Chapter 6

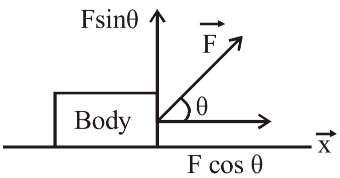
Work, Energy and Power

WORK

Work Done by a Constant Force

Work done (W) by a force $\vec{F}$ in displacing a body through a displacement x is given by

$W = \vec{F}.\vec{x} = Fx \cos\theta$



Where θ is the angle between the applied force $\vec{F}$ and displacement $\vec{x}$.

The **S.I. unit** of work is joule, **CGS unit** is erg and its **dimensions** are $[ML^2T^{-2}]$.

1 joule = 10^7 erg

(a) When $\theta = 0°$ then $W = Fx$

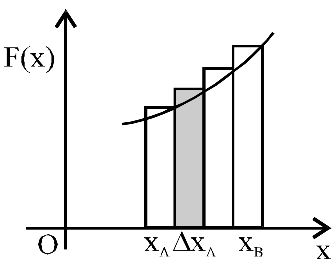
(b) When θ is between 0 and π/2 then
$W = Fx \cos\theta$ = **positive**

(c) When $\theta = \pi/2$ then $W = Fx \cos 90° = 0$ **(zero)**
Work done by centripetal force is zero as in this case angle $\theta = 90°$

(d) ∴ When θ is between $\pi/2$ and π then
$W = Fx \cos\theta$ = **negative**

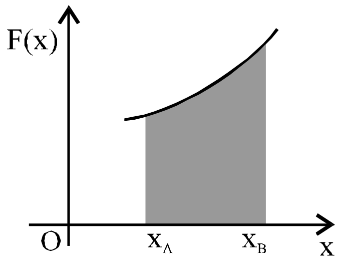
Work Done by a Variable Force

When the force is an arbitrary function of position, we need the techniques of calculus to evaluate the work done by it. The figure shows F_x as function of the position x. We begin by replacing the actual variation of the force by a series of small steps.



The area under each segment of the curve is approximately equal to the area of a rectangle. The height of the rectangle is a constant value of force, and its width is a small displacement Δx. Thus, the step involves an amount of work $\Delta W_n = F_n \Delta x_n$. The total work done is approximately given by the sum of the areas of the rectangles.

i.e., $W \approx \sum F_n \Delta x_n$.



As the size of the steps is reduced, the tops of the rectangle more closely trace the actual curve shown in figure. If the limit $\Delta x \to 0$, which is equivalent to letting the number of steps tend to infinity, the discrete sum is replaced by a continuous integral.

$$W = \lim_{\Delta x_n \to 0} \sum F_n \Delta x_n = \int F_x dx$$

Thus, the work done by a force F_x from an initial point A to final point B is $W_{A\to B} = \int_{x_A}^{x_n} F_x dx$

The work done by a variable force in displacing a particle from x_1 to x_2

$$W = \int_{x_1}^{x_2} F dx = \text{area under force displacement graph}$$

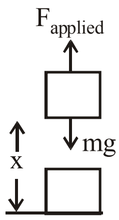
⚠ **CAUTION :** When we find work, we should be cautious about the question, work done by which force? Let us take an example to understand this point. Suppose you are moving a body up without acceleration.

Work done by applied force

$W_{app} = \vec{F}_{app} \cdot \vec{x} = F_{app} x$

Work done by gravitational force

$\overrightarrow{W}_{grav} = \vec{F}_g \, \vec{x} = -mgx$



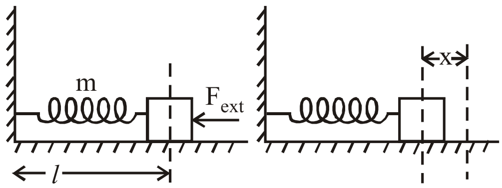
ENERGY

It is the capacity of doing work. Its units and dimensions are same as that of work.

Potential Energy

The energy possessed by a body by virtue of its position or configuration is called potential energy. Potential energy is defined only for conservative forces. It does not exist for non conservative forces.

(a) Elastic potential energy (Potential energy of a spring) : Let us consider a spring, its one end is attached to a rigid wall and other is fixed to a mass m. We apply an external force $\vec{F}_{ext.}$ on mass m in the left direction, so that the spring is compressed by a distance x.

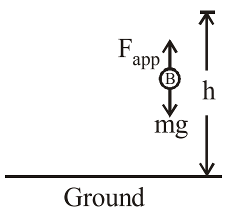


If spring constant is k, then energy stored in spring is given by

P.E. of compressed spring = ½kx²

Now if the external force is removed, the mass m is free to move then due to the stored energy in the spring, it starts oscillating

(b) Gravitational potential energy : When a body is raised to some height, above the ground, it acquires some potential energy, due to its position. The potential energy due to height is called gravitational potential energy. Let us consider a ball B, which is raised by a height h from the ground.



In doing so, we do work against gravity and this work is stored in the ball B in the form of gravitational potential energy and is given by

$W = F_{app}.\ h = mgh$ = *gravitational potential energy* ...(i)

Further if ball B has gravitational P.E. (potential energy) U_o at ground and at height h, U_h, then

$$U_h - U_o = mgh \quad ...(ii)$$

If we choose $U_o = 0$ at ground (called reference point) then absolute gravitational P.E of ball at height h is

$$U_h = mgh \quad ...(iii)$$

In general, if two bodies of masses m_1 and m_2 are separated by a distance r, then the gravitational potential energy is

$$U = -G\frac{m_1 m_2}{r}$$

Kinetic Energy

The energy possessed by a body by virtue of its motion is called kinetic energy.

The kinetic energy E_k is given by

$$E_k = \tfrac{1}{2} mv^2 \quad ...(i)$$

Where m is mass of body, which is moving with velocity v. We know that linear momentum (p) of a body which is moving with a velocity v is given by

$$p = mv \quad ...(ii)$$

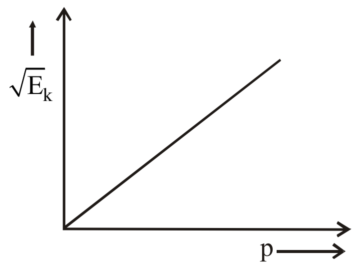
So from eqs. (i) and (ii), we have

$$E_k = \frac{p^2}{2m} \quad ...(iii)$$

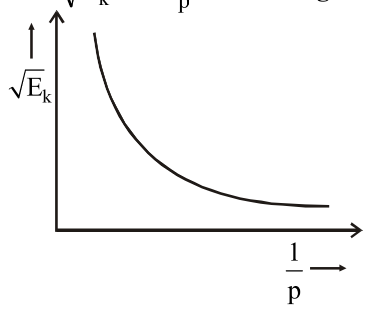
This is the **relation between momentum and kinetic energy.**

The graph between $\sqrt{E_k}$ and p is **a straight line**

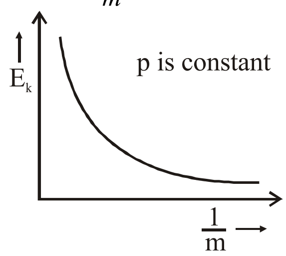
$\because p = \sqrt{2m\,E_k}$



The graph between $\sqrt{E_k}$ and $\frac{1}{p}$ is a **rectangular hyperbola**



The graph between E_k and $\frac{1}{m}$ is a **rectangular hyperbola**



Keep in Memory

1. Work done by the conservative force in moving a body in a closed loop is zero.

   Work done by the non-conservative force in moving a body in a closed loop is non-zero.

2. If the momenta of two bodies are equal then the kinetic energy of lighter body will be more.

   $$\because \quad p_1 = p_2 \text{ or } \sqrt{2m_1E_1} = \sqrt{2m_2E_2}$$

$\therefore \quad \frac{E_1}{E_2} = \frac{m_2}{m_1}$

3. If the kinetic energies of two bodies are same then the momentum of heavier body will be more.

$\because E_1 = E_2 \qquad \therefore \frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}$

WORK-ENERGY THEOREM

Let a number of forces acting on a body of mass m have a resultant force $\vec{F}_{ext.}$ And by acting over a displacement x (in the direction of $\vec{F}_{ext.}$), $\vec{F}_{ext.}$ does work on the body, and there by changing its velocity from u (initial velocity) to v (final velocity). Kinetic energy of the body changes.

So, work done by force on the body is equal to the change in kinetic energy of the body.

$$\boxed{W = \tfrac{1}{2} mv^2 - \tfrac{1}{2} mu^2}$$

This expression is called **Work energy (W.E.) theorem.**

LAW OF CONSERVATION OF MECHANICAL ENERGY

The sum of the potential energy and the kinetic energy is called the total mechanical energy.

The total mechanical energy of a system remains constant if only conservative forces are acting on a system of particles and the work done by all other forces is zero.

i.e., $\Delta K + \Delta U = 0$

or $K_f - K_i + U_f - U_i = 0$

or $K_f + U_f = K_i + U_i = \text{constant}$

VARIOUS FORMS OF ENERGY : THE LAW OF CONSERVATION OF ENERGY

Energy is of many types – mechanical energy, sound energy, heat energy, light energy, chemical energy, atomic energy, nuclear energy etc.

In many processes that occur in nature energy may be transformed from one form to other. Mass can also be transformed into energy and vice-versa. This is according to Einstein's **mass-energy equivalence relation,** $E = mc^2$.

In dynamics, we are mainly concerned with purely mechanical energy.

Law of Conservation of Energy :

The study of the various forms of energy and of transformation of one kind of energy into another has led to the statement of a very important principle, known as the law of conservation of energy.

"Energy cannot be created or destroyed, it may only be transformed from one form into another. As such the total amount of energy never changes".

Keep in Memory

1. Work done against friction on horizontal surface $= \mu\, mgx$ and work done against force of friction on inclined plane $= (\mu mg \cos\theta)\, x$ where μ = coefficient of friction.

2. If a body moving with velocity v comes to rest after covering a distance 'x' on a rough surface having coefficient of friction μ, then (from work energy theorem), $2\mu\, gx = v^2$. Here retardation is $\vec{a} = -\mu\vec{g}$

3. Work done by a centripetal force is always zero.

4. Potential energy of a system decreases when a conservative force does work on it.

5. If the speed of a vehicle is increased by n times, then its stopping distance becomes n^2 times and if momentum is increased by n times then its kinetic energy increases by n^2 times.

6. Stopping distance of the vehicle $= \frac{\text{Kinetic energy}}{\text{Stopping force}}$

7. Two vehicles of masses M_1 and M_2 are moving with velocities u_1 and u_2 respectively. When they are stopped by the same force, their stopping distance are in the ratio as follows :

Since the retarding force F is same in stopping both the vehicles. Let x_1 and x_2 are the stopping distances of vehicles of masses M_1 & M_2 respectively, then

$$\frac{F.x_1 (\text{work done in stopping the mass } M_1)}{F.x_2 (\text{work done in stopping the mass } M_2)}$$

$$= \frac{\tfrac{1}{2} M_1 u_1^2}{\tfrac{1}{2} M_2 u_2^2} = \frac{E_{k_1}}{E_{k_2}} \qquad \text{....(i)}$$

where u_1 and u_2 are initial velocity of mass M_1 & M_2 respectively & final velocity of both mass is zero.

$$\Rightarrow \frac{x_1}{x_2} = \frac{E_{k_1}}{E_{k_2}} \qquad \text{....(ii)}$$

Let us apply a retarding force F on M_1 & M_2, a_1 & a_2 are the decelerations of M_1 & M_2 respectively. Then from third equation of motion $\left(v^2 = u^2 + 2ax\right)$:

$$0 = u_1^2 - 2a_1x_1 \Rightarrow a_1 = \frac{u_1^2}{2x_1} \qquad \text{....(iii a)}$$

$$\text{and } 0 = u_2^2 - 2a_2x_2 \Rightarrow a_2 = \frac{u_2^2}{2x_2} \qquad \text{....(iii b)}$$

If t_1 & t_2 are the stopping time of vehicles of masses M_1 & M_2 respectively, then from first equation of motion

$$(v = u + at)\ 0 = u_1 - a_1t_1 \Rightarrow t_1 = \frac{u_1}{a_1} \qquad \text{....(iv a)}$$

$$\text{and } 0 = u_2 - a_2t_2 \Rightarrow t_2 = \frac{u_2}{a_2} \qquad \text{....(iv b)}$$

Then by rearranging equation (i), (iii) & (iv), we get

$$\frac{t_1}{t_2}=\frac{u_1/a_1}{u_2/a_2}=\left(\frac{x_1}{x_2}\right)\times\left(\frac{u_2}{u_1}\right)$$

$$\Rightarrow \frac{t_1}{t_2}=\frac{(½M_1u_1^2)}{(½M_2u_2^2)}\times\frac{u_2}{u_1}=\left(\frac{M_1u_1}{M_2u_2}\right)$$

(a) If $u_1 = u_2 \Rightarrow \frac{t_1}{t_2}=\frac{M_1}{M_2}$

(b) If $M_1 = M_2 \Rightarrow \frac{t_1}{t_2}=\frac{u_1}{u_2}$

(c) If $M_1u_1 = M_2u_2 \Rightarrow t_1 = t_2$

and $\frac{x_1}{x_2}=\frac{(M_1u_1)^2}{(M_2u_2)^2}\times\frac{M_2}{M_1}\Rightarrow\frac{x_1}{x_2}=\frac{M_2}{M_1}$

(d) Consider two vehicles of masses M_1 & M_2 respectively.
If they are moving with same velocities, then the ratio of their stopping distances by the application of same retarding force is given by

$\frac{x_1}{x_2}=\frac{M_1}{M_2}$ and let $M_2 > M_1$ then $x_1 < x_2$

$\Rightarrow$ lighter mass will cover less distance then the heavier mass
And the ratio of their retarding times are as follows :

$\frac{t_1}{t_2}=\frac{M_1}{M_2}$ i.e $\frac{x_1}{x_2}=\frac{t_1}{t_2}$

8. If kinetic energy of a body is doubled, then its momentum becomes $\sqrt{2}$ times, $E_k=\frac{p^2}{2m}\Rightarrow p\propto\sqrt{E_k}$

9. If two bodies of masses m_1 and m_2 have equal kinetic energies, then their velocities are inversely proportional to the square root of the respective masses. i.e.

$$\frac{1}{2}m_1v_1^2=\frac{1}{2}m_2v_2^2 \text{ then } \frac{v_1}{v_2}=\sqrt{\frac{m_2}{m_1}}$$

10. (a) The spring constant of a spring is inversely proportional to the no. of turns i.e. $K\propto\frac{1}{n}$ or kn = const.
(b) Greater the no. of turns in a spring, greater will be the work done i.e. $W \propto n$
(c) The greater is the elasticity of the spring, the greater is the spring constant.

11. **Spring constant :** The spring constant of a spring is inversely proportional to length i.e., $K\propto\frac{1}{l}$ or Kl = constant.

(a) If a spring is divided into n equal parts, the spring constant of each part = nK.
(b) If spring of spring constant K_1, K_2, K_3 .... are connected in series, then effective force constant

$$\frac{1}{K_{eff}}=\frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3}+......$$

(c) If spring of spring constant K_1, K_2, K_3........... are connected in parallel, then effective spring constant $K_{eff}=K_1+K_2+K_3+$............

POWER

Power of the body is defined as the time rate of doing work by the body.

The average power P_{av} over the time interval Δt is defined by

$$P_{av}=\frac{\Delta W}{\Delta t} \quad ...(i)$$

And the instantaneous power P is defined by

$$P=\lim_{\delta t\to 0}\frac{\Delta W}{\Delta t}=\frac{dW}{dt} \quad ...(ii)$$

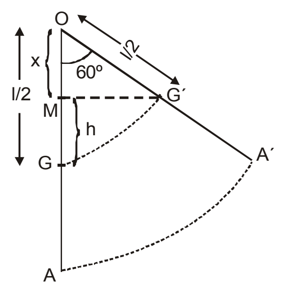
Power is a scalar quantity
The **S.I. unit** of power is joule per second
1 joule/sec = 1watt
The **dimensions** of power are $[ML^2T^{-3}]$

$$P=\frac{dW}{dt}=\frac{d}{dt}(\vec{F}.\vec{S})=\vec{F}.\frac{d\vec{S}}{dt}=\vec{F}.\vec{v}$$

(force is constant over a small time interval)
So instantaneous power (or instantaneous rate of working) of a man depends not only on the force applied to body, but also on the instantaneous velocity of the body.

Example 1.
A metre stick of mass 600 mg, is pivoted at one end and displaced through an angle of 60°. The increase in its potential energy is (g = 10 ms^{-2})
(a) 1.5 J *(b) 15 J*
(c) 150 J *(d) 0.15 J*

Solution : (a)



The C.G. of stick rises from G to G′.
$\therefore$ Increase in P.E. = mgh

$$= mg\,(\ell/2-x)=\frac{mg\ell}{2}(1-\cos 60^\circ)$$

$$=\frac{0.6\times10\times1}{2}\left[1-\frac{1}{2}\right]=1.5\text{ J}$$

Example 2.

A block of mass M slides along the sides of a flat bottomed bowl. The sides of the bowl are frictionless and the base has a coefficient of friction 0.2. If the block is released from the top of the side which is 1.5 m high, where will the block come to rest if, the length of the base is 15 m ?

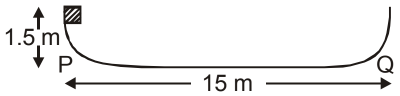
(a) 1 m from P (b) Mid point of flat part PQ

(c) 2 m from P (d) At Q

Solution : (b)

P.E. of the block at top of side = 1.5 mg.

This is wasted away in doing work on the rough flat part,

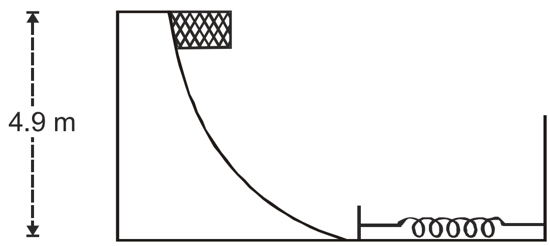


$\therefore$ 1.5 mg = μ mg.x or $x = \dfrac{1.5}{\mu} = 7.5\,\text{m}.$

i.e, the block comes to rest at mid-point of PQ.

Example 3.

Fig. given below shows a smooth curved track terminating in a smooth horizontal part. A spring of spring constant 400 N/m is attached at one end to the wedge fixed rigidly with the horizontal part. A 40 g mass is released from rest at a height of 4.9 m on the curved track. Find the maximum compression of the spring.



