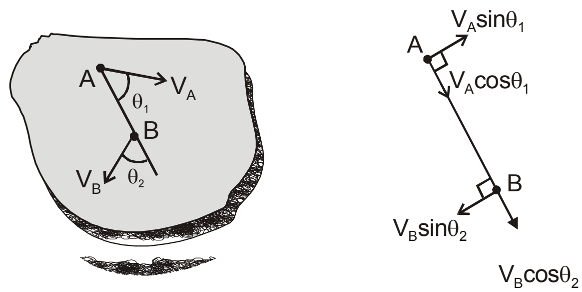
PHYSICS

Crash Course for
JEE Main 2020

ROTATIONAL MOTION

RIGID BODY DYNAMICS

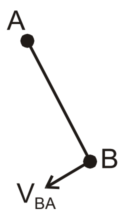
1. RIGID BODY :

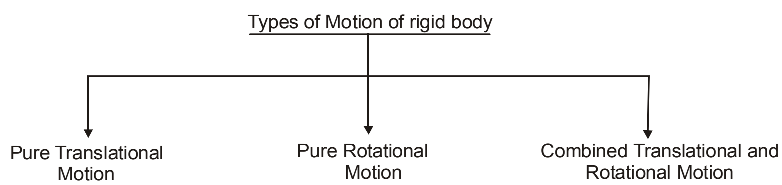


If the above body is rigid

$$V_A \cos\theta_1 = V_B \cos\theta_2$$

V_{BA} = relative velocity of point B with respect to point A.





2. MOMENT OF INERTIA (I) :

Definition : Moment of Inertia is defined as the capability of system to oppose the change produced in the rotational motion of a body.

Moment of Inertia is a scalar positive quantity.

$$I = mr_1^2 + m_2r_2^2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$= I_1 + I_2 + I_3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

SI units of Moment of Inertia is Kgm^2.

Moment of Inertia of :

2.1 A single particle : $I = mr^2$

where m = mass of the particle

r = perpendicular distance of the particle from the axis about which moment of Inertia is to be calculated

2.2 For many particles (system of particles) :

$$I = \sum_{i=1}^{n} m_i r_i^2$$

2.3 For a continuous object :

$$I = \int dmr^2$$

where dm = mass of a small element

r = perpendicular distance of the particle from the axis

2.4 For a larger object :

$$I = \int dI_{element}$$

where dI = moment of inertia of a small element

3. TWO IMPORTANT THEOREMS ON MOMENT OF INERTIA :

3.1 Perpendicular Axis Theorem

[Only applicable to plane lamina (that means for 2-D objects only)].

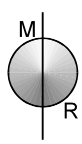
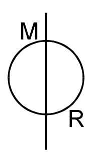
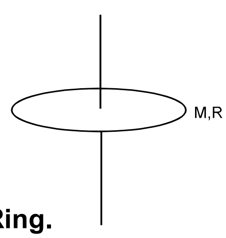
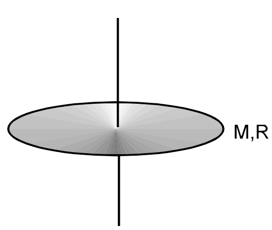
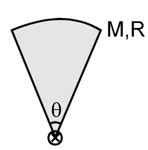
$$I_z = I_x + I_y \qquad \text{(when object is in x-y plane).}$$

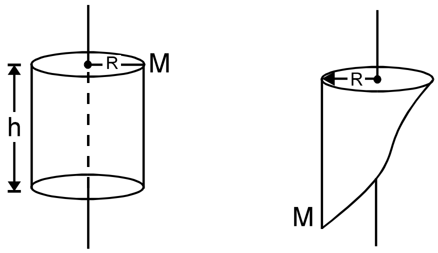
3.2 Parallel Axis Theorem

(Applicable to any type of object):

$$I_{AB} = I_{cm} + Md^2$$

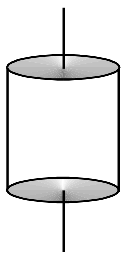
List of some useful formula :

Object	Moment of Inertia
 **Solid Sphere**	$\frac{2}{5}MR^2$ (Uniform)
 **Hollow Sphere**	$\frac{2}{3}MR^2$ (Uniform)
 **Ring.**	MR^2 (Uniform or Non Uniform)
 **Disc** 	$\frac{MR^2}{2}$ (Uniform)



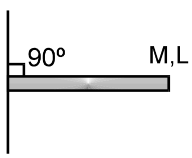
MR^2 (Uniform or Non Uniform)

Hollow cylinder

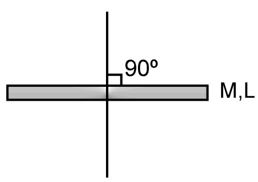


$\frac{MR^2}{2}$ (Uniform)

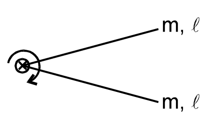
Solid cylinder



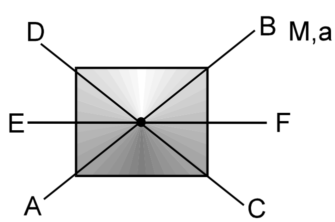
$\frac{ML^2}{3}$ (Uniform)



$\frac{ML^2}{12}$ (Uniform)

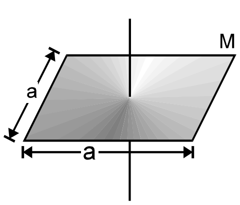


$\frac{2m\ell^2}{3}$ (Uniform)



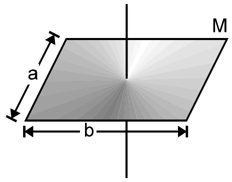
$I_{AB} = I_{CD} = I_{EF} = \frac{Ma^2}{12}$ (Uniform)

Square Plate



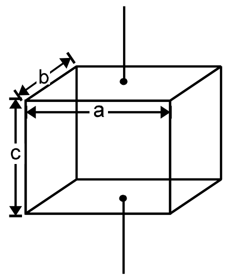
$\frac{Ma^2}{6}$ (Uniform)

Square Plate



$I = \frac{M(a^2 + b^2)}{12}$ (Uniform)

Rectangular Plate



$\frac{M(a^2 + b^2)}{12}$ (Uniform)

