Heat Transfer

Heat may be transported from one point to another by any of three possible mechanisms : conduction, convection, and radiation. We study the rate of energy transfer between bodies due to temperature difference between them.

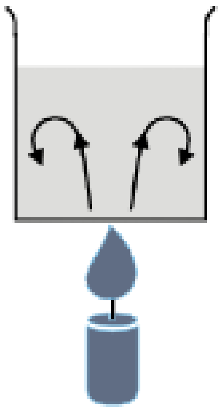
Convection

Convection is the process in which heat is carried from place to place by the bulk movement of a fluid. In liquid and gases, the atoms or molecules can move from point to point. The transfer of heat that accompanies mass transport is called convection.

In forced convection, a fan or pump sets up fluid currents. For examples, a fan blows air, or a pump circulates water in a hot-water heating system in a house.

In free convection, it occurs because the density of a fluid varies with its temperature.

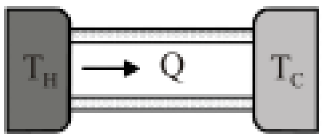
An example of convection currents in a pan of water being heated on a gas burner. The currents distribute the heat from the burning gas to all parts of the water. The direction of convection current is opposite to acceleration due to gravity as shown in figure.



In convection, heat transfer accompanies the movement of a fluid

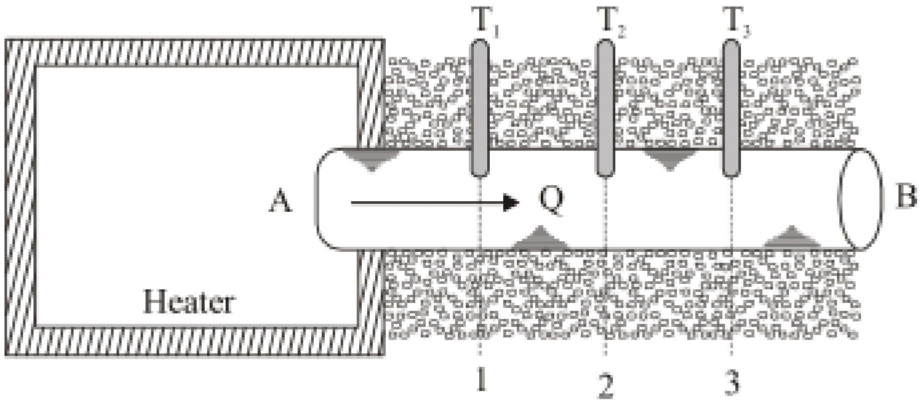
Conduction

A rod whose ends are in thermal contact with a hot reservoir at temperature T_H and a cold reservoir at temperature T_C. The sides of the rod are covered with insulation meterial, so the transport of heat is along the rod, not through the sides. The molecules at the hot reservoir have greater vibrational energy. This energy is trransferred by collisions to the atoms at the end face of the rod. These atoms in turn transfer enery to their neighbors futher along the rod. Such transfer of heat through a substance is called conduction as shown in figure.

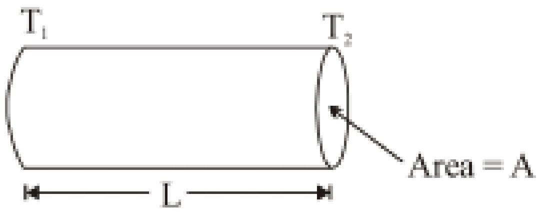


Heat is conducted through an insulated bar whose ends are in thermal contact with two reservoirs

Steady and Transient State :



Consider a metal rod AB, with one end A inserted into a chamber containing a heater with other end B left free and exposed to the surrounding as shown in figure. The rod is thermally insulated sideways with some bad conductor of heat say cotton. Three thermometers are installed in the rod at three distinct sections numbered (1), (2) and (3). Initially, the enitre system is at the room temperature and the three thermometers show the same room temperature. The heater is then switched on. The end A first gets heated up and simultaneously heat is conducted to the adjacent sections towards end B. Due to heat absorption at each sections. The corresponding temperatures start rising with $T_1 > T_2 > T_3$. Such a state, encountered initially is known as a transient state. In this state, the heat coming through end A, is continuously absorbed at each sections with a temperature rise as time elapses. After some time when the temperature of end B becomes equal to that of surrounding and thus becomes constant. Similarly, the temperature of each of the sections of the rod (for example 1, 2, 3) becomes constant or steady. But these steady values at different sections are different.