Solution :

According to the law of conservation of energy,

$$mgh = \frac{1}{2}kx^2$$

where x is maximum compression.

$$\therefore \quad x = \sqrt{\left(\frac{2mgh}{k}\right)}$$

$$\text{or} \quad x = \sqrt{\left\{\frac{2\times(0.4\,\text{kg})\times(9.8\,\text{m/s}^2)0\times(4.9\,\text{m})}{(400\ \text{N/m})}\right\}} = 9.8\,\text{cm}.$$

Example 4.

The K.E. of a body decreases by 19%. What is the percentage decrease in momentum?

Solution :

As $p = \sqrt{2m\,K.E}$

$$\frac{p_f - p_i}{p_i}\times 100 = \frac{\sqrt{K.E_f} - \sqrt{K.E_i}}{\sqrt{K.E_i}}\times 100$$

$$= \left(\frac{\sqrt{81} - \sqrt{100}}{\sqrt{100}}\right)\times 100$$

$$= -10\%$$

Example 5.

A particle of mass m is moving along a circular path of constant radius R. The centripetal acceleration varies as $a = K^2 Rt^2$, where K is a constant and t is the time elapsed. What is the power delivered to the particle by the force acting on it?

Sol. For circular motion, $a_c = v^2/R$ here $K^2Rt^2 = \dfrac{v^2}{R}$ or $v^2 = K^2R^2t^2$

Now, $KE = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m.K^2R^2t^2$

Work done in time t = W = ΔK

(from work energy theorem)

$$\therefore \Delta K = \frac{m}{2}(K^2R^2t^2) - 0 = \frac{m}{2}K^2R^2t^2$$

$$\text{Power} = \frac{dW}{dt} = \frac{m}{2}K^2.R^2.2t = mK^2R^2t$$

Example 6.

An electron of mass 9.0 × 10^{-28} g is moving at a speed of 1000 m/sec. Calculate its kinetic energy if the electron takes up this speed after moving a distance of 10 cm from rest. Also calculate the force in kg weight acting on it.

Solution :

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}\times 9\times 10^{-31}\,(10^3)^2 = 4.5\times 10^{-25}\,\text{J};$$

From $v^2 - u^2 = 2as$, $v^2 = 2as$ $\because u = 0$

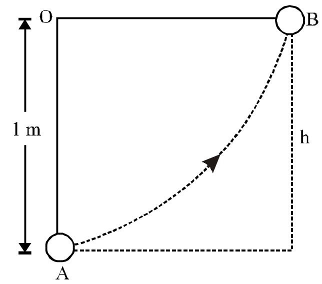
$$\therefore\ a = \frac{v^2}{2s} = \frac{(10^3)^2}{2\times 10^{-1}}$$

$$F = ma = 9\times 10^{-31}\,(0.5\times 10^7)\,\text{N} = \frac{4.5\times 10^{-24}}{9.8}\ \text{kg wt.}$$

$$= 0.46\times 10^{-24}\ \text{kg wt.}$$

Example 7.

The bob of a simple pendulum of length 1 m is drawn aside until the string becomes horizontal. Find the velocity of the bob, after it is released, at the equilibrium position.



Solution :

When the bob is raised from A to B the height through which it is raised is the length of the pendulum. h = 1m

Taking A as the standard level.

P.E. at B = mgh = m × 9.8 × 1 = (9.8) m joule, where m is the mass of the bob.

Since it is at rest at B it has no K.E.

When the bob reaches A after it is released from B, its energy at A is kinetic one. P.E. at A is zero.

If v be the velocity at A, from the law of conservation of energy

K.E. at A = P.E. at B

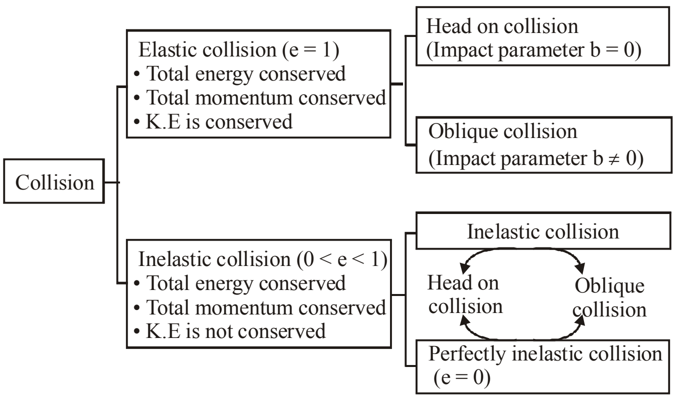
$$\frac{1}{2}mv^2 = mgh \quad \text{or} \quad v^2 = 2gh$$

$$\Rightarrow \qquad v = \sqrt{2gh} = \sqrt{2\times 9.8\times 1} = \sqrt{196} = 4.427 \text{ m/s}$$

The bob has a velocity 4.427 m/s at A.

COLLISION

Collision between two bodies is said to take place if either of two bodies come in physical contact with each other or even when path of one body is affected by the force exerted due to the other.



(1) Elastic collision : *The collision in which both the momentum and kinetic energy of the system remains conserved is called elastic collision.*

Forces involved in the interaction of elastic collision are conservative in nature.

(2) Inelastic collision : *The collision in which only the momentum of the system is conserved but kinetic energy is not conserved is called inelastic collision.*

Perfectly inelastic collision is one in which the two bodies stick together after the collision.

Forces involved in the interaction of inelastic collision are non-conservative in nature.

Coefficient of Restitution (or coefficient of resilience) :

It is the ratio of velocity of separation after collision to the velocity of approach before collision.

i.e., $e = |v_1 - v_2| / |u_1 - u_2|$

Here u_1 and u_2 are the velocities of two bodies before collision and v_1 and v_2 are the velocities of two bodies after collision.

1. $0 < e < 1$ (Inelastic collision)

   Collision between two ivory balls, steel balls or quartz ball is nearly elastic collision.
2. For perfectly elastic collision, e = 1
3. For a perfectly inelastic collision, e = 0

Oblique Elastic Collision :

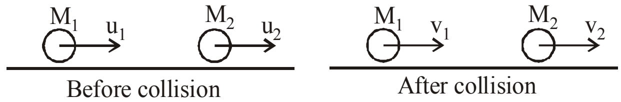
When a body of mass m collides obliquely against a stationary body of same mass then after the collision the angle between these two bodies is always 90°.

Elastic Collision in One Dimension (Head on)

Let two bodies of masses M_1 and M_2 moving with velocities u_1 and u_2 along the same straight line, collide with each other. Let $u_1 > u_2$. Suppose v_1 and v_2 respectively are the velocities after the elastic collision, then:

According to law of conservation of momentum

$$M_1u_1 + M_2u_2 = M_1v_1 + M_2v_2 \qquad ...(1)$$



From law of conservation of energy

$$\frac{1}{2}M_1u_1^2 + \frac{1}{2}M_2u_2^2 = \frac{1}{2}M_1v_1^2 + \frac{1}{2}M_2v_2^2 \qquad ...(2)$$

$$u_1 - u_2 = -(v_1 - v_2) \qquad ...(3)$$

Relative velocity of a body before collision   Relative velocity of a body after collision

Solving eqs. (1) and (2) we get,

$$v_1 = \frac{(M_1 - M_2)u_1}{(M_1 + M_2)} + \frac{2M_2u_2}{(M_1 + M_2)} \qquad ...(4)$$

$$v_2 = \frac{(M_2 - M_1)u_2}{(M_1 + M_2)} + \frac{2M_1u_1}{(M_1 + M_2)} \qquad ...(5)$$

From eqns. (4) and (5), it is clear that :

(i) If $M_1 = M_2$ and $u_2 = 0$ then $v_1 = 0$ and $v_2 = u_1$. Under this condition the first particle comes to rest and the second particle moves with the velocity of first particle after collision. In this state there occurs maximum transfer of energy.

(ii) If $M_1 >> M_2$ and ($u_2 = 0$) then, $v_1 = u_1$, $v_2 = 2u_1$ under this condition the velocity of first particle remains unchanged and velocity of second particle becomes double that of first.

(iii) If $M_1 << M_2$ and ($u_2 = 0$) then $v_1 = -u_1$ and $v_2 = \frac{2M_1}{M_2}u_1 \approx 0$

under this condition the second particle remains at rest while the first particle moves with the same velocity in the opposite direction.

(iv) If $M_1 = M_2 = M$ but $u_2 \neq 0$ then $v_1 = u_2$ i.e., the particles mutually exchange their velocities.

(v) If second body is at rest i.e., $u_2 = 0$, then fractional decrease in kinetic energy of mass M_1, is given by

$$\frac{E_{k_1} - E'_{k_1}}{E_{k_1}} = 1 - \frac{v_1^2}{u_1^2} = \frac{4M_1M_2}{(M_1 + M_2)^2}$$

Inelastic Collision :

Let two bodies A and B collide inelastically. Then from law of conservation of linear momentum

$M_1u_1 + M_2u_2 = M_1v_1 + M_2v_2$ ...(i)

$$\text{Coefficient of restitution} = -\left(\frac{\text{velocity of separation}}{\text{velocity of approach}}\right)$$

$$e = -\frac{(v_1 - v_2)}{(u_1 - u_2)} \quad \text{...(ii)}$$

From eqns.(i) and (ii), we have,

$$v_1 = \left(\frac{M_1 - eM_2}{M_1 + M_2}\right)u_1 + \left(\frac{M_2(1+e)}{M_1 + M_2}\right)u_2 \quad \text{...(iii)}$$

$$v_2 = \left(\frac{(1+e)M_1}{M_1 + M_2}\right)u_1 + \left(\frac{M_2 - eM_1}{M_1 + M_2}\right)u_2 \quad \text{...(iv)}$$

Loss in kinetic energy $(-\Delta E_k)$ = initial K.E. – final K.E

$$\Rightarrow -\Delta E_k = \left[\frac{1}{2}M_1u_1^2 + \frac{1}{2}M_2u_2^2\right] - \left[\frac{1}{2}M_1v_1^2 + \frac{1}{2}M_2v_2^2\right]$$

$$\Rightarrow -\Delta E_k = \frac{1}{2}\left(\frac{M_1M_2}{M_1 + M_2}\right)(e^2 - 1)(u_1 - u_2)^2 \quad \text{...(v)}$$

Negative sign indicates that the final kinetic energy is less than initial kinetic energy.

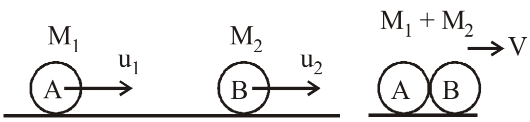
Perfectly Inelastic Collision

In this collision, the individual bodies A and B move with velocities u_1 and u_2 but after collision move as a one single body with velocity v.

So from law of conservation of linear momentum, we have

$M_1u_1 + M_2u_2 = (M_1 + M_2)V$ ...(i)

$$\text{or } V = \left(\frac{M_1u_1 + M_2u_2}{M_1 + M_2}\right) \quad \text{...(ii)}$$



Before collision $(u_1 > u_2)$ After collision

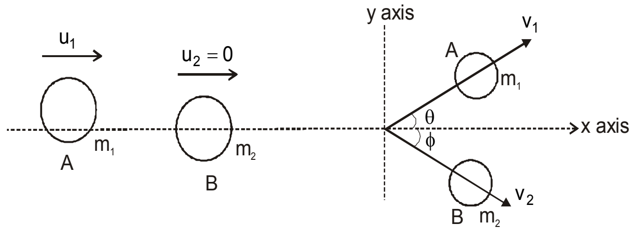
And loss in kinetic energy, $-\Delta E_k$ = total initial K.E – total final K.E

$$= \frac{1}{2}M_1u_1^2 + \frac{1}{2}M_2u_2^2 - \frac{1}{2}(M_1 + M_2)\left(\frac{M_1u_1 + M_2u_2}{M_1 + M_2}\right)^2$$

$$\text{or, } -\Delta E_k = \frac{1}{2}\frac{M_1M_2}{(M_1 + M_2)}(u_1 - u_2)^2 \quad \text{...(iii)}$$

Oblique Collision :

This is the case of collision in two dimensions. After the collision, the particles move at different angle.



Before collision After collision

We will apply the principle of conservation of momentum in the mutually perpendicular direction.

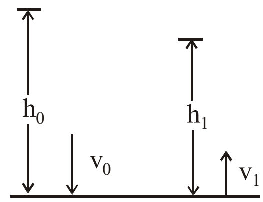
Along x-axis, $m_1u_1 = m_1v_1\cos\theta + m_2v_2\cos\phi$

Along y-axis, $0 = m_1v_1\sin\theta - m_2v_2\sin\phi$

Keep in Memory

1. Suppose, a body is dropped from a height h_0 and it strikes the ground with velocity v_0. After the (inelastic) collision let it rise to a height h_1. If v_1 be the velocity with which the body rebounds, then the coefficient of restitution.

$$e = \frac{v_1}{v_0} = \left(\frac{2gh_1}{2gh_0}\right)^{\frac{1}{2}} = \left(\frac{h}{h_0}\right)^{\frac{1}{2}}$$

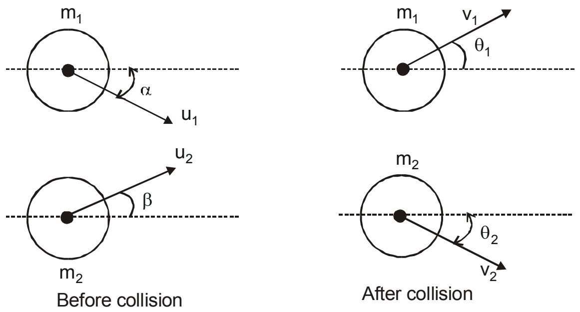


2. If after n collisions with the ground, the velocity is v_n and the height to which it rises be h_n, then

$$e^n = \frac{v_n}{v_o} = \left(\frac{h_n}{h_o}\right)^{\frac{1}{2}}$$

3. When a ball is dropped from a height h on the ground, then after striking the ground n times , it rises to a height $h_n = e^{2n}h_o$ where e = coefficient of restitution.
4. If a body of mass m moving with velocity v, collides elastically with a rigid ball, then the change in the momentum of the body is $2mv$.
   (i) If the collision is elastic then we can conserve the energy as
   $$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$
   (ii) If two particles having same mass and moving at right angles to each other collide elastically then after the collision they also move at right angles to each other.
   (iii) If a body A collides elastically with another body of same mass at rest obliquely, then after the collision the two bodies move at right angles to each other, i. e.
   $$(\theta + \phi) = \frac{\pi}{2}$$
5. In an elastic collision of two equal masses, their kinetic energies are exchanged.

6. When two bodies collide obliquely, their relative velocity resolved along their common normal after impact is in constant ratio to their relative velocity before impact (resolved along common normal), and is in the opposite direction.



$$\frac{v_1 \sin\theta_1 - v_2 \sin\theta_2}{u_1 \sin\alpha - u_2 \sin\beta} = -e$$

Example 8.

A body of mass m moving with velocity v collides head on with another body of mass 2m which is initially at rest. What will be the ratio of K.E. of colliding body before and after collision?

Solution :

$mv + 2m \times 0 = mv_1 + 2mv_2$;

$v = v_1 + 2v_2$ or $v_2 = \dfrac{v - v_1}{2}$;

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2$$

$$v^2 = v_1^2 + 2v_2^2 = v_1^2 + 2\left(\frac{v - v_1}{2}\right)^2$$

$v^2 = v_1^2 + \dfrac{v_1^2 + v^2 - 2vv_1}{2}$ or $2v^2 = 2v_1^2 + v_1^2 + v^2 - 2vv_1$

or $3v_1^2 - 2vv_1 - v^2 = 0$; $v_1 = -\dfrac{v}{3}$

$$\frac{\text{K.E. of colliding body before collision}}{\text{K.E. of colliding body after collision}} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}m\left(\frac{v}{3}\right)^2} = 9 : 1$$

Example 9.

A bullet of mass m moving horizontally with velocity v hits a block of wood of mass M, resting on a smooth horizontal plane. Find the fraction of energy of the bullet dissipated in the collision itself (assume collision to be inelastic).

Solution :

Applying the law of conservation of momentum, we have,
$m v = (m + M) v_1$

$$v_1 = \left(\frac{mv}{m + M}\right)$$

$$\text{Loss of K.E.} = \frac{1}{2}mv^2 - \frac{1}{2}(m + M)v_1^2$$

$$= \frac{1}{2}m v^2 - \frac{1}{2}(m + M)\left(\frac{m v}{M + m}\right)^2$$

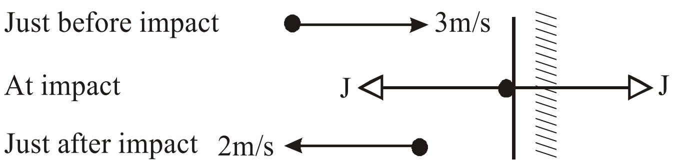
$$= \frac{1}{2}m v^2\left[1 - \frac{m}{m + M}\right] = \frac{1}{2}m v^2\left[\frac{M}{m + M}\right]$$

$$\text{Fraction of K.E. dissipated} = \frac{\text{Loss of K.E.}}{\text{Initial K.E.}} = \left(\frac{M}{m + M}\right)$$

Example 10.

A smooth sphere of mass 0.5 kg moving with horizontal speed 3 m/s strikes at right angles a vertical wall and bounces off the wall with horizontal speed 2 m/s. Find the coefficient of restitution between the sphere and the wall and the impulses exerted on the wall at impact.

Solution :



e = separation speed : approach speed = 2 : 3

Therefore the coefficient of restitution is 2/3.

Using impulse = change in momentum for the sphere we have : $= 0.5 \times 2 - 0.5\,(-3) = 2.5$

The equal and opposite impulse acting on the wall is therefore 2.5 N s.

CONCEPT MAP

WORK, ENERGY AND POWER

Work (W)
Work done by a constant force
$W = Fs\cos\theta = \vec{F}.\vec{s}$
Work done by a variable force
$W = \int_{s_1}^{s_2} Fds\cos\theta = \int_{s_1}^{s_2} \vec{F}.\vec{ds}$

Types of work

Positive work
When force (F) and displacement (s) are in same direction
$W_{Positive}$ when $0 < \theta < \pi/2$

Negative work
When force (F) and displacement (s) are in opposite direction
$W_{(Negative)}$ when $\pi/2 < \theta < \pi$

Zero work
When force and displacement (s) perpendicular to each other
W_{Zero} when $\theta = \pi/2$

Conservative force
If work done by the force is path independent and depends on position only e.g., spring force

Non-conservative force
If the work done by the force is path dependent e.g., force of friction, drag force, etc.

