Statistics

MEASURES OF CENTRAL TENDENCY

It is that single value which may be taken as the most suitable representative of the data. This single value is known as the average. Averages are generally, the central part of the distribution and therefore, they are also called the measures of **Central Tendency.**

It can be divided into two groups :

(a) Mathematical Average :

i. Arithmetic mean or mean

ii. Geometric mean

iii. Harmonic mean

(b) Positional Average :

i. Median

ii. Mode or positional average

ARITHMETIC MEAN

1. Individual observation or unclassified data :

If $x_1, x_2, ...x_n$ be n observations, then their arithmetic mean is given by

$$\bar{x} = \frac{x_1 + x_2 + ... + x_n}{n} \text{ or } \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

2. Arithmetic mean of discrete frequency distribution :

Let $x_1, x_2, ..., x_n$ be n observation and let $f_1, f_2, ..., f_n$ be their corresponding frequencies, then their mean

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + ... + f_nx_n}{f_1 + f_2 + ... + f_n} \text{ or } \bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

Short cut method : If the values of x or (and) f are large the calculation of arithmetic mean by the previous formula used, is quite tedious and time consuming. In such case we take the deviation from an arbitrary point A.

$$\bar{x} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

where A = Assumed mean

$d_i = x_i - A$ = deviation for each term

Step deviation method : Sometimes during the application of shortcut method of finding the mean, the deviation d_i are divisible by a common number h (say). In such case the arithmetic is reduced to a great extent taken by

$$u_i = \frac{x_i - A}{h}, \ i = 1, 2, ...n$$

$$\therefore \text{ mean } \bar{x} = A + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

3. Weighted arithmetic mean :

If $w_1, w_2, w_3, ..., w_n$ are the weight assigned to the values $x_1, x_2, x_3, ..., x_n$ respectively, then the weighted average is defined as -

$$\text{Weighted A.M.} = \frac{w_1x_1 + w_2x_2 + ... + w_nx_n}{w_1 + w_2 + ... + w_n}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

4. Combined mean :

If $\bar{x}_1, \bar{x}_2, ..., \bar{x}_k$ are the mean of k series of sizes $n_1, n_2, ..., n_k$ respectively then the mean of the composite series is given by

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + ...n_k\bar{x}_k}{n_1 + n_2 + ... + n_k}$$

5. Properties of Arithmetic Mean :

(i) In a statistical data, the sum of the deviation of items from A.M. is always zero.

$$\text{i.e. } \sum_{i=1}^{n}(x_i - \bar{x}) = 0 \text{ or } \sum_{i=1}^{n} x_i - n\bar{x}$$

$$\text{or } n\bar{x} - n\bar{x} = 0 \qquad \left(\because \bar{x} = \frac{\sum x_i}{n}\right)$$

(ii) In a statistical data, the sum of squares of the deviation of items from A.M. is least i.e.

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 \quad \text{is least}$$

(iii) If each of the n given observation be doubled, then their mean is doubled.

(iv) If $\overline{x}$ is the mean of $x_1, x_2, ..., x_n$. The mean of $ax_1, ax_2, ..., ax_n$ is $a\overline{x}$ where a is any number different from zero.

(v) Arithmetic mean is independent of origin i.e. it is not effected by any change in origin.

SOLVED EXAMPLE

Example-1

In a class of 40 boys and 30 girls, the average age is 16 years. If the mean age of boys is 1 year more than the mean age of girls, then find the mean age of girls.

Sol. Let mean age of 30 girls is x years so that of 40 boys is $(x + 1)$ years.

$$\therefore 16 = \frac{40(x+1)+30x}{40+30} = \frac{4x+4+3x}{7}$$

$\therefore\ 7x + 4 = 112$

$7x = 108$

$$x = 15\frac{3}{7} \text{ years}$$

Example-2

If n persons donate each rupees 1, 2, 4, 8.........2^{n-1} respectively then the mean donation per person is

Sol. $\overline{x} = \dfrac{1+2+2^2+2^3+......+2^{n-1}}{n}$ this is a G.P. with common ratio as 2,

$$= \frac{(2^n-1)}{n.(2-1)} = \frac{2^n-1}{n} \text{ rupees}$$

Example-3

Find the mean of first n natural numbers whose frequencies are equal to the corresponding numbers.

Sol. Here $x_1 = 1, x_2 = 2, x_3 = 3 \ldots x_n = n$

also $f_1 = 1, f_2 = 2, f_3 = 3 \ldots f_n = n$

$$\text{so } \overline{X} = \frac{x_1f_1 + x_2f_2 + x_3f_3 + \ldots + x_nf_n}{f_1 + f_2 + \ldots + f_n}$$

$$= \frac{1.1 + 2.2 + 3.3 + \ldots + n.n}{1 + 2 + 3 + ... + n}$$

$$= \frac{n(n+1)(2n+1).2}{6.n(n+1)} = \frac{2n+1}{3}$$

Example-4

The mean heights of team A and team B are 6'2" and 5'10" respectively. One tallest member from team A was transferred to B so that average height of team B rise by 0.5". Then the shortest-member from team B was transferred to A so that now both teams have same average height. Find the heights of members transferred respectively. Each team consists of 6 members.

Sol. Total height of 6 members of team A = 6'2" × 6 = 37'

Total height of 6 members of team B = 5'10" × 6 = 35'

Let one member of height h' is transferred from team A to B

Now total height of 5 members of team A remaining = 37' – h'

Total height of 7 members of team B now = 35' + h'

$$\text{Mean height of team } B = \frac{35'+h'}{7} = 5'10.5''$$

$$\therefore\ h' = 7(5'\ 10.5'') - 35' = 73.5''$$

$$= 6' \text{ and } \frac{1}{8}''$$

Let one member transferred from B to A of height H'

So mean height of 6 members of team $B = \dfrac{35+h-H}{6}$ feet

So mean height of 6 members of team $A = \dfrac{37-h+H}{6}$ feet

$$\frac{35+h-H}{6} = \frac{37-h+H}{6}$$

$2h - 2H = 2$

$$H = h - 1 = 5' \text{ and } \frac{1}{8}''$$

So the members transferred were of height 6 feet and $\frac{1}{8}$ inch, 5 feet and $\frac{1}{8}$ inch respectively.

Example-5

Mean of 25 observations was found to be 78.4. But later on it was found that 96 was misread as 69. The correct mean is

(A) 79.48 (B) 76.54 (C) 81.32 (D) 78.4

Sol. We know that the mean is given by

$$\overline{x} = \frac{\sum x}{n} \text{ or } \sum x = n\overline{x}$$

Here $\overline{x} = 78.4, n = 25$

$\therefore$ Sx = 25 ′ 78.4 = 1960

But this Sx is incorrect as 96 was misread as 69

correct Sx = 1960 – 69 = 1987

$$\therefore \text{ correct mean} = \frac{1987}{25} = 79.48$$

Example-6

If the frequencies of first four numbers out of 1, 2, 4, 6, 8 are 2, 3, 3 , 2 respectively, then the frequency of 8 if their AM is 5, is

(A) 4 (B) 5 (C) 6 (D) none of these

Sol. Here mean A = 5

Let the frequency of 8 be x. Then by the formula

$$A = \frac{\Sigma xf}{\Sigma f}$$

$$5 = \frac{1.2 + 2.3 + 4.3 + 6.2 + 8.x}{2 + 3 + 3 + 2 + x} = \frac{32 + 8x}{10 + x}$$

or 18 = 3x;

$\therefore$ x = 6.

Example-7

In a family, there are 8 men, 7 women and 5 children whose mean ages separately are respectively 24, 20 and 6 years. The mean age of the family is

(A) 17.1 years (B) 18.1 years

(C) 19.1 years (D) none of these

Sol. Here we have three collections for which $A_1 = 24$, $n_1 = 8$, $A_2 = 20$, $n_2 = 7$ and $A_3 = 6$, $n_3 = 5$. Their combined mean is the required mean.

By the formula $A = \dfrac{n_1A_1 + n_2A_2 + n_3A_3}{n_1 + n_2 + n_3}$

$$\therefore \quad A = \frac{8 \times 24 + 7 \times 20 + 5 \times 6}{8 + 7 + 3}$$

$$= \frac{192 + 140 + 30}{20} = \frac{362}{20} = 18.1$$

GEOMETRIC MEAN

1. **Individual data :** If $x_1, x_2, x_3, \ldots, x_n$ are n values of a variate x, none of them being zero, then the geometric mean G is defined as-

$$G = (x_1 x_2 x_3 \ldots\ldots x_n)^{1/n}$$

or $$G = \text{antilog}\left(\frac{\log x_1 + \log x_2 + \ldots\ldots + \log x_n}{n}\right)$$

or $$G = \text{antilog}\left(\frac{1}{n}\sum_{i=1}^{n} \log x_i\right)$$

2. **Geometric Mean of grouped data** :

Let $x_1, x_2, \ldots\ldots, x_n$ be n observation and let $f_1, f_2, \ldots\ldots, f_n$ be their corresponding frequency then their Geometric Mean is

$$G = \left(x_1^{f_1} x_2^{f_2} \ldots\ldots x_n^{fn}\right)^{1/N} \qquad \text{where } N = \sum_{i=1}^{n} f_i$$

$$\therefore G = \text{antilog}\left(\frac{\sum_{i=1}^{n} f_i \log x_i}{\sum_{i=1}^{n} f_i}\right)$$

SOLVED EXAMPLE

Example-8

The GM of the series 1, 2, 4, 8, 16, ….. 2^n is

Sol. $$GM = \sqrt[n]{1.2.4.8.16\ldots\ldots 2^n}$$

$$= \sqrt{2^1.2^2.2^3 2^4 \ldots\ldots 2^n}$$

$$= \left(2^{1+2+3+4+\ldots\ldots+n}\right)^{1/n}$$

$$= \left(2^{n(n+1)/2}\right)^{1/n} = 2^{n+1/2}$$

Example-9

The geometric mean of the numbers 7, 7^2, 7^3,……, 7^n is

Sol. $$GM = \sqrt[n]{7.7^2, 7^3 \ldots\ldots 7^n}$$

$$= \left[7^{1+2+3+\ldots\ldots+n}\right]^{1/n} = \left[7^{n(n+1)/2}\right]^{1/n} = 7^{n+1/2}$$

HARMONIC MEAN

Harmonic mean is reciprocal of mean of reciprocal.

1. **Individual observation** :

The H.M. of $x_1, x_2, \ldots\ldots, x_n$ of n observation is given by

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots\ldots + \frac{1}{x_n}} \qquad \text{i.e. } H = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$$

2. **H.M. of grouped data** :

Let $x_1, x_2, \ldots\ldots, x_n$ be n observation and let $f_1, f_2, \ldots\ldots, f_n$ be their corresponding frequency then H.M. is

$$H = \frac{\sum_{i=1}^{n} f_i}{\sum_{i=1}^{n}\left(\frac{f_i}{x_i}\right)}$$

Relation between A.M. , G.M., and H.M.

A.M. $\geq$ G.M. $\geq$ H.M.

Equality sign holds only when all the observations in the series are same.

MEDAIN

Median is the middle most or the central value of the variate in a set of observations, when the observations are arranged either in ascending or in descending order of their magnitudes. It divides the arranged series in two equal parts.

1. **Median of an individual series :**

Let n be the number of observations-

(i) arrange the data in ascending or descending order.

(ii) **(a) If n is odd then-**

$$\text{Median (M) = Value of } \left(\frac{n+1}{2}\right)^{th} \text{ observation}$$

(b) If n is even then

Median (M) = mean of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observation

i.e. $M = \dfrac{\left(\frac{n}{2}\right)^{th}\text{observation} + \left(\frac{n}{2}+1\right)^{th}\text{observation}}{2}$

2. Median of the discrete frequency distribution:

Algorithm to find the median :

Step-I : Find the cumulative frequency (C. F.)

