i^r]$

$= i\left[\dfrac{-i - 1}{i - 1}\right] = \dfrac{-i^2 - i}{i - 1}$

$= \dfrac{1 - i}{i - 1} = -1$

14. If $z = x + iy$, where $i = \sqrt{-1}$, then
what does the equation
$z\bar{z} + |z|^2 + 4(z + \bar{z}) - 48 = 0$
represent?
(a) Straight line
(b) Parabola
(c) Circle
(d) Pair of straight lines

⊙ (**c**) Given,　　$z = x + iy$

$\therefore$　　　$\bar{z} = x - iy$

$\therefore$　　　$z + \bar{z} = 2x$

and　　$|z|^2 = x^2 + y^2$

$\therefore$　　$z\bar{z} + |z|^2 + 4(z + \bar{z}) - 48 = 0$

$(x + iy)(x - iy) + x^2 + y^2 + 4(2x) - 48 = 0$

$x^2 + y^2 + x^2 + y^2 + 8x - 48 = 0$

$2x^2 + 2y^2 + 8x - 48 = 0$

$x^2 + y^2 + 4x - 24 = 0$

which represents circle.

Hence, option (c) is correct.

15. Which one of the following is a
square root of $2a + 2\sqrt{a^2 + b^2}$,
where $a, b \in \mathbb{R}$?
(a) $\sqrt{a + ib} + \sqrt{a - ib}$
(b) $\sqrt{a + ib} - \sqrt{a - ib}$
(c) $2a + ib$
(d) $2a - ib$, where $i = \sqrt{-1}$

⊙ (**a**) $2a + 2\sqrt{a^2 + b^2}$

$= 2a + ib - ib + 2\sqrt{a^2 - i^2 b^2}$

$= (a + ib) + (a - ib)$
$\qquad + 2\sqrt{(a + ib)(a - ib)}$

$= (\sqrt{a + ib} + \sqrt{a - ib})^2$

Hence, square root of

$2a + 2\sqrt{a^2 + b^2} = \sqrt{a + ib} + \sqrt{a - ib}$

16. If $\sin\theta$ and $\cos\theta$ are the roots of the
equation $ax^2 + bx + c = 0$, then
which one of the following is
correct?
(a) $a^2 + b^2 - 2ac = 0$
(b) $-a^2 + b^2 + 2ac = 0$
(c) $a^2 - b^2 + 2ac = 0$
(d) $a^2 + b^2 + 2ac = 0$

⊙ (**c**) Given, equation $ax^2 + bx + c = 0$ ...(i)

$\because$ Roots are $\sin\theta$ and $\cos\theta$

$\therefore$　　$\sin\theta + \cos\theta = -\dfrac{b}{a}$

and　　$\sin\theta \cdot \cos\theta = \dfrac{c}{d}$

On squaring both sides, we get

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \dfrac{b^2}{a^2}$

$1 + 2\dfrac{c}{a} = \dfrac{b^2}{a^2}$

$a(a + 2c) = b^2$

$\Rightarrow$　$a^2 - b^2 + 2ac = 0$

$\therefore$ Option (c) is correct.

17. If $C(n, 4), C(n, 5)$ and $C(n, 6)$ are in
AP, then what is the value of n?
(a) 7　　(b) 8　　(c) 9　　(d) 10

⊙ (**a**) $^nC_4 \cdot {}^nC_5$ and $^nC_6$ are in AP.

$\Rightarrow 2 \cdot {}^nC_5 = {}^nC_4 + {}^nC_6$

$\dfrac{2n!}{5!(n-5)!} = \dfrac{n!}{4!(n-4)!} + \dfrac{n!}{6!\,(n-6)!}$

$\dfrac{2(n!)}{5(4!)(n-5)\,(n-6)!}$

$= \dfrac{n!}{4!(n-6)!}\left[\dfrac{1}{(n-4)\,(n-5)} + \dfrac{1}{6\times 5}\right]$

$\dfrac{2}{5(n-5)} = \dfrac{1}{(n-4)\,(n-5)} + \dfrac{1}{30}$

$\dfrac{2n-8-5}{5(n^2 - 9n + 20)} = \dfrac{1}{30}$

$30(2n - 13) = 5n^2 - 45n + 100$

$5n^2 - 105n + 490 = 0$

$n^2 - 21n + 98 = 0$

$(n - 14)\,(n - 7) = 0$

$n = 14$ or $n = 7$

$\therefore$ Option (a) is correct.

18. How many 4-letter words (with or
without meaning) containing two
vowels can be constructed using
only the letters (without repetition)
of the word 'LUCKNOW'?
(a) 240　　　　(b) 200
(c) 150　　　　(d) 120

⊙ (**a**) In LUCKNOW, there are 2 vowels
and 5 consonants.

$\therefore$ 4 letter words $= {}^5C_2 \cdot {}^2C_2 \cdot 4!$

$= 10 \times 1 \times 24 = 240$

$\therefore$ Option (a) is correct.

19. Suppose 20 distinct points are
placed randomly on a circle. Which
of the following statements is/are
correct?
  1. The number of straight lines that
     can be drawn by joining any two
     of these points is 380.
  2. The number of triangles that can
     be drawn by joining any three of
     these points is 1140.
Select the correct answer using the
code given below.
(a) 1 only　　　　(b) 2 only
(c) Both 1 and 2　　(d) Neither 1 nor 2

⊙ (**b**) Given, that there are 20 distinct
points on a circle and we have to draw a
straight line by joining any two of these
points.

Hence, number of straight lines

$= {}^{20}C_2 = \dfrac{20 \times 19}{2} = 190$

$\therefore$ Statement (1) is wrong.

and Number of triangle

$= {}^{20}C_3 = \dfrac{20 \times 19 \times 18}{1 \times 2 \times 3} = 1140$

$\therefore$ Statement (2) is correct.

Hence, option (b) is correct.

20. How many terms are there in the

expansion of $\left(\dfrac{a^2}{b^2} + \dfrac{b^2}{a^2} + 2\right)^{21}$

where $a \neq 0, b \neq 0$?
(a) 21　　　　(b) 22
(c) 42　　　　(d) 43

⊙ (**d**) $\left(\dfrac{a^2}{b^2} + \dfrac{b^2}{a^2} + 2\right)^{21}$

$\Rightarrow \left[\left(\dfrac{a}{b} + \dfrac{b}{a}\right)^2\right]^{21} = \left(\dfrac{a}{b} + \dfrac{b}{a}\right)^{42}$

Since, we know that number of terms in
the expansion of $(a + b)^n = n + 1$
Hence, total number of terms
$\qquad\qquad = 42 + 1 = 43$

$\therefore$ Option (d) is correct.

21. For what values of k is the system
of equations $2k^2 x + 3y - 1 = 0$,
$7x - 2y + 3 = 0, 6kx + y + 1 = 0$
consistent?

(a) $\dfrac{3 \pm \sqrt{11}}{10}$　　(b) $\dfrac{21 \pm \sqrt{161}}{10}$

(c) $\dfrac{3 \pm \sqrt{7}}{10}$　　(d) $\dfrac{4 \pm \sqrt{11}}{10}$

⊙ (**b**) Given equations,

$2k^2 x + 3y - 1 = 0$

$7x - 2y + 3 = 0$

$6kx + y + 1 = 0$

For consistency, determinant formed by the equations

$$\begin{vmatrix} 2k^2 & 3 & -1 \\ 7 & -2 & 3 \\ 6k & 1 & 1 \end{vmatrix} = 0$$

$2k^2(-2-3) - 3(7-18k)$
$\qquad -1(7+12k) = 0$
$-10k^2 - 21 + 54k - 7 - 12k = 0$
$-10k^2 - 42k - 28 = 0$
$5k^2 - 21k + 14 = 0$
$k = \dfrac{21 \pm \sqrt{441 - 280}}{10}$
$k = \dfrac{21 \pm \sqrt{161}}{10}$

Hence, option (b) is correct.

22. The inverse of a matrix A is given by $\begin{bmatrix} -2 & 1 \\ 3 & -\dfrac{1}{2} \\ 2 & -2 \end{bmatrix}$

What is A equal to?

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$

⊙ (**a**) $A = \begin{bmatrix} -2 & 1 \\ 3 & -\dfrac{1}{2} \\ 2 & -2 \end{bmatrix}$

$\therefore \ |A| = (-2)\left(-\dfrac{1}{2}\right) - \dfrac{3}{2} = -\dfrac{1}{2} \neq 0$

$A_{11} = -\dfrac{1}{2}, A_{12} = -\dfrac{3}{2}$

$A_{21} = -1, A_{22} = -2$

$\therefore \text{adj } A = \begin{bmatrix} -\dfrac{1}{2} & -1 \\ -\dfrac{3}{2} & -2 \end{bmatrix}$

$\therefore A^{-1} = \dfrac{1}{|A|} \text{adj } A = \dfrac{1}{-\dfrac{1}{2}} \begin{bmatrix} -\dfrac{1}{2} & -1 \\ -\dfrac{3}{2} & -2 \end{bmatrix}$

$= 2 \begin{bmatrix} \dfrac{1}{2} & 1 \\ \dfrac{3}{2} & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Hence, option (a) is correct.

23. What is the period of the function $f(x) = \ln(2 + \sin^2 x)$?

(a) $\dfrac{\pi}{2}$ (b) π (c) 2π (d) 3π

⊙ (**b**) $f(x) = \ln(2 + \sin^2 x)$

$\because$ Period of $\sin^2 x$ is π.
and $f(\pi + x) = \log\{2 + \sin^2(\pi + x)\}$
$\qquad = \log\{2 + \sin^2 x\}$
$\qquad = f(x)$

Hence, period of $\ln(2 + \sin^2 x) = \pi$
$\therefore$ Option (b) is correct.

24. If $\sin(A + B) = 1$ and $2\sin(A - B) = 1$, where $0 < A, B < \dfrac{\pi}{2}$, then what is $\tan A : \tan B$ equal to?

(a) 1 : 2 (b) 2 : 1
(c) 1 : 3 (d) 3 : 1

⊙ (**d**) Given, $\sin(A + B) = 1$
and $\quad 2\sin(A - B) = 1$
$\qquad 0 < A, B < \dfrac{\pi}{2}$

$\because \qquad \sin(A + B) = 1 = \sin\dfrac{\pi}{2}$

$\Rightarrow \qquad A + B = \dfrac{\pi}{2}$ ...(i)

$2\sin(A - B) = 1$

$\Rightarrow \quad \sin(A - B) = \dfrac{1}{2} = \sin\dfrac{\pi}{6}$

$\Rightarrow \qquad A - B = \dfrac{\pi}{6}$ ...(ii)

Now, adding Eq. (i) and Eq. (ii), we get

$2A = \dfrac{\pi}{2} + \dfrac{\pi}{6} = \dfrac{4\pi}{6}$

$A = \dfrac{\pi}{3}, B = \dfrac{\pi}{6}$

$\therefore \ \tan A : \tan B = \tan\dfrac{\pi}{3} : \tan\dfrac{\pi}{6}$

$= \sqrt{3} : \dfrac{1}{\sqrt{3}} = 3 : 1$

$\therefore$ Option (d) is correct.

25. Consider a regular polygon with 10 sides. What is the number of triangles that can be formed by joining the vertices which have no common side with any of the sides of the polygon?

(a) 25 (b) 50
(c) 75 (d) 100

⊙ (**b**) Given number of sides $(n) = 10$

Number of triangles which have no common side with any of the sides of the polygon $= \dfrac{n(n-4)(n-5)}{3!}$

$\therefore$ Number of triangles

$= \dfrac{10(10-4)(10-5)}{6}$

$= \dfrac{10 \times 6 \times 5}{6}$

$= 50$

Hence, option (b) is correct.

26. Consider all the real roots of the equation $x^4 - 10x^2 + 9 = 0$.

What is the sum of the absolute values of the roots?

(a) 4 (b) 6
(c) 8 (d) 10

⊙ (**c**) Given equation,

$x^4 - 10x^2 + 9 = 0$
Let $y = x^2$
$\therefore \qquad y^2 - 10y + 9 = 0$
$(y - 9)(y - 1) = 0$
$\Rightarrow \qquad y = 9 \quad \text{or} \quad y = 1$
$\qquad x^2 = 9 \quad \text{or} \quad y = 1$
$\qquad x^2 = 9 \quad \text{or} \quad x^2 = 1$
$\qquad x = \pm 3, x = \pm 1$
$\therefore \quad \text{Sum} = |3| + |-3| + |1| + |-1|$
$\qquad = 8$

Hence, option (c) is correct.

27. Consider the expansion of $(1 + x)^n$. Let p, q, r and s be the coefficients of first, second, nth and $(n + 1)$th terms respectively. What is $(ps + qr)$ equal to?

(a) $1 + 2n$ (b) $1 + 2n^2$
(c) $1 + n^2$ (d) $1 + 4n$

⊙ (**c**) Given, $(1 + x)^n$

In the above expansion, $(r + 1)$th term

$T_{r+1} = {}^nC_r x^r$

$\therefore \qquad T_1 = {}^nC_0 x^0, T_2 = {}^nC_1 x'$

$\qquad p = 1$ ...(i)

$\therefore \qquad {}^nC_1 = q$

$\qquad n = q$ ...(ii)

$T_n = {}^nC_{n-1} x^{n-1},$
$T_{n+1} = {}^nC_n x^n$

$\therefore \qquad r = n$ ...(iii)

$\therefore \qquad s = 1$ ...(iv)

$\therefore \quad (ps + qr) = 1 \cdot 1 + n \cdot n = 1 + n^2$

$\qquad = (1 + n^2)$

Hence, option (c) is correct.

28. Let $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \dfrac{3\pi}{2}$ for $0 \le x, y, z \le 1$. What is the value of $x^{1000} + y^{1001} + z^{1002}$?

(a) 0 (b) 1
(c) 3 (d) 6

⊙ (**c**) Let $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \dfrac{3\pi}{2}$

Which is only possible when,

$\sin^{-1} x = \dfrac{\pi}{2}, \ \sin^{-1} y = \dfrac{\pi}{2},$

$\sin^{-1} z = \dfrac{\pi}{2}$

$\Rightarrow \quad x = 1, y = 1, z = 1$

$\therefore \ x^{1000} + y^{1001} + z^{1002}$

$\qquad = 1 + 1 + 1 = 3$

Hence, option (c) is correct.

29. Let $\sin x + \sin y = \cos x + \cos y$ for all $x, y \in \mathbb{R}$. What is $\tan\left(\dfrac{x}{2} + \dfrac{y}{2}\right)$ equal to?

(a) 1 (b) 2
(c) $\sqrt{2}$ (d) $2\sqrt{2}$

⊙ **(a)** Given that,

$\sin x + \sin y = \cos x + \cos y \ \forall \ x \in \mathbb{R}$

$2\sin\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)$

$= 2\cos\left(\dfrac{x+y}{2}\right)\cdot\cos\left(\dfrac{x-y}{2}\right)$

$\Rightarrow \dfrac{2\sin\left(\dfrac{x+y}{2}\right)\cdot\cos\left(\dfrac{x-y}{2}\right)}{2\cos\left(\dfrac{x+y}{2}\right)\cdot\cos\left(\dfrac{x-y}{2}\right)} = 1$

$\tan\left(\dfrac{x+y}{2}\right) = 1 \ $ or $\ \tan\left(\dfrac{x}{2} + \dfrac{y}{2}\right) = 1$

Hence, option (a) is correct.

30. Let $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

and $(mI + nA)^2 = A$, where m, n are positive real numbers and I is the identity matrix. What is $(m + n)$ equal to?

(a) 0 (b) $\dfrac{1}{2}$

(c) 1 (d) $\dfrac{3}{2}$

⊙ **(d)** Let $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ and $(mI + nA)^2 = A$

where, I is Identify matrix

$\because mI + nA = m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + n\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

$= \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} + \begin{bmatrix} 0 & 2n \\ -2n & 0 \end{bmatrix}$

$= \begin{bmatrix} m & 2n \\ -2n & m \end{bmatrix}$

$\therefore (mI + nA)^2 = A$

$\begin{bmatrix} m & 2n \\ -2n & m \end{bmatrix}\begin{bmatrix} m & 2n \\ -2n & m \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

$\begin{bmatrix} m^2 - 4n^2 & 4mn \\ -4mn & m^2 - 4n^2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

$\Rightarrow \ 4mn = 2 \ $ and $\ m^2 - 4n^2 = 0$

$mn = \dfrac{1}{2} \ $ and $\ m = \pm\,2n$

When, $m = 2n$

$(2n)(n) = \dfrac{1}{2}$

$n = \pm\dfrac{1}{2} \Rightarrow m = \pm 1$

$\therefore \ m + n = 1 + \dfrac{1}{2} = \dfrac{3}{2}$

Hence, option (d) is correct.

31. What is the value of the following?

$\cot\left[\sin^{-1}\left(\dfrac{3}{5}\right) + \cot^{-1}\left(\dfrac{3}{2}\right)\right]$

(a) $\dfrac{6}{17}$ (b) $\dfrac{7}{16}$

(c) $\dfrac{16}{7}$ (d) $\dfrac{17}{6}$

⊙ **(a)** $\cot\left[\sin^{-1}\dfrac{3}{5} + \cot^{-1}\dfrac{3}{2}\right]$

$\Rightarrow \cot\left[\cot^{-1}\left(\dfrac{\sqrt{1 - \left(\dfrac{3}{5}\right)^2}}{\dfrac{3}{5}}\right) + \cot^{-1}\dfrac{3}{2}\right]$

$[\because \sin^{-1}x = \cot^{-1}\left(\dfrac{\sqrt{1 - x^2}}{x}\right)$

and $\cot x + \cot^{-1}y = \cot^{-1}\left(\dfrac{xy - 1}{x + y}\right)]$

$\Rightarrow \cot\left[\cot^{-1}\left(\dfrac{4}{3}\right) + \cot^{-1}\left(\dfrac{3}{2}\right)\right]$

$\Rightarrow \cot\left[\cot^{-1}\left(\dfrac{\dfrac{4}{3}\times\dfrac{3}{2} - 1}{\dfrac{4}{3} + \dfrac{3}{2}}\right)\right]$

$\Rightarrow \dfrac{\dfrac{1}{17}}{6} = \dfrac{6}{17}$

Hence, option (a) is correct.

32. Let $4\sin^2 x = 3$, where $0 \le x \le \pi$. What is $\tan 3x$ equal to?

(a) -2 (b) -1
(c) 0 (d) 1

⊙ **(c)** Given that $4\sin^2 x = 3, 0 \le x \le \pi$

$\therefore \quad \sin^2 x = \dfrac{3}{4}$

$\Rightarrow \quad \sin x = \dfrac{\sqrt{3}}{2} = \sin\dfrac{\pi}{3} \text{ or } \sin\dfrac{2\pi}{3}$

$\therefore \quad x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

$\therefore \quad \tan 3x = \tan\dfrac{3\pi}{3}$

$= \tan\pi = 0$

Also $\quad \tan 3x = \tan 3\left(\dfrac{2\pi}{3}\right)$

$= \tan 2\pi = 0$

$\therefore$ Option (c) is correct.

33. Let p, q and 3 be respectively the first, third and fifth terms of an AP. Let d be the common difference. If the product (pq) is minimum, then what is the value of d?

(a) 1 (b) $\dfrac{3}{8}$

(c) $\dfrac{9}{8}$ (d) $\dfrac{9}{4}$

⊙ **(c)** Given that first term of AP $= p$

$\Rightarrow \quad a = p \quad \quad\quad$ ...(i)

Where, a denotes first term.

and $\quad a_3 = q, a_5 = 3$

$\Rightarrow \quad a + 2d = q \quad\quad$ ...(ii)

$\quad\quad a + 4d = 3 \quad\quad$ ...(iii)

$\therefore \quad pq = a(a + 2d)$

$= (3 - 4d)(3 - 4d + 2d)$

$= (3 - 4d)(3 - 2d)$

$= 9 - 18d + 8d^2$

Let $\quad f = 9 - 18d + 8d^2$

$f' = 0 - 18 + 16d$

$= -18 + 16d$

For maxima and minima

$f' = 0$

$\Rightarrow \ -18 + 16d = 0$

$\Rightarrow \quad d = \dfrac{18}{16} = \dfrac{9}{8}.$

Now, $\quad f'' = 16 \text{ (Positive)}$

So, f will be maximum at $d = \dfrac{9}{8}.$

Hence, option (c) is correct.

34. Consider the following statements in respect of the roots of the equation $x^3 - 8 = 0$

1. The roots are non-collinear.
2. The roots lie on a circle of unit radius.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

⊙ **(a)** $\quad\quad x^3 - 8 = 0$

$\Rightarrow (x - 2)(x^2 + 2x + 4) = 0$

$\quad\quad x = 2, 2\omega, 2\omega^2$

Where, $\omega = \dfrac{-1 + \sqrt{3}i}{2}$

Hence, roots are non-collinear and will lie on a circle of 2 unit radius.

