GUIDED REVISION

PHYSICS **GR # CIRCULAR MOTION + WORK, POWER & ENERGY**

SECTION-I

Single Correct Answer Type **12 Q. [3 M (–1)]**

1. A long horizontal rod has a bead which can slide along its length and is initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration, α. If the coefficient of friction between the rod and bead is μ, and gravity is neglected, then the time after which the bead starts slipping is :– **[IIT-JEE 2000]**

(A) $\sqrt{\frac{\mu}{\alpha}}$ (B) $\frac{\mu}{\sqrt{\alpha}}$ (C) $\frac{1}{\sqrt{\mu\alpha}}$ (D) infinitesimal

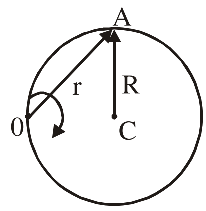
2. A ring of radius r and mass per unit length m rotates with an angular velocity ω in free space. The tension in the ring is :

(A) zero (B) $\frac{1}{2}m\omega^2r^2$ (C) $m\omega^2r^2$ (D) $mr\omega^2$

3. A uniform rod of mass m and length ℓ rotates in a horizontal plane with an angular velocity ω about a vertical axis passing through one end. The tension in the rod at a distance x from the axis is :

(A) $\frac{1}{2}m\omega^2x$ (B) $\frac{1}{2}m\omega^2\frac{x^2}{\ell}$ (C) $\frac{1}{2}m\omega^2\ell\left(1-\frac{x}{\ell}\right)$ (D) $\frac{1}{2}\frac{m\omega^2}{\ell}[\ell^2-x^2]$

4. A particle A moves along a circle of radius R=50 cm so that its radius vector r relative to the point O (figure) rotates with the constant angular velocity ω=0.40 rad/s. Then modulus of the velocity of the particle, and the modulus of its total acceleration will be



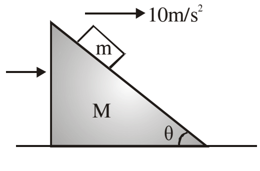
(A) v= 0.4 m/s, a = 0.4 m/s^2 (B) v = 0.32 m/s, a = 0.32 m/s^2
(C) v = 0.32 m/s, a = 0.4 m/s^2 (D) v = 0.4 m/s, a = 0.32 m/s^2

5. Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of the drum in revolutions per minute(rpm) to ensure proper mixing is close to : (Take the radius of the drum to be 1.25 m and its axle to be horizontal):

[JEE Main (Online) - 2016]

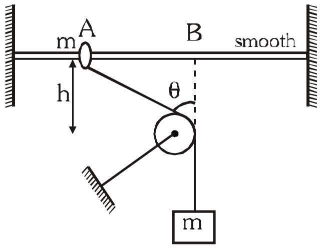
(A) 8.0 (B) 0.4 (C) 1.3 (D) 27.0

6. In the figure shown all the surfaces are frictionless, and mass of the block, m = 1 kg. The block and wedge are held initially at rest. Now wedge is given a horizontal acceleration of 10 m/s^2 by applying a force on the wedge, so that the block does not slip on the wedge. Then work done by the normal force in ground frame on the block in √3 seconds is :



(A) 30J (B) 60 J (C) 150 J (D) 100 $\sqrt{3}$ J

7. A ring of mass m slides from rest on the smooth rod as shown in the figure, due to the block of mass m. Pulley and string are massless. Then find the speed of ring when the string become straight. (Given $\theta = 60°$)

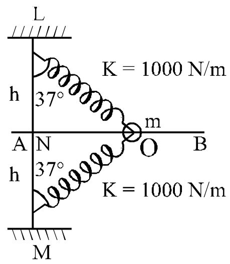


(A) $4\sqrt{gh}$ (B) $\sqrt{2gh}$ (C) $2\sqrt{2gh}$ (D) $8\sqrt{gh}$

8. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to- **[AIEEE - 2003]**

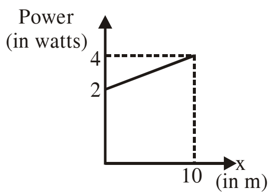
(A) $t^{3/4}$ (B) $t^{3/2}$ (C) $t^{1/4}$ (D) $t^{1/2}$

9. A bead of mass 5kg is free to slide on the horizontal rod AB. They are connected to two identical springs of natural length h m as shown. If initially bead was at O & M is vertically below L then, velocity of bead at point N will be



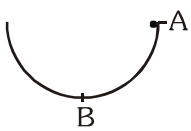
(A) 5h m/s (B) 40h/3 m/s (C) 8h m/s (D) none of these

10. A particle 'A' of mass $\frac{10}{7}$ kg is moving in the positive x–direction. Its initial position is x = 0 & initial velocity is 1 m/s. The velocity at x = 10m is : (use the graph given)



(A) 4 m/s (B) 2 m/s (C) $3\sqrt{2}\ m/s$ (D) 100/3 m/s

11. A small ball can move in a vertical plane along a semi–circle of radius r without friction. At what speed is the ball to launch from point A so that its acceleration is 3g at point B ?



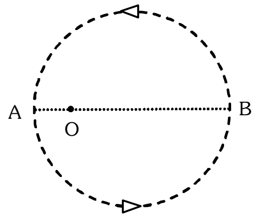
(A) $(3gr)^{1/2}$ (B) $(2gr)^{1/2}$ (C) $(gr)^{1/2}$ (D) $2(gr)^{1/2}$

12. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8\ ms^{-2}$:- **[JEE-Main-2016]**

(A) 12.89×10^{-3} kg (B) 2.45×10^{-3} kg (C) 6.45×10^{-3} kg (D) 9.89×10^{-3} kg

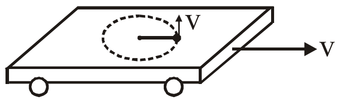
Multiple Correct Answer Type 8 Q. [4 M (–1)]

13. An object moves counter–clockwise along the circular path shown. As it moves along the path, its acceleration vector continuously points towards point O. In the figure, line AB is a diameter.



(A) The object speeds up from A to B and slows down from B to A.
(B) The object slows down from A to B and speeds up from B to A.
(C) The object has maximum speed at A and minimum speed at B.
(D) The object has minimum speed at A and maximum speed at B.

14. On a train moving along east with a constant speed v, a boy revolves a bob with string of length ℓ on smooth surface of a train, with equal constant speed v relative to train. Mark the correct option(s).



(A) Maximum speed of bob is 2 v in ground frame.

(B) Tension in string connecting bob is $\frac{4mv^2}{\ell}$ at an instant.

(C) Tension in string is $\frac{mv^2}{\ell}$ at all the moments.

(D) Minimum speed of bob is zero in ground frame.