Step-II : Find $\frac{N}{2}$, where $N = \sum_{i=1}^{n} f_i$

Step-III : See the cumulative frequency (C.F.) just greater than $\frac{N}{2}$ and determine the corresponding value of the variable.

Step-IV : The value obtained in step III is the median.

3. Median of grouped data or continuous series:

Let the no. of observations be n

(i) Prepare the cumulative frequency table

(ii) Find the median class i.e. the class in which the $\left(\frac{N}{2}\right)^{th}$ observation lies

(iii) The median value is given by the formulae

$$\text{Median (M)} = \ell + \left[\frac{\left(\frac{N}{2}\right) - F}{f}\right] \times h$$

N = total frequency = Σf_i

ℓ = lower limit of median class

f = frequency of the median class

F = cumulative frequency of the class preceding the median class

h = class interval (width) of the median class

4. Properties of Median :

(A) The sum of the absolute value of deviations of the items from median is minimum

(B) It is a positional average and it is not influenced by the position of the items.

SOLVED EXAMPLE

Example-10

The median of the items 6, 10, 4, 3, 9, 11, 22, 18, is

(A) 9 (B) 10 (C) 9.5 (D) 11

Sol. Let s arrange the items in ascending order 3, 4, 6, 9,9 10, 11, 18, 22.

In this data the number of items is n = 8, which is even.

$\therefore$ Median = M = average of $\left(\frac{n}{2}\right)$ th and $\left(\frac{n}{2}+1\right)$ th terms.

= Average of $\left(\frac{8}{2}\right)$ th and $\left(\frac{8}{2}+1\right)$ th terms

= Average of 4^{th} and 5^{th} terms

$= \frac{9+10}{2} = \frac{19}{2} = 9.5$

Example-11

The marks obtained by 20 students in a particular subject are 96, 88, 99, 87, 92, 86, 93, 94, 96, 97. Find the median mark.

Sol. We first arrange the data in ascending order as follows 86, 87, 88, 92, 93, 94, 96, 96, 97, 99

So there are altogether 10 observations. Hence the number of observations is even.

Then the median is the average of $\frac{10}{2}$ th and $\left(\frac{10}{2}+1\right)$ th

i.e. 5th and 6th observations.

That is Average of 93 and 94

i.e. $\frac{93+94}{2} = 93.5$

Example-12

In a class of 20 students, the marks obtained by them in a test are grouped as follows

Marks	No. of students
0	1
1	3
4	2
5	6
7	2
9	3
11	1
15	1
16	1
	20

Find the median marks.

Sol.

x	f	Σf
0	1	1
1	3	4
4	2	6
5	6	12
7	2	14
9	3	17
11	1	18
15	1	19
16	1	20

Here $n = 20$, so $\frac{n}{2} = 10$ and $\frac{n}{2} + 1 = 11$

Size corresponding to $\frac{n}{2}$ and $\left(\frac{n}{2}+1\right)$ is both '5'

Hence median is 5.

Example-13

The marks obtained by 60 students in a certain examination are given below

Marks	No. of Students
10 – 20	04
20 – 30	05
30 – 40	11
40 – 50	06
50 – 60	05
60 – 70	08
70 – 80	09
80 – 90	07
90 – 100	05

Calculate the median mark.

Sol. We calculate the cumulative frequency from the following table:

Marks – Obtained	Frequency (No. of students) (f)	(Cumulative Frequency) (f)
10 – 20	04	04
20 – 30	05	09
30 – 40	11	20
40 – 50	06	26
50 – 60	05	31
60 – 70	08	39
70 – 80	09	48
80 – 90	07	55
90 – 100	05	60

$n = 60$

Here we have $n = 60$, $\frac{n}{2} = 30$

We see that 30 lies in the class 50 - 60 with cumulative frequency 31.

Thus the median class 50 - 60

$\therefore l_{med}$ = lower limit of the median class i.e. the class in which $\frac{n}{2}$ th or $\frac{n+1}{2}$ th observation lies = 50

f_{med} = frequency of the median class = 05

Cf_{-1} = 26 (cumulative frequency of the class preceding the median class)

c = class size = 60 – 50 = 10

$$\text{Median} = l_{\text{med}} + \frac{\frac{n}{2} - c_{f-1}}{f_m} \times c$$

$$= 50 + \frac{30-26}{5} \times 10 = 50 + 8 = 58$$

MODE

Mode is that value in a series which occurs most frequently. In a frequency distribution, mode is that variate which has the maximum frequency.

1. **Computation of Mode** :

Mode for individual series : In the case of individual series, the value which is repeated maximum number of times is the mode of the series.

2. **Mode for grouped data (discrete frequency distribution series)**

In the case of discrete frequency distribution, mode is the value of the variate corresponding to the maximum frequency.

3. **Mode for continuous frequency distribution** :

(i) First find the model class i.e. the class which has maximum frequency. The model class can be determined either by inspecting or with the help of grouping data.

(ii) The mode is given by the formula

$$\text{Mode} = \ell + \frac{f_m - f_{m-1}}{2f_m - f_{m-1} - f_{m+1}} \times h$$

where $\ell \to$ lower limit of the model class

$h \to$ width of the model class

$f_m \to$ frequency of the model class

$f_{m-1} \to$ frequency of the class preceding model class

$f_{m+1} \to$ frequency of the class succeeding model class

(iii) In case the model value lies in a class other than the one containing maximum frequency (model class) then we use the following formula :-

$$\text{Mode} : \ell + \frac{f_{m+1}}{f_{m-1} + f_{m+1}} \times h$$

4. **Properties of Mode** : It is not effected by presence of extremely large or small items.

Relationship between mean, mode and median :

(i) In symmetrical distribution
Mean = Mode = Median

(ii) In skew (moderately symmetrical) distribution
Mode = 3 median – 2 mean

SOLVED EXAMPLE

Example-14

The median of a set of 11 distinct numbers is 20.5. If each of largest 5 observations is increased by 2 and each of the smallest observations is decreased by 3 then. Choose the correct statement.

(1) Mean remains unchanged
(2) Median remains unchanged
(3) Mean increases
(4) Median decreases

Sol. Median is the middle i.e. 6th member in ascending order or descending order of given numbers. When the left or right numbers are changed the middle number remains same. Hence median is unchanged.

Example-15

Calculate the mode from the following data:

Marks	No. of Students
0 - 10	04
10 - 20	12
20 - 30	24
30 - 40	36
40 - 50	20
50 - 60	16
60 - 70	08
70 - 80	05

Sol.

Value (Class)	Frequency
0 - 10	04
10 - 20	12
20 - 30	24
30 - 40	36
40 - 50	20
50 - 60	16
60 - 70	08
70 - 80	05
	$n = 125$

We choose the highest frequency class as the modal class.

So here 30 - 40 is the modal class.

Now

l = lower limit of modal class = 30

h = width of the modal class = 10

f_1 = frequency of the class preceding the modal class = 24

f_2 = frequency of the class following the modal class = 20

f = frequency of the modal class = 36

$$\text{So mode} = l + \frac{(f - f_1)}{(2f - f_1 - f_2)} \times h$$

$$= 30 + \frac{36 - 24}{72 - 24 - 20} \times 10$$

$$= 30 + \frac{12}{28} \times 10$$

$$= 30 + \frac{30}{7} = 34.28$$

(a) Symmetric Distribution A distribution is a symmetric distribution if values of mean, mode and median coincide.

Mean = Model = Median

(b) Skewed Distribution A distribution which is not symmetric is called a skewed distribution. In a moderately asymmetric the interval between the mean and median is approximately one-third of the interval between the mean and the mode. We have the following empirical relation between them.

Mean - Mode = 3(Mean - Median)

$\Rightarrow$ Mode = 3Median - 2Mean (Approximately)

Example-16

If the value of the mode and mean is 60 and 66 respectively then the value of median is

(A) 60 (B) 64 (c) 68 (D) none of these

Sol. Median = $\frac{1}{3}$ [Mode + 2 Mean]

$$= \frac{1}{3}[60 + 2 \times 66] = 64$$

MEASURES OF DISPERSION

Dispersion is the measure of the variations. The degree to which numerical data tend to spread about an average value is called the dispersion of the data.

The measures of dispersion commonly used are:

(i) Range
(ii) Quartile deviation or the semi- interquartile range
(iii) Mean Deviation
(iv) Standard Deviation

(i) Range

The range is the difference between the greatest and the least observations of the distribution. Thus if A and B are the greatest and the smallest observations respectively in a distribution, then its range = A - B.

The co-efficient of range (or scatter) = $\frac{A - B}{A + B}$

(ii) Quartile Deviation

If Q_1 and Q_3 be the lower and upper quartiles, then quartile deviation or semi inter quartile range Q is given by $Q = \frac{1}{2}(Q_3 - Q_1)$

Co-efficient of quartile deviation

$$= \frac{(Q_3 - Q_1)/2}{(Q_3 + Q_1)/2} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

(iii) Mean Deviation : Mean deviation is defined as the arithmetic mean of the absolute deviations of all the values taken about any central value.

i. Mean deviation of individual observations:

If $x_1, x_2, ..., x_n$ are n values of a variable x, then the mean deviation from an average A (median or AM) is given by

$$\text{M.D.} = \frac{1}{n}\sum_{i=1}^{n} |x_i - A|$$

$$= \frac{1}{n}\sum |d_i|, \text{ where } d_i = x_i - A$$

ii. Mean deviation of a discrete frequency distribution:

If $x_1, x_2, ..., x_n$ are n observation with frequencies $f_1, f_2, ..., f_n$, then mean deviation from an average A is given by -

$$\text{Mean Deviation} = \frac{1}{N}\sum f_i |x_i - A|$$

where $N = \sum_{i=1}^{n} f_i$

iii. Mean deviation of a grouped or continuous frequency distribution:

For calculating mean deviation of a continuous frequency distribution the procedure is same as for a discrete frequency distribution. The only difference is that here we have to obtain the mid-point of the various classes and take the deviations of these mid points from the given central value (median or mean)

SOLVED EXAMPLE

Example-17

An A.P. 2, 5, 8......, 21 terms is given. Find the mean deviation of A.P. from its mean.