Power (P) $= \dfrac{\text{work (w)}}{\text{time } (t)}$
Instantaneous power
$P_{inst} = \vec{F}.\dfrac{d\vec{s}}{dt} = \vec{F}.\vec{v} = Fv\cos\theta$
1 hp = 746 watt
1 kilowatt hour (1 kw h) = 3.6×10^6 J

Energy
Capacity of doing work

Types of mechanical energy

Kinetic energy
$E_K = \dfrac{1}{2}mv^2 = \dfrac{p^2}{2m}$

Potential energy
$E_P = mgh$

Potential energy of a spring $U = \dfrac{1}{2}kx^2$
K = spring or force constant

Law of Conservation of Energy
It can neither be created nor destroyed only be transformed from one form to another

Work-energy theorem
Change in kinetic energy = work done by net force on the body
i.e., $\dfrac{1}{2}mv^2 - \dfrac{1}{2}mu^2 = W$

Collision
When exchange of momentum takes place between two physical bodies due to their mutual interaction

Types of Collision

Perfectly elastic collision
Total mechanical/kinetic energy conserved
Momentum conserved, $e = 1$

Perfectly inelastic collision
Total energy and linear momentum conserved. Kinetic energy not conserved, coefficient of restitution $e = 0$

Coefficient of restitution
$e = \dfrac{v_2 - v_1}{u_1 - u_2} = \dfrac{\text{velocity of separation}}{\text{velocity of approach}}$

EXERCISE - 1
Conceptual Questions

1. The magnitude of work done by a force :
   (a) depends on frame of reference
   (b) does not depend on frame of reference
   (c) cannot be calculated in non-inertial frames.
   (d) both (a) and (b)
2. Work done by a conservative force is positive if
   (a) P.E. of the body increases
   (b) P.E. of the body decreases
   (c) K.E. of the body increases
   (d) K.E. of the body decreases
3. A vehicle is moving with a uniform velocity on a smooth horizontal road, then power delivered by its engine must be
   (a) uniform (b) increasing
   (c) decreasing (d) zero
4. Which of the following force(s) is/are non-conservative?
   (a) Frictional force (b) Spring force
   (c) Elastic force (d) All of these
5. A ball of mass *m* and a ball *B* of mass 2*m* are projected with equal kinetic energies. Then at the highest point of their respective trajectories.
   (a) P.E. of *A* will be more than that of *B*
   (b) P.E of *B* will be more than that of *B*
   (c) P.E of *A* will be equal to that of *B*
   (d) can't be predicted.
6. In case of elastic collision, at the time of impact.
   (a) total K.E. of colliding bodies is conserved.
   (b) total K.E. of colliding bodies increases
   (c) total K.E. of colliding bodies decreases
   (d) total momentum of colliding bodies decreases.
7. The engine of a vehicle delivers constant power. If the vehicle is moving up the inclined plane then, its velocity,
   (a) must remain constant
   (b) must increase
   (c) must decrease
   (d) may increase, decrease or remain same.
8. A ball projected from ground at a certain angle collides a smooth inclined plane at the highest point of its trajectory. If the collision is perfectly inelastic then after the collision, ball will
   (a) come to rest
   (b) move along the incline
   (c) retrace its path.
9. The vessels A and B of equal volume and weight are immersed in water to depth h. The vessel A has an opening at the bottom through which water can enter. If the work done in immersing A and B are W_A and W_B respectively, then
   (a) $W_A = W_B$ (b) $W_A < W_B$
   (c) $W_A > W_B$ (d) $W_A \gtrless W_B$
10. A metallic wire of length L metre extends by ℓ metre when stretched by suspending a weight Mg from it. The mechanical energy stored in the wire is
    (a) $2\,Mg\,\ell$ (b) $Mg\,\ell$
    (c) $\frac{Mg\,\ell}{2}$ (b) $\frac{Mg\,\ell}{4}$
11. A body of mass m accelerates uniformly from rest to v_1 in time t_1. As a function of t, the instantaneous power delivered to the body is
    (a) $\frac{m v_1 t}{t_2}$ (b) $\frac{m v_1^2 t}{t_1}$
    (c) $\frac{m v_1 t^2}{t_1}$ (d) $\frac{m v_1^2 t}{t_1^2}$
12. A block is acted upon by a force, which is inversely proportional to the distance covered (x). The work done will be proportional to
    (a) x (b) $x^{1/2}$
    (c) x^2 (d) None of these
13. A small body is projected in a direction inclined at 45º to the horizontal with kinetic energy K. At the top of its flight, its kinetic energy will be
    (a) Zero (b) K/2
    (c) K/4 (d) $K/\sqrt{2}$
14. A motor cycle is moving along a straight horizontal road with a speed v_0. If the coefficient of friction between the tyres and the road is μ, the shortest distance in which the car can be stopped is
    (a) $\frac{v_0^2}{2\mu g}$ (b) $\frac{v^2}{\mu}$
    (c) $\left(\frac{v_0}{\mu g}\right)^2$ (d) $\frac{v_0}{\mu g}$
15. Consider the following two statement:
    I. Linear momentum of a system of particles is zero.
    II. Kinetic energy of a system of particles is zero.
    Then
    (a) I implies II but II does not imply I.
    (b) I does not imply II but II implies I.
    (c) I implies II and II implies I.
    (d) I does not imply II and II does not imply I.
16. Which of the following must be known in order to determine the power output of an automobile?
    (a) Final velocity and height
    (b) Mass and amount of work performed
    (c) Force exerted and distance of motion
    (d) Work performed and elapsed time of work

17. The work done in stretching a spring of force constant k from length ℓ_1 and ℓ_2 is

(a) $k(\ell_2^2 - \ell_1^2)$ (b) $\frac{1}{2}k(\ell_2^2 - \ell_1^2)$

(c) $k(\ell_2 - \ell_1)$ (d) $\frac{k}{2}(\ell_2 + \ell_1)$

18. If the force acting on a body is inversely proportional to its velocity, then the kinetic energy acquired by the body in time t is proportional to

(a) t^0 (b) t^1

(c) t^2 (d) t^4

19. The engine of a truck moving along a straight road delivers constant power. The distance travelled by the truck in time t is proportional to

(a) t (b) t^2

(c) $\sqrt{t}$ (d) $t^{3/2}$

20. A bullet of mass 'a' and velocity 'b' is fired into a large block of wood of mass 'c'. The bullet gets embedded into the block of wood. The final velocity of the system is

(a) $\frac{b}{a+b} \times c$ (b) $\frac{a+b}{c} \times a$

(c) $\frac{a}{a+c} \times b$ (d) $\frac{a+c}{a} \times b$

21. A ball is dropped from a height h. If the coefficient of restitution be e, then to what height will it rise after jumping twice from the ground?

(a) e h/2 (b) 2 e h

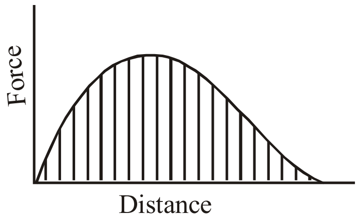
(c) e h (d) e^4 h

22. A ball of mass m moving with a constant velocity strikes against a ball of same mass at rest. If e = coefficient of restitution, then what will be the ratio of velocity of two balls after collision?

(a) $\frac{1-e}{1+e}$ (b) $\frac{e-1}{e+1}$

(c) $\frac{1+e}{1-e}$ (d) $\frac{2+e}{e-1}$

23. Which one of the following physical quantities is represented by the shaded area in the given graph?



(a) Torque (b) Impulse

(c) Power (d) Work done

24. A particle of mass m_1 moving with velocity v collides with a mass m_2 at rest, then they get embedded. Just after collision, velocity of the system

(a) increases (b) decreases

(c) remains constant (d) becomes zero

25. A mass m_1 moves with a great velocity. It strikes another mass m_2 at rest in a head on collision. It comes back along its path with low speed, after collision. Then

(a) $m_1 > m_2$ (b) $m_1 < m_2$

(c) $m_1 = m_2$ (d) cannot say

EXERCISE - 2

Applied Questions

1. A particle describe a horizontal circle of radius 0.5 m with uniform speed. The centripetal force acting is 10 N. The work done in describing a semicircle is

(a) zero (b) 5 J

(c) 5π J (d) 10π J

2. A cord is used to lower vertically a block of mass M, a distance d at a constant downward acceleration of g/4. The work done by the cord on the block is

(a) $Mg\frac{d}{4}$ (b) $3Mg\frac{d}{4}$

(c) $-3Mg\frac{d}{4}$ (d) Mg d

3. A boy pushes a toy box 2.0 m along the floor by means of a force of 10 N directed downward at an angle of 60º to the horizontal. The work done by the boy is

(a) 6 J (b) 8 J

(c) 10 J (d) 12 J

4. A particle moving in the xy plane undergoes a displacement of $\vec{s} = (2\hat{i} + 3\hat{j})$ while a constant force $\vec{F} = (5\hat{i} + 2\hat{j})$ N acts on the particle. The work done by the force F is

(a) 17 joule (b) 18 joule

(c) 16 joule (d) 15 joule

5. A simple pendulum 1 metre long has a bob of 10 kg. If the pendulum swings from a horizontal position, the K.E. of the bob, at the instant it passes through the lowest position of its path is

(a) 89 joule (b) 95 joule

(c) 98 joule (d) 85 joule

6. A particle moves under the effect of a force F = cx from x = 0 to x = x_1, the work done in the process is

(a) cx_1^2 (b) $\frac{1}{2}cx_1^2$

(c) $2cx_1^2$ (d) zero

7. A motor of 100 H.P. moves a load with a uniform speed of 72 km/hr. The forward thrust applied by the engine on the car is
(a) 1111 N (b) 3550 N
(c) 2222 N (d) 3730 N

8. Two bodies A and B having masses in the ratio of 3 : 1 possess the same kinetic energy. The ratio of linear momentum of B to A is
(a) 1 : 3 (b) 3 : 1
(c) $1:\sqrt{3}$ (d) $\sqrt{3}:1$

9. When a U^{238} nucleus, originally at rest, decays by emitting an α-particle, say with speed of v m/sec, the recoil speed of the residual nucleus is (in m/sec.)
(a) – 4 v/234 (b) – 4 v/238
(c) 4 v/238 (d) – v/4

10. Calculate the K.E and P.E. of the ball half way up, when a ball of mass 0.1 kg is thrown vertically upwards with an initial speed of 20 ms^{-1}.
(a) 10 J, 20 J (b) 10 J, 10 J
(c) 15 J, 8 J (d) 8 J, 16 J

11. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is
(a) 16 J (b) 8 J
(c) 32 J (d) 24 J.

12. If the linear momentum is increased by 5%, the kinetic energy will increase by
(a) 50% (b) 100%
(c) 125% (d) 10%

13. A cord is used to lower vertically a block of mass M, through a distance d at a constant downward acceleration of g/8. Then the work done by the cord on the block is
(a) Mg d/8 (b) 3 Mg d/8
(c) Mg d (d) – 7 mg d/8

14. A force $\vec{F} = (5\vec{i} + 3\vec{j} + 2\vec{k})N$ is applied over a particle which displaces it from its origin to the point $\vec{r} = (2\vec{i} - \vec{j})m$. The work done on the particle in joule is
(a) +10 (b) +7
(c) –7 (d) +13

15. A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is
(a) 18.75 J (b) 25.00 J
(c) 6.25 J (d) 12.50 J

16. Two solid rubber balls A and B having masses 200 & 400 gm respectively are moving in opposite direction with velocity of A equal to 0.3 m/sec. After collision the two balls come to rest when the velocity of B is
(a) 0.15 m/sec (b) 1.5 m/sec
(c) –0.15 m/sec (d) None of these

17. A bomb of mass 9 kg explodes into the pieces of masses 3 kg and 6 kg. The velocity of mass 3 kg is 16 m/s. The kinetic energy of mass 6 kg in joule is
(a) 96 (b) 384
(c) 192 (d) 768

18. A long string is stretched by 2 cm and the potential energy is V. If the spring is stretched by 10 cm, its potential energy will be
(a) V / 25 (b) V/5
(c) 5 V (d) 25 V

19. When the kinetic energy of a body is increased to three times, then the momentum increases
(a) 6 times (b) 1.732 times
(c) $\sqrt{2}$ times (d) 2 times

20. Two bodies of masses 2 m and m have their KE in the ratio 8 : 1. What is the ratio of their momenta ?
(a) 8 : 1 (b) 4 : 1
(c) 2 : 1 (d) 1 : 1

21. A body of mass 5 kg initially at rest explodes into 3 fragments with mass ratio 3 : 1 : 1. Two of fragments each of mass 'm' are found to move with a speed 60 m/s in mutually perpendicular directions. The velocity of third fragment is
(a) $60\sqrt{2}$ (b) $20\sqrt{3}$
(c) $10\sqrt{2}$ (d) $20\sqrt{2}$

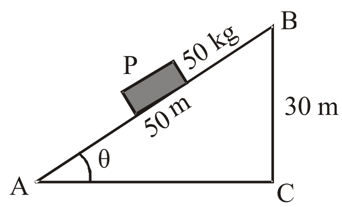
22. A machine, which is 75% efficient, uses 12 J of energy in lifting up a 1 kg mass through a certain distance. The mass is then allowed to fall through that distance. The velocity at the end of its fall is (in m/s)
(a) $\sqrt{24}$ (b) $\sqrt{12}$
(c) $\sqrt{18}$ (d) $\sqrt{9}$

23. A body accelerates uniformly from rest to a velocity of 1 ms^{-1} in 15 seconds. The kinetic energy of the body will be $\frac{2}{9}$ J when 't' is equal to [Take mass of body as 1 kg]
(a) 4s (b) 8s
(c) 10s (d) 12s

24. A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1}. The man holding it can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most?
(a) 2 (b) 4
(c) 1 (d) 3

25. A crane is used to lift 1000 kg of coal from a mine 100 m deep. The time taken by the crane is 1 hour. The efficiency of the crane is 80%. If g = 10 ms^{-2}, then the power of the crane is
(a) 10^4 W (b) 10^5 W
(c) $\frac{10^4}{36\times 8}$ W (d) $\frac{10^5}{36\times 8}$ W

26. In figure, a carriage P is pulled up from A to B. The relevant coefficient of friction is 0.40. The work done will be
(a) 10 kJ
(b) 23 kJ
(c) 25 kJ
(d) 28 kJ



27. A neutron with velocity v strikes a stationary deuterium atom, its K.E. changes by a factor of

(a) $\frac{15}{16}$ (b) $\frac{1}{2}$

(c) $\frac{2}{1}$ (d) None of these

28. A body moves a distance of 10 m along a straight line under the action of a force of 5 newtons. If the work done is 25 joules, the angle which the force makes with the direction of motion of body is

(a) 0º (b) 30º

(c) 60º (d) 90º

29. A sphere of mass 8m collides elastically (in one dimension) with a block of mass 2m. If the initial energy of sphere is E. What is the final energy of sphere?

(a) 0.8 E (b) 0.36 E

(c) 0.08 E (d) 0.64 E

30. Johnny and his sister Jane race up a hill. Johnny weighs twice as much as jane and takes twice as long as jane to reach the top . Compared to Jane

(a) Johnny did more work and delivered more power.

(b) Johnny did more work and delivered the same amount of power.

(c) Johnny did more work and delivered less power

(d) Johnny did less work and johnny delivered less power.

31. A body of mass m moving with velocity v makes a head on elastic collision with another body of mass 2m which in initially at rest. The loss of kinetic energy of the colliding body (mass m) is

(a) $\frac{1}{2}$ of its initial kinetic energy

(b) $\frac{1}{9}$ of its initial kinetic energy

(c) $\frac{8}{9}$ of its initial kinetic energy

(d) $\frac{1}{4}$ of its initial kinetic energy

32. In the non-relativistic regime, if the momentum, is increase by 100% , the percentage increase in kinetic energy is

(a) 100 (b) 200

(c) 300 (d) 400

33. A body is dropped from a height of 20m and rebounds to a height 10m. The loss of energy is

(a) 10% (b) 45%

(c) 50% (d) 75%

34. A moving body with a mass m_1 and velocity u strikes a stationary body of mass m_2. The masses m_1 and m_2 should be in the ratio m_1/m_2 so as to decrease the velocity of the first body to 2u/3 and giving a velocity of u to m_2 assuming a perfectly elastic impact. Then the ratio m_1/m_2 is

(a) 5 (b) 1 / 5

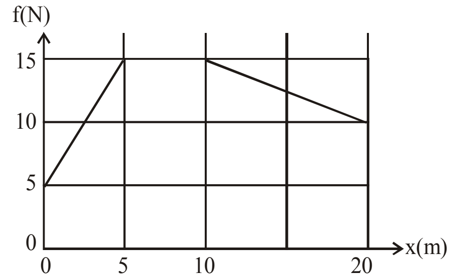
(c) 1/25 (d) 25.

35. A body of mass m is suspended from a massless spring of natural length ℓ. It stretches the spring through a vertical distance y. The potential energy of the stretched spring is

(a) $mg(\ell + y)$ (b) $\frac{1}{2}mg(\ell + y)$

(c) $\frac{1}{2}mgy$ (d) mgy

36. Figure here shows the frictional force versus displacement for a particle in motion. The loss of kinetic energy in travelling over s = 0 to 20 m will be



(a) 250 J (b) 200 J

(c) 150 J (d) 10 J

37. Ten litre of water per second is lifted from a well through 10 m and delivered with a velocity of 10 ms^{-1}. If g = 10 ms^{-2} , then the power of the motor is

(a) 1 kW (b) 1.5 kW

(c) 2 kW (d) 2.5 kW

38. A nucleus ruptures into two nuclear parts which have their velocity ratio equal to $2:1$. The ratio of their respective nuclear sizes (nuclear radii)is

(a) 1 : 2 (b) $1:\sqrt{2}$

(c) $1 : 2^{1/3}$ (d) 1 : 8

39. The rest energy of an electron is 0.511 MeV. The electron is accelerated from rest to a velocity 0.5 c. The change in its energy will be

(a) 0.026 MeV (b) 0.051 MeV

(c) 0.08 MeV (d) 0.105 MeV

40. A one-ton car moves with a constant velocity of 15 ms^{-1} on a rough horizontal road. The total resistance to the motion of the car is 12% of the weight of the car. The power required to keep the car moving with the same constant velocity of 15ms^{-1} is

[Take g = 10 ms^{-2}]

(a) 9 kW (b) 18 kW

(c) 24 kW (d) 36 kW

41. Hail storms are observed to strike the surface of the frozen lake at 30^0 with the vertical and rebound at 60^0 with the vertical. Assume contact to be smooth, the coefficient of restitution is

(a) $e = \frac{1}{\sqrt{3}}$ (b) $e = \frac{1}{3}$

(c) $e = \sqrt{3}$ (d) $e = 3$

42. A body starts from rest and acquires a velocity V in time T. The work done on the body in time t will be proportional to

(a) $\frac{V}{T}t$ (b) $\frac{V^2}{T}t^2$

(c) $\frac{V^2}{T^2}t$ (d) $\frac{V^2}{T^2}t^2$

43. A ball is allowed to fall from a height of 10 m. If there is 40% loss of energy due to impact, then after one impact ball will go up to

(a) 10m (b) 8m

(c) 4m (d) 6m

44. The potential energy of a conservative system is given by $U = ay^2 - by$, where y represents the position of the particle and a as well as b are constants. What is the force acting on the system ?