15. An ant travels along a long rod with a constant velocity $\vec{u}$ relative to the rod starting from the origin. The rod is kept initially along the positive x-axis. At t = 0, the rod also starts rotating with an angular velocity ω (anticlockwise) in x-y plane about origin. Then

(A) the position of the ant at any time t is $\vec{r} = ut[\cos \omega t\hat{i} + \sin \omega t\hat{j}]$

(B) the speed of the ant at any time t is $u\sqrt{1+\omega^2 t^2}$

(C) the magnitude of the tangential acceleration of the ant at any time t is $\frac{\omega^2 tu}{\sqrt{1+\omega^2 t^2}}$

(D) the speed of the ant at any time t is $\sqrt{1+2\omega^2 t^2 u}$

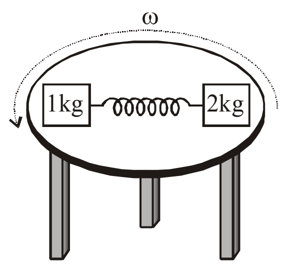
16. On a circular turn table rotating about its center horizontally with uniform angular velocity ω rad/s placed two blocks of mass 1 kg and 2 kg, on a diameter symmetrically about center. Their separation is 1m and friction is sufficient to avoid slipping. The spring between them as shown is stretched and applied force of 5N. If f_1 and f_2 are values of friction on 1 kg & 2kg block respectively:-

(A) For ω = 2 rad/s, f_1 = 3N & f_2 = 1N

(B) For ω = 3 rad/s, f_1 = 0.5 N & f_2 = 4N

(C) For ω = $\sqrt{10}$ rad/s, f_1 = 0 & f_2 = 5N

(D) For ω = $\sqrt{10}$ rad/s, f_1 = 0 & f_2 = 0N



17. Two particles move on a circular path (one just inside and the other just outside) with angular velocities ω and 5ω starting from the same point. Then

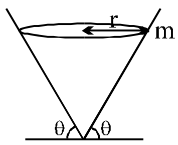
(A) they cross each other at regular intervals of time $\frac{2\pi}{4\omega}$ when their angular velocities are oppositely directed.

(B) they cross each other at points on the path subtending an angle of 60° at the centre if their angular velocities are oppositely directed.

(C) they cross at intervals of time $\frac{\pi}{3\omega}$ if their angular velocities are oppositely directed.

(D) they cross each other at points on the path subtending 90° at the centre if their angular velocities are in the same sense.

18. A ball of mass 'm' is rotating in a circle of radius 'r' with speed v inside a smooth cone as shown in figure. Let N be the normal reaction on the ball by the cone, then choose the correct option.



(A) $N = mg\cos\theta$

(B) $g\sin\theta = \frac{v^2}{r}\cos\theta$

(C) $N\sin\theta - \frac{mv^2}{r} = 0$

(D) none of these

19. A particle of mass 2 kg is projected with an initial speed u = 10 m/sec at an angle $\theta = 30^{\circ}$ with the horizontal

(A) Total work done on the particle during the first half of the total time of flight of the particle is (–25) J.

(B) Total work done on the particle during the total time of flight of the particle is 0 J.

(C) Average power delivered to the particle during the first half of the flight is (–50) watt.

(D) The radius of curvature of the trajectory of the particle at the highest point of the projectile is 7.5m.

20. Which of the following is/are conservative force(s)?

(A) $\vec{F} = 2r^3\hat{r}$

(B) $\vec{F} = -\frac{5}{r}\hat{r}$

(C) $\vec{F} = \frac{3(x\hat{i} + y\hat{j})}{(x^2+y^2)^{3/2}}$

(D) $\vec{F} = \frac{3(y\hat{i} + x\hat{j})}{(x^2+y^2)^{3/2}}$

Linked Comprehension Type (2 Para × 2Q, 1 Para × 3 Q.) [3 M(-1)]
(Single Correct Answer Type)

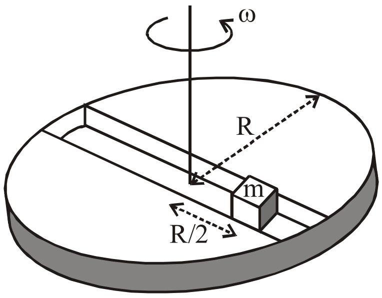
Paragraph for Question No. 21 and 22

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in an inertial frame of reference is

$$\vec{F}_{rot} = \vec{F}_{in} + 2m\left(\vec{\upsilon}_{rot} \times \vec{\omega}\right) + m\left(\vec{\omega} \times \vec{r}\right) \times \vec{\omega},$$

where $\vec{\upsilon}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the centre of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis $(\vec{\omega} = \omega\hat{k})$. A small block of mass m is gently placed in the slot at $\vec{r} = (R/2)\hat{i}$ at $t = 0$ and is constrained to move only along the slot. **[IIT-JEE Advanced-2016]**



21. The distance r of the block at time t is :

(A) $\frac{R}{4}(e^{2\omega t} + e^{-2\omega t})$ (B) $\frac{R}{2}\cos 2\omega t$ (C) $\frac{R}{2}\cos \omega t$ (D) $\frac{R}{4}(e^{\omega t} + e^{-\omega t})$

22. The net reaction of the disc on the block is :

(A) $-m\omega^2 R\cos \omega t\hat{j} - mg\hat{k}$

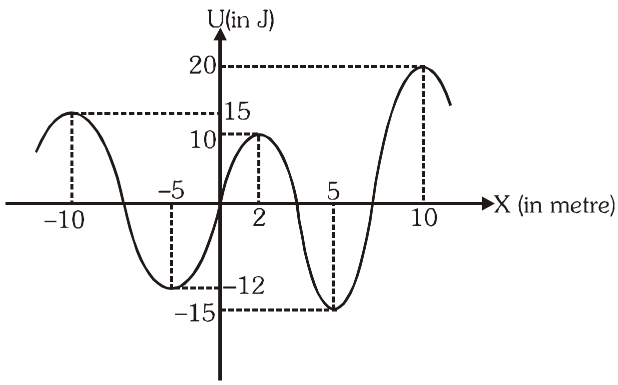
(B) $m\omega^2 R\sin \omega t\hat{j} - mg\hat{k}$

(C) $\frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j} + mg\hat{k}$

(D) $\frac{1}{2}m\omega^2 R(e^{2\omega t} - e^{-2\omega t})\hat{j} + mg\hat{k}$

Paragraph for Question No. 23 and 24

In the figure the variation of potential energy of a particle of mass m = 2kg is represented w.r.t. its x-coordinate. The particle moves under the effect of this conservative force along the x-axis.



23. If the particle is released at the origin then :

(A) It will move towards positive x-axis.

(B) It will move towards negative x-axis.

(C) It will remain stationary at the origin.

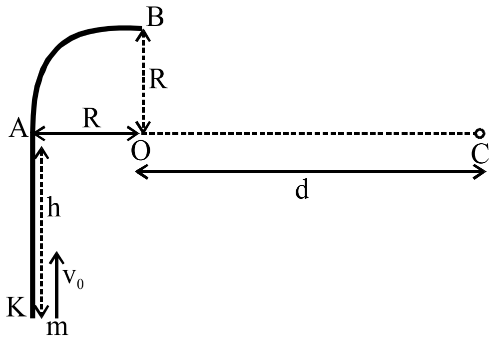
(D) Its subsequent motion cannot be decided due to lack of information.