Sol. $\Sigma x_i = 2 + 5 + 8 \text{ .... } + 21$ terms

$$= \frac{21}{2}[4+20\times 3] = 21 \times 32 = 672$$

$$\therefore \overline{x} = \frac{672}{21} = 32$$

$$\Sigma |x_i - \overline{x}| = 2d[1+2+3+..........+10]$$

$$= 2\times 3\left[\frac{10\times 11}{2}\right] = 330$$

$$\therefore \text{M.D.} = \frac{330}{21} = \frac{110}{7}$$

VARIANCE AND STANDARD DEVIATION

The variance of a variate x is the arithmetic mean of the squares of all deviations of x from the arithmetic mean of the observations and is denoted by var (x) or σ^2

The positive square root of the variance of a variate x is known as standard deviation i.e. standard deviation $= +\sqrt{\text{var}(x)}$

(i) Variance of Individual observations :

If $x_1, x_2, ..., x_n$ are n values of a variable x, then by definition

$$\text{var}(x) = \frac{1}{n}\left[\sum_{i=1}^{n}(x_i - \overline{x})^2\right] = \sigma^2 \qquad ...(i)$$

$$\text{or var}(x) = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \overline{x}^2 \qquad ...(ii)$$

If the values of variable x are large, the calculation of variance from the above formulae is quite tedious and time consuming. In that case, we take deviation from an arbitrary point A (say) then

$$\text{var}(x) = \frac{1}{n}\sum_{i=1}^{n} d_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} d_i\right)^2 \qquad ...(iii)$$

(ii) Variance of a discrete frequency distribution:

If $x_1, x_2, ..., x_n$ are n observations with frequencies $f_1, f_2, ..., f_n$ then $\text{var}(x) = \frac{1}{N}\left\{\sum_{i=1}^{n} f_i (x_i - \overline{x})^2\right\}$ ...(i)

$$\text{or var}(x) = \frac{1}{N}\sum_{i=1}^{n} f_i x_i^2 - \overline{x}^2 \qquad ...(ii)$$

If the values of x or f are large, we take the deviations of the values of variable x from an arbitrary point A. (say)

$\therefore d_i = x_i - A ; i = 1, 2, ...n$

$$\therefore \text{Var}(x) = \frac{1}{N}\left(\sum_{i=1}^{n} f_i d_i^2\right) - \left(\frac{1}{N}\sum_{i=1}^{n} f_i d_i\right)^2 \qquad ...(iii)$$

where $N = \sum_{i=1}^{n} f_i$

Sometime $d_i = x_i - A$ are divisible by a common number h (say)

then

$$u_i = \frac{x_i - A}{h} = \frac{d_i}{h}, \ i = 1, 2, ..., n$$

then

$$\text{var}(x) = h^2\left[\frac{1}{N}\sum_{i=1}^{n} f_i u_i^2 - \left(\frac{1}{N}\sum_{i=1}^{n} f_i u_i\right)^2\right] \qquad ...(iv)$$

(iii) Variance of a grouped or continuous frequency distribution :

In a grouped or continuous frequency distribution any of the formulae discussed in discrete frequency distribution can be used.

SOLVED EXAMPLE

Example-18

Find the variance of first *n* natural numbers.

Sol. We have

$$s^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2$$

$$= \frac{1}{n}(1^2 + 2^2 + \ldots + n^2) - \left(\frac{1}{n}.(1+2+...+)\right)^2$$

$$= \frac{1}{n}.\frac{n(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{n^2 - 1}{12}$$

Example-19

Find the standard deviation of first *n* natural numbers.

Sol. Sum of first *n* natural numbers $= \frac{n(n+1)}{2}$

$$\overline{x} = \frac{n+1}{2}$$

$$\sigma = \sqrt{\frac{(1-\overline{x})^2 + (2-\overline{x})^2 + ...+(n-\overline{x})^2}{n}}$$

$$= \sqrt{\frac{\Sigma n^2 + n\overline{x}^2 - 2\overline{x}(\Sigma n)}{n}}$$

$$= \sqrt{\frac{\frac{n(n+1)(2n+1)}{6} + \frac{n \times (n+1)^2}{4} - (n+1)\frac{n(n+1)}{2}}{n}}$$

$$= \sqrt{\frac{n(n+1)}{12n}(4n+2+3n+3-6n-6)}$$

$$= \sqrt{\frac{n}{12}\frac{(n+1)(n-1)}{n}} = \sqrt{\frac{(n^2-1)}{12}}$$

Example-20

A student obtained the mean and standard deviation of 100 observations as 40 and 5.1 respectively. It was later found that he had wrongly copied down an observation as 50 instead of 40. The correct mean and standard deviation are

(A) 39.9, 6 (B) 36.4, 5

(C) 39.9, 5 (D) none of these

Sol. Let the variable be x. Here n = 100, incorrect A = 40, incorrect s = 5.1.

From formula $A = \frac{\sum x}{n}$, $\sum x = nA$.

$\therefore$ incorrect Sx = 100 ´ 40 = 4000

$\therefore$ correct Sx = incorrect Sx – 50 + 40

= 4000 – 50 + 40 = 3990

Again, from $s^2 + A^2 = \frac{\sum x^2}{n}$,

we get $Sx^2 = n(s^2 + A^2)$

$\therefore$ incorrect $Sx^2 = 100\{(5.1)^2 + (40)^2\}$

$= 100(26.01 + 1600) = 162601$

$\therefore$ correct Sx^2 = incorrect $Sx^2 - 50^2 + 40^2$

$= 162601 - 2500 + 1600$

$= 161701$

$\therefore$ correct mean $A = \frac{\sum x}{n} = \frac{3900}{100} = 39.9$

Correct $(\sigma^2 + A^2) = \frac{\sum x^2}{n} = \frac{161701}{100} = 1617.01$

$\therefore$ correct $s^2 = 1617.01 - (\text{correct } A^2)$

$= 1617.01 - (39.9)^2$

$= 1617.01 - 1592.01 = 25$

MEAN AND OF VARIANCE OF BINOMIAL DISTRIBUTION

If the frequencies of the values 0, 1, 2,...., n of a variate are represented by the following coefficients of a binomial :

$q^n, {}^nC_1 q^{n-1}p, {}^nC_2 q^{n-2} p^2, ..., p^n$

where p is the probability of the success of the experiment (variate), q is the probability of its failure and p + q = 1 i.e. distribution is a binomial distribution : then

$P(x = r) = {}^nC_r q^{n-r} p^r$

mean $\bar{x} = \sum p_i x_i = np$

Variance $\sigma^2 = npq = \bar{x} q$

EXERCISE-I

Arithmetic Mean

Q.1 If the mean of 3, 4, x, 7, 10 is 6, then the value of x is-

(1) 4 (2) 5
(3) 6 (4) 7

Q.2 The A.M. of the series 1, 2, 4, 8, 16,...., 2n is-

(1) $\frac{2^n-1}{n}$ (2) $\frac{2^{n+1}-1}{n+1}$

(3) $\frac{2^n+1}{n}$ (4) $\frac{2^n-1}{n+1}$

Q.3 Mean of the first n terms of the A.P. a + (a + d) + (a + 2d) + ... is

(1) $a+\frac{nd}{2}$ (2) $a+\frac{(n-1)d}{2}$

(3) a + (n – 1)d (4) a + nd

Q.4 If the mean of n observations $1^2, 2^2, 3^2,....., n^2$ is $\frac{46n}{11}$, then n is equal to

(1) 11 (2) 12 (3) 23 (4) 22

Q.5 Following table shows the weight of 12 students:

Weight (in kgs.)	67	70	72	73	75
No. of students	4	3	2	2	1

(1) 70.25 kg. (2) 70.50 kg
(3) 70.75 kg. (4) None of these

Q.6 A factory employs 100 workers of whom 60 work in the first shift and 40 work in the second shift. The average wage of all the 100 workers is Rs.38. If the average wage of 60 workers of the first shift is Rs.40, then the average wage of the remaining 40 workers of the second shift is-

(1) 35 (2) 40
(3) 45 (4) None of these

Q.7 If $\overline{x}$ is the mean of a set of n observations $x_1, x_2, x_3,....., x_n$ then $\sum_{i=1}^{n}(x_i-\overline{x})$ is equal to

(1) M.D. about mean (2) S.D.
(3) 0 (4) None of these

Q.8 The mean of a set of observations is $\overline{x}$. If each observation is divided by α, α ≠ 0, and then is increased by 10 then the mean of the new set is-

(1) $\frac{\overline{x}}{\alpha}$ (2) $\frac{\overline{x}+10}{\alpha}$

(3) $\frac{\overline{x}+10\alpha}{\alpha}$ (4) a $\overline{x}$ + 10

Q.9 If the mean of first n natural numbers is equal to $\frac{n+7}{3}$, then n is equal to -

(1) 10 (2) 11
(3) 12 (4) None of these

Q.10 The mean of first three terms is 14 and mean of next two terms is 18. The mean of all the five terms is-

(1) 14.5 (2) 15.0 (3) 15.2 (4) 15.6

Q.11 The weighted AM of first n natural numbers whose weight are equal to the corresponding numbers is equal to

(1) 2n + 1 (2) $\frac{1}{2}(2n+1)$

(3) $\frac{1}{3}(2n+1)$ (4) $\frac{2n+1}{6}$

Q.12 The mean of distribution, in which the values of X are 1, 2,….., n the frequency of each being unity is

(1) $\frac{n(n+1)}{2}$ (2) $\frac{n}{2}$

(3) $\frac{n+1}{2}$ (4) $\frac{n(n-1)}{2}$

Geometric Mean & Harmonic Mean

Q.13 Geometric mean of the numbers $2, 2^2, 2^3, ..., 2^n$ is

(1) $2^{2/n}$ (2) $2^{n/2}$

(3) $2^{\frac{n-1}{2}}$ (4) $2^{\frac{n+1}{2}}$

Q.14 The harmonic mean of 4, 8, 16 is-

(1) 6.4 (2) 6.7 (3) 6.85 (4) 7.8

Median

Q.15 The median of 10, 14, 11, 9, 8, 12, 6 is

(1) 10 (2) 12
(3) 14 (4) 11

Q.16 If a variable takes the discrete values $\alpha + 4$, $\alpha-\frac{7}{2}, \alpha-\frac{5}{2}, \alpha-3, \alpha-2$, $\alpha+\frac{1}{2}, \alpha-\frac{1}{2}, \alpha+5 (\alpha>0)$, then the median is –

(1) $\alpha-\frac{5}{4}$ (2) $\alpha-\frac{1}{2}$ (3) $\alpha-2$ (4) $\alpha+\frac{5}{4}$

Q.17 The median of the items 6, 10, 4, 3, 9, 11, 22, 18 is-
(1) 9 (2) 10 (3) 9.5 (4) 11

Q.18 Find the median from the following distribution

Marks(x)	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of students	10	20	30	50	40	30

is

(1) 30 (2) 45
(3) 36 (4) 40

Mode

Q.19 Mode of the distribution

Marks	4	5	6	7	8
No. of students	3	5	10	6	1

is

(1) 6 (2) 10
(3) 8 (4) None of these

Q.20 The mode of the following distribution

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of student	2	18	30	45	35	20	6	3

is

(1) 36 (2) 37.3 (3) 38 (4) 39

Q.21 If median = (mode + 2 mean)M, then M is equal to-
(1) 3 (2) 1/3 (3) 2 (4) None of these

Dispersion

Q.22 The standard deviation of a variate x is σ. The standard deviation of the variable $\frac{ax+b}{c}$; a, b, c are constants, is-

(1) $\left(\frac{a}{c}\right)\sigma$ (2) $\left|\frac{a}{c}\right|\sigma$

(3) $\left(\frac{a^2}{c^2}\right)\sigma$ (4) None of these

Q.23 The scores of a batsman in ten innings are: 38, 70, 48, 34, 42, 55, 63, 46, 54, 44, then mean deviation about the median is :
(1) 8.4 (2) 8.5
(3) 8.6 (4) 8.8

Q.24 Mean deviation from the mean for the observations –1, 0, 4 is

(1) $\sqrt{\frac{14}{3}}$ (2) $\frac{2}{3}$

(3) 2 (4) None of these

Q.25 The S.D. of 7 scores 1, 2, 3, 4, 5, 6, 7 is
(1) 4 (2) 2
(3) $\sqrt{7}$ (4) None of these

Q.26 Let σ be the standard deviation of n observations. Each of the n observations is multiplied by a constant c. Then the standard deviation of the resulting numbers is-
(1) σ (2) $|c|.\sigma$
(3) $\sigma\sqrt{c}$ (4) None of these

Q.27 The mean of 100 items is 50 and their SD is 4, the sum of all the items and also the sum of the squares of the items is
(1) 5000, 251600
(2) 4000, 251600
(3) 5000, 26100
(4) 3000, 26100

Q.28 The mean and SD of distributions of 100 and 150 items are 50, 5 and 40, 6 respectively. The mean and SD of all the 250 items taken together are
(1) 44, 6.46 (2) 42, 7.46
(3) 44, 7.46 (4) 42, 6.46

Q.29 The variance of first n natural numbers is

(1) $\frac{n^2+1}{12}$ (2) $\frac{n^2-1}{12}$

(3) $\frac{(n+1)(2n+1)}{6}$ (4) $\frac{(n+1)(2n+1)}{2}$

Q.30 MD of the series a, a + d, a + 2d, ….., a + 2nd from its mean is

(1) $\frac{(n+1)d}{2n+1}$ (2) $\frac{nd}{2n+1}$

(3) $\frac{n(n+1)d}{2n+1}$ (4) $\frac{(2n+1)d}{n(n+1)}$

EXERCISE-II

Q.1 The mean weight of 9 items is 15. If one more item is added to the series, the mean becomes 16. The value of 10th item is
(1) 35 (2) 30 (3) 25 (4) 20

Q.2 In the frequency distribution of the discrete data given below, the frequency k against value 0 is missing.