Hence, option (a) is correct.

35. Let the equation $\sec x \cdot \csc x = p$ have a solution, where p is a positive real number. What should be the smallest value of p?

(a) $\dfrac{1}{2}$ (b) 1

(c) 2
(d) Minimum does not exist

⊙ **(c)** $\quad \sec x \cdot \csc x = p$

$\Rightarrow \quad \dfrac{1}{\sin x \cdot \cos x} = p$

$\Rightarrow \quad \dfrac{2}{2\sin x \cos x} = p$

$\quad\quad \dfrac{2}{\sin 2x} = p$

Where, $\sin 2x \in [-1, 1]$

If $\sin 2x = 1$

Then $p = 2$ will be the smallest value.
Hence, option (c) is correct.

36. For what value of θ, where

$0 < \theta < \dfrac{\pi}{2}$, does $\sin\theta + \sin\theta\cos\theta$

attain maximum value?

(a) $\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{3}$

(c) $\dfrac{\pi}{4}$ (d) $\dfrac{\pi}{6}$

⊙ (**b**) Let $P = \sin\theta + \sin\theta \cdot \cos\theta$

∴ $\dfrac{dP}{d\theta} = \cos\theta + \cos^2\theta - \sin^2\theta$

For maxima-minima $= \dfrac{dP}{d\theta} = 0$

$\cos\theta + \cos^2\theta - \sin^2\theta = 0$
$\cos + \cos^2\theta - 1 + \cos^2\theta = 0$
$2\cos^2\theta + \cos\theta - 1 = 0$
$(\cos\theta + 1)(2\cos\theta - 1) = 0$

$\Rightarrow \cos\theta = -1$ or $\cos\theta = \dfrac{1}{2}$

$\Rightarrow \quad \theta = \pi$ or $\theta = \dfrac{\pi}{3}$

$\theta = \pi$ can be neglected as $\theta \in \left(0, \dfrac{\pi}{2}\right)$.

∴ $\theta = \dfrac{\pi}{3}$

Hence, option (b) is correct.

37. Consider the following statements
in respect of sets.
1. The union over intersection of
   sets is distributive.
2. The complement of union of two
   sets is equal to intersection of
   their complements.
3. If the difference of two sets is
   equal to empty set, then the two
   sets must be equal.
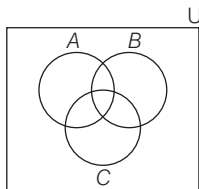Which of the above statements are
correct ?

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

⊙ (**a**) Since, we know that distributive
property for sets A, B and C.

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
and $(A \cup B)' = A' \cap B'$

(By De Morgan's Law)

Also, if $A - B = \phi$

$\Rightarrow$ We cannot say $A = B$
e.g., if $A = \phi$ and $B = \{1, 2\}$
$\Rightarrow A - B = \phi$ and $A \ne B$
∴ Option (a) is correct.

38. Consider three sets X, Y and Z
having 6, 5 and 4 elements
respectively. All these 15 elements
are distinct. Let $S = (X - Y) \cup Z$.
How many proper subsets does S
have?

(a) 255 (b) 256
(c) 1023 (d) 1024

⊙ (**c**) Given, $n(X) = 6, n(Y) = 5, n(Z) = 4$

$S = (X - Y) \cup Z$

Since, all 15 elements are different.

Hence, $n(X - Y) = 6$
and $n(S) = 6 + 4 = 10$

$\Rightarrow$ Number of proper subsets of S

$= 2^{10} - 1$
$= 1024 - 1$
$= 1023$

∴ Option (c) is correct.

39. Consider the following statements
in respect of relations and functions.
1. All relations are functions but all
   functions are not relations.
2. A relation from A to B is a
   subset of Cartesian product
   $A \times B$.
3. A relation in A is a subset of
   Cartesian product $A \times A$.
Which of the above statements are
correct?

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

⊙ (**b**) Since, we know that relations can be
function iff every element has unique
image.

Hence, first statement is wrong.

If $R : A \to A$ then $R \subseteq A \times A$
and if $R : A \to B$ then $R \subseteq A \times B$

Hence, 2nd and 3rd statements are
correct.

∴ Option (b) is correct.

40. If $\log_{10} 2\log_2 10 + \log_{10}(10^x) = 2$,

then what is the value of x?

(a) 0 (b) 1
(c) $\log_2 10$ (d) $\log_5 2$

⊙ (**b**) Given that,

$\log_{10} 2 \cdot \log_2 10 + \log_{10}(10)^x = 2$

$\log_{10} 2 \times \dfrac{1}{\log_{10} 2} + x\log_{10} 10 = 2$

$1 + x = 2$

$\Rightarrow \qquad x = 1$

∴ Option (b) is correct.

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41. Let ABC be a triangle.
If $\cos 2A + \cos 2B + \cos 2C = -1$,
then which one of the following is
correct?

(a) $\sin A \sin B \sin C = 0$
(b) $\sin A \sin B \cos C = 0$
(c) $\cos A \sin B \sin C = 0$
(d) $\cos A \cos B \cos C = 0$

⊙ (**d**) Given that, ABC is a triangle and

$\cos 2A + \cos 2B + \cos 2C = -1$

$\Rightarrow 1 + \cos 2A + \cos 2B + \cos 2C = 0$

$\Rightarrow 2\cos^2 A + 2\cos\left(\dfrac{2B + 2C}{2}\right)$

$\cdot \cos\left(\dfrac{2B - 2C}{2}\right) = 0$

$\Rightarrow 2\cos^2 A + 2\cos(B + C)$

$\cdot \cos(B - C) = 0$

$\{\because A + B + C = 180\}$

$\Rightarrow 2\cos^2 A + 2\cos(180° - A)$

$\cdot \cos(B - C) = 0$

$\Rightarrow 2\cos^2 A - 2\cos A \cdot \cos(B - C) = 0$

$\Rightarrow 2\cos A[\cos A - \cos(B - C)] = 0$

$\Rightarrow 2\cos A[\cos(180° - (B + C)]$

$- \cos(B - C)] = 0$

$\Rightarrow -2\cos A[\cos(B + C) + \cos(B - C)] = 0$

$\Rightarrow -2\cos A\left(2\cos\dfrac{B + C + B - C}{2}\right.$

$\left.\cdot \cos\dfrac{B + C - B + C}{2}\right) = 0$

$-4\cos A \cdot \cos B \cdot \cos C = 0$

$\Rightarrow \qquad \cos A \cdot \cos B \cdot \cos C = 0$

∴ Option (d) is correct.

42. What is the value of the following
determinant?

$\begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$

(a) -1
(b) 0
(c) $2\tan A \sin B \sin C$
(d) $-2\tan A \sin B \sin C$

⊙ (**b**) Let $\Delta = \begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$

$\Delta = \cos C \,[0 + \sin B \tan A]$

$- \tan A \,[\sin B \cos C - 0]$

$= \tan A \sin B \cos C$

$- \tan A \sin B \cos C$

∴ $\Delta = 0$

Hence, option (b) is correct.

43. Suppose set A consists of first 250 natural numbers that are multiples of 3 and set B consists of first 200 even natural numbers. How many elements does $A \cup B$ have?

(a) 324   (b) 364
(c) 384   (d) 400

⊙ (**c**) Given that, A consists of first 250 natural numbers that are multiple of 3.

∴ $A = \{3, 6, 9, 12, ..., 750\}$,
     $n(A) = 250$

Set B consists of first 200 even natural numbers.

∴ $B = \{2, 4, 6, 8, ..., 400\}$
∴ $A \cap B = \{6, 12, ..., 750\}$
∵ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
     $= 250 + 200 - 66$
     $n(A \cup B) = 384$

Hence, option (c) is correct.

44. Let S_k denote the sum of first k terms of an AP. What is $\dfrac{S_{30}}{S_{20} - S_{10}}$ equal to?

(a) 1   (b) 2
(c) 3   (d) 4

⊙ (**c**) Let's take first K terms are first K natural numbers.

∴ $S_K = \dfrac{K(K+1)}{2}$

Consider $\dfrac{S_{30}}{S_{20} - S_{10}} = \dfrac{\dfrac{30(31)}{2}}{\dfrac{20(21)}{2} - \dfrac{10(11)}{2}}$

     $= \dfrac{930}{310} = 3$

∴ Option (c) is correct.

45. If the roots of the equation $4x^2 - (5k+1)x + 5k = 0$ differ by unity, then which one of the following is a possible value of k?

(a) -3 (b) -1
(c) $-\dfrac{1}{5}$ (d) $-\dfrac{3}{5}$

⊙ (**c**) Given equation,
$4x^2 - (5K+1)x + 5K = 0$ ...(i)
Let the roots are α and β.
$$\alpha + \beta = \dfrac{-(-(5K+1))}{4}$$
$$= \dfrac{5K+1}{4}$$
$$\alpha \cdot \beta = \dfrac{5K}{4}$$

Given that, $\alpha - \beta = 1$
$$\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = 1$$
$$\left(\dfrac{5K+1}{4}\right)^2 - 4\left(\dfrac{5K}{4}\right) = 1$$

$$\dfrac{25K^2 + 1 + 10K}{16} = 1 + 5K$$
$$25K^2 + 10K + 1 = 80K + 16$$
⇒ $25K^2 - 70K - 15 = 0$
     $5K^2 - 14K - 3 = 0$
     $5K^2 - 15K + K - 3 = 0$
     $5K(K-3) + 1(K-3) = 0$
     $(K-3)(5K+1) = 0$
⇒ $K = 3 \text{ or } -\dfrac{1}{5}$

Hence, option (c) is correct.

46. Consider the digits 3, 5, 7, 9. What is the number of 5-digit numbers formed by these digits in which each of these four digits appears?

(a) 240   (b) 180
(c) 120   (d) 60

⊙ (**a**) Given digits are 3, 5, 7, 9.
Since, the number of ways to find 5-digit numbers = 5!
but using 3, 5, 7, 9 every time one-digit will be repeated.
Hence number of 5-digit numbers with digit 3 repeated $= \dfrac{5!}{2!}$

Number of 5-digit numbers with digit 5 repeated $= \dfrac{5!}{2!}$

Number of 5-digit numbers with digit 7 repeated $= \dfrac{5!}{2!}$

Number of 5-digit numbers with digit 9 repeated $= \dfrac{5!}{2!}$

∴ Total 5-digit numbers
$$= \dfrac{5!}{2!} + \dfrac{5!}{2!} + \dfrac{5!}{2!} + \dfrac{5!}{2!}$$
$$= 4 \times \left(\dfrac{5 \times 4 \times 3 \times 2!}{2!}\right)$$
$$= 4 \times \left(\dfrac{5 \times 4 \times 3 \times 2!}{2!}\right)$$
$$= 240$$

Hence, option (a) is correct.

47. How many distinct matrices exist with all four entries taken from $\{1, 2\}$?

(a) 16   (b) 24
(c) 32   (d) 48

⊙ (**a**) Given digits are 1, 2.

Let matrix $= \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

∵ Each entries can filled with 2 ways.
Therefore, number of distinct matrices
     $= 2 \times 2 \times 2 \times 2$
     $= 16$

Hence, option (a) is correct.

48. If $i = \sqrt{-1}$, then how many values does i^{-2n} have for different $n \in \mathbb{Z}$?

(a) One   (b) Two
(c) Four   (d) Infinite

⊙ (**b**) Given that, $i = \sqrt{-1}$

To find $(i)^{-2n}$

Let $i = r(\cos\theta + i\sin\theta)$
⇒ $r = 1, \theta = \dfrac{\pi}{2}$

∴ $i = \left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right)$

∴ $(i)^{-2n} = \left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right)^{-2n}$

$$= \cos\left(\dfrac{-2n\pi}{2}\right) + i\sin\left(\dfrac{-2n\pi}{2}\right)$$
$$= \cos(n\pi) - i\sin(n\pi)$$
$$= (-1)^n = \begin{cases} -1; & \text{if } n \text{ is odd} \\ 1; & \text{if } n \text{ is even} \end{cases}$$

∴ Option (b) is correct.

49. If $x = \dfrac{a}{b-c}, y = \dfrac{b}{c-a}, z = \dfrac{c}{a-b}$, then what is the value of the following ?

$$\begin{vmatrix} 1 & -x & x \\ 1 & 1 & -y \\ 1 & z & 1 \end{vmatrix}$$

(a) 0   (b) 1
(c) abc (d) $ab + bc + ca$

⊙ (**a**) Given, $x = \dfrac{a}{b-c}, y = \dfrac{b}{c-a}, z = \dfrac{c}{a-b}$

∴ $\begin{vmatrix} 1 & -x & x \\ 1 & 1 & -y \\ 1 & z & 1 \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$= \begin{vmatrix} 1 & -x & x \\ 0 & 1+x & -y-x \\ 0 & z+x & 1-x \end{vmatrix}$

$= (1+x)(1-x) - (-y-x)(z+x)$
$= 1 - x^2 + x^2 + (y+z)x + yz$
$$= 1 + \left(\dfrac{b}{c-a} + \dfrac{c}{a-b}\right)\left(\dfrac{a}{b-c}\right)$$
$$+ \left(\dfrac{b}{c-a} \times \dfrac{c}{a-b}\right)$$
$$= 1 + \left(\dfrac{ab - b^2 + c^2 - ac}{(a-b)(c-a)}\right)\left(\dfrac{a}{b-c}\right)$$
$$+ \left(\dfrac{bc}{(c-a)(a-b)}\right)$$
$$= 1 + \dfrac{(b-c)(a-b-c)a}{(a-b)(c-a)(b-c)}$$
$$+ \dfrac{bc}{(c-a)(a-b)}$$
$$= \dfrac{(a-b)(c-a) + a^2 - ab - ac + bc}{(a-b)(c-a)}$$

$$= \frac{ac - a^2 - bc + ab + a^2 - ab - ac + bc}{(a - b)(c - a)}$$

$= 0$

Hence, option (a) is correct.

50. Consider the following in respect of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

1. Inverse of A does not exist

2. $A^3 = A$

3. $3A = A^2$

Which of the above are correct ?
(a) 1 and 2 only   (b) 2 and 3 only
(c) 1 and 3 only   (d) 1, 2 and 3

⊙ (**c**) Given matrix, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

∵ $|A| = 1(1 - 1) - 1(1 - 1) + 1(1 - 1) = 0$
∴ A^{-1} doesn't exist.

Now, $A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 3A$

and $A^3 = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9 & 9 & 9 \\ 9 & 9 & 9 \\ 9 & 9 & 9 \end{bmatrix} \neq A$

Hence, option (c) is correct.

Directions (Q.Nos. 51 and 52)
Consider the following for the next two questions that follow.
A circle is passing through the points $(5, -8), (-2, 9)$ and $(2,1)$.

51. What are the coordinate of the centre of the circle?
(a) $(-2, -50)$ (b) $(-50, -20)$
(c) $(-24, -58)$ (d) $(-58, -24)$

⊙ (**d**) Given that, circle is passing through the points $(5, -8), (-2, 9)$ and $(2, 1)$.
Let the equation of circle be
$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)
If Eq. (i) passes through $(5, -8)$
∴ $25 + 64 + 10g - 16f + c = 0$
⇒ $10g - 16f + c + 89 = 0$ ...(ii)
If Eq. (i) passes through $(-2, 9)$
$4 + 81 - 4g + 18f + c = 0$
$-4g + 18f + c + 85 = 0$ ...(iii)

If Eq. (i) passes through $(2, 1)$
⇒ $4 + 1 + 4g + 2f + c = 0$
      $4g + 2f + c + 5 = 0$ ...(iv)
On solving Eqs. (ii), (iii) and (iv)
Eqs. (ii) – Eq. (iii)
⇒ $14g - 34f + 4 = 0$
⇒ $7g - 17f + 2 = 0$ ...(v)
Eq. (iv) – Eq. (iii)
⇒ $8g - 16f - 80 = 0$
      $g - 2f - 10 = 0$ ...(vi)
Eq. (v) $- 7 \times$ Eq. (vi)
      $-3f + 72 = 0$
       $f = 24$
From Eq. (vi)
     $g = 2f + 10$
     $g = 58$
From Eq. (iv) $c = -4g - 2f - 5$
      $= -232 - 48 - 5$
      $c = -285$
∴   Centre $= (-g, -f)$
      $= (-58, -24)$
∴ Option (d) is correct.

52. If r is the radius of the circle, then which one of the following is correct?
(a) $r < 10$ (b) $10 < r < 30$
(c) $30 < r < 60$ (d) $r > 60$

⊙ (**d**) Since, the centre of the above circle
      $= (-58, -24)$
$g = 58, f = 24$ and $c = -285$
∴ Radius $= \sqrt{g^2 + f^2 - c}$
    $= \sqrt{(58)^2 + (24)^2 - (-285)}$
    $= \sqrt{3364 + 576 + 285}$
    $= \sqrt{4225}$
    $r = 65$ unit.
∴ Option (d) is correct.

Directions (Q.Nos. 53 and 54)
Consider the following for the next two questions that follow.
The two vertices of an equilateral triangle are $(0, 0)$ and $(2, 2)$.

53. Consider the following statements.
1. The third vertex has atleast one irrational coordinate.
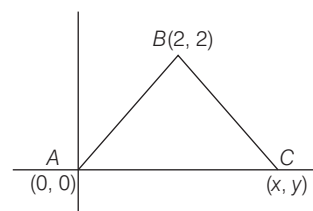2. The area is irrational.
Which of the above statements is/are correct?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

⊙ (**c**) Let vertex of $A = (0, 0)$
      $B = (2, 2)$

∵ $\triangle ABC$ is equilateral triangle.
∴ $AB = BC$
⇒ $\sqrt{(2 - 0)^2 + (2 - 0)^2}$
    $= \sqrt{(2 - x)^2 + (2 - y)^2}$
⇒ $8 = 4 + x^2 - 4x + 4 + y^2 - 4y$
⇒ $x^2 + y^2 - 4x - 4y = 0$ ...(i)
and $AB = AC$
⇒ $\sqrt{(2 - 0)^2 + (2 - 0)^2}$
    $= \sqrt{(x - 0)^2 + (y - 0)^2}$
⇒ $8 = x^2 + y^2$ ...(ii)
and $AC = BC$
⇒ $\sqrt{x^2 + y^2} = \sqrt{(x - 2)^2 + (y - 2)^2}$
⇒ $x^2 + y^2 = x^2 + y^2 - 4x - 4y + 8$
⇒ $x + y = 2$ ...(iii)
From Eqs. (ii) and (iii)
      $8 = x^2 + (2 - x)^2$
$x^2 + 4 + x^2 - 4x = 8$
   $x^2 - 2x - 2 = 0$
      $x = \frac{2 \pm \sqrt{4 + 8}}{2}$
      $= \frac{1 \pm \sqrt{3}}{2}$
Hence, third vertex atleast one irrational coordinate.
⇒ Area will also be irrational.
Hence, option (c) is correct.

54. The difference of coordinates of the third vertex is
(a) 0   (b) $\sqrt{3}$
(c) $2\sqrt{2}$ (d) $2\sqrt{3}$

⊙ (**d**) Since, $x = \frac{1 \pm \sqrt{3}}{2}$
and $y = 2 - x$
      $y = 2 - \frac{1 \pm \sqrt{3}}{2}$
      $y = \frac{3 \pm \sqrt{3}}{2}$
If $x = \frac{1 + \sqrt{3}}{2}, y = \frac{3 - \sqrt{3}}{2}$
∴ $|x - y| = 2\sqrt{3}$
Hence, option (d) is correct.
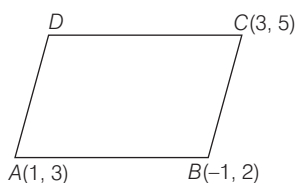
Consider the following for the questions that follow.

The coordinates of three consecutive vertices of a parallelogram ABCD are A(1, 3), B(– 1, 2) and C(3, 5).

55. What is the equation of the diagonal *BD*?
(a) $2x - 3y + 2 = 0$
(b) $3x - 2y + 5 = 0$
(c) $2x - 3y + 8 = 0$
(d) $3x - 2y - 5 = 0$

⊘ (*c*) Given, vertices of parallelogram are
$A = (1, 3), B = (– 1, 2), C = (3, 5)$

ABCD is a parallelogram, then

Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{1 + 3}{2}, \frac{3 + 5}{2}\right) = \left(\frac{-1 + x}{2}, \frac{2 + y}{2}\right)$$

$$\Rightarrow \quad \frac{-1 + x}{2} = \frac{4}{2} \Rightarrow x = 5$$

and $\quad \frac{2 + y}{2} = \frac{8}{2} \Rightarrow y = 6$

∴ Point $= (5, 6)$

∴ Equation of *BD*,

where $B = (– 1, 2)$ and $D = (5, 6)$

$$y - 2 = \frac{6 - 2}{5 - (-1)}(x + 1)$$

$$y - 2 = \frac{4}{6}(x + 1)$$

$$6y - 12 = 4x + 4$$

$$\Rightarrow \quad 2x - 3y + 8 = 0$$

∴ Option (c) is correct.