24. If the particle is released at $x = 2 + \Delta$ where $\Delta \to 0$ (it is positive) then its maximum speed in subsequent motion will be-

(A) $\sqrt{22}$ m/s (B) $\sqrt{25}$ m/s (C) $\sqrt{24}$ m/s (D) $\sqrt{23}$ m/s

Paragraph for Questions Nos. 25 to 27

A point like object of mass m starts from point K as shown in the figure. It slides inside along the full length of the smooth track of radius R, and then moves freely and travels to point C. [The track is kept in vertical plane]



25. Determine the vertical initial velocity of the point like object.

(A) $v_0 = \sqrt{2g(h+R)+\frac{d^2g}{R}}$

(B) $v_0 = \sqrt{2g(h-R)+\frac{d^2g}{2R}}$

(C) $v_0 = \sqrt{2g(h+R)+\frac{d^2g}{2R}}$

(D) $v_0 = \sqrt{2g(h+R)+\frac{dg}{2R}}$

26. What is the minimum possible distance OC = d, necessary for the object to slide along the entire length of the track ?

(A) $d_{min} = R\sqrt{2}$ (B) $d_{min} = \sqrt{3}R$ (C) $d_{min} = \frac{R}{\sqrt{2}}$ (D) $d_{min} = \frac{R}{\sqrt{3}}$

27. Find the normal force exerted by the track at point A.

(A) $F_A = mg\left(\frac{d^2}{2R^2}-2\right)$

(B) $F_A = mg\left(\frac{d^2}{2R^2}+2\right)$

(C) $F_A = mg\left(\frac{3d^2}{R^2}+2\right)$

(D) $F_A = mg\left(\frac{d}{3R^2}+2\right)$

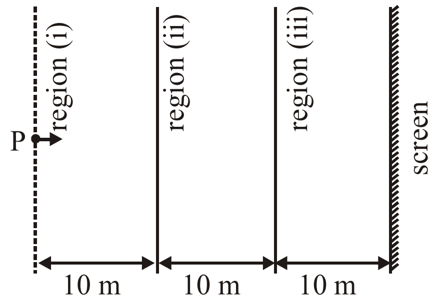
SECTION-II

Numerical Answer Type Question **7 Q. [3(0)]**

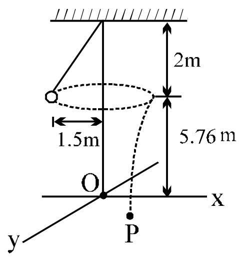
(upto second decimal place)

1. A thin circular loop of radius R rotates about its vertical diameter with an angular frequency ω. Show that a small bead on the wire loop remains at its lowermost point for $\omega \le \sqrt{g/R}$. What is the angle made by the radius vector joining the centre to the bead with the vertical downward direction for $\omega = \sqrt{2g/R}$? Neglect friction.

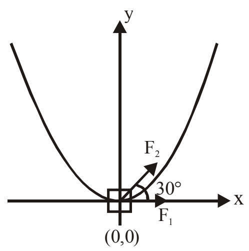
2. A small particle initially at point p starts moving from rest. The whole space where particle will move is divided into three regions as shown in figure. In region (i) particle accelerates through ($5\ m/s^2$) where direction of acceleration is along the normal of the screen while in region (ii) the acceleration acts in such a way that it is always perpendicular to the direction of motion resulting the particle to move on a circular track having radius $\frac{20}{\sqrt{3}}$ m. There is uniform acceleration in region (iii) in such a manner that velocity of particle become thrice (without change in direction) when it just reach the screen. Find the average speed of a particle (in m/s).



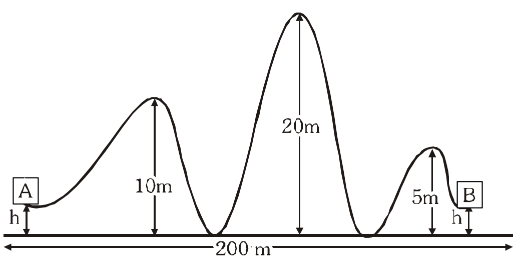
3. A particle suspended from the ceiling by inextensible light string is moving along a horizontal circle of radius 1.5 m as shown. The string traces a cone of height 2 m. The string breaks and the particle finally hits the floor (which is xy plane 5.76 m below the circle) at point P. Find the distance OP in cm.



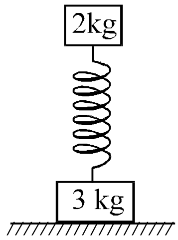
4. A sleeve of mass 2 kg at origin can move on wire of parabolic shape $x^2 = 4y$. Two forces $F_1 = 6N$ and $F_2 = 8N$ are applied on the sleeve. F_1 is constant and is in x-direction. F_2 is constant in direction and magnitude. Body is displaced from origin to $x = 4$, then net work done by F_1 and F_2 is



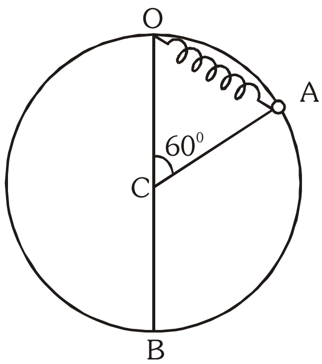
5. A point object of mass 2 kg is moved from point A to point B very slowly on a curved path by applying a tangential force on a curved path as shown in figure. Then find the work done by external force in moving the body. Given that $\mu_s = 0.3$, $\mu_k = 0.1$. [$g = 10\ m/s^2$]



6. The ends of spring are attached to blocks of mass 3kg and 2kg. The 3kg block rests on a horizontal surface and the 2kg block which is vertically above it is in equilibrium producing a compression of 1cm of the spring. The 2kg mass must be compressed further by at least ________, so that when it is released, the 3 kg block may be lifted off the ground.



7. A particle of mass 5 kg is free to slide on a smooth ring of radius r = 20 cm fixed in a vertical plane. The particle is attached to one end of a spring whose other end is fixed to the top point O of the ring. Initially the particle is at rest at a point A of the ring such that $\angle OCA = 60^0$, C being the centre of the ring. The natural length of the spring is also equal to r = 20 cm. After the particle is released and slides down the ring the contact force between the particle & the ring becomes zero when it reaches the lowest position B. Determine the force constant of the spring.



SECTION-III

Numerical Grid Type (Ranging from 0 to 9) 2 Q. [4 M (0)]

1. A particle is moving along a circular path of radius R in such a way that at any instant magnitude of radial acceleration & tangential acceleration are equal. If at t = 0 velocity of particle is V_0. Find the speed of the particle after time $t = \frac{R}{2V_0}$

2. A block of mass $m = \frac{1}{3}$ kg is kept on a rough horizontal plane. Friction coefficient is $\mu = 0.75$. The work done by minimum force required to drag the block along the plane by a distance 5 m, is W joule, then find the value of W.

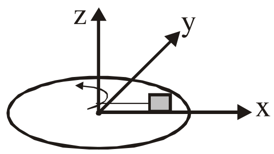
SECTION-IV

Matrix Match Type (4 × 5) **1 Q. [8 M (for each entry +2(0)]**

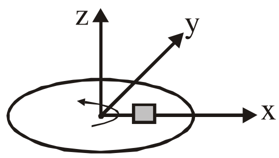
1. Mark the matching options for situations shown in respective figures of column-I at the instant, when particle / car is located on the x-axis as shown :

Column I

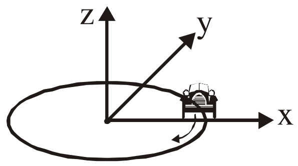
(A) Block attached to string is moving along a circle on rough surface



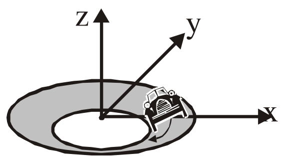
(B) Block is placed on a disc rotating with non-uniform angular velocity as shown below. There is no slipping between block & disc.