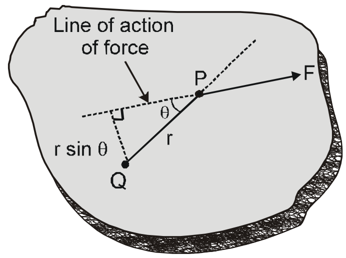
Cuboid

4. RADIUS OF GYRATION :

$$I = MK^2$$

5. TORQUE :

$\vec{\tau} = \vec{r} \times \vec{F}$

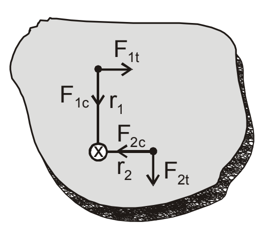


5.5 Relation between 'τ' & 'α ' (for hinged object or pure rotation)

$\vec{\tau}_{ext})_{Hinge} = I_{Hinge}\ \vec{\alpha}$

Where $\vec{\tau}_{ext})_{Hinge}$ = net external torque acting on the body about Hinge point

I_{Hinge} = moment of Inertia of body about Hinge point



$F_{1t} = M_1 a_{1t} = M_1 r_1 \alpha$

$F_{2t} = M_2 a_{2t} = M_2 r_2 \alpha$

$\tau_{resultant} = F_{1t}\, r_1 + F_{2t}\, r_2 + \ldots\ldots..$

$= M_1\, \alpha\, r_1^2 + M_2\, \alpha\, r_2^2 + \ldots\ldots\ldots\ldots$

$\tau_{resultant})_{external} = I\,\alpha$

Rotational Kinetic Energy = $\frac{1}{2}.I.\omega^2$

$\vec{P} = M\vec{v}_{CM} \quad \Rightarrow \quad \vec{F}_{external} = M\vec{a}_{CM}$

Net external force acting on the body has two parts tangential and centripetal.

$\Rightarrow \quad F_C = ma_C = m\frac{v^2}{r_{CM}} = m\omega^2 r_{CM} \quad \Rightarrow \quad F_t = ma_t = m\alpha r_{CM}$

6. ROTATIONAL EQUILIBRIUM :

For translational equilibrium.

$\Sigma F_x = 0$ .... (i)

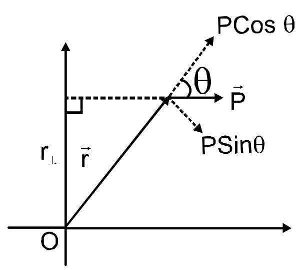
and $\Sigma F_y = 0$ .... (ii)

The condition of rotational equilibrium is

$\Sigma \Gamma_z = 0$

7. ANGULAR MOMENTUM ($\vec{L}$)

7.1 Angular momentum of a particle about a point.



$\vec{L} = \vec{r} \times \vec{P} \quad \Rightarrow \quad L = rpsin\theta$

$|\vec{L}| = r_\perp \times P$

$|\vec{L}| = P_\perp \times r$

7.3 Angular momentum of a rigid body rotating about fixed axis :

$\vec{L_H} = I_H\vec{\omega}$

L_H = angular momentum of object about axis H.

I_H = Moment of Inertia of rigid object about axis H.

ω = angular velocity of the object.

7.4 Conservation of Angular Momentum

Angular momentum of a particle or a system remains constant if $\tau_{ext} = 0$ about that point or axis of rotation.

7.5 Relation between Torque and Angular Momentum

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

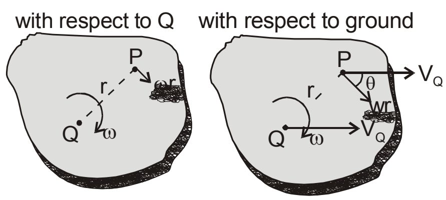
Torque is change in angular momentum

7.6 Impulse of Torque :

$$\int \tau dt = \Delta J \qquad\qquad \Delta J \rightarrow \text{Change in angular momentum.}$$

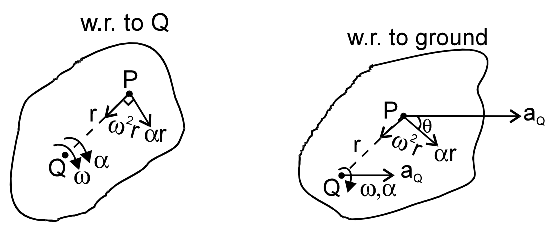
For a rigid body, the distance between the particles remain unchanged during its motion i.e. $r_{P/Q}$ = constant

For velocities



$$V_P = \sqrt{V_Q^2 + (\omega r)^2 + 2\, V_Q\, \omega r \cos\theta}$$

For acceleration :



θ, ω, α are same about every point of the body (or any other point outside which is rigidly attached to the body).

Dynamics :

$$\vec{\tau}_{cm} = I_{cm}\vec{\alpha}\ ,\ \vec{F}_{ext} = M\vec{a}_{cm}$$

$\vec{P}_{system} = M\vec{v}_{cm}$,

Total K.E. $= \frac{1}{2}Mv_{cm}^2 + \frac{1}{2} I_{cm}\,\omega^2$

Angular momentum axis AB = $\vec{L}$ about C.M. + $\vec{L}$ of C.M. about AB

$$\vec{L}_{AB} = I_{cm}\vec{\omega} + \vec{r}_{cm} \times M\vec{v}_{cm}$$

SECTION-1
SINGLE CHOICE TYPE QUESTION

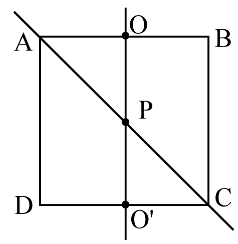
Q.1 Three bodies have equal masses m. Body A is solid cylinder of radius R, body B is a square lamina of side R, and body C is a solid sphere of radius R. Which body has the smallest moment of inertia about an axis passing through their centre of mass and perpendicular to the plane (in case of lamina)

(A) A (B) B (C) C (D) A and C both

Q.2 For the same total mass which of the following will have the largest moment of inertia about an axis passing through its centre of mass and perpendicular to the plane of the body

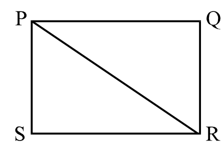
(A) a disc of radius a (B) a ring of radius a
(C) a square lamina of side 2a (D) four rods forming a square of side 2a

Q.3 Let I_1 and I_2 be the moment of inertia of a uniform square plate about axes APC and OPO' respectively as shown in the figure. P is centre of square. The ratio $\frac{I_1}{I_2}$ of moment of inertia is



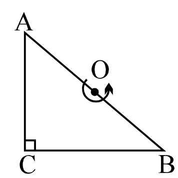
(A) $\frac{1}{\sqrt{2}}$ (B) 2 (C) $\frac{1}{2}$ (D) 1

Q.4 Moment of inertia of a rectangular plate about an axis passing through P and perpendicular to the plate is I. Then moment of PQR about an axis perpendicular to the plane of the plate:



