Chapter 2

Compound Stresses, Shear Force and Bending Moments

CHAPTER HIGHLIGHTS

- *Compound Stresses*
- *Stresses on an Inclined or Oblique Plane*
- *Direct Stresses Acting in Two Mutually Perpendicular Planes*
- *Mohr's Circle*
- *Construction of Mohr's Circle for Complex Stresses*
- *Measurement of Stresses in a Plane Making an Angle θ with the Plane at Which p_x Acts*
- *Shear Force and Bending Moment in Beams and Cantilevers*
- *Shear Force and Bending Moment*
- *Relation Between Load Intensity, Shear Force and Bending Moment*
- *Shear Force and Bending Moment Diagrams*
- *Cantilever Subjected to Uniformly Distributed Load*
- *Cantilever Subjected to Uniformly Varying Load*
- *Simply Supported Beam with Concentrated Load*
- *Simply Supported Beam with Uniformly Distributed Load*
- *Simply Supported Beam with Uniformly Varying Load*

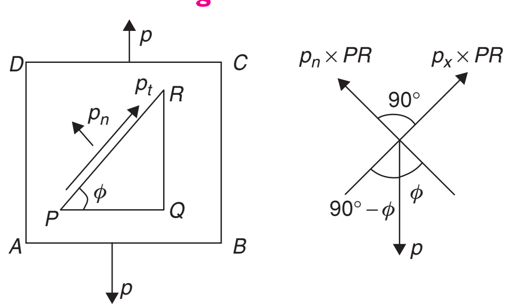
Compound Stresses

A stressed material may be subjected to stresses of complex nature. If we consider a plane it may have direct and shear stresses. Stress can be analyzed by analytical or graphical method.

Stresses on an Inclined or Oblique Plane

For analyzing stresses a general plane inclined at an angle to the known plane is considered.

Direct Stress Acting in One Direction Only



Consider a rectangular block *ABCD* subjected to the axial forces P. Consider a wedge *PQR* with angle ϕ inside the block.

Thickness of the block = t

Area of cross section perpendicular to the forces

$A = AB \times t$

$A = AB$ for unit thickness

Stress on $AB = p = \dfrac{P}{A}$.

For equilibrium of the wedge, forces resolved in any direction will be zero.

Resolving forces perpendicular to PR,

$$p_n\, PR = p \cdot PQ \cos\phi$$

$$p_n = p \cdot \frac{PQ}{PR} \cos\phi$$

$$p_n = p \cos^2\phi$$

Resolving force along PR,

$$p_t \cdot PR = p \cdot PQ \sin\phi$$

$$p_t = p \cdot \frac{PQ}{PR} \sin\phi$$

$$p_t = p \cdot \cos\phi \sin\phi = \frac{p}{2} \sin 2\phi$$

Normal stress will be maximum when $\cos 2\theta = 1$.

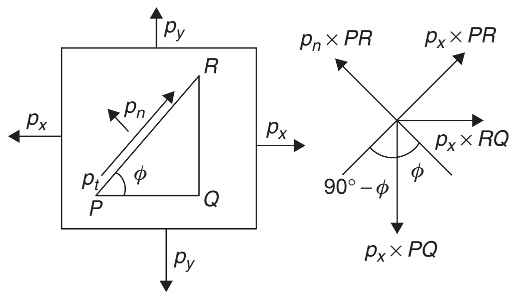
or $\cos\phi = \pm 1$

or $\phi = 0°$ or $180°$

Shear stress will be maximum when $\sin 2\phi = \pm 1$
or $2\phi = 90°$ or $270°$
or $\phi = 45°$ or $135°$
It can be seen that maximum shear stress is equal to half of maximum normal stress.

Resultant stress $= \sqrt{p_n^2 + p_t^2} = p\cos\phi$.

Direct Stress Acting in Two Mutually Perpedicular Planes



Considering equilibrium of the wedge PQR and resolving forces perpendicular to PR,

$$p_n\, PR = p_y\, PQ \cos\phi + p_x\, RQ \sin\phi$$
$$p_n = p_y \cos^2\phi + p_x \sin^2\phi$$
$$p_n = p_y \cos^2\phi + p_x \sin^2\phi$$
$$= p_y \frac{(1+\cos 2\phi)}{2} + p_x \frac{(1-\cos 2\phi)}{2}$$
$$= \frac{1}{2}[p_y + p_y \cos 2\phi + p_x - \cos 2\phi]$$
$$= \frac{1}{2}[(p_x + p_y) + (p_y - p_x)\cos 2\phi]$$
$$= \frac{1}{2}[(p_x + p_y) + (p_y - p_x)\cos 2\phi]$$

Resolving forces along PR,

$$p_t\, PR = p_y\, PQ \sin\phi - p_x\, RQ \cos\phi$$
$$p_t = p_y \cos\phi \sin\phi - p_x \sin\phi \cos\phi$$
$$= (p_y - p_x)(\sin\phi\cos\phi) = \frac{1}{2}(p_y - p_x)\sin 2\phi$$

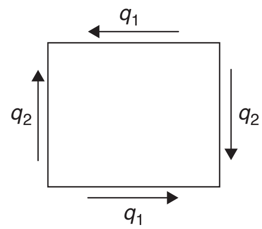
p_t is maximum when $2\phi = 90°$ or $\phi = 45°$

when $\phi = 45°$, $p_n = \frac{1}{2}(p_x + p_y)$

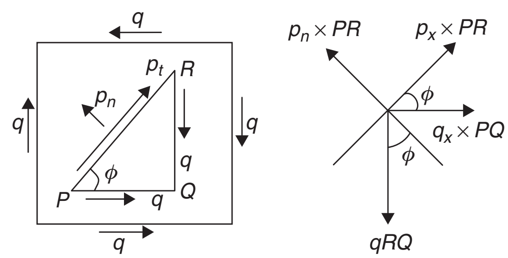
$$p_t = \frac{1}{2}(p_y - p_x)$$

Subjected to Pure Shear Only

In a square or rectangular block in equilibrium, shear stress in one plane is accompanied by an equal and opposite stress, called complimentary shear stress.



$q_1 = q_2 = q$



Resolving all forces perpendicular to PR,

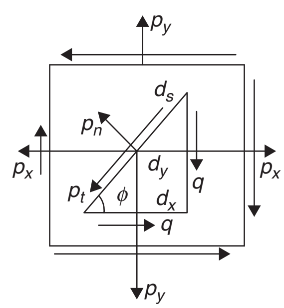
$$p_n\, PR = q\, PQ \sin\phi + q\, RQ \cos\phi$$
$$p_n = q \cos\phi \sin\phi + q \sin\phi \cos\phi$$
$$= 2q \sin\phi \cos\phi = q \sin 2\phi$$

Resolving all forces parallel to PR,

$$p_t\, PR = -q\, PQ \cos\phi + q\, RQ \sin\phi$$
$$p_t = -q \cos^2\phi + q \sin^2\phi$$
$$= q(\sin^2\phi - \cos^2\phi) = -q \cos 2\phi$$

When $\phi = 45°$,
$p_n = q$ and $p_t = 0$.