(C) Car moving on ground along a circular horizontal track at a constant speed



(D) Car moving on ground at constant speed along a circular banked track.



Column II

(P) Force due to friction may have non-zero x-component

(Q) Force due to friction may have non-zero y-component.

(R) Force due to friction may have non-zero z-component.

(S) Force due to friction may be zero.

(T) Centripetal force is only due to friction.

Subjective Type **9 Q. [4 M (0)]**

1. A 70 kg man stands in contact against the inner wall of a hollow cylindrical drum of radius 3 m rotating about its vertical axis. The coefficient of friction between the wall and his clothing is 0.15. What is the minimum rotational speed of the cylinder to enable the man to remain stuck to the wall (without falling) when the floor is suddenly removed ?

2. A stone is launched upward at 45° with speed v_0. A bee follows the trajectory of the stone at a constant speed equal to the initial speed of the stone.

(i) Find the radius of curvature at the top point of the trajectory.

(ii) What is the acceleration of the bee at the top point of the trajectory? For the stone, neglect the air resistance.

3. Two blocks of mass m_1=10kg and m_2=5kg connected to each other by a massless inextensible string of length 0.3m are placed along a diameter of a turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10rad/sec about a vertical axis passing through its centre. The masses are placed along the diameter of the table on either side of the centre O such that m_1 is at a distance of 0.124 m from O. The masses are observed to be at rest with respect to an observer on the turn table.

(i) Calculate the frictional force on m_1

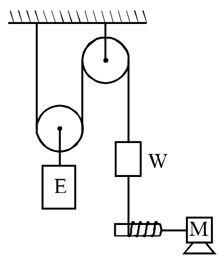
(ii) What should be the minimum angular speed of the turn table so that the masses will slip from this position.

(iii) How should the masses be placed with the string remaining taut, so that there is no frictional force acting on the mass m_1.

4. The elevator E has a mass of 3000 kg when fully loaded and is connected as shown to a counterweight W of mass 1000 kg. Determine the power in kilowatts delivered by the motor

(a) when the elevator is moving down at a constant speed of 3 m/s,

(b) when it has an upward velocity of 3 m/s and a deceleration of 0.5 m/s^2.



5. The P.E. of a particle oscillating on x-axis is given as $U = 20 + (x - 2)^2$ here U is in Joules & x is in meters. Total mechanical energy of particle is 36 J

(i) Find the mean position

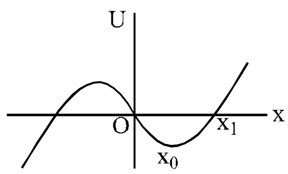
(ii) Find the max. K.E. of the particle

6. A particle is confined to move along the +x axis under the action of a force F(x) that is derivable from the potential energy $U(x) = ax^3 - bx$.

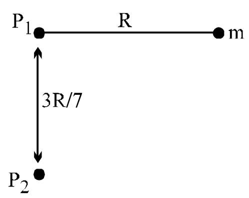
(i) Find the expression for F(x)

(ii) When the total energy of the particle is zero, the particle can be trapped with in the interval x=0 to $x = x_1$. For this case find the values of x_1.

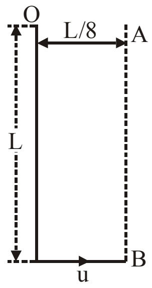
(iii) Determine the maximum kinetic energy that the trapped particle has in its motion. Express all answers in terms a and b. At what value of x will the kinetic energy be maximum ?



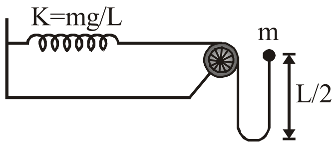
7. A simple pendulum consists of a bob of mass m and a string of length R suspended from a peg P_1 on the wall. A second peg P_2 is fixed vertically below the first one at a distance $3R/7$ from it. The pendulum is drawn aside such that the string is horizontal and released. Calculate the maximum height (with respect to the lowest point) to which it rises



8. A particle is suspended vertically from a point O by an inextensible massless string of length L. A vertical line AB is at a distance $\frac{L}{8}$ from O as shown in figure. The object is given a horizontal velocity u. At some point, its motion ceases to be circular and eventually the object passes through the line AB. At the instant of crossing AB, its velocity is horizontal. Find u.



9. The ball of mass m is connected to an elestic string force constant of spring constant K = mg/L through an inextensible string as shown. Find



(a) The maximum velocity of mass m during fall.

(b) The maximum potential energy stored in the spring during the fall.

ANSWER KEY	GR # CIRCULAR MOTION + WORK, POWER & ENERGY

SECTION-I

Single Correct Answer Type — **12 Q. [3 M (–1)]**

1. Ans. (A)	**2. Ans. (C)**	**3. Ans. (D)**	**4. Ans. (D)**
5. Ans. (D)	**6. Ans. (C)**	**7. Ans. (B)**	**8. Ans. (B)**
9. Ans. (A)	**10. Ans. (A)**	11. **Ans. (C)**	**12. Ans. (A)**

Multiple Correct Answer Type — **8 Q. [4 M (–1)]**

13. Ans. (B,C)	**14. Ans. (A,C,D)**	**15. Ans. (A, B, C)**	**16. Ans. (A, B, C)**
17. Ans. (B, C, D)	**18. Ans. (B, C)**	**19. Ans. (A,B,C,D)**	**20. Ans. (A,B,C)**

Linked Comprehension Type — **(2 Para × 2Q, 1 Para × 3 Q.) [3 M(-1)]**

(Single Correct Answer Type)

21. Ans. (D)	**22. Ans. (C)**	**23. Ans. (B)**	**24. Ans. (B)**
25. Ans. (C)	**26. Ans. (A)**	**27. Ans. (B)**	

SECTION-II

Numerical Answer Type Question — **7 Q. [3(0)]**

(upto second decimal place)

1. Ans. $\theta = 60°$	**2. Ans. 10**	**3. Ans. 390 cm**	**4. Ans. 67.7 J**
5. Ans. 400 J	**6. Ans. 2.5cm**	**7. Ans. 500 N/m**	