(A) about P = I/2 (B) about R = I/2
(C) about P > I/2 (D) about R > I/2

Q.5 Find the moment of inertia of a plate cut in shape of a right angled triangle of mass M, side AC = BC = a about an axis perpendicular to the plane of the plate and passing through the mid point of side AB

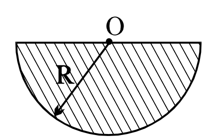


(A) $\frac{Ma^2}{12}$ (B) $\frac{Ma^2}{6}$ (C) $\frac{Ma^2}{3}$ (D) $\frac{2Ma^2}{3}$

Q.6 A thin uniform rod of mass M and length L has its moment of inertia I_1 about its perpendicular bisector. The rod is bend in the form of a semicircular arc. Now its moment of inertia through the centre of the semi circular arc and perpendicular to its plane is I_2. The ratio of $I_1 : I_2$ will be _______

(A) < 1 (B) > 1 (C) = 1 (D) can't be said

Q.7 Moment of inertia of a thin semicircular disc (mass = M & radius = R) about an axis through point O and perpendicular to plane of disc, is given by :



(A) $\frac{1}{4}MR^2$ (B) $\frac{1}{2}MR^2$ (C) $\frac{1}{8}MR^2$ (D) MR^2

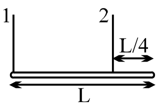
Q.8 Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to

(A) I (B) $I\sin^2\theta$ (C) $I\cos^2\theta$ (D) $I\cos^2(\theta/2)$

Q.9 A weightless rod is acted on by upward parallel forces of 2N and 4N ends A and B respectively. The total length of the rod AB = 3m. To keep the rod in equilibrium a force of 6N should act in the following manner:

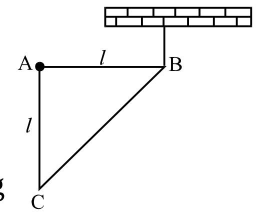
(A) Downwards at any point between A and B.
(B) Downwards at mid point of AB.
(C) Downwards at a point C such that AC = 1m.
(D) Downwards at a point D such that BD = 1m.

Q.10 A heavy rod of length L and weight W is suspended horizontally by two vertical ropes as shown. The first rope is attached to the left end of the rod while the second rope is attached a distance L/4 from right end. The tension in the second rope is:



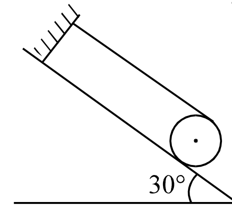
(A) (1/2)W (B) (1/4)W (C) (1/3)W (D) (2/3)W (E) W

Q.11 A right triangular plate ABC of mass m is free to rotate in the vertical plane about a fixed horizontal axis through A. It is supported by a string such that the side AB is horizontal. The reaction at the support A in equilibrium is:



(A) $\frac{mg}{3}$ (B) $\frac{2mg}{3}$ (C) $\frac{mg}{2}$ (D) mg

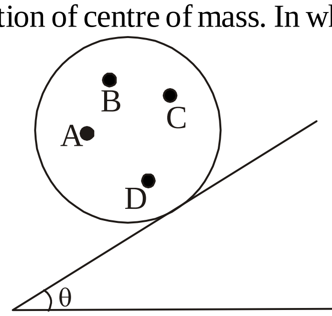
Q.12 A thin hoop of weight 500 N and radius 1 m rests on a rough inclined plane as shown in the figure. The minimum coefficient of friction needed for this configuration is



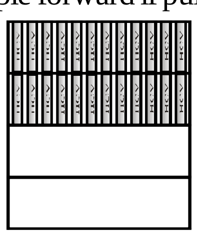
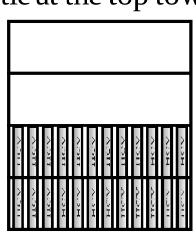
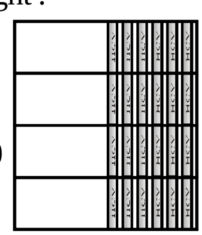
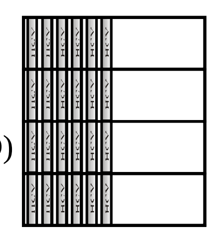
(A) $\frac{1}{3\sqrt{3}}$ (B) $\frac{1}{\sqrt{3}}$

(C) $\frac{1}{2}$ (D) $\frac{1}{2\sqrt{3}}$

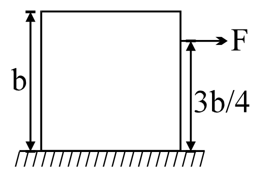
Q.13 A non uniform sphere can be kept on a rough inclined plane so that it is in equilibrium. In the figure below the dots represents location of centre of mass. In which one of the positions can sphere be in equilibrium?



Q.14 Same number of books are placed in four book cases as shown. Which bookcase is most likely to topple forward if pulled a little at the top towards right :

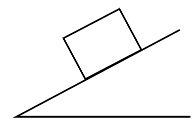
(A)  (B)  (C)  (D) 

Q.15 A uniform cube of side 'b' and mass M rest on a rough horizontal table. A horizontal force F is applied normal to one of the face at a point, at a height 3b/4 above the base. What should be the coefficient of friction (μ) between cube and table so that is will tip about an edge before it starts slipping?



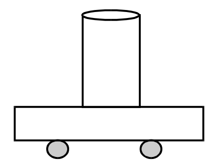
(A) $\mu > \frac{2}{3}$ (B) $\mu > \frac{1}{3}$ (C) $\mu > \frac{3}{2}$ (D) none

Q.16 A homogeneous cubical brick lies motionless on a rough inclined surface. The half of the brick which applies greater pressure on the plane is :



(A) left half
(B) right half
(C) both applies equal pressure
(D) the answer depend upon coefficient of friction

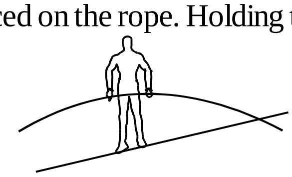
Q.17 A uniform 2 kg cylinder rests on a laboratory cart as shown. The coefficient of static friction between the cylinder and the cart is 0.5. If the cylinder is 4 cm in diameter and 10 cm in height, which of the following is closest to the minimum acceleration of the cart needed to cause the cylinder to tip over?