Subjected to Two-dimensional Stress System



Considering a wedge of sides ds, dy and dz and unit thickness at the plane inclined at angle ϕ for equilibrium, $\Sigma F_n = 0$ and $\Sigma F_t = 0$

$\Sigma F_n = p_n\, ds - p_x\, d_y \sin\phi - py\, dx \cos\phi - q\, dy \cos\phi - q\, dx \sin\phi = 0$

i.e., $p_n\, ds = p_x\, ds \sin^2\phi + p_y ds \cos^2\phi + q\, ds \sin\phi \cos\phi + q\, ds \cos\phi \sin\phi$

i.e., $p_n = p_x \sin^2\phi + p_y \cos^2\phi + 2q \sin\phi \cos\phi$

$$= p_x \frac{(1-\cos 2\phi)}{2} + p_y \frac{(1+\cos 2\phi)}{2} + q\sin 2\phi$$

i.e., $p_n = \frac{1}{2}(p_x + p_y) - \frac{1}{2}(p_x - p_y)\cos 2\phi + q \sin 2\phi$

$\Sigma F_t = p_t\, d_s - (px\, dy + q\, dx)\cos\phi + (p^y dx + qdy)\sin\phi = 0$

i.e., $p_t\, ds = p_x\, ds \sin\phi \cos\phi + q\, ds \cos^2\phi - p_y\, ds \sin\phi \cos\phi - q\, ds \sin^2\phi = 0$ $p_t = \sin\phi \cos\phi\,(p_x - p_y) + q(\cos^2\phi - \sin^2\phi)$

i.e., $p_t = \frac{1}{2}(p_x - p_y)\sin 2\phi + q\cos 2\phi$

To find the angle ϕ_p at which the maximum normal stress occurs, set $\left.\frac{d(p_n)}{d\phi}\right|_{\phi=\phi_p} = 0$

$$\left.\frac{d(p_n)}{d\phi}\right|_{\phi=\phi_p} = (p_x - p_y)\sin 2\phi_p + 2q\cos 2\phi_p = 0$$

or $(p_x - p_y)\sin 2\phi_p = -2q\cos 2\phi$

or $\tan 2\phi_p = \frac{-2q}{p_x - p_y}$

The planes defined by the angles ϕ_p are called *principal planes*. The normal stresses on these planes are called principal stresses.

It can be seen that it is a plane at which shear stress is equal to zero.

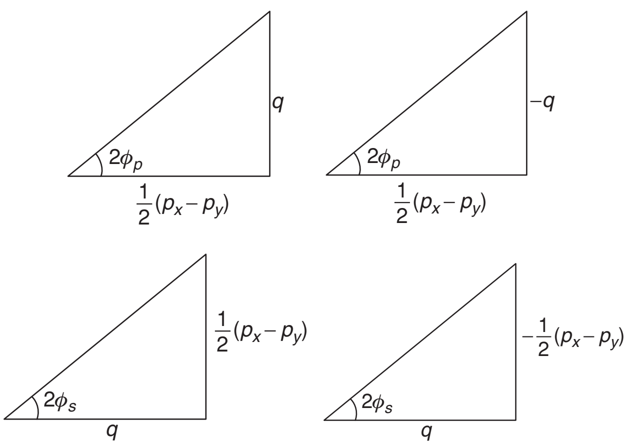
Maximum and minimum values of shear stress occur on planes defined by

$$\left.\frac{d(p_t)}{d\phi}\right|_{\phi=\phi_s} = 0$$

$$\left.\frac{d(p_t)}{d\phi}\right|_{\phi=\phi_s} = (p_x - p_y)\cos 2\phi_s - 2q\sin 2\phi_s = 0$$

$$\tan 2\phi_s = \frac{(p_x - p_y)}{2q} = \frac{\frac{1}{2}(p_x - p_y)}{q}$$

The stresses of maximum and minimum values may be graphically represented as follows:



It can be seen that angles $2\phi_p$ and $2\phi_s$ differ by 90°. So the planes defined by ϕ_p and ϕ_s differ by 45°.

That is the planes of maximum shear stress are oriented 45° from the planes of maximum principal stress .

Mohr's Circle

Mohr's circle is used for analysing stresses graphically.

Squaring and adding expressions for normal stress and shear stress leads to the equation of a circle. This principle is used in Mohr's circle.

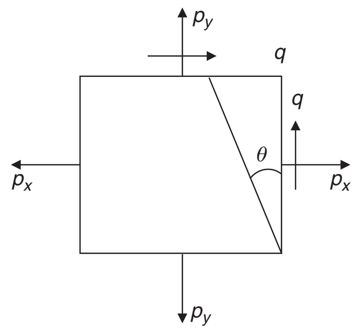
Construction of Mohr's Circle for Complex Stresses

It can be seen that radius of the Mohr's circle represents the maximum shear stress.

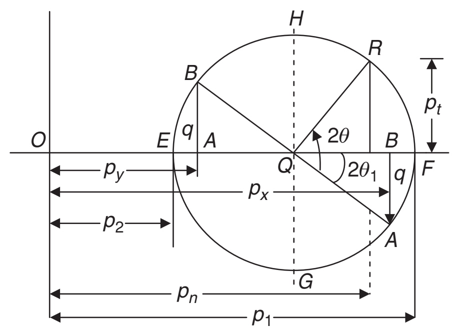
The following sign conventions should be observed while constructing the Mohr's circle:

(a) Tensile stress is to be treated as positive and compressive stress negative. Positive normal stresses are to be plotted to the right of the origin and negative normal stresses to the left of the origin.
(b) Shear stress producing clockwise moment in element is treated as positive and should be drawn above the x-axis.

Measurement of Stresses on a Plane Making an angle–with the Plane at which p_x Acts

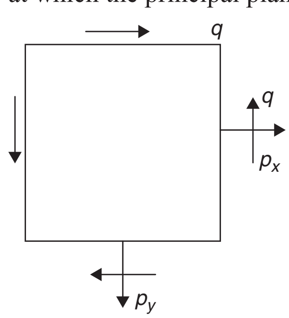


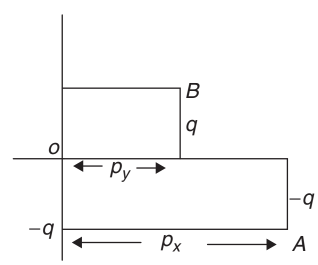
The plane makes angle θ in the anticlockwise direction from plane at which p_x acts. As per the sign convention, radial vector will be above x-axis in positive direction.



Line AB is the reference line representing p_x, p_y and q. Line QR is drawn at an angle twice θ (i.e., 2θ). Coordinates of R give the values of normal stress p_x and shear stress p_t on the plane.

In the diagram, $\angle AQF = 2\theta_1$ represents the position of the principal plane. θ_1 is the angle from the reference plane at which the principal plane is situated.



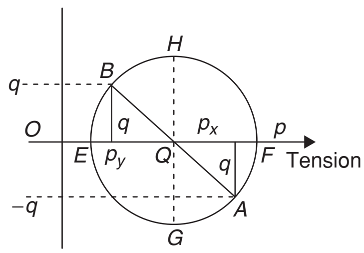


Mohr's circle is drawn on a set of axes representing normal stress (x-direction) and shear stress (y-direction).