SECTION-III

Numerical Grid Type (Ranging from 0 to 9) — **2 Q. [4 M (0)]**

1. Ans. $2V_0$ **2. Ans. 8**

SECTION-IV

Matrix Match Type (4 × 5) — **1 Q. [8 M (for each entry +2(0)]**

1. Ans. (A) Q, (B) P,Q,T (C) P,T (D) P,R,S

Subjective Type — **9 Q. [4 M (0)]**

1. Ans. $\sqrt{\frac{200}{9}}$

2. Ans. (i) $\frac{V_0^2}{2g}$, (ii) 2g

3. Ans. (i) 36N, (ii) 11.66rad/sec, (iii) 0.1m, 0.2m

4. Ans. (a) –30 kW, 19.5 kW

5. Ans. (i) x =2, (ii) 16 J

6. Ans. $F = -3ax^2 + b,\ x = \sqrt{\frac{b}{a}},\ K_{max} = \frac{2b}{3}\sqrt{\frac{b}{3a}}$

7. Ans. 27R/28

8. Ans. $u = \sqrt{gL\left(2+\frac{3\sqrt{3}}{2}\right)}$

9. Ans. (a) $\sqrt{3gL}$, (b) mgL $[2+\sqrt{3}]$

SOLUTIONS
SECTION-I

Single Correct Answer Type **12 Q. [3 M (–1)]**

1. Ans. (A)

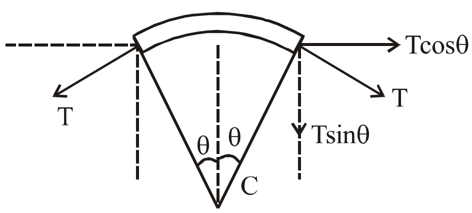
Sol. $\omega = \alpha t \Rightarrow f_s = m\omega^2\ell$

$N = m\alpha\ell \Rightarrow \mu m\alpha\ell = m\omega^2\ell$

$\mu\alpha = \alpha^2 t \Rightarrow t = \sqrt{\dfrac{\mu}{\alpha}}$

2. Ans. (C)

Sol.



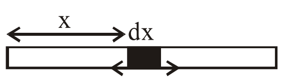
For small θ, $\sin\theta = \theta$

$2T\theta = 2mr\theta\omega^2 r$

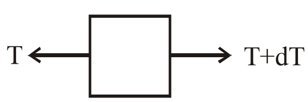
$T = m\omega^2 r^2$

3. Ans. (D)

Sol.



Tension is zero at $x = \ell$



$$-dT = \left[\frac{m}{\ell}\right]\omega^2 x \cdot dx$$

$$\int_0^T dT = \int_\ell^x \frac{m}{\ell}\omega^2 x dx \Rightarrow T = \frac{m\omega^2}{2\ell}[\ell^2 - x^2]$$

4. Ans. (D)

Sol. About centre of circle :

$\omega_C = 2\omega = 0.8$ rad/sec

$v = \omega_C \cdot R = 0.4$ m/s

$a_C = \omega_C^2 R = 0.32$ m/sec

5. **Ans. (D)**

Sol. $R\omega^2 = g$

$$\omega = \sqrt{\frac{g}{R}} = \sqrt{\frac{10}{1.25}} = 2.82$$

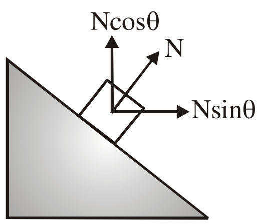
$$f = \frac{2.82}{2\pi}$$

$$\frac{2.82}{2\pi} \times 60$$

$= 26.9 \approx 27$

6. **Ans. (C)**

Sol.



$N \sin \theta = ma = 10$

$$\Delta x = \frac{1}{2}at^2 = 15$$

$W = N \sin \theta \cdot \Delta x = 150$ J

7. **Ans. (B)**

Sol. $$mg\left(\frac{h}{\cos\theta} - h\right) = \frac{1}{2}mv^2$$

8. **Ans. (B)**

Sol. Power = constant

$$\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = \text{constant}$$

$2v \times a = \text{constant}$

$$v \cdot \frac{dv}{dt} = k_1$$

$$\int v dv = \int k_1 dt$$

$v^2 = k_1 t$

$v \propto \sqrt{t}$

$$\frac{dx}{dt} = k_2 t^{1/2}$$

$$\int dx = k_2 \int t^{1/2} \cdot dt$$

$x \propto t^{3/2}$

9. Ans. (A)

Sol. $PE_0 = 2\left(\frac{1}{2}k(\Delta x)^2\right)$

Let total length of spring after extension be x.

$x \cos 37° = h$

$x = \frac{5h}{4} \Rightarrow \Delta x = \frac{h}{4}$

$PE_0 = 2\left(\frac{1}{2}k\left(\frac{h}{4}\right)^2\right) = \frac{125h^2}{2}$ $KE_0 = 0$

$PE_N = 0$ $KE_N = 1/2\ mV^2$

$\frac{1}{2}mv^2 = \frac{125h^2}{2}$ $v = 5h$ m/s

10. Ans. (A)

Sol. $P = F \cdot v = mav = mv^2 \cdot \frac{dv}{ds}$

$\int P ds = \int mv^2 dv$

$\Rightarrow 30 = \frac{10}{7} \times \frac{1}{3}\left[v^3\right]_1^v$

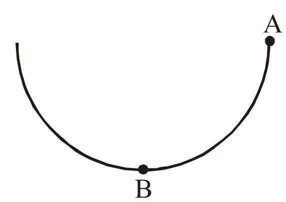
$\Rightarrow 63 = v^3 - 1 \Rightarrow v = 4$ m/s

11. Ans. (C)

Sol. $a_B = \frac{v_B^2}{R} = 3g$

$v_B^2 = 3gR$

Applying COE at A & B



$mgR + \frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 = \frac{1}{2}m \times 3gR$

$\frac{1}{2}mv_A^2 = \frac{mgR}{2}$

$v_A = \sqrt{gR}$

12. Ans. (A)

Sol. Work done against gravity = (mgh) 1000

in lifting 1000 times

$= 10 \times 9.8 \times 10^3$

$= 9.8 \times 10^4$ Joule

20% efficiency is to converts fat into energy.

$[20\% \text{ of } 3.8 \times 10^7 \text{ J}] \times (m) = 9.8 \times 10^4$

(Where m is mass)

$m = 12.89 \times 10^{-3}$ kg

Multiple Correct Answer Type 8 Q. [4 M (–1)]

13. Ans. (B,C)

Sol. From B to A acceleration makes acute angle with velocity.

14. Ans. (A,C,D)

Sol. Circular motion is in train's frame. In this frame sped v is constant.

$$\vec{v}_{bob,ground} = \vec{v}_{b,T} + \vec{v}_T$$

$$v_{max} = v + v = 2v$$

$$v_{min} = -v + v = 0$$

$$T = \frac{mv^2}{\ell}$$

15. Ans. (A, B, C)

Sol. $\vec{r} = ut[\cos\omega t\hat{i} + \sin\omega t\hat{j}]$

$$\Rightarrow \vec{v} = \frac{d\vec{r}}{dt} = u[\cos\omega t\hat{i} + \sin\omega t\hat{j}] + ut[-\omega\sin\omega t\hat{i} + \omega\cos\omega t\hat{j}]$$

$$|a_1| = \frac{d|\vec{v}|}{dt} = \frac{\omega^2 tu}{\sqrt{1+\omega^2 t^2}}$$

16. Ans. (A, B, C)

Sol. $5 + f_1 = 1 \times \omega^2 \times \frac{1}{2}$

$$5 + f_2 = 2 \times \omega^2 \times \frac{1}{2}$$

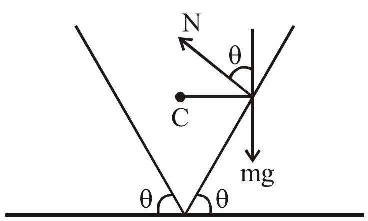
depending on sign of f_1 and f_2 they can be inward or outward.