(A) 2 m/s^2 (B) 4 m/s^2 (C) 5 m/s^2 (D) 6 m/s^2
(E) the cylinder would slide at all of these accelerations

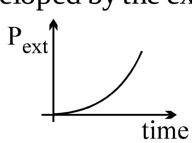
Q.18 A heavy seesaw (i.e., not massless) is out of balance. A light girl sits on the end that is tilted downward, and a heavy body sits on the other side so that the seesaw now balances. If they both move forward so that they are one-half their original distance from the pivot point (the fulcrum) what will happen to the seesaw ?
(A) The side the body is sitting on will tilt downward
(B) The side the girl is sitting on will once again tilt downward
(C) Nothing ; the seesaw will still be balanced
(D) It is impossible to say without knowing the masses and the distances

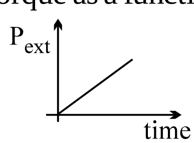
Q.19 A tightrope walker in a circus holds a long flexible pole to help stay balanced on the rope. Holding the pole horizontally and perpendicular to the rope helps the performer.

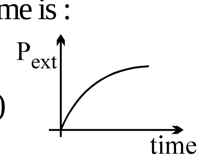


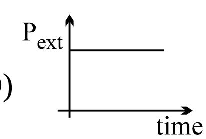
(A) by lowering the overall centre-of-gravity
(B) by increasing the rotation inertia
(C) in the ability to adjust the centre-of-gravity to be over the rope
(D) in achieving the centre of gravity to be under the rope

Q.20 A rod is hinged at its centre and rotated by applying a constant torque starting from rest. The power developed by the external torque as a function of time is :

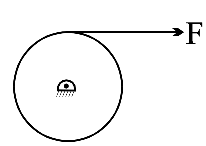
(A) 

(B) 

(C) 

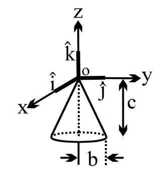
(D) 

Q.21 A pulley is hinged at the centre and a massless thread is wrapped around it. The thread is pulled with a constant force F starting from rest. As the time increases,



(A) its angular velocity increases, but force on hinge remains constant
(B) its angular velocity remains same, but force on hinge increases
(C) its angular velocity increases and force on hinge increases
(D) its angular velocity remains same and force on hinge is constant

Q.22 A solid cone hangs from a frictionless pivot at the origin O, as shown. If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors, and a, b, and c are positive constants, which of the following forces F applied to the rim of the cone at a point P results in a torque τ on the cone with a negative component τ_Z?



(A) F = a$\hat{k}$, P is (0,b,–c) (B) F = –a$\hat{k}$, P is (0,–b,–c)

(C) F = a$\hat{j}$, P is (–b,0,–c) (D) None

Q.23 A man, sitting firmly over a rotating stool has his arms streched. If he folds his arms, the work done by the man is

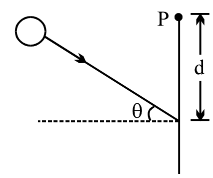
(A) zero (B) positive

(C) negative (D) may be positive or negative.

Q.24 A particle of mass 2 kg located at the position $(\hat{i}+\hat{j})$ m has a velocity $2(+\hat{i}-\hat{j}+\hat{k})$ m/s. Its angular momentum about z-axis in kg-m^2/s is:

(A) zero (B) +8 (C) 12 (D) –8

Q.25 A ball of mass m moving with velocity v, collide with the wall elastically as shown in the figure. After impact the change in angular momentum about P is:



(A) 2 *mvd* (B) 2 *mvd* cosθ (C) 2 *mvd* sin θ (D) zero

Q.26 Two uniform spheres of mass M have radii R and 2R. Each sphere is rotating about a fixed axis through a diameter. the rotational kinetic energies of the spheres are identical. What is the ratio of the magnitude of the angular momenta of these spheres? That is, $\dfrac{L_{2R}}{L_R} =$

(A) 4 (B) $2\sqrt{2}$ (C) 2 (D) $\sqrt{2}$ (E) 1

Q.27 A spinning ice skater can increase his rate of rotation by bringing his arms and free leg closer to his body. How does this procedure affect the skater's angular momentum and kinetic energy?

(A) angular momentum remains the same while kinetic energy increases

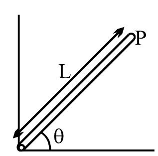
(B) angular momentum remains the same while kinetic energy decreases

(C) both angular momentum and kinetic energy remain the same

(D) angular momentum increases while kinetic energy remains the same

(E) both angular momentum and kinetic energy increase

Q.28 A uniform flag pole of length L and mass M is pivoted on the ground with a frictionless hinge. The flag pole makes an angle θ with the horizontal. The moment of inertia of the flag pole about one end is (1/3)ML^2. If it starts falling from the position shown in the accompanying figure, the linear acceleration of the free end of the flag pole — labeled P – would be:

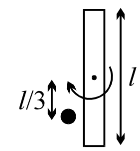


(A) (2/3) gcosθ (B) (2/3) g (C) g (D) (3/2) gcosθ (E) (3/2) g

Q.29 A mass *m* is moving at speed *v* perpendicular to a rod of length *d* and mass *M* = 6*m* which pivots around a frictionless axle running through its centre. It strikes and sticks to the end of the rod. The moment of inertia of the rod about its centre is $Md^2/12$. Then the angular speed of the system right after the collision is

(A) 2*v*/*d* (B) 2*v*/(3*d*) (C) *v*/*d* (D) 3*v*/(2*d*)

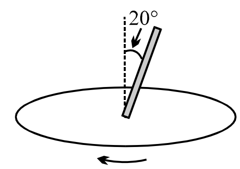
Q.30 A uniform rod of length l and mass M rotating about a fixed vertical axis on a smooth horizontal table. It elastically strikes a particle placed at a distance $l/3$ from its axis and stops. Mass of the particle is



(A) 3M (B) $\frac{3M}{4}$ (C) $\frac{3M}{2}$ (D) $\frac{4M}{3}$

Question No. 31 & 32 (2 questions)

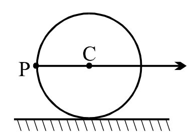
A uniform rod is fixed to a rotating turntable so that its lower end is on the axis of the turntable and it makes an angle of 20° to the vertical. (The rod is thus rotating with uniform angular velocity about a vertical axis passing through one end.) If the turntable is rotating clockwise as seen from above.



