Chapter 2 – Pythagoras Theorem

Practice set 2.1

1. Identify, with reason, which of the following are Pythagorean triplets.
(i)(3, 5, 4)
(ii)(4, 9, 12)
(iii)(5, 12, 13)
(iv) (24, 70, 74)
(v)(10, 24, 27)
(vi)(11, 60, 61)

Solution:
(i)(3, 5, 4)
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
Here, 9+16 = 25
$5^2 = 3^2+4^2$
The square of largest number is equal to sum of squares of the other two numbers.
∴ (3, 5, 4) is a Pythagorean triplet.

(ii)(4, 9, 12)
$4^2 = 16$
$9^2 = 81$
$12^2 = 144$
Here $4^2+9^2 \neq 12^2$
The square of largest number is not equal to sum of squares of the other two numbers.
∴ (4, 9, 12) is not a Pythagorean triplet.

(iii)(5, 12, 13)
$5^2 = 25$
$12^2 = 144$
$13^2 = 169$
Here $5^2+ 12^2 = 13^2$
The square of largest number is equal to sum of squares of the other two numbers.
∴ (5, 12, 13) is a Pythagorean triplet.

(iv) (24, 70, 74)
$24^2 = 576$
$70^2 = 4900$
$74^2 = 5476$
Here, $24^2 + 70^2 = 74^2$
The square of largest number is equal to sum of squares of the other two numbers.
$\therefore$ (24, 70, 74) is a Pythagorean triplet.

(v)(10, 24, 27)
$10^2 = 100$
$24^2 = 576$
$27^2 = 729$
Here $10^2+24^2 \neq 27^2$
The square of largest number is not equal to sum of squares of the other two numbers.
$\therefore$ (10, 24, 27) is not a Pythagorean triplet.

(vi) (11, 60, 61)
$11^2 = 121$
$60^2 = 3600$
$61^2 = 3721$
Here $11^2+60^2 = 61^2$
The square of largest number is equal to sum of squares of the other two numbers.
$\therefore$ (11, 60, 61) is a Pythagorean triplet.

2. In figure 2.17, $\angle$MNP = 90°, seg NQ $\perp$ seg MP, MQ = 9, QP = 4, find NQ.

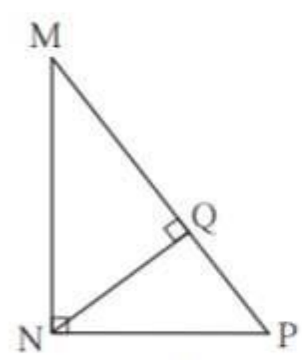


Fig. 2.17

Solution:
In ΔMNP, $\angle$MNP = 90°, seg NQ $\perp$ seg MP
MQ = 9, QP = 4 [Given]
NQ = $\sqrt{(MQ \times QP)}$ [Theorem of geometric mean]
$\therefore$ NQ = $\sqrt{(9 \times 4)} = \sqrt{36} = 6$
Hence NQ = 6 units.

3. In figure 2.18, ∠QPR = 90°, seg PM ⊥ seg QR and Q-M-R, PM = 10, QM = 8, find QR.

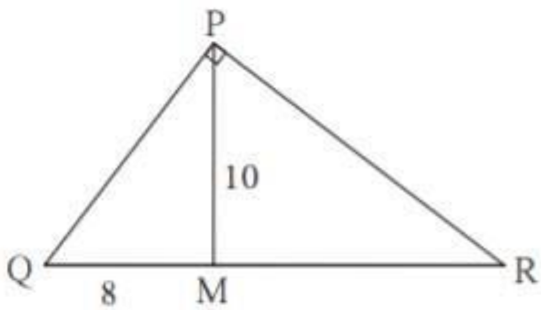


Fig. 2.18

Solution:

In ΔPQR , ∠QPR = 90˚

seg PM ⊥ seg QR [Given]

$PM^2 = QM \times MR$ [Theorem of geometric mean]

$10^2 = 8 \times MR$

MR = 100/8 = 12.5

QR = QM + MR

QR = 8+12.5 = 20.5

Hence, measure of QR is 20.5 units.