Consider a portion of the rod of cross sectional area A as shown in figure. Let the temperatures of the two sections separated by a length L be T_1 and T_2 respectively (with $T_1 > T_2$).

Temperature gradient (fall in temperature per unit length) along the length of the rod will be $\frac{T_1 - T_2}{L}$.

Experiments show that the conduction rate (energy transferred per unit time) is given by: Fourier's Law of Heat Conduction

$$H = \frac{\partial Q}{\partial t} = KA\frac{d(-T)}{dx}$$

(Where K : Thermal conductivity of material

H : Thermal current

$\frac{dT}{dx}$: Temperature gradient

A : cross-sectional area of heat path)

The reciprocal of thermal conductivity (K) is called thermal resistivity or thermal specific resistance. Substances having high values of K are good conductors of heat.

Temperature distribution along a conductor :

In order to study conduction in more detail consider figure (i), which shows a metal bar AB whose ends have been soldered into the walls of two metal tanks H and C. Tanks H contains boiling water and C contains ice-water. Heat flows along the bar from A to B and when conditions are steady the temperature θ of the bar is measured at points along its length.

The curve in the upper part of the figure shows how the temperature falls along the bar, less and less steeply from the hot end to the cold. So the temperature gradient decreases from the hot end to the cold.

The figure (ii) shows how the temperature varies along the bar, if the bar is well lagged with a bad conductor, such as asbestos or wool. It now falls uniformly from the hot to the cold end, so the temperature gradient along the bar is constant.

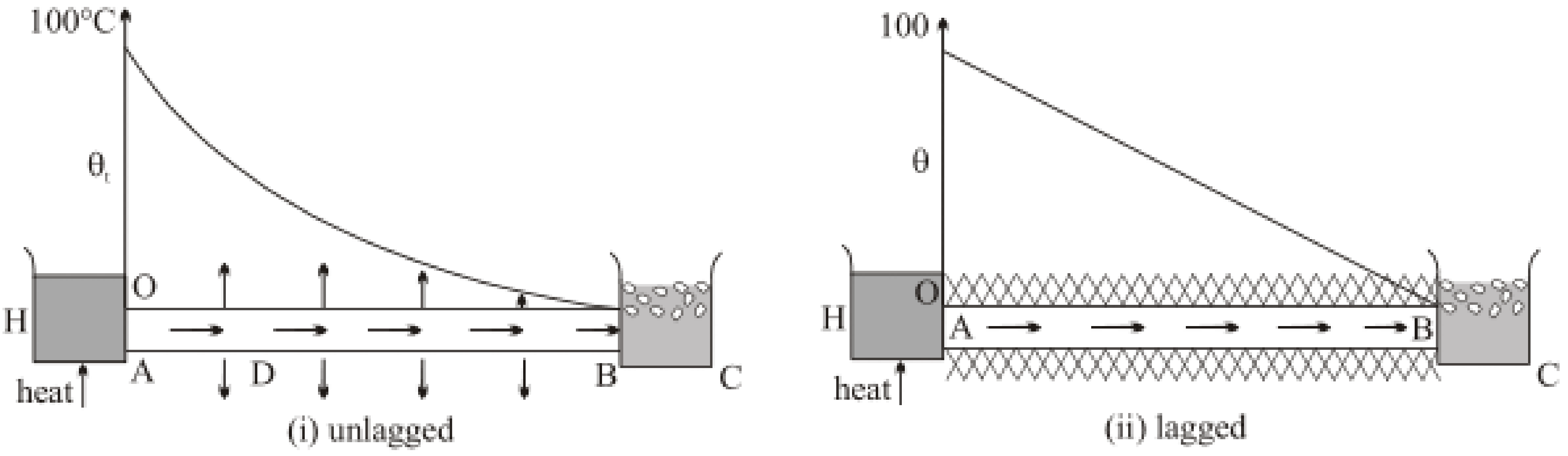
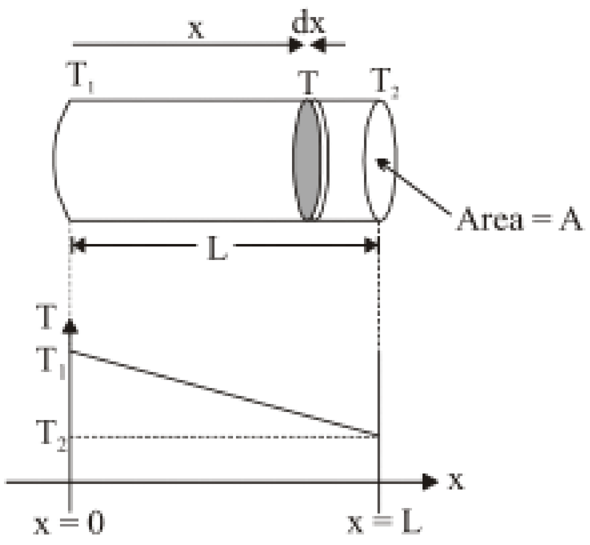