Q.31 What is the direction of the rod's angular momentum vector (calculated about its lower end)?
(A) vertically downwards (B) down at 20° to the horizontal
(C) up at 20° to the horizontal (D) vertically upwards

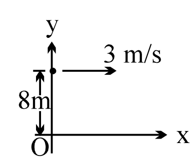
Q.32 Is there a torque acting on it, and if so in what direction?
(A) yes, vertically (B) yes, horizontally
(C) yes at 20° to the horizontal (D) no

Q.33 A disc of radius R is rolling purely on a flat horizontal surface, with a constant angular velocity. The angle between the velocity and acceleration vectors of point P is



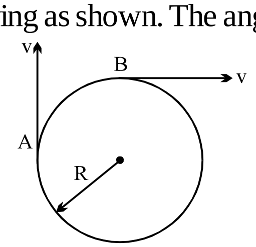
(A) zero (B) 45° (C) 135° (D) $\tan^{-1}(1/2)$

Q.34 A particle starts from the point (0m, 8m) and moves with uniform velocity of $3\,\hat{i}$ m/s. After 5 seconds, the angular velocity of the particle about the origin will be :



(A) $\frac{8}{289}$ rad/s (B) $\frac{3}{8}$ rad/s (C) $\frac{24}{289}$ rad/s (D) $\frac{8}{17}$ rad/s

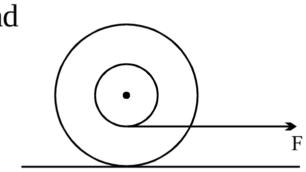
Q.35 Two points of a rigid body are moving as shown. The angular velocity of the body is:



(A) $\frac{\upsilon}{2R}$ (B) $\frac{\upsilon}{R}$ (C) $\frac{2\upsilon}{R}$ (D) $\frac{2\upsilon}{3R}$

Q.36 A yo-yo is released from your hand with the string wrapped around your finger. If you hold your hand still, the acceleration of the yo-yo is
(A) downward, much greater than g (B) downward, much less than g
(C) upward, much less than g (D) upward, much greater than g
(E) downward, at g

Q.37 Inner and outer radii of a spool are r and R respectively. A thread is wound over its inner surface and placed over a rough horizontal surface. Thread is pulled by a force F as shown in fig. then in case of pure rolling



(A) Thread unwinds, spool rotates anticlockwise and friction act leftwards
(B) Thread winds, spool rotates clockwise and friction acts leftwards
(C) Thread winds, spool moves to the right and friction act rightwards
(D) Thread winds, spool moves to the right and friction does not come into existence.

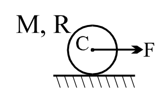
Q.38 A body kept on a smooth horizontal surface is pulled by a constant horizontal force applied at the top point of the body. If the body rolls purely on the surface, its shape can be :
(A) thin pipe (B) uniform cylinder (C) uniform sphere (D) thin spherical shell

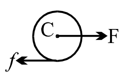
Q.39 A solid sphere with a velocity (of centre of mass) v and angular velocity ω is gently placed on a rough horizontal surface. The frictional force on the sphere:
(A) must be forward (in direction of v) (B) must be backward (opposite to v)
(C) cannot be zero (D) none of the above

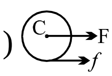
Question No. 40 to 43 (4 questions)

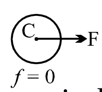
In the following problems, indicate the correct direction of friction force acting on the cylinder, which is pulled on a rough surface by a constant force F.

Q.40 A cylinder of mass M and radius R is pulled horizontally by a force F. The friction force can be given by which of the following diagrams



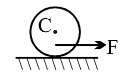
(A) 

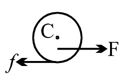
(B) 

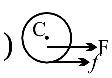
(C) 

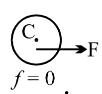
(D) cannot be interpreted

Q.41 A cylinder is pulled horizontally by a force F acting at a point below the centre of mass of the cylinder, as shown in figure. The friction force can be given by which of the following diagrams



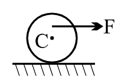
(A) 

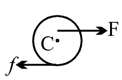
(B) 

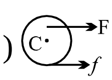
(C) 

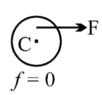
(D) cannot be interpreted

Q.42 A cylinder is pulled horizontally by a force F acting at a point above the centre of mass of the cylinder, as shown in figure. The friction force can be given by which of the following diagrams



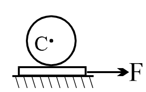
(A) 

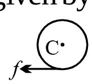
(B) 

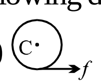
(C) 

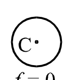
(D) cannot be interpreted

Q.43 A cylinder is placed on a rough plank which in turn is placed on a smooth surface. The plank is pulled with a constant force F. The friction force can be given by which of the following diagrams



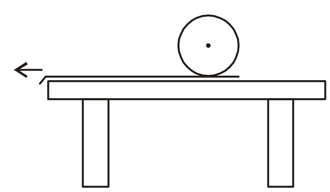
(A) 

(B) 

(C) 

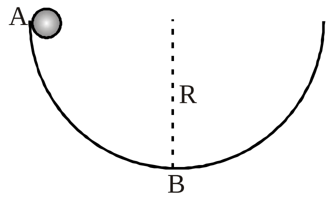
(D) canot be interpreted

Q.44 A thin tabecloth covers a horizontal table and a unifirm body of round shape lies on top of it. The tabecloth is pulled from under the body, and friction causes the body to slide and rotate. What is the body's final motion on the table?
(Assume that the table is so large that the body does not fall off it.)



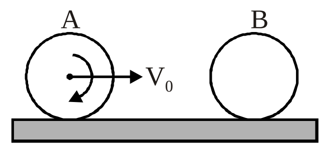
(A) Body will finally roll towards left (B) Body will finally roll towards right
(C) Body will finally come to rest (D) Any of the above is possible depending on shape of body

Q.45 A small sphere A of mass m and radius r rolls without slipping inside a large fixed hemispherical bowl of radius R (>> r) as shown in figure. If the sphere starts from rest at the top point of the hemisphere find the normal force exerted by the small sphere on the hemisphere when it is at the bottom B of the hemisphere.



(A) $\frac{10}{7}$mg (B) $\frac{17}{7}$mg (C) $\frac{5}{7}$ mg (D) $\frac{7}{5}$mg

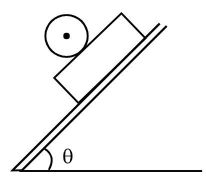
Q.46 A hollow smooth uniform sphere A of mass 'm' rolls without sliding on a smooth horizontal surface. It collides head on elastically with another stationary smooth solid sphere B of the same mass m and same radius. The ratio of kinetic energy of 'B' to that of 'A' just after the collision is :



(A) 1 : 1 (B) 2 : 3 (C) 3 : 2 (D) None

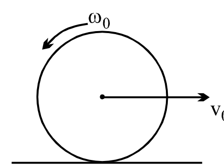
Q.47 A plank of mass M is placed over smooth inclined plane and a sphere is also placed over the plank. Friction is sufficient between sphere and plank. If plank and sphere are released from rest, the frictional force on sphere is:

(A) up the plane (B) down the plane (C) horizontal (D) zero



Q.48 A uniform circular disc placed on a rough horizontal surface has initially a velocity v_0 and an angular velocity ω_0 as shown in the figure. The disc comes to rest after moving some distance in the direction of motion. Then $\frac{v_0}{r\omega_0}$ is

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) 2

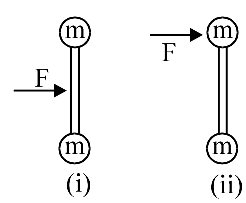


Q.49 The moment of inertia of a solid cylinder about its axis is given by $(1/2)MR^2$. If this cylinder rolls without slipping, the ratio of its rotational kinetic energy to its translational kinetic energy is

(A) 1 : 1 (B) 2 : 2 (C) 1 : 2 (D) 1 : 3

Q.50 A force F is applied to a dumbbell for a time interval, t, first as in (i) and then as in (ii). In which case does the dumbbell acquire the greater centre-of-mass speed?