4. See figure 2.19. Find RP and PS using the information given in □ PSR.

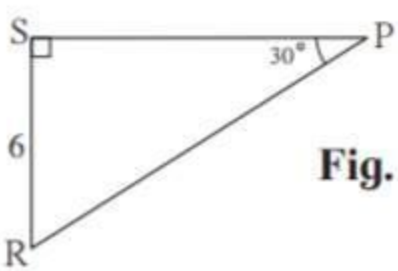


Fig. 2.19

Solution:

In ΔPSR , ∠P = 30˚ , ∠S = 90˚

∠R = 180 - (90+30) = 60˚ [Angle Sum property of triangle]

ΔPSR is a 30˚ - 60˚ - 90˚ triangle.

SR = (½)×PR [side opposite to 30˚]

6 = (½)× PR

PR = 12

$PS = \sqrt{(PR^2 - SR^2}$ [Pythagoras theorem]

$PS = \sqrt{(12^2-6^2)}$

$PS = \sqrt{(144-36)}$

$PS = \sqrt{108}$

$PS = 6\sqrt{3}$

Hence, RP = 12 units and PS = $6\sqrt{3}$ units.

5. For finding AB and BC with the help of information given in figure 2.20, complete following activity.

AB = BC ….____

∴ ∠BAC = _____

∴ AB = BC = ____× AC

= ____ $\times\sqrt{8}$

= ____ $\times 2\sqrt{2}$

= ____

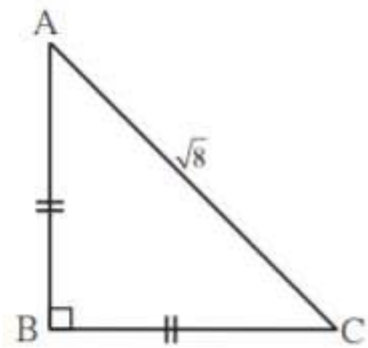


Fig. 2.20

Solution:

AB = AC [Given]

∴ ∠BAC = ∠BCA [Angles opposite to equal sides of an isosceles triangle are equal]

∴ AB = BC = $1/\sqrt{2}\times$ AC [By 45° - 45°-90° theorem]

$= (1/\sqrt{2}) \times\sqrt{8}$

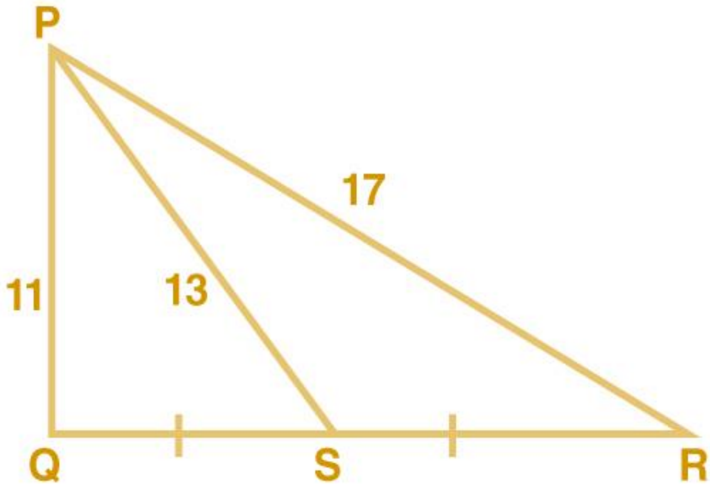
$= (1/\sqrt{2}) \times 2\sqrt{2}$

$= 2$

Practice set 2.2

1. In $\triangle PQR$, point S is the midpoint of side QR. If PQ = 11,PR = 17, PS =13, find QR.

Solution:



Given, S is the midpoint of QR.

$\therefore$ PS is the median.

$PQ^2+PR^2 = 2\ PS^2+2SR^2$ [By Apollonius theorem]

$11^2+17^2 = 2\times13^2+2\times SR^2$

$121+289 = 2\times169+2\times SR^2$

$2SR^2 = 121+289-338$

$2SR^2 = 72$

$SR^2 = 72/2 = 36$

SR = 6

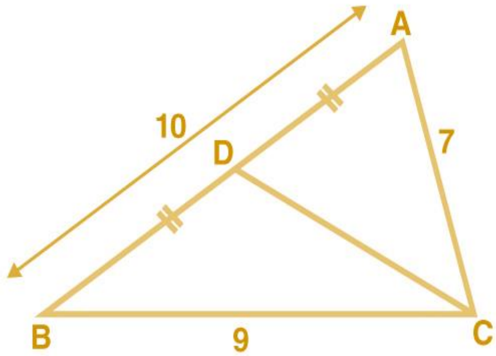
Since S is the midpoint of QR, QR = 2SR

$QR = 2\times6 = 12$

Hence QR = 12 units.