56. What is the area of the parallelogram?
(a) 1 sq. unit
(b) $\frac{3}{2}$ sq. units
(c) 2 sq. units
(d) $\frac{5}{2}$ sq. units

⊘ (*c*) The vertices of parallelogram are
$A(1, 3), B(– 1, 2), C(3, 5)$ and $D(5, 6)$.
∴ Area = | Area of $\triangle ABC$ + Area of $\triangle ACD$ |

$= \frac{1}{2}| 1(2 - 5) - 1(5 - 3) + 3(3 - 2)|$

$+ \frac{1}{2}| 1(5 - 6) + 3(6 - 3) + 5(3 - 5)|$

$= \frac{1}{2}|-3 - 2 + 3 + (-1) + 9 - 10|$

$= 2$ sq. units

∴ Option (c) is correct.

Consider the following for the next two questions that follow.

The equations of the sides AB, BC and CA of a triangle ABC are x – 2 = 0 , y + 1 = 0 and x + 2y – 4 = 0 respectively.

57. What is the equation of the altitude through *B* on *AC*?
(a) $x - 3y + 1 = 0$ (b) $x - 3y + 4 = 0$
(c) $2x - y + 4 = 0$ (d) $2x - y - 5 = 0$

⊘ (*d*) Equation of $AB \Rightarrow x - 2 = 0$ ...(i)

Equation of $BC \Rightarrow y + 1 = 0$ ...(ii)

Equation of $AC \Rightarrow x + 2y - 4 = 0$ ...(iii)
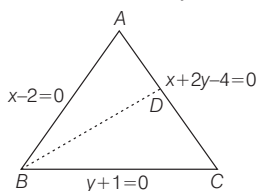
On solving Eq. (i) and Eq. (ii), we get
$$x = 2, y = - 1$$
∴ $\quad B = (2, - 1)$

Slope of $AC = \left(\frac{- \text{coefficient of } x}{\text{coefficient of } y}\right)$

$$m_1 = - \frac{1}{2}$$

∴ Slope of altitude $BD = \frac{-1}{m_1} = \frac{-1}{-\frac{1}{2}} = 2$

∴ Equation of altitude *BD* drawn from *B* on *AC* having slope 2.

$$y + 1 = 2(x - 2)$$
$$y + 1 = 2x - 4$$
$$\Rightarrow \quad 2x - y - 5 = 0$$

Hence, option (d) is correct.

58. What are the coordinates of circumcentre of the triangle?
(a) (4, 0)   (b) (2, 1)
(c) (0, 4)   (d) (2, – 1)

⊘ (*a*) Slope of line *AB*

$$\Rightarrow \frac{- \text{coefficient of } x}{\text{coefficient of } y} = \frac{-1}{0} = \infty$$

Slope of line $BC = - \frac{0}{1} = 0$

∴ Angle between *AB* and *BC*

$$= \left| \frac{\infty - (0)}{1 + \infty \cdot (0)} \right|$$

$\Rightarrow \quad \tan \theta = \infty$ $\quad \left| \theta = \frac{\pi}{2} \right|$

∵ $\triangle ABC$ is right angled triangle.

∴ Circumcentre will lie on Hypotenuse
AC i.e. $x + 2y - 4 = 0$ at mid point.
Equation of $AB : x - 2 = 0$ ...(i)
Equation of $AC : x + 2y - 4 = 0$ ...(ii)
Equation of $BC : y + 1 = 0$. ..(iii)

On solving Eqs. (i) and (ii)
$$x = 2, y = 1$$
∴ $\quad A = (2, 1)$
On solving Eqs. (ii) and (iii)
$$y = - 1, x = 6$$
∴ $\quad c = (6, - 1)$
∴ Circumcentre will be mid-point of

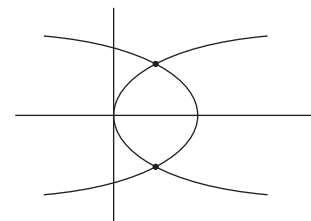$$AC = \left(\frac{2 + 6}{2}, \frac{1 - 1}{2}\right) = (4, 0)$$

∴ Option (a) is correct.

Consider the following for the next two questions that follow.

The two ends of the latus rectum of a parabola are (– 2, 4) and (– 2, – 4).

59. What is the maximum number of parabolas that can be drawn through these two points as end points of latusrectum?
(a) Only one   (b) Two
(c) Four   (d) Infinite

⊘ (*b*) The maximum number of parabolas that can be drawn through

These two points as end points of latusrectum = two
∴ Option (b) is correct.
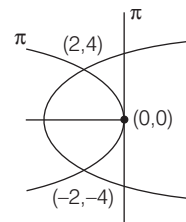
60. Consider the following statements in respect of such parabolas
1. One of the parabolas passes through the origin (0, 0).
2. The focus of one of the parabolas lies at (– 2, 0).
Which of the above statements is/are correct?
(a) 1 only   (b) 2 only
(c) Both 1 and 2   (d) Neither 1 nor 2

⊘ (*a*) Let parabola-1 passes through origin.

It's equation will be
$$y^2 = - 4ax$$

Whose leading points of latusrectum will be $(-a, 2a)$ and $(-a, -2a)$
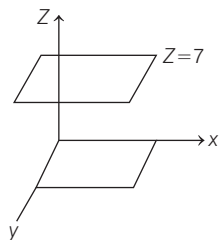
∴ $a = 2$

∴   Focus $= (-2, 0)$

Hence, option (a) is correct.

61. The locus of a point $P(x, y, z)$ which moves in such a way that $z = 7$ is a

(a) line parallel to X-axis
(b) line parallel to Y-axis
(c) line parallel to Z-axis
(d) plane parallel to xy-plane

⊘ (**d**) Since, point moves in a plane $z = 7$ which will be parallel to xy-plane.

Hence, option (d) is correct.

62. Consider the following statements

1. A line in space can have infinitely many direction ratios.
2. It is possible for certain line that the sum of the squares of direction cosines can be equal to sum of its direction cosines.

Which of the above statements is/are correct?

(a) 1 only   (b) 2 only
(c) Both 1 and 2   (d) Neither 1 nor 2

⊘ (**c**) Since, we know that A line in space can have infinitely many direction ratio and also it is possible for certain line that the sum of the squares of direction cosine can be equal to sum of its direction cosines.

For example, (1, 0, 0) is the direction cosines for X-axis.
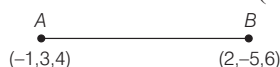
∴ $l = 1$, $m = 0$, $n = 0$, then

$l^2 + m^2 + n^2 = 1^2 + 0^2 + 0^2$

Hence, option (c) is correct.

63. The xy-plane divides the line segment joining the points $(-1, 3, 4)$ and $(2, -5, 6)$.

(a) internally in the ratio 2 : 3
(b) internally in the ratio 3 : 2
(c) externally in the ratio 2 : 3
(d) externally in the ratio 2 : 1

⊘ (**c**) Since, we know that xy-plane divides the line segment joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio $\left(-\dfrac{z_1}{z_2}\right)$.

Hence, $-\dfrac{4}{6} = -\dfrac{2}{3}$

where (−) indicates externally division.

Hence, option (c) is correct.

64. The number of spheres of radius r touching the coordinate axes is

(a) 4   (b) 6
(c) 8   (d) infinite

⊘ (**c**) Since, we know that the number of spheres of radius r touching the coordinate axes is 8.

Hence, option (c) is correct.

65. $ABCDEFGH$ is a cuboid with base $ABCD$. Let $A(0, 0, 0)$, $B(12, 0, 0)$, $C(12, 6, 0)$ and $G(12, 6, 4)$ be the vertices. If α is the angle between AB and AG. β is the angle between AC and AG, then what is the value of $\cos 2\alpha + \cos 2\beta$?

(a) $\dfrac{40}{49}$ (b) $\dfrac{64}{49}$

(c) $\dfrac{120}{49}$ (d) $\dfrac{160}{49}$

⊘ (**b**) Given, $ABCDEFGH$ is a cuboid.

∵ Angle between AB and AG = α

d.r's of $AB = (12 - 0, 0 - 0, 0 - 0)$

         $= (12, 0, 0)$

d.r's of $AG = (12 - 0, 6 - 0, 4 - 0)$

         $= (12, 6, 4)$

∴ $\cos\alpha = \dfrac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

      $= \dfrac{144 + 0 + 0}{\sqrt{12^2}\sqrt{12^2 + 6^2 + 4^2}}$

$\cos\alpha = \dfrac{144}{12 \times 14} = \dfrac{6}{7}$

Now, d.r's of $AC = (12, 6, 0)$

d.r's of $AG = (12, 6, 4)$

∴ $\cos\beta = \dfrac{144 + 36}{\sqrt{180} \times 14}$

      $= \dfrac{180}{\sqrt{180} \times 14} = \dfrac{\sqrt{180}}{14}$

∴ $\cos 2\alpha + \cos 2\beta$

     $= 2\cos^2\alpha - 1 + 2\cos^2\beta - 1$

     $= 2\left[\left(\dfrac{6}{7}\right)^2 + \left(\dfrac{\sqrt{180}}{14}\right)^2\right] - 2$

     $= 2\left(\dfrac{36}{49} + \dfrac{180}{196}\right) - 2$

     $= \dfrac{72}{49} + \dfrac{90}{49} - 2$

     $= \dfrac{162 - 98}{49} = \dfrac{64}{49}$

Hence, option (b) is correct.

66. Let $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be unit vectors such that $\mathbf{a} \times \mathbf{b}$ is perpendicular to $\mathbf{c}$. If θ is the angle between $\mathbf{a}$ and $\mathbf{b}$, then which of the following is/are correct?

1. $\mathbf{a} \times \mathbf{b} = \sin\theta\, \mathbf{c}$
2. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0$

Select the correct answer using the code given below.

(a) only   (b) 2 only
(c) Both 1 and 2   (d) Neither 1 nor 2

⊘ (**c**) Given, that $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be unit vectors such that $\mathbf{a} \times \mathbf{b}$ is perpendicular to $\mathbf{c}$.

angle between $\mathbf{a}$ and $\mathbf{b} = \theta$

∴ $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta \cdot \mathbf{c}$

{∵ $\mathbf{a} \times \mathbf{b}$ is the vector perpendicular to $\mathbf{a}$ and $\mathbf{b}$}

        $= 1 \cdot 1 \cdot \sin\theta \cdot \mathbf{c}$

        $= \sin\theta\, \mathbf{c}$

Since, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are lying on the same plane.

∴ $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0$

Hence, option (c) is correct.

67. If $\mathbf{a} + 3\mathbf{b} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}}$ and $2\mathbf{a} + \mathbf{b} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}}$, then what is the angle between $\mathbf{a}$ and $\mathbf{b}$?

(a) 0   (b) $\dfrac{\pi}{6}$

(c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$

⊘ (**d**) Given, $\mathbf{a} + 3\mathbf{b} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}}$ ...(i)

and $2\mathbf{a} + \mathbf{b} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ ...(ii)

Eq. (i) × 2 − Eq. (ii)

$(2\mathbf{a} + 6\mathbf{b}) - (2\mathbf{a} + \mathbf{b}) = 2(3\hat{\mathbf{i}} - \hat{\mathbf{j}}) - (\hat{\mathbf{i}} - 2\hat{\mathbf{j}})$

               $5\mathbf{b} = 5\hat{\mathbf{i}}$

∴ $\mathbf{b} = \hat{\mathbf{i}}$

From Eq. (i)

        $\mathbf{a} = (3\hat{\mathbf{i}} - \hat{\mathbf{j}}) - 3\mathbf{b}$

        $\mathbf{a} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 3\hat{\mathbf{i}}$

        $\mathbf{a} = -\hat{\mathbf{j}}$

∴ $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$

where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

     $(-\hat{\mathbf{j}}) \cdot (\hat{\mathbf{i}}) = 1 \cdot 1 \cdot \cos\theta$

           $0 = \cos\theta$

∴ $\theta = \dfrac{\pi}{2}$

Hence, option (d) is correct.

68. If $(\mathbf{a} + \mathbf{b})$ is perpendicular to $\mathbf{a}$ and magnitude of $\mathbf{b}$ is twice that of $\mathbf{a}$, then what is the value of $(4\mathbf{a} + \mathbf{b}) \cdot \mathbf{b}$ equal to?

(a) 0   (b) 1
(c) $8|\mathbf{a}|^2$ (d) $8|\mathbf{b}|^2$

(a) Given, $\mathbf{a} + \mathbf{b}$ is perpendicular to $\mathbf{a}$.

$\therefore \qquad (\mathbf{a} + \mathbf{b}) \cdot \mathbf{a} = 0$

$\Rightarrow \qquad |\mathbf{a}|^2 + \mathbf{b} \cdot \mathbf{a} = 0$

$\qquad \mathbf{a} \cdot \mathbf{b} = -|\mathbf{a}|^2$

and $\qquad |\mathbf{b}| = 2|\mathbf{a}|$

$\therefore \qquad (4\mathbf{a} + \mathbf{b}) \cdot \mathbf{b} = 4\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{b}$

$\qquad = 4(-|\mathbf{a}|^2) + |\mathbf{b}|^2$

$\qquad = -4|\mathbf{a}|^2 + (2|\mathbf{a}|)^2$

$\qquad = -4|\mathbf{a}|^2 + 4|\mathbf{a}|^2$

$\qquad = 0$

Hence, option (a) is correct.

69. Let $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be three vectors such $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are coplanar. Which of the following is/are correct?

1. $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$ is coplanar with $\mathbf{a}$ and $\mathbf{b}$

2. $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$ is perpendicular to $\mathbf{a} \times \mathbf{b}$

Select the correct answer using the code given below.
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

(c) Given that $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are coplanar.

$\Rightarrow \qquad \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0 = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a})$

$\qquad = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$

$\Rightarrow \qquad [\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = [\mathbf{b}\ \mathbf{c}\ \mathbf{a}] = [\mathbf{c}\ \mathbf{a}\ \mathbf{b}] = 0$

$\because \qquad (\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = -\mathbf{c} \times (\mathbf{a} \times \mathbf{b})$

$\qquad = -[(\mathbf{c} \cdot \mathbf{b})\mathbf{a} - (\mathbf{c} \cdot \mathbf{a})\mathbf{b}]$

$\qquad = -[[\mathbf{c}\ \mathbf{b}\ \mathbf{a}] - [\mathbf{c}\ \mathbf{a}\ \mathbf{b}]]$

$\qquad = 2[\mathbf{c}\ \mathbf{a}\ \mathbf{b}] = 0$

Hence, $\{(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}\} \cdot (\mathbf{a} \times \mathbf{b}) = 0$

$\Rightarrow (\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$ is perpendicular to $\mathbf{a} \times \mathbf{b}$. and coplanar with $\mathbf{a}$ and $\mathbf{b}$.

Hence, option (c) is correct.

70. If the position vectors of A and B are $(\sqrt{2} - 1)\hat{\mathbf{i}} - \hat{\mathbf{j}}$ and $\hat{\mathbf{i}} + (\sqrt{2} + 1)\hat{\mathbf{j}}$ respectively, then what is the magnitude of $\mathbf{AB}$?
(a) $2\sqrt{2}$
(b) $3\sqrt{2}$
(c) $2\sqrt{3}$
(d) $3\sqrt{3}$

(c) Given that, $\mathbf{OA} = (\sqrt{2} - 1)\hat{\mathbf{i}} - \hat{\mathbf{j}}$

and $\qquad \mathbf{OB} = \hat{\mathbf{i}} + (\sqrt{2} + 1)\hat{\mathbf{j}}$

$\therefore \quad \mathbf{AB} = \mathbf{OB} - \mathbf{OA}$

$\qquad = (1 - \sqrt{2} + 1)\hat{\mathbf{i}} + (\sqrt{2} + 1 + 1)\hat{\mathbf{j}}$

$\qquad \mathbf{AB} = (2 - \sqrt{2})\hat{\mathbf{i}} + (\sqrt{2} + 2)\hat{\mathbf{j}}$

$\therefore |\mathbf{AB}| = \sqrt{(2 - \sqrt{2})^2 + (2 + \sqrt{2})^2}$

$\qquad = \sqrt{4 + 2 - 4\sqrt{2} + 4 + 2 + 4\sqrt{2}}$

$\qquad = \sqrt{12} = 2\sqrt{3}$

Hence, option (c) is correct.

71. If $y = (1+x)(1+x^2)(1+x^4)(1+x^8)$ $(1 + x^{16})$, then what is $\dfrac{dy}{dx}$ at $x = 0$ equal to?
(a) 0
(b) 1
(c) 2
(d) 4

(b) Given, $y = (1 + x)(1 + x^2)(1 + x^4)$
$\qquad\qquad\qquad\qquad (1 + x^8)(1 + x^{16})$

$\therefore \dfrac{dy}{dx} = (1 + x)(1 + x^2)(1 + x^4)$
$\qquad\qquad\qquad (1 + x^8) \cdot (16x^{15})$
$+ (1 + x)(1 + x^2)(1 + x^4)(8x^7)(1 + x^{16})$
$+ (1 + x)(1 + x^2)(4x^3)(1 + x^8)(1 + x^{16})$
$+ (1 + x)(2x)(1 + x^4)(1 + x^8)(1 + x^{16})$
$+ (1)(1 + x^2)(1 + x^4)(1 + x^8)(1 + x^{16})$

$\therefore \left.\dfrac{dy}{dx}\right|_{x=0} = 0 + 0 + 0 + 0 + 1 = 1$

Hence, option (b) is correct.

72. If $y = \cos x \cdot \cos 4x \cdot \cos 8x$, then what is $\dfrac{1}{y}\dfrac{dy}{dx}$ at $x = \dfrac{\pi}{4}$ equal to?
(a) -1 \quad (b) 0 \quad (c) 1 \quad (d) 3

(a) Given, $y = \cos x \cdot \cos 4x \cdot \cos 8x$

$\therefore \log y = \log \cos x + \log \cos 4x$
$\qquad\qquad\qquad\qquad + \log \cos 8x$

On differentiating w.r.t 'x'.

$\dfrac{1}{y} \cdot \dfrac{dy}{dx} = \dfrac{1}{\cos x}(-\sin x) + \dfrac{1}{\cos 4x}$
$\qquad (-4\sin 4x) + \dfrac{1}{\cos 8x}(-8\sin 8x)$

$\qquad = -\tan x - 4\tan 4x - 8\tan 8x$

$\therefore \left(\dfrac{1}{y}\dfrac{dy}{dx}\right)_{at\ x = \frac{\pi}{4}} = -\tan\dfrac{\pi}{4} - 4\tan \pi$
$\qquad\qquad\qquad\qquad - 8\tan 2\pi$
$\qquad = -1 - 0 - 0 = -1$

Hence, option (a) is correct.

73. Let $f(x)$ be a polynomial function such that $fof(x) = x^4$. What is $f'(1)$ equal to?
(a) 0 \quad (b) 1 \quad (c) 2 \quad (d) 4

(c) Given, $f(x)$ be a polynomial such that
$\qquad\qquad fof(x) = x^4$

To find $\qquad f'(1) = ?$

$\because \qquad fof(x) = x^4 \Rightarrow f(x) = x^2$

$\therefore \qquad f'(x) = 2x$

$\Rightarrow \qquad f'(1) = 2 \times 1 = 2$

Hence, option (c) is correct.

74. What is $\displaystyle\lim_{n \to \infty} \dfrac{a^n + b^n}{a^n - b^n}$ where $a > b > 1$, equal to?
(a) -1 \qquad (b) 0
(c) 1
(d) Limit does not exist

(c) Given, $\displaystyle\lim_{n \to \infty} \dfrac{a^n + b^n}{a^n - b^n}$ where $a > b > 1$

$\Rightarrow \displaystyle\lim_{n \to \infty} \dfrac{a^n\left[1 + \left(\dfrac{b}{a}\right)^n\right]}{a^n\left[1 - \left(\dfrac{b}{a}\right)^n\right]}$ $\qquad \left[\because \dfrac{b}{a} < 1\right]$

$\therefore \qquad = \dfrac{1 + 0}{1 - 0} = 1$

$\therefore \qquad \left(\dfrac{b}{a}\right)^\infty = 0$

Hence, option (c) is correct.