(A) (i)
(B) (ii)
(C) there is no difference
(D) the answer depends on the rotational inertia of the dumbbell



Q.51 A hoop and a solid cylinder have the same mass and radius. They both roll, without slipping, on a horizontal surface. If their kinetic energies are equal

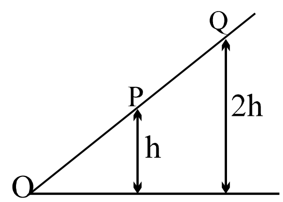
(A) the hoop has a greater translational speed than the cylinder

(B) the cylinder has a greater translational speed than the hoop

(C) the hoop and the cylinder have the same translational speed

(D) the hoop has a greater rotational speed than the cylinder

Q.52 A ball rolls down an inclined plane, figure. The ball is first released from rest from P and then later from Q. Which of the following statement is/ are correct?



(i) The ball takes twice as much time to roll from Q to O as it does to roll from P to O.

(ii) The acceleration of the ball at Q is twice as large as the acceleration at P.

(iii) The ball has twice as much K.E. at O when rolling from Q as it does when rolling from P.

(A) i, ii only (B) ii, iii only (C) i only (D) iii only

Q.53 One ice skater of mass m moves with speed 2v to the right, while another of the same mass m moves with speed v toward the left, as shown in figure I. Their paths are separated by a distance b. At $t = 0$, when they are both at $x = 0$, they grasp a pole of length b and negligible mass. For $t > 0$, consider the system as a rigid body of two masses m separated by distance b, as shown in figure II. Which of the following is the correct formula for the motion after $t = 0$ of the skater initially at $y = b/2$?

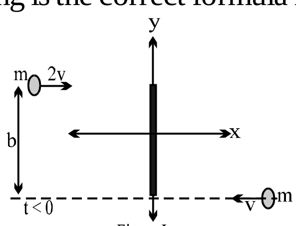


Figure I

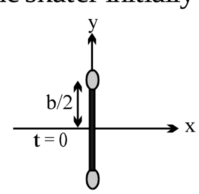


Figure II

(A) $x = 2vt, y = b/2$

(B) $x = vt + 0.5b\sin(3vt/b), y = 0.5b\cos(3vt/b)$

(C) $x = 0.5vt + 0.5b\sin(3vt/b), y = 0.5b\cos(3vt/b)$

(D) $x = 0.5vt + 0.5b\sin(6vt/b), y = 0.5b\cos(6vt/b)$

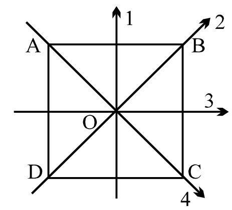
SECTION-2

ONE OR MORE THAN ONE OPTION MAY BE CORRECT

Q.1 A rigid object is rotating in a counterclockwise sense around a fixed axis. If the rigid object rotates through more than 180° but less than 360°, which of the following pairs of quantities can represent an initial angular position and a final angular position of the rigid object.
Which of the sets can only occur.

(A) 3 rad, 6 rad (B) – 1 rad, 1 rad (C) 1 rad, 5 rad (D) –1 rad, 2.5 rad

Q.2 ABCD is a square plate with centre O. The moments of inertia of the plate about the perpendicular axis through O is I and about the axes 1, 2, 3 & 4 are I_1, I_2, I_3 & I_4 respectively. It follows that :



(A) $I_2 = I_3$ (B) $I = I_1 + I_4$
(C) $I = I_2 + I_4$ (D) $I_1 = I_3$

Q.3 A body is in equilibrium under the influence of a number of forces. Each force has a different line of action. The minimum number of forces required is

(A) 2, if their lines of action pass through the centre of mass of the body.
(B) 3, if their lines of action are not parallel.
(C) 3, if their lines of action are parallel.
(D) 4, if their lines of action are parallel and all the forces have the same magnitude.

Q.4 If a person sitting on a rotating stool with his hands outstretched, suddenly lowers his hands, then his

(A) Kinetic energy will decrease (B) Moment of inertia will decrease
(C) Angular momentum will increase (D) Angular velocity will remain constant

Q.5 A block of mass m moves on a horizontal rough surface with initial velocity v. The height of the centre of mass of the block is h from the surface. Consider a point A on the surface.

(A) angular momentum about A is mvh initially
(B) the velocity of the block decreases at time passes.
(C) torque of the forces acting on block is zero about A
(D) angular mometum is not conserved about A.

Q.6 A paritcle falls freely near the surface of the earth. Consider a fixed point O (not vertically below the particle) on the ground.

(A) Angular momentum of the particle about O is increasing .
(B) Torque of the gravitational force on the particle about O is decreasing.
(C) The moment of inertia of the particle about O is decreasing .
(D) The angular velocity of the particle about O is increasing.

Q.7 The torque $\vec{\tau}$ on a body about a given point is found to be equal to $\vec{A} \times \vec{L}$ where $\vec{A}$ is a constant vector and $\vec{L}$ is the angular momentum of the body about that point. From this it follows that

(A) $d\vec{L}/dt$ is perpendicular to $\vec{L}$ at all instants of time
(B) the components of $\vec{L}$ in the direction of $\vec{A}$ does not change with time
(C) the magnitude of $\vec{L}$ does not change with time
(D) $\vec{L}$ does not change with time

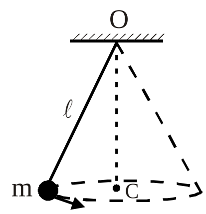
Q.8 A man spinning in free space changes the shape of his body, eg. by spreading his arms or curling up. By doing this, he can change his

(A) moment of inertia (B) angular momentum (C) angular velocity (D) rotational kinetic energy

Q.9 A ring rolls without slipping on the ground. Its centre C moves with a constant speed u. P is any point on the ring. The speed of P with respect to the ground is ν.