Each set of a normal stress and shear stress can be represented by a point.

Points representing $(p_x, -q)$ and $(p_y, +q)$ are marked as A and B.

Line joining A and B passes through point Q on the horizontal axis. A circle is drawn through A and B with Q as centre. This is Mohr's circle.



$$OQ = \frac{p_x + p_y}{2}$$

Radius = $QB = QA$

$$= \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2}$$

Points E and F represent principal planes where the shear stress is zero.

$\angle AQF = 2\phi_p$ = Angle at which the principal plane exists

$\angle AQH = 2\phi_s$ = Angle at which the plane of maximum shear stress exists

Classification of Beams

A structural member on which forces act at right angles to its axis is called a beam. Beam can be classified depending upon the types of supports as follows:

Cantilever If one end of the beam is fixed and the other end is free, it is called a cantilever.

Simply supported beam When both ends of the beam are supported, it is called a simply supported beam.

Fixed beam When both ends are rigidly fixed, it is called a fixed beam.

Over hanging beam In over hanging beams, supports are not provided at the ends.

Continuous beam If more than two supports are provided, it is called a continuous beam.

Shear Force and Bending Moment in Beams and Cantilevers

Statically Determinate Beam

In statically determinate beams, the reaction at supports can be determined by applying the equation of static equilibrium. The values of reactions are not affected by the deformation of the beam.

Types of loading are

1. Point load or concentrated loads
2. Uniformly distributed loads
3. Uniformly varying loads

Shear Force and Bending Moment

Shear force is the force that is trying to shear off a section of the beam and is obtained by the algebraic sum of all the forces and reactions acting normal to the axis of the beam acting either to the left or right of the section.

Bending moment acting at a section of a beam is the moment that is trying to bend it and is obtained by the algebraic sum of all the moments and reactions about the section, either to the right or left of the section.

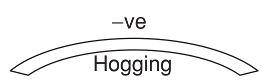
Shear force is treated as positive if it leads to move the left portion upward in relation to the right portion.

Bending moment is treated as positive if tries to sag the beam. The moment will be clockwise if the left portion of the beam is considered.

Sign Conventions

Positive bending moment produces concavity upwards

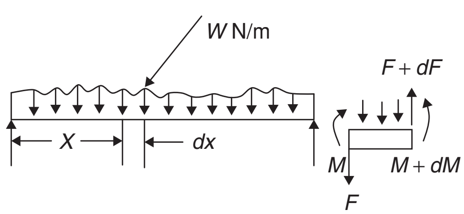
Negative bending moment produces convexity upwards.



Point of Contraflexure

Bending moment in a beam varies depending up on the loads. Bending moment at a point may be positive, negative or zero. The point at which bending moment changes its sign is called point of contraflexure. Bending moment is zero at this point. At point of contraflexure beam curvature is changed from sagging to hogging or vice versa.

Relation Between Load Intensity, Shear Force and Bending Moment



Considering an elemental length dx

The shear force F acts on the left side of the element and at the right side it is $F + dF$.

The bending moment M acts on the left side of the element and at the right side it is $M + dM$.

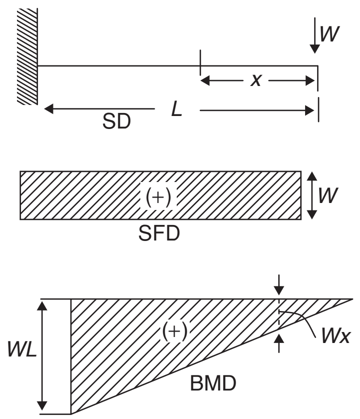
Since dx is very small applied load may be taken as uniform and equal to W N/m.

Taking moment about the right face and neglecting small quantity of higher order we get $\frac{dM}{dx} = -F$.

Shear Force and Bending Moment Diagrams

Cantilever subjected to central concentrated load

SD – Space Diagram
SFD – Shear Force Diagram
BMD – Bending Moment Diagram

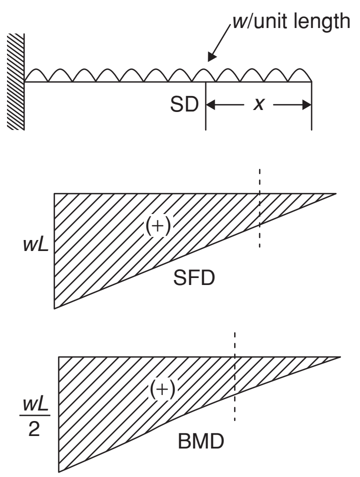


Shear force is constant throughout the beam. Bending moment varies linearly.

$F = W$

$M = Wx$

Cantilever subjected to uniformly distributed load



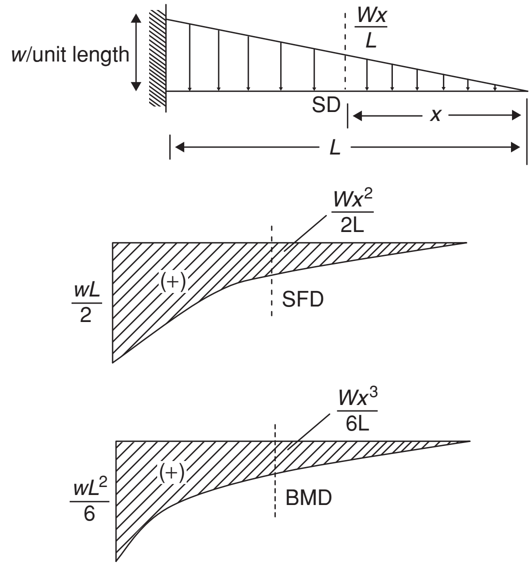
Shear force has got linear variation.

$F = Wx$

Bending moment varies parabolically

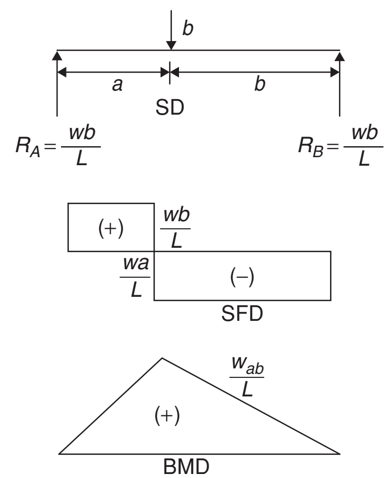
$$M = \frac{Wx^2}{2}$$

Cantilever subjected to uniformly varying load

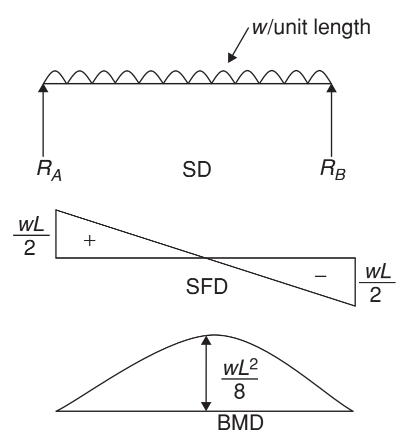


Shear force has a parabolic variation and bending moment has a cubic variation.