75. Let $f(x) = \begin{cases} 1 + \dfrac{x}{2k}, & 0 < x < 2 \\ kx, & 2 \le x < 4 \end{cases}$

If $\displaystyle\lim_{x \to 2} f(x)$ exists, then what is the value of k?
(a) -2
(b) -1
(c) 0
(d) 1

(d) Let $f(x) \begin{cases} 1 + \dfrac{x}{2k}; & 0 < x < 2 \\ kx; & 2 \le x < 4 \end{cases}$

$\because \displaystyle\lim_{x \to 2} f(x)$ exists

$\Rightarrow \qquad \displaystyle\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+}(kx)$

$\qquad \displaystyle\lim_{n \to 2^-}\left(1 + \dfrac{x}{2k}\right) = \lim_{x \to 2^+}(kx)$

$\qquad 1 + \dfrac{2}{2k} = 2k$

$\qquad \dfrac{2}{2k} = 2k - 1$

$\qquad 2 = 4k^2 - 2k$

$\qquad 4k^2 - 2k - 2 = 0$

$\qquad 2k^2 - k - 1 = 0$

$\qquad (2k + 1)(k - 1) = 0$

$\Rightarrow \qquad k = 1 \text{ or } k = -\dfrac{1}{2}$

Hence, option (d) is correct.

76. Consider the following statements in respect of $f(x) = |x| - 1$:
1. $f(x)$ is continuous at $x = 1$.
2. $f(x)$ is differentiable at $x = 0$.
Which of the above statements is/are correct?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

(a) Given, $f(x) = |x| - 1$

Since, modulus function is continuous.

$\Rightarrow f(x)$ is continuous at $x = 1$

and $|x|$ is not differentiable if $x = 0$

$\therefore f(x) = |x| - 1$ is not differentiable at $x = 0$

Hence, statement (1) is correct and (2) is false.

Hence, option (a) is correct.

77. If $f(x) = \dfrac{[x]}{[x]}$, $x \neq 0$,

where [ ] denotes the greatest integer function, then what is the right-hand limit of $f(x)$ at $x = 1$?
(a) – 1
(b) 0
(c) 1
(d) Right-hand limit of $f(x)$ at $x = 1$ does not exist

⊚ (**c**) Given that, $f(x) = \dfrac{[x]}{|x|}$, $x \neq 0$

$$= \lim_{x \to 1^+} \dfrac{[x]}{|x|}$$

$x = 1 + h$, where $h \to 0$

∴ $\quad \lim_{h \to 0} = \dfrac{[1+h]}{[1+h]} = \dfrac{1}{|1+0|} = 1$

Hence, option (c) is correct.

78. Consider the following statements in respect of the function.

$$f(x) = \sin\left(\dfrac{1}{x^2}\right), \quad x \neq 0.$$

1. It is continuous at $x = 0$, if $f(0) = 0$.
2. It is continuous at $x = \dfrac{2}{\sqrt{x}}$.

Which of the above statements is/are correct?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

⊚ (**b**) Given that, $f(x) = \sin\left(\dfrac{1}{x^2}\right)$, $x \neq 0$

At $x = 0$,

LHL $\lim_{x \to 0^-} \sin\left(\dfrac{1}{x^2}\right)$

$\qquad$ = value in between – 1 and + 1

RHL $\lim_{x \to 0^+} \sin\left(\dfrac{1}{x^2}\right)$

$\qquad$ = value in between – 1 and + 1

∴ Limit doesn't exist $\Rightarrow f(x)$ is not continuous at $x = 0$.

At $x = \dfrac{2}{\sqrt{\pi}}$,

$\lim_{x \to \frac{2}{\sqrt{\pi}}} \sin\left(\dfrac{1}{x^2}\right) = \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$

Also $\quad f\left(\dfrac{2}{\sqrt{\pi}}\right) = \sin\left(\dfrac{1}{\left(\frac{2}{\sqrt{\pi}}\right)^2}\right) = \dfrac{1}{\sqrt{2}}$

∴ $f(x)$ is continuous at $x = \dfrac{2}{\sqrt{\pi}}$

Hence, option (b) is correct.

79. What is the range of the function $f(x) = 1 - \sin x$ defined on entire real line?
(a) (0, 2)
(b) [0, 2]
(c) (– 1, 1)
(d) [– 1, 1]

⊚ (**b**) Given that, $f(x) = 1 - \sin x$

Since, the range of $\sin x$ is [– 1, 1].
$\qquad$ – 1 ≤ sin x ≤ 1
$\qquad$ – 1 ≤ – sin x ≤ 1
$\qquad$ 1 – 1 ≤ 1 – sin x ≤ 1 + 1
$\qquad$ 0 ≤ 1 – sin x ≤ 2
∴ Range = [0, 2]
Hence, option (b) is correct.

80. What is the slope of the tangent of $y = \cos^{-1}(\cos x)$ at $x = -\dfrac{\pi}{4}$?

(a) – 1
(b) 0
(c) 1
(d) 2

⊚ (**a**) Given that, $y = \cos^{-1}(\cos x)$

Since range of $\cos^{-1} x$ is [0, π].
∴ $\qquad y = \cos^{-1}(\cos x) = -x$,
if $\qquad x \in (-\pi, 0)$
∵ $\qquad x = -\dfrac{\pi}{4}$
∴ $\qquad y = -x$
$\Rightarrow \quad \dfrac{dy}{dx} = -1$
∴ Slope of tangent = – 1
Hence, option (a) is correct.

81. What is the integral of $f(x) = 1 + x^2 + x^4$ with respect to x^2?

(a) $x + \dfrac{x^3}{3} + \dfrac{x^5}{5} + C$

(b) $\dfrac{x^3}{3} + \dfrac{x^5}{5} + C$

(c) $x^2 + \dfrac{x^4}{4} + \dfrac{x^6}{6} + C$

(d) $x^2 + \dfrac{x^4}{2} + \dfrac{x^6}{3} + C$

⊚ (**d**) Given function, $f(x) = 1 + x^2 + x^4$

∴ Integral of $f(x)$ w.r.t x^2.

$\qquad = \int (1 + x^2 + x^4) \cdot 2x \, dx$

$\qquad = \int (2x + 2x^3 + 2x^5) dx$

$\qquad = x^2 + \dfrac{x^4}{2} + \dfrac{x^6}{3} + C$

Hence, option (d) is correct.

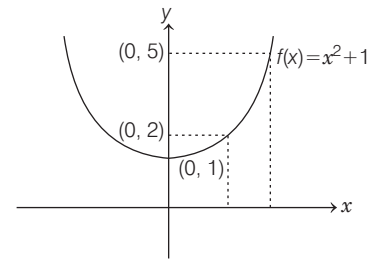**82.** Consider the following statements in respect of the function $f(x) = x^2 + 1$ in the interval (1, 2).

1. The maximum value of the function is 5.
2. The minimum value of the function is 2.

Which of the above statements is/are correct?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

⊚ (**c**) Given function,

$f(x) = x^2 + 1 \qquad$ in (1, 2)
$\Rightarrow \qquad y = x^2 + 1$
$\qquad x^2 = (y - 1)$

Which is the equation of parabola with vertex (0, 1).

At $\qquad x = 1$
$\qquad f(1) = 1^2 + 1 = 2$
$\qquad f(2) = 2^2 + 1 = 5$

Hence, maximum value of the function in (1, 2) is 5 and minimum value is 2.
Hence, option (c) is correct.

83. If $f(x)$ satisfies $f(1) = f(4)$, then what is $\int_1^4 f'(x) \, dx$ equal to?

(a) – 1
(b) 0
(c) 1
(d) 2

⊚ (**b**) $\qquad f(1) = f(4)$

∴ $\int_1^4 f'(x)dx = [f(x)]_1^4 = f(4) - f(1)$

$\qquad = f(1) - f(1) = 0$

84. What is $\int_0^{\frac{\pi}{2}} e^{\ln(\cos x)} dx$ equal to?

(a) – 1
(b) 0
(c) 1
(d) 2

⊚ (**c**) Let $I = \int_0^{\frac{\pi}{2}} e^{\ln(\cos x)} dx = \int_0^{\frac{\pi}{2}} (\cos x) dx$

$\qquad = [\sin x]_0^{\frac{\pi}{2}} = \sin\dfrac{\pi}{2} - \sin 0 = 1$

Hence, option (c) is correct.

85. If $\int \sqrt{1 - \sin 2x} \, dx = A$ $\sin x + B\cos x + C$, where $0 < x < \dfrac{\pi}{4}$, then which one of the following is correct?
(a) $A + B = 0$
(b) $A + B - 2 = 0$
(c) $A + B + 2 = 0$
(d) $A + B - 1 = 0$

⊚ (**b**) Given that,

$\int \sqrt{1 - \sin 2x} \, dx = A\sin x + B\cos x + C,$

where $0 \leq x \leq \dfrac{\pi}{4}$.

Let

$I = \int \sqrt{\cos^2 x + \sin^2 x - 2\sin x \cdot \cos x} \, dx$

$I = \int \sqrt{(\cos x - \sin x)^2} \, dx$

$\qquad \{\because \cos x > \sin x \text{ when } 0 < x < \dfrac{\pi}{4}\}$

$I = \int (\cos x - \sin x)\, dx$

$I = \sin x + \cos x + C$

$\quad = A \sin x + B \cos x + C$

$\therefore \qquad A = 1, B = 1$

$\therefore\ A + B - 2 = 1 + 1 - 2 = 0$

Hence, option (b) is correct.

86. What is the order of the differential equation of all ellipses whose axes are along the coordinate axes?

(a) 1   (b) 2

(c) 3   (d) 4

⊚ (**b**) Since, the equation of ellipse
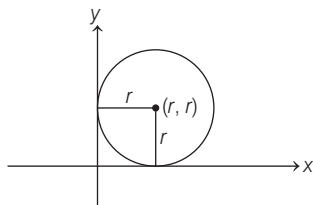
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$\therefore$ There are 2 variable a and b.

$\therefore$ Order of the differential equation = 2

Hence option (d) is correct.

87. What is the degree of the differential equation of all circles touching both the coordinate axes in the first quadrant?

(a) 1   (b) 2

(c) 3   (d) 4

⊚ (**b**) If r be the radius of circle.

Since, the circle touching both the coordinate axes in the first quadrant.

$\therefore$ Centre = (r, r) and radius = r

$\therefore$ Equation of circle

$$(x - r)^2 + (y - r)^2 = r^2$$

$x^2 + y^2 - 2xr - 2yr + r^2 = 0 \qquad \ldots(i)$

$2x + 2yy' - 2r - 2ry' = 0$

$\qquad r(1 + y') = x + yy'$

$$\qquad r = \frac{x + yy'}{1 + y'}$$

Putting the value of r in Eq. (i)

$x^2 + y^2 - 2x\dfrac{(x + yy')}{1 + y'} - 2y\dfrac{(x + yy')}{1 + y'}$

$\qquad + \left(\dfrac{x + yy'}{1 + y'}\right)^2 = 0$

$(1 + y')^2 x^2 + (1 + y')^2\, y^2 - 2x(x + yy')$

$\qquad (1 + y') - 2y(x + yy')(1 + y')$

$\qquad\qquad + (x + yy')^2 = 0$

$(1 + y')^2\,(x^2 + y^2) - 2(x - y)(x + yy')$

$\qquad (1 + y') + (x + yy')^2 = 0$

Hence, the degree of the differential equation is 2.

88. What is the differential equation of $y = A - \dfrac{B}{x}$?

(a) $xy_2 + y_1 = 0$ (b) $xy_2 + 2y_1 = 0$

(c) $xy_2 - 2y_1 = 0$ (d) $2xy_2 + y_1 = 0$

⊚ (**b**) Given, $y = A - \dfrac{B}{x}$

On differentiating w.r.t 'x'

$$\frac{dy}{dx} = 0 - B\left(-\frac{1}{x^2}\right) = \frac{B}{x^2}$$

$$x^2\frac{dy}{dx} = B$$

On differentiating again w.r.t 'x'.

$$x^2 \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot(2x) = 0$$

$\Rightarrow \qquad\qquad x(xy_2 + 2y_1) = 0$

$\Rightarrow \qquad\qquad xy_2 + 2y_1 = 0$

Hence, option (b) is correct.

89. What is $\displaystyle\int_0^\pi \ln\left(\tan\frac{x}{2}\right) dx$ equal to?

(a) 0   (b) $\dfrac{1}{2}$ (c) 1   (d) 2

⊚ (**a**) Let $I = \displaystyle\int_0^\pi \ln\left(\tan\frac{x}{2}\right) dx \qquad \ldots(i)$

$$I = \int_0^\pi \ln\left(\cot\left(\frac{\pi - x}{2}\right)\right) dx$$

$$I = \int_0^\pi \ln\left(\cot\left(\frac{x}{2}\right)\right) dx \qquad \ldots(ii)$$

Adding Eq. (i) and Eq. (ii)

$$2I = \int_0^\pi \left\{\ln\left(\tan\frac{x}{2}\right) + \ln\left(\cot\frac{x}{2}\right)\right\} dx$$

$$= \int_0^\pi \ln\left(\tan\frac{x}{2}\cdot\cot\frac{x}{2}\right) dx = \int_0^\pi \ln(1)\, dx$$

$2I = 0$

$\therefore \qquad I = 0$

Hence, option (a) is correct.

90. Where does the tangent to the curve $y = e^x$ at the point $(0, 1)$ meet X-axis?

(a) $(1, 0)$ (b) $(-1, 0)$

(c) $(2, 0)$ (d) $\left(-\dfrac{1}{2}, 0\right)$

⊚ (**b**) Given curve, $y = e^x$

$$\therefore \qquad\qquad \frac{dy}{dx} = e^x$$

$$\left(\frac{dy}{dx}\right)_{at(0, 1)} = e^0 = 1$$

$\therefore$ Equation of tangent at $(0, 1)$.

$$y - 1 = \left(\frac{dy}{dx}\right)_{at(0, 1)} (x - 0)$$

$y - 1 = x$

Since, $(-1, 0)$ satisfies above equation.

Hence, option (b) is correct.

91. Consider the following statements in respect of the function

$$f(x) = x + \frac{1}{x}.$$

1. The local maximum value of $f(x)$ is less than its local minimum value.

2. The local maximum value of $f(x)$ occurs at $x = 1$.

Which of the above statements is/are correct?

(a) 1 only   (b) 2 only

(c) Both 1 and 2   (d) Neither 1 nor 2

⊚ (**a**) Given, $f(x) = x + \dfrac{1}{x}$

$$f'(x) = 1 - \frac{1}{x^2} \text{ and } f''(x) = \frac{2}{x^3}$$

For critical points $f'(x) = 0$

$$1 - \frac{1}{x^2} = 0$$

$$x = \pm 1$$

At $x = 1, \quad f''(x) = 2 > 0$

$\Rightarrow f(x)$ is minimum at $x = 1$

$\Rightarrow f(1) = 2$

At $\quad x = -1\, f''(x) = -2 < 0$

$\Rightarrow f(x)$ is maximum at $x = -1$

$\Rightarrow f(-1) = -2$

Hence, statement (1) is correct and (2) is false.

$\therefore$ Option (a) is correct.

92. What is the maximum area of a rectangle that can be inscribed in a circle of radius 2 units?

(a) 4 sq. units   (b) 6 sq. units

(c) 8 sq. units   (d) 16 sq. units

⊚ (**c**) Let x and y be the length and breadth of rectangle respectively.
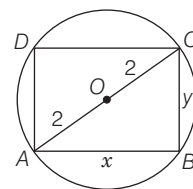
In $\triangle ABC$,

$$x^2 + y^2 = 16$$

$\Rightarrow \qquad\qquad y = \sqrt{16 - x^2}$

$\therefore$ Area of rectangle, $A = xy$

$$A = x\sqrt{16 - x^2}$$

$$\frac{dA}{dx} = \sqrt{16 - x^2} + \frac{x}{2\sqrt{16 - x^2}}(-2x)$$

$$= \frac{16 - x^2 - x^2}{\sqrt{16 - x^2}} = \frac{16 - 2x^2}{\sqrt{16 - x^2}}$$

For maximum A,

$$\frac{dA}{dx} = 0$$

$\Rightarrow \dfrac{16 - 2x^2}{\sqrt{16 - x^2}} = 0 \quad \Rightarrow \quad 16 - 2x^2 = 0$

$\Rightarrow \qquad x^2 = 8 \quad \Rightarrow \quad x = \pm 2\sqrt{2}$

Now, $\dfrac{d^2A}{dx^2} = \dfrac{-4x \cdot \sqrt{16 - x^2} - (16 - 2x^2)\dfrac{1}{2\sqrt{16 - x^2}}(-2x)}{16 - x^2}$

$= \dfrac{-4x(16 - x^2) + x(16 - 2x^2)}{(16 - x^2)^{3/2}}$

$= \dfrac{-3x(16 - x^2)}{(16 - x^2)^{3/2}}$

$\left(\dfrac{d^2A}{dx^2}\right)_{\text{at } x = 2\sqrt{2}} = \dfrac{-3(2\sqrt{2})(16 - 8)}{(16 - 8)^{3/2}}$ (Negative)

$\therefore \qquad y = \sqrt{16 - (2\sqrt{2})^2} = 2\sqrt{2}$

Area $= 2\sqrt{2} \times 2\sqrt{2} = 8$ sq. units

Hence, A is maximum at $x = 2\sqrt{2}$.

93. What is $\displaystyle\int \dfrac{dx}{x(x^2 + 1)}$ equal to?

(a) $\dfrac{1}{2}\ln\left(\dfrac{x^2}{x^2 + 1}\right) + C$

(b) $\ln\left(\dfrac{x^2}{x^2 + 1}\right) + C$

(c) $\dfrac{3}{2}\ln\left(\dfrac{x^2}{x^2 + 1}\right) + C$

(d) $\dfrac{1}{2}\ln\left(\dfrac{x^2 + 1}{x^2}\right) + C$

⊚ (**a**) Let $I = \displaystyle\int \dfrac{dx}{x(x^2 + 1)}$

$\therefore \dfrac{1}{x(x^2 + 1)} = \dfrac{A}{x} + \dfrac{Bx + C}{x^2 + 1}$

(by using partial fraction)

$\dfrac{1}{x(x^2 + 1)} = \dfrac{Ax^2 + A + Bx^2 + Cx}{x(x^2 + 1)}$

$1 = (A + B)x^2 + Cx + A$

$\Rightarrow A + B = 0, C = 0, A = 1$

$\therefore \qquad B = -A = -1$

$\therefore I = \displaystyle\int \dfrac{dx}{x(x^2 + 1)} = \int\left(\dfrac{1}{x} - \dfrac{x}{x^2 + 1}\right)dx$

$= \ln x - \dfrac{1}{2}\ln(x^2 + 1) + C$

$= \dfrac{1}{2}(2 \ln x) - \dfrac{1}{2}\ln(x^2 + 1) + C$

$= \dfrac{1}{2}(\ln x^2 - \ln(x^2 + 1)) + C$

$= \dfrac{1}{2}\ln\left(\dfrac{x^2}{x^2 + 1}\right) + C$

Hence, option (a) is correct.

94. What is the derivative of e^{e^x} with respect to e^x?

(a) e^{e^x}

(b) e^x

(c) $e^{e^x}e^x$

(d) ee^x

⊚ (**a**) Let $y_1 = e^{e^x}$ and $y_2 = e^x$

$\therefore \quad \dfrac{dy_1}{dx} = e^{e^x} \cdot e^x, \dfrac{dy_2}{dx} = e^x$

$\Rightarrow \quad \dfrac{dy_1}{dy_2} = \dfrac{e^{e^x} \cdot e^x}{e^x} = e^{e^x}$

∴ Option (a) is correct.

95. What is the condition that $f(x) = x^3 + x^2 + kx$ has no local extremum?

(a) $4k < 1$

(b) $3k > 1$

(c) $3k < 1$

(d) $3k \le 1$

⊚ (**b**) Given that, $f(x) = x^3 + x^2 + kx$

∵ $f(x)$ has no local extremum.

$\Rightarrow \qquad f'(x) \ne 0$

$\Rightarrow \quad 3x^2 + 2x + k \ne 0$

for no extremum, $D < 0$

$\Rightarrow \quad (2)^2 - 4(3)(k) < 0$

$\Rightarrow \qquad 4 - 12k < 0$

$\qquad\qquad 3k > 1$

∴ Option (b) is correct.

96. If $f(x) = 2^x$, then what is $\displaystyle\int_2^{10} \dfrac{f'(x)}{f(x)}\,dx$ equal to?

(a) $4\ln 2$

(b) $\ln 4$

(c) $\ln 5$

(d) $8\ln 2$

⊚ (**d**) Given, $f(x) = 2^x$

$\therefore \displaystyle\int_2^{10} \dfrac{f'(x)}{f(x)}\,dx = [\ln f(x)]_2^{10} = [\ln 2^x]_2^{10}$

$= [x \ln 2]_2^{10}$

$= 10 \ln 2 - 2 \ln 2$

$= 8 \ln 2$

∴ Option (d) is correct.