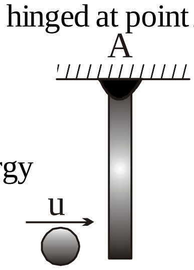
(A) $0 \le \nu \le 2u$
(B) $\nu = u$, if CP is horizontal
(C) $\nu = u$, if CP makes an angle of 30° with the horizontal and P is below the horizontal level of C.
(D) $\nu = \sqrt{2}u$, if CP is horizontal

Q.10 A small ball of mass m suspended from the celling at a point O by a thread of length ℓ moves along a horizontal circle with a constant angular velocity ω.



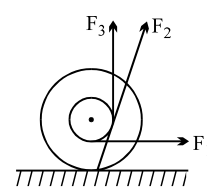
(A) angular momentum is constant about O
(B) angular momentum is constant about C
(C) vertical component of angular momentum about O is constant
(D) Magnitude of angular momentum about O is constant

Q.11 In the given figure a ball strikes a uniform rod of same mass elastically and rod is hinged at point A. Then which of the statement(s) is / are correct?



(A) linear momentum of system (ball + rod) is conserved.
(B) angular momentum of system (ball + rod) about hinged point A is conserved.
(C) kinetic energy of system (ball + rod) before the collision is equal to kinetic energy of system just after the collision
(D) linear momentum of ball is conserved.

Q.12 A yo-yo is resting on a perfectly rough horizontal table. Forces F_1, F_2 and F_3 are applied separately as shown. The correct statement is



(A) when F_3 is applied the centre of mass will move to the right.
(B) when F_2 is applied the centre of mass will move to the left.
(C) when F_1 is applied the centre of mass will move to the right.
(D) when F_2 is applied the centre of mass will move to the right.

Q.13 If a cylinder is rolling down the incline with sliding.

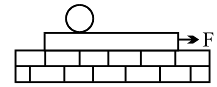
(A) after some time it may start pure rolling (B) after sometime it will start pure rolling
(C) it may be possible that it will never start pure rolling (D) none of these

Q.14 Which of the following statements are correct.

(A) friction acting on a cylinder without sliding on an inclined surface is always upward along the incline irrespective of any external force acting on it.
(B) friction acting on a cylinder without sliding on an inclined surface is may be upward may be downwards depending on the external force acting on it.
(C) friction acting on a cylinder rolling without sliding may be zero depending on the external force acting on it.
(D) nothing can be said exactly about it as it depends on the friction coefficient on inclined plane.

Q.15 A plank with a uniform sphere placed on it rests on a smooth horizontal plane. Plank is pulled to right by a constant force F. If sphere does not slip over the plank. Which of the following is correct.

(A) Acceleration of the centre of sphere is less than that of the plank.
(B) Work done by friction acting on the sphere is equal to its total kinetic energy.
(C) Total kinetic energy of the system is equal to work done by the force F
(D) None of the above



Q.16 A hollow sphere of radius R and mass m is fully filled with non viscous liquid of mass m. It is rolled down a horizontal plane such that its centre of mass moves with a velocity v. If it purely rolls

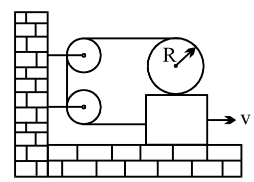
(A) Kinetic energy of the sphere is $\frac{5}{6}mv^2$

(B) Kinetic energy of the sphere is $\frac{4}{5}mv^2$

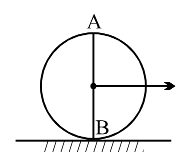
(C) Angular momentum of the sphere about a fixed point on ground is $\frac{8}{3}mvR$

(D) Angular momentum of the sphere about a fixed point on ground is $\frac{14}{5}mvR$

Q.17 In the figure shown, the plank is being pulled to the right with a constant speed v. If the cylinder does not slip then:
(A) the speed of the centre of mass of the cylinder is 2v.
(B) the speed of the centre of mass of the cylinder is zero.
(C) the angular velocity of the cylinder is v/R.
(D) the angular velocity of the cylinder is zero.

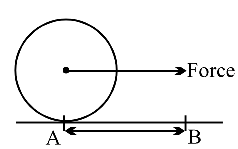


Q.18 A uniform disc is rolling on a horizontal surface. At a certain instant B is the point of contact and A is at height 2R from ground, where R is radius of disc.
(A) The magnitude of the angular momentum of the disc about B is thrice that about A.
(B) The angular momentum of the disc about A is anticlockwise.
(C) The angular momentum of the disc about B is clockwise
(D) The angular momentum of the disc about A is equal to that about B.



Q.19 A wheel of radius r rolling on a straight line, the velocity of its centre being v. At a certain instant the point of contact of the wheel with the grounds is M and N is the highest point on the wheel (diametrically opposite to M). The incorrect statement is:
(A) The velocity of any point P of the wheel is proportional to MP.
(B) Points of the wheel moving with velocity greater than v form a larger area of the wheel than points moving with velocity less than v.
(C) The point of contact M is instantaneously at rest.
(D) The velocities of any two parts of the wheel which are equidistant from centre are equal.

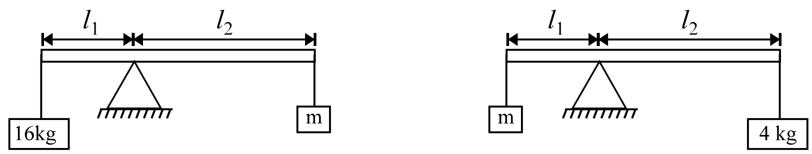
Q.20 A disc of circumference s is at rest at a point A on a horizontal surface when a constant horizontal force begins to act on its centre. Between A and B there is sufficient friction to prevent slipping, and the surface is smooth to the right of B. AB = s. The disc moves from A to B in time T. To the right of B,



(A) the angular acceleration of the disc will disappear, linear acceleration will remain unchanged
(B) linear acceleration of the disc will increase
(C) the disc will make one rotation in time T/2
(D) the disc will cover a distance greater than s in further time T.

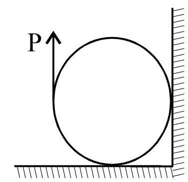
EXERCISE-I

Q.1 In an experiment with a beam balance on unknown mass m is balanced by two known mass m is balanced by two known masses of 16 kg and 4 kg as shown in figure.

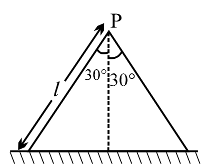


Find the value of the unknown mass m.