2. In $\triangle ABC$, AB = 10, AC = 7, BC = 9 then find the length of the median drawn from point C to side AB

Solution:



Let CD is the median drawn from C to AB.
Given AB = 10
AD = (1/2)×AB   [D is the midpoint of side AB]
AD = 10/2 = 5
Since CD is the median
$AC^2+BC^2 = 2CD^2+2AD^2$ [Apollonius theorem]
$\therefore\ 7^2+9^2 = 2\ CD^2+2\times5^2$
$2\ CD^2 = 7^2+9^2-2\times5^2$
$2\ CD^2 = 80$
$CD^2 = 40$
Taking square roots on both sides
CD = 2√10
Hence, the length of median drawn from point C to side AB is 2√10 units.

3. In the figure 2.28 seg PS is the median of △PQR and PT⊥QR. Prove that,
(1) $PR^2 = PS^2 + QR \times ST + (QR/2)^2$

(ii) $PQ^2 = PS^2 - QR \times ST + (QR/2)^2$

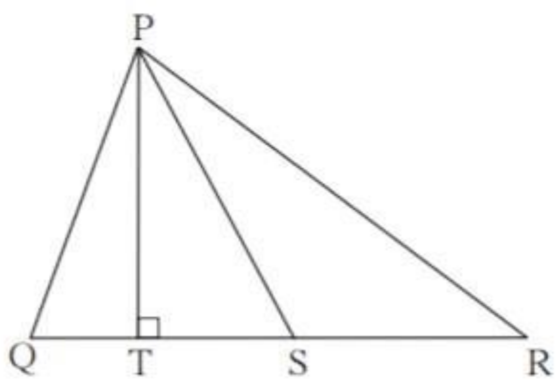
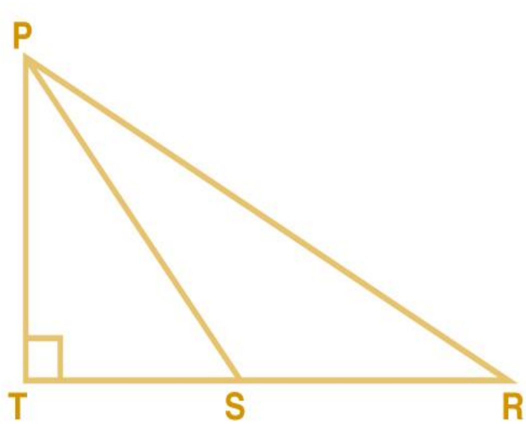


Fig. 2.28

Solution:



(1) $QS = \frac{1}{2} QR$ ……(i) [S is the midpoint of QR]

$SR = \frac{1}{2} QR$ ……(ii)

$\therefore$ $QS = SR$ [From (i) and (ii)]

$PT \perp QR$ [Given]

$\angle PSR$ is an obtuse angle. [From figure]

$\therefore$ $PR^2 = SR^2+PS^2+2SR\times ST$ …..(iii) [Application of Pythagoras theorem]

Substitute $SR = \frac{1}{2} QR$ in (iii)

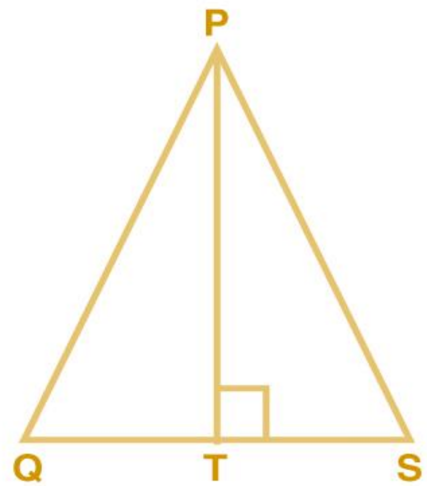
$PR^2 =[(\frac{1}{2})QR]^2+PS^2+2(1/2)QR\times ST$

$PR^2 =[(\frac{1}{2})QR]^2+PS^2+QR\times ST$

$PR^2 = PS^2 + QR\times ST + (QR/ 2)^2$

Hence proved.