97. If $\displaystyle\int_{-2}^{0} f(x)\,dx = k$, then

$\displaystyle\int_{-2}^{0} |f(x)|\,dx$ is

(a) less than k

(b) greater than k

(c) less than or equal to k

(d) greater than or equal to k

⊚ (**d**) Given, $\displaystyle\int_{-2}^{0} f(x)\,dx = k$

To find $\displaystyle\int_{-2}^{0} |f(x)|\,dx$

Let $f(x) = x$

$\therefore \displaystyle\int_{-2}^{0} x\,dx = \left[\dfrac{x^2}{2}\right]_{-2}^{0} = -2 = k$

$\therefore \displaystyle\int_{-2}^{0} |x|\,dx = -\int_{-2}^{0} x\,dx$

$= -(-2) = 2 \ge k$

∴ Option (d) is correct.

98. If the function $f(x) = x^2 - kx$ is monotonically increasing in the interval $(1, \infty)$, then which one of the following is correct?

(a) $k < 2$

(b) $2 < k < 3$

(c) $3 < k < 4$

(d) $k > 4$

⊚ (**a**) Let the function $f(x) = x^2 - kx$ is monotonically increasing in $(1, \infty)$.

$\Rightarrow \qquad f'(x) \ge 0$

$\Rightarrow \qquad 2x - k \ge 0$

$\Rightarrow k \le 2x$ in $(1, \infty)$ at lower value

at $\qquad\qquad x = 1$

$\qquad\qquad k < 2$

Hence, option (a) is correct.

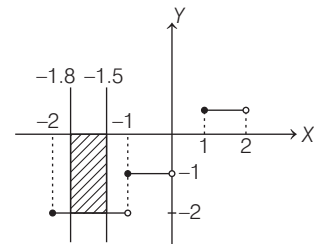**99.** What is the area bounded by $y = [x]$, where $[\cdot]$ is the greatest integer function, the X-axis and the lines $x = -1.5$ and $x = -1.8$?

(a) 0.3 sq. unit

(b) 0.4 sq. unit

(c) 0.6 sq. unit

(d) 0.8 sq. unit

⊚ (**c**) Given, $y = [x]$

$\therefore \qquad$ Area $= \displaystyle\int_{-1.8}^{-1.5} [x]\,dx = \int_{-1.8}^{-1.5} (-2)\,dx$

$= -2(x)_{-1.8}^{-1.5}$

$= -2(-1.5 + 1.8) = -0.6$

$\therefore \qquad$ Area $= 0.6$ sq. unit

Hence, option (c) is correct.

100. The tangent to the curve $x^2 = y$ at $(1, 1)$ makes an angle θ the with the positive direction of X-axis. Which one of the following is correct?

(a) $\theta < \dfrac{\pi}{6}$

(b) $\dfrac{\pi}{6} < \theta < \dfrac{\pi}{4}$

(c) $\dfrac{\pi}{4} < \theta < \dfrac{\pi}{3}$

(d) $\dfrac{\pi}{3} < \theta < \dfrac{\pi}{2}$

⊚ (**d**) Given, curve $y = x^2$

$\qquad\qquad \dfrac{dy}{dx} = 2x$

$\qquad\qquad \dfrac{dy}{dx}\Big|_{\text{at}(1, 1)} = 2 \times 1 = 2$

$\Rightarrow \qquad \tan\theta = 2$

∵ $\tan\dfrac{\pi}{3} = \sqrt{3} = 1.732$ and $\tan\dfrac{\pi}{2} = \infty$

$\therefore \qquad \dfrac{\pi}{3} < \theta < \dfrac{\pi}{2}$

Hence, option (d) is correct.

101. Consider the following relations for two events E and F.

1. $P(E \cap F) \geq P(E) + P(F) - 1$
2. $P(E \cup F) = P(E) + P(F) + P(E \cap F)$
3. $P(E \cup F) \leq P(E) + P(F)$

Which of the above relations is/are correct?

(a) 1 only   (b) 3 only
(c) 1 and 3 only   (d) 1, 2 and 3

⊘ (**c**) Let E and F be two events.

Then, $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
                                                    ...(i)

or $P(E \cup F) \leq P(E) + P(F)$

$P(E \cup F) \leq 1$

$- P(E \cap F) \geq -1$

$\Rightarrow P(E) + P(F) - P(E \cup F) \geq P(E) + P(F) - 1$

$\Rightarrow \qquad P(E \cap F) \geq P(E) + P(F) - 1$

Hence, option (c) is correct.

102. If $P(A / B) < P(A)$, then which one of the following is correct?

(a) $P(B \mid A) < P(B)$ (b) $P(B \mid A) > P(B)$
(c) $P(B \mid A) = P(B)$ (d) $P(B \mid A) > P(A)$

⊘ (**a**) If $P\left(\dfrac{A}{B}\right) < P(A)$

$\because \qquad P\left(\dfrac{A}{B}\right) = \dfrac{P(A \cap B)}{P(B)} < P(A)$

$\Rightarrow \dfrac{P(A \cap B)}{P(A)} < P(B)$

$\Rightarrow \qquad P\left(\dfrac{B}{A}\right) < P(B)$

Hence, option (a) is correct.

103. When the measure of central tendency is available in the form of mean, which one of the following is the most reliable and accurate measure of variability?

(a) Range
(b) Mean deviation
(c) Standard deviation
(d) Quartile deviation

⊘ (**c**) When the measure of central tendency is available in the form of mean then, we know that Standard Deviation is the most reliable and accurate measure of variability.

Hence, option (c) is correct.

104. A problem is given to three students A, B and C, whose probabilities of solving the problem independently are $\dfrac{1}{2}$, $\dfrac{3}{4}$ and p, respectively. If the probability that the problem can be solved is $\dfrac{29}{32}$, then what is the value of p ?

(a) $\dfrac{2}{5}$ (b) $\dfrac{2}{3}$

(c) $\dfrac{1}{3}$ (d) $\dfrac{1}{4}$

⊘ (**d**) Given, $P(A) = \dfrac{1}{2}$, $P(B) = \dfrac{3}{4}$, $P(C) = p$

$\because$ Probability that the problem can not be solved $= P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C})$

$= \left(1 - \dfrac{1}{2}\right)\left(1 - \dfrac{3}{4}\right)(1 - p)$

$= \dfrac{1}{2} \times \dfrac{1}{4}(1 - p)$

$= \dfrac{1 - p}{8}$

$\therefore$ Probability that the problem can be solved

$= 1 -$ Probability that the problem cannot be solved

$\dfrac{29}{32} = 1 - \dfrac{(1 - p)}{8}$

$\dfrac{1 - p}{8} = 1 - \dfrac{29}{32}$

$\dfrac{1 - p}{8} = \dfrac{3}{32}$

$1 - p = \dfrac{3}{4}$

$\therefore \qquad p = \dfrac{1}{4}$

Hence, option (d) is correct.

105. In a cricket match a batsman hits a six 8 times out of 60 balls he plays. What is the probability that on a ball played he does not hit a six?

(a) $\dfrac{2}{3}$ (b) $\dfrac{1}{15}$ (c) $\dfrac{2}{15}$ (d) $\dfrac{13}{15}$

⊘ (**d**) Since, the batsman hits a six 8 times out of 60 balls.

The batsman could not hit sixes in (60-8) balls.

$\therefore$ Probability that on a ball played he does not hit six $= \dfrac{52}{60}$

$p = \dfrac{13}{15}$

Hence, option (d) is correct.

Directions (Q. Nos. 106 and 107)
Consider the following for the questions that follow.
Two regression lines are given as
$3x - 4y + 8 = 0$ and $4x - 3y - 1 = 0$

106. Consider the following statements.

1. The regression line of y on x is

$y = \dfrac{3}{4}x + 2$

2. The regression line of x on y is

$x = \dfrac{3}{4}y + \dfrac{1}{4}$

Which of the above statements is/are correct?

(a) 1 only   (b) 2 only
(c) Both 1 and 2   (d) Neither 1 nor 2

⊘ (**c**) Two regression lines are

$3x - 4y + 8 = 0$, $4x - 3y - 1 = 0$

for finding the regression line of y on x

$3x - 4y + 8 = 0$

$4y = 3x + 8$

$y = \left(\dfrac{3}{4}\right)x + 2 \qquad$ ...(i)

and the regression line of x on y:

$4x - 3y - 1 = 0$

$4x = 3y + 1$

$x = \dfrac{3}{4}y + \dfrac{1}{4} \qquad$ ...(ii)

Hence, option (c) is correct.

107. Consider the following statements.

1. The coefficient of correlations

r is $\dfrac{3}{4}$

2. The means of x and y are 3 and 4 respectively.

Which of the above statements is/are correct?

(a) 1 only   (b) 2 only
(c) Both 1 and 2   (d) Neither 1 nor 2

⊘ (**a**) Since, regression line of y on x

$\Rightarrow \qquad y = \dfrac{3}{4}x + 2$

$\therefore \qquad b_{xy} = \dfrac{3}{4}$

and regression line of x on y

$\Rightarrow \qquad x = \dfrac{3}{4}y + \dfrac{1}{4}$

$\therefore \qquad b_{yx} = \dfrac{3}{4}$

$\therefore$ Coefficient of correlations

$r = \sqrt{b_{xy} \times b_{yx}} = \sqrt{\dfrac{3}{4} \times \dfrac{3}{4}}$

$r = \dfrac{3}{4}$

Means of x and y are nothing but the solution of regression lines

$3x - 4y + 8 = 0$

and $4x - 3y - 1 = 0$

$3x - 4y = -8 \qquad$ ...(i)

$4x - 3y = 1 \qquad$ ...(ii)

Eq. (i) $\times 4 -$ Eq. (ii) $\times 3$

$12x - 16y = -32$

$12x - 9y = 3$

$7y = 35 \Rightarrow y = 5$

$\therefore \qquad 4x = 1 + 3 \times 5$

$x = 4$

$\therefore$ Statement (2) is wrong.

Hence, option (a) is correct.

Consider the following for the questions that follow.

The marks obtained by 60 students in a certain subject out of 75 are given below.

Marks	Number of students
15-20	4
20-25	5
25-30	11
30-35	6
35-40	5
40-45	8
45-50	9
50-55	6
55-60	4
60-65	2

108. What is the median?
- (a) 35
- (b) 38
- (c) 39
- (d) 40

⊙ (*c*)

Marks	Frequency	Cumulative frequency
15-20	4	4
20-25	5	9
25-30	11	20
30-35	6	26 = Cf
35-40	5	31
40-45	8	39
45-50	9	48
50-55	6	54
55-60	4	58
60-65	2	60
	N = 60	

$\therefore \frac{N}{2} = 30 \Rightarrow$ model class will be 35 − 40.

$\therefore$ lower limit (l) = 35

$$h = 40 - 35 = 5$$

$\therefore$ Median $= l + \dfrac{\frac{N}{2} - C.f}{f} \times h$

$$= 35 + \frac{30 - 26}{5} \times 5 = 39$$

Hence, option (c) is correct.

109. What is the mode?
- (a) 27.27
- (b) 27.73
- (c) 27.93
- (d) 28.27

Marks	Frequency
15-20	4
20-25	5 → f_0
25-30	11 → f_1
30-35	6 → f_2
35-40	5

Marks	Frequency
40-45	8
45-50	9
50-55	6
55-60	4
60-65	2

⊙ (*b*) Highest frequency is given for class 25-30.

∴ Model class will be 25-30.

∴ $l = 25, h = 5$

∴ $f_1 = 11, f_0 = 5, f_2 = 6$

∴ Mode $= l + \dfrac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$$= 25 + \frac{11 - 5}{22 - 5 - 6} \times 5$$

$$= 25 + \frac{6}{11} \times 5 = \frac{275 + 30}{11}$$

$$= 27.73$$

Hence, option (b) is correct.

110. What is the mean of natural numbers contained in the interval [15, 64]?
- (a) 36.8
- (b) 38.3
- (c) 39.5
- (d) 40.3

⊙ (*c*) Mean of natural numbers contained in [15, 64].

$$= \frac{15 + 16 + 17 + ... + 64}{50}$$

$$= \frac{\sum_{n=1}^{64} n - \sum_{r=1}^{14} r}{50}$$

$$= \frac{\frac{64 \times 65}{2} - \frac{14 \times 15}{2}}{50}$$

$$= \frac{2080 - 105}{50} = 39.5$$

Hence, option (c) is correct.

111. For the set of numbers $x, x, x + 2, x + 3, x + 10$ where x is a natural number, which of the following is/are correct?
1. Mean > Mode
2. Median > Mean

Select the correct answer using the code given below.
- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

⊙ (*a*) Given data $x, x, x + 2, x + 3, x + 10$ where $x \in N$.

∴ Mean $= \dfrac{x + x + x + 2 + x + 3 + x + 10}{5}$

$$= \frac{5x + 15}{5} = x + 3$$

∴ Mode $= x$

Median $= \left(\dfrac{5 + 1}{2}\right)^{th}$ term $= 3^{rd}$ term

$$= x + 2$$

∴ Mean > Mode and Median < Mean

Hence, correct option is (a).

112. The mean of 10 observations is 5.5. If each observation is multiplied by 4 and subtracted from 44, then what is the new mean?
- (a) 20
- (b) 22
- (c) 34
- (d) 44

⊙ (*b*) Given that, the mean of 10 observation is 5.5.

∵ Mean $= \dfrac{x_1 + x_2 + ... + x_{10}}{10}$

$$5.5 = \frac{x_1 + x_2 + ... + x_{10}}{10}$$

∴ $\sum_{i=1}^{10} x_i = 55$

Also, given that new observations are obtained by multiplying by 4 and subtracting from 44.

Hence, new mean $= 44 - 4 \times 5.5$

$$= 44 - 22 = 22$$

Hence, correct option is (b).

113. If g is the geometric mean of 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, then which one of the following is correct?
- (a) $8 < g < 16$
- (b) $16 < g < 32$
- (c) $32 < g < 64$
- (d) $g > 64$

⊙ (*c*) Given that, geometric mean of 2, 4, 8, 16, 32, 64, 126, 256, 512, 1024 is g.

∴ $g = \sqrt[10]{\begin{array}{c}2 \times 4 \times 8 \times 16 \times 32 \times 64 \\ \times 128 \times 256 \times 512 \times 1024\end{array}}$

$$= (2^{1 + 2 + 3 + ... + 10})^{\frac{1}{10}}$$

$$g = (2^{55})^{\frac{1}{10}} \Rightarrow g = (2)^{\frac{11}{2}}$$

∴ $2^5 < g < 2^6 \Rightarrow 32 < g < 64$

Hence, option (c) is correct.

114. If the harmonic mean of 60 and x is 48, then what is the value of x?
- (a) 32
- (b) 36
- (c) 40
- (d) 44

⊙ (*c*) Given, harmonic mean of 60 and x is 48.

∵ $H = \dfrac{2ab}{a + b}$

$$48 = \frac{2 \times 60 \times x}{60 + x}$$

$$2880 + 48x = 120x$$

$$72x = 2880$$

$$x = 40$$

Hence, option (c) is correct.

115. What is the mean deviation of first 10 even natural numbers?

(a) 5 (b) 5.5
(c) 10 (d) 10.5

(**a**) Mean deviation of first 10 even natural numbers

Since, mean $(\bar{x}) = \dfrac{2 + 4 + 6 + \dots 20}{10}$

$= \dfrac{2(10 \times 11)}{20} = 11$

$\therefore$ Mean deviation

$= \dfrac{|2 - 11| + |4 - 11| + |6 - 11| + \dots + |20 - 11|}{10}$

$= \dfrac{9 + 7 + 5 + 3 + 1 + 1 + 3 + 5 + 7 + 9}{10}$

$= 5$

Hence, option (a) is correct.

116. If $\displaystyle\sum_{i=1}^{10} x_i = 110$ and $\displaystyle\sum_{i=1}^{10} x_i^2 = 1540,$ then what is the variance?

(a) 22 (b) 33
(c) 44 (d) 55

(**b**) Given, $\displaystyle\sum_{i=1}^{10} x_i = 110$

and $\displaystyle\sum_{i=1}^{10} x_i^2 = 1540$

$\therefore$ Variance $= \dfrac{\Sigma x_i^2}{n} - \left(\dfrac{\Sigma x_i}{n}\right)^2$

$= \dfrac{1540}{10} - \left(\dfrac{110}{10}\right)^2$

$= 154 - 121$

$= 33$

Hence, option (b) is correct.

117. 3-digit numbers are formed using the digits 1, 3, 7 without repetition of digits. A number is randomly selected. What is the probability that the number is divisible by 3?

(a) 0 (b) $\dfrac{1}{3}$
(c) $\dfrac{1}{4}$ (d) $\dfrac{1}{8}$

(**a**) Let 3-digit numbers using the digits without repetition 1, 3, 7 are 3!.

Since, the sum of the digits
$= 1 + 3 + 7 = 11$
which is not divisible by 3.

$\therefore P$ (number of divisible by 3) $= \dfrac{0}{3!}$

$= 0$

Hence, option (a) is correct.

118. What is the probability that the roots of the equation $x^2 + x + n = 0$ are real, where $n \in N$ and $n < 4$?

(a) 0 (b) $\dfrac{1}{4}$
(c) $\dfrac{1}{3}$ (d) $\dfrac{1}{2}$

(**a**) Given, equation $x^2 + x + n = 0$, where $n \in N, n < 4$

$\therefore \quad n \in \{1, 2, 3\}$

Since, above equation is quadratic. So, for each value of n, we have two roots.

$\therefore$ Total number of roots $= 6$

When $n = 1$

$x^2 + x + 1 = 0$

$x = \dfrac{-1 \pm \sqrt{1 - 4}}{2} = \dfrac{-1 \pm \sqrt{3}i}{2}$

when $n = 2,$

$x^2 + x + 2 = 0$

$\Rightarrow \quad x = \dfrac{-1 \pm \sqrt{1 - 8}}{2} = \dfrac{-1 \pm \sqrt{7}i}{2}$

and $n = 3, x^2 + x + 3 = 0$

$\Rightarrow \quad x = \dfrac{-\pm \sqrt{11}i}{2}$

There are no real roots.

$\therefore P$ (roots are real) $= \dfrac{0}{6} = 0$

Hence, option (a) is correct.

119. If A and B are two events such that $P(\text{not } A) = \dfrac{7}{10}, P(\text{not } B) = \dfrac{3}{10}$ and $P(A \mid B) = \dfrac{3}{14}$, then what is $P(B \mid A)$ equal to?

(a) $\dfrac{11}{14}$ (b) $\dfrac{9}{14}$
(c) $\dfrac{1}{4}$ (d) $\dfrac{1}{2}$

(**d**) Given, $P(\text{not } A) = \dfrac{7}{10}, P(\text{not } B) = \dfrac{3}{10}$

$P\left(\dfrac{A}{B}\right) = \dfrac{3}{14}$

$\therefore \quad P\left(\dfrac{B}{A}\right) = \dfrac{P(A \cap B)}{P(A)} \quad$ ...(i)

$\because \quad P(\text{not } A) = \dfrac{7}{10}$

$\therefore \quad P(A) = 1 - \dfrac{7}{10} = \dfrac{3}{10}$

$P(\text{not } B) = \dfrac{3}{10}$

$\therefore \quad P(B) = 1 - \dfrac{3}{10} = \dfrac{7}{10}$

$P\left(\dfrac{A}{B}\right) = \dfrac{P(A \cap B)}{P(B)}$

$\dfrac{3}{14} = \dfrac{P(A \cap B)}{\dfrac{7}{10}}$

$\therefore \quad P(A \cap B) = \dfrac{3}{20}$

Eq. (i) $\Rightarrow P\left(\dfrac{B}{A}\right) = \dfrac{\dfrac{3}{20}}{\dfrac{3}{10}} = \dfrac{1}{2}$

Hence, option (d) is correct.

120. Seven white balls and three black balls are randomly placed in a row. What is the probability that no two black balls are placed adjacently?

(a) $\dfrac{7}{15}$ (b) $\dfrac{8}{15}$
(c) $\dfrac{11}{15}$ (d) $\dfrac{13}{15}$

(**a**) There are 10 balls among which 7 are white and 3 are black.

$\therefore$ Number of ways to arrange 10 balls $= 10!$

If we put the balls in such a way that no two black balls are placed adjacently.

$\therefore$ Number of arrangements $= 7! \times {}^8P_3$

$\therefore \quad P = \dfrac{7! \times 8!}{5! \times 10!}$

$= \dfrac{6 \times 7}{9 \times 10} = \dfrac{7}{15}$

Hence, option (a) is correct.