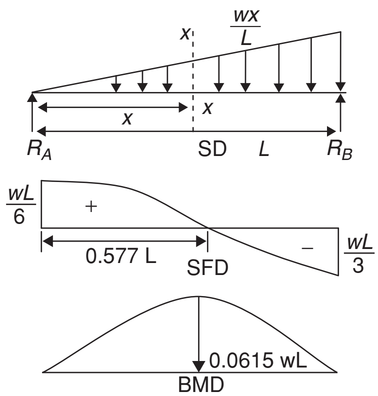
Simply supported beam with concentrated load



Simply supported beam with uniformly distributed load



Simply supported beam with uniformly varying load



Total load $= \dfrac{wL}{2}$

This acts at the centroid $\left(\dfrac{L}{3}\right)$

$$R_A \cdot L - \frac{wL}{2}\frac{L}{3} = 0$$

$$R_A = \frac{wL}{6}$$

$$R_B = \frac{wL}{2} - \frac{wL}{6} = \frac{wL}{3}$$

Total load on LHS of xx

$$\frac{wx}{L} \times \frac{x}{2} = \frac{wx^2}{2L}$$

$$F = R_A - \frac{wx^2}{2L} = \frac{wL}{6} - \frac{wx^2}{2L}$$

$$Alx = 0,\ F = \frac{wL}{6}$$

$$Alx = L,\ F = -\frac{wL}{3}$$

Moment at section xx

$$M = \frac{wLx}{6} - \frac{wx^2}{2L}\frac{x}{3}$$

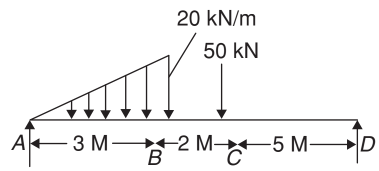
Maximum value of moment occurs at $x = \dfrac{L}{\sqrt{3}}$.

Some Important Points

1. Algebraic sum of all forces (including reactions) is zero.
2. Algebraic sum of all moments about any point in zero.
3. Moment at hinged joint is zero.
4. Moment is zero at the free end of a beam.
5. Shear force and bending moment are maximum at the fixed end of a cantilever.
6. Moment is zero at simply supported ends.

Solved Examples

Example 1: Determine the shear force and bending moment variation for the simply supported beam as shown in figure. Indicate values of salient points.

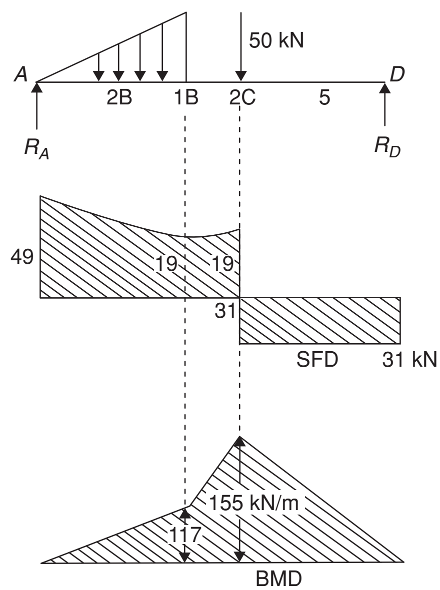


Solution: Shear force is taken as +ve if it tends to move the left portion upward.

If the moment on the left side is clockwise, it is treated as +ve.

Total value of the uniformly varying load on AB is

$$\frac{20 \times 3}{2} = 30 \text{ kN}$$

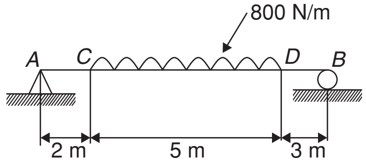


$R_A + R_D = 30 + 50 = 80$
Taking moments about A
$30 \times 2 + 50 \times 5 - R_D \times 10 = 0$

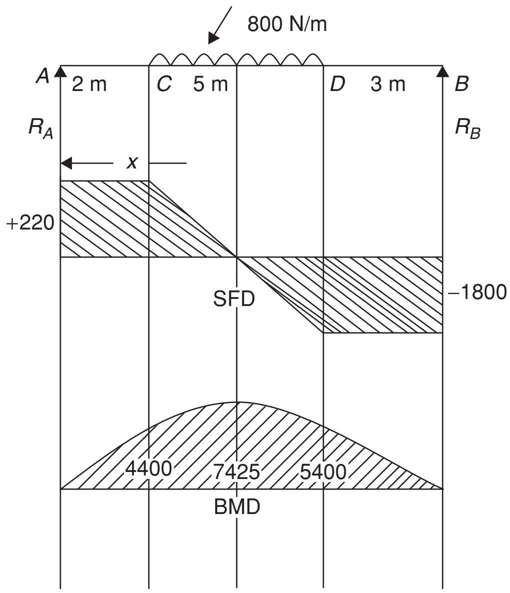
$$R_D = \frac{310}{10} = 31 \text{ kN}$$

$\therefore\quad R_A = 80 - 31 = 49$ kN
Shear force at A, $F_A = R_A = 49$ kN
Moment at A, $M_A = 0$ (as simply supported)
(Also $M_A = 30 \times 2 + 50 \times 5 - 31 \times 10 = 0$)
Shear force at B, $F_B = 49 - 30 = 19$ kN
$M_B = 49 \times 3 - 30 \times 1 = 117$ kNm
F_C (left) $49 - 30 = 19$ kN
$M_c = 49 \times 5 - 30 \times 3 = 155$ kNm
F_c (right) $= 19 - 50 = -31$ kN
$F_D = R_D = 31$ kN
$M_D = 0$

Example 2: Determine the shear force and bending moment variation for a simply supported beam as shown in the figure.



Solution:



Taking moment about A

$R_B \times 10 = (800 \times 5) \times 4.5 = 1800$ N

$R_A = (800 \times 5) - 1800 = 2200$ N

Shear force at $A = R_A = +2200$ N

Portion AC

Measuring x from A and taking all these forces to the left of section.

Shear force $F = +R_A = +2200$ N. (constant)

Bending moment $M = R_A x = 2200x$

$$M_A = 0$$
$$M_C = 2200 \times 2 = 4400 \text{ Nm}$$

Portion CD

Measuring x from A

$$F = +2200 - (x - 2) \times 800$$
$$= +2200 - 800x + 1600$$
$$= +3800 - 800x \text{ (Linear variation)}$$
$$F = 0$$

When $800x = 3800$

or $x = \dfrac{3800}{800} = 4.75$ M

$$M = R_A \times x - (x-2)800 \times \frac{(x-2)}{2}$$
$$= R_A x - (x-2)^2 \times 400$$

$F_D = +3800 - 800 \times 7 = -1800$ N

$M_D = 2200 \times 7 - (7 - 2)^2 \times 400 = 5400$ N

Maximum bending moment occurs when

$$F = 0$$

i.e., at $x = 4.75$m

So, $M_{max} = 2200 \times 4.75 - (4.75 - 2)^2\ 400$

$= 7425$ Nm

Portion DB

Taking x from B and considering the right hand side forces.