(a) $-ay$ (b) $-by$

(c) $2ay - b$ (d) $b - 2ay$

45. An automobile engine of mass M accelerates and a constant power p is applied by the engine. The instantaneous speed of the engine will be

(a) $[Pt/M]^{1/2}$ (b) $[2Pt/M]^{1/2}$

(c) $[Pt/2M]^{1/2}$ (d) $[Pt/4M]^{1/2}$

46. A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest assuming that it faces constant resistance to motion ?

(a) 2.0 cm (b) 3.0 cm

(c) 1.0 cm (d) 1.5 cm

47. A bomb of mass 16kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velolcity of the 12 kg mass is $4ms^{-1}$. The kinetic energy of the other mass is

(a) 144 J (b) 288 J

(c) 192 J (d) 96 J

48. Given that a force $\hat{F}$ acts on a body for time t, and displaces the body by $\hat{d}$. In which of the following cases, the speed of the body must not increase?

(a) $F > d$ (b) $F < d$

(c) $\hat{F} = \hat{d}$ (d) $\hat{F} \perp \hat{d}$

49. A body is attached to the lower end of a vertical helical spring and it is gradually lowered to its equilibrium position. This stretches the spring by a length x. If the same body attached to the same spring is allowed to fall suddenly, what would be the maximum stretching in this case?

(a) x (b) 2x

(c) 3x (d) x/2

50. A bag of mass M hangs by a long thread and a bullet (mass m) comes horizontally with velocity V and gets caught in the bag. Then for the combined (bag + bullet) system

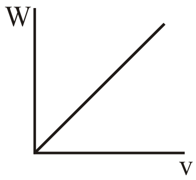
(a) momentum $= \frac{mvM}{M+m}$

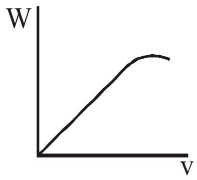
(b) kinetic energy $= \frac{mV^2}{2}$

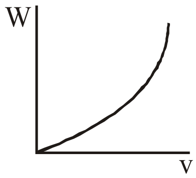
(c) momentum $= \frac{mV(M+m)}{M}$

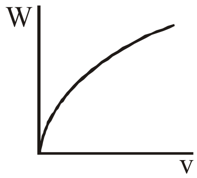
(d) kinetic energy $= \frac{m^2 v^2}{2(M+m)}$

51. A particle, initially at rest on a frictionless horizontal surface, is acted upon by a horizontal force which is constant in magnitude and direction. A graph is plotted of the work done on the particle W, against the speed of the particle v. If there are no other horizontal forces acting on the particle, the graph would look like

(a) 

(b) 

(c) 

(d) 

52. A particle of mass m moving eastward with a speed v collides with another particle of the same mass moving northwards with the same speed. If two particles coalesce on collision, the new particle of mass 2 m will move in the north-east direction with a velocity

(a) $v/2$ (b) $v\sqrt{2}$

(c) $v/\sqrt{2}$ (d) None of these

53. A small block of mass m is kept on a rough inclined surface of inclination θ fixed in an elevator. The elevator goes up with a uniform velocity v and the block does not slide on the wedge. The work done by the force of friction on the block in time t as seen by the observer on the inclined p lane will be

(a) zero (b) $mgvt \cos^2\theta$

(c) $mgvt \sin^2\theta$ (d) $mgvt \sin 2\theta$

54. A nucleus moving with a velocity $\vec{v}$ emits an α-particle. Let the velocities of the α-particle and the remaining nucleus be $\vec{v}_1$ and $\vec{v}_2$ and their masses be m_1 and m_2.

(a) $\vec{v}$, $\vec{v}_1$ and $\vec{v}_2$ must be parallel to each other

(b) None of the two of $\vec{v}$, $\vec{v}_1$ and $\vec{v}_2$ should be parallel to each other

(c) $m_1\vec{v}_1 + m_2\vec{v}_2$ must be parallel to $(m_1 + m_2)\vec{v}$.

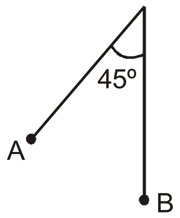
(d) None of these

55. A shell is fired from a cannon with a velocity V at an angle θ with the horizontal direction. At the highest point in its path, it explodes into two pieces of equal masses. One of the pieces retraces its path to the cannon. The speed of the other piece immediately after the explosion is

(a) $3V\cos\theta$ (b) $2V\cos\theta$

(c) $\frac{3}{2}V\cos\theta$ (d) $V\cos\theta$

56. The bob A of a simple pendulum is released when the string makes an angle of 45° with the vertical. It hits another bob B of the same material and same mass kept at rest on the table. If the collision is elastic



(a) both A and B rise to the same height
(b) both A and B come to rest at B
(c) both A and B move with the velocity of A
(d) A comes to rest and B moves with the velocity of A

57. Two masses m_a and m_b moving with velocities v_a and v_b in opposite direction collide elastically and after the collision m_a and m_b move with velocities V_b and V_a respectively. Then the ratio m_a/m_b is

(a) $\frac{V_a - V_b}{V_a + V_b}$ (b) $\frac{m_a + m_b}{m_a}$
(c) 1 (d) $\frac{1}{2}$

58. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to
(a) x (b) e^x
(c) x^2 (d) $\log_e x$

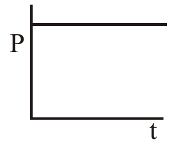
59. A particle of mass m_1 moving with velocity v strikes with a mass m_2 at rest, then the condition for maximum transfer of kinetic energy is
(a) $m_1 >> m_2$ (b) $m_2 >> m_2$
(c) $m_1 = m_2$ (d) $m_1 = 2m_2$

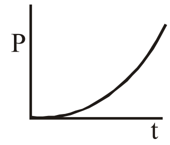
60. Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block. The velocity of the centre of mass is
(a) 30 m/s (b) 20 m/s
(c) 10 m/s (d) 5 m/s

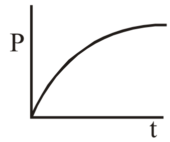
61. A mass *m* is moving with velocity v collides inelastically with a bob of simple pendulum of mass m and gets embedded into it. The total height to which the masses will rise after collision is

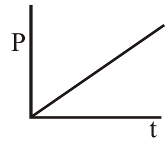
(a) $\frac{v^2}{8g}$ (b) $\frac{v^2}{4g}$
(c) $\frac{v^2}{2g}$ (d) $\frac{2v^2}{g}$

62. A motor drives a body along a straight line with a constant force. The power P developed by the motor must vary with time t according to

(a) 

(b) 

(c) 

(d) 

63. A glass marble dropped from a certain height above the horizontal surface reaches the surface in time t and then continues to bounce up and down. The time in which the marble finally comes to rest is

(a) $e^n t$ (b) $e^2 t$
(c) $t\left[\frac{1+e}{1-e}\right]$ (d) $t\left[\frac{1-e}{1+e}\right]$

64. A weight suspended from the free end of a vertically hanging spring produces an extension of 3 cm. The spring is cut into two parts so that the length of the longer part is $\frac{2}{3}$ of the original length, If the same weight is now suspended from the longer part of the spring, the extension produced will be
(a) 0.1 cm (b) 0.5 cm
(c) 1 cm (d) 2 cm

65. A 10 m long iron chain of linear mass density 0.8 kg m^{-1} is hanging freely from a rigid support. If $g = 10\ ms^{-2}$, then the power required to left the chain upto the point of support in 10 second
(a) 10 W (b) 20W
(c) 30 W (d) 40 W

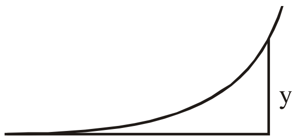
66. A smooth sphere of mass M moving with velocity u directly collides elastically with another sphere of mass m at rest. After collision, their final velocities are V and v respectively. The value of v is

(a) $\frac{2uM}{m}$ (b) $\frac{2um}{M}$
(c) $\frac{2u}{1+\frac{m}{M}}$ (d) $\frac{2u}{1+\frac{M}{m}}$

67. The kinetic energy of particle moving along a circle of radius R depends upon the distance covered S and is given by K = aS where a is a constant. Then the force acting on the particle is

(a) $\frac{aS}{R}$ (b) $\frac{2(aS)^2}{R}$
(c) $\frac{aS^2}{R^2}$ (d) $\frac{2aS}{R}$

68. A ramp is constructed in parabolic shape such that the height y of any point on its surface is given in terms of the point's horizontal distance x from the bottom of the ramp be $y = x^2/2L$. A block of granite is to be set on the ramp; the coefficient of static friction is 0.80. What is the maximum x coordinate at which the block can be placed on the ramp and remain at rest, if L = 10 m?



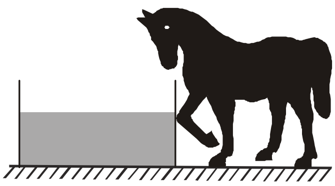
(a) 8m (b) 8.4m
(c) 9m (d) 9.4m

69. The force F acting on a body moving in a circle of radius r is always perpendicular to the instantaneous velocity v. The work done by the force on the body in half rotation is
(a) Fv (b) $F \cdot 2\pi r$
(c) Fr (d) 0

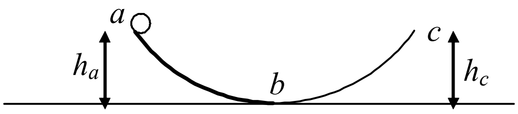
70. The negative of the distance rate of change of potential energy is equal to
(a) force acting on the particle in the direction of displacement
(b) acceleration of the particle, perpendicular to displacement
(c) power
(d) impulse.

71. n small balls each of mass m impinge elastically each second on a surface with velocity v. The force experienced by the surface will be
(a) $\frac{1}{2}$mnv (b) 2 mnv
(c) mnv (d) 2 mnv

72. A horse drinks water from a cubical container of side 1 m. The level of the stomach of horse is at 2 m from the ground. Assume that all the water drunk by the horse is at a level of 2m from the ground. Then minimum work done by the horse in drinking the entire water of the container is
(Take $\rho_{water} = 1000$ kg/m^3 and g = 10 m/s^2) –
(a) 10 kJ
(b) 15 kJ
(c) 20 kJ
(d) zero



73. The ball rolls down without slipping (which is at rest at a) along ab having friction. It rolls to a maximum height h_c where bc has no friction. K_a, K_b and K_c are kinetic energies at a, b and c.
Which of the following is correct ?



(a) $K_a = K_c, h_a = h_c$ (b) $K_b > K_c, h_a = h_c$
(c) $K_b > K_c, h_a < h_c$ (d) $K_b > K_c, h_a > h_c$

74. A body falls freely under gravity. Its velocity is v when it has lost potential energy equal to U. What is the mass of the body ?
(a) U^2/v^2 (b) $2U^2/v^2$
(c) $2U/v^2$ (d) U/v^2

75. If v be the instantaneous velocity of the body dropped from the top of a tower, when it is located at height h, then which of the following remains constant ?
(a) $gh + v^2$ (b) $gh + \frac{v^2}{2}$
(c) $gh - \frac{v^2}{2}$ (d) $gh - v^2$

76. The coefficient of friction between the tyres and the road is μ. A car is moving with momentum p. What will be the stopping distance due to friction alone ? The mass of the car is m.
(a) $p^2/2\mu g$ (b) $p^2/2m\mu g$
(c) $p^2/2m^2\mu g$ (d) $p^2/2mg$

77. A particle moves in the X–Y plane under the influence of a force $\vec{F}$ such that its instantaneous momentum is
$\vec{p} = \hat{i}2\cos t + \hat{j}2\sin t$.
What is the angle between the force and instantaneous momentum ?
(a) 0° (b) 45°
(c) 90° (d) 180°

78. A particle of mass 10 kg moving eastwards with a speed 5 ms^{-1} collides with another particle of the same mass moving north-wards with the same speed 5 ms^{-1}. The two particles coalesce on collision. The new particle of mass 20 kg will move in the north-east direction with velocity
(a) 10 ms^{-1} (b) 5 ms^{-1}
(c) $(5/\sqrt{2})$ms^{-1} (d) none of these

79. A metal ball of mass 2 kg moving with a velocity of 36 km/h has a head on collision with a stationary ball of mass 3 kg. If after the collision, the two balls move together, the loss in kinetic energy due to collision is
(a) 140 J (b) 100 J
(c) 60 J (d) 40 J

80. A force acts on a 30 gm particle in such a way that the position of the particle as a function of time is given by $x = 3t - 4t^2 + t^3$, where x is in metres and t is in seconds. The work done during the first 4 seconds is
(a) 576mJ (b) 450mJ
(c) 490mJ (d) 530mJ

81. A rubber ball is dropped from a height of 5m on a plane, where the acceleration due to gravity is not shown. On bouncing it rises to 1.8 m. The ball loses its velocity on bouncing by a factor of
(a) $\frac{16}{25}$ (b) $\frac{2}{5}$
(c) $\frac{3}{5}$ (d) $\frac{9}{25}$

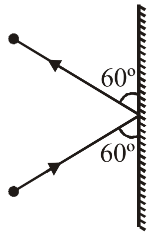
82. A 3 kg ball strikes a heavy rigid wall with a speed of 10 m/s at an angle of 60°. It gets reflected with the same speed and angle as shown here. If the ball is in contact with the wall for 0.20s, what is the average force exerted on the ball by the wall?

(a) 150N

(b) Zero

(c) $150\sqrt{3}$N

(d) 300N



83. A bomb of mass 1 kg is thrown vertically upwards with a speed of 100 m/s. After 5 seconds it explodes into two fragments. One fragment of mass 400 gm is found to go down with a speed of 25 m/s. What will happen to the second fragment just after the explosion? ($g = 10$ m/s^2)
(a) It will go upward with speed 40 m/s
(b) It will go upward with speed 100 m/s
(c) It will go upward with speed 60 m/s
(d) It will also go downward with speed 40m/s

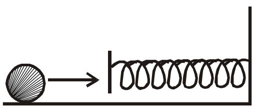
84. In a simple pendulum of length l the bob is pulled aside from its equilibrium position through an angle θ and then released. The bob passes through the equilibrium position with speed

(a) $\sqrt{2g\ell(1+\cos\theta)}$ (b) $\sqrt{2g\ell\sin\theta}$

(c) $\sqrt{2g\ell}$ (d) $\sqrt{2g\ell(1-\cos\theta)}$

85. A stationary particle explodes into two particles of masses m_1 and m_2 which move in opposite directions with velocities v_1 and v_2. The ratio of their kinetic energies E_1/E_2 is
(a) m_1v_2/m_2v_1 (b) m_2/m_1
(c) m_1/m_2 (d) 1

86. A mass of 0.5 kg moving with a speed of 1.5 m/s on a horizontal smooth surface, collides with a nearly weightless spring of force constant k = 50 N/m. The maximum compression of the spring would be



(a) 0.5m (b) 0.15m
(c) 0.12m (d) 1.5m

87. A ball moving with velocity 2 m/s collides head on with another stationary ball of double the mass. If the coefficient of restitution is 0.5, then their velocities (in m/s) after collision will be
(a) 0, 1 (b) 1, 1
(c) 1, 0.5 (d) 0, 2

88. A body projected vertically from the earth reaches a height equal to earth's radius before returning to the earth. The power exerted by the gravitational force is greatest
(a) at the highest position of the body
(b) at the instant just before the body hits the earth
(c) it remains constant all through
(d) at the instant just after the body is projected

89. A mass m moving horizontally (along the x-axis) with velocity v collides and sticks to mass of 3m moving vertically upward (along the y-axis) with velocity 2v. The final velocity of the combination is

(a) $\frac{1}{4}v\hat{i}+\frac{3}{2}v\hat{j}$ (b) $\frac{1}{3}v\hat{i}+\frac{2}{3}v\hat{j}$

(c) $\frac{2}{3}v\hat{i}+\frac{1}{3}v\hat{j}$ (d) $\frac{3}{2}v\hat{i}+\frac{1}{4}v\hat{j}$

90. The potential energy of particle in a force field is $U=\frac{A}{r^2}-\frac{B}{r}$, where A and B are positive constants and r is the distance of particle from the centre of the field. For stable equilibrium, the distance of the particle is
(a) $B/2A$ (b) $2A/B$
(c) A/B (d) B/A

91. A uniform force of $(3\hat{i}+\hat{j})$ newton acts on a particle of mass 2 kg. The particle is displaced from position $(2\hat{i}+\hat{k})$ meter to position $(4\hat{i}+3\hat{j}-\hat{k})$ meter. The work done by the force on the particle is
(a) 6 J (b) 13 J
(c) 15 J (d) 9 J

92. An explosion breaks a rock into three parts in a horizontal plane. Two of them go off at right angles to each other. The first part of mass 1 kg moves with a speed of 12 ms^{-1} and the second part of mass 2 kg moves with speed 8 ms^{-1}. If the third part flies off with speed 4 ms^{-1} then its mass is
(a) 5 kg (b) 7 kg
(c) 17 kg (d) 3 kg

93. If the kinetic energy of a body is increased by 300%, the momentum of the body is increased by
(a) 300% (b) 200%
(c) 100% (d) 50%

94. If the mass of the body is halved and velocity gets doubled then final kinetic energy would be ....of initial.
(a) same (b) four times
(c) double (d) eight times

95. A train of weight 10^7 N is running on a level track with uniform speed of 36 km h^{-1}. The frictional force is 0.5 kg per quintal. If g = 10 m/s^2, then power of engine is
(a) 500 kW (b) 50 kW
(c) 5 kW (d) 0.5 kW

DIRECTIONS (Qs. 96 to 100): Each question contains Statement-1 and Statement-2. Choose the correct answer (ONLY ONE option is correct) from the following.

(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

96. **Statement-1 :** A quick collision between two bodies is more violent than slow collision, even when initial and final velocities are identical.
Statement -2 : The rate of change of momentum determines that the force is small or large.

97. **Statement -1 :** If collision occurs between two elastic bodies their kinetic energy decreases during the time of collision.

Statement -2 : During collision intermolecular space decreases and hence elastic potential energy increases.

98. **Statement -1 :** A work done by friction is always negative.

Statement -2 : If frictional force acts on a body its K.E. may decrease.

99. **Statement -1 :** An object of mass m is initially at rest. A constant force F acts on it. Then the velocity gained by the object during a fixed displacement is proportional to $1/\sqrt{m}$.