PAPER II : English Language and General Studies

Part A (English Language)

Directions (Q. Nos. 1-10) *Each item in this section consists of a sentence with an underlined word followed by four words (a), (b), (c) and (d). Select the option that is nearest in meaning to the underlined word.*

1. He is essentially a <u>lowbrow</u> person.
(a) coarse   (b) proud
(c) passionate   (d) pathetic

⊙ (*a*) 'Lowbrow' means not highly cultured and lacking in refinement. Hence, 'coarse' is its correct synonym.

2. His <u>nostalgia</u> is deep.
(a) excitement   (b) longing
(c) happiness   (d) ability

⊙ (*b*) 'Nostalgia' means a sentimental longing for a period in the past. Hence, option (b) is the correct answer.

3. His <u>truancy</u> is detrimental.
(a) loyalty   (b) integrity
(c) honesty   (d) absenteeism

⊙ (*d*) 'Truancy' refers to the problem or situation of children being absent from school regularly without permission. Hence, 'absenteeism' is its correct synonym.

4. He is a <u>maleficent</u> person.
(a) generous   (b) cunning
(c) criminal   (d) friendly

⊙ (*c*) 'Maleficent' means working or productive of harm or evil. Hence, 'criminal' is nearest in meaning to maleficent.

5. His <u>solemnity</u> is celebrated.
(a) greed   (b) desire
(c) trust   (d) dignity

⊙ (*d*) 'Solemnity' refers to the state of being serious and dignified. Hence 'dignity' is its correct synonym.

6. His <u>bounty</u> is limitless.
(a) benevolence   (b) gallantry
(c) nepotism   (d) chivalry

⊙ (*a*) 'Bounty' means generosity. Hence, 'benevolence' is its correct synonym.

7. The <u>Holocaust</u> was experienced by millions.
(a) celebration   (b) destruction
(c) construction   (d) beautiful

⊙ (*b*) 'Holocaust' refers to destruction that occurred on a massive scale. Hence, option (b) is the correct answer.

8. His <u>aversion</u> is known to all.
(a) hospitality   (b) hostility
(c) humility   (d) humbleness

⊙ (*b*) 'Aversion' refers to a strong dislike or disinclination towards something. Hence, 'hostility' is its correct synonym.

9. To be able to <u>decipher</u> something is wonderful.
(a) decode   (b) encode
(c) simulate   (d) animate

⊙ (*a*) 'Decipher' means to convert a coded text or signal into normal language. Hence, 'decode' is its correct synonym.

10. It is the <u>opportune</u> time to think about investment in the real estate sector.
(a) honorary   (b) appropriate
(c) difficult   (d) unsuitable

⊙ (*b*) 'Opportune' time means a time that is suitable for doing something. Hence 'appropriate' is its correct synonym.

Directions (Q. Nos. 11-20) *Each question in this section has a sentence with three parts labelled (a), (b) and (c). Read each sentence to find out whether there is any error in any part. If you find no error, your responses should be indicated as (d).*

11. Nursery classes (a)/ for the children (b) / starts on April every year. (c) / No error (d)

⊙ (c) Part (c) contains the error. Replace 'on' with 'in' to make the sentence error free.

12. The pollution level is very (a) / high in our city, and (b) / becomes a health hazard. (c) / No error (d)

⊙ (c) Part (c) contains the error. Replace 'becomes' with 'has become' to make the sentence error free.

13. Indian is a country of linguistic (a) / and cultural diversity with (b) / multiple ethnic and social groups. (c) / No error (d)

⊙ (d) The given sentence is error free.

14. Why does the lapse rate (a) / effect the tendency (b) / of air to rise ? (c) / No error (d)

⊙ (b) Part (b) contains the error. Replace 'effect' with 'affect' to make the sentence grammatically correct.

15. The thorax is the upper part (a) /of your body from the neck down (b) / to the bottom of the ribs and diaphragm. (c) / No error (d)

⊙ (b) Part (b) contains the error. Replace 'from' with 'extending from' to make the sentence error free.

16. The practical uses of astrology (a) / have seldom been seriously (b) / studied outside India. (c) / No error (d)

⊙ (d) The given sentence is error free and grammatically correct.

17. She collected (a) / some armful of medicines and (b) / poured them down a drain. (c) / No error (d)

⊙ (c) Part (c) contains the error. Replace 'a' with 'the' to make the sentence error free.

18. In many countries, the United Nations (a) / or other internationally organised (b) / peacekeeping forces are trying to keep the peace. (c) / No error (d)

⊙ (b) Part (b) contains the error. Replace 'or' with 'and' to make the sentence error free.

19. Early cities rely on food and water (a) / from the surrounding countryside, (b) / but today cities often depend on distant sources. (c) / No error (d)

⊙ (a) Part (a) contains the error. Replace 'early cities rely' with 'earlier cities relied' to make the sentence grammatically correct.

20. Climate change, whether it is (a) / natural or human-induced, (b) / is stressful for the people. (c) / No error (d)

⊙ (c) Part (c) contains the error. Replace 'for' with 'to' to make the sentence error free.

Directions (Q. Nos. 21-30) *Each question in this section consists of a sentence with an underlined word followed by four words (a), (b), (c) and (d). Select the option that is opposite in meaning to the underlined word.*

21. His opinion is <u>lopsided</u>.
    (a) partial   (b) crooked
    (c) unequal   (d) balanced

⊙ (**d**) 'Lopsided' means not equally balanced. Hence, 'balanced' is its correct antonym.

22. His work is <u>praiseworthy</u>.
    (a) admirable   (b) condemnable
    (c) commendable   (d) creditable

⊙ (**b**) 'Praiseworthy' means deserving approval and admiration. Hence, 'condemnable' meaning blamable is opposite in meaning to praiseworthy.

23. His deeds had <u>retrograde</u> results .
    (a) progressive   (b) negative
    (c) retreating   (d) reverse

⊙ (**a**) 'Retrograde' means to revert to an earlier and inferior conditions. Hence, 'progressive' meaning forward-moving is its correct antonym.

24. He always offers <u>palatable</u> solutions.
    (a) acceptable   (b) agreeable
    (c) unacceptable   (d) pleasant

⊙ (**c**) 'Palatable' means acceptable or satisfactory. Hence, 'unacceptable' is its correct antonym.

25. His views on the subject are <u>microscopic</u>.
    (a) broad-based   (b) minute
    (c) precise   (d) fine

⊙ (**a**) 'Microscopic' means concerned with minute details. Hence, 'broad-based' is opposite in meaning to 'microscopic'.

26. He is always <u>obdurate</u> in his behaviour towards other people.
    (a) flexible   (b) callous
    (c) insensible   (d) obnoxious

⊙ (**a**) 'Obdurate' means stubbornly refusing to change one's opinion. Hence, 'flexible' is its correct antonym.

27. Mohan always had <u>profound</u> respect for Sohan.
    (a) deep   (b) extreme
    (c) sincere   (d) superficial

⊙ (**d**) 'Profound' means very great or intense. Hence, 'superficial' is its correct antonym. 'Superficial' means Seeming to have a particular quality, although this is not true or real.

28. We were living in <u>turbulent</u> times.
    (a) destructive   (b) unstable
    (c) calm   (d) stormy

⊙ (**c**) 'Turbulent' means involving a lot of sudden changes, arguments ore violence. Hence, 'calm' is its correct antonym.

29. Amit is a <u>dogmatic</u> person.
    (a) assertive   (b) amenable
    (c) bold   (d) rigid

⊙ (**b**) 'Dogmatic' means of a person completely certain of their bailiffs and expecting other people to accept them. Hence, 'amenably is its correct antonym as its means willing to accept what someone says or does without arguing.

30. Sachin is very <u>fickle</u> in his behaviour.
    (a) stable   (b) capricious
    (c) mercurial   (d) vacillating

⊙ (**a**) 'Fickle' means frequently changing one's loyalties and affections. Hence, 'stable' is its correct antonym.

Directions (Q. Nos. 31-40) *Each of the following items in this section consists of a sentence, the parts of which have been jumbled. These parts have been labelled P, Q, R and S. Given below each sentence are four sequences namely (a), (b), (c) and (d), You are required to re-arrange the jumbled parts of the sentence and mark your answer carefully.*

31. symbolic of peace, and prosperity (P)/the Gupta Empire in India is (Q)/represented through numismatic archaeological (R) / and literary evidences of multiple kinds (S)
    The correct sequence should be
    (a) QPRS   (b) QSRP
    (c) PSQR   (d) RSQP

⊙ (**a**) QPRS

32. the transportation system so that it (P) / and not just the affluent section of society (Q) / provides mobility to everyone (R) / there are many ways to restructure (S)
    The correct sequence should be :
    (a) PQRS   (b) RQPS
    (c) SPRQ   (d) QRPS

⊙ (**c**) SPRQ

33. and accurately displays spatial (P) / relationships between landforms and water bodies (Q) / the globe shows Earth's spherical shape (R) / and comparative distance between locations (S)
    The correct sequence should be :
    (a) PQRS   (b) RPQS
    (c) SPQR   (d) QRPS

⊙ (**b**) RPQS

34. the function of the capillaries (P) / and other material to all the cells in (Q) / is to take nutrients, oxygen (R) / the body and to take away their waste material (S)

The correct sequence should be :
    (a) PRQS   (b) RPQS
    (c) SPQR   (d) QRPS

⊙ (**a**) PRQS

35. like mine (P) / even to someone with (Q) / a tin ear (R) / her singing sounded pretty melodious (S)
    The correct sequence should be
    (a) PRQS   (b) RPQS
    (c) SPQR   (d) QRPS

⊙ (**d**) QRPS

36. The sorry state of affairs in the modern factories (P) / the Government of India (Q) / under the British was pro-capitalist (R) / though it took some half-hearted steps to mitigate (S)
    The correct sequence should be :
    (a) PRQS   (b) QRSP
    (c) SPQR   (d) QPSR

⊙ (**b**) QRSP

37. of extreme poverty among its people (P) / economic policies was the prevalence (Q)/a major characteristic of British rule in India (R) / and the net result of British (S)
    The correct sequence should be :
    (a) PRQS   (b) QRSP
    (c) RSQP   (d) QPSR

⊙ (**c**) RSQP

38. you can't judge (P) / very intelligent, but (Q) / a book by its cover (R) / he doesn't look (S)
    The correct sequence should be :
    (a) SQPR   (b) QRSP
    (c) RSQP   (d) QPSR

⊙ (**a**) SQPR

39. membership and partisanship as well as (P) / can be found in the decline in party (Q) / evidence of a crisis in party politics (R) / in the rise of antiparty groups and movements (S)
    The correct sequence should be :
    (a) RQPS   (b) QRSP
    (c) RSQP   (d) QPSR

⊙ (**a**) RQPS

40. government, market and voluntary organisations (P) / mixture of contributions from (Q) / the provision of public goods (R) / and services can be regarded as a complex (S)
    The correct sequence should be
    (a) RQPS   (b) QRSP
    (c) RSQP   (d) QPSR

⊙ (**c**) RSQP

41. Once-over
(a) To do something quickly
(b) To do something elaborately
(c) To do something perforce
(d) To do something for others

⊘ (*a*) The phrase 'once-over' means to review, inspect or examine something in a quick fashion.
Hence, option (a) is the correct answer.

42. A bag of bones
(a) An extremely fat person
(b) An extremely thin person
(c) A wealthy person
(d) A healthy person

⊘ (*b*) The idiom 'a bag of bones' means a very thin person or animal.
Hence, option (b) is the correct answer.

43. Muddy the waters
(a) To spread dirt around
(b) To make a situation easy
(c) To make one's life happy
(d) To make the situation complicated

⊘ (*d*) The idiom 'muddy the waters' means to make a situation more complicated or confusing than it was before.

44. To level with
(a) To be honest
(b) To be on a plain
(c) To cheat someone
(d) To take revenge

⊘ (*a*) The phrase 'to level with' someone means to tell someone the truth, especially when it may be unpleasant. Hence, 'To be honest' gives its correct meaning.

45. Show your hand
(a) To tell people about the past
(b) To tell people about the future
(c) To tell people about them
(d) To tell people about your plans

⊘ (*d*) The idiom 'show your hands' means to allow people to know about your intentions.
Hence, option (d) is the correct answer.

46. To pull up
(a) To lift heavy weights
(b) To check oneself
(c) To shorten the distance
(d) Too short of destination

⊘ (*b*) The phrase 'pull up' means to life cense someone to stop ore pause: check. So, 'To check' oneself given its correct meaning.

47. Be out of action
(a) To get punishment
(b) To get reward

(c) To become dysfunctional
(d) To become superlative

⊘ (*c*) The idiom 'be out of action' means broken or not working. Hence, option (c) given its correct meaning.

48. Learning from the school of hard knocks
(a) Learning things from experience
(b) Learning things from others
(c) Learning things from environment
(d) Learning things through difficult experiences

⊘ (*d*) The idiom 'leaning form the school of hard knacks' means leaning thing through difficult experiences.

49. Raise some hackles
(a) To annoy
(b) To make someone happy
(c) To do a great task
(d) To sit idle

⊘ (*a*) The idiom 'raise some hackles' means to annoy or upset someone. Hence, option (a) is the correct answer.

50. A man of letters
(a) An excellent artist
(b) A scholar in literature
(c) An outstanding sportsman
(d) A known novelist

⊘ (*b*) 'A man of letters' means a man, usually a writer, who knows a lot about literature. Hence, 'A Scholar in literature gives its correct meaning.

Part B (General Studies)

51. Girth of stem of a plant increases due to division of cells in
(a) apical meristem only
(b) lateral meristem only
(c) apical and intercalary meristem
(d) both apical and lateral meristem

⊘ (*b*) The girth of the stem of a plant increases due to the division of cells in lateral meristem only. Lateral meristems are known as secondary meristems because they are responsible for secondary growth or increase in stem girth and thickness.

52. Different varieties of the same gene are called
(a) Genotypes   (b) Sib pairs
(c) Alleles   (d) Isomers

⊘ (*c*) Different varieties of the same gene are called alleles because an allele is a variant form of a gene. Some genes have a variety of different forms, which are located at the same position, or genetic locus, on a chromosome.

Alleles contribute to the organism's phenotype, which is the outward appearance of the organism. Some alleles are dominant or recessive.

53. Which one of the following viruses used to be responsible for highly infectious disease smallpox?
(a) Adenovirus   (b) Variola virus
(c) Aichi virus   (d) Coxsackie virus

⊘ (*b*) Variola virus was used to be responsible for the highly infectious disease smallpox.

The disease is spread through person-to-person contact, most often from inhalation of air droplets expelled through the infected person. The disease was eradicted worldwide in 1976 through vaccination.

54. Buoyancy is a/an
(a) upward pressure
(b) downward pressure
(c) downward force
(d) upward force

⊘ (*d*) Buoyancy or upthrust, is an upward force exerted by a fluid that opposes the weight of a partially or fully immersed object. In a column of fluid, pressure increases with depth as a result of the weight of the overlying fluid.

55. Weight and mass of an object are defined with Newton's laws of motion. Which among the following is true?
(a) Weight is a constant of proportionality
(b) Mass is a constant of proportionality
(c) Mass is not a constant of proportionality
(d) Weight is a universal constant.

⊘ (*b*) The weight of an object is defined as the force of gravity on the object and may be calculated as the mass times the acceleration of gravity, $w = mg$. Since the weight is a force, its SI unit is the newton. Thus mass is a constant of proportionality.

56. Fundamental laws of physics require
 (a) conservation of energy and non-conservation of charge
 (b) conservation of charge and non-conservation of linear momentum
 (c) conservation of charge and non-conservation of energy
 (d) conservation of energy, momentum and charge

⊘ (*d*) Fundamental laws of physics require conservation of energy, momentum and Charge. With respect to classical physics, conservation laws include conservation of energy, mass (or matter), linear momentum, angular momentum, and electric charge. Most conservation laws are exact, or absolute, in the sense that they apply to all possible processes.

57. Work is said to be one Joule when a force of
 (a) 4 N moves an object by 25 cm
 (b) 2 N moves an object by 1 m
 (c) 1 N moves an object by 1 cm
 (d) 1 N moves an object by 50 cm

⊘ (*a*) One joule is defined as the amount of work done when a force of one newton is exerted through a distance of one meter. Thus force of 4 N moves an object by 25 cm resulting in 1 joule of work.

58. A uniform motion of a car along a circular path experiences
 (a) a change in speed due to a change in its direction of motion
 (b) a change in velocity due to a change in its direction of motion
 (c) a change in momentum due to no change in its direction of motion
 (d) a constant momentum due to a change in its direction of motion

⊘ (*b*) A uniform motion of a car along with a circular path experiences a change in velocity due to a change in its direction of motion.

Since velocity is a vector, changes in direction constitute changes in velocity. A change in velocity is known as acceleration. The change in velocity due to circular motion is known as centripetal acceleration.

59. Three equal resistors are connected in parallel configuration in a closed electrical circuit. Then the total resistance in the circuit becomes
 (a) one-third of the individual resistance
 (b) two-third of the individual resistance
 (c) equal to the individual resistance
 (d) three times of the individual resistance

⊘ (*a*) One-third of the individual resistance because if three or more resistors each with the same value are connected in parallel, then the equivalent resistance will be equal to $\frac{R}{n}$ where R is the value of the resistor and n is the number of individual resistances in the combination.

60. Rutherford's alpha-particle (α) scattering experiment was responsible for the discovery of which one of the following?
 (a) Electron (b) Proton
 (c) Atomic Nucleus (d) Neutron

⊘ (*c*) Rutherford's alpha-particle experiment was responsible for the discovery Of Atomic Nucleus. Ernest Rutherford's most famous experiment is the gold foil experiment. A beam of alpha particles was aimed at a piece of gold foil. Most alpha particles passed through the foil, but a few were scattered backward. This showed that most of the atom is empty space surrounding a tiny nucleus.

61. What is the maximum number of electrons in the M-Shell?
 (a) 6 (b) 8
 (c) 18 (d) 32

⊘ (*c*) Maximum number of electrons in the M-Shell is 18. Each shell has a fixed number of electrons. The formula to determine the number of electrons in a shell is 2(n)2.

For the M shell principle, the quantum number is 3.

So, the maximum number of electrons is = 2(3)2

Maximum number of electrons =18.

62. Chlorine occurs in nature in two isotopic forms of masses 35 u and 37 u in the ratio of 3 : 1 respectively. What is the average atomic mass of the Chlorine atom?
 (a) 36.1 u (b) 35.5 u
 (c) 36.5 u (d) 35.1 u

⊘ (*c*) The average atomic mass for an element is calculated by summing the masses of the element's isotopes, each multiplied by its natural abundance on Earth.

Average atomic mass $= \dfrac{P_1 x_1 + P_2 x_2}{P_1 + P_2}$

$= \dfrac{(35 \times 1) + (37 \times 3)}{3 + 1} = 36.5\, \text{u}$

63. Which one of the following elements' isotopes is used in the treatment of cancer?
 (a) Iodine (b) Sodium
 (c) Cobalt (d) Uranium

⊘ (*c*) Cobalt therapy is the medical use of gamma rays from the radioisotope cobalt-60 to treat conditions such as cancer. Beginning in the 1950s, cobalt-60 was widely used in external beam radiotherapy (teletherapy) machines, which produced a beam of gamma rays that was directed into the patient's body to kill tumor tissue.

64. To protect steel and iron from rusting, a thin layer of which one of the following metals is applied?
 (a) Magnesium (b) Zinc
 (c) Aluminium (d) Lead

⊘ (*b*) Zinc metal present on the surface of iron and steel forms a thin protective layer of basic zinc carbonate due to the reaction between zinc, oxygen, CO_2 and moisture in the air. The standard reduction potential of Zinc is less than iron, or Zinc is more reactive than Iron. Hence, Iron will not undergo corrosion.

65. Cinnabar is an ore of which one of the following?
 (a) Copper (b) Zinc
 (c) Mercury (d) Manganese

⊘ (*c*) Cinnabar is a toxic ore of mercury, composed of mercury sulfide (HgS). It is the only important ore of mercury. It has a bright red color that has caused people to use it as a pigment, and carve it into jewelry and ornaments for thousands of years in many parts of the world.

66. Imagine a current-carrying straight conductor with magnetic field of lines in anti-clockwise direction. Then the direction of current is determined by
 (a) the Right-Hand Thumb rule and it would be in the downward direction.
 (b) the Left-Hand Thumb rule and it would be in the downward direction.
 (c) the Right-Hand Thumb rule and it would be in the upward direction.
 (d) the Left-Hand Thumb rule and it would be in the upward direction.

⊘ (*c*) The right-hand thumb rule is used to find the direction of the magnetic field around a current carrying a straight conductor. Thus, a current-carrying straight conductor with the magnetic field of lines in the anti-clockwise direction would be in the upward direction according to the right-hand thumb rule.