Figure : Temperature fall along lagged and unlagged bars

The difference between the temperature distributions is due to the fact that, when the bar is unlagged, heat escapes from its sides, by convection in the surrounding air, figure (i). The arrows in the figure represent the heat escaping per second from the surface of the bar, and the heat flowing per second along its length. The heat flowing per second along the length decreases from the hot end to the cold. But when the bar is lagged, the heat escaping from its sides is negligible, and the flow per second is now constant along the length of the bar, figure (ii).

Steady State Heat Conduction :



Temperature variation
along length of rod

At steady state, energy transferred through one cross-section of the rod during a certain time interval is equal to the energy transferred by at the other cross-section of the rod during the same time interval.

$$H = \frac{\Delta Q}{\Delta t} = KA\left(\frac{\Delta T}{\Delta x}\right) = KA\left(\frac{T_1 - T_2}{L}\right)$$

Temperature distribution across the rod :

Let at distance x we take element of length dx having a cross-sectional area A and temperature T (As shown in figure). In steady state, rate of heat flow H remains constant

$$H = -KA\frac{dT}{dx}$$

$$\int_{T_1}^{T} dT = -\int_{0}^{x}\frac{H}{KA}dx$$

$$T - T_1 = -\frac{Hx}{KA} \qquad \left(\because \frac{H}{KA} = \frac{T_1 - T_2}{L}\right)$$

$$T = T_1 - \frac{x}{L}(T_1 - T_2)$$

The variation has been plotted above.

Thermal Resistance :

The heat transfer by conduction due to temperature difference has an analogy with flow of electric current through a wire when a potential difference is applied. In that case, electrical resistance is defined as

$$R = \frac{v}{i}$$

Similarly, thermal resistance is defined as

$$R = \frac{(T_1 - T_2)}{H}$$

For a rod having length L , area of cross-section A and thermal conductivity K,

$$R = \frac{(T_1 - T_2)}{H}$$

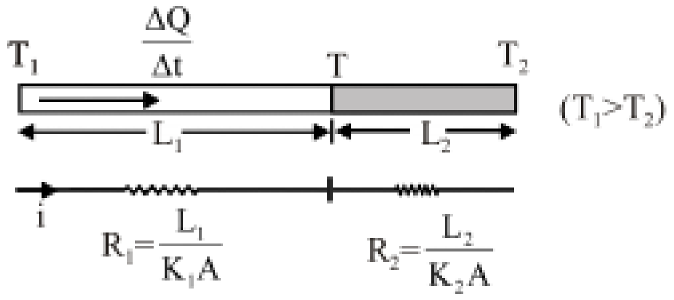
$$= \frac{(T_1 - T_2)}{KA(T_1 - T_2)/L}$$

$$R = \frac{L}{KA}$$

Having calculated the thermal resistance, we can now apply the results of series combination and parallel combination of resistors. It has been explained below.

Composite Rods :

Series Connection : If same heat current are flowing both the rods in steady state, they are said to be in series.



$$(\because\ R_{eq} = R_1 + R_2 = \Sigma R)$$

Where A - cross-section area of rods

T - Temperature at the juction or Interface temperature

K_1 & K_2 - Themal conductivities of rods having lenghts L_1 and L_2 respectively.