Variable x :	0	1	2	3	4	5
Frequency f :	k	20	40	40	20	4

If the mean is 2.5, then the missing frequency k will be
(1) 0 (2) 1 (3) 3 (4) 4

Q.3 If mean of n item is $\overline{x}$. If each r^{th} item is increased by 2r. Then new mean will be
(1) $\overline{x}$ (2) $\overline{x}+\frac{n}{2}$
(3) $\overline{x}+\frac{n+2}{2}$ (4) $\overline{x}+n+1$

Q.4 The SD of 15 items is 6 and if each item is decreases by 1, then standard deviation will be
(1) 5 (2) 7 (3) $\frac{91}{15}$ (4) 6

Q.5 The sum of squares of deviations for 10 observations taken from mean 50 is 250. The coefficient of variation is
(1) 10% (2) 40% (3) 50% (4) none of these

Q.6 Mean of n items is x. If these n items are increased by $1^2, 2^2, 3^2, \ldots, n^2$ successively, then mean gets increased by
(1) $\frac{(n+1)(2n+1)}{6}$ (2) $\frac{n(n+1)(2n+1)}{6}$
(3) $\frac{n^2}{2}$ (4) remains same

Q.7 The mean value of the median and mean of the odd divisors of 360 is
(1) 13 (2) 7 (3) 6 (4) 10

Q.8 A sample of 35 observations has the mean 80 and s.d. as 4. A second sample of 65 observation from the same population has mean 60 and s.d. 3. The s.d. of the combined sample is
(1) 5.85 (2) 5.58 (3) 10.12 (4) None of these

Q.9 The mean and S.D. of the marks of 200 candidates were found to be 40 and 15 respectively. Later, it was discovered that a score of 40 was wrongly read as 50. The correct mean and S.D. respectively are
(1) 14.98, 39.95 (2) 39.95, 16.12
(3) 39.95, 224.5 (4) none of these

Q.10 The average of n numbers $x_1, x_2, x_3, \ldots\ldots, x_n$ is M. If x_1 is replaced by x, then the new average is
(1) $M-x_1+x$ (2) $\frac{(n-1)M-x_1+x}{n}$
(3) $\frac{(n+1)M-x_1+x}{n}$ (4) $\frac{nM-x_1+x}{n}$

Q.11 The variance of α, β and γ is 9, then variance of 5α, 5β and 5γ is
(1) 45 (2) 9/5 (3) 5/9 (4) 225

Q.12 The mean of n items is $\overline{x}$. If each item is successively increased by $3, 3^2, 3^3, \ldots\ldots, 3^n$, then new mean equals
(1) $\overline{x}+\frac{3^{n+1}}{n}$ (2) $\overline{x}+3\frac{(3^n-1)}{2n}$
(3) $\overline{x}+\frac{3^n}{n}$ (4) $\overline{x}+3\frac{(3^n-1)}{2n}$

Q.13 In a class of 100 students there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class is 72, then what is the average of the girls ?
(1) 73 (2) 65 (3) 68 (4) 74

Q.14 The median of a set of 9 distinct observation is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set
(1) is increased by 2
(2) is decreased by 2
(3) is two times the original median
(4) remains the same as that of the original set

Q.15 In an experiment with 15 observations on x, the following results were available
$$\sum x^2 = 2830, \sum x = 170$$
One observation that was 20, we found to be wrong and was replaced by the correct value 30. Then, the corrected variance is
(1) 78.00 (2) 188.66 (3) 177.33 (4) 8.33

Q.16 In a series of 2n observations, half of them equal a and remaining half equal – a. If the standard deviation of the observations is 2, then $|a|$ equals

(1) $\frac{1}{n}$ (2) $\sqrt{2}$ (3) 2 (4) $\frac{\sqrt{2}}{n}$

Q.17 If the standard deviation of the observations – 5, – 4, – 3, – 2, – 1, 0, 1, 2, 3, 4, 5 is $\sqrt{10}$. The standard deviation of observations 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25 will be

(1) $\sqrt{10}+20$ (2) $\sqrt{10}+10$

(3) $\sqrt{10}$ (4) None of these

Q.18 Median of ${}^{2n}C_0$, ${}^{2n}C_1$, ${}^{2n}C_2$, ${}^{2n}C_3$,........, ${}^{2n}C_n$ (where n is even) is

(1) ${}^{2n}C_{\frac{n}{2}}$

(2) ${}^{2n}C_{\frac{n+1}{2}}$

(3) ${}^{2n}C_{\frac{n-1}{2}}$

(4) None of these

EXERCISE-III

NUMERICAL VALUE BASED

Q.1 If the mean of n observation $1^2, 2^2, 3^2, ..., n^2$ is $\frac{46n}{11}$, then n is equal to

Q.2 If the mean deviation of number 1, 1 + d, 1 + 2d, ..., 1 + 100d from their mean is 255, then the d is equal to

Q.3 The means and variance of n observations $x_1, x_2, x_3, ...x_n$ are 5 and 0 respectively. If $\sum_{i=1}^{n} x_i^2 = 400$, then the value of n is equal to

Q.4 The mean of 30 given numbers, when it is given that the mean of 10 of them is 12 and the mean of the remaining 20 is 9, is equal to

Q.5 The algebraic sum of the deviation of 20 observations measured from 30 is 2. Then, mean of observations is

Q.6 If the average of the numbers 148, 146, 144, 142, ... in AP, be 125, then the total numbers in the series will be

Q.7 A group of 10 items has arithmetic mean 6. If the arithmetic mean of 4 of these items is 7.5, then the mean of the remaining items is

Q.8 Mean of 100 observation is 45. If it was later found that two observations 19 and 31 were incorrectly recorded as 91 and 13. The correct mean is

Q.9 In a class of 50 students, 10 have failed and their average marks are 28. The total marks obtained by the entire class are 2800. The average marks of those who have passed, are

EXERCISE-IV

JEE-MAIN PREVIOUS YEAR'S

Q.1 If the standard deviation of the number 2, 3, *a* and 11 is 3.5, then which of the following is true?

[JEE Main-2016]

(1) $3a^2-26a+55=0$ (2) $3a^2-32a+84=0$

(3) $3a^2-34a+91=0$ (4) $3a^2-23a+44=0$

Q.2 If $\sum_{i=1}^{9}(x_1-5)=9$ and $\sum_{i=1}^{9}(x_i-5)^2=45$, then the standard deviation of the 9 items $x_1, x_2 ....x_9$ is -

[JEE Main - 2018]

(1) 4 (2) 2

(3) 3 (4) 9

Q.3 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is : **[JEE Main - 2019 (January)]**

(1) 16 (2) 22 (3) 20 (4) 18

Q.4 A data consists of n observations : $x_1, x_2, ...x_n$. If $\sum_{i=1}^{n}(x_i+1)^2 = 9n$ and $\sum_{i=1}^{n}(x_i-1)^2 = 5n$, then the standard deviation of this data is :

[JEE Main - 2019 (January)]

(1) 5 (2) $\sqrt{5}$ (3) $\sqrt{7}$ (4) 2

Q.5 The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is:

[JEE Main - 2019 (January)]

(1) 10 : 3 (2) 4 : 9 (3) 5 : 8 (4) 6 : 7

Q.6 If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5 are 10 and 3, respectively, then the variance of 6 observations $x_1, x_2, ...,x_3$ and –50 is equal to

[JEE Main - 2019 (January)]

(1) 509.5 (2) 586.5 (3) 582.5 (4) 507.5

Q.7 A bag contains 30 white balls and 10 red balls. 16 balls are drawn one by one randomly from the bag with replacement. If X be the number of white balls drawn, then $\left(\frac{\text{mean of X}}{\text{standard deviation of X}}\right)$ is equal to :

[JEE Main - 2019 (January)]

(1) 4 (2) $4\sqrt{3}$ (3) $3\sqrt{2}$ (4) $\frac{4\sqrt{3}}{3}$

Q.8 If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observations is:

[JEE Main - 2019 (January)]

(1) 30 (2) 51 (3) 50 (4) 31

Q.9 The mean and the variance of five observations are 4 and 5.20, respectively. If three of the observations are 3 , 4 and 4; then the absolute value of the difference of the other two observations, is :

[JEE Main - 2019 (January)]

(1) 7 (2) 5 (3) 1 (4) 3

Q.10 The mean and vriance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is : **[JEE Main - 2019(April)]**

(1) 40 (2) 49 (3) 48 (4) 45

Q.11 A student scores the following marks in five tests : 45, 54, 41, 57, 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is

[JEE Main- 2019(April)]

(1) $\frac{10}{\sqrt{3}}$ (2) $\frac{100}{\sqrt{3}}$ (3) $\frac{100}{3}$ (4) $\frac{10}{3}$

Q.12 If the standard deviation of the numbers –1, 0, 1 k is $\sqrt{5}$ where k > 0, then k is equal to

[JEE Main - 2019(April)]

(1) $2\sqrt{\frac{10}{3}}$ (2) $2\sqrt{6}$ (3) $4\sqrt{\frac{5}{3}}$ (4) $\sqrt{6}$

Q.13 The mean and the median of the following ten numbers in increasing order 10, 22, 26, 29, 34, x 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to :-

[JEE Main - 2019(April)]

(1) $\frac{7}{3}$ (2) $\frac{9}{4}$ (3) $\frac{7}{2}$ (4) $\frac{8}{3}$

Q.14 If for some $x \in R$, the frequency distribution of the marks obtained by 20 students in a test is :

[JEE Main - 2019(April)]

Marks	2	3	5	7
Frequencey	$(x+1)^2$	$2x-5$	x^2-3x	x

then the mean of the marks is :

(1) 2.8 (2) 3.2 (3) 3.0 (4) 2.5

Q.15 If both the mean and the standard deviation of 50 observations $x_1, x_2....,x_{50}$ are equal to 16, then the mean of $(x_1-4)^2, (x_2-4)^2....... (x_{50}-4)^2$ is :

[JEE Main - 2019(April)]

(1) 525 (2) 380 (3) 480 (4) 400

Q.16 If the data $x_1, x_2, ..., x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all these is 2,000; then the standard deviation of this data is :

[JEE Main - 2019(April)]

(1) 4 (2) 2 (3) $\sqrt{2}$ (4) $2\sqrt{2}$

Q.17 If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then (m + n) is equal to _________.