17. Ans. (B, C, D)

Sol. For same sense : $5\omega t = \omega t + 2\pi$

18. Ans. (B, C)

Sol.

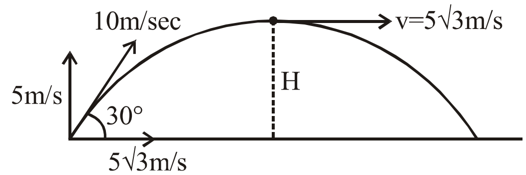


$$N\cos\theta = mg$$

$$N\sin\theta = \frac{mv^2}{r}$$

19. Ans. (A,B,C,D)

Sol. (A) $W = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$



$$W = \frac{1}{2}\times 2\times\left(75-100\right) = -25\text{ J}$$

(B) Change in KE = work done = 0

(C) $t = \frac{u\sin\theta}{g} = \frac{5}{10} = \frac{1}{2}\text{sec}$

$$P_{avg} = \frac{W}{\text{time}} = \frac{-25}{\frac{1}{2}} = -50\,\text{watt}$$

(D) $R_C = \frac{v_T^2}{a_e} = \frac{\left(5\sqrt{3}\right)^2}{10} = 7.5\text{m}$

20. Ans. (A,B,C)

Sol. $\vec{F} = \frac{3\vec{r}}{r^3} = \frac{3\hat{r}}{r^2}$

$\vec{F}$ is a constant force and constant force is always conservative.

Linked Comprehension Type (2 Para × 2Q, 1 Para × 3 Q.) [3 M(-1)]
(Single Correct Answer Type)

21. Ans. (D)

Sol. Force on block along slot = $m\omega^2 r$ = ma = $m\left(\frac{vdv}{dr}\right)$

$$\int_0^v vdv = \int_{R/2}^{r}\omega^2 rdr$$

$$\frac{v^2}{2} = \frac{\omega^2}{2}\left(r^2 - \frac{R^2}{4}\right) \Rightarrow v = \omega\sqrt{r^2 - \frac{R^2}{4}} = \frac{dr}{dt}$$

$$\Rightarrow \int_{R/4}^{r}\frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \int_0^t \omega dt$$

$$\ell n\left(\frac{r+\sqrt{r^2-\frac{R^2}{4}}}{\frac{R}{2}}\right) - \ell n\left(\frac{R/2+\sqrt{\frac{R^2}{4}-\frac{R^2}{4}}}{\frac{R}{2}}\right) = \omega t$$

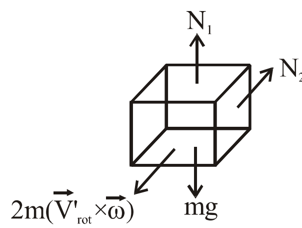
$$\Rightarrow r+\sqrt{r^2-\frac{R^2}{4}}=\frac{R}{2}e^{\omega t}$$

$$\Rightarrow r^2-\frac{R^2}{4}=\frac{R^2}{4}e^{2\omega t}+r^2-2r\frac{R}{2}e^{\omega t}$$

$$\Rightarrow r=\frac{\frac{R^2}{4}e^{2\omega t}+\frac{R^2}{4}}{Re^{\omega t}}=\frac{R}{4}\left(e^{\omega t}+e^{-\omega t}\right)$$

22. Ans. (C)

Sol. $\vec{N}_1=mg\hat{k}$



$\vec{N}_2=2m\left(V'_{rot}\times\vec{\omega}\right)\hat{j}\quad=2m\left[\frac{\omega R}{4}\left(e^{\omega t}-e^{-\omega t}\right)\right]\omega\hat{j}$

$=\frac{1}{2}m\omega^2R\left(e^{\omega t}-e^{-\omega t}\right)\hat{j}$

Total reaction on block = $\vec{N}_1+\vec{N}_2$

$=\frac{1}{2}m\omega^2R\left(e^{\omega t}-e^{-\omega t}\right)\hat{j}+mg\hat{k}$

23. Ans. (B)

Sol. If the particle is released at the origin, it will try to go in the direction of force. Here $\frac{dU}{dx}$ is positive and hence force is in negative x-direction, as a result it will move towards –ve x-axis.

24. Ans. (B)

Sol. When the particle is released at $x=2+\Delta$ it will reach the point of least possible potential energy (–15J) where it will have maximum kinetic energy. $\therefore \frac{1}{2}mv_{max}^2=25\Rightarrow v_{max}=5m/s$

25. Ans. (C)

Sol. Applying WET

$-mg\left(h+R\right)=\frac{1}{2}mv_B^2-\frac{1}{2}mv_0^2$ ... (i)

Considering motion from B to C

$v_B\times\sqrt{\frac{2R}{g}}=d\Rightarrow v_B=d\sqrt{\frac{g}{2R}}$

Putting in equation (i) we get option (C)

26. Ans. (A)

Sol. Particle should not loose contact any where on the circular track.

Consdring N = 0 at highest point.

$$\frac{mv^2}{R} = mg \Rightarrow d = \sqrt{2}R$$

27. Ans. (B)

Sol. $v_0^2 - v_A^2 = 2gh$

$$N_A = \frac{mv_A^2}{R} = \frac{m\left(v_0^2 - 2gh\right)}{R}$$

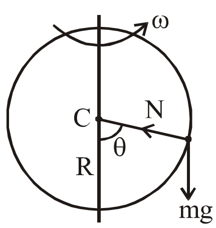
SECTION-II

Numerical Answer Type Question 7 Q. [3(0)]
(upto second decimal place)

1. Ans. $\theta = 60°$

Sol. Consider the free-body diagram of the bead when the radius vector joining the centre of the wire makes an angle θ with the vertical downward direction. We have $mg = N\cos\theta$ and $mR\sin\theta\omega^2 = N\sin\theta$. These equations give $\cos\theta = g/R\omega^2$. Since $\cos\theta \le 1$, the bead remains at its lowermost point for $\omega \le \sqrt{\frac{g}{R}}$.

For $\omega = \sqrt{\frac{2g}{R}}$, $\cos\theta = \frac{1}{2}$ i.e. $\theta = 60°$



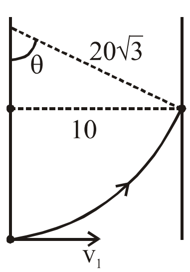
2. Ans. 10

Sol. $V_{AV} = \frac{\text{Total distance}}{\text{total time}} = \frac{10 + \frac{\pi}{3}\frac{20}{\sqrt{3}} + 20}{2 + \frac{2\pi}{3\sqrt{3}} + 1} = 10$ m/s. = 10 m/s

(I) $v_1^2 = 2 \times 5 \times 10$

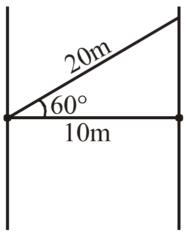
$v_1 = 10$ m/sec

(II)

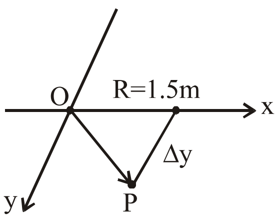


$$\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\text{arc} = \frac{20}{\sqrt{3}} \times \frac{A}{3}$$

(III) 

3. Ans. 0390 cm

Sol. 