$F = -RB$

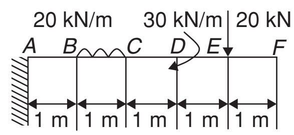
$= -1800$ N (constant)

$M = R_B \times x = 1800x$

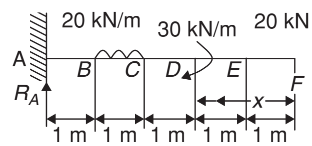
$M_B = 0$

$M_D = 1800 \times 3 = 5400$ Nm

Example 3: Draw shear force and bending moment variation for the cantilever beam loaded as shown in the figure.



Solution:



$$R_A = 20 \times 1 + 20 = 40 \text{ kN}$$

Measuring x from F

Portion FE

$F_E = 0$ up to E

$M_F = 0$ up to E

Portion ED

$F = 20$ constant

$M = [20\ (x \times 1)]$ linear

$F_E = 20$ kN

$M_E = [20\ (1 - 1)] = 0$

$M_P = -20\ (2 - 1) = -20$

Portion DC

$F = 20$ constant

$M = -[20\ (x - 1) + 30]$ linear

$F_D = 20$ kN

$M_D = [20\ (x^2 - 1) + 30]$

$= -[20 + 30]$

$= -50$ kNm

$F_c = 20$

$M_c = -[20\ (3 - 1) + 30]$

$= -50$ kNm

$F_c = 20$

$M_c = [20\ (3 - 1) + 30]$

$= (20 \times 2 + 30)$

$= (40 + 30) = -70$ kN/m

Portion CB

$F = 20 + (x - 3)\ 20$ linear

$$M = \left[20(x-1)+30+\frac{(x-3)^2\,20}{2}\right] \text{ parabolic}$$

$F_c = 20 + 0 = 20$ kN

$M_c = [40 + 70 + 0] = -70$ kNm

$F_B = 20 + 1 \times 20 = 40$

$$M_B = \left[20(4-1)+30+\frac{(4-3)^2\,20}{2}\right]$$

$= [60 + 30 + 10]$

$= -100$ kNm

Portion BA

$F = 20 + 20 = 40$ constant

$M = -[20\ (x - 1) + 30 + 20\ (x - 3.5)]$ linear

$F_B = 40$

$M_B = -(20 - 3 + 30 + 20 - 0.5)$

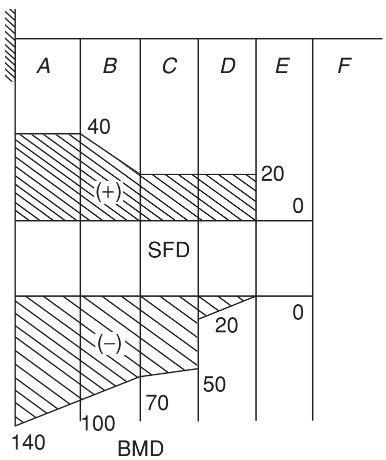
$= (60 + 30 + 10) \quad = -100$

$F_A \ = 40$

$M_A = (20 - 4 + 30 + 20 - 1.5)$

$= -(80 + 30 + 30)$

$= -140$ kNm



Example 4: Show that sum of normal stresses in any two mutually perpendicular directions is constant.

Solution: Equation for normal stress is

$$p_n = \frac{p_x + p_y}{2} + \frac{p_x - p_y}{2}\cos 2\theta + q\sin 2\theta$$

on a plane at angle $\theta + 90°$,

$$p_n 1 = \frac{p_x + p_y}{2} + \frac{p_x - p_y}{2} \times \cos(2\theta + 180) + q\sin(2\theta + 180)$$

$$= \frac{p_x + p_y}{2} - \frac{p_x - p_y}{2}\cos 2\theta - q\sin 2\theta$$

By adding, $p_n + p_n = p_x + p_y =$ constant

Example 5: The components of stresses on a rectangular element are

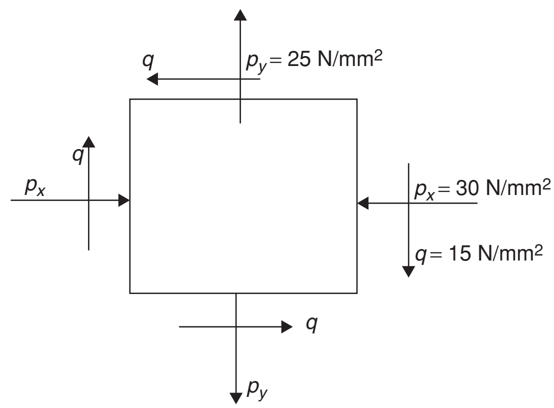
$$p_x = -30 \text{ N/mm}^2$$

$$p_y = +25 \text{ N/mm}^2$$

$$q = +15 \text{ N/mm}^2$$

Determine the magnitude of the two principal stresses and the angle between p_x and the major principal stress.

Solution:



To draw the Mohr's circle first draw a horizontal line representing the normal stress and then a vertical line, representing the shearing stress. The point of intersection of these lines is the origin O, the point from where the stress values are plotted.

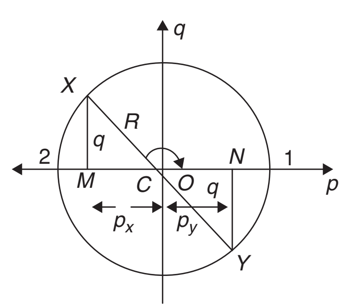
Locate the point X such that $OM = p_x$

$$= -30 \text{ N/mm}^2 \text{ and } M_x = q = +15 \text{ N/mm}^2$$

Similarly, locate point Y such that

$$\text{ON} = p_y = +25 \text{ N/mm}^2$$

$$NY = q = -15 \text{ N/mm}^2$$



Draw the line XY and locate the midpoint C, the centre of Mohr's circle. With C as centre and radius equal to CX or CY, draw a circle, which is the Mohr's circle.

$$OC = \frac{p_x + p_y}{2}$$

$$= \frac{-30+25}{2} = -2.5 \text{ N/mm}^2$$

Radius of Mohr's circle = R

$$= \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2}$$

$$= \sqrt{\left(\frac{-30-25}{2}\right)^2 + 15^2} = 31.33 \text{ N/mm}^2$$

Draw the horizontal diameter passing through the centre O. Locate the extreme points 1 and 2 on this diameter. Then O 1 is the maximum principal stress and O 2 is the minor principal stress.

Now,

$p_1 = O\,1$ = the maximum principal stress

$p_2 = O\,2$ = the minor principal stress

From Mohr's circle, we have

$p_1 = OC + R = 2.5 + 31.33$

$= 28.82$ N/mm^2 (tensile)

$p_2 = OC - R = -2.5 - 31.33$

$= -33.82$ N/mm^2 (compressive)

$$\tan\alpha = \frac{XM}{CM} = \frac{15}{OM - OC}$$

$$= \frac{15}{30-2.5} = 0.545$$

$\therefore\ \theta = 28.6°$

$\angle XC1 = 2\,\phi_p = 180 - 28.6° = 151.4°$

$\therefore\ \phi_p = 75.7°$

Here the major principal stress

$p_1 = 28.82$ N/mm^2 (tensile) acts on a plane making an angle $\phi_p = 75.7°$ in the clockwise direction from the diameter xy to the diameter 1–2.

i.e., principal planes lie at an angle ϕ_p from the x–direction.