[JEE Main-2020 (January)]

Q.18 If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then x.y is equal to_____________. **[JEE Main-2020 (January)]**

Q.19 The mean and the standard deviation (s, d) of 10 observations are 20 and 2 respectively. Each of these 10 observations is multiplied by p and then reduced by q, where $p \neq 0$ and $q \neq 0$. If the new mean and new s.d. become half of their original values, then q is equal to

[JEE Main-2020 (January)]

(1) – 20 (2) 10 (3) – 10 (4) – 5

Q.20 The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is :

[JEE Main-2020 (January)]

(1) 3.99 (2) 4.02 (3) 4.01 (4) 3.98

Q.21 Let the observations $x_i (1 \le i \le 10)$ satisfy the equations, $\sum_{i=1}^{10}(x_i-5)=10$ and $\sum_{i=1}^{10}(x_i-5)^2=40$. If μ and λ are the mean and the variance of the observations, $x_1-3, x_2-3, ... x_{10}-3$, then the ordered pair (μ, λ) is equal to: **[JEE Main-2020 (January)]**

(1) (3, 3) (2) (3, 6) (3) (6, 6) (4) (6, 3)

Q.22 If the variance of the terms in an increasing A.P., $b_1, b_2, b_3, ..., b_{11}$ is 90, then the common difference of this A.P. is ________. **[JEE Main-2020 (September)]**

Q.23 Let $X = \{x \in N : 1 \le x \le 17\}$ and $Y \{ax + b : x \in X\}$ and $a, b \in R, a > 0\}$. If mean and variance of elements of Y are 17 and 216 respectively then a + b is equal to : **[JEE Main-2020 (September)]**

(1) 7 (2) –27 (3) 9 (4) –7

Q.24 Let x_i $(1 \le i \le 10)$ be ten observations of a random variable X. If $\sum_{i=1}^{10}(x_i - p) = 3$ and $\sum_{i=1}^{10}(x_i - p)^2 = 9$ where $0 \ne 0$ $p \in R$, then the standard deviation of these observations is **[JEE Main-2020 (September)]**

(1) $\frac{7}{10}$ (2) $\frac{9}{10}$ (3) $\sqrt{\frac{3}{5}}$ (4) $\frac{4}{5}$

Q.25 For the frequency distribution : **[JEE Main-2020 (September)]**

Variate (x) : $x_1\ x_2\ x_3 \ldots x_{15}$

Frequency (f) : $f_1\ f_2\ f_3 \ldots f_{15}$

Where $0 < x_1 < x_2 < x_3 < \ldots\ldots < x_{15} = 10$ and $\sum_{i=1}^{15} f_i > 0$, then standard deviation cannot be :

(1) 1 (2) 6 (3) 2 (4) 4

Q.26 If the variance of the following frequency distribution: **[JEE Main-2020 (September)]**

Class :	10 – 20	20 – 30	30 – 40
Frequency :	2	x	2

is 50, then x is equal to ________.

Q.27 The mean and variance of 8 observations are 10 and 13. 5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is : **[JEE Main-2020 (September)]**

(1) 9 (2) 3 (3) 7 (4) 5

Q.28 If the mean and the standard deviation of the data 3, 5, 7, a, b are 5 and 2 respectively, then a and b are the roots of the equation : **[JEE Main-2020 (September)]**

(1) $x^2 - 20x + 18 = 0$ (2) $2x^2 - 20x + 19 = 0$

(3) $x^2 - 10x + 18 = 0$ (4) $x^2 - 10x + 19 = 0$

Q.29 The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2, 4, 10, 12, 14, then the absolute difference of the remaining two observations is : **[JEE Main-2020 (September)]**

(1) 2 (2) 4 (3) 3 (4) 1

Q.30 Consider the data on x taking the values 0, 2, 4, 8, ..., 2^n with frequencies ${}^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$ then n is equal to ______ . **[JEE Main-2020 (September)]**

Q.31 If $\sum_{i=1}^{n}(x_i - a) = n$ and $\sum_{i=1}^{n}(x_i - a)^2 = na$, $(n, a > 1)$ Then the standard deviation of n observations $x_1, x_2, \ldots\ldots x_n$ is : **[JEE Main-2020 (September)]**

(1) $a - 1$ (2) $n\sqrt{a-1}$

(3) $\sqrt{n(a-1)}$ (4) $\sqrt{a-1}$

ANSWER KEY

EXERCISE-I

Q.1 (3)	**Q.2** (2)	**Q.3** (2)	**Q.4** (1)	**Q.5** (1)	**Q.6** (1)	**Q.7** (3)	**Q.8** (3)	**Q.9** (2)	**Q.10** (4)
Q.11 (3)	**Q.12** (3)	**Q.13** (4)	**Q.14** (3)	**Q.15** (1)	**Q.16** (1)	**Q.17** (3)	**Q.18** (3)	**Q.19** (1)	**Q.20** (1)
Q.21 (2)	**Q.22** (2)	**Q.23** (3)	**Q.24** (3)	**Q.25** (2)	**Q.26** (2)	**Q.27** (1)	**Q.28** (3)	**Q.29** (2)	**Q.30** (3)

EXERCISE-II

Q.1 (3)	**Q.2** (4)	**Q.3** (4)	**Q.4** (4)	**Q.5** (1)	**Q.6** (1)	**Q.7** (4)	**Q.8** (3)	**Q.9** (2)	**Q.10** (4)
Q.11 (4)	**Q.12** (2)	**Q.13** (2)	**Q.14** (4)	**Q.15** (1)	**Q.16** (3)	**Q.17** (3)	**Q.18** (1)		

EXERCISE-III

Q.1 [11]	**Q.2** [10.1]	**Q.3** [16]	**Q.4** [10]	**Q.5** [30.1]	**Q.6** [24]	**Q.7** [5]	**Q.8** [44.46]	**Q.9** [63]

EXERCISE-IV

JEE-MAIN
PREVIOUS YEAR'S

Q.1 (2)	**Q.2** (2)	**Q.3** (3)	**Q.4** (2)	**Q.5** (2)	**Q.6** (4)	**Q.7** (2)	**Q.8** (4)	**Q.9** (1)	**Q.10** (3)
Q.11 (1)	**Q.12** (2)	**Q.13** (1)	**Q.14** (1)	**Q.15** (4)	**Q.16** (2)	**Q.17** [18]	**Q.18** [54]	**Q.19** (1)	**Q.20** (1)
Q.21 (1)	**Q.22** (3)	**Q.23** (4)	**Q.24** (2)	**Q.25** (2)	**Q.26** (4)	**Q.27** (3)	**Q.28** (4)	**Q.29** (1)	**Q.30** [06]
Q.31 (4)									

EXERCISE (Solution)

EXERCISE-I

Q.1 **(3)**

$$\bar{x} = \frac{3+4+x+7+10}{5} = 6$$

$$x + 24 = 30$$

$$\therefore\ x = 6$$

Q.2 **(2)**

$$\bar{x} = \frac{1+2+4.....+2^n}{n+1}$$

$$\therefore\ \bar{x} = \frac{\left(\dfrac{2^{n+1}-1}{2-1}\right)}{n+1} = \frac{2^{n+1}-1}{n+1}$$

Q.3 **(2)**

$$\bar{x} = \frac{(a)+(a+d)+(a+2d).....n \text{ terms}}{n}$$

$$\therefore\ \bar{x} = \frac{na + d(1+2+...+(n+1))}{n}$$

$$\bar{x} = a + \frac{n(n-1)}{2}.\frac{d}{n}$$

$$\therefore\ \bar{x} = a + \frac{(n-1)}{2}d$$

Q.4 **(1)**

$$\bar{x} = \frac{1^2+2^2+...+n^2}{n}$$

$$\therefore\ \bar{x} = \frac{n(n+1)(2n+1)}{6n}$$

$$\therefore\ \bar{x} = \frac{(n+1)(2n+1)}{6} = \frac{46n}{11} \text{(given)}$$

$$\Rightarrow 22n^2 + 33n + 11 = 276\,n$$

$$\therefore 22n^2 - 243n + 11 = 0$$

$$\therefore\ n^2 - \left(\frac{243}{22}\right)n + \frac{1}{2} = 0$$

$$\Rightarrow (n-11)\left(n - \frac{1}{22}\right) = 0$$

but n is integer

$$\therefore\ n = 11$$

Q.5 **(1)**

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{4.67+3.70+2.72+2.73+1.75}{4+3+2+2+1}$$

$$= \frac{268+210+144+146+75}{12}$$

$$= \frac{843}{12} = 70.25 \text{ kg}$$

Q.6 **(1)**

100 workers

$n_1 = 60; \bar{x}_1 = 40$		$n_2 = 40;\ \bar{x}_2 = ?$
60 workers	+	40 workers

$n = 100;\ \ \bar{x} = 38$

$$\because\ n\bar{x} = n_1\bar{x}_1 + n_2\bar{x}_2$$

$$\bar{x}_2 = \frac{n\bar{x} - n_1\bar{x}_1}{n_2} = \frac{3800-2400}{40} = 35$$

Q.7 **(3)**

$$\sum(x_i - \bar{x}) = \sum x_i - n\bar{x} = n\bar{x} - \bar{x}\cdot n = 0$$

Q.8 **(3)**

Let

Data	Mean
x_i	$\bar{x}$
$\dfrac{x_i}{\alpha}$	$\dfrac{\bar{x}}{\alpha}$
$\dfrac{x_i}{\alpha}+10$	$\dfrac{\bar{x}}{\alpha}+10$

$$\therefore\ \text{Mean} = \frac{\bar{x}+10\alpha}{\alpha}$$

Q.9 **(2)**

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{\Sigma x}{n} = \frac{n(n+1)}{2n}$$

$$\therefore\ \bar{x} = \frac{(n+1)}{2} = \frac{(n+7)}{3} \text{ (Given)}$$

$$\therefore\ n = 11$$

Q.10 **(4)**

$\because\ n\bar{x} = n_1\bar{x}_1 + n_2\bar{x}_2$

$\therefore\ \bar{x} = \dfrac{3.14 + 2.18}{5} = \dfrac{78}{5} = 15.6$

Q.11 **(3)**

x_i	w_i	$x_i w_i$
0	0	0
1	1	1^2
2	2	2^2
3	3	3^2
4	4	4^2
⋮	⋮	⋮
n	n	n^2

$$\frac{\sum x_i w_i}{\sum w_i} = \frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}} = \boxed{\frac{2n+1}{3}}$$

Q.12 **(3)**

x_i	f_i
1	1
2	1
3	1
:	:
n	1

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1+2+3+....n}{n}$$

$$= \frac{n(n+1)}{2n}$$

$$= \left(\frac{n+1}{2}\right)$$

Q.13 **(4)**

$G = (x_1 . x_2 . x_3 ....)^{1/n}$

$G = (2^1 .2^2.........2^n)^{1/n}$

$G = (2^{1+2\ ....+n})^{1/n}$

$G = 2^{\frac{n(n+1)}{2}\frac{1}{n}} = 2^{\frac{n+1}{2}}$

Q.14 **(3)**

$$H = \frac{3}{(1/4)+(1/8)+(1/16)}$$

$$\therefore H = \frac{3\times16}{4+2+1} = \frac{48}{7}$$

$\therefore = 6.85$

Q.15 **(1)**

Arranged Data in increasing order is

$6, 8, 9, \boxed{10}, 11, 12, 14$

Median

Q.16 **(1)**

Data arranged in ascending order is

$\alpha - \frac{7}{2}, \alpha - 3, \alpha - \frac{5}{2}, \alpha - 2, \alpha - \frac{1}{2},$

$\alpha + \frac{1}{2}, \alpha + 4, \alpha + 5$

$$\therefore \text{Median} = \frac{(\alpha - 2) + \left(\alpha - \frac{1}{2}\right)}{2} = \alpha - \frac{5}{4}$$

Q.17 **(3)**

Data in ascending order is

3, 4, 6, 9, 10, 11, 18, 22

$\therefore \text{Median} = \dfrac{9+10}{2} = 9.5$

Q.18 **(3)**

Marks (x)	frequency	Cumulative frequency
0 – 10	10	10
10 – 20	20	30
20 – 30	30	60
30 – 40	50	110
40 – 50	40	150
50 – 60	30	180