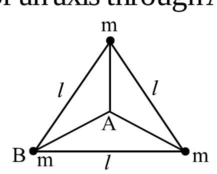
Q.2 Figure shows a vertical force applied tangentially to a uniform cylinder of weight F_g. The coefficient of static friction between the cylinder and both surfaces is 0.500. In terms of F_g, find the maximum force **P** that can be applied that does not cause the cylinder to rotate.



Q.3 An inverted "V" is made up of two uniform boards each weighing 200 N. Each side has the same length and makes an angle 30° with the vertical as shown in figure. What is the magnitude of the static frictional force that acts on each of the lower end of the V ?

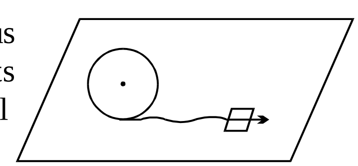


Q.4 Three equal masses m are rigidly connected to each other by massless rods of length l forming an equilateral triangle, as shown above. The assembly is to be given an angular velocity ω about an axis perpendicular to the triangle. For fixed ω, what is the ratio of the kinetic energy of the assembly for an axis through B compared with that for an axis through A.

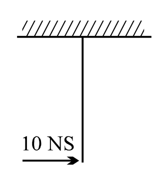


Q.5 A particle of mass 10kg is moving with a uniform speed of 6m/sec. in x–y plane along the line 3y = 4x + 10, what is the magnitude of its angular momentum about the origin in kg – m^2/s?

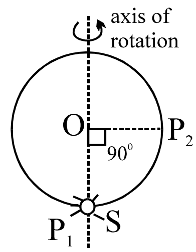
Q.6 A block of mass m is attached to a pulley disc of equal mass m, radius r by means of a slack string as shown. The pulley is hinged about its centre on a horizontal table and the block is projected with an initial velocity of 5 m/s. Find the velocity when the string becomes taut.



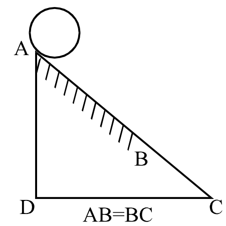
Q.7 A thin uniform straight rod of mass 2 kg and length 1 m is free to rotate about its upper end when at rest. It receives an impulsive blow of 10 Ns at its lowest point, normal to its length as shown in figure. Find the kinetic energy of rod just after impact.



Q.8 A uniform ring is rotating about vertical axis with angular velocity ω initially. A point insect (S) having the same mass as that of the ring starts walking from the lowest point P_1 and finally reaches the point P_2(as shown in figure). What is the final angular velocity of the ring?

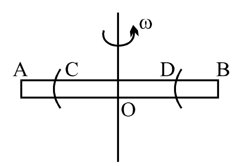


Q.9 Portion AB of the wedge shown in figure is rough and BC is smooth. A solid cylinder rolls without slipping from A to B. Find the ratio of translational kinetic energy to rotational kinetic energy, when the cylinder reaches point C.

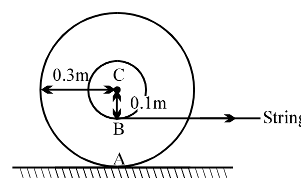


Q.10 A ring of radius R rolls without sliding with a constant velocity. Find the radius of curvature of the path followed by any particle of the ring at the highest point of its path.

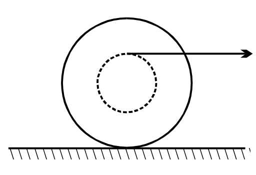
Q.11 A rigid horizontal smooth rod AB of mass 0.75 kg and length 40 cm can rotate freely about a fixed vertical axis through its mid point O. Two rings each of mass 1 kg are initially at rest at a distance of 10 cm from O on either side of the rod. The rod is set in rotation with an angular velocity of 30 radians per second. Find the velocity of each ring along the length of the rod in m/s when they reach the ends of the rod



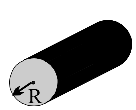
Q.12 A wheel is made to roll without slipping, towards right, by pulling a string wrapped around a coaxial spool as shown in figure. With what velocity the string should be pulled so that the centre of the wheel moves with a velocity of 3 m/s?

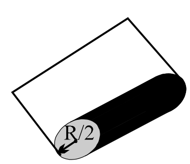


Q.13 A spool of inner radius R and outer radius 3R has a moment of inertia = MR^2 about an axis passing through its geometric centre, where M is the mass of the spool. A thread wound on the inner surface of the spool is pulled horizontally with a constant force = Mg. Find the acceleration of the point on the thread which is being pulled assuming that the spool rolls purely on the floor.



Q.14 A carpet of mass 'M' made of inextensible material is rolled along its length in the form of a cylinder of radius 'R' and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. The horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to R/2 will be:





ANSWER KEY

SECTION-1

ONLY ONE OPTION IS CORRECT.

Q.1	B	Q.2	D	Q.3	D	Q.4	C	Q.5	B	Q.6	A	Q.7	B
Q.8	A	Q.9	D	Q.10	D	Q.11	B	Q.12	D	Q.13	C	Q.14	C
Q.15	A	Q.16	A	Q.17	B	Q.18	B	Q.19	C	Q.20	B	Q.21	A
Q.22	C	Q.23	B	Q.24	D	Q.25	B	Q.26	C	Q.27	A	Q.28	D
Q.29	B	Q.30	B	Q.31	B	Q.32	B	Q.33	B	Q.34	C	Q.35	B
Q.36	B	Q.37	B	Q.38	A	Q.39	D	Q.40	A	Q.41	A	Q.42	D
Q.43	B	Q.44	C	Q.45	B	Q.46	C	Q.47	D	Q.48	A	Q.49	C
Q.50	C	Q.51	B	Q.52	D	Q.53	C						

SECTION-2

ONE OR MORE THAN ONE OPTION MAY BE CORRECT

Q.1	C, D	Q.2	A,B,C, D	Q.3	B, C,D	Q.4	B	Q.5	A,B,D
Q.6	A,C,D	Q.7	A,B,C	Q.8	A, C, D	Q.9	A, C, D	Q.10	B, C, D
Q.11	B, C	Q.12	C	Q.13	A, C	Q.14	B, C	Q.15	A,B,C
Q.16	C	Q.17	B, C	Q.18	A,B, C	Q.19	D	Q.20	B,C,D

EXERCISE–I

Q.1	8 kg	Q.2	(3/8) F_g	Q.3	$\frac{100}{\sqrt{3}}$ N	Q.4	2
Q.5	120	Q.6	10/3 m/s	Q.7	75 J	Q.8	$\frac{\omega}{3}$
Q.9	5	Q.10	4R	Q.11	3	Q.12	2 m/s
Q.13	16 m/s^2	Q.14	$v = \sqrt{\frac{14gR}{3}}$				