67. The device used to produce electric current is known as
 (a) motor (b) generator
 (c) ammeter (d) galvanometer

(*b*) The device used to produce electric current is called a Generator. Electric generator, also called a dynamo, is any machine that converts mechanical energy to electricity for transmission and distribution over power lines to domestic, commercial, and industrial customers. Generators also produce the electrical power required for automobiles, aircraft, ships, and trains.

68. Myopia is a defect in human vision where an image of a
(a) nearby object is focused beyond the retina.
(b) nearby object is focused before the retina.
(c) distant object is focused before the retina.
(d) distant object is focused beyond the retina.

(*c*) Nearsightedness (myopia) is a common vision condition in which you can see objects near to you clearly, but objects farther away are blurry. It occurs when the shape of your eye causes light rays to bend (refract) incorrectly, focusing images in front of your retina instead of on your retina.

69. Tyndall effect is a phenomenon of
(a) scattering of light by the colloidal particles
(b) refraction of light by the colloidal particles
(c) dispersion of light by dust particles
(d) refraction of light by dust particles

(*a*) Tyndall effect, also called Tyndall phenomenon, scattering of a beam of light by a medium containing small suspended particles. For e.g., smoke or dust in a room, which makes visible a light beam entering a window. The effect is named for the 19th-century British physicist John Tyndall, who first studied it extensively.

70. Twinkling of stars is primarily due to the atmospheric
(a) refraction   (b) reflection
(c) polarisation   (d) dispersion

(*a*) The twinkling of a star is due to the atmospheric refraction of starlight. The starlight, on entering the earth's atmosphere, undergoes refraction continuously before it reaches the earth. The atmospheric refraction occurs in a medium of gradually changing refractive index.

71. Dingko Singh, an Asian Games gold medallist, who died recently, was associated with which one of the following games?
(a) Swimming   (b) Boxing
(c) Archery   (d) Gymnastics

(*b*) Dingko Singh was an Indian boxer who won the gold medal at 1998 Asian Games in Bangkok. He was from the North-eastern Indian state of Manipur. He was awarded the Padma Shri by the Government of India in 2013.

72. Who among the following is the lone Indian Cricketer included in the ICC Hall of Fame special inductions to mark the inaugural ICC World Test Championship Final?
(a) Dilip Vengsarkar
(b) Ravi Shastri
(c) Mohinder Amarnath
(d) Vinoo Mankad

(*d*) Vinoo Mankad is the lone Indian Cricketer included in the ICC Hall of Fame special inductions to mark the inaugural ICC World Test Championship Final. The BCCI commemorated Vinoo Mankad's contribution to Indian cricket, naming the domestic Under-19 One day tournament as the Vinoo Mankad Trophy.

73. Which one of the following nations is not a permanent member of G7?
(a) India   (b) Canada
(c) France   (d) Italy

(*a*) India is not the permanent member of G7. The Group of Seven is an inter-governmental political forum consisting of Canada, France, Germany, Italy, Japan, the United Kingdom and the United States.

74. Operation Olivia, an initiative to protect Olive Ridley turtles, is undertaken by
(a) Indian Navy
(b) Indian Coast Guard
(c) Ministry of Environment Forest and Climate Change
(d) Ministry of Earth Sciences

(*b*) Operation Olivia, particles. an initiative to protect Olive Ridley turtles, is undertaken by Indian Coast Guard. Operation Olivia was started by the Indian Coast Guard (ICG), first in early 1980s. This operation helps in protecting Olive Ridley turtles every year when they start nesting along Odisha coast for breeding in months of November to December.

75. Who among the following is the winner of French Open 2021 Men's Singles Final?
(a) Novak Djokovic
(b) Rafael Nadal
(c) Stefanos Tsitsipas
(d) Daniil Medvedev

(*a*) Novak Djokovic is the winner of the French Open 2021 Men's Singles Final.

Novak Djokovic defeated Stefanos Tsitsipas in the final, 6–7, 2–6, 6–3, 6–2, 6–4 to win the Men's Singles tennis title at the 2021 French Open.He is from Serbia and is currently (as of November, 2021) the No.1 Ranked tennis player in the world.

76. Economist Rebeca Grynspan, who recently became the first woman to be appointed as the head of the United Nations Conference on Trade and Development (UNCTAD), hails from
(a) Germany   (b) Poland
(c) Costa Rica   (d) Mexico

(*c*) Economist Rebeca Grynspan, who recently became the first woman to be appointed as the head of the United Nations Conference on Trade and Development (UNCTAD), hails from Costa Rica. UNCTAD is a permanent intergovernmental body established by the United Nations General Assembly in 1964. Its headquarters are located in Geneva, Switzerland. It also have offices in New York and Addis Ababa.

77. 'Belt and Road Initiative' is a foreign policy initiative of
(a) China   (b) USA
(c) Canada   (d) Japan

(*a*) Belt and Road Initiative' is a foreign policy initiative of China. It is also known as One Belt One Road or OBOR and is a global infrastructure development strategy adopted by the Chinese government in 2013 to invest in nearly 70 countries and international organizations.

78. Which one of the following is the theme of the International Day of Yoga 2021?
(a) Yoga at home and yoga with family
(b) Yoga for climate action
(c) Yoga for well-being
(d) Yoga for peace.

(*c*) Yoga for well-being is the theme of the International Day of Yoga 2021. The International Day of Yoga has been celebrated annually on 21st June since 2015, following its inception in the United Nations General Assembly in 2014. Yoga is a physical, mental and spiritual practice which originated in India.

79. Kenneth Kaunda, who died recently at the age of 97, was the former President of
(a) Brazil   (b) Mexico
(c) Zambia   (d) South Sudan

(*c*) Kenneth Kaunda, who died recently at the age of 97, was the former President of Zambia. He was the first

President of Zambia from 1964 to 1991. He was at the forefront of the struggle for independence from British rule. Zambia, in southern Africa, is a landlocked country of rugged terrain and diverse wildlife, with many parks and safari areas.

80. The maiden Indian Navy - European Union Naval Force (IN-EUNAVFOR) Exercise (2021) was conducted in
(a) Gulf of Mannar
(b) Gulf of Aden
(c) Gulf of Khambhat
(d) Gulf of Aqaba

⊘ (**b**) Indian Navy - European Union Naval Force (IN-EUNAVFOR) Exercise (2021) was conducted in Gulf of Aden. Along with Indian Navy, other naval forces are from Italy, Spain and France.The naval exercise included advanced air defence and anti-submarine exercises, tactical manoeuvres, Search & Rescue, and other maritime security operations.

81. The pH value of Milk of Magnesia is approximately
(a) Zero   (b) 7
(c) 10   (d) 14

⊘ (**c**) pH value of Milk of Magnesia is approximately 10. It is a base and is used to neutralize the excess acid in the stomach.

82. Which one of the following compounds is used in 'black and white' photography?
(a) AgF   (b) AgBr
(c) AgCl   (d) Ag_2SO_4

⊘ (**b**) AgBr (Silver Bromide) is used in black and white photography. Its use in black and white photography is an example of Photochemical reaction. Silver chloride is sensitive to light and breaks down to bromide form metallic silver, which appears black.

83. What is the number of water molecules present in a Ferrous Sulphate crystal?
(a) 1   (b) 3
(c) 5   (d) 7

⊘ (**d**) 7 water molecules are present in a Ferrous Sulphate crystal known as the water of crystallization. The colour in ferrous sulphate is due to the presence of water molecule in it.

84. What is the colour of the precipitate obtained by passing CO_2 gas through lime water?
(a) Green   (b) Blue
(c) White   (d) Brown

⊘ (**c**) White colour precipitate is obtained by passing CO_2 (carbon dioxide) gas through lime water. Carbon dioxide reacts with limewater (a solution of calcium hydroxide, $Ca(OH)_2$, to form a white precipitate (appears milky) of calcium carbonate, $CaCO_3$. Adding more carbon dioxide results in the precipitate dissolving to form a colourless solution of calcium hydrogencarbonate.

85. Which one of the following pairs of elements is liquid at room temperature and at normal pressure?
(a) Gallium and Bromine
(b) Mercury and Bromine
(c) Gallium and Mercury
(d) Gallium and Caesium

⊘ (**b**) The two liquid elements at room temperature and at normal pressure are Mercury (symbol Hg and atomic number 80) and Bromine (symbol Br and atomic number 35). Mercury is the only metal that is a liquid at room temperature.

86. Which one of the following methods can be used to separate anthracene from a mixture of salt and anthracene?
(a) Distillation   (b) Sublimation
(c) Evaporation   (d) Chromatography

⊘ (**b**) Sublimation is the conversion between the solid and the gaseous phases of matter, with no intermediate liquid stage. Thus, Sublimation can be used to separate anthracene from a mixture of salt and anthracene.

87. Shoots of plant show upward movement and it can be designated to be
(a) Negatively phototropic
(b) Positively chemotropic
(c) Positively hydrotropic
(d) Negatively geotropic

⊘ (**d**) The tendency of plant stems and other parts to grow upwards. 'This is called negative geotropism because the plant is growing away from the force of gravity. A negative geotropism is a turning away from the earth, such as by a plant stem that grows upward.

88. Which one of the following statements about the process of photosynthesis is correct ?
(a) Chemical energy is converted into light energy.
(b) Carbon dioxide is oxidised to form carbohydrate.
(c) Water molecule splits into hydrogen and oxygen.
(d) Light energy is directly used to split water.

⊘ (**d**) Photosynthesis is a process used by plants and other organisms to convert light energy into chemical energy that, through cellular respiration, can later be released to fuel the organism's activities. Here, light energy is directly used to split water.

89. How are evergreen plants with woody stems having naked seed classified?
(a) Angiosperms
(b) Monocotyledons
(c) Pteridophytes
(d) Gymnosperms

⊘ (**d**) Gymnosperms are woody plants, either shrubs, trees, or, rarely, vines (some gnetophytes). They differ from flowering plants in that the seeds are not enclosed in an ovary but are exposed (naked seeds) within any of a variety of structures, the most familiar being cones.

90. Which one of the following tissues is known as basic packing tissue and found" in xylem and phloem?
(a) Collenchyma   (b) Parenchyma
(c) Sclerenchyma   (d) Vessels

⊘ (**b**) Parenchyma forms the bulk of plant ground tissue, where they may be specialised to function in photosynthesis, storage, or transport. Parenchyma is integral to vascular tissue, where it provides a route of exchange for materials within and between the xylem and the phloem.

91. Which one from among the following planets is largest in size?
(a) Earth   (b) Venus
(c) Mars   (d) Mercury

⊘ (**a**) Size of the planets in order from smallest to largest is Mercury, Mars, Venus, Earth, Neptune, Uranus, Saturn, and Jupiter. Thus, Earth is the bigger in size as compared to Mercury,Mars and Venus.

92. Match List I with List-II and select the correct answer using the codes given below the lists :

List I (Place)	List II (Feature)
A. Mumbai	1. Queen of the Arabian Sea
B. Visakha-patnam	2. Biggest port of India
C. Chennai	3. Land-locked harbour
D. Kochi	4. Oldest port on the Eastern Coast