Example 6: At a point in a material, the principal stresses are 800 N/cm^2 and 300 N/cm^2 where both are tensile. Find the normal, tangential and resultant stresses on a plane inclined at 50° to the major principal plane.

Solution: $p_1 = 800$ N/cm^2 (tensile)

$p_2 = 300$ N/cm^2 (tensile)

Angle with major principal plane = 50º

Let p_n = Normal stress at the point

$$p_n = \frac{p_1 + p_2}{2} + \frac{p_1 - p_2}{2}\cos 2\phi$$

$$= \frac{800+300}{2} + \frac{800-300}{2}\cos 100°$$

$$= 550 + 250\,(-0.1736)$$

$$= 506.6 \text{ N/cm}^2$$

Let q be the tangential stress at the point

$$q = \frac{p_1 - p_2}{2}\sin 2\theta$$

$$= \frac{800-300}{2}\sin 100$$

$$= 250(0.9848) = 246.20 \text{ N/cm}^2$$

$$\text{Resultant stress} = p_r = \sqrt{p_n^2 + q^2}$$

$$= \sqrt{(506.6)^2 + (246.20)^2}$$

$$= 563.26 \text{ N/cm}^2.$$

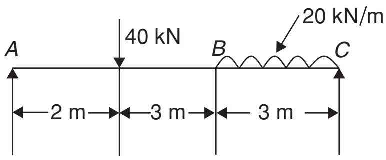
Exercises

Practice Problems I

1. At a certain point in an elastic material, on planes at right angles to each other, direct stresses of 120 N/mm^2 (tensile) and 100 N/mm^2 (compressive) are acting. The major principal stress is not to exceed 150 N/mm^2. The maximum shearing stress that may be subjected on the given planes is
   (A) 92.2 N/mm^2 (B) 81.74 N/mm^2
   (C) 86.6 N/mm^2 (D) 140 N/mm^2
2. A simply supported beam of uniform moment of inertia has a span 5 m carrying a point load of 50 kN at a point 1.5 m from the left hand support.
   Determine the reaction at both ends.
   (A) 45 kN, 20 kN (B) 40 kN, 48 kN
   (C) 35 kN, 15 kN (D) 32 kN, 12 kN
3. A cantilever beam having 5 m length is loaded such that it develops a shearing force of 40 kN and bending moment of 40 kNm in a section 2 m from the free end. Maximum shearing force and maximum bending moment are 100 kN and 250 kNm, respectively. The load on the beam is
   (A) 50 kN concentrated load at the end
   (B) 40 kN concentrated load at the en
   (C) 10 kN concentrated load at the free end and 4 kN/m UDL over the entire length
   (D) 20 kN/m UDL over the entire length

4. If failure in shear along 45° planes is to be avoided, then a material subjected to uniaxial tension should have its shear strength equal to at least the
(A) tensile strength.
(B) compressive strength.
(C) half the difference between the tensile and the compressive strength.
(D) half the tensile strength.

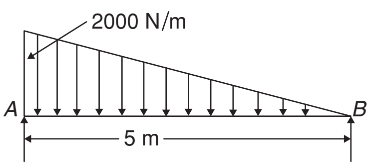
Direction for questions 5 and 6: A two span continuous beam is loaded as shown in the figure. EI is constant throughout.



5. Determine the support moments at A and C.
(A) 80 and 100 kNm
(B) zero kNm and zero kNm
(C) 120 kNm and 150 kNm
(D) 150 kNm and 150 kNm

6. Determine the support reactions at A and C.
(A) 52.6 kN, 80.2 kN
(B) 45.5 kN, 76.1 kN
(C) 48.9 kN, 71.1 kN
(D) 55.2 kN, 83.6 kN

Direction for questions 7 to 10: A simply supported beam is loaded as shown in the figure.



7. Determine the reaction at A.
(A) 4356 N (B) 3666 N
(C) 2867 N (D) 3333 N

8. Determine the bending moment at a distance 3 m from B
(A) 2852 Nm (B) 3201 Nm
(C) 3684 Nm (D) 3940 Nm

9. The position of maximum bending moment from B is
(A) 2.89 m (B) 2.62 m
(C) 3.17 m (D) 2.31 m

10. Determine the maximum bending moment.
(A) 2816 Nm (B) 3624 Nm
(C) 3208 Nm (D) 2677 Nm

11. The principal stresses in the wall of a vessel are 100 MPa and 50 MPa. The normal stress in a plane the normal of which makes an angle of 30° with the direction of maximum principal stress will be
(A) 62.5 MPa (B) 87.5 MPa
(C) 100 MPa (D) 50 MPa

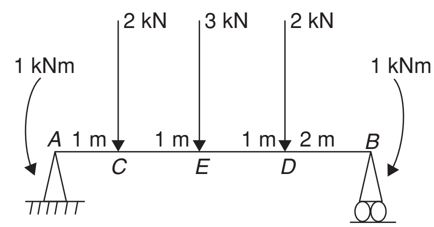
12. A circular bar is subjected to an axial pull of 20 kN. If the maximum allowable shear stress on any section is 5000 N/cm^2, the diameter of the bar will be
(A) 1.482 cm (B) 2.022 cm
(C) 1.596 cm (D) 1.624 cm

Direction for questions 13 and 14: At a point in a material the principal stresses are 1000 N/cm^2 and 400 N/cm^2 both tensile.

13. The normal stress on a plane inclined at 48° to the major principal plane is
(A) 668.6 N/cm^2 (B) 762.8 N/cm^2
(C) 572.4 N/cm^2 (D) 802.5 N/cm^2

14. The resultant stress on the same plane will be
(A) 692.6 N/cm^2 (B) 703.4 N/cm^2
(C) 653.2 N/cm^2 (D) 732.2 N/cm^2

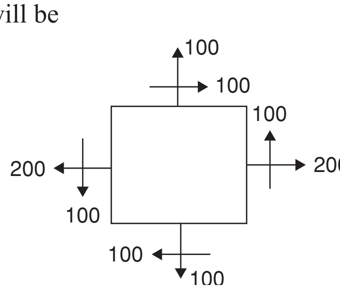
15.



For the simply supported beam loaded as shown in the figure the bending moment at E will be
(A) 6.3 kNm (B) 6.5 kNm
(C) 5.4 kNm (D) 7.6 kNm

Practice Problems 2

1. Maximum shear stress in respective units in the figure shown will be



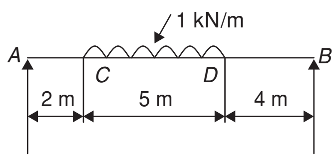
(A) $25\sqrt{5}$ (B) $50\sqrt{5}$
(C) $100\sqrt{5}$ (D) $200\sqrt{5}$

2. Mohr's circle will become a point when the body is subjected to
(A) pure shear
(B) equal axial stresses on two mutually perpendicular planes without shear stresses.
(C) uniaxial stress only.
(D) equal axial stresses on two mutually perpendicular planes.