In steady state, heat current is constant throughout the rods.

$$i = \frac{\Delta Q}{\Delta t} = \frac{T_1 - T}{R_1} = \frac{T - T_2}{R_2}$$

$\therefore\ T_1 - T = iR_1$ ...(i)

$T - T_2 = iR_2$ ...(ii)

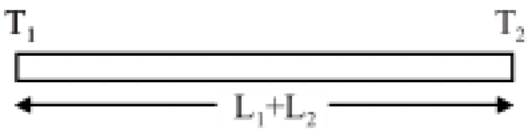
From (i) & (ii)

$$\frac{T_1 - T_2}{R_1 + R_2} = i \quad \text{and} \quad T = \frac{(T_1 R_2 + T_2 R_1)}{R_1 + R_2}$$

$$i = \frac{\Delta T}{R_{eq}}, \text{ in series } R_{eq} = R_1 + R_2$$

Equivalent conductivity of composite Rods (K_{eq}) :

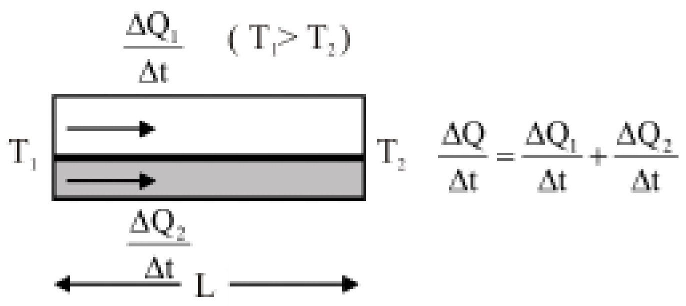
If this rod is replaced by a single rod, then $i = (T_1 - T_2)/R_{eq}$

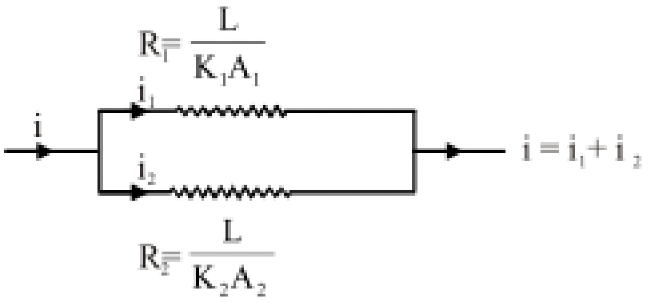


$$\therefore \quad R_{eq} = R_1 + R_2 = \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} = \frac{L_1 + L_2}{K_{eq} A}$$

$$K_{eq} = \frac{L_1 + L_2}{\dfrac{L_1}{K_1} + \dfrac{L_2}{K_2}}$$

Parallel Connection : If the tow rods have the same temperature difference across it, they are said to be in parallel.





$$\left(\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \Sigma \frac{1}{R} \right)$$

$$i_1 = \frac{T_1 - T_2}{R_1}, \; i_2 = \frac{T_1 - T_2}{R_2}$$

$$\therefore \; i = i_1 + i_2 = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

In parallel, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

If the two rods are repleaced by a single rod, then K_{eq} will be

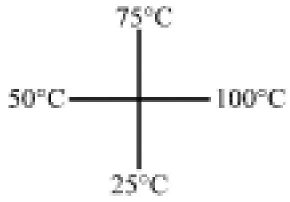
$$K_{eq} = \frac{L}{R_{eq}(A_1 + A_2)} \text{ and } i = \frac{T_1 - T_2}{R_{eq}}$$

Thus, the heat current in thermal resistances in terms of total thermal current is given by :

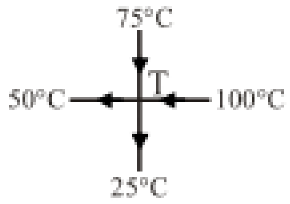
$$i_1 = \left(\frac{R_2}{R_1 + R_2}\right) \times i \qquad \text{and} \qquad i_2 = \left(\frac{R_1}{R_1 + R_2}\right) \times i$$

Illustration :

Two identical rods are joined at their middle points. The ends are maintained at constant temperatures as indicated. The temperature of the junction is _______?