Statement -2 : For a given force and displacement velocity is always inversely proportional to root of mass.

100. **Statement -1 :** Mechanical energy is the sum of macroscopic kinetic & potential energies.

Statement - 2 : Mechanical energy is that part of total energy which always remain conserved.

EXERCISE - 3
Exemplar & Past Years NEET/AIPMT Questions

Exemplar Questions

1. An electron and a proton are moving under the influence of mutual forces. In calculating the change in the kinetic energy of the system during motion, one ignores the magnetic force of one on another. This is, because
(a) the two magnetic forces are equal and opposite, so they produce no net effect
(b) the magnetic forces do not work on each particle
(c) the magnetic forces do equal and opposite (but non-zero) work on each particle
(d) the magnetic forces are necessarily negligible

2. A proton is kept at rest. A positively charged particle is released from rest at a distance *d* in its field. Consider two experiments; one in which the charged particle is also a proton and in another, a positron. In the same time *t*, the work done on the two moving charged particles is
(a) same as the same force law is involved in the two experiments
(b) less for the case of a positron, as the positron moves away more rapidly and the force on it weakens
(c) more for the case of a positron, as the positron moves away a larger distance
(d) same as the work done by charged particle on the stationary proton.

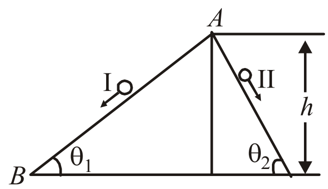
3. A man squatting on the ground gets straight up and stand. The force of reaction of ground on the man during the process is
(a) constant and equal to *mg* in magnitude
(b) constant and greater than *mg* in magnitude
(c) variable but always greater than *mg*
(d) at first greater than *mg* and later becomes equal to *mg*

4. A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200N and is directly opposed to the motion. The work done by the cycle on the road is
(a) +2000 J (b) – 200 J
(c) zero (d) – 20,000 J

5. A body is falling freely under the action of gravity alone in vaccum. Which of the following quantities remain constant during the fall?
(a) Kinetic energy
(b) Potential energy
(c) Total mechanical energy
(d) Total linear momentum

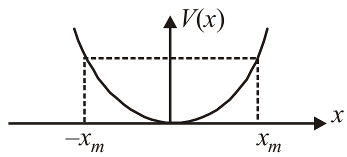
6. During inelastic collision between two bodies, which of the following quantities always remain conserved?
(a) Total kinetic energy
(b) Total mechanical energy
(c) Total linear momentum
(d) Speed of each body

7. Two inclined frictionless tracks, one gradual and the other steep meet at *A* from where two stones are allowed to slide down from rest, one on each track as shown in figure. Which of the following statement is correct?



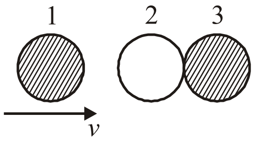
(a) Both the stones reach the bottom at the same time but not with the same speed
(b) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II
(c) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I
(d) Both the stones reach the bottom at different times and with different speeds

8. The potential energy function for a particle executing linear SHM is given by $V(x) = \frac{1}{2}kx^2$ where k is the force constant of the oscillator (Fig.). For $k = 0.5$ N/m, the graph of $V(x)$ versus x is shown in the figure. A particle of total energy E turns back when it reaches $x = \pm x_m$. If V and K indicate the PE and KE, respectively of the particle at $x = +x_m$, then which of the following is correct?

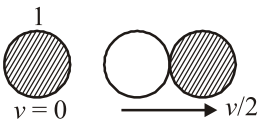


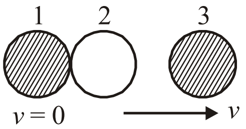
(a) $V = O,\ K = E$ (b) $V = E,\ K = O$
(c) $V < E,\ K = O$ (d) $V = O,\ K < E$

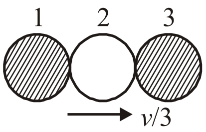
9. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed v as shown in figure.

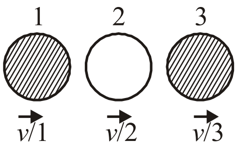


If the collision is elastic, which of the following (figure) is a possible result after collision?

(a) 

(b) 

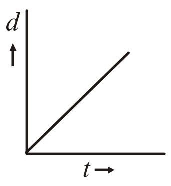
(c) 

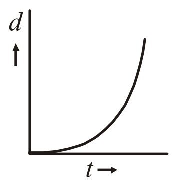
(d) 

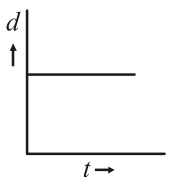
10. A body of mass 0.5 kg travels in a straight line with velocity $v = a\,x^{3/2}$ where $a = 5\ \text{m}^{-1/2}\text{s}^{-1}$. The work done by the net force during its displacement from $x = 0$ to $x = 2$ m is

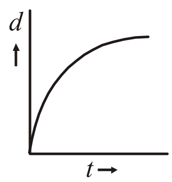
(a) 15 J (b) 50 J
(c) 10 J (d) 100 J

11. A body is moving unidirectionally under the influence of a source of constant power supplying energy. Which of the diagrams shown in figure correctly shown the displacement-time curve for its motion?

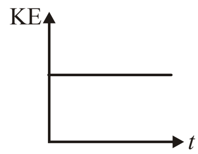
(a) 

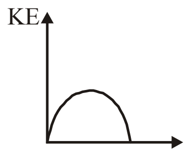
(b) 

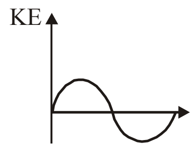
(c) 

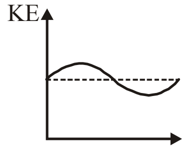
(d) 

12. Which of the diagrams shown in figure most closely shows the variation in kinetic energy of the earth as it moves once around the sun in its elliptical orbit?

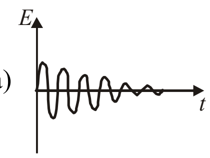
(a) 

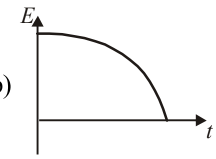
(b) 

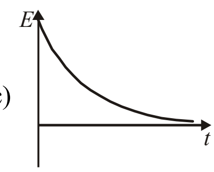
(c) 

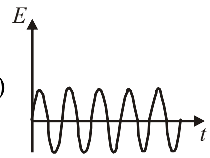
(d) 

13. Which of the diagrams shown in figure represents variation of total mechanical energy of a pendulum oscillating in air as function of time?

(a) 

(b) 

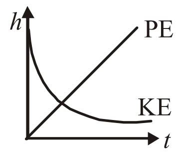
(c) 

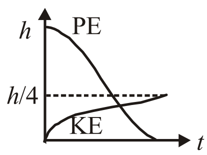
(d) 

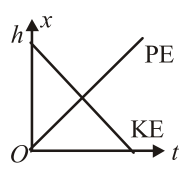
14. A mass of 5 kg is moving along a circular path of radius 1 m. If the mass moves with 300 rev/min, its kinetic energy would be

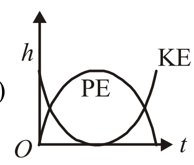
(a) $250\,\pi^2$ (b) $100\,\pi^2$
(c) $5\,\pi^2$ (d) 0

15. A raindrop falling from a height h above ground, attains a near terminal velocity when it has fallen through a height $(3/4)h$. Which of the diagrams shown in figure correctly shows the change in kinetic and potential energy of the drop during its fall up to the ground?

(a) 

(b) 

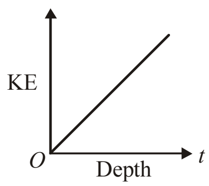
(c) 

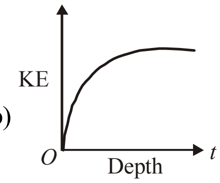
(d) 

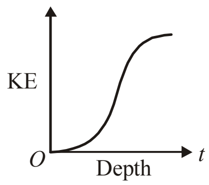
16. In a shotput event an athlete throws the shotput of mass 10 kg with an initial speed of 1 m s^{-1} at 45° from a height 1.5 m above ground. Assuming air resistance to be negligible and acceleration due to gravity to be 10 m s^{-2}, the kinetic energy of the shotput when it just reaches the ground will be

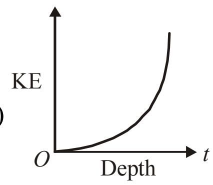
(a) 2.5 J (b) 5.0 J
(c) 52.5 J (d) 155.0 J

17. Which of the diagrams in figure correctly shows the change in kinetic energy of an iron sphere falling freely in a lake having sufficient depth to impart it a terminal velocity?

(a) 

(b) 

(c) 

(d) 

18. A cricket ball of mass 150 g moving with a speed of 126 km/h hits at the middle of the bat, held firmly at its position by the batsman. The ball moves straight back to the bowler after hitting the bat. Assuming that collision between ball and bat is completely elastic and the two remain in contact for 0.001s, the force that the batsman had to apply to hold the bat firmly at its place would be

(a) 10.5 N (b) 21 N
(c) 1.05×10^4 N (d) 2.1×10^4 N

NEET/AIPMT (2013-2017) Questions

19. A uniform force of $(3\hat{i} + \hat{j})$ newton acts on a particle of mass 2 kg. The particle is displaced from position $(2\hat{i} + \hat{k})$ meter to position $(4\hat{i} + 3\hat{j} - \hat{k})$ meter. The work done by the force on the particle is ***[2013]***

(a) 6 J (b) 13 J
(c) 15 J (d) 9 J

20. An explosion breaks a rock into three parts in a horizontal plane. Two of them go off at right angles to each other. The first part of mass 1 kg moves with a speed of 12 ms^{-1} and the second part of mass 2 kg moves with speed 8 ms^{-1}. If the third part flies off with speed 4 ms^{-1} then its mass is

(a) 5 kg (b) 7 kg ***[2013]***
(c) 17 kg (d) 3 kg

21. A person holding a rifle (mass of person and rifle together is 100 kg) stands on a smooth surface and fires 10 shots horizontally, in 5 s. Each bullet has a mass of 10 g with a muzzle velocity of 800 ms^{-1}. The final velocity acquired by the person and the average force exerted on the person are ***[NEET Kar. 2013]***

(a) –1.6 ms^{-1}; 8 N (b) –0.08 ms^{-1}; 16 N
(c) – 0.8 ms^{-1}; 8 N (d) –1.6 ms^{-1}; 16 N

22. A particle with total energy E is moving in a potential energy region $U(x)$. Motion of the particle is restricted to the region when ***[NEET Kar. 2013]***

(a) $U(x) > E$ (b) $U(x) < E$
(c) $U(x) = O$ (d) $U(x) \leq E$

23. One coolie takes 1 minute to raise a suitcase through a height of 2 m but the second coolie takes 30 s to raise the same suitcase to the same height. The powers of two coolies are in the ratio of ***[NEET Kar. 2013]***

(a) 1 : 2 (b) 1 : 3
(c) 2 : 1 (d) 3 : 1

24. A body of mass (4m) is lying in x-y plane at rest. It suddenly explodes into three pieces. Two pieces, each of mass (m) move perpendicular to each other with equal speeds (v). The total kinetic energy generated due to explosion is :

(a) mv^2 (b) $\frac{3}{2}mv^2$ ***[2014]***
(c) $2\,mv^2$ (d) $4\,mv^2$

25. A particle of mass m is driven by a machine that delivers a constant power of k watts. If the particle starts from rest the force on the particle at time t is ***[2015]***

(a) $\sqrt{mk}\ t^{-1/2}$ (b) $\sqrt{2mk}\ t^{-1/2}$
(c) $\frac{1}{2}\sqrt{mk}\ t^{-1/2}$ (d) $\sqrt{\frac{mk}{2}}t^{-1/2}$

26. Two similar springs P and Q have spring constants K_P and K_Q, such that $K_P > K_Q$. They are stretched, first by the same amount (case a,) then by the same force (case b). The work done by the springs W_P and W_Q are related as, in case (a) and case (b), respectively ***[2015]***

(a) $W_P = W_Q$; $W_P = W_Q$
(b) $W_P > W_Q$; $W_Q > W_P$
(c) $W_P < W_Q$; $W_Q < W_P$
(d) $W_P = W_Q$; $W_P > W_Q$

27. A block of mass 10 kg, moving in x direction with a constant speed of 10 ms^{-1}, is subject to a retarding force F = 0.1 × J/m during its travel from x = 20 m to 30 m. Its final KE will be:

(a) 450 J (b) 275 J ***[2015]***
(c) 250 J (d) 475 J

28. The heart of man pumps 5 litres of blood through the arteries per minute at a pressure of 150 mm of mercury. If the density of mercury be 13.6×10^3 kg/m^3 and g = 10m/s^2 then the power of heart in watt is : ***[2015 RS]***

(a) 2.35 (b) 3.0
(c) 1.50 (d) 1.70

29. A ball is thrown vertically downwards from a height of 20 m with an initial velocity v_0. It collides with the ground loses 50 percent of its energy in collision and rebounds to the same height. The initial velocity v_0 is : ***[2015 RS]***
(Take $g = 10\ ms^{-2}$)
(a) $20\ ms^{-1}$ (b) $28\ ms^{-1}$
(c) $10\ ms^{-1}$ (d) $14\ ms^{-1}$

30. On a frictionless surface a block of mass M moving at speed v collides elastically with another block of same mass M which is initially at rest. After collision the first block moves at an angle θ to its initial direction and has a speed $\frac{v}{3}$. The second block's speed after the collision is : ***[2015 RS]***
(a) $\frac{3}{4}v$ (b) $\frac{3}{\sqrt{2}}v$
(c) $\frac{\sqrt{3}}{2}v$ (d) $\frac{2\sqrt{2}}{3}v$

31. Two particles A and B, move with constant velocities $\vec{v}_1$ and $\vec{v}_2$. At the initial moment their position vectors are $\vec{r}_1$ and $\vec{r}_2$ respectively. The condition for particles A and B for their collision is: ***[2015 RS]***
(a) $\vec{r}_1.\vec{v}_1 = \vec{r}_2.\vec{v}_2$ (b) $\vec{r}_1 \times \vec{v}_1 = \vec{r}_2 \times \vec{v}_2$
(c) $\vec{r}_1 - \vec{r}_2 = \vec{v}_1 - \vec{v}_2$ (d) $\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} = \frac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|}$

32. A body of mass 1 kg begins to move under the action of a time dependent force $\vec{F}=(2t\hat{i}+3t^2\hat{j})$ N, where $\hat{i}$ and $\hat{j}$ are unit vectors alogn x and y axis. What power will be developed by the force at the time t? ***[2016]***
(a) $(2t^2 + 3t^3)$W (b) $(2t^2 + 4t^4)$W
(c) $(2t^3 + 3t^4)$ W (d) $(2t^3 + 3t^5)$W

33. A particle of mass 10 g moves along a circle of radius 6.4 cm with a constant tangential acceleration. What is the magnitude of this acceleration if the kinetic energy of the particle becomes equal to 8×10^{-4} J by the end of the second revolution after the beginning of the motion ? ***[2016]***
(a) $0.1\ m/s^2$ (b) $0.15\ m/s^2$
(c) $0.18\ m/s^2$ (d) $0.2\ m/s^2$

34. Consider a drop of rain water having mass 1 g falling from a height of 1 km. It hits the ground with a speed of 50 m/s. Take 'g' constant with a value $10\ m/s^2$.
The work done by the (i) gravitational force and the (ii) resistive force of air is ***[2017]***
(a) (i) 1.25 J (ii) –8.25 J
(b) (i) 100 J (ii) 8.75 J
(c) (i) 10 J (ii) – 8.75 J
(d) (i) – 10 J (ii) –8.25 J

Hints & Solutions

EXERCISE - 1

1. (a) 2. (b) 3. (d) 4. (a) 5. (c)
6. (c) 7. (a) 8. (b)

9. (b) Because water enters into the vessel A, it becomes heavier. Gravity helps it sink. External work required for immersing A is obviously less than that for immersing B.

10. (c) Weight Mg moves the centre of gravity of the spring through a distance $\frac{(0+\ell)}{2} = \ell/2$

$\therefore$ Mechanical energy stored = Work done = Mg $\ell/2$.

11. (d) From $v = u + at$, $v_1 = 0 + at_1$

$\therefore \quad a = \frac{v_1}{t_1}$

$F = ma = m\,v_1/t_1$

Velocity acquired in t sec = at = $\frac{v_1}{t_1}t$

Power $= F \times v = \frac{m\,v_1}{t_1} \times \frac{v_1\,t}{t_1} = \frac{m\,v_1^2 t}{t_1^2}$

12. (d) $W = F \times s$

$W \propto \frac{1}{x}(x) \;\therefore\; W \propto x^0$

13. (b) At the top of flight, horizontal component of velocity $= u \cos 45° = u/\sqrt{2}$

$\therefore \quad K.E. = \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2}\left(\frac{mu^2}{2}\right) = \frac{1}{2}K.$

14. (a) From $v^2 - u^2 = 2as$

$0 - v_0^2 = 2(\mu g)s \qquad \therefore \; s = \frac{v_0^2}{2\mu g}$

15. (b) If $\vec{L} = 0 \Rightarrow$ K.E may or may not be zero.

If K.E $= 0 \;\Rightarrow \vec{L} = 0$.

16. (d) Power is defined as the rate of doing work. For the automobile, the power output is the amount of work done (overcoming friction) divided by the length of time in which the work was done.

17. (b) $W = \frac{1}{2}k\ell_2^2 - \frac{1}{2}k\ell_1^2 = \frac{1}{2}k(\ell_2^2 - \ell_1^2)$

18. (b) $F \propto \frac{1}{v}$ (given)

Then $W = E_k = F.s$

Become = $E_k \propto \frac{s}{v} \Rightarrow E_k \propto \frac{s}{s/t}$

$\Rightarrow \; E_k \propto t$

19. (d) $\frac{mv^2}{t} = \frac{ms^2}{t^3}$

since both P and m are constants

$\therefore \quad \frac{s^2}{t^3} = \text{constant}$

20. (c) $v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{a \times b + 0}{a + c} = \frac{a\,(b)}{a + c}$.

21. (d) As $e^n = \left(\frac{h_n}{h_0}\right)^{1/2}$

$\therefore \quad h_n = e^{2n} h_0 = e^{2\times 2}\,h = e^4\,h.$

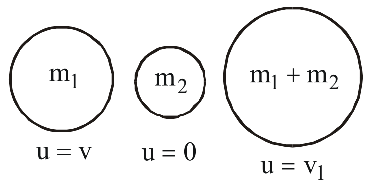
22. (a) As $u_2 = 0$ and $m_1 = m_2$, therefore from $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$ we get $u_1 = v_1 + v_2$

Also, $e = \frac{v_2 - v_1}{u_1} = \frac{v_2 - v_1}{v_2 + v_1} = \frac{1 - v_1/v_2}{1 + v_1/v_2}$,

which gives $\frac{v_1}{v_2} = \frac{1-e}{1+e}$

23. (d) Work done $= \int F dx$

24. (b)



applying conservation of momentum

$m_1 v + m_2\,(0) = (m_1 + m_2)v_1 \;\Rightarrow v_1 = \frac{m_1}{(m_1 + m_2)}v$

as $m_1 < (m_1 + m_2)$ so velocity decreases.