3. In Mohr's circle of stresses, the distance of the centre of the circle from the shearing stress axis is given by the expression
   (A) $p_x - p_y$ (B) $p_x + p_y$
   (C) $\frac{p_x + p_y}{2}$ (D) $\frac{p_x - p_y}{2}$

4. In a simply supported beam of span L, an external moment M acts at a distance 'a' from the left support. The shear force at any section will be
   (A) $\frac{Ma}{L}$ (B) $\frac{M(L-a)}{L}$
   (C) $\frac{M}{L}$ (D) $\frac{M}{2L}$

5. For the beam loaded as shown in the figure



   the shear force at point D will be
   (A) 2.236 N (B) 2.045 N
   (C) 1.976 N (D) 1.864 N

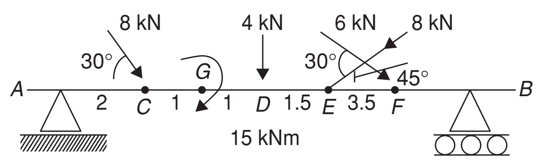
Direction for questions 6 and 7: A beam AB, 10 m long simply supported at the left hand end A and at a point C, 2 m from the extreme right hand end B. It carries two concentrated loads of 5 kN each at 3 m and 7 m from A and a uniformly distributed load of 1 kN per m over the portion in between the concentrated loads. An upward force FB is applied at B to make the reaction at C equal to zero.

6. Determine the upward force F_B applied at B.
   (A) 6 kN (B) 8 kN
   (C) 7 kN (D) 5 kN

7. Determine the reaction at A.
   (A) 6 kN (B) 8 kN
   (C) 7 kN (D) 5 kN

8. A simply supported beam of span 5 m carries a uniformly increasing load from zero at left hand support to 1 kN/m at the other support. Maximum bending moment will be
   (A) 1500 Nm (B) 1550 Nm
   (C) 1600 Nm (D) 1650 Nm

Direction for questions 9 and 10: A horizontal beam AB 10 m long is hinged at A and simply supported at B. The beam is loaded as shown in the figure.

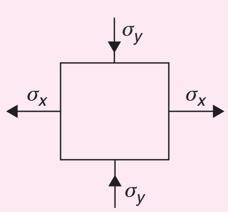


9. The value of reaction force at A will be
   (A) 6.47 kN (B) 34.78 kN
   (C) 8.58 kN (D) 9.49 kN

10. The maximum bending moment will be
   (A) 31.82 kNm (B) 34.78 kNm
   (C) 33.17 kNm (D) 38.25 kNm

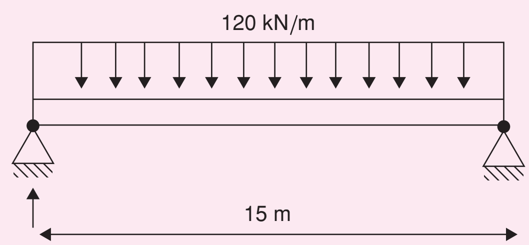
Previous Years' Questions

1. The figure shows the state of stress at a certain point in a stressed body. The magnitudes of normal stresses in the X and Y direction are 100 MPa and 20 MPa, respectively. The radius of Mohr's stress circle representing this state of stress is **[2004]**



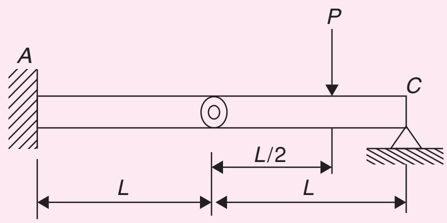
   (A) 120 (B) 80
   (C) 60 (D) 40

2. A steel beam of breadth 120 mm and height 750 mm is loaded as shown in the figure. Assume E_{steel} = 200GPa.



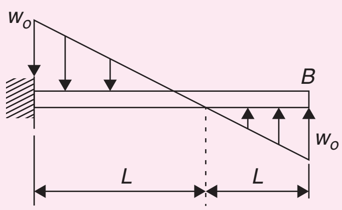
   The beam is subjected to a maximum bending moment of **[2004]**
   (A) 3375 kN/m
   (B) 4750 kN/m
   (C) 6750 kN/m
   (D) 8750 kN/m

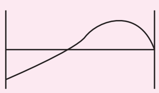
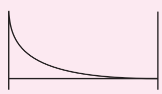
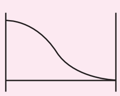
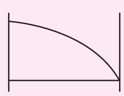
3. A beam is made up of two identical bars AB and BC, by hinging them together at B. The end A is built-in (cantilevered) and the end C is simply supported. With the load P acting as shown, the bending moment at A is **[2005]**



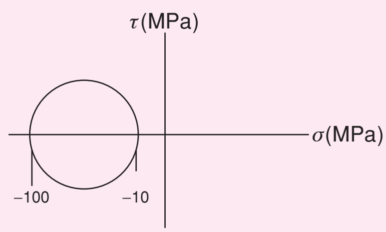
(A) zero
(B) $\frac{PL}{2}$
(C) $\frac{3PL}{2}$
(D) Indeterminate

4. A cantilever beam carries the anti-symmetric load shown, where w_o is the peak intensity of the distributed load. Qualitatively, the correct bending moment diagram for this beam is **[2006]**



(A) 
(B) 
(C) 
(D) 

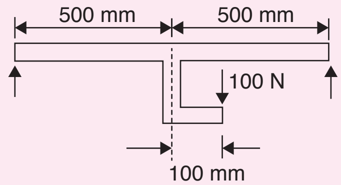
5. The Mohr's circle of plane stress for a point in a body is shown. The design is to be done on the basis of the maximum shear stress theory for yielding. Then, yielding will just begin if the designer chooses a ductile material whose yield strength is **[2006]**



(A) 45 MPa
(B) 50 MPa
(C) 90 MPa
(D) 100 MPa

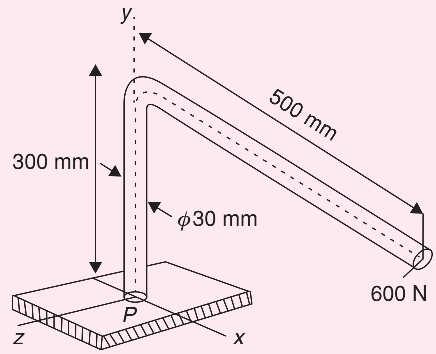
6. A simply supported beam of span length 6 m and 75 mm diameter carries a uniformly distributed load of 1.5 kN/m. What is the maximum value of bending moment? **[2007]**
(A) 9 kN/m
(B) 13.5 kN/m
(C) 81 kN/m
(D) 125 kN/m

7. In a simply-supported beam loaded as shown in the following the maximum bending moment in Nm is **[2007]**



(A) 25
(B) 30
(C) 35
(D) 60

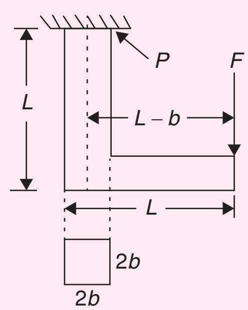
Direction for questions 8 and 9: A machine frame shown in the figure below is subjected to a horizontal force of 600 N parallel to z-direction.