N = 180

then $\dfrac{N}{2} = 90^{th}$ item

Median in Class = 30 – 40, $l = 30$, C. = 60, f = 50 and i = 10

$$\textbf{Median} = l + \frac{\left(\frac{N}{2} - C\right)}{f} \times i = 36$$

Q.19 **(1)**

Highest frequency data is 6

$\therefore$ mode = 6

Q.20 **(1)**

Mode class = 30 – 40

$$\text{Mode} = 30 + \frac{45-30}{90-30-35} \times 10$$

$$= 30 + \frac{150}{25}$$

$$= 30 + 6 = 36$$

Q.21 **(2)**

For skew distribution

$\because$ mode = 3(median) – 2(mean)

$$\therefore \text{Median} = \frac{1}{3}\ (\text{Mode} + 2\ (\text{mean}))$$

$$\therefore M = \frac{1}{3}$$

Q.22 (2)

Data	S.D.
x	σ
$\frac{ax}{c}$	$\left\lvert\frac{a}{c}\right\rvert\sigma$
$\frac{ax}{c}+\frac{b}{c}$	$\left\lvert\frac{a}{c}\right\rvert\sigma$

Q.23 (3)

Arrange data is in increasing order is

34, 38, 42, 44, 46, 48, 54, 55, 63, 70

$$\therefore \text{Median} = \frac{46+48}{2} = 47$$

x_i	$D_i = x_i$ – median	$\lvert d_i \rvert$
34	–13	13
38	–9	9
42	–5	5
44	–3	3
46	–1	1
48	+1	1
54	+7	7
55	+8	8
63	+16	16
70	+23	23

$$\text{M.D.} = \frac{\sum \lvert d_i \rvert}{n} = \frac{86}{10} = 8.6$$

Q.24 **(3)**

$$\bar{x} = \frac{-1+0+4}{3} = 1$$

$$\therefore \text{M.D.} = \frac{\sum\lvert x_i - \bar{x}\rvert}{n}$$

$$= \frac{\lvert -1-1\rvert + \lvert 0-1\rvert + \lvert 4-1\rvert}{3} = \frac{2+1+3}{3} = 2$$

Q.25 **(2)**

$$\text{S.D.}: \sigma = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$= \sqrt{\frac{1^2+2^2.....+7^2}{7} - \left(\frac{1+2+.....7}{7}\right)^2}$$

$$= \sqrt{\frac{7.8.15}{6.7} - \left(\frac{7.8}{2.7}\right)^2} = \sqrt{20-16} = \sqrt{4} = 2$$

Q.26 **(2)**

When each data is multiplied by 'c' then S.D. all gets multiplied $\lvert c\rvert$.

$\therefore$ new S.D. = $\lvert c\rvert.\sigma$

Q.27 **(1)**

$\Sigma x = n\bar{x} = 100 \times 50 = 5000$

$$\text{S.D.} = \sqrt{\sigma^2}$$

$$4 = \sqrt{\sigma^2}$$

$$= \sqrt{\frac{1}{n}\Sigma x_i^2 - \bar{x}^2}$$

$$= \sqrt{\frac{\Sigma x^2}{100} - (50)^2}$$

$$16 = \frac{\Sigma x_i^2}{100} - 2500$$

$$(16+2500)\cdot 100 = \Sigma x_i^2$$

$$\boxed{251600 = \Sigma x_i^2}$$

Q.28 **(3)**

$n_1 = 100$	$n_2 = 150$
$\bar{x}_1 = 50$	$\bar{x}_2 = 110$
$\sigma_1^2 = 5$	$\sigma_2^2 = 6$

$$n\bar{x} = n_1\bar{x}_1 + n_2\bar{x}_2$$
$$= 100 \times 50 + 150 \times 40$$
$$= 5000 + 6000$$

$$\bar{x} = \frac{11000}{250} = 44$$

$$\sigma^2 = n_1\frac{(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$$

$$d_1 = 50 - 44 = 6$$
$$d_2 = 40 - 44 = -4$$

$$\sigma^2 = 100\frac{(25+36)+150(36+16)}{250}$$

$$= \frac{6100 + 7800}{250} = 55.6$$

$$\sigma = \sqrt{55.6} = 7.46$$

Q.29 **(2)**

$$\sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$$

$$= \frac{1^2 + 2^2 + 3^2 + ... + n^2}{n} - \left(\frac{1+2+3+....+n}{n}\right)^2$$

$$= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n(n+1)}{2n}\right)^2$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{n^2(n+1)^2}{4n^2}$$

$$= \frac{n^2 - 1}{12}$$

Q.30 (3)

Mean of series is

$$\bar{x} = \frac{a + (a+d) + (a+2d) + ... + (a+2nd)}{(2n+1)}$$

$$\bar{x} = a + nd$$

$$\therefore \quad \sum_{i=0}^{2n} |x_i - \bar{x}| \Rightarrow \frac{2d(n)(n+1)}{2} \Rightarrow n(n+1)d$$

$$\therefore \quad \text{Mean deviation} = \frac{n(n+1)d}{(2n+1)}$$

EXERCISE-II

Q.1 **(3)** Let $x_1, x_2, x_3 \ldots\ldots x_9$ are 9 items they

$$\Rightarrow \frac{x_1 + x_2 + x_3 + \ldots\ldots + x_9}{9} = 15$$

$$\therefore \quad \sum_{i=1}^{9} x_i = 135$$

if one more item x_{10} is added then

$$\sum_{i=1}^{10} x_i = 135 + x_{10}$$

$$\therefore \quad \text{mean } \bar{x} = \frac{135 + x_0}{10} = 16$$

$$\therefore \quad x_0 = 160 - 135 = 25$$

Q.2 **(4)**

Given : Mean ($\bar{x}$) = 2.5 and discrete data

Variable x :	0	1	2	3	4	5
Frequency f :	k	20	40	40	20	4

We know that

$$\Sigma f = k + 20 + 40 + 40 + 20 + 4 = 124 + k$$

We also know that

$$\Sigma fx = (0 \times k) + (1 \times 20) + (2 \times 40) + (3 \times 40) + 4(4 \times 20) + (5 \times 4)$$
$$= 0 + 20 + 80 + 20 = 320$$

Therefore, mean ($\bar{x}$)

$$= \frac{\Sigma f(x)}{\Sigma f} = \frac{320}{124+k} \quad \text{or } 2.5 = \frac{320}{124+k}$$

$$\text{or} \quad 124 + k = \frac{320}{2.5} = 128 \quad \text{or} \quad k = 128 - 124 = 4$$

Q.3 (4)

$$\frac{x_1 + x_2 + x_3 + \ldots\ldots + x_n}{n} = \bar{x}$$

$$\therefore \text{ New mean is } \bar{x}'_B = \frac{\sum_{r=1}^{x}(x_i + 2r)}{n}$$

$$\Rightarrow \quad \bar{x} + \frac{2x(x+1)}{2x}$$

$$\bar{x}' \Rightarrow \bar{x} + (n+1)$$

Q.4 (4)

If variable is decreased by 1 then there is no effect on standard deviation.

Q.5 (1)

$$\sum (x_i - \bar{x})^2 = 250$$

$$\therefore \sigma^2 = \frac{250}{10} = 25$$

$\therefore\ \sigma^2 = 25 \Rightarrow \sigma = 5$

or coefficient of variation is $= \frac{\sigma}{\overline{x}} \times 100 = \frac{5}{50} \times 100 =$ 10%

Q.6 (1)

Given $\frac{x_1 + x_2 + x_3 + \ldots\ldots + x_n}{n} = \overline{x}$

then new mean is

$$\frac{x_1 + x_2 + x_3 + \ldots + x_n + 1^2 + 2^2 + \ldots + n^2}{n}$$

$$= \overline{x} + \frac{(n+1)(2n+1)}{6}$$

Q.7 (4)

Odd divisors of 360 are

$360 = 2^3 \times 3^2 \times 5$

$\Rightarrow$ 1, 3, 5, 9, 15, 45

so mean $= \frac{1+3+5+9+15+45}{6} = 13$

median $= \frac{5+9}{2} = 7$ mean of 13 and 7 $= \frac{13+7}{2} = 10$

Q.8 (3)

$n_1 = 35,\ \overline{x}_1 = 80,\quad \sigma_1 = 4$

$n_2 = 65,\ \overline{x}_2 = 60,\quad \sigma_2 = 4$

$$\sigma^2 = \frac{n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_1^2\right)}{n_1 + n_2}$$

countioned mean $= \frac{35 \times 80 + 65 \times 60}{100} = 67$

$\therefore\ d_1 = 80 - 67 = 13,\quad d_2 = 60 - 67 = -7$

$$\therefore\ \sigma^2 = \frac{35(16+169) + 65(9+49)}{100}$$

$\sigma^2 \Rightarrow 102.45$

$\therefore\ \sigma = \sqrt{102.45} = 10.12$

Q.9 (2)

Number of candidates = 200

mean = 40

s.d. = 15

Correct data = 40

Incorrect data = 50

(i) Incorrect mean $= \frac{\sum x_i}{200} = 40$

Incorrect $\sum x_i = 8000$

correct sum $\sum x_i = 8000 - 50 + 40 = 7990$

correct mean $= \frac{\sum x_i}{200} = 39.95$

(ii) Incorrect s.d. $= \sqrt{\frac{1}{n}\sum x_i^2 - \left(\frac{1}{n}\sum x_i\right)^2}$

$$\Rightarrow \left[(\text{s.d.})^2 + \left(\frac{1}{n}\sum x_i\right)^2\right] n = \sum x_i^2$$

$\Rightarrow (225 + 1600) \times 200 = \sum x_i^2 =$ Incorrected sum

$\Rightarrow \sum x_i^2 = 365000$

$\Rightarrow$ corrected sum $\Rightarrow \sum x_i^2 - (50)^2 + (40)^2$

corrected $\sigma^2 = \left[\frac{364100}{200} - (39.95)^2\right] = 1820.5 - 1560.25 = 260$

$\sigma = 16.12$]

Q.10 (4)

Ginve : Numbers $x_1, x_2, x_3, \ldots\ldots x_n$ and their average = M. We know that the average of n numbers

$$M = \frac{x_1 + x_2 + x_3 + \ldots\ldots + x_n}{n}$$

or $\frac{nM + x}{n} = \frac{x_1 + x_2 + x_3 + \ldots\ldots + x_n + x}{n}$

or $\frac{nM + x}{n} = \frac{x_1}{n} + \frac{x_2 + x_3 + \ldots\ldots + x_n + x}{n}$

or $\frac{nM + x - x_1}{n} = \frac{x_2 + x_3 + \ldots\ldots + x_n + x}{n}$

or new average

$$= \left(\frac{x_2 + x_3 + \ldots\ldots + x_n + x}{n}\right) = \frac{nM - x_1 + x}{n}$$

Q.11 (4)

Data are multipled by 5 then variance sould be 25 times so new variance = 25 × 9 = 225]

Q.12 (2)

Mean = n items is $\bar{x}$

New mean

$$= \frac{x_1 + x_2 + \ldots\ldots x_n}{n} + \frac{3 + 3^2 + 3^3 + \ldots\ldots 3^n}{n}$$

$$\Rightarrow \bar{x} + \frac{3(3^n - 1)}{2n}$$

Q.13 (2)

Since the total number students = 100 and the number of boys = 70

$\therefore$ Number of girls = (100 – 70) = 30

Now, the total marks of 100 students = 100 × 72 = 7200

And total marks of 70 boys = 70 × 75 = 5250

$\Rightarrow$ Total marks of 30 girls = 7200 – 5250 = 1950

$\therefore$ Average marks of 30 girls = $\frac{1950}{30} = 65$

Q.14 (4)

Since the median is the 5th and the increase is made in the last four terms, hence the median remains unchanged.