Sol. *Let junction temperature be T*



Accourding to kirchouff's junction law,

Net input thermal current is equal to net output thermal current on a junction. i.e.

$$\sum\left(\frac{\Delta Q}{\Delta t}\right)_{in} = \sum\left(\frac{\Delta Q}{\Delta t}\right)_{out}$$

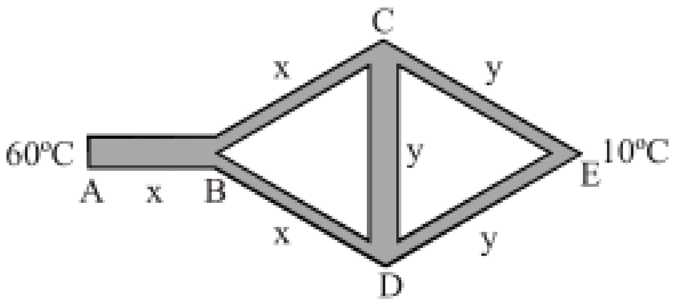
$$\frac{(100 - T)}{(R/2)} + \frac{(75 - T)}{(R/2)} = \frac{(T - 50)}{(R/2)} + \frac{(T - 25)}{(R/2)}$$

$175 + 75 = 4T$

$T = 62.5°C$

Illustration :

Three rods of material x and three of material y are connected as shown in figure. All the rods are identical in length and cross-sectional area. If the end A is maintained at 60ºC and the junction E at 10ºC, calculate the temperature of the junction B. The thermal conductivity of x is 800 W/n – ºC and that of y is 400W/m –ºC.



Sol. *It is clear from the symmetry of the figure that the points C and D are equivalent in all respect and hence, they are at the same temperature, say T. No heat will flow through the rod CD. We can, therefore neglect this rod in further analysis. (Treated as balance wheat stone bridge)*

Let L and A be the length and the area of cross-section of each rod. The thermal resistnces of AB, BC and BD are equal. Each has a value

$$R_1 = \frac{1}{K_x}\frac{L}{A} \qquad ...(i)$$

Similarly, thermal resistances of CE and DE are equal, each having a value

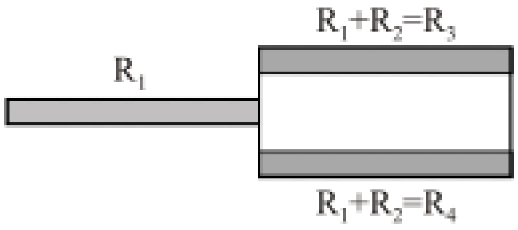
$$R_2 = \frac{1}{K_y}\frac{L}{A} \qquad ...(ii)$$

As the rod CD has no effect, we can say that the rods BC and CE are joined in series. Their equivalent thermal resistance is

$$R_3 = R_{BC} + R_{CE} = R_1 + R_2.$$

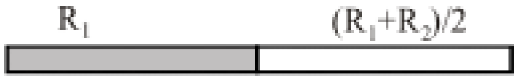
Also, the rods BD ad DE together have an equivalent thermal resitance

$$R_4 = R_{BD} + R_{DE} = R_1 + R_2.$$



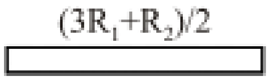
The resistances R_3 and R_4 are joined in parallel and hence their equivalent thermal resistance is given by

$$\frac{1}{R_5} = \frac{1}{R_3} + \frac{1}{R_4} = \frac{2}{R_3} \qquad \text{or} \qquad R_5 = \frac{R_3}{2} = \frac{R_1 + R_2}{2}$$



This resistance R_5 is connected in series with AB. Thus, the total arrangement is equivalent to a thermal resistance.

$$R = R_{AB} + R_5 = R_1 + \frac{R_1 + R_2}{2} = \frac{3R_1 + R_2}{2}$$



The heat current through A is

$$i = \frac{T_A - T_E}{R} = \frac{2(T_A - T_E)}{3R_1 + R_2} \qquad ...(i)$$

This current passes through the rod AB. We have

$$i = \frac{T_A - T_B}{R_{AB}} \qquad ...(ii)$$

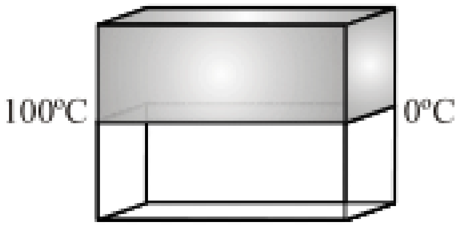
by using (i) and (ii) we get

$$T_A - T_B = \frac{2K_y(T_A - T_E)}{K_x + 3K_y} = \frac{2\times 400}{800 + 3\times 400} = 20^\circ C$$