8. The normal and shear stresses in MPa at point P are respectively **[2007]**
(A) 67.9 and 56.6
(B) 56.6 and 67.9
(C) 67.9 and 0.0
(D) 0.0 and 56.6

9. The maximum principal stress in MPa and the orientation of the corresponding principal plane in degrees are respectively **[2007]**
(A) –32.0 and –29.52
(B) 100.0 and 60.48
(C) –32.0 and 60.48
(D) 100.0 and –29.52

10. For the component loaded with a force F as shown in the figure, the axial stress at the corner point P is **[2008]**



(A) $\frac{F(3L-b)}{4b^3}$
(B) $\frac{F(3L+b)}{4b^3}$
(C) $\frac{F(3L-4b)}{4b^3}$
(D) $\frac{F(3L-2b)}{4b^3}$

11. A two-dimensional fluid element rotates like a rigid body. At a point within the element, the pressure is 1 unit. Radius of the Mohr's circle, characterizing the state of stress at that point is **[2009]**

(A) 0.5 unit (B) 0 unit
(C) 1 unit (D) 2 unit

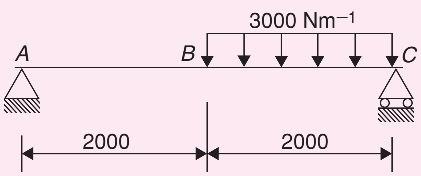
12. If the principal stresses in a plane stress problem are σ_1 = 100 MPa, σ_2 = 40 MPa, the magnitude of the maximum shear stress (in MPa) will be **[2009]**

(A) 60 (B) 50
(C) 30 (D) 20

13. The state of plane-stress at a point is given by σ_x = −200 MPa, σ_y = 100 MPa and σ_{xy} = 100 MPa. The maximum shear stress in MPa is **[2010]**

(A) 111.8 (B) 150.1
(C) 180.3 (D) 223.6

14. A massless beam has a loading pattern as shown in the figure. The beam is of rectangular cross-section with a width of 30 mm and height of 100 mm.



The maximum bending moment occurs at **[2010]**

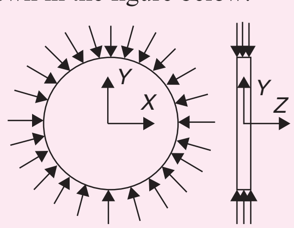
(A) Location B
(B) 2675 mm to the right of A
(C) 2500 mm to the right of A
(D) 3225 mm to the right of A

15. The state of stress at a point under plane stress condition is σ_{xx} = 40 MPa, σ_{yy} = 100 MPa and τ_{xy} = 40 MPa. The radius of the Mohr's circle representing the given state of stress in MPa is **[2012]**

(A) 40 (B) 50
(C) 60 (D) 100

16. The state of stress at a point is given by σ_x = –6 MPa, σ_y = 4 MPa, and τ_{xy} = –8 MPa. The maximum tensile stress (in MPa) at the point is _____. **[2014]**

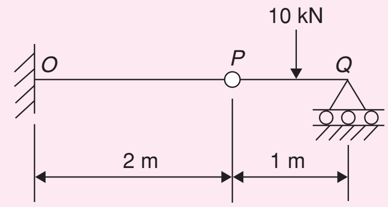
17. A thin plate of uniform thickness is subject to pressure as shown in the figure below: **[2014]**



Under the assumption of plane stress, which one of the following is correct?

(A) Normal stress is zero in z-direction
(B) Normal stress is tensile in z-direction
(C) Normal stress is compressive in z-direction
(D) Normal stress varies in z-direction

18.



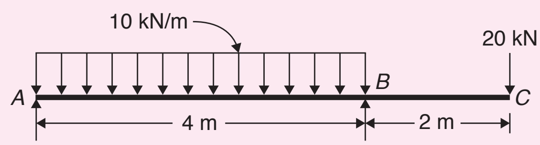
A cantilever beam OP is connected to another beam PQ with a pin joint as shown in the figure. A load of 10 kN is applied at the mid-point of PQ. The magnitude of bending moment (in kN-m) at fixed end O is: **[2015]**

(A) 2.5 (B) 5
(C) 10 (D) 25

19. In a plane stress condition, the components of stress at a point are σ_x = 20 MPa, σ_y = 80 MPa and τ_{xy} = 40 MPa. The maximum shear stress (in MPa) at the point is: **[2015]**

(A) 20 (B) 25
(C) 50 (D) 100

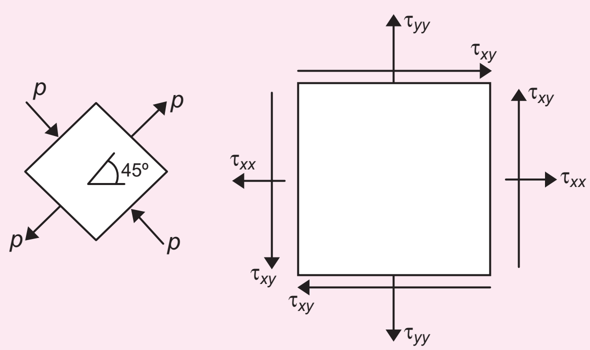
20. For the overhanging beam shown in figure, the magnitude of maximum bending moment (in kN-m) is ______. **[2015]**



21. A shaft with a circular cross-section is subjected to pure twisting moment. The ratio of the maximum shear stress to the largest principal stress is: **[2016]**

(A) 2.0 (B) 1.0 (C) 0.5 (D) 0

22. The state of stress at a point on an element is shown in figure (a). The same state of stress is shown in another coordinate system in figure (b).



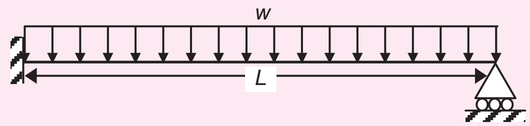
(a) (b)

The components (τ_{xx}, τ_{yy}, τ_{xy}) are given by **[2016]**

(A) $\left(\frac{p}{\sqrt{2}}, -\frac{p}{\sqrt{2}}, 0\right)$ (B) $(0, 0, p)$

(C) $\left(p, -p, \frac{p}{\sqrt{2}}\right)$ (D) $\left(0, 0, \frac{p}{\sqrt{2}}\right)$

23. A beam of length L is carrying a uniformly distributed load w per unit length. The flexural rigidity of the beam is EI. The reaction at the simple support at the right end is: **[2016]**



(A) $\frac{wL}{2}$ (B) $\frac{3wL}{8}$

(C) $\frac{wL}{4}$ (D) $\frac{wL}{8}$

Answer Keys

Exercises

Practice Problems 1

1. C **2.** C **3.** D **4.** D **5.** B **6.** C **7.** D **8.** B **9.** A **10.** C
11. B **12.** C **13.** A **14.** D **15.** C

Practice Problems 2

1. B **2.** B **3.** C **4.** C **5.** B **6.** C **7.** B **8.** C **9.** D **10.** B

Previous Years' Questions

1. C **2.** A **3.** B **4.** C **5.** C **6.** None **7.** B **8.** A **9.** D **10.** D
11. B **12.** C **13.** A **14.** C **15.** B **16.** 8.4 to 8.5 **17.** A **18.** C **19.** C
20. 40 **21.** B **22.** B **23.** B