(ii) $PT\perp QS$ (Given)



$\angle PSQ$ is an acute angle [From figure]

$PQ^2 = QS^2+PS^2-2QS\times ST$ [Application of Pythagoras theorem]

$PR^2 = [(1/2) QR]^2+PS^2-2(1/2)QR\times ST$

$PR^2 = (QR/2)^2+PS^2-QR\times ST$

$PR^2 = PS^2-QR\times ST+ (QR /2)^2$

Hence proved.

Problem set 2

1. Some questions and their alternative answers are given. Select the correct alternative.
(1) Out of the following which is the Pythagorean triplet?
(A) (1, 5, 10)
(B) (3, 4, 5)
(C) (2, 2, 2)
(D) (5, 5, 2)

Solution:
(A) (1, 5, 10)
Here $1^2+5^2 \neq 10^2$
The square of largest number is not equal to sum of squares of the other two numbers. So (1, 5, 10) is not a Pythagorean triplet.

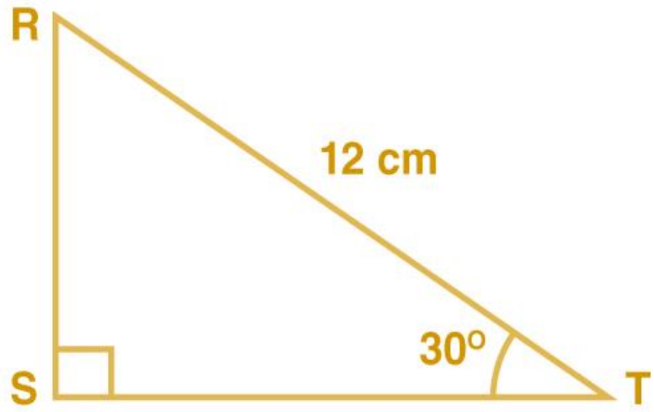
(B) (3, 4, 5)
Here $3^2+4^2 = 5^2$
The square of largest number is equal to sum of squares of the other two numbers. So (3, 4, 5) is a Pythagorean triplet.

(C) (2, 2, 2)
Here $2^2+2^2 \neq 2^2$
The square of largest number is not equal to sum of squares of the other two numbers.
So (2, 2, 2) is not a Pythagorean triplet.

(D) (5, 5, 2)
Here $2^2+5^2 \neq 5^2$
The square of largest number is not equal to sum of squares of the other two numbers.
So (5, 5, 2) is not a Pythagorean triplet.
Hence, option B is the correct answer.

3. In ΔRST, ∠S = 90°, ∠T = 30°, RT = 12 cm then find RS and ST.

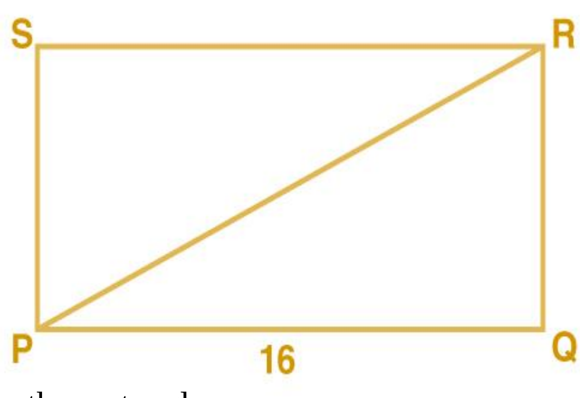
Solution:



Given ∠S = 90˚, ∠T = 30˚
∠R = 180-(90+30) = 60˚ [Sum of angles of triangle is equal to 180˚]
ΔRST is a 30˚ - 60˚- 90˚ triangle
RS = ½ RT [Side opposite to 30˚]
RS = ½ ×12 = 6
ST = √3/2 RT [side opposite to 60˚]
ST = (√3/2) ×12
ST = 6√3
Hence, RS = 6 cm and ST = 6√3cm.