25. (b) $v_1 = \frac{(m_1 - m_2)\,u_1}{m_1 + m_2}$ As v_1 is negative and less than u_1, therefore, $m_1 < m_2$.

EXERCISE - 2

1. (a) $W = F\,s \cos 90° = $ zero
2. (c) As the cord is trying to hold the motion of the block, work done by the cord is negative.

$W = -M\,(g - a)\,d = -M\left(g - \frac{g}{4}\right)d = \frac{-3Mgd}{4}$

3. (c) $W = F\,s \cos\theta = 10 \times 2 \cos 60° = 10$ J.
4. (c) $W = \vec{F}.\vec{s} = (5\hat{i} + 2\hat{j}).(2\hat{i} + 3\hat{j}) = 10 + 6 = 16$ J.

5. (c) From horizontal position to lowest position, height through which the bob falls = ℓ

$\therefore$ At lowest position, $v = \sqrt{2\ell g}$

K.E. at lowest point

$$= \frac{1}{2}mv^2 = \frac{1}{2}m(2\ell g) = m\ell g = 10 \times 1 \times 9.8 = 98\,J.$$

6. (b) $W = \int_0^{x_1} F\,dx = \int_0^{x_1} c\,x\,dx = \left[\frac{1}{2}cx^2\right]_0^{x_1}$

$$= \frac{1}{2}c(x_1^2 - 0) = \frac{1}{2}cx_1^2$$

7. (d) Forward thrust, $F = \frac{P}{v} = \frac{100 \times 746}{20} = 3730\,N.$

8. (c) As $\frac{1}{2}m_A v_A^2 = \frac{1}{2}m_B v_B^2$

$$\frac{v_A}{v_B} = \sqrt{\frac{m_B}{m_A}};$$

$$\frac{P_B}{P_A} = \frac{m_B v_B}{m_A v_A} = \frac{m_B}{m_A}\sqrt{\frac{m_A}{m_B}} = \sqrt{\frac{m_B}{m_A}} = \frac{1}{\sqrt{3}}$$

9. (a) From $m_1 v_1 + m_2 v_2 = 0$,

$$v_2 = \frac{-m_1 v_1}{m_2} = -\frac{4}{234}$$

10. (b) Total energy at the time of projection

$$= \frac{1}{2}mv^2 = \frac{1}{2} \times 0.1(20)^2 = 20J$$

Half way up, P.E. becomes half the P.E. at the top i.e.

$$P.E. = \frac{20}{2} = 10J \;\therefore\; K.E. = 20 - 10 = 10J.$$

11. (b) Workdone, $W = \frac{1}{2}k\left(x_2^2 - x_1^2\right)$

$$= \frac{1}{2}k\left[(0.15)^2 - (0.05)^2\right]$$

$$= \frac{1}{2} \times 800 \times 0.02 = 8J$$

12. (d) As $E = \frac{p^2}{2m}$ $\quad\therefore\; \frac{dE}{E} = 2\left(\frac{dp}{p}\right) = 2 \times 5\% = 10\%$

13. (d) $T = M(g - g/8) = \frac{7}{8}Mg$

Work done by the cord = $T \times d \cos 180^\circ$

$$= \frac{7}{8}Mgd(-1) = -7Mgd/8.$$

14. (b) $W = \vec{F}.\vec{x} = (5\hat{i} + 3\hat{j} + 2\hat{k}).(2\hat{i} - \hat{j}) = 10 - 3 = 7$ joules

15. (a) $W_1 = \frac{1}{2} \times 5 \times 10^3 (0.05)^2$

$$\Rightarrow W_2 = \frac{1}{2} \times 5 \times 10^3 (0.10)^2$$

$$\therefore\; \Delta W = \frac{1}{2} \times 5 \times 10^3 \times 0.15 \times 0.05 = 18.75J.$$

16. (a) $m_1 = 0.2$ kg, $m_2 = 0.4$ kg, $v_1 = 0.3\,m/s$, $v_2 = ?$

Applying law of conservation of momentum

$$m_1 v_1 - m_2 v_2 = \frac{0.2 \times 0.3}{0.4} = 0.15\,m/s.$$

17. (c) $v_2 = \frac{-m_1 v_1}{m_2} = \frac{-3}{6} \times 16 = -8\,m/s$

$$E_2 = \frac{1}{2}m_2 v_2^2 = \frac{1}{2} \times 6(-8)^2 = 192\,J.$$

18. (d) $V = \frac{1}{2}k(x)^2 = \frac{1}{2}k(2)^2$ or $k = \frac{2V}{4} = \frac{V}{2}$

$$V' = \frac{1}{2}k(10)^2 = \frac{1}{2} \times \left(\frac{V}{2}\right)(10)^2 = 25V$$

19. (b) If P = momentum, K = kinetic energy, then

$$P_1^2 = 2mK_1,\; P_2^2 = 2mK_2$$

$$\therefore \left(\frac{P_2}{P_1}\right)^2 = \frac{K_2}{K_1} = \frac{3K_1}{K_1} = 3 \;\therefore\; \frac{P_2}{P_1} = \sqrt{\frac{3}{1}} = 1.732$$

20. (b) $\frac{p_1}{p_2} = \sqrt{\frac{2m_1K_1}{2m_2K_2}}$

21. (d) Applying the principle of conservation of linear momentum, we get

$$3m \times v = \sqrt{(m \times 60)^2 + (m \times 60)^2} = m \times 60\sqrt{2}$$

$$v = 20\sqrt{2}\,m/s$$

22. (c) Energy stored (E) $= \frac{75}{100} \times (12) = 9\,J$

As $E = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times 9}{1}} = \sqrt{18}\,m/sec$$

23. (c) The uniform acceleration is $a = \frac{1-0}{15} = \frac{1}{15}\,ms^{-2}$

Let v be the velocity at kinetic energy $\frac{2}{9}J$

therefore $\frac{1}{2} \times 1 \times v^2 = \frac{2}{9}$ or $v = \frac{2}{3}\,ms^{-1}$

Using v = u + at

$$\frac{2}{3} = 0 + \frac{1}{15} \times t \Rightarrow t = 10s$$

24. (d) Let n be the number of bullets that the man can fire in one second.
$\therefore$ change in momentum per second $= n \times mv = F$
[m= mass of bullet, v = velocity] ($\because$ F is the force)

$$\therefore n = \frac{F}{mv} = \frac{144 \times 1000}{40 \times 1200} = 3$$

25. (d) Power supplied $= \frac{mgh}{t}$

Power used by crane $= \frac{mgh}{t} \times \frac{100}{80}$

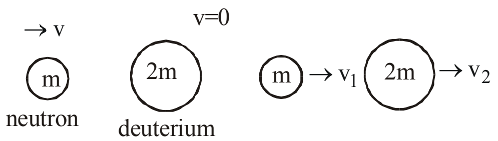
$$= \frac{1000 \times 10 \times 100}{3600} \times \frac{100}{80} = \frac{10^5}{36 \times 8} \text{W}$$

26. (b) Work done against gravity
$W_g = 50 \times 10 \times 30 = 15$ kJ
Work done against friction

$$W_f = \mu mg \cos\theta \times s = 0.4 \times 50 \times 10 \times \frac{4}{5} \times 50 = 8 \text{ kJ}$$

Total work done $= W_g + W_f = 15 \text{ kJ} + 8 \text{ kJ} = 23 \text{ kJ}$

27. (d) Let mass of neutron = m
then mass of deuterium = 2m
[$\because$ it has double nuclides thus has neutron].
Let initial velocity of neutron = v and final velocities of neutron and deuterium are v_1 and v_2 respectively.



Applying conservation of momentum

$$mv + 2m(0) = mv_1 + 2mv_2$$

$$\Rightarrow v = v_1 + 2v_2 \text{..........(i)}$$

applying conservation of energy

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}2m.v_2^2$$

$$\Rightarrow v^2 = v_1^2 + 2v_2^2 \text{......(ii)}$$

from (i) and (ii), $v^2 = (v - 2v_2)^2 + 2v_2^2$

$$\Rightarrow \quad v^2 = v^2 + 4v_2^2 - 4v_2 v + 2v_2^2$$

$$6v_2^2 - 4v_2 v = 0$$

$$\Rightarrow \quad v_2 = \frac{2v}{3} \;\&\; v_1 = -\frac{v}{3}$$

now fractional change in kinetic energy

$$= \frac{K_i - K_f}{K_i} = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \frac{v^2 - \frac{v^2}{9}}{v^2} = \frac{8}{9}$$

28. (c) $W = F s \cos\theta,\ \cos\theta = \frac{W}{Fs} = \frac{25}{5 \times 10} = \frac{1}{2},\ \theta = 60°.$

29. (b) For elastic collision in one dimension

$$v_1 = \frac{2m_2u_2}{m_1 + m_2} + \frac{(m_1 - m_2)u_1}{(m_1 + m_2)}$$

As mass 2m, is at rest, So $u_2 = 0$

$$\Rightarrow \quad v_1 = \frac{(8m - 2m)u}{8m + 2m} = \frac{3}{5}u$$

Final energy of sphere = $(K.E.)_f$

$$= \frac{1}{2}(8m)\left(\frac{3u}{5}\right)^2 = \frac{1}{2}(8m)u^2 \times \left(\frac{3}{5}\right)^2$$

$$= \frac{9}{25}E = 0.36\,E$$

30. (b) The work is done against gravity so it is equal to the change in potential energy. $W = E_p = mgh$
For a fixed height, work is proportional to weight lifted. Since Johnny weighs twice as much as Jane he works twice as hard to get up the hill.
Power is work done per unit time. For Johnny this is $W/\Delta t$. Jane did half the work in half the time, $(1/2\ W)/(1/2\ \Delta t) = W/\Delta t$ which is the same power delivered by Johnny.

31. (c) Fraction of energy transferred $= \frac{4 \times 2}{(1+2)^2} = \frac{8}{9}$

32. (c) $E = \frac{p^2}{2m} \Rightarrow \frac{E_1}{E_2} = \frac{p_1^2}{p_2^2} \Rightarrow E_2 = E_1 \times 4$

$$\therefore \ E_2 - E_1 = 3E_1$$

33. (c) Since the new height gained is half, therefore there is 50% loss of energy.

34. (a) $m_1u = m_1\frac{2u}{3} + m_2u$ (By condition of linear momentum)

$$\Rightarrow \quad \frac{1}{3}m_1u = m_2v \qquad \text{...... (i)}$$

Also $e = \frac{|v_1 - v_2|}{|u_2 - u_1|}$

$$\Rightarrow \ v - \frac{2u}{3} = u \ \Rightarrow \ v = \frac{5}{3}u \qquad \text{...... (ii)}$$

From (i) and (ii), $\frac{1}{3}m_1u = \frac{5}{3}m_2u \Rightarrow \frac{m_1}{m_2} = 5$

35. (c) At Equilibrium , $ky = mg \Rightarrow k = \frac{mg}{y}$

$$U = \frac{1}{2}\left(\frac{mg}{y}\right)y^2 = \frac{1}{2}mgy$$

36. (a) Loss in K.E = Area under the curve

37. (b) In this case, $P = \dfrac{mgh + \frac{1}{2}mv^2}{t}$

$$\Rightarrow P = \frac{m}{t}\left[gh + \frac{v^2}{2}\right]$$

$$\Rightarrow P = \frac{10}{1}\left[10\times10 + \frac{10\times10}{2}\right]W = 1500W$$

38. (c) By conservation of linear momentum, $\dfrac{m_1}{m_2} = \dfrac{v_2}{v_1}$

or $\dfrac{\rho_x \frac{4}{3}\pi r_1^3}{\rho_x \frac{4}{3}\pi r_2^3} = \dfrac{1}{2} \Rightarrow \dfrac{r_1}{r_2} = \dfrac{1}{2^{1/3}}$

39. (c) $mc^2 = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}c^2 = \dfrac{m_0}{\sqrt{1-\frac{(0.5c)^2}{c^2}}}c^2$

$$= \frac{m_0}{\sqrt{1-0.25}}c^2 = \frac{m_0}{\sqrt{0.75}}c^2 = 1.15m_0c^2$$

Change in energy $= 1.15\ m_0c^2 - m_0c^2$
$= 0.15 \times 9.1 \times 10^{-31} \times (3 \times 10^8)^2$
$= 12.285 \times 10^{-15}$ J

$$= \frac{12.285\times10^{-15}}{1.6\times10^{-13}}\text{MeV}$$

$= 0.07678$ MeV

40. (b) $F = \dfrac{12}{100}\times1000\times10\ N = 1200\ N$

$P = Fv = 1200\ N \times 15\ ms^{-1} = 18$ kW.

41. (b) Components of velocity before and after collision parallel to the plane are equal, So
$v \sin 60° = u \sin 30°$ ....(1)
Components of velocity normal to the plane are related to each other
$v \cos 60° = e\, u\, (\cos 30°)$ ....(2)

$\Rightarrow$ $\cot 60° = e \cot 30° \Rightarrow e = \dfrac{\cos 60°}{\cot 30°}$

$\Rightarrow$ $e = \dfrac{\frac{1}{\sqrt{3}}}{\sqrt{3}} \Rightarrow e = \dfrac{1}{3}$.

42. (d) Work done on the body is gain in the kinetic energy. Acceleration of the body is a = V/T.

Velocity acquired in time t is $v = at = \dfrac{V}{T}t$

K.E. acquired $\propto v^2$. That is work done $\propto \dfrac{V^2t^2}{T^2}$

43. (d) Kinetic energy of ball when reaching the ground $= mgh = mg \times 10$

Kinetic energy after the impact

$$= \frac{60}{100}\times mg\times10 = 6mg$$

If the ball rises to a height h, then $mgh = 6$ mg. Hence, h = 6 m.

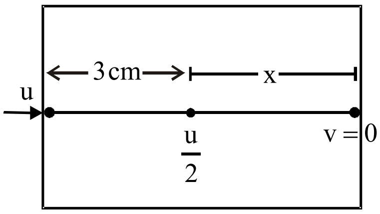
44. (d) $F = -\dfrac{dU}{dy} = b - 2ay$

45. (b) $Fv = P$. or $M\dfrac{dv}{dt}v = P$

That is $\int v\,dv = \int \dfrac{P}{M}dt$

Hence $v = [2Pt/M]^{1/2}$

46. (c)



Case I :

$$\left(\frac{u}{2}\right)^2 - u^2 = 2.a.3$$

or $-\dfrac{3u^2}{4} = 2.\ a.\ 3 \Rightarrow a = -\dfrac{u^2}{8}$

Case II :

$$0 - \left(\frac{u}{2}\right)^2 = 2.\ a.\ x \quad \text{or} \quad -\frac{u^2}{4} = 2.\left(-\frac{u^2}{8}\right)\times x$$

$\Rightarrow$ x = 1 cm

Alternative method : Let K be the initial energy and F be the resistive force. Then according to work-energy theorem,
$W = \Delta K$

i.e., $3F = \dfrac{1}{2}mv^2 - \dfrac{1}{2}m\left(\dfrac{v}{2}\right)^2$

$$3F = \frac{1}{2}mv^2\left(1-\frac{1}{4}\right)$$

$$3F = \frac{3}{4}\left(\frac{1}{2}mv^2\right) \quad ...(1)$$

and $Fx = \dfrac{1}{2}m\left(\dfrac{v}{2}\right)^2 - \dfrac{1}{2}m(0)^2$

i.e., $\dfrac{1}{4}\left(\dfrac{1}{2}mv^2\right) = Fx$ ...(2)

Comparing eqns. (1) and (2)
F = Fx
or x = 1 cm

47. (b) Let the velocity and mass of 4 kg piece be v_1 and m_1 and that of 12 kg piece be v_2 and m_2.
Applying conservation of linear momentum
$m_2v_2 = m_1v_1$

$$\Rightarrow v_1 = \frac{12\times 4}{4} = 12\ \text{ms}^{-1}$$

$$\therefore \text{K.E.}_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}\times 4\times 144 = 288\ \text{J}$$

48. (d) Velocity will increase when force is along the direction of displacement i.e. $\hat{F} = \hat{d}$.

49. (b) When body is lowered gradually, its weight acts at C.G. of the spring. When same body is allowed to fall freely, the same weight acts at lower end of the spring. In the latter case, original length (L) of spring is double. As $\Delta L \propto L$, therefore, ΔL becomes twice in second case i.e. 2x.

50. (d) If V is velocity of combination (bag + bullet), then from principle of conservation of linear momentum

$$(m+M)\,V = m\,v \text{ or } V = \frac{m\,v}{(m+M)}$$

$$\text{K.E.} = \frac{1}{2}(m+M)\,V^2 = \frac{m^2\,v^2}{2(m+M)}$$

51. (c) Workdone W = $[ML^2\,T^{-2}]$

It shows that $W \propto (LT^{-1})^2$ i.e. $W \propto v^2$.
$\therefore$ graph between W & v is a parabola.
Alternatively :
According to work energy theorem

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}mv^2 \quad (\because u = 0)$$

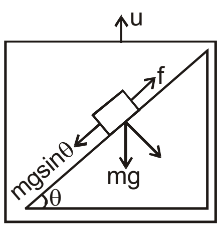
$\Rightarrow W \propto v^2$

52. (c) Applying principle of conservation of linear momentum

$$(2m)\,V = \sqrt{(m\,v)^2 + (m\,v)^2} = m\,v\sqrt{2}$$

$$\therefore \quad V = \frac{v\sqrt{2}}{2} = \frac{v}{\sqrt{2}}$$

53. (a) Since block does not slide on wedge so displacement is zero & hence work done by force is zero.



54. (c) If mass of nucleus is m, mass of α particle is m_1 & mass of remaining nucleus is m_2, then from the law of conservation of momentum.

$$\underset{\substack{\text{initial momentum}\\ \text{of nucleus}}}{m\vec{v}} = \underset{\substack{\text{momentum of}\\ \alpha\text{ particle}}}{m_1\vec{v}_1} + \underset{\substack{\text{momentum of}\\ \text{remaining nucleus}}}{m_2\vec{v}_2}$$

55. (a) Let M be the mass of shell. Applying law of conservation of linear momentum

$$MV\cos\theta = \left(-\frac{MV}{2}\cos\theta\right) + \frac{M}{2}v$$

$$\text{ie, } MV\cos\theta + \frac{M}{2}V\cos\theta = \frac{M}{2}\ v$$

or $v = 3\,V\cos\theta$.

