ACTIVITY 5

To draw the graph of $\sin^{-1} x$, using the graph of $\sin x$ and demonstrate the concept of mirror reflection (about the line $y = x$).

Material Required

Cardboard, white chart paper, scale, coloured pens, gum, pencil, eraser, cutter, some nails and some thin wires.

Method of Construction

1. Take a cardboard of suitable dimensions, say, 40 cm × 40 cm.
2. On the cardboard, fix a white chart paper of size 30 cm × 30 cm (say) using gum.
3. On the paper, draw two lines, perpendicular to each other and name them $X'OX$ and YOY' as rectangular axes [see given Fig.].
4. Graduate the axes approximately as shown in figure by taking unit on X - axis = 1.25 times the unit of Y-axis.
5. Mark the points (approximately) $\left(\frac{\pi}{6}, \sin\frac{\pi}{6}\right), \left(\frac{\pi}{4}, \sin\frac{\pi}{4}\right), \ldots, \left(\frac{\pi}{2}, \sin\frac{\pi}{2}\right)$ in the coordinate plane.

   Also, fix nail at each point and name them N_1, N_2, N_3, N_4.
6. Repeat the above process on the other side of the X - axis, marking the points $\left(\frac{-\pi}{6}, \sin\frac{-\pi}{6}\right), \left(\frac{-\pi}{4}, \sin\frac{-\pi}{4}\right), \ldots, \left(\frac{-\pi}{2}, \sin\frac{-\pi}{2}\right)$ approximately and fix nails on these points as N_1', N_2', N_3', N_4'. At O, also fix a nail.

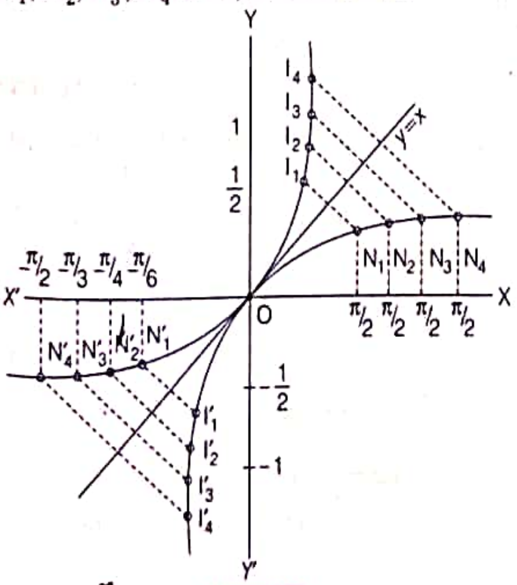

Figure 5.1

7. With the help of wire join the nail on both sides of X-axis to get the graph of $\sin x$ from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$.
8. Draw the graph of the line $y = x$ by plotting the points (1, 1), (2, 2), (3, 3), ... etc. At these points fix a wire.
9. From the nails N_1, N_2, N_3, N_4, draw perpendicular on the line $y = x$ and produce these lines such that length of perpendicular on both sides of the line $y = x$ are equal. Name them I_1, I_2, I_3, I_3 and fix nails at these points
10. Similarly, repeat the above activity on the other side of X - axis and fix nails at I_1', I_2', I_3', I_4'.
11. Join the nails on both sides of the line $y = x$ by a tight wire. The curve so obtain shows the graph of $y = \sin^{-1} x$.

Demonstration

Put a mirror on the line $y = x$. The image of the graph of $\sin x$ in the mirror will represent the graph of $\sin^{-1} x$ showing that $\sin^{-1} x$ is mirror reflection of $\sin x$ and vice versa.

Observation

The image of point N_1 in the mirror (the line $y = x$) is ....

The image of point N_2 in the mirror (the line $y = x$) is ....

The image of point N_3 in the mirror (the line $y = x$) is ....

The image of point N_4 in the mirror (the line $y = x$) is ....

The image of point N_1' in the mirror (the line $y = x$) is ....

The image of point N_2' in the mirror (the line $y = x$) is ....

The image of point N_3' in the mirror (the line $y = x$) is ....

The image of point N_4' in the mirror (the line $y = x$) is ....

The image of the graph of $\sin x$ in $y = x$ is the graph of .... , and the image of the graph of $\sin^{-1} x$ in $y = x$ is the graph of ....

Application

In the same way we can draw the graph of other inverse trigonometric functions i.e. $\cos^{-1} x$, $\tan^{-1} x$ etc.

VIVA-VOCE

1 What is the domain of $\sin^{-1} x$?

Ans Domain of $\sin^{-1} x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

2 What is the range of $\sin^{-1} x$?

Ans Range of $\sin^{-1} x$ is $[-1, 1]$.

3 What is the relation between the graphs of a function and its inverse.

Ans The graphs of a function and its inverse are mirror images of each other in the line $y = x$.

4 Evaluate : $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Ans Let $\theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\Rightarrow \sin\theta = -\frac{\sqrt{3}}{2}$

$\Rightarrow \sin\theta = \sin\left(\frac{-\pi}{3}\right)$

$\Rightarrow \theta = \frac{-\pi}{3}$

5 Write the difference between maximum and minimum value of $\sin^{-1} x$ for $x \in [-1, 1]$.

Ans The minimum and maximum values of $\sin^{-1} x$ are $\frac{-\pi}{2}$ and $\frac{\pi}{2}$ respectively.

$\therefore$ Required difference $= \frac{\pi}{2} - \left(\frac{-\pi}{2}\right) = \pi$

6 If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then write the value of $x + y + z$

Ans We know that, the maximum value of $\sin^{-1} x$ for $x \in [-1, 1]$ is $\frac{\pi}{2}$.

$\therefore \sin^{-1} x \le \frac{\pi}{2}, \sin^{-1} y \le \frac{\pi}{2}, \sin^{-1} z \le \frac{\pi}{2}$ for all

$\therefore x, y, z \in [-1, 1]$

$\therefore \sin^{-1} x + \sin^{-1} y + \sin^{-1} z \le \frac{3\pi}{2}$

$\therefore \sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$

$\Rightarrow \sin^{-1} x = \sin^{-1} y = \sin^{-1} z = \frac{\pi}{2}$

$\Rightarrow x + y + z = 1 + 1 + 1 = 3$

7 Find the domain of $f(x) = \sin^{-1} x + \sin x$.

Ans The domain of $\sin^{-1} x$ is $[-1, 1]$ and that of $\sin x$ is R. Therefore, domain of $f(x) = \sin^{-1} x + \sin x$ is $[-1,1] \cap R = [-1, 1]$