Q.15 (1)

Given : N = 15

$\Sigma x^2 = 2830, \Sigma x = 170$

One observation 20 was replaced by 30, then

$\Sigma x^2 = 2830 - 400 + 900 = 3330$

$\Sigma x = 170 - 20 + 30 = 180$

$\therefore$ Variance, σ^2

$$= \frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2 = \frac{3330}{15} - \left(\frac{180}{15}\right)^2$$

$= 222 - 144 = 78$

Q.16 (3)

In the 2n observations, half of them are equal to a and the remaining half are equal to –a. Then, then mean of the total 2n observations is equal to zero.

$$\therefore \quad \text{S.D.} = \sqrt{\frac{\Sigma(x - \bar{x})^2}{N}}$$

$$\Rightarrow \quad 2 = \sqrt{\frac{\Sigma x^2}{2n}}$$

$$\Rightarrow \quad 4 = \frac{\Sigma x^2}{2n} = \frac{2na^2}{2n}$$

$\Rightarrow \quad a^2 = 4$

$\Rightarrow \quad |a| = 2$

Q.17 (3)

Each variable is increased by 20 so there will be no effect on standard deviation.

Q.18 (1)

Total number of terms = n + 1 = odd

$$\therefore \text{ median} = \frac{n + 1 + 1}{2}$$

$$\Rightarrow \left(\frac{n}{2} + 1\right)^{th} \text{ term}$$

$$\Rightarrow {}^{2n}C_{\frac{n}{2}}$$

EXERCISE-III

NUMERICAL TYPE

Q.1 [11]

Mean of $1^2, 2^2, 3^2, \ldots\ldots, n^2$ is

$$\frac{1^2 + 2^2 + 3^2 + \ldots..n^2}{n} = \frac{\Sigma n^2}{n}$$

$$\frac{46n}{11} = \frac{n(n+1)(2n+1)}{6n}$$

$\Rightarrow 22n^2 + 33n + 11 - 276n = 0$

$\Rightarrow (n - 11)(22n - 1) = 0 \Rightarrow n = 11$ and $n \neq \frac{1}{22}$

Q.2 [10.1]

$$\bar{x} = \frac{\text{sum of quantities}}{n} = \frac{\frac{n}{2}(a+1)}{n}$$

$$= \frac{1}{2}[1 + 1 + 100d] = 1 + 50d$$

$$\therefore \text{ MD} = \frac{1}{n}\Sigma|x_i - \bar{x}|$$

$$\Rightarrow 255 = \frac{1}{101}[50d + 49d + \ldots.. + d + 0 + d + 49d + 50d]$$

$$= \frac{2d}{101}\left[\frac{50 \times 51}{2}\right]$$

$$\Rightarrow d = \frac{255 \times 101}{50 \times 51} = 10.1$$

Q.3 [16]

$\overline{x} = 5$

Variance $= \frac{1}{n}\Sigma x_i^2 - \left(\overline{x}^2\right)$

$0 = \frac{1}{n} \cdot 400 - 25$

$\Rightarrow n = \frac{400}{25} = 16$

Q.4 [10]

Given that, $n_1 = 10, \overline{x}_1 = 12, n_2 = 20, \overline{x}_2 = 9$

$\therefore \overline{x} = \frac{n_1\overline{x}_1 + n_2\overline{x}_2}{n_1 + n_2} = \frac{10 \times 12 + 20 \times 9}{10 + 20}$

$= \frac{120 + 180}{30} = \frac{300}{30} = 10$

Q.5 30.1

Given that, $\Sigma_{i=1}^{20}\left(x_i - 30\right) = 2$

$\Rightarrow \sum_{i=1}^{20} x_i - \sum_{i=1}^{20}(30) = 2$

$\Rightarrow \overline{x} = \frac{20.30}{20} + \frac{2}{20} = 30 + 0.1 = 30.1$

Q.6 [24]

Given series is 148, 146, 144, 142, ... whose first term and common difference is

$a = 148, d = (146 - 148) = -2$

$S_n = \frac{n}{2}\left[2a + (n+1)d\right] = 125$ (given)

$\Rightarrow 125n = \frac{n}{2}\left[2 \times 148 + (n-1) \times (-2)\right]$

$\Rightarrow n^2 - 24n = 0 \Rightarrow n(n - 24) = 0$

$\Rightarrow n = 24\ (n \neq 0)$

Q.7 [5]

Given that, $n_1 = 4,\ \overline{x} = 7.5,\ n_1 + n_2 = 10,\ \overline{x} = 6$

$\therefore\ 6 = \frac{4 \times 7.5 + 6 \times \overline{x}_2}{10} \Rightarrow 60 = 30 + 6\overline{x}_2$

$\Rightarrow x_2 = \frac{30}{6} = 5$

Q.8 [44.46]

Total of corrected observations

$= 4500 - (91 + 13) + (19 + 31) = 4446$

$\therefore$ Mean $= \frac{4446}{100} = 44.46$

Q.9 [63]

Total marks of 10 failed students $= 28 \times 10 = 280$

and Total marks of 50 students $= 2800$

$\therefore$ Total marks of 40 passed students

$= 2800 - 280 = 2520$

$\therefore$ Average marks of 40 passed students $= \frac{2520}{40} = 63$

EXERCISE-IV

JEE-MAIN

PREVIOUS YEAR'S

Q.1 [2]

$\overline{x} = 4 + \frac{a}{4}$, then $\frac{49}{4} = \frac{\left(2+\frac{a}{4}\right)^2 + \left(1+\frac{a}{4}\right)^2 + \left(4-\frac{3a}{4}\right)^2 + \left(7-\frac{a}{4}\right)^2}{4}$

$\Rightarrow 3a^2 - 32a + 84 = 0$

Q.2 (2)

Given $\sum_{i=1}^{9}\left(x_i - 5\right) = 9 \Rightarrow \Sigma x_i = 54$ .....(i)

Also, $\sum_{i=1}^{9}\left(x_i - 5\right)^2 = 45$

$\Rightarrow \Sigma x_i^2 - 10.\Sigma x_i + 9(25) = 45$

$\Rightarrow \Sigma x_i^2 360$ (using (1)).....(ii)

As, variance $= \frac{\Sigma x_i^2}{9} - \left(\frac{\Sigma x_i}{9}\right)^2$

$= \frac{360}{9} - \left(\frac{54}{9}\right)^2$

$= 40 - 36 = 4$

Hence standard deviation is 2 (As standared deviation $= \sqrt{\text{variance}}$

Q.3 (3)

Let 5 students are x_1, x_2, x_3, x_4, x_5

Given $\overline{x} = \frac{\sum x_i}{5} = 150 \Rightarrow \sum_{i=1}^{5} = 750$ ....(i)

$\frac{\sum x_i^2}{5} - (\overline{x})^2 = 18 \Rightarrow \frac{\sum x_i^2}{5} - (150)^2 = 18$

$\Rightarrow \sum x_i^2 = (22500 + 18) \times 5$

$\Rightarrow \sum_{i=1}^{5} x_i^2 = 112590$ .....(2)

Height of new student = 156 (Let x_6)
Then $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 750 + 156$

$$\overline{x}_{new} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = \frac{906}{6} = 151 \quad ....(3)$$

Variance (new) $= \frac{\sum x_i^2}{6} - \left(\overline{x}_{new}\right)^2$

from equation (2) and (3)
variance (new)

$$= \frac{112590 + (156)^2}{6} - (151)^2 = 22821 - 22801 = 20$$

Q.4 (2)

$$\sum (x_i + 1)^2 = 9n \quad ....(1)$$

$$\sum (x_i - 1)^2 = 5n \quad ....(2)$$

$$(1) + (2) \Rightarrow \sum \left(x_i^2 - 1\right) = 7n$$

$$\Rightarrow \frac{\sum x_i^2}{n} = 6$$

$$(1) . (2) \Rightarrow 4\sum x_i = 4n$$

$$\Rightarrow \sum x_i = n \Rightarrow \frac{\sum x_i}{n} = 1 \Rightarrow \text{variance} = 6 - 1 = 5$$

$\Rightarrow$ standard diviation $= \sqrt{5}$

Q.5 (2)

$$\mu = \frac{1 + 3 + 8 + x + y}{5}$$

$25 = 12 + x + y \Rightarrow x + y = 13 \quad .....(1)$

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$$

$$9.2 = \frac{1 + 9 + 64 + x^2 + y^2}{5} - 25$$

$34.2 \times 5 = 74 + x^2 + y^2$
$171 = 74 + x^2 + y^2$
$97 = x^2 + y^2 \quad .....(2)$
$(x + y)^2 = x^2 + y^2 + 2xy$
$169 - 97 = 2xy \Rightarrow xy = 36$
$T = 4, 9$

So ratio is $\frac{4}{9}$ or $\frac{9}{4}$

Q.6 (4)
$\Sigma x = 50$

$$(3)^2 = \frac{1}{5}\left(\sum x^2 - \frac{\left(\sum x\right)^2}{5}\right)$$

$$9 = \frac{1}{5}\left(\Sigma x^2 - \frac{2500}{2}\right)$$

$\therefore \Sigma x^2 = 545$

Now variable $= \frac{1}{6}\left(3045 - \frac{0}{6}\right) = 507.5$

Q.7 (2)
There are 30 white balls and 10 red balls

$$P(\text{white ball}) = \frac{30}{40} = \frac{3}{4} = p$$

$$\Rightarrow \quad q = \frac{1}{4}$$

$$\frac{\text{mean of } (x)}{\text{standard deviation of } (x)} = \frac{np}{\sqrt{npq}}$$

$$= \sqrt{\frac{np}{q}} = \sqrt{\frac{16 \times \left(\frac{3}{4}\right)}{\frac{1}{4}}} = 4\sqrt{3}$$

Q.8 (4)

$$\text{Given } \sum_{i=1}^{50} (x_i - 30) = 50$$

$$\Rightarrow \sum x_i = 30(50) + 50 \Rightarrow \frac{\sum x_i}{50} = 31$$

Q.9 (1)
Mean $\overline{x} = 4, \sigma^2 = 5.2, n = 5, x_1 = 3x_2 = 4 = x_3$

$\Sigma x_i = 20$

$x_4 + x_5 = 9 \quad ...(i)$

$$\frac{\Sigma x_i^2}{x} - (\overline{x})^2 = \sigma^2 \Rightarrow \Sigma x_i^2 = 106$$

$x_4^2 + x_5^2 = 65 \quad ...(ii)$

Using (i) and (ii) $(x_4 - x_5)^2 = 49$

$|x_4 - x_5| = 7$

Q.10 (3)
Let 7 observations be $x_1, x_2, x_3, x_4, x_5, x_6, x_7$

$$\overline{x} = 8 \Rightarrow \sum_{i=1}^{7} x_i = 56 \quad ....(1)$$

Also $\sigma^2 = 16$

$$\Rightarrow \quad 16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - (\overline{x})^2$$

$\Rightarrow \quad 16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - 64$

$\Rightarrow \quad \left(\sum_{i=1}^{7} x_i^2\right) = 560$ ....(2)

Now, $x_1 = 2, x_2 = 4, x_3 = 10, x_4 = 12, x_5 = 14$
$\Rightarrow \quad x_6 + x_7 = 14$ (from (1))
& $x_6^2 + x_7^2 = 100$ (from (2))
$\therefore \quad x_6^2 + x_7^2 = (x_6 + x_7)^2 - 2x_6x_7 \Rightarrow x_6 . x_7 = 48$

