

Chapter 12

Introduction To Three Dimensional Geometry

Exercise 12.3

Question 1: Find the coordinates of the point which divides the line segment joining the points $(-2, 3, 5)$ and $(1, -4, 6)$ in the ratio (i) 2:3 internally, (ii) 2:3 externally.

Answer 1:

(i) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio m: n is

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right)$$

Let R (x, y, z) be the point that divides the line segment joining points $(-2, 3, 5)$ and $(1, -4, 6)$ internally in the ratio 2:3

$$x = \frac{2(1) + 3(-2)}{2+3}, y = \frac{2(-4) + 3(3)}{2+3} \text{ and } z = \frac{2(6) + 3(5)}{2+3}$$

$$\text{i.e., } x = \frac{-4}{5}, y = \frac{1}{5}, \text{ and } z = \frac{27}{5}$$

Thus, the coordinates of the required point are $\left(\frac{-4}{5}, \frac{1}{5}, \frac{27}{5} \right)$

(ii) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) externally in the ratio m: n is

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n} \right)$$

Let R (x, y, z) be the point that divides the line segment joining points $(2, 3, 5)$ and $(1, -4, 6)$ externally in the ratio 2:3

$$x = \frac{2(1) - 3(-2)}{2-3}, y = \frac{2(-4) - 3(3)}{2-3}, \text{ and } z = \frac{2(6) - 3(5)}{2-3}$$

i.e., $x = -8$, $y = 17$, and $z = 3$

Thus, the coordinates of the required point are $(-8, 17, 3)$.

Question 2: Given that P $(3, 2, -4)$, Q $(5, 4, -6)$ and R $(9, 8, -10)$ are collinear. Find the ratio in which Q divides PR.

Answer 2:

Let point Q $(5, 4, -6)$ divide the line segment joining points P $(3, 2, -4)$ and R $(9, 8, -10)$ in the ratio $k:1$.

Therefore, by section formula,

$$\begin{aligned}(5, 4, -6) &= \left(\frac{k(9)+3}{k+1}, \frac{k(8)+2}{k+1}, \frac{k(-10)-4}{k+1} \right) \\&= \frac{9k+3}{k+1} = 5 \\&= 9k + 3 = 5k + 5 \\&= 4k = 2 \\&= k = \frac{2}{4} = \frac{1}{2}\end{aligned}$$

Thus, point Q divides PR in the ratio 1:2.

Question 3: Find the ratio in which the YZ-plane divides the line segment formed by joining the points $(-2, 4, 7)$ and $(3, -5, 8)$.

Answer 3:

Let the YZ plane divide the line segment joining points $(-2, 4, 7)$ and $(3, -5, 8)$ in the ratio $k:1$.

Hence, by section formula, the coordinates of point of intersection are given by

$$\left(\frac{k(3)-2}{k+1}, \frac{k(-5)+4}{k+1}, \frac{k(8)+7}{k+1} \right)$$

On the YZ plane, the x-coordinate of any point is zero

$$= \frac{3k-2}{k+1} = 0$$

$$= 3k - 2 = 0$$

$$= k = \frac{2}{3}$$

Thus, the YZ plane divides the line segment formed by joining the given points in the ratio 2:3.

Question 4: Using section formula, show that the points A (2, -3, 4), B (-1, 2, 1) and C ($0, \frac{1}{3}, 2$) are collinear.

Answer 4:

The given points are A (2, -3, 4), B (-1, 2, 1), and C ($0, \frac{1}{3}, 2$).

Let P be a point that divides AB in the ratio k:1.

Hence, by section formula, the coordinates of P are given by

$$\left(\frac{k(-1)+2}{k+1}, \frac{k(2)-3}{k+1}, \frac{k(1)+4}{k+1} \right)$$

Now, we find the value of k at which point P coincides with point C.

By taking, $\frac{-k+2}{k+1} = 0$ we obtain k = 2.

For k = 2, the coordinates of point p is ($0, \frac{1}{3}, 2$).

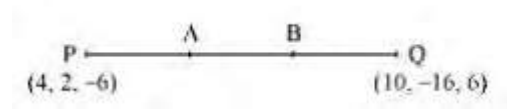
i.e., C ($0, \frac{1}{3}, 2$) is a point that divides AB externally in the ratio 2:1 and is the same as point P.

Hence, points A, B, and C are collinear.

Question 5: Find the coordinates of the points which trisect the line segment joining the points P (4, 2, -6) and Q (10, -16, 6).

Answer 5:

Let A and B be the points that trisect the line segment joining points P (4, 2, -6) and Q (10, -16, 6)



Point A divides PQ in the ratio 1:2. Therefore, by section formula, the coordinates of point A are given by

$$\left(\frac{1(10)+2(4)}{1+2}, \frac{1(-16)+2(2)}{1+2}, \frac{1(6)+2(-6)}{1+2} \right) = (6, -4, -2)$$

Point B divides PQ in the ratio 2:1. Therefore, by section formula, the coordinates of point B are given by

$$\left(\frac{2(10)+1(4)}{2+1}, \frac{2(-16)+1(2)}{2+1}, \frac{2(6)+1(-6)}{2+1} \right) = (8, -10, 2)$$

Thus, (6, -4, -2) and (8, -10, 2) are the points that trisect the line segment joining points P (4, 2, -6) and Q (10, -16, 6).