4. Find the diagonal of a rectangle whose length is 16 cm and area is 192 sq.cm.

Solution:

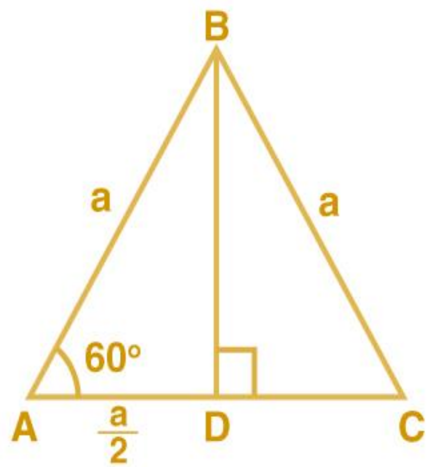


Let PQRS be the rectangle.
Let length be PQ = 16 cm
Area of rectangle = Length × Breadth
Area of rectangle PQRS = PQ×QR
∴ 192 = 16×QR
QR = 192/16 = 12cm

Now in ΔPQR, $\angle Q = 90^\circ$ [Angles of a rectangle are 90°]
$PR^2 = PQ^2+QR^2$ [Pythagoras theorem]
$PR^2 = 16^2+12^2$
$PR^2 = 256+144 = 400$
$PR = 20$
Hence, the diagonal of the rectangle is 20cm long.

5*. Find the length of the side and perimeter of an equilateral triangle whose height is √3 cm.

Solution:



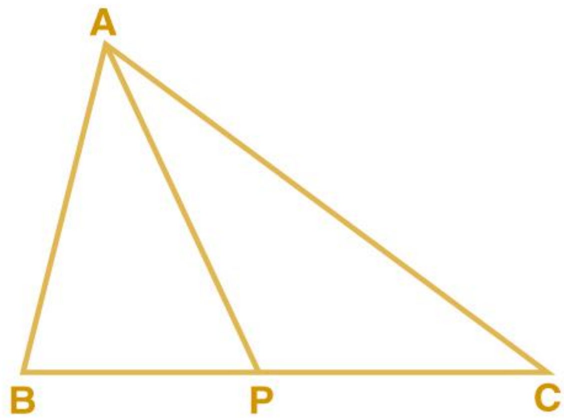
Let ABC be the equilateral triangle with side a.
Let BD be height of the triangle.
Since ΔABC is equilateral, BD is a perpendicular bisector.
∴ $AD = a/2$
$BD = \sqrt{3}$ [given height = $\sqrt{3}$]
$AB = a$
Applying Pythagoras theorem in ΔABD
$AB^2 = AD^2+BD^2$
$a^2 = (a/2)^2+ (\sqrt{3})^2$
$a^2 = (a^2/4)+3$
$(3/4)\ a^2 = 3$

$a^2 = 4$
$a = 2$
Hence, length of side of equilateral triangle is 2 cm.
∴ Perimeter = $3\times2 = 6$

[Perimeter of equilateral triangle = 3×side]
Hence, perimeter of equilateral triangle is 6 cm.

6. In $\triangle ABC$ seg AP is a median. If BC = 18, $AB^2 + AC^2 = 260$ Find AP.

Solution:



Given AP is the median.

$\therefore$ PC = BC/2

PC = 18/2 =9

$AB^2+AC^2 = 2AP^2+2PC^2$ [Apollonius theorem]

$260 = 2AP^2+2\times9^2$

$2AP^2 = 260-2\times9^2$

$2AP^2 = 260-162$

$AP^2 = 68/2 = 49$

Taking square roots on both sides

AP = 7

Hence AP = 7 units.

7*. $\triangle ABC$ is an equilateral triangle. Point P is on base BC such that PC = 1/3 BC, if AB = 6 cm find AP.

Solution:

Given $\triangle ABC$ is an equilateral triangle.

PC = (1/3) BC

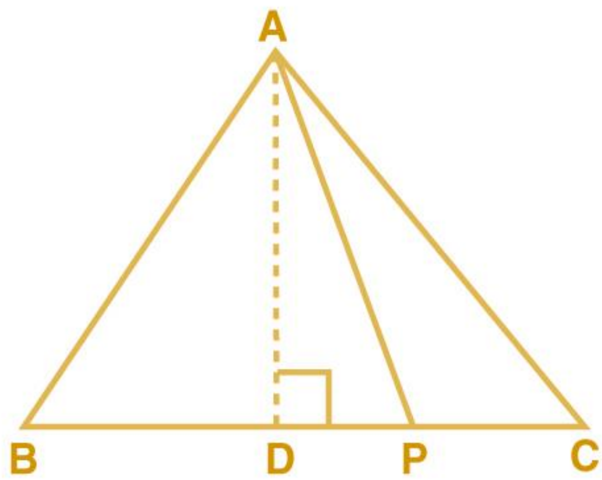
$\therefore$ PC = $(1/3)\times6$

[BC = 6, side of equilateral triangle]