$\Delta y = v \times t$

$t = \sqrt{\frac{2h}{g}} = 1.1$ sec

$v = \sqrt{rg \tan\theta} = 3.3$ m/sec

4. Ans. 67.7 J

Sol. $W_1 = \vec{F}_1.\vec{d}_1 = (6)(4) = 24N$

$W_2 = \vec{F}_2.\vec{d}_2$ where $\vec{F}_2 = 8\cos 30i + 8\sin 30j = \left(4\sqrt{3}\hat{i} + 4\hat{j}\right)$, $\vec{s} = (4\hat{i} + 4\hat{j})$

$\Rightarrow W_2 = 43.7$ J therefore $W_{net} = 67.7$ J

5. Ans. 400 J

Sol. $\Delta E = W_{fr} + W_g + W_{ext}$

$= -(0.1 \times 20 \times 200) + 0 + W_{ext} = 400$ J

6. Ans. 2.5cm

Sol. $kx_0 = mg \Rightarrow k = 2000$ N/m [$x_0 = 1$ cm in inequlibrium]

to lift 3 kg mass, extension in the spring must be, $y = \frac{30}{2000} = \frac{3}{200}$ m

Let further compresion in spring is x.

$$W_s + W_{mg} = \Delta KE \Rightarrow -\frac{1}{2}k\left[y^2 - \left(x + \frac{1}{100}\right)^2\right] - mg\left[x + \frac{1}{100} + y\right] = 0$$

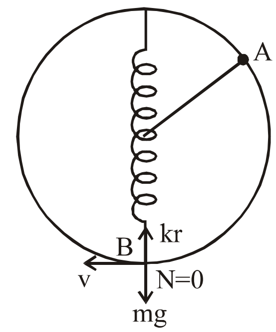
$$\Rightarrow -1000\left[\frac{9}{40000} - \left(x + \frac{1}{100}\right)^2\right] = 20\left[\frac{3}{200} + \left(x + \frac{1}{100}\right)\right]$$

$$\Rightarrow -\frac{9}{800} + 50\left(x + \frac{1}{100}\right)^2 = \frac{3}{200} + \left(x + \frac{1}{100}\right)$$

$$\Rightarrow 50\left(x+\frac{1}{100}\right)^2-\left(x+\frac{1}{100}\right)-\frac{21}{800}=0 \Rightarrow x = 2.5\text{cm}$$

7. **Ans.** 500 N/m

Sol. Initial length of spring = r (relaxed)



$$kr - mg = \frac{mv^2}{r} \qquad \text{... (i)}$$

COE at A & B

$$mgr\,(1 + \cos 60°) - \frac{1}{2}kr^2 = \frac{1}{2}mv^2 \Rightarrow mv^2 = 3\,mgr$$

From equaiton (i)

kr – mg = 3mg

kr = 4 mg

$$k = \frac{2mg}{r} = 500$$

SECTION-III

Numerical Grid Type (Ranging from 0 to 9) 2 Q. [4 M (0)]

1. **Ans.** $2V_0$

Sol. $\frac{dv}{dt} = \frac{v^2}{R}$

$$\int_{v_0}^{v}\frac{dv}{v^2} = \frac{1}{R}\int_0^{\frac{R}{2v_0}} dt$$

$$-\frac{1}{v}+\frac{1}{v_0}=\frac{1}{v_0}$$

$v = 2v_0$

2. **Ans. 8**

Sol. $F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$ when $\theta = \tan^{-1}(\mu) \Rightarrow W = (F_{min})\, S \cos\theta = \frac{\mu mgS}{(1+\mu^2)} = 8$ joule

SECTION-IV

Matrix Match Type (4 × 5) 1 Q. [8 M (for each entry +2(0)]

1. **Ans. (A) Q, (B) P,Q,T (C) P,T (D) P,R,S**

Sol. (A) Friction along tangent only.

(B) Friction provide centripetal and tangential acceleration.

(C) Friction provide only centripetal acceleration.

(D) If speed is not equal to $\sqrt{rg\tan\theta}$ then friction is along the incline of road to avoid slipping & if speed is equal to $\sqrt{rg\tan\theta}$, no friction.

Subjective Type **9 Q. [4 M (0)]**

1. **Ans.** $\sqrt{\frac{200}{9}}$

Sol. The horizontal force N by the wall on the man provides the needed centripetal force : $N = mR\omega^2$. The frictional force f (vertically upwards) balances the weight mg. The man remains stuck to the wall after the floor is removed if $mg = f < \mu N$ i.e. $mg < \mu mR\omega^2$. The minimum angular speed of rotation of the cylinder is $w_{min} = \sqrt{\frac{g}{\mu R}} = \sqrt{\frac{200}{9}}$

2. **Ans.** (i) $\frac{V_0^2}{2g}$, (ii) 2g

Sol. (i) $a_n = \frac{v^2}{R}$

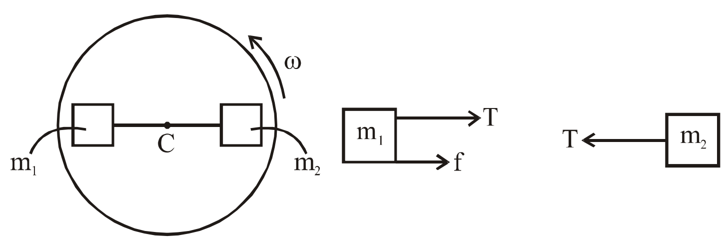
$$R = \frac{v_0^2 \cos^2\theta}{g} = \frac{v_0^2}{2g}$$

(ii) $\frac{v_0^2}{2g} = \frac{v_0^2}{a}$

$a = 2g$

3. **Ans.** (i) 36N, (ii) 11.66rad/sec, (iii) 0.1m, 0.2m

Sol. (i)



$T + f = m_1\omega^2 r_1$ ... (i)

$T = m_2\omega^2 r_2$ ... (ii)

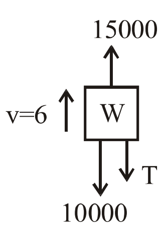
$\Rightarrow f = (m_1r_1 - m_2r_2)\,\omega^2 = 36$ N

(ii) $\mu m_1 g = (m_1r_1 - m_2r_2)\omega^2$

(iii) $0 = (m_1r_1 - m_2r_2)\omega^2$

4. **Ans.** (a) –30 kW, 19.5 kW

Sol.