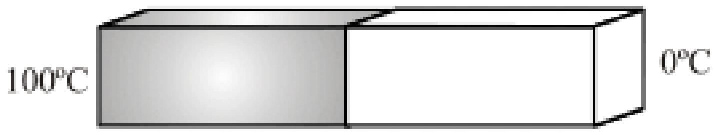
$$T_B = T_A - 20^\circ C = 40^\circ C$$

Illustration :

Two identical rectangular rods of metal are welded as shown in figure (1) and 20 J of heat flows through the rods in 1 min. How long would it take for 20 J heat to flow through the rods if they are welded as shown in figure (2).



(Figure - 1)



(Figure - 2)

Sol. *Let R be the thermal resistance of each rod.*

$\therefore$ *In first case* $\frac{1}{R_1} = \frac{1}{R} + \frac{1}{R}$ *or* $R_1 = \frac{R}{2}$

So the rate of flow of heat in this situation will be

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta T}{R_1} = \frac{100 - 0}{R/2} = \frac{20}{60}$$

$R = 600\ ^\circ C/W$

Now for case (2)

$R_2 = R + R = 600 + 600 = 1200\ ^\circ C/W$

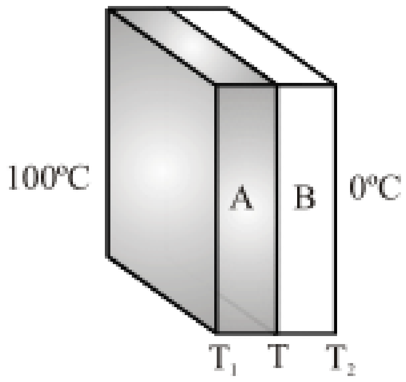
$$\therefore \quad \frac{\Delta Q}{\Delta t} = \frac{\Delta T}{R_2}$$

$$\frac{20}{t} = \frac{100}{1200}$$

$t = 240$ *sec.*

Illustration :

Two parallel plates A and B are joined together to form a compound plate (in figure). The thicknesses of the plates are 4.0 cm and 2.5 cm respectively and the area of cross-section is 100 cm² for each plate. The thermal conductivities are K_A = 200 W/m–° for the plate A and K_B = 400 W/m–°C for the plate B. The outer surface of the plate A is maintained at 100°C and the outer surface of the plate B is maintained at 0°C. Find (a) the rate of heat flow through any cross-section, (b) the temperature at the interface and (c) the equivalent thermal conductivity of the compound plate.



Sol. (a) *Let the temperature of the interface be T.*

The area of cross-section of each pate is A = 100 cm² = 0.01m². The thicknesses are x_A = 0.04 m and x_B = 0.025 m

The thermal resistance of the plate A is

$$R_A = \frac{x_A}{K_A A}$$

and that of the plate B is

$$R_B = \frac{x_B}{K_B A}$$

The equivalent thermal resistance is

$$R_{eq} = R_A + R_B = \frac{1}{A}\left(\frac{x_A}{K_A} + \frac{x_B}{K_B}\right) \qquad \text{... (i)}$$

Thus, $$\frac{\Delta Q}{\Delta t} = \frac{T_1 - T_2}{R_{eq}} = \frac{A(T_1 - T_2)}{x_A / K_A + x_B / K_B}$$

$$= \frac{(0.01\,m^2)\,(100\,°C)}{(0.04\,m)/(200\,W/m-°C) + (0.025m)/(400\,W/m-°C)} = 3810\ W.$$