PC = 2

Construction:

Draw segment $AD\perp BC$



In ΔADC, $\angle C = 60^\circ$

$\angle D = 90^\circ$

$\angle CAD = 30^\circ$

ΔADC is a 30°- 60°- 90°triangle

$AD = (\sqrt{3}/2) \times AC$ [Side opposite to 60°]

$AD = (\sqrt{3}/2) \times 6$ □ AD

$=3\sqrt{3}$cm

$DC = (1/2)\ BC$ [$AD \perp BC$]

$DC = (1/2) \times 6 = 3$cm

$DC = DP+PC$ [D-P-C]

$3 = DP + 2$

$DP = 1$

In ΔADP, $\angle D = 90^\circ$

Applying Pythagoras theorem

$AP^2 = AD^2+DP^2$

$AP^2 = (3\sqrt{3})^2+1^2$

$AP^2 = 28$

$AP = 2\sqrt{7}$ cm

Hence, $AP = 2\sqrt{7}$cm.

8. From the information given in the figure 2.31, prove that PM = PN = √3×a

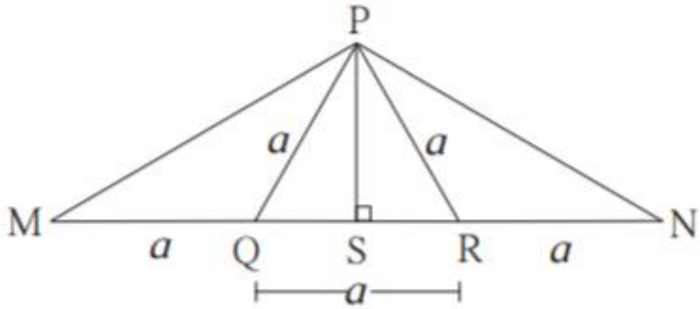
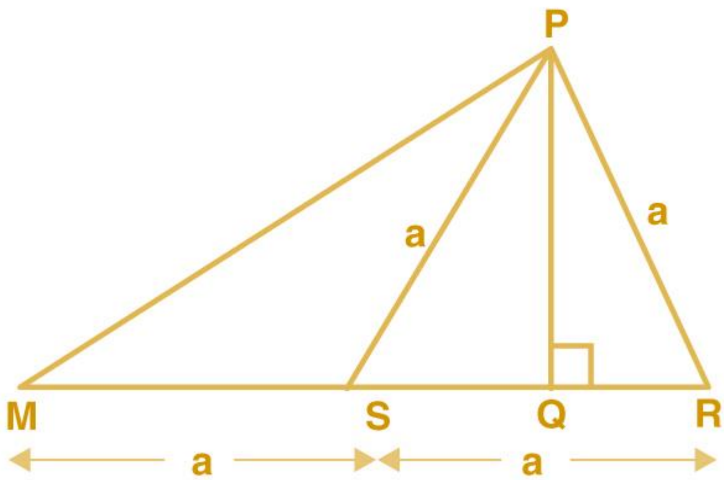


Fig. 2.31

Solution:



In ΔPRM

Given MQ = QR = a

Q is the midpoint of MR.

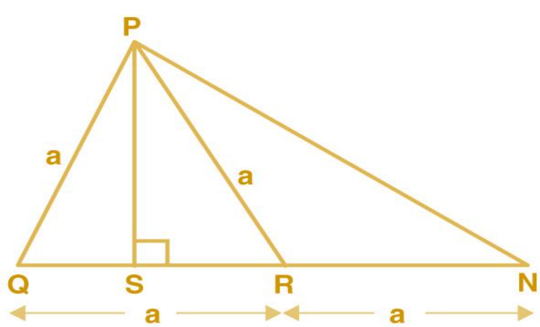
PQ is the median.