Q.11 (1)
Let x be the 6th observation
$\Rightarrow 45 + 54 + 41 + 57 + 43 + x = 48 \times 6 = 288$
$\Rightarrow x = 48$

$$\text{variance} = \left(\frac{\Sigma x_i^2}{6} - (\bar{x})^2\right)$$

$$\Rightarrow \text{variance} = \frac{14024}{6} - (48)^2$$

$$= \frac{100}{3}$$

$$\Rightarrow \text{standard deviation} = \frac{10}{\sqrt{3}}$$

Q.12 (2)

$$S.D = \sqrt{\frac{\Sigma\left(x - \bar{x}\right)^2}{n}}$$

$$\bar{x} = \frac{\Sigma x}{4} = \frac{-1+0+1+k}{4} = \frac{k}{4}$$

Now

$$\sqrt{5} = \sqrt{\frac{\left(-1-\frac{k}{4}\right)^2 + \left(0-\frac{k}{4}\right)^2 + \left(1+\frac{k}{4}\right)^2 + \left(k-\frac{k}{4}\right)^2}{4}}$$

$$\Rightarrow 5 \times 4 = 2\left(1+\frac{k}{16}\right)^2 + \frac{5k^2}{8}$$

$$\Rightarrow 18 = \frac{3k^2}{4}$$

$\Rightarrow k^2 = 24$
$\Rightarrow k = 2\sqrt{6}$

Q.13 (1)

$$\frac{34 + x}{2} = 35$$

$x = 36$

$$42 = \frac{10+22+26+29+34+36+42+67+70+y}{10}$$

$420 - 336 = y \Rightarrow y = 84$

$$\frac{y}{x} = \frac{84}{36} = \frac{7}{3}$$

Q.14 (1)
$\Sigma f_i = 20 = 2x^2 + 2x - 4$
$\Rightarrow \quad x^2 + 2x - 24 = 0$
$x = 3, -4$ (rejected)

$$\bar{x} = \frac{\sum x_i f_i}{\sum f_i} = 2.8$$

Q.15 (4)

$$\text{Mean } (\mu) = \frac{\sum x_i}{50} = 16$$

$$\text{standard deviation } (\sigma) = \sqrt{\frac{\sum x_i^2}{50} - (\mu)^2} = 16$$

$$\Rightarrow (256) \times 2 = \frac{\sum x_i^2}{50}$$

$\Rightarrow$ New mean

$$= \frac{\sum (X_i - 4)^2}{50} = \frac{\sum x_i^2 + 16 \times 50 - 8\sum x_i}{50}$$

$= (256) \times 2 + 16 - 8 \times 16 = 400$

Q.16 (2)
$x_1 + ... + x_4 = 44$
$x_5 + ... + x_{10} = 96$
$\bar{x} = 14$, $\Sigma x_i = 140$

$$\text{Variance} = \frac{\Sigma x_i^2}{n} - \bar{x}^2 = 4$$

Standard deviation = 2

Q.17 [18]
$\text{Var}(1, 2, \ldots\ldots, n) = 10$

$$\Rightarrow \frac{1^2 + 2^2 + \ldots\ldots + n^2}{n} - \left(\frac{1 + 2 + \ldots\ldots + n}{n}\right)^2 = 10$$

$$\Rightarrow \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 = 10$$

$\Rightarrow n^2 - 1 = 120 \Rightarrow n = 11$
$\text{Var}(2, 4, 6, \ldots\ldots, 2m) = 16 \Rightarrow \text{Var}(1, 2, \ldots\ldots, m) = 4$
$\Rightarrow m^2 - 1 = 48 \Rightarrow m = 7 \Rightarrow m + n = 18.$

Q.18 [54]

mean $\bar{x} = \dfrac{3+7+9+12+13+20+x+y}{8} = 10$

$\Rightarrow x + y = 16$ ....(i)

variance $\sigma^2 = \dfrac{\sum (x_i)^2}{8} - (\bar{x})^2 = 25$

$\dfrac{9+49+81+144+169+400+x^2+y^2}{8} - 100 = 25$

$\Rightarrow \quad x^2 + y^2 = 148$ ....(ii)

$(x+y)^2 = (16)^2 \Rightarrow x^2 + y^2 + 2xy = 256 \Rightarrow xy = 54$

Q.19 (1)

If each observation is multiplied with p and then q is subtracted

New mean $\bar{x}_1 = p\bar{x} - q$

$\Rightarrow 10 = p(20) - q$ ……..(1)

and new standard deviations

$\Rightarrow \sigma_2 = |\rho|\sigma_1 \Rightarrow 1 = |p|(2) \Rightarrow |p| = \dfrac{1}{2} \Rightarrow p = \pm\dfrac{1}{2}$

If $p = \dfrac{1}{2}$, then $q = 0$ [from equation (1)]

If $p = \dfrac{-1}{2}$, then $q = -20$.

Q.20 (1)

$\dfrac{\Sigma x_i}{20} = 10$ ....(i)

$\dfrac{\Sigma x_i^2}{20} - 100 = 4$ ....(ii)

$\Sigma x_i^2 = 104 \times 20 = 2080$

Actual mean $= \dfrac{200-9+11}{20} = \dfrac{202}{20}$

Variance $= \dfrac{2080-81+121}{20} - \left(\dfrac{202}{20}\right)^2$

$= \dfrac{2120}{20} - (10.1)^2 = 106 - 102.01 = 3.99$

Q.21 (1)

Mean $(x_i - 5) = \dfrac{\Sigma(x_i-5)}{10} = 1$

$\therefore \quad \lambda = \{\text{Mean}(x_i - 5)\} + 2 = 3$

$\mu = \text{var}(x_i - 5) = \dfrac{\Sigma(x_i-5)^2}{10} - \dfrac{\Sigma(x_i-5)}{10} = 3$

Q.22 (3)

$$\text{Variance} = \frac{\sum_{i=1}^{11} b_i^2}{11} - \left(\frac{\sum_{i=1}^{11} b_i}{11}\right)^2$$

$$= \frac{\sum_{i=0}^{11}(b_1+rd)^2}{11} - \left(\frac{\sum_{i=0}^{10}(b_1+rd)}{11}\right)^2$$

$$= \frac{11b_1^2 + 2b_1d\left(\frac{10\times11}{2}\right) + d^2\left(\frac{10\times11\times21}{6}\right)}{11}$$

$$- \left(\frac{11b_1 + \frac{10\times11}{2}d}{11}\right)^2$$

$= (b_1^2 + 10b_1d + 35d^2) - (b_1 + 5d)^2 = 10d^2$

$\because$ Variance $= 90$

$\Rightarrow 10d^2 = 90 \Rightarrow d = 3$

Q.23 (4)

$\because \ \bar{x} = \dfrac{\sum_{r=1}^{17} r}{17} = \dfrac{17\times18}{17\times2} = 9$

$\because \ \bar{y} = a\bar{x} + b = 17 \Rightarrow 9a + b = 17$ .....(i)

$\because \ \text{Var}(x) = \dfrac{\sum_{r=1}^{17} r^2}{17} - (\bar{x})^2 = \dfrac{17\times18\times35}{17\times6} - 9^2 = 24$

and Var (y) $= a^2$. Var(x) $\Rightarrow 24a^2 = 216 \Rightarrow a = 3$ ....(2)

Clearly $b = -10$

Hence $a + b = -7$

Q.24 (2)

Here $n = 10$

So, S.D. $= \sqrt{\dfrac{\sum(x_i-p)^2}{n} - \left(\dfrac{\sum x_i - p}{n}\right)^2}$

$= \sqrt{\dfrac{9}{10} - \dfrac{9}{100}} = \sqrt{\dfrac{90-9}{100}} = \sqrt{\dfrac{9}{10}}$

Q.25 (2)

If variate varries from a to b then variance

$$\text{var}(x) \le \left(\frac{b-a}{2}\right)^2 \Rightarrow \text{var}(x) < \left(\frac{10-0}{2}\right)^2 \Rightarrow \text{var}(x) < 25$$

$\Rightarrow$ Standard deviation < 5

Clearly standard deviation cann't be 6.

Q.26 (4)

x_i	15	25	35
f_i	2	x	2

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{30+70+25x}{4+x} = 25$$

$$\text{Given } \sigma^2 = 50 = \frac{\sum f_i x_i}{\sum f_i} - (\bar{x})^2$$

$$\Rightarrow 50 = \frac{450+625x+2450}{4+x} - 625$$

$$\Rightarrow 675 = \frac{2900+625x}{4+x} \Rightarrow 50x = 200$$

$\therefore x = 4$

Q.27 (3)

Let the two remaining observations be x and y.

$$\because \ \bar{x} = 10 = \frac{5+7+10+12+14+15+x+y}{8}$$

$\Rightarrow x + y = 17$ .....(1)

$\because$ var (x) = 13.5

$$= \frac{25+49+100+144+196+225+x^2+y^2}{8} - (10)^2$$

$\Rightarrow x^2 + y^2 = 169$ ....(2)

From (1) and (2)

(x, y) = (12, 5) or (5, 12)

So $|x - y| = 7$

Q.28 (4)

$$\text{Mean} = \frac{3+5+7+a+b}{5} = 5 \Rightarrow a + b = 10$$

$$\text{Variance} = \frac{3^2+5^2+7^2+a^2+b^2}{5} - (5)^2 = 4$$

$\Rightarrow a^2 + b^2 = 62 \ \Rightarrow (a+b)^2 - 2ab = 62 \qquad \Rightarrow ab = 19$

So, a and b are the roots of the equation

$x^2 - 10x + 19 = 0$

Q.29 (1)

Let two remaining observations are x, y

$$\text{So } \bar{x} = \frac{2+4+10+12+14+x+y}{7} = 8 \qquad \text{(given)}$$

$\Rightarrow x + y = 14$ .... (1)

$$\text{Now also } \sigma^2 = \frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2 = 16 \qquad \text{(given)}$$

$$= \frac{4+16+100+144+196+x^2+y^2}{7} - 64 = 16$$

$\Rightarrow 460 + x^2 + y^2 = (16 + 64) \times 7$

$\Rightarrow x^2 + y^2 = 100$ .... (2)

Now $(x + y)^2 = x^2 + y^2 + 2xy \Rightarrow xy = 48$ .....(3)

Now $(x - y)^2 = (x + y)^2 - 4xy = 196 - 192 = 4$

$\Rightarrow x - y = 2 \ \Rightarrow |x - y| = 2$

Q.30 [6]

$$\text{Mean } \frac{\sum X_i f_i}{\sum f_i}$$

$$= \frac{0.{}^nC_0 + 2.{}^nC_1 + 2^2.{}^nC_2 + ... + 2^n.{}^nC_n}{{}^nC_0 + {}^nC_1 + ... + {}^nC_n +}$$

For finding sum of numerator consider

$(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ... + {}^nC_nx^n$ .....(1)

Put $x = 2 \Rightarrow 3^n - 1 = 2.{}^nC_1 + 2^2.{}^nC_2 + ... + 2^n.{}^nC_n$

For sum of denominator

Put x = 1 in (1)

$2^n = {}^nC_0 + {}^nC_1 + ... + {}^nC_n$

$$\therefore \ \frac{3^n - 1}{2^n} = \frac{728}{2^n} \Rightarrow 3n = 728 \Rightarrow n = 6$$

Q.31 (4)

$$\text{Standard deviation} = \sqrt{\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2}$$

$$= \sqrt{\frac{\sum (x_i - a)^2}{N} - \left(\frac{\sum (x_i - a)}{N}\right)^2}$$

$$= \sqrt{\frac{na}{n} - \left(\frac{n}{n}\right)^2} = \sqrt{a-1}$$