(b) *We have* $$\frac{\Delta Q}{\Delta t} = \frac{A(T - T_2)}{x_B / K_B}$$

or, $$3810\ W = \frac{(0.01\,m^2)\,(T - 0°C)}{(0.025\,m)\ /\ (400\,W/m-°C)}$$

or, $T = 25°C$

(c) *If K is the equivalent thermal conductivity of the compound plate, its thermal resistance is*

$$R_{eq} = \frac{1}{A}\frac{x_A + x_B}{K_{eq}}$$

Comparing with (i),

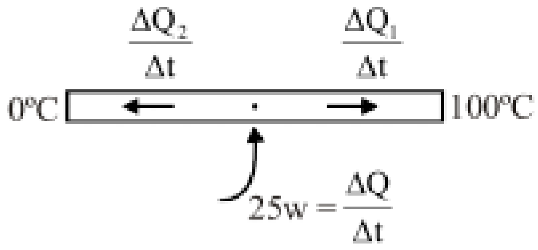
$$\frac{x_A + x_B}{K_{eq}} = \frac{x_A}{K_A} + \frac{x_B}{K_B}$$

or, $$K_{eq} = \frac{x_A + x_B}{x_A / K_A + x_B / K_B} = 248\ W/m\text{–}°C$$

Illustration :

The ends of copper rod of length 1 m and area of cross section 1 cm^2 are maintained at 0°C and 100°C. At the centre of the rod there is a source of heat of power 25 W. Thermal conductivity of copper is 400 W/m-K. In steady state, the temperature at the section on rod at which source is supplying heat, will be__________?

Sol.



Net thermal current supplied by source $\left(\frac{\Delta Q}{\Delta t}\right)$ *then*

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta Q_1}{\Delta t} + \frac{\Delta Q_2}{\Delta t}$$

$$25 = \frac{kA}{0.5}(T - 100) + \frac{kA}{0.5}(T - 0)$$

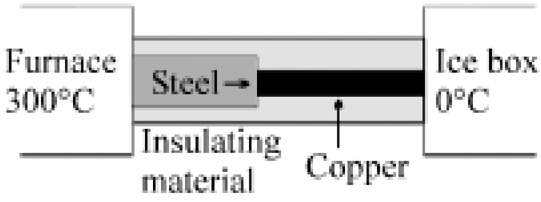
$$\frac{25 \times 0.5}{400 \times 1 \times 10^{-4}} = 2T - 100$$

$$\therefore 2T = \frac{1250}{4} + 100$$

$T = 206.25$ ºC

Illustration :

What is the temperature of the steel-copper junction in the steady state of the system shown in the figure. Length of the steel rod = 25 cm, length of the copper rod = 50 cm, temperature of the furnace = 300 °C, temperature of the other end = 0°C. The area of cross section of the steel rod is twice that of the copper rod. (Thermal conductivity of steel = 50 J s^{-1} m^{-1} K^{-1} and of copper = 400 J s^{-1} m^{-1} K^{-1})



Sol. *Let temperature be T, in steady state series connection heat transfered* $\left(\frac{\Delta Q}{\Delta t}\right)$ *through each rod is same.*

$$\frac{\Delta Q}{\Delta t} = \frac{k_1A_1(T_1 - T)}{L_1} = \frac{k_2A_2(T - T_2)}{L_2}$$

$$300 - T = \left(\frac{L_1}{L_2}\right)\left(\frac{k_2}{k_1}\right)\left(\frac{A_2}{A_1}\right)(T - 0)$$

$$300 - T = 2T$$

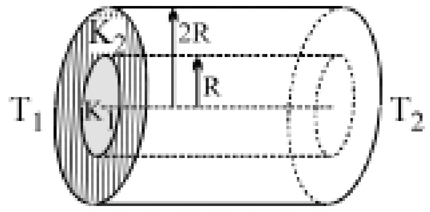
$$T = 100°C$$

Practice Exercise

Q.1 Three identical metal rods A, B and C are placed end to end and a temperature difference is maintained between the free ends of A and C. If the thermal conductivity of B (K_B) is twice that of C (K_C) and half that of A (K_A), (K_A = 49 w/mK)calculate the effective thermal conductivity of the system ?

Q.2 Two identical rectangular rods of metal are welded end to end in series between temperatures of 0° C and 100°C and 10J of heat is conducted (in a steady state process) through the rods in 2.0min. How long would it take for 10J to be conducted through the rods if they are welded together in parallel across the same temperatures?

Q.3 A composite cylinder is made of two materials having thermal conductivities K_1 and K_2 as shown. Temperature of the two flat faces of cylinder are maintained at T_1 and T_2. For what ratio K_1/K_2 the heat current through the two materials will be same. Assume steady state and the rod is lagged (insulated from the curved surface).



Answers

Q.1 21 w/mK   Q.2 30 secs.   Q.3 $\frac{K_1}{K_2} = 3$