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        A  B  C  D              A  B  C  D
(a)     2  4  3  1      (b)     2  3  4  1
(c)     1  3  4  2      (d)     1  4  3  2
```

⊙ (**b**) The correct matching is
A-2,B-3,C-4,D-1.

Mumbai Port is India's largest port by size and shipping traffic. Located in West Mumbai on the western coast of India, the Mumbai Port is situated in a natural harbor.

Visakhapatnam port is the land locked port in India and the only major port of Andhra Pradesh. It is India's third largest state-owned port by volume of cargo handled and largest on the Eastern Coast.

Chennai is the oldest port on the eastern coast of India.It is formerly known as Madras Port, is the second largest container port of India, behind Mumbai's Nhava Sheva. The port is the largest one in the Bay of Bengal.

Kochi port is known as the 'Queen of Arabian Sea' as it was an important trade center for spices on the western coast of India from the 14th century.

93. Which one of the following is a crater lake in India?
(a) Lonar lake
(b) Sambhar lake
(c) Chilika lake
(d) Vembanad lake

⊙ (**a**) Lonar Lake, also known as Lonar crater, is a notified National Geo-heritage Monument situated in the state of Maharashtra and is a crater lake. It was created due to a meteorite. It is the only major hoverback in the basalt.

94. Fine-grained bed of ephemeral lake in a desert is also known as
(a) Playa   (b) Oasis
(c) Drumlin   (d) Natural levee

⊙ (**a**) Playa is a fine-grained bed of ephemeral lake in a desert. A playa is a dry, vegetation-free, flat area at the lowest part of an undrained desert basin. It is a location where ephemeral lakes form during wet periods, and is underlain by stratified clay, silt, and sand, and commonly, soluble salts.

95. Which one of the following factors does not affect the distribution of groundwater?
(a) Amount of precipitation
(b) Rate of evaporation
(c) Ability of the ground surface to allow water to infiltrate into the groundwater system
(d) Distance from the sea

⊙ (**d**) The factors affecting the occurrence and distribution of ground water includes eight criteria; land use, soil, slope, rainfall, elevation, lithology, lineament density and drainage density.

Thus amount of precipitation (rainfall), Rate of evaporation and the infiltration of the water to the ground (drainage density) are relevant factors. Distance from sea has no bearing on distribution of ground water.

96. The Constitution (35th Amendment) Act of 1974 is related to which one of the following States?
(a) Mizoram
(b) Sikkim
(c) Nagaland
(d) Arunachal Pradesh

⊙ (**b**) 35th amendment to the constitution is related to the state of sikkim.It gives effect to the wishes of the people of Sikkim for strengthening Indo-Sikkim co-operation and inter-relationship, the Bill seeks to amend the Constitution to provide for the terms and conditions of association to Sikkim with the Union.

97. Which one of the following best describes the electoral system of India?
(a) First-Past-the-Post System
(b) Proportional Representation
(c) Mixed System
(d) General Ticket

⊙ (**a**) First past the post system is the electoral system of India. In a first-past-the-post electoral system , voters cast their vote for a candidate of their choice, and the candidate who receives the most votes wins (even if the top candidate gets less than 50%, which can happen when there are more than two popular candidates.

98. Which one of the following is a non-justiciable right?
(a) Right to adequate livelihood
(b) Right against exploitation
(c) Right of accused
(d) Right to life and personal liberty

⊙ (**a**) Right to adequate means of livelihood under Article 39 is a non-justiciable right mentioned in part IV as Directive principal of state policy.

Right against exploitation (Article 23), Right of accused(Article 20) and Right to life and perosnal liberty (Article 21) are all fundamental rights enshrined in part III of the Constitution and are justiciable in nature under Article 32 of the Indian Constitution.

99. Which one of the following is not a writ?
(a) Mandamus   (b) Habeas Corpus
(c) Certiorari   (d) Severability

⊙ (**d**) Severabilty is not a writ. There are 5 writs mentioned under Article 32 viz, Habeas Corpus; Mandamus; Prohibition; Certiorari; Quo-Warranto. Under these Writs, Judiciary can protect the rights of the aggrieved parties.

100. Which one of the following is not a part of Fundamental Rights?
(a) Right to education
(b) Right to establish educational institutions by minorities
(c) Right to be conferred with titles
(d) Right against untouchability

⊙ (**c**) Right to be conferred with titles is not a fundamental rights as Article 18(1) abolishes all titles. It prohibits the State to confer titles on anybody whether a citizen or a non-citizen.

Military and academic distinctions are, however, exempted from the prohibition.

101. Which one of the following is not a function of the Constitution of India?
(a) To ensure participation of good people in politics.
(b) To guarantee a set of rights to citizens.
(c) To define the power of the different organs of government.
(d) To create conditions for a just society.

⊙ (**a**) To ensure participation of good people in politics is not a function of the constitution of India as it provides everyone an opportunity as a freedom to participate in democratic politics.

Rights of the citizens are mentioned in Part III, Separation of powers of executive, judiciary and legislature is also defined in the constitution and Directive principles of state policy mentions provisions for a just society.

102. Which one of the following statements about biodiversity is not correct?
(a) The term 'biodiversity' was coined by Walter G. Rosen in 1986.
(b) The term 'biodiversity hotspots' was coined by Norman Myers in 1988.
(c) The regions having richest biodiversity are called 'biodiversity hotspots'.
(d) More than 100 hotspots of biodiversity are identified in the world.

(**d**) Statement (d) is incorrect. There are 36 biodiversity hotspots in the world. These regions have very high level of species richness or high level of biodiversity concentration. The term biodiversity was given by walter G. Rosen in 1986, while 'Biodiversity Hotspot' term was joined by Norman Myers in 1988.

103. The Earth's atmosphere is mainly heated by which one of the following?
(a) Short wave solar radiation
(b) Reflected solar radiation
(c) Long wave terrestrial radiation
(d) Scattered solar radiation

(**c**) Earth's atmosphere is mainly heated by Long wave terrestrial radiation.The insolation received by the earth is in the form of shortwaves. The earth after being heated itself becomes a radiating body and it radiates energy to the atmosphere in the long waveform.

104. Which one of the following is the correct sequence of layers as we move from the Earth's surface upwards?
(a) Troposphere, Stratosphere, Thermosphere, Mesosphere
(b) Troposphere, Stratosphere, Mesosphere, Thermosphere
(c) Thermosphere, Mesosphere, Stratosphere, Troposphere
(d) Stratosphere, Mesosphere, Troposphere, Thermosphere

(**b**) Moving upward from ground level, the correct sequence of atmospheric layers are the troposphere (upto 12 km), stratosphere (located between 12-50 km), mesosphere (located between 50-80 km), thermosphere (80-700 km) and exosphere (located between 700-10000 km).

105. Which one of the following is not a soil forming factor?
(a) Parent material
(b) Topography
(c) Climate
(d) Human habitation

(**d**) Human habitation is not a soil forming factor. The whole soil, from the surface to its lowest depths, develops naturally as a result of five factors. The five factors are: 1. parent material, 2. relief or topography, 3. organisms (including humans), 4. climate, and 5. time.

106. Which one of the following best describes the Lithosphere?
(a) Upper and lower mantle
(b) Crust and core
(c) Crust and upper mantle
(d) Lower mantle and core

(**c**) The lithosphere includes the crust and, below, the uppermost layer of the mantle; it floats on the weaker asthenosphere.The lithosphere is the outer solid shell of the Earth. As the cooling surface layer of the Earth's convective system, the lithosphere thickens over time.

107. Which one of the following countries has the maximum time difference from Greenwich Mean Time (GMT) ?
(a) India   (b) Nepal
(c) Sri Lanka   (d) Bhutan

(**d**) Bhutan (GMT+6) has the maximum time difference from the Greenwhich Mean Time. India (GMT+5:30), Nepal (GMT +5:45) and Sri Lanka (GMT+5:30) are all behind Bhutan and closer to GMT.

108. Which one of the following is not a feature of the Constitution of India?
(a) It provides a set of basic rules.
(b) It specifies the power of the Government.
(c) It is the supreme law of the land.
(d) It specifies the supremacy of the judiciary.

(**d**) Primacy of judiciary is not one of the provisions of the Indian constitution. It provides a set of rights in Part III of the constitution, it also provides a basic set of rules in the form separation of powers between different organs of the government and is the supreme law of the land and nothing in contravention of the constitution can be done in the country.

109. Which one of the following statements about the Attorney General of India is not correct?
(a) He has the right of audience only in the Supreme Court of India.
(b) He shall receive such remuneration as the President may determine.
(c) He shall be qualified to be appointed as a Judge of the Supreme Court.
(d) He shall give advice to the Government of India on all legal matters.

(**a**) Statement (a) is not correct.
The Attorney General for India is the Indian government's chief legal advisor, and is its principal Advocate before the Supreme Court of India.
The Attorney General has the right of audience in all Courts in India as well as the right to participate in the proceedings of the Parliament, though not to vote.

110. Who among the following described the Directive Principles of State Policy as the novel feature of the Constitution of India?
(a) Jawaharlal Nehru
(b) Rajendra Prasad
(c) S.N. Mukherjee
(d) B.R.Ambedkar

(**d**) Dr. B.R. Ambedkar described the Directive Principles of State Policy as the novel feature of the Constitution of India. Directive Principle of State Policy's ("DPSP") are a set of social and economic obligation imposed on the Government (Union & State) to establish a welfare society.

111. Cornea in human eye
(a) is a light sensitive screen.
(b) is a muscular diaphragm.
(c) contains blood vessels.
(d) is composed of proteins and cells.

(**d**) Cornea in human eye is composed of protein and cells. The cornea is made up of cellular and acellular components. The cellular components include the epithelial cells, keratocytes, and endothelial cells. The acellular component includes collagen and glycosaminoglycans. The epithelial cells are derived from epidermal ectoderm.

112. Power of a lens of focal length 25 cm is
(a) + 2.5 Dioptre   (b) + 3 Dioptre
(c) + 4 Dioptre   (d) + 5 Dioptre

(**c**) Power of a lens of focal length 25 cm is +4 Dioptre.
First we will convert the focal length in meters.
$\Rightarrow$ $25 \text{ cm} = 0.25 \text{ m}$
Now, substituting the value of focal length in the formula of power of lens we get
$$P = 10.25 \text{ m}^{-1}$$
$\Rightarrow$ $P = 4 \text{ m}^{-1}$
m-1 is also expressed as diopters or D
As it is a convex lens so its power is positive. Thus, the power of the lens is +4D.

113. What is the total number of chambers in the stomach of domestic animals like cattle, buffalo, goat and sheep ?
(a) Four   (b) Two
(c) One   (d) Three

(**a**) There are four chambers in stomach of domestic animals like cattle, buffalo, goat and sheep. Ruminant stomachs have four compartments: the rumen, the reticulum, the omasum and the abomasum.

114. Intestinal bacteria are main source of which one of the following vitamins?
(a) Vitamin E   (b) Vitamin C
(c) Vitamin B_{12} (d) Vitamin A

⊙ (**c**) Intestinal bacteria are main source of Vitamin B_{12}. Vitamin B_{12} is readily absorbed in the last part of the small intestine (ileum), which leads to the large intestine. However, to be absorbed, the vitamin must combine with intrinsic factor, a protein produced in the stomach.

115. Which one of the following hormones is responsible for the development of female sexual characters?
(a) Prolactin   (b) Estrogen
(c) Oxytocin   (d) Progesterone

⊙ (**b**) Estrogen is a hormone responsible for development of female sexual characters. It acts on organs in the body (including the breast and uterus) and as a chemical messenger in the brain, controlling key aspects of the reproductive system, including childbirth and lactation, and aspects of human behaviour.

116. Spherical mirror formula relating an object distance 'u', image distance 'v' and focal length of mirror 'f' may be applied to a plane mirror when
(a) focal length goes to infinity.
(b) focal length goes to zero.
(c) image distance goes to zero.
(d) image distance goes to infinity.

⊙ (**a**) Spherical mirror formula relating an object distance u, image distance v and focal length of mirror f may be applied to a plane mirror when focal length goes to infinity. For plane Mirror focal length is infinite. Thus power = 1/focal length results in power being Zero.

117. Nuclear energy is generated by
(a) nuclear fission and its expression was proposed by Einstein.
(b) nuclear fission and its expression was proposed by Rutherford.
(c) nuclear fusion and its expression was proposed by Bohr.
(d) nuclear fusion and its expression was proposed by Heisenberg.

⊙ (**b**) Nuclear energy is generated by nuclear fission and its expression was proposed by Rutherford. A nuclear reactor is driven by the splitting of atoms, a process called fission, where a particle (a 'neutron') is fired at an atom, which then fissions into two smaller atoms and some additional neutrons.

118. Reverberation is a phenomenon associated with a
(a) multiple refraction of sound.
(b) multiple reflection of sound.
(c) single refraction of sound.
(d) single reflection of sound.

⊙ (**b**) Reverberation is a phenomenon associated with a multiple reflection of sound. Reverberation is the phenomenon of persistence of sound after it has been stopped as a result of multiple reflections from surfaces such as furniture, people, air, etc.

These reflections build up with each reflection and decay gradually as they are absorbed by the surfaces of objects in the space enclosed.

119. Which among the following is true for propagation of sound waves?
(a) Sound can travel in vacuum and it is a transverse wave in air.
(b) Sound cannot travel in vacuum and it is a longitudinal wave in air.
(c) Sound can travel in vacuum and it is a longitudinal wave in air.
(d) Sound cannot travel in vacuum and it is a transverse wave in air.

⊙ (**b**) Statement (b) is true. Sound cannot travel in vacuum and it is a longitudinal wave in air. Sound propagates through air or other mediums as a longitudinal wave, in which the mechanical vibration constituting the wave occurs along the direction of propagation of the wave.

120. A tennis ball is thrown in the vertically upward direction and the ball attains a maximum height of 20 m. The ball was thrown approximately with an upward velocity of
(a) 8 m/s   (b) 12 m/s
(c) 16 m/s   (d) 20 m/s

⊙ (**d**) In this question
$$H\ (max) = 20m$$
and $$g = 10\ m/s^2.$$
According to formula
$$Height(max) = \frac{u^2}{g}$$
Hence, the answer will be 20 m/s.

121. An object of mass 2000 g possesses 100 J kinetic energy. The object must be moving with a speed of
(a) 10.0 m/s   (b) 11.1 m/s
(c) 11.2 m/s   (d) 12.1 m/s

⊙ (**a**) In this question,
Mass (M) = 2000 g
Kinetic energy (K) = 100 J

According to formula,
$$K = \frac{1}{2}mv^2$$
$$100 = \frac{1}{2} \times 2000 \times v^2$$
$$100 = 1000 \times v^2$$
$$v^2 = 100$$
$$v = 10.0\ m/s$$
Hence, the correct answer is 10.0 m/s

122. Which one of the following ions is not iso-electronic with F^-?
(a) O^{2-} (b) Na^+
(c) Ne   (d) N^-

⊙ (**d**) N negative is not iso-electronic with F^- because Isoelectronic species are known as atoms or ions that have the same number of electrons.
In isoelectronic species, there the number of electrons would be equal but the elements would be different. In other words, ions and atoms which are having equal numbers of electrons are called the isoelectronic species.

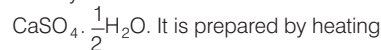**123.** What is the total number of covalent bonds in methanol?
(a) 3   (b) 4
(c) 5   (d) 6

⊙ (**c**) There are total number of 5 covalent bonds in methanol as carbon is attached to 3 H and 1 OH with 4 covalent bonds. O and H in OH are bonded with 1 covalent bond. So there are total 5 covalent bonds.

124. Which one of the following is the chemical formula of Plaster of Paris?
(a) $CaSO_4 \cdot \frac{1}{2}H_2O$

(b) $CaSO_4 \cdot 2H_2O$
(c) $CaSO_4 \cdot 5H_2O$
(d) $CaSO_4 \cdot 4H_2O$

⊙ (**a**) Plaster of paris is calcium sulphate hemihydrate. Its chemical formula is: $CaSO_4 \cdot \frac{1}{2}H_2O$. It is prepared by heating gypsum to a temperature of 100°C in a kiln; it loses 3/4th of its water of crystallisation and forms plaster of paris.

**125.** The unit of the ratio between thrust and impulse is same as that of
(a) frequency
(b) speed
(c) wavelength
(d) acceleration

⊙ (**a**) Thrust is force. Impulse is the integral of a force, F, over the time interval, t, for which it acts.

Thus the ratio of thrust i.e F and Impulse would give time inverse which is equivalent to Frequency.

126. Which one of the following figures correctly shows the path of a ray of light through a glass prism?

(a)   (b)

(c)   (d)
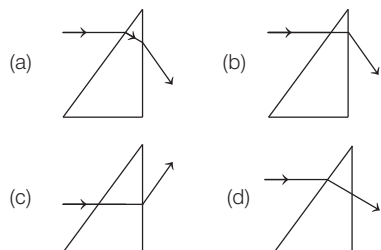
⊙ (**a**) Figure a correctly shows the path of a ray of light through a glass prism as it shows the rays moving towards normal while going from rarer to denser medium and away from normal while moving from denser to rarer medium.

127. When a light beam falls on a triangular glass prism, a band of colours is obtained. Which one of the following statements is correct in this regard?
(a) Red light bends the most, as the refractive index of glass for red light is greatest.
(b) Red light bends the most, as the refractive index of glass for red light is lowest.
(c) Violet light bends the most, as the refractive index of glass for violet light is greatest.
(d) Violet light bends the most, as the refractive index of glass for violet light is lowest.

⊙ (**c**) Each beam of light, with its own particular wavelength (or color), is slowed differently by the glass.The light waves are refracted as they enter and leave the prism. The shorter the wavelength of the light, the more it is refracted.Violet light is refracted the most as it has the shortest wavelength causing the coloured light to spread out to form a spectrum.

128. The image of an object formed by a plane mirror is
(a) erect, real and larger.
(b) erect, virtual and same size.
(c) inverted, virtual and same size.
(d) inverted, real and smaller.

⊙ (**b**) The image formed by a plane mirror is always virtual (meaning that the light rays do not actually come from the image), erect (upright), and of the same shape and size as the object it is reflecting.

A virtual image is a copy of an object formed at the location from which the light rays appear to come.

129. Which one of the following is not a conservative force?
(a) Frictional force
(b) Electric force
(c) Gravitational force
(d) Spring force

⊙ (**a**) Frictional forces are not conservative forces because the amount of work done by friction depends on the path. One can associate a potential energy with a conservative force but not with a non-conservative force. A conservative force is one for which the work done is independent of path.

130. A negative work is done when an applied force F and the corresponding displacement S are
(a) perpendicular to each other.
(b) parallel to each other.
(c) anti-parallel to each other.
(d) equal in magnitude.

⊙ (**c**) A negative work is done when an applied force and the corresponding displacement S are anti-parallel to each other. The work W that a force F does on an object is the product of the magnitude F of the force, times the magnitude d of the displacement, times the cosine of the angle θ between them. In symbols, $W = fd \cos \theta$.

So, if angle is 180 degree, W will be negative.

131. Which of the following statements about the Indian Councils Act of 1861 is/are correct?
1. It enlarged the Governor General's Council for the purpose of making laws.
2. The Governor General was not authorised to increase the number of members.
Select the correct answer using the codes given below:
(a) Only 1   (b) Only 2
(c) Both 1 and 2   (d) Neither 1 nor 2

⊙ (**a**) Statement 1 is correct. Indian Councils Act of 1861 enlarged the Governor General's Council for the purpose of making laws. For the executive functions of the Council, a fifth member was added. Now there were five members for home, military, law, revenue, and finance. (A sixth member for public works was added in 1874.)

132. Which one of the following statements about Tattvabodhini Patrika is correct?
(a) It promoted a systematic study of India's past in the Bengali language.
(b) It promoted a systematic study of India's past through Sanskrit sources.
(c) It promoted a systematic study of India's past through Persian sources.
(d) It promoted a systematic study of India's past through Western sources.

⊙ (**a**) Statement a is correct. Tattvabodhini Patrika promoted a systematic study of India's past in the Bengali language. On 6 October 1839 Debendranath Tagore established Tattvaranjini Sabha which was shortly thereafter renamed the Tattwabodhini (Truth-seekers) Sabha. The objective of Tattvabodhini Sabha was to encourage religious inquiries and disseminate the essence of Upanishads.

133. In which one of the following years did the British demarcate a large area of land as Damin-i-koh for settling the Santhals?
(a) 1810   (b) 1793
(c) 1885   (d) 1832

⊙ (**d**) It was in the year 1832 that the British demarcated a large area of land as Damin-i-Koh for settling the Santhals. Damin-i Koh is in the area of Sahebganj, Pakur, and Godda districts in the Jharkhand. British persuaded the local tribe of Rajmahal Hills i.e. Santhals to live in the foothills of Rajmahal by giving land to them. "Damin-i-Koh" is a Persian term that means 'skirts of the hills'.

134. The Tropic of Cancer does not pass through which one of the following States?
(a) Manipur   (b) West Bengal
(c) Gujarat   (d) Jharkhand

⊙ (**a**) The tropic of cancer does not pass through the state of Manipur.The Tropic of Cancer passes through eight states in India: Gujarat (Jasdan), Rajasthan (Kalinjarh), Madhya Pradesh (Shajapur), Chhattisgarh (Sonhat), Jharkhand (Lohardaga), West Bengal (Krishnanagar), Tripura (Udaipur) and Mizoram (Champhai).

135. Which of the following pairs of crop and product is/are correctly matched?
1. Food crop   Ragi
2. Cash crop   Jute
3. Plantation crop   Coconut
Select the correct answer using the codes given below:
(a) Only 1   (b) 2 and 3
(c) 1, 2 and 3   (d) Only 3

⊙ (**c**) All are correctly matched.

'Ragi' also known as Finger Millet, is a robust, popular food crop and marvelous grain crop in India. It is called dryland crops. Jute is also known as the golden fiber and is India's major cash crop. Jute fiber is obtained from the inner bark of the jute plant. It is used in making gunny bags, mats, ropes, yarn, carpets and other artifacts.

The term plantation crop refers to those crops which are cultivated on an extensive scale in contiguous areas, owned and managed by an individual or a company. The crops include tea, coffee, rubber, cocoa, **coconut**, arecanut, oil palm, palmyrah and cashew.

136. Which one of the following coalfields is not located in Jharkhand?
(a) Jharia　　　(b) Ramgarh
(c) Deogarh　　(d) Umaria

⊙ (**d**) Umaria Coalfield is located in Umaria district in the Indian state of Madhya Pradesh in the valley of the Umrer River, a tributary of the Son River.

137. Which one of the following is the longest parallel of latitude?
(a) Tropic of Cancer
(b) Tropic of Capricorn
(c) Arctic Circle
(d) Equator

⊙ (**d**) Equator is the longest parallel of latitude. The Equator is at 0°, and the North Pole and the South Pole are at 90° north and 90° south, respectively. The Equator is the longest circle of latitude and is the only circle of latitude which also is a great circle.

138. The periodic rise and fall of ocean water in response to gravitational forces is called
(a) Current　　(b) Waves
(c) Tides　　　(d) Tsunami

⊙ (**c**) The periodic rise and fall of ocean water in response to gravitational forces is called Tides. Tides are long-period waves that move through the oceans in response to the forces exerted by the moon and sun. Tides originate in the oceans and progress toward the coastlines where they appear as the regular rise and fall of the sea surface.

139. Which of the following statements about the first Indian Factory Act passed in 1881 is/are correct?
1. The Act dealt primarily with the problem of child labour.
2. The Act laid down that children between 7 years and 12 years of age would not work for more than 9 hours a day.

Select the correct answer using the code given below:
(a) Only 1　　(b) Only 2
(c) Both 1 and 2　(d) Neither 1 nor 2

⊙ (**c**) Both 1 and 2 are correct.In 1875, a committee was appointed to inquire the conditions of all factories .After effect of this committee , factory act 1881 was adopted. The act prohibited the employment of children under the age of 7. Children between the ages of 7-12 were to work for a maximum of 9 hours. It was passed during viceroyship of Lord Rippon.

140. Which of the following statements about Mahatma Gandhi's anti-untouchability campaign is/are correct?
1. The All India Harijan Sevak Sangh was founded for this purpose.
2. The campaign was to root out untouchability, since it did not enjoy the sanction of the Hindu Shastras.

Select the correct answer using the codes given below:
(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

⊙ (**a**) Statement 1 is correct. In 1932, Gandhi founded the Harijan Sevak Sangh as part of his efforts to eradicate the concept of untouchability from India's caste system.

Statement 2 is incorrect-Untouchability did enjoy the sanction of shastras as it is based on the caste-based system mentioned as varna system in Rig Veda dividing society into four classes.

141. The founders of the Paramhansa Mandali founded in Maharashtra believed in which one of the following?
(a) Social distancing along casts lines
(b) Glorification of caste system
(c) Forbidding widow remarriage
(d) One God and in breaking caste rules

⊙ (**d**) The founders of the Paramhansa Mandali founded in Maharashtra believed in One God and in breaking caste rules.

Paramahansa Mandali was a secret socio-religious group, established in 1849, in Bombay and is closely related to Manav Dharma Sabha which was founded in 1844. It was started by Durgaram Mehtaji, Dadoba Pandurang, and a group of his friends.

142. Which one of the following statements about the ishtahars issued 'during the Revolt of 1857 is correct?
(a) They glorified the Muslim rule in India.
(b) They glorified the co-existence of different communities under the Mughal Empire.
(c) They glorified the message of Islam.
(d) They glorified the role of Queen Victoria.

⊙ (**b**) Statement b is correct. Ishtahars glorified the co-existence of different communities under the Mughal Empire. During the revolt only a few proclamations and 'ishtahars' (notification) were issued by rebel leaders to propagate their ideas and persuade people to join the revolt. Proclamation issued by rebel leaders appealed to all sections of the population irrespective of cast and creed. The rebellion was seen as a war in which both Hindus and Muslims were equally to lose or gain. It was remarkable that during the uprising, the religious division between Hindus and Muslims was hardly noticeable despite the attempt of the British government.

143. During the Industrial Revolution, who among the following designed the 'flying shuttle loom?
(a) Samuel Crompton
(b) Edmund Cartwright
(c) John Kay
(d) Richard Arkwright

⊙ (**c**) John Kay was an English inventor whose most important creation was the flying shuttle, which was a key contribution to the Industrial Revolution. Through this machine one person could operate a shuttle across a very wide loom, which greatly increased the rate of cloth production.

144. Match List I with List II and select the correct answer using the codes given below the lists :

	List I (Dam)		List II (State)
A.	Hirakud	1.	Gujarat
B.	Panchet	2.	Bihar
C.	Kosi	3.	Odisha
D.	Ukai	4.	Jharkhand

Codes
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     A B C D              A B C D
(a)  3 4 2 1       (b)    3 2 4 1
(c)  1 2 4 3       (d)    1 4 2 3
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⊙ (**a**) The correct matching is A-3, B-4, C-2, D-1.

Hirakud Dam Project is a multipurpose scheme intended for flood control, irrigation and power generation. The dam is built across river Mahanadi at about 15 km upstream of Sambalpur town in the state of Odisha.

Panchet dam is built across the Damodar River in Dhanbad District's Panchet area in Jharkhand.

Koshi High Dam, is a multipurpose project proposed to be constructed on the Saptakoshi River of Nepal. The project is primarily aimed to control floods in south-east Nepal and northern Bihar of India.

Ukai dam is the largest multipurpose project so far completed in Gujarat state. The Ukai dam is located across Tapi River near Ukai village of Fort-Songadh taluka in Surat district.

145. Which one of the following features is the result of erosion and deposition work of a river?
(a) Pothole
(b) Oxbow lake
(c) Levee
(d) Rapid

⊙ (**a**) Pothole is the result of erosion and deposition work of a river. Pothole is a cylindrical, bowl-shaped, or irregular hollow that is usually deeper than wide.

It is formed in the rocky bed of a stream by either the grinding action of sediment whirled around by stream eddies or the force of fast flowing water. Potholes usually have spirally grooved surfaces.

146. Which one of the following is not a feature of agriculture in India?
(a) Subsistence agriculture
(b) Pressure of population on agriculture
(c) Dependence upon Monsoon
(d) Predominance of cash crops

⊙ (**d**) Predominance of cash crops is not a feature of agriculture in India.

Contrary to staple and food crops, cash crops form a small part of Indian agriculture and are entirely sold and exported. Indian agriculture is subsistence-based, dependent on Monsoon rains and 1.25 billion people are dependent on only 2.4 percent of arable land in the world.

147. Which one of the following is not a specified Sub-Mission of the National Livestock Mission?
(a) Livestock development
(b) Pig development in North-Western region
(c) Fodder and feed development
(d) Skill development, technology transfer and extension

⊙ (**b**) Pig development in the north-western region is not a specified Sub-Mission of the National Livestock Mission. National Livestock Mission scheme is implemented with the following three Sub-Missions: Sub-Mission on Breed Development of Livestock & Poultry, Sub-Mission on Feed and Fodder development, Sub-Mission on Extension and Innovation.

148. Who among the following wrote the famous text 'A Discourse on Political Economy'?
(a) Montesquieu
(b) Voltaire
(c) Rousseau
(d) Adam Smith

⊙ (**c**) Jean-Jacques Rousseau wrote the famous text 'A Discourse on Political Economy' in 1775. A revolutionary and controversial work, this book is a classic of political theory and a key source of democratic ideals.

149. Which of the following statements about the resolution adopted in the Karachi session of the Indian National Congress with regard to fundamental rights and economic policy is/are correct?

1. It was opposed by Ambalal Sarabhai.
2. Its 20 points included demands for civil liberties and adult suffrage.

Select the correct answer using the codes given below :
(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

⊙ (**b**) Option 2 is correct. The Karachi Resolution was passed by the Indian National Congress at its 1931 Karachi session.

Elections on the basis of Universal Adult Franchise and Basic civil rights of freedom of speech, Freedom of Press, Freedom of assembly, Freedom of association were included in the 20 points put forward in the resolution.

150. Which of the following statements about the Law Commission headed by Lord Macaulay is/are correct ?

1. It attempted to codify the laws.
2. It was opposed to uniform system of Courts.

Select the correct answer using the codes given below:
(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

⊙ (**a**) Option 1 is correct. Lord Macaulay headed the first such Law Commission established in 1834 under the Charter Act of 1833. It recommended codification of the Penal Code, the Criminal Procedure Code, and a few other matters.

The statement (2) is incorrect as it supported the idea of uniform system of courts.