56. (d) As bob B is of same material and same mass as the bob A, therefore, on elastic collision, their velocities are exchanged. Bob A comes to rest and B moves with the velocity of A.

57. (c) As velocities are exchanged on perfectly elastic collision, therefore masses of two objects must be equal.

$$\therefore \frac{m_a}{m_b} = 1 \text{ or } m_a = m_b.$$

58. (c) $a = -kx \Rightarrow \frac{dv}{dt} = -kx$

Also $\frac{dx}{dt} = v$ or $dt = \frac{dx}{dv}$

$$\therefore \frac{vdv}{dx} = -kx \Rightarrow \int_{v_1}^{v_2} v\,dv = -\int_0^x kxdx$$

$$\left(v_2^2 - v_1^2\right) = -\frac{kx^2}{2} \Rightarrow \frac{1}{2}m\left(v_2^2 - v_1^2\right) = \frac{1}{2}m\left(\frac{-kx^2}{2}\right)$$

$\therefore \Delta K\ \alpha\ x^2$

59. (c) $K_i = \frac{1}{2}m_1u_1^2$,

$K_f = \frac{1}{2}m_1v_1^2,\ v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1$

Fractional loss

$$\frac{K_i - K_f}{K_i} = \frac{\frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2}{\frac{1}{2}m_1u_1^2}$$

$$= 1 - \frac{v_1^2}{u_1^2} = 1 - \frac{(m_1 - m_2)^2}{(m_1 + m_2)^2} = \frac{4m_1m_2}{(m_1 + m_2)^2}$$

$m_2 = m$; $m_1 = nm$ $\qquad 1 - \frac{K_f}{K_i} = \frac{4n}{(1+n)^2}$

Energy transfer is maximum when $K_f = 0$

$$\frac{4n}{(1+n)^2} = 1 \Rightarrow 4n = 1 + n^2 + 2n \Rightarrow n^2 + 1 - 2n = 0$$

$(n-1)^2 = 0 \qquad n = 1$ ie. $m_2 = m$, $m_1 = m$

Transfer will be maximum when both masses are equal and one is at rest.

60. (c) $V_c = \dfrac{10\times14+4\times0}{10+4} = 10 m/s$; since spring force is internal force.

61. (a) For inelastic collision, linear momentum is conserved

$$\Rightarrow mv_1 = 2mv_2 \Rightarrow v_2 = \frac{v_1}{2}$$

Loss in K.E. = Gain in P.E.

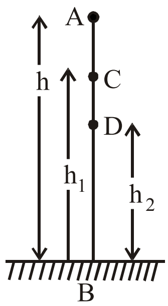
$$= \frac{1}{2}mv_1^2 - \frac{1}{2}(2m)v_2^2 = 2mgh$$

$$\Rightarrow 4\text{ mgh} = mv_1^2 - \frac{mv_1^2}{2} = \frac{mv_1^2}{2} = \frac{mv^2}{2}$$

$$\Rightarrow h = \frac{v^2}{8g}$$

62. (d) $P = F\times v \Rightarrow P = F\,a\,t \quad \therefore P \propto t$

63. (c) $t_{AB} = \sqrt{\dfrac{2h}{g}}$



$$t_{BC} + t_{CB} = 2\sqrt{\frac{2h_1}{g}}$$

$$= 2\sqrt{\frac{2e^2h}{g}} = 2e\sqrt{\frac{2h}{g}}$$

$$t_{BD} + t_{DB} = 2e^2\sqrt{\frac{2h}{g}}$$

$\therefore$ Total time taken by the body in coming to rest

$$= \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}} + 2e^2\sqrt{\frac{2h}{g}} + ...$$

$$= \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}}\,[1+e+e^2+.........]$$

$$= \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}}\times\frac{1}{1-e} = \sqrt{\frac{2h}{g}}\left[\frac{1+e}{1-e}\right] = t\left(\frac{1+e}{1-e}\right)$$

64. (d) Initially, $3k = mg$ or $k = \dfrac{mg}{3}$

New force constant of longer part

$$k' = \frac{3}{2}k = \frac{3}{2}\times\frac{mg}{3} = \frac{mg}{2}$$

Finally, $k'y = mg$

$$y = \frac{mg}{k'} = \frac{mg}{mg}\times 2 = 2cm$$

65. (d) $m = 10\times0.8kg = 8kg$

height of iron chain = 5m

$$P = \frac{mgh}{t} = \frac{8\times10\times5}{10}W = 40W$$

66. (c) By law of conservation of momentum,

$Mu = MV + mv$ ....(i)

Also $e = \dfrac{|v_1 - v_2|}{|u_1 - u_2|} \Rightarrow Mu = Mv - MV$ ....(ii)

From (i) and (ii), $2Mu = (M+m)v$

$$\Rightarrow v = \frac{2uM}{M+m} \Rightarrow v = \frac{2u}{1+\frac{m}{M}}$$

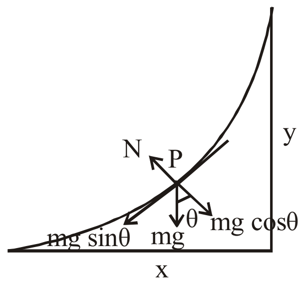
67. (d) Centripetal force $= \dfrac{mv^2}{R}$

$$= \left(\frac{1}{2}mv^2\right)\frac{2}{R} = \frac{2K}{R} = \frac{2aS}{R}$$

68. (a) As the block is at rest at P.

$mg\sin\theta = \mu mg\cos\theta$

$$\mu = \tan\theta = \text{slope} = \frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2}{2L}\right) = \frac{x}{L}$$



$$\therefore \frac{x}{L} = 0.8$$

$\Rightarrow x = 0.8\times 10 = 8\,m$

69. (d) Work done by centripetal force is zero.

70. (a) $F = -\dfrac{dU}{dx}$

71. (c) The change in momentum in the ball after the collision with surface is $m(0-v) = -mv$

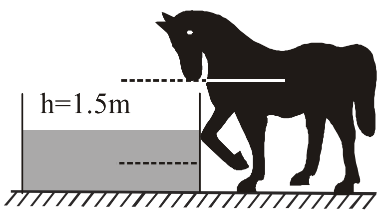
Since n balls impinge elastically each second on the surface, then rate of change of momentum of ball per second is

mvn (consider magnitude only)

Now According to Newton's second law

rate of change of momentum per second of ball = force experienced by surface.

72. (d)



The mass of water is $m = 1\times10^3$ kg

$\therefore$ The increase in potential energy of water is

$= mgh = (1\times10^3)(10)(1.5) = 15$ kJ

73. (d)

74. (c) $U = (1/2)Mv^2$

75. (b) P. E + K.E = constant, mass being constant
$gh + v^2/2$ = constant

76. (c) $K = p^2/2m = \mu mgx$

Hence, $x = p^2/2m^2\mu g$.

77. (c) $\vec{F} = \frac{d\vec{p}}{dt} = -\hat{i}2\sin t + \hat{j}2\cot t$

Hence $\vec{F}.\vec{p} = 0$, hence angle between $\vec{F}$ and $\vec{p}$ is 90°

78. (c) Here $\hat{i}mv + \hat{j}mv = 2m\vec{V}$

That is $\vec{V} = \frac{v}{2}(\hat{i} + \hat{j})$

Hence $V = \frac{v}{2} \times \sqrt{2} = \frac{v}{\sqrt{2}}$. Hence v = 5 ms^{-1}

79. (c) Apply conservation of momentum,
$m_1v_1 = (m_1 + m_2)v$

$$v = \frac{m_1 v_1}{(m_1 + m_2)}$$

Here v_1 = 36 km/hr = 10 m/s,
m_1 = 2 kg, m_2 = 3 kg

$$v = \frac{10 \times 2}{5} = 4 \text{ m/s}$$

$$\text{K.E. (initial)} = \frac{1}{2} \times 2 \times (10)^2 = 100 \text{ J}$$

$$\text{K.E. (Final)} = \frac{1}{2} \times (3+2) \times (4)^2 = 40 \text{ J}$$

Loss in K.E. = 100 – 40 = 60 J
Alternatively use the formula

$$-\Delta E_k = \frac{1}{2}\frac{m_1m_2}{(m_1 + m_2)}(u_1 - u_2)^2$$

80. (a) $x = 3t - 4t^2 + t^3$ $\quad \frac{dx}{dt} = 3 - 8t + 3t^2$

$$\text{Acceleration} = \frac{d^2x}{dt^2} = -8 + 6t$$

Acceleration after 4 sec = –8 + 6 × 4 = 16
Displacement in 4 sec = 3 ×4 – 4 × 4^2 + 4^3 = 12 m
∴ Work = Force × displacement
= Mass × acc. × disp. = 3 × 10^{-3} × 16 × 12 = 576 mJ

81. (b) According to principle of conservation of energy
Potential energy = kinetic energy

$$\Rightarrow mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}$$

If h_1 and h_2 are initial and final heights, then

$$\Rightarrow v_1 = \sqrt{2gh_1}, v_2 = \sqrt{2gh_2}$$

Loss in velocity, $\Delta v = v_1 - v_2 = \sqrt{2gh_1} - \sqrt{2gh_2}$

∴ fractional loss in velocity

$$= \frac{\Delta v}{v_1} = \frac{\sqrt{2gh_1} - \sqrt{2gh_2}}{\sqrt{2gh_1}} = 1 - \sqrt{\frac{h_2}{h_1}}$$

$$= 1 - \sqrt{\frac{1.8}{5}} = 1 - \sqrt{0.36} = 1 - 0.6 = 0.4 = \frac{2}{5}$$

82. (c) Change in momentum along the wall
= mv cos60° – mv cos 60° = 0
Change in momentum perpendicular to the wall
= mv sin60° – (– mv sin60°) = 2mv sin60°

$$\therefore \text{Applied force} = \frac{\text{Change in momentum}}{\text{Time}}$$

$$= \frac{2\,mv\sin 60^\circ}{0.20} = \frac{2 \times 3 \times 10 \times \sqrt{3}}{2 \times 0.20}$$

$$= 50 \times 3\sqrt{3} = 150\sqrt{3} \text{ newton}$$

83. (b) Speed of bomb after 5 second,
v = u – gt = 100 –10×5 = 50m/s
Momentum of 400 g fragment

$$= \frac{400}{1000} \times (-25) \text{ (downward)}$$

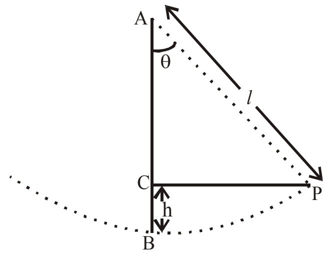
$$\text{Momentum of 600g fragment} = \frac{600}{1000}v$$

Momentum of bomb just before explosion
= 1 × 50 = 50
From conservation of momentum
Total momentum just before collision = Total momentum just after collision

$$\Rightarrow 50 = -\frac{400}{1000} \times 25 + \frac{600}{1000}v$$

⇒ v = 100 m/s (upward)

84. (d) If l is length of pendulum and θ be angular amplitude then height.



h = AB – AC $= l - l\cos\theta = l(1 - \cos\theta)$

At extreme position, potential energy is maximum and kinetic energy is zero; At mean (equilibrium) position potential energy is zero and kinetic energy is maximum, so from principle of conservation of energy.

(KE + PE) at P = (KE + PE) at B

$$0 + mgh = \frac{1}{2}mv^2 + 0$$

$$\Rightarrow v = \sqrt{2gh} = \sqrt{2g\ell(1 - \cos\theta)}$$

85. (b) From law of conservation of momentum, before collision and after collision linear momentum (p) will be same.
or initial momentum = final momentum.

$$E = \frac{p^2}{2m}$$

According to question,

$$\frac{E_1}{E_2} = \frac{p_1^2}{2m_1} \times \frac{2m_2}{p_2^2} \Rightarrow \frac{E_1}{E_2} = \frac{m_2}{m_1} \quad [p_1 = p_2]$$

86. (b) $\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \Rightarrow mv^2 = kx^2$ or $0.5 \times (1.5)^2 = 50 \times x^2$

$\therefore x = 0.15\,m$

87. (a) Clearly $v_1 = 2\ ms^{-1}, v_2 = 0$
$m_1 = m$ (say), $m_2 = 2m$
$v_1' = ?, v_2' = ?$

$$e = \frac{v_1' - v_2'}{v_2 - v_1} \quad ....(i)$$

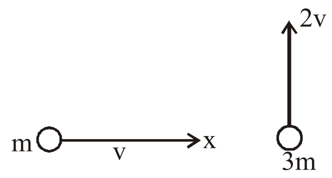
By conservation of momentum,
$2m = mv_1' + 2mv_2'$ ... (ii)

From (i), $0.5 = \frac{v_2' - v_1'}{2}$

$\therefore \quad v_2' = 1 + v_1'$
From (ii), $2 = v_1' + 2 + 2v_1'$
$\Rightarrow \quad v_1 = 0$ and $v_2 = 1\ ms^{-1}$

88. (b) Power exerted by a force is given by
P = F.v
When the body is just above the earth's surface, its velocity is greatest. At this instant, gravitational force is also maximum. Hence, the power exerted by the gravitational force is greatest at the instant just before the body hits the earth.

89. (a) As the two masses stick together after collision, hence it is inelastic collision. Therefore, only momentum is conserved.



$\therefore mv\hat{i} + 3m(2v)\hat{j} = (4m)\vec{v}$

$$\vec{v} = \frac{v}{4}\hat{i} + \frac{6}{4}v\hat{j}$$

$$= \frac{v}{4}\hat{i} + \frac{3}{2}v\hat{j}$$

90. (b) For equilibrium

$$\frac{dU}{dr} = 0 \Rightarrow \frac{-2A}{r^3} + \frac{B}{r^2} = 0$$

$$r = \frac{2A}{B}$$

for stable equilibrium

$\frac{d^2U}{dr^2}$ should be positive for the value of r.

here $\frac{d^2U}{dr^2} = \frac{6A}{r^4} - \frac{2B}{r^3}$ is +ve value for

$r = \frac{2A}{B}$ So

91. (d) Given : $\vec{F} = 3\hat{i} + \hat{j}$

$$\vec{r_1} = (2\hat{i} + \hat{k}),\ \vec{r_2} = (4\hat{i} + 3\hat{j} - \hat{k})$$

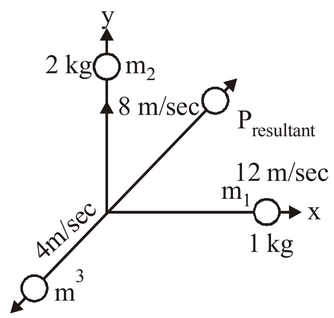
$$\vec{r} = \vec{r_2} - \vec{r_1} = (4\hat{i} + 3\hat{j} - \hat{k}) - (2\hat{i} + \hat{k})$$

or $\vec{r} = 2\hat{i} + 3\hat{j} - 2\hat{k}$

So work done by the given force $w = \vec{f}.\vec{r}$

$$= (3\hat{i} + \hat{j}).(2\hat{i} + 3\hat{j} - 2\hat{k}) = 6 + 3 = 9J$$

92. (a)



$$P_{resultant} = \sqrt{12^2 + 16^2}$$

$$= \sqrt{144 + 256} = 20$$

$m_3v_3 = 20$ (momentum of third part)

or, $m_3 = \frac{20}{4} = 5\,kg$

93. (c) $p = \sqrt{2mK}$

$$p' = \sqrt{2m[K + 3K]} = 2p$$

$$\frac{\Delta p \times 100}{p} = \frac{2p - p}{p} \times 100 = 100\%$$

94. (c) Let m and v be the mass and the velocity of the body. Then initial K.E.,

$$K_i = \frac{1}{2}mv^2$$

Now, mass $= \frac{m}{2}$

velocity $= 2v$

$\therefore$ Final K.E. $= \frac{1}{2}\left(\frac{m}{2}\right)(2v)^2$

$= \frac{1}{2}(2mv^2)$

$= 2\left(\frac{1}{2}mv^2\right)$

$= 2K_i$

95. (a) Power of engine $=$ Force $\times$ velocity $= Fv$

Here, mass of engine $= \frac{10^7}{10}$ kg $= 10^6$ kg

F = frictional force

= 0.5 kgf per quintal

= (0.5 × 10) N per quintal

= 5 N per quintal

$= \left(\frac{5\times10^6}{100}\right)N$

$= (5 \times 10^4)\,N$

and $v = 36$ km h^{-1}

$= \frac{36\times1000}{60\times60}kmh^{-1} = 10\text{ ms}^{-1}$

$\therefore$ Power of engine $= (5 \times 10^4 \times 10)$W

$= 5 \times 10^5$ W

$= 500$ kW

96. (b) 97. (b)

98. (a) When frictional force is opposite to velocity, kinetic energy will decrease.

99. (c) 100. (d)

EXERCISE - 3

Exemplar Questions

1. **(b)** When electron and proton are moving under influence of their mutual forces, then according to the flemings left hand rule, the direction of force acting on a charge particle is perpendicular to the direction of motion.

In magnetic field, work-done $= F.\,s.\cos\theta$

$= F\,.\,s.\cos 90° = 0.$

So magnetic forces do not work on moving charge particle.

2. **(c)** Forces between two protons is same as that of between proton and a positron.

As positron is much lighter than proton, it moves away through much larger distance compared to proton.

Work done = Force × Distance

As forces are same in case of proton and positron but distance moved by positron is larger, hence, work done on positron will be more than proton.

3. **(d)** When the man squatting on the ground he is tilted somewhat, hence he also has to apply frictional force besides his weight.

R (reactional force) = friction force $(f) + mg$

i.e. $R > mg$

When the man does not squat and gets straight up in that case friction $(f) \approx 0$

R (Reactional force) $\approx mg$

Hence, the reaction force (R) is larger when squatting and become equal to mg when no squatting.

4. **(c)** According to the question, work done by the frictional force on the cycle is :

$= 200 \times 10 = -2000$ J

As the road is not moving, hence work done by the cycle on the road is zero.

5. **(c)** As the body is falling freely under gravity and no external force act on body in vaccum so law of conservation, the potential energy decreases and kinetic energy increases because total mechanical energy (PE + KE) of the body and earth system will be remain constant.

6. **(c)** According to the question, consider the two bodies as system, the total external force on the system will be zero.

Hence, in an inelastic collision KE does not conserved but total linear momentum of the system remain conserved.

7. **(c)** As the (inclined surface) are frictionless, hence, mechanical energy will be conserved. As both the tracks having common height, h (and no external force acts on system).

KE & PE of stone I at top = KE + PE at bottom of I.