T = force applied by motor

(a) P= F·v = –T × 6

= –5000 × 6

= –30 kW

5. **Ans.** (i) x =2, (ii) 16 J

Sol. At mean position, U = minimum

U_{min} = 20 j, at x = 2m, KE_{max} = 36 – 20 = 16 J

6. **Ans.** $F = -3ax^2 + b$, $x = \sqrt{\frac{b}{a}}$, $K_{max} = \frac{2b}{3}\sqrt{\frac{b}{3a}}$

Sol. (i) $F = -\frac{dU}{dx} = b - 3ax^2$

(ii) U = 0

$x = \sqrt{\frac{b}{a}} = x_1$

(iii) for KE = max., PE = min

$\frac{dU}{dx} = 0 \Rightarrow x = \sqrt{\frac{b}{3a}}$

7. **Ans. 27R/28**

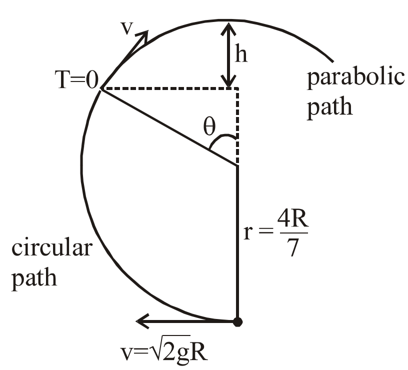
Sol. Where T = 0, $v = \sqrt{gr\cos\theta}$

Applying COE

$mgR = \frac{1}{2}$ mgr cos θ + mgr (1 + cos θ)

$R = \frac{3}{2}r\cos\theta + r$

putting $r = \frac{4R}{7}$ we get $\cos\theta = \frac{1}{2}$ & θ = 60°

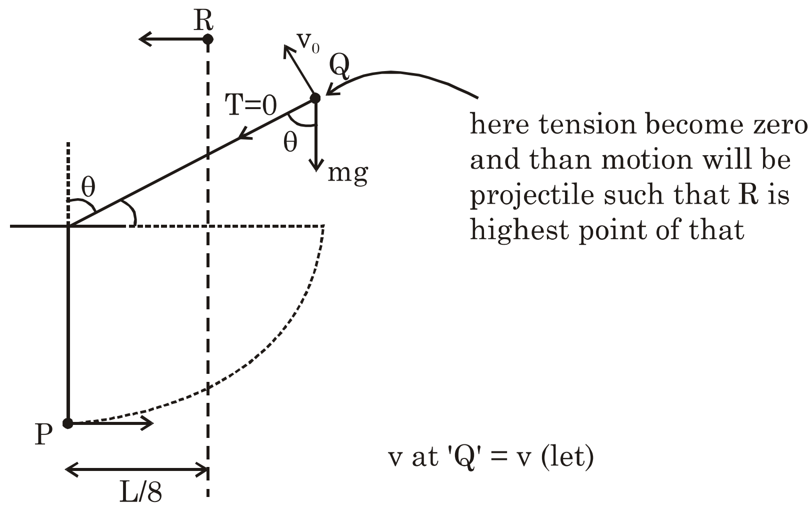


Maximum height (w.r.t. lowest point)

$$h_{max} = \frac{4R}{7} + \frac{4R}{7}\cos\theta + \frac{v^2\sin^2\theta}{2g} = \frac{27R}{28}$$

8. **Ans.** $u = \sqrt{gL\left(2 + \frac{3\sqrt{3}}{2}\right)}$

Sol. When particle is crossing AB line, it is given that velocity of particle at that instant is horizontal hence, that will be highest point of projectile motion



v at 'Q' = v (let)

Circular motion equation $\Rightarrow mg\cos\theta + T = \dfrac{mv^2}{L}$ where (T = 0) ... (i)

Work energy theorem $\Rightarrow -mg\left(L + L\cos\theta\right) = \dfrac{1}{2}mv^2 - \dfrac{1}{2}mu^2$ .... (ii)

$$\frac{\text{Range}}{2} = \left(L\sin\theta - \frac{L}{8}\right) \qquad \text{... (iii)}$$

Where range $= \dfrac{2v\sin\theta}{g} v\cos\theta$ ....(iv)

Solve (i), (ii), (iii) & (iv)

$Lg\cos\theta = v^2$ (from .... (i))

$v^2 = u^2 - 2gL\ (1 + \cos v)$ ... from (ii)

$Lg\cos\theta = u^2 - 2gL - 2gL\cos\theta$

$u^2 = 2gL + 3gL\cos\theta$ ... (iv)

$$\frac{2v^2\sin\theta\cos\theta}{2g} = L\sin\theta - \frac{L}{8} \qquad \{v^2 = L\,g\cos\theta \text{ put}\}$$

$$\frac{2Lg\cos\theta \cdot \sin\theta\cos\theta}{2g} = L\sin\theta - \frac{L}{8}$$

$$\cos^2\theta \cdot \sin\theta = \sin\theta - \frac{1}{8}$$

$$\left(1 - \sin^2\theta\right)\cdot \sin\theta = \sin\theta - \frac{1}{8}$$

$$\sin\theta - \sin^3\theta = \sin\theta - \frac{1}{8}$$

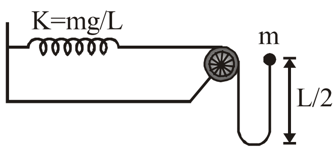
$$\sin^3\theta = \frac{1}{8}$$

$$\sin\theta = \frac{1}{2} \qquad\qquad \theta = 30°$$

$$\Rightarrow \text{ from ... (iv) } u = \sqrt{2g\ell + 3gL\frac{\sqrt{3}}{2}}$$

9. **Ans.** (a) $\sqrt{3gL}$, (b) mgL [2 + $\sqrt{3}$]

Sol.



(a) maximum velocity will be at m.p. i.e. when elongation in spring will be $\frac{mg}{k}$.

Work energy $W_g + W_{sp} = \Delta KE$

$$mg\left(\frac{L}{2}+\frac{L}{2}+\frac{mg}{k}\right)-\frac{1}{2}k\left\{\left(\frac{mg}{k}\right)^2-o^2\right\}=\frac{1}{2}mv_{max}^2-0$$

$$mgL+\frac{m^2g^2}{k}-\frac{1}{2}\frac{m^2g^2}{k}=\frac{1}{2}mv_{max}^2$$

$$2gL+\frac{mg^2}{k}=v_{max}^2 \qquad\qquad k=\frac{mg}{L}$$

$$2gL+\frac{mg^2}{mg/L}=v_{max}^2$$

$$\sqrt{3gL}=v_{max}$$

(b) When elongation in spring will be maximum, potential energy in the spring will be maximum means when velocity of block become zero.

Work energy theorem

$$mg\left(L+x_{max}\right)-\frac{1}{2}k\left(x_{max}^2-0\right)=0-0$$

$$mgL+mgx_{max}=\frac{1}{2}\times\frac{mg}{L}\cdot x_{max}^2$$

$$L+x_{max}=\frac{x_{max}^2}{2L}$$

$$x^2_{max}-2L\cdot x_{max}-2L^2=0$$

quadratic

$$x_{max} = \frac{2L \pm \sqrt{4L^2 + 4 \times 2L^2}}{2}$$

$$= \frac{2L + 2L\sqrt{3}}{2} = L + L\sqrt{3}$$

$$(P.E.)_{max} = \frac{1}{2}kx_{max}^2$$

$$= \frac{1}{2} \times \frac{mg}{L} \times \left[L\left(1+\sqrt{3}\right)\right]^2$$

$$= \frac{1}{2}\frac{mg}{L}L^2\left(1+3+2\sqrt{3}\right)$$

$$= mgL\left(2+\sqrt{3}\right)$$