$PR^2+PM^2 = 2PQ^2+2QM^2$ [Apollonius theorem]

$a^2+PM^2 = 2a^2+2a^2$

$PM^2 = 3a^2$

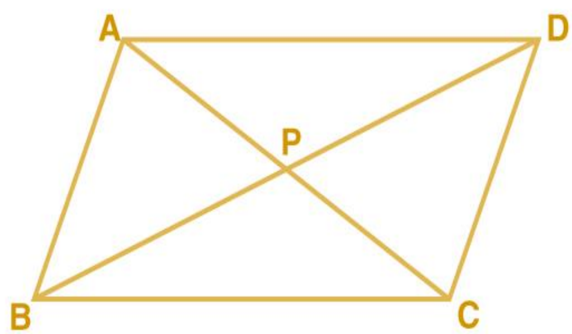
$PM = \sqrt{3}a$………… (i)



In ΔPQN
Given. $NR = QR = a$
R is the midpoint of QN.
PR is the median.
$PN^2+PQ^2 = 2PR^2+2RN^2$ [Apollonius theorem]
$PN^2+a^2 = 2a^2+2a^2$
$PN^2 = 3a^2$
$PN = \sqrt{3}a$…………..(ii)
From (i) and (ii) $PM = PN = \sqrt{3}\times a$
Hence proved.

9. Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.

Solution:
Construction:
Draw a parallelogram ABCD. Let diagonals AC and BD meet at P.

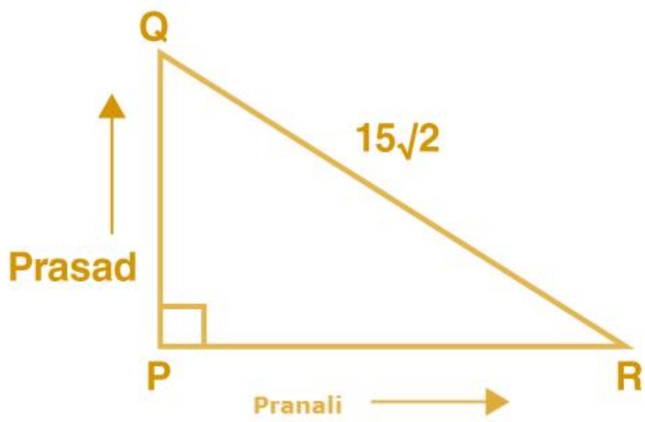


To prove: $AC^2+BD^2 = AB^2+BC^2+CD^2+DA^2$
$AB = CD$ [Opposite sides of parallelogram are equal]
$BC = DA$ [Opposite sides of parallelogram are equal]
Since diagonals of a parallelogram bisect each other,
$AP = ½ AC$
$BP = ½ BD$
P is the midpoint of diagonals AC and BD.
In ΔABC, BP is the median.
$AB^2+BC^2 = 2AP^2+2BP^2$ [Apollonius theorem]
$AB^2+BC^2 = 2[(1/2) AC]^2+2[(1/2)BD]^2$
$AB^2+BC^2 =AC^2/2+BD^2/2$
$2(AB^2+BC^2) = AC^2+BD^2$

$2AB^2+2BC^2 = AC^2+BD^2$
$AB^2+ AB^2+ BC^2+BC^2 = AC^2+BD^2$
$AB^2+ CD^2+ BC^2+DA^2 = AC^2+BD^2$
$AC^2+BD^2 = AB^2+BC^2+CD^2+DA^2$
Hence proved.

10. Pranali and Prasad started walking to the East and to the North respectively, from the same point and at the same speed. After 2 hours distance between them was 15√2 km. Find their speed per hour.

Solution:



Distance between Pranali and Prasad after 2 hours = $15\sqrt{2}$ km
Since they travel at same speed, they have covered same distance.
Construction: Draw a triangle PQR such that PQ = PR = x and QR = $15\sqrt{2}$

$\angle P = 90^\circ$
In ΔPQR , $PQ^2+PR^2 = QR^2$ [Pythagoras theorem]
$x^2+x^2 = (15\sqrt{2})^2$
$2x^2 = 2\times225$
$x^2 = 225$
$x = 15$
∴ Distance covered by them is 15 km.
Given time = 2 hours
Speed = Distance / time
Speed = 15/2 = 7.5km/hr
∴ Speed of Pranali and Prasad is 7.5km/hr.