From conservation of mechanical energy,

$$0+\frac{1}{2}mv_1^2 = mgh+0$$

$\Rightarrow v_1 = \sqrt{2gh}$ similarly $v_2 = \sqrt{2gh}$

Hence, speed is same for both stones.

For stone I, acceleration along inclined plane $a_1 = g\sin\theta_1$

Similarly, for stone II $a_2 = g\sin\theta_2$

$\sin\theta_1 < \sin\theta_2$ Thus, $\theta_2 > \theta_1$ hence $a_2 > a_1$.

a_2 is greater than a_1 and both length for track II is also less hence, stone II reaches earlier than stone I.

8. **(b)** Total Mechanical energy is E = PE + KE at any instant. When particle is at $x = x_m$ i.e., at extreme position, partical returns back and its velocity become zero for an instant. Hence, at $x = x_m$; $x = 0$, K.E. = 0.

From Eq. (i),

$E = \text{PE} + 0 = \text{PE} = V(x_m) = \frac{1}{2}kx_m^2$

but at mean position at origin $V(x_m) = 0$.

9. **(b)** If two bodies of equal masses collides elastically, their velocities are interchanged.

When ball 1 collides with ball-2, then velocity of ball-1, v_1 becomes zero and velocity of ball-2, v_2 becomes v, i.e., similarly then its own all momentum is mV.

So, $v_1 = 0 \Rightarrow v_2 = v,\ P_1 = 0,\ P_2 = mV$

Now ball 2 collides to ball 3 and its transfer it's momentum is mV to ball 3 and itself comes in rest.

So, $v_2 = 0 \Rightarrow v_3 = v,\ P_2 = 0,\ P_3 = mV$

So, ball 1 and ball 2, become in rest and ball 3 move with velocity v in forward direction.

10. (b) As we know that,

$$\text{W.D.} = \int_{x_1}^{x_2} \vec{F}\cdot\overrightarrow{dx} = \int_{x_1}^{x_2} m\vec{a}_0\cdot\overrightarrow{dx}$$

As given that,
$m = 0.5$ kg, $a = 5\ \text{m}^{-1/2}\ \text{s}^{-1}$,
work done $(W) = ?$

$v = ax^{3/2}$
We also know that,
Acceleration,

$$a_0 = \frac{dv}{dt} = v\cdot\frac{dv}{dx} = ax^{3/2}\frac{d}{dx}(ax^{3/2})$$

$$= ax^{3/2}\times a\times\frac{3}{2}\times x^{1/2} = \frac{3}{2}a^2x^2$$

Now, Force $= ma_0 = m\frac{3}{2}a^2x^2$

From (i),

$$\text{Work done} = \int_{x=0}^{x=2} Fdx$$

$$= \int_0^2\left[\frac{3}{2}ma^2x^2\right]dx$$

$$= \frac{3}{2}ma^2\times\left(\frac{x^3}{3}\right)_0^2$$

$$= \frac{1}{2}ma^2\times 8$$

$$= \frac{1}{2}\times(0.5)\times(25)\times 8 = 50\ \text{J}$$

11. (b) As given that power = constant
As we know that power (P)

$$P = \frac{dW}{dt} = \frac{\vec{F}\cdot\overrightarrow{dx}}{dt} = \frac{F\ dx}{dt}$$

As the body is moving unidirectionally.
Hence, $F\cdot dx = Fdx\cos 0° = Fdx$

$$P = \frac{Fdx}{dt} = \text{constant}\quad (\because P = \text{ constant by question})$$

Now, by dimensional formula
$F\cdot v = 0$
$[F]\ [v]$ = constant
$[\text{MLT}^{-2}]\ [\text{LT}^{-1}]$ = constant
$[\text{ML}^2\text{T}^{-3}]$ = constant

$$\text{L}^2 = \frac{\text{T}^3}{\text{M}}$$

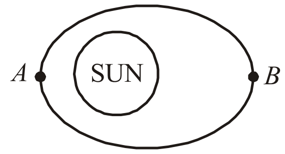
(As mass of body constant)

$$\text{L}^2 \propto \text{T}^3 \Rightarrow \text{L}\propto\text{T}^{3/2} \Rightarrow \text{Displacement } (d)\propto t^{3/2}$$

Verifies the graph (b).

12. (d) The speed of earth around the sun can never be zero or negative, so the kinetic energy of earth cannot be zero and negative.

So, option (b) and (c) represents wrongly the variation in kinetic energy of earth.



When the earth is closest to the sun, speed of the earth is maximum, hence, KE is maximum. When the earth is farthest from the sun speed is minimum hence, KE is minimum.

So, K.E. of earth increases (B to A) and then decrease, variation is correctly represented by option (d).

13. (c) When a pendulum oscillates in air, due to air resistance the force of friction acts between bob of pendulum and air, so it will lose energy continuously in overcoming. Therefore, total mechanical energy (KE + PE) of the pendulum decreases continuously with time and finally becomes zero.

Sum of KE and PE can never be negative. So, option (a) and (d) are incorrect. Hence option (c) is verifies.

14. (a) As given that, mass $(m) = 5$ kg,
$n = 300$ revolution
Radius $(R) = 1$ m
$t = 60$ sec

$$\omega = \left(\frac{2\pi n}{t}\right) = (300\times 2\times\pi)\ \text{rad}/60\text{s}$$

$$= \frac{600\times\pi}{60}\ \text{rad/s} = 10\ \pi\ \text{rad/s}$$

linear speed $(v) = \omega R = (10\pi\times 1)$
$v = 10\pi$ m/s

$$\text{KE} = \frac{1}{2}mv^2$$

$$= \frac{1}{2}\times 5\times(10\pi)^2$$

$$= 100\pi^2\times 5\times\frac{1}{2}$$

$$= 250\pi^2\text{J}$$

So, verifies the option (a).

15. (b) P.E. is maximum when drop start falling at $t = 0$ as it fall is P.E. decrease gradually to zero. So, it rejects the graph (a), (c) and (d).

K.E. at $t = 0$ is zero as drop falls with zero velocity, its velocity increases (gradually), hence, first KE also increases. After sometime speed (velocity) is constant this is called terminal velocity, so, KE also become constant. It happens when it falls $\left(\frac{3}{4}\right)$ height or remains at $\left(\frac{4}{4}\right)$ from ground, then PE decreases continuously as the drop is falling continuously.

The variation in PE and KE is best represented by (b).

16. (d) As given that, $h = 1.5$ m, $v = 1$ m/s, $m = 10$ kg, $g = 10\ \text{ms}^{-2}$
By the law of conservation of mechanical energy as no force acts on shotput after thrown.
$(PE)_i + (KE)_i = (PE)_f + (KE)_f$

$$mgh_i + \frac{1}{2}mv_i^2 = 0 + (KE)_f$$

$$(KE)_f = mgh_i + \frac{1}{2}mv_i^2$$

Total energy when it reaches ground, so

$$(KE)_f = 10 \times 10 \times 1.5 + \frac{1}{2} \times 10 \times (1)^2$$

$E = 150 + 5 = 155$ J.

17. (b) First velocity of the iron sphere
$V = \sqrt{2gh}$ after sometime its velocity becomes constant, called terminal velocity. Hence, according first KE increases and then becomes constant due to resistance of sphere and water which is represented by (b).

18. (c) As given that,

$$m = 150\ \text{g} = \frac{150}{1000}\ \text{kg} = 0.15\ \text{kg}$$

Δt = time of contact = 0.001 s

$$u = 126\ \text{km/h} = \frac{126 \times 1000}{60 \times 60}\ \text{m/s}$$

$$= 126 \times \frac{5}{18} = 35\ \text{m/s}$$

$$v = -126\ \text{km/h} = -126 \times \frac{5}{18} = -35\ \text{m/s}$$

So, final velocity is acc. to initial force applied by batsman.
So, change in momentum of the ball

$$\Delta p = m(v - u) = \frac{3}{20}(-35 - 35)\ \text{kg-m/s}$$

$$= \frac{3}{20}(-70) = -\frac{21}{2}$$

As we know that, force

$$F = \frac{\Delta p}{\Delta t} = \frac{-21/2}{0.001}\ \text{N} = -1.05 \times 10^4\ \text{N}$$

Hence negative sign shown that direction of force will be opposite to initial velocity which taken positive direction. Hence verify the option (c).

NEET/AIPMT (2013-2017) Questions

19. (d) Given : $\vec{F} = 3\hat{i} + \hat{j}$

$$\vec{r_1} = \left(2\hat{i} + \hat{k}\right),\ \vec{r_2} = \left(4\hat{i} + 3\hat{j} - \hat{k}\right)$$

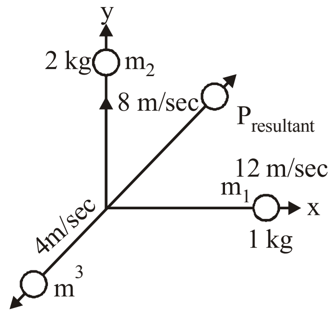
$$\vec{r} = \vec{r_2} - \vec{r_1} = \left(4\hat{i} + 3\hat{j} - \hat{k}\right) - \left(2\hat{i} + \hat{k}\right)$$

or $\vec{r} = 2\hat{i} + 3\hat{j} - 2\hat{k}$

So work done by the given force $w = \vec{f}.\vec{r}$

$$= \left(3\hat{i} + \hat{j}\right).\left(2\hat{i} + 3\hat{j} - 2\hat{k}\right) = 6 + 3 = 9\text{J}$$

20. (a)



$$P_{resultant} = \sqrt{12^2 + 16^2}$$

$$= \sqrt{144 + 256} = 20$$

$m_3v_3 = 20$ (momentum of third part)

or, $m_3 = \frac{20}{4} = 5$ kg

21. (c) According to law of conservation of momentum
$MV + mnv = 0$

$$\Rightarrow V = \frac{-mNv}{M} = \frac{-0.01\ \text{kg} \times 10 \times 800\ \text{m/s}}{100}$$

$\Rightarrow -0.8$ m/s
According to work energy theorem,
Average work done = Change in average kinetic energy

i.e., $F_{av} \times S_{av} = \frac{1}{2}mV_{rms}^2$

$$\Rightarrow \frac{F_{av}V_{max}t}{2} = \frac{1}{2}m\frac{V_{rms}^2}{2}$$

$\Rightarrow F_{av} = 8$ N

22. (d) As the particle is moving in a potential energy region.
$\therefore$ Kinetic energy ≥ 0
And, total energy E = K.E. + P.E.
$\Rightarrow U(x) \leq E$

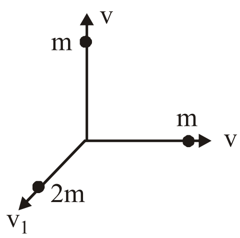
23. (a) $\because$ Power $P = \frac{w}{t}$

$$\Rightarrow \frac{P_1}{P_2} = \frac{t_2}{t_1} = \frac{30s}{1\ \text{minute}} = \frac{30s}{60s} = \frac{1}{2}$$

(t_1 = 1 minute; t_2 = 30 second given)

24. (b) By conservation of linear momentum

$$2mv_1 = \sqrt{2}mv \Rightarrow v_1 = \frac{v}{\sqrt{2}}$$



As two masses of each of mass m move perpendicular to each other.
Total KE generated

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(2m)v_1^2$$

$$= mv^2 + \frac{mv^2}{2} = \frac{3}{2}mv^2$$

25. (d) As we know power $P = \frac{dw}{dt}$

$$\Rightarrow \quad w = Pt = \frac{1}{2}mV^2$$

So, $v = \sqrt{\frac{2Pt}{m}}$

Hence, acceleration $a = \frac{dV}{dt} = \sqrt{\frac{2P}{m}} \cdot \frac{1}{2\sqrt{t}}$

Therefore, force on the particle at time 't'

$$= ma = \sqrt{\frac{2Km^2}{m}} \cdot \frac{1}{2\sqrt{t}} = \sqrt{\frac{Km}{2t}} = \sqrt{\frac{mK}{2}}\, t^{-1/2}$$

26. (b) As we know work done in stretching spring

$$w = \frac{1}{2}kx^2$$

where k = spring constant
x = extension

Case (a) If extension (x) is same,

$$W = \frac{1}{2}K\,x^2$$

So, $W_P > W_Q (\because K_P > K_Q)$

Case (b) If spring force (F) is same $W = \frac{F^2}{2K}$

So, $W_Q > W_P$

27. (d) From, $F = ma$

$$a = \frac{F}{m} = \frac{0.1x}{10} = 0.01x = V\frac{dV}{dx}$$

So, $\int_{v_1}^{v_2} vdV = \int_{20}^{30} \frac{x}{100}dx$

$$-\frac{V^2}{2}\Bigg|_{V_1}^{V_2} = \frac{x^2}{200}\Bigg|_{20}^{30} = \frac{30\times30}{200} - \frac{20\times20}{200}$$

$$= 4.5 - 2 = 2.5$$

$$\frac{1}{2}m\left(V_2^2 - V_1^2\right) = 10 \times 2.5\text{ J} = -25\text{J}$$

Final K.E.

$$= \frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 - 25 = \frac{1}{2}\times10\times10\times10 - 25$$

$$= 500 - 25 = 475\text{ J}$$

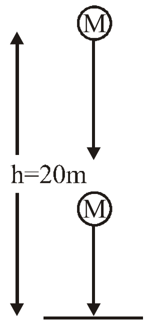
28. (d) Power $\vec{F}.\vec{V} = PA\vec{V} = \rho ghAV$

$$\left[\because P = \frac{F}{A} \text{ and } P = \rho gh\right]$$

$$= 13.6 \times 10^3 \times 10 \times 150 \times 10^{-3} \times 0.5 \times 10^{-3}/60$$

$$= \frac{102}{60} = 1.70\text{ watt}$$

29. (a) When ball collides with the ground it loses its 50% of energy



$$\therefore \frac{KE_f}{KE_i} = \frac{1}{2} \Rightarrow \frac{\frac{1}{2}mV_f^2}{\frac{1}{2}mV_i^2} = \frac{1}{2}$$

or $\frac{V_f}{V_i} = \frac{1}{\sqrt{2}}$

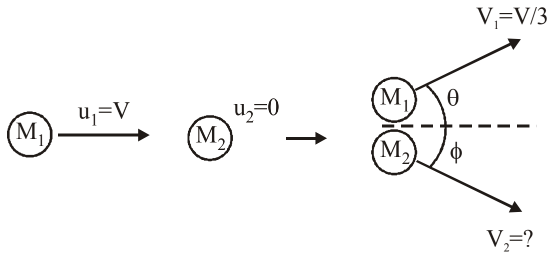
or, $\frac{\sqrt{2gh}}{\sqrt{V_0^2 + 2gh}} = \frac{1}{\sqrt{2}}$

or, $4gh = V_0^2 + 2gh$

$\therefore V_0 = 20\text{ms}^{-1}$

30. (d) Here, $M_1 = M_2$ and $u_2 = 0$

$u_1 = V, \quad V_1 = \frac{V}{3}; \quad V_2 = ?$



From figure, along x-axis,

$M_1u_1 + M_2u_2 = M_1V_1\cos\theta + M_2V_2\cos\phi$ ...(*i*)

Along y-axis

$0 = M_1V_1\sin\theta - M_2V_s\sin\phi$ ...(*ii*)

By law of conservation of kinetic energy

$$\frac{1}{2}M_1u_1^2 + \frac{1}{2}M_2u_2^2 = \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 \quad ...(iii)$$

Putting $M_1 = M_2$ and $u_2 = 0$ in equation (*i*), (*ii*) and (*iii*) we get

$$\theta + \phi = \frac{\pi}{2} = 90^\circ$$

and $u_1^2 = V_1^2 + V_2^2$

$$V^2 = \left(\frac{V}{3}\right)^2 + V_2^2 \quad \left[\because u_1 = V \text{ and } V_1 = \frac{V}{3}\right]$$

or, $V^2 - \left(\frac{V}{3}\right)^2 = V_2^2$

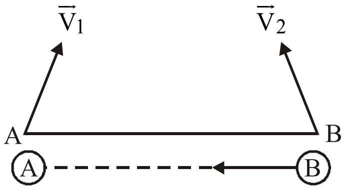
$$V^2 - \frac{V^2}{9} = V_2^2$$

or $V_2^2 = \frac{8}{9}V^2 \Rightarrow V_2 = \frac{2\sqrt{2}}{3}V$

31. (d) For collision $\vec{V}_{B/A}$ should be along

$\overline{B \to A}\left(\vec{r}_{A/B}\right)$

So, $\frac{\vec{V}_2 - \vec{V}_1}{|V_2 - V_1|} = \frac{\vec{r}_1 - \vec{r}_2}{|r_1 - r_2|}$



32. (d) Given force $\vec{F} = 2t\hat{i} + 3t^2\hat{j}$

According to Newton's second law of motion,

$$m\frac{d\vec{v}}{dt} = 2t\hat{i} + 3t^2\hat{j} \qquad (m = 1\text{ kg})$$

$$\Rightarrow \int_0^{\vec{v}} d\vec{v} = \int_0^t \left(2t\hat{i} + 3t^2\hat{j}\right)dt$$

$$\Rightarrow \vec{v} = t^2\hat{i} + t^3\hat{j}$$

Power $P = \vec{F}\cdot\vec{v} = (2t\,\hat{i} + 3t^2\hat{j})\cdot(t^2\hat{i} + t^3\hat{j})$

$= (2t^3 + 3t^5)$W

33. (a) Given: Mass of particle, $M = 10g = \frac{10}{1000}$ kg

radius of circle $R = 6.4$ cm

Kinetic energy E of particle $= 8 \times 10^{-4}$J

acceleration $a_t = ?$

$$\frac{1}{2}mv^2 = E \Rightarrow \frac{1}{2}\left(\frac{10}{1000}\right)v^2 = 8 \times 10^{-4}$$

$\Rightarrow v^2 = 16 \times 10^{-2}$

$\Rightarrow v = 4 \times 10^{-1} = 0.4$ m/s

Now, using

$v^2 = u^2 + 2a_t s (s = 4\pi R)$

$$(0.4)^2 = 0^2 + 2a_t\left(4\times\frac{22}{7}\times\frac{6.4}{100}\right)$$

$$\Rightarrow a_t = (0.4)^2 \times \frac{7\times100}{8\times22\times6.4} = 0.1\text{ m/s}^2$$

34. (c) From work-energy theorem,

$W_g + W_a = \Delta K.E$

or, $mgh + W_a = \frac{1}{2}mv^2 - 0$

$$10^{-3}\times10\times10^3 + W_a = \frac{1}{2}\times10^{-3}\times(50)^2$$

$\Rightarrow W_a = -8.75$ J

which is the work done due to air resistance

Work done due to gravity = mgh

$= 10^{-3} \times 10 \times 10